Towns, density and fractality

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Fractals are fashionable because they produce fascinating images and because the models relate them to the dynamics of complex systems. It is also normal that they give rise to urban research since the urban fabric is an irregular and fragmented form, the infrastructure networks are ramified and the systems of cities are hierarchical. Self-similarity, or the reproduction of the same structures at different spatial scales, seems to be a characteristic of the geographical organization of towns. Theories like central places theory have tried for a long time to explain this. To introduce the fractal reference into urban models is perhaps an opportunity to take into account the specific features of the spatial organization of towns and to find new and more dynamic expressions for describing and understanding it.

By exploring what fractals can offer to the analysis of urban density, and taking the risk of apppearing rather naive or trivial to the mathematicians, the modalities of the transfer of this concept to the town are investigated here and the new information they provide is considered in relation to previous methods.

1. Urban density and fractal dimension

Density is a very old measuring instrument and has been used extensively for the analysis and models of towns or urban systems. It is a measurement of the intensity of the occupation of space. It is a synthetic indicator and in many respects revealing, as many urban features are correlated with it. It provides an ideal description of the state of a population in a given area. Systematic links have been established between density, distance to the town centre, size of the towns, position of the towns in the urban system (or as Bussiere and Stoval (1981) called it "the demographical time"). However, the use of this measuring tool questions the model of spatial distribution to which it refers implicitly. Would'nt fractal geometry be a more suitable reference ? Are the corresponding measurement tools and models able to build descriptive tools of urban space which are more relevant than those based on the density concept?

1.1. Inadequacy of the density concept

from ground efficiency to the appropriation of space

From the point of view of urban space, the concept of density calls forth several basic criticisms. It is an analogous measurement in its construction to efficiency indices (yields), like quintals of wheat per hectare, as the number of people per square kilometer are counted. In this way, its conceptual efficiency is limited to the situations where there is a real ecological relationship between an area and a population which develops its resources, between the soil and the human population that it can support. This relationship has a meaning when agrarian economies are described and compared, but as towns by definition have non agricultural activities, the connotation of efficiency that the density indicator presents is not applicable.

An indicator of the occupation of urban space which would be constructed on the model of other social indicators should include the population as a denominator and not as a numerator and would, for example, be a measurement of the surface available per inhabitant. This indicator is more satisfying from this point of view, but would raise problems of definition and interpretation : how is the space allocated to each inhabitant to be measured? is it a matter of surface of the owned property, or of the space which is privately rented, i.e. the dwelling space ? Between whom and in what way should the public town space be allocated, is it the space which is actually used by the town dwellers, the space where they wander regularly or occasionally, the space that they know ? Between day and night, holiday time and working hours, the distribution of the population changes just as it does with time and, according to the itineraries within the town, the inhabitant's time/space changes its configuration (Hägerstrand, 1970). The significance of the indicator becomes clearer as the possibilities of the measuring tools are reduced by the diversity and the imprecision of urban practices.

Le Bras (1993) rightly criticized the so-called measurements of space occupation of the type : « 80 % of the population lives on 20 % of the territory », as this does not mention the level of the aggregation of observations to which it refers (here for example it should rather be said that one fifth of French communes among the most populated bring together the four fifths of the total population). Brunet (1990) pushes the image to the absurd to underline the arbitrariness and the difficulty of this concept of reference space indicating that «according to the Paris underground standards, the world's population would fit into the Territoire de Belfort » (which is the smallest French department in the eastern part of France).

No proportionality between population and surface area.

Whether a number of inhabitants is compared with a surface area unit or the number of square meters available to a resident or a passerby, these indicators of the intensity of human presence in a town have the same defect, their implicit reference, as a means, to the notion of homogeneity. In chemistry and physics, density is a measurement that is characteristic of an homogeneous distribution of particles for a body at equilibrium in a closed system. This type of reference is not particularly well suited to the study of urban systems which are always open and where heterogenenty is the rule.

The distribution of geographical densities are always heterogeneous. Whatever the scale of observation of the urban phenomenon, measuring instruments have established a fundamental heterogeneity of the spatial distribution of the indicators measuring urban mass : people, built up area, activities or flows in the area. For a long time Clark (1950) has observed that in towns the distribution of the resident population densities is organized with a strongly decreasing gradient from the centre to the periphery, according to a negative exponential model or negative power function of the distance to the centre. This model has maintained its descriptive power even if the competition from tertiary employment for the most accessible central units has brought about the formation of a central "crater" in the surface representing the densities in three dimensions. Neither is the model completely put into question by the recent evolution which, due to the residential deconcentration in the centre, the densification of the suburbs and the spread of periurban areas, has considerably reduced the gradients of density distributions in most of the large cities in the world and even in Europe in small towns with a population of 20,000.

The territory covered by urban networks is unequally occupied by the extremely hierarchical system of cities. Here again, the model used to analyse this territory is not particularly well adapted because it refers to the notion of uniformity. Thus, the measurements aiming at testing the shape of the distribution of the pattern of the towns in a territory, have used as their references the Poisson distribution (Dacey, 1967), which implies an equal probability of occupation by towns and does not take into account the characteristic grouping effetcs of urban networks. As well, the attempts at using spectral analysis to characterize the components expressing the hierarchical organization of urban networks (Dacey, 1967 ; Cauvin et al., 1985) were confronted with large irregularities in this organization. The attempts to identify the Christallerian models or a possible combination of the three principles, market, transport and administration, in the same urban network, were not rewarding.

For a town or a system of towns, the underlying homogeneity hypothesis leads to the rejection of the use of the normal references to analyse these structures. Density implicitly suggests a relationship of the linear type, a proportionality between population and surface area. Usually this relationship is not of a linear form. When it is determined empirically, the relationship between the population and the surface of the units of urban administrative boundaries often takes the shape of a power function with the exponent being less than one and in general about two thirds (Haggett, 1973). In other words, the most populated units have smaller areas than the less populated areas or low density administrative units. According to Reymond (1981) this rule can be explained intuitively by the obligation of the population, which is like a volume, to adapt itself to a surface. This results from a "spacing" process that is required to manage the competition for space between the habitat and human activities. It is because of these systematic variations that concentration measurements provide different results according to the geogaphical scale used for their calculation (Isard, 1960 ; Le Bras, 1993). Other authors have used the concept of allometry, to measure for instance the relationship between the population and the surface of cities (Nordbeck, 1971) or to evaluate the differential growth of cities in an urban system (Villeneuve, Ray, 1975), but, whereas measurements are always possible, the transposition of the allometric concept from biology to geography is not straightforward (Pumain, 1982).

Intuitive utility of the reference to the fractal model

All the methods and models mentioned above describe the population or urban activities in relation to a support space containing them and euclidean geometry is describing their properties. A long time ago, Harvey (1969, 20) has emphasized the usefulness of a relative space of reference which would be defined by historical and social practice : « the activities and the objects themselves define their spatial field of operation ». Several authors, either analysing space perception (Hägerstrand, 1970 ; Cauvin et al., 1985 ; Muller, 1982), looking for better adapted cartographic representations (Tobler, 1976 ; Rimbert, 1986), or accounting theoretical research (Reymond, 1981 ; Brunet, 1990), have underlined the non-euclidean character of geographical space, which is genuinely heterogeneous and anisotropic.

In the case of intra-urban space, reference to an homogeneous space is hardly suitable since the general pattern is of extremely contrasted distributions which have steep gradients expressing and inducing highly polarized areas from center to periphery, with either a single or multiple centres. It is the same in the case of systems of cities for which the geometrical models of the central places theories appear to be incompatible, in their reference to an homogeneous population, with the gradients inevitably created by the way space is occupied by towns (Frankhauser, 1993). The self-similarity of geographical space has, for example, been interpreted by Philbrick (1957) as a result of an almost systematic alternance of polarization effects, which differentiate a centre from its periphery, and similarity effects, which define homogeneous regions at all geographical levels : parcels of homogeneous land-use are polarized by a farm, the grouping of several farm units forms a small homogeneous agricultural region which is polarized by a market town, several agricultural regions become the periphery of a small regional capital which joins with other similar capitals in a region polarized by a metropolis, and so on.

This functionalist and static interpretation, enriched by taking account of the speed of movements between the different areas at different levels and their differentiated historical evolution should enable a genuine transfer of the fractal concept to intra and interurban space (Pumain, 1993). Moreover, the distribution of urban hierarchies seems to be described more efficiently by statistical models like Pareto functions (still knowned by Zipf's (1949) term of "rank-size rule") or Gibrat's (1931) lognormal distribution, which appear to be compatible with fractal geometry. Indeed, the multifractal generators used by Le Bras (1993) to simulate the spatial distribution of demographic growth, which are not explicitly linked in his book to any territorial processes, have in fact an analogy with a stochastic process of spatial growth distribution which gives rise to lognormal distributions of population sizes.

As the distribution of urban densities, whatever the scale, is never homogeneous, it could be useful to replace the density model by a fractal reference which from the beginning contains this trivial information. First it would make possible to compare directly the levels of heterogeneity or to integrate this property into urban models. Second, it can be hoped that it would help to discover something new. If, instead of treating the hierarchization phenomena as a residual from the adjustment to an homogeneous model, those divergences were eleminated by including directly this already known essential component, one could reveal other characteristics which have hitherto been hidden. If space occupation by towns resembles the images produced by reference models using fractal geometry, researchers could then try to understand why it is like that, by imagining plausible processes which simulate the genesis of such configurations. "Plausible" means compatible with urban theory or socio-economic theories concerning the formation and the growth or towns. Reference to a fractal model has the advantage, compared with the density model, of more directly referring to a dynamic or even an evolutive concept of town networks (Pumain, 1996).

This conceptual revolution changes the centre of interest as far as densities are concerned: it is no longer the intensity of space occupation which will be considered, but its structure, created from an underlying order which is espressed at different geographical scales and which involves the articulation of these scales. This implicit order is accompanied by chance or random variables and implies that the degree of irregularity found at the different scales are approximately equal : « in fractal items chance must have the same importance at all scales. This would imply that it is meaningless to speak of microscopic and macroscopic levels » (Mandelbrot, 1977: 44). This observation by Mandelbrot requires that much more attention should be given to the fractality of towns. In the organization of town systems, the absence of discontinuity between dimensional or functional levels would to be seen as the rule contray to what the theory of central places predicts. However, at least concerning built up areas, a discontinuity seems to exist between the built up area of towns and of their periphery (Frankhauser, 1993). This observation could be considered as a theoretical justification for the use of multifractals by referring to different genetic processes between the towns and the systems of towns.

1.2. Fractal dimension and intraurban density

Fractal geometry has already been used by several authors to describe the morphology of towns (Frankhauser, 1993 ; Batty and Longley, 1994) or more generally the spatial organization of population on a territory (Arlinghaus, 1985 ; Le Bras, 1993).

These works have already presented the mathematical notion of fractals which will not be recalled here. The characteristic of fractal items, which has been used the most often, is the fractal dimension. Several definitions of fractal dimension have been put forward. These mathematical definitions are sometimes rather formal and not always meaningful. For a given fractal structure they usually give the same value, but this is not always the case.

One method that is often used to evaluate the fractal dimensions consists in calculating the mass of fractal structure included within the area, i.e. a square or a circle that is slowly enlarged. M, which is the structural mass included in the area, is equivalent to the side or to the radius R of the area increased to the power of D, with D being the fractal dimension. D is thus determined by the relation :

$$D = \log(M)/\log(R) \qquad (1)$$

The measurement, called mass, can be a built up area or any other scalar quantity attached to a support i.e. an available surface. Before applying it to urban data, one has to test the analytical problems posed by weighting the mass by a quantity such as the height of buildings or the number of inhabitants.

In practice, the side or the radius of the area on the x axis is represented on a graph with two logarithmic coordinates and the mass measurement (for example the number of structural items) on the y axis. The fractal dimension D is the slope of the straight line adjusting the cloud of points representing the mass-radius relationship on a given scale :

$$\log (M) = \log K - D \log(R) \qquad (2)$$

where K is a constant.

The different methods used to measure the fractal dimension are not equivalent. The radial analysis is a relevant tool to study the fractal behaviour from a previously defined centre. This is a local analysis while the graph method or the dilatation method, measuring fractality over all the places, give a global information on the whole spatial distribution (Frankhauser, 1993).

The measurements of fractal dimension are carried out from the slope of an adjustment between the mass and surface and are obviously not independent of the models expressing density variations. One of the density expressions p(R) at a distance R from the centre of a town is a powerful negative function of the distance R from the centre. On a graph with two logarithmic coordinates, the observations are adjusted by a decreasing straight line whose slope   is representing the density gradient :

$$\log p(R) = \log K - \quad \log R \qquad (3)$$

Batty and Kim (1992) demonstrated the following relationship between the fractal dimension D and the density gradient   :

$$D + \quad = 2 \qquad (4)$$

According to these authors, the negative power function of the distance from the centre is a better model than the exponential function. It has the property of not being modified by a change in scale and it can be deduced "naturally" from a maximization process of entropy (or minimum information). All that needs to be stipulated is a function of the logarithm of the distance – this would appear to be more in agreement with the observations concerning the perception of geographical space than the linear effect hypothesis used by Wilson (1970) to derive the exponential model. Thus, the value of the fractal dimension can be interpreted as an expression of density gradient which will remain more or less constant on a given surface. For example, a fractal dimension can be determined by radial analysis and counting the M(R) number of places occupied at a distance R from a centre, by estimating the slope of the line which adjusts the observations according to the following equation :

$$\log M(R) = \log K - D \log R \quad (5)$$

2 Fractality and urban morphology.

As fractal analysis concerns multiscalar phenomena, attention must be given to the cartographic scale of the maps representing urban areas. The number of pixels analysed on the digitized map, the resolution level of the original map and the minimum dimension of the structural items must be compatible. The first measurements of the fractal dimension of urban built up areas (Frankhauser, 1993 ; Batty and Longley, 1994) mainly dealt with large cities and comparable representations, but only on small scale maps, i.e. the urban constructions were highly generalized. Thus, we decided to use documents offering the finest possible resolution, i.e. able to show the texture of the buildings inside urban blocks. We discovered that the measurements of fractal dimensions for the same town were sensitive to the cartographic scale.

Maps with a meso scale (1/50,000 for experiments carried out in the Lons-le-Saunier region) could not be used for a fine detailed study to the extent that the width of the routes are exaggerated and that an heterogeneity is introduced into the representation of the habitat (simplification and homogeneization of the most dense urban buildings). Thus, comparative analysis were carried out with several measurement methods using IGN maps with a scale of 1/25,000 for the town of Besançon (at four dates : 1958, 1973, 1980 and 1990), the urban district of Montbéliard (in 1913, 1954 and 1986) and the Dole region. Moreover, the Besançon municipality provided a Geographical Information System, bringing together the cadastral survey data and aerial photographs for 1993, with a resolution of 15 meters (Figure 1).

The average slope : a measurement of the dilution of the built up area.

Despite the size of the towns that were studied, which was much smaller than that of the large metropolises previously studied in the literature (Batty and Longley, 1994 ; Frankhauser, 1993), all the experiments had the same type of scalar behaviour of the built up structure. As the precision of the 1/25,000 maps was greater, the values of the fractal dimensions were a little lower (approximately 1.80 for all the town centres) than those calculated from maps with a smaller scale, which were over 1.9 (see Frankhauser, 1993).

A proper adjustment of the curves implied a hierarchial organization of the distribution of the built up area in such a way that in the peripheral area, the share of the surface that remained empty was always larger than in the centre.

Inflexions of curves and segmentation of the built up area.

In many agglomerations of various size there were two different fractal behaviours from the centre towards the periphery of the urbanized area. A first break was generally clear between the town centre and its periphery, a second appeared between the first peripheral ring, still of urban character, and the peri-urban rural areas. For example, on Figure 2 representing a radial analysis of Besançon, there is a clear weakening of the curve at a distance of approximately 400 m from the counting centre. Inside the first circle, the fractal dimension corresponding to the built up area inside the loop of the Doubs river (grey square on Figure 1) is 1.8, whereas between 400 m and the center, the fractal dimension of the built up area of the suburbs is only approximately 1.6.

Thus, a first slight weakening of the curve which occurs at about 100 m from the centre can be interpreted. The weakening of the fractal curve near the centre reflects the reduction of the built up area observed inside the urban blocks.

Over a period of time, the fractal dimension of the urban periphery of built up areas tends to increase and becomes closer to the values observed in the town centre. These values do not vary very much and always remain below 2 because there is not a total densification inside the urban area. Thus, in Besançon, the radial dimension of the centre went from 1.68 in 1958 to 1.70 in 1973 and 1990.

In order to precisely locate the weakening points on the curves, the variations of their slope were systematically calculated (Figure 3). A smoothing procedure using a Gaussian convolution of the curve of the slopes led to the indentification of the inflexions and this could lead to the possibility of an automatic segmentation of the data according to their fractal dimension. For example, the smoothed curve of the calculated slopes on the built-up area of Moscou (Figure 4) identifies the transition between the city and its periphery.

Fluctuations in the slope values

For interpreting correctly the possible variations in the slopes of empirical curves used to estimate the fractal dimensions, a certain number of tests were carried out concerning the fluctuations of these slopes measured on theoretical fractals. As a result, it is important to note that the magnitude of the local fluctuations is smaller in the measurements carried out on actual towns than for those carried out on regular fractals. The models that can be used as a reference to analyse the urban structures would therefore be more likely stochastic fractals. Then the problem consists in determining to what extent the local fluctuations are the sign of a significant and interpretable modification of the structure. The fluctuations are larger in small than in large towns whose structure is more regular and whose spatial organization is stronger. This observation is in agreement with the hypothesis of a self organization of these structures.

These measures provide a tool for evaluating the level of organizational homogeneity in an urban built-up area or the regularity of density gradients. The sensitivity of this measurement has been compared with that of more classical tools such as the variogram or other methods of mathematical morphology (Voiron-Canicio, 1993 ; Guérin-Pace, 1993).

3 Fractality and urban networks

The following results are given as an indication. The application of the measurements of fractal dimension at the scale of the population patterns poses a number of problems that still need to be solved. The objective is no longer to make a precise analysis of the built up area, but to characterize the degree of hierarchical organisation of the spatial pattern of an urban system.

As it is a question of groups of scattered points in the space, unlike the built up area of a town which is much more compact, a global analysis method like the dilatation analysis could be well adapted to a local analysis with a strongly fluctuating curve.

Two preliminary studies provide examples (François, 1995) carried out using two different scales. The first concerned the representation on a small scale map of the urban network of France. Two types of analysis were made, one on the pattern of the towns located as simple dots, and the other where the point representing each town is weighted by the town's population at a given date. The second analysis was carried out on maps with a bigger scale, i.e. 1/25,000 and 1/50,000, on the distribution of built-up areas in rural regions.

3.1. Analysis of town patterns

The fractal dimension of the pattern of French towns over 10 000 inhabitants (Figure 5) is calculated by radial analysis. One considers that each town represents a mass unit and one counts the total mass in a circle, whose radius increases by 20 km from a centre. Different centres were used as starting points for the counting in order to test the sensitivity of the measurement to the choice of these points and to detect any multifractality in the network. The minimum threshold of the size of the towns selected was modifed (these were urban agglomerations, which were considered to be relevant for analysing the urban network at this scale).

Table 3.1 : Radial dimension of the town pattern (1990)

Position of the counting centre	Towns over 10,000 inhabitants	Towns over 50,000 inhabitants	Towns over 100,000 inhabitants
Paris	1.34	1.30	1.32
Lyon	1.50	1.49	1.47
Marseille	1.33	1.30	1.21
Centre of France	1.70	1.62	1.92

The results in table 3.1 show a tendency towards fractality, as the estimated dimension differs from the value of 2 which would correspond to an homogeneous pattern. However, the French urban network in general can hardly be characterized as a unique fractal object to the extent that the dimension varies according to the counting centre. It can be noted that the degree of dilution or the centre-periphery gradient of the pattern density that this parameter represents, is higher, i.e. that the value of the fractal dimension is lower, when the measurement is taken from an eccentric position like Paris or Marseille, while the more central positions like Lyon and especially at the barycentre of the territory provide an image which tends more towards homogeneity.

When each point representing a town is weighted by the number of its population, each individual is taken as a mass unit. It is not too unrealistic to consider them all to be gathered together at one point since the counting operation is carried out by circles with a radius of 20 km : this rarely exceeds the diameter occupied by an agglomeration. The results are then very different.

Table 3.2 : Radial dimension of urban population (distribution centred in Paris)

date	Towns over 10,000 inhabitants	towns over 50,000 inhabitants	Towns over 100,000 inhabitants
1831	0.63	0.50	0.44
1911	0.42	0.36	0.32
1954	0.40	0.35	0.30
1990	0.46	0.40	0.36

The calculated dimensions reflect the particularities of the population distribution on the French territory which decreased with a strong gradient from the Parisian centre towards the peripheries (Figure 6). The fact that nearly one fifth of the French population is concentrated in the first circle and that this number continues to weigh heavily during the whole cumulative calculation mode explains that the dimension is lower than one and that expresses a strong peripheral dilution effect. The evolution of the dimension values is consistent with what is known of the urban population variation over time (Pumain, 1982 ; Guérin-Pace, 1993). This shows a trend to increase the inequalities of the spatial distribution until after Second World War (a reduction in the radial dimension from 1831 to 1954), followed by a more recent deconcentration phase (the values increased in 1990).

The dimension measurements carried out from counting centres other than Paris do not have the same magnitude.

Table 3.3 : Radial dimension of the population
(towns over 10,000 inhabitants)

Counting centre / Dates	Lyon	Marseille	Centre of France
1831	1.49	1.09	1;77
1911	1.47	1.07	1.87
1954	1.32	1.10	1.90
1990	1.25	1.04	1.92

The dimensions are still below 2, they all represent a space filling process which differs from homogeneity. The degree of peripheral dilution is, however, less marked (thus the fractal dimension is higher) as a large part of the population (including Paris) is at a greater distance from the counting centre. The temporal variations in the dimensions can also reflect contradictory evolutions according to the observation point that is selected. The interpretation of the radial dimension cannot be dissociated from the counting point position.

On a larger scale, digitized maps of the distribution of the built up area were analysed for the region of Lons-le-Saunier. The results go in the same direction as those set out above.

Conclusion

It would be wrong to say that fractals offer immediate solutions for analysing urban densities. This new formulation is still at the stage where the application potentials are greater than the actual achievements. In the case of urban hierarchial organizations, either built up areas, intraurban densities or regional urban networks, it would be in any case more appropriate to speak of scalar behaviour as in physics rather than fractals as in mathematics.

The experiments related here have brought insights about very important details in the application of fractal methods, which are not only of practical interest but which again demonstrate, if needed, how carefully the model has to be precised when transposed in any urban context. According to the scale and range of the analysis, the choice of a type of measurement and the significance of the results require each time a specific interpretation.

A large part of research until now has concerned the simulation of fractal growth or has been focused on producing or reproducing plausible images. Among others, Batty and Longley (1994) have constructed several models using different fractal generators and have shown that these generators gave rise to images that were more or less plausible. However, generally the study of fractals is separated from urban theory. Frankhauser (1993) has put forward several urban built up area growth models simulating concrete mechanisms such as: the alternating development of constructions and of communication networks; the integration into the main expanding urbanised areas of former satellite villages; the blocking of construction on certain free plots of land ; each of these could lead to fractal structures. At another scale of study, Haag (1994) established linkages between a dynamic migration model and the development of a rank-size distribution in an urban system, whose slopes reveal a multifractal structure. Much progresses have to be made in this direction where the description of fractal structures can be rooted in a theory of urban dynamics.

Besides its contribution to theoretical research, the simulation of constructed fractal urban forms which are equivalent to their level of internal organization, could lead to the development of urban reference models, useful for experimenting, at least on fictitious situations, measures for town and country planning. If fractal structures correspond more to real structures, they are a priori more viable structures that those which would be planned according to the rules of traditional geometry.

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Titles and captions of figures

Figure 1 : Built up area in Besançon
source: GIS maps of the municipality

Figure 2 : Radial analysis of the centre of Besançon
(Y axis): Number of occupied pixels (log)
(X axis): Distance to the center in meters (log)

Figure 3 : Fluctuations in the curve slope of a radial analysis of Besançon
(Y axis) Slope value
(X axis) Distance to the centre in meters

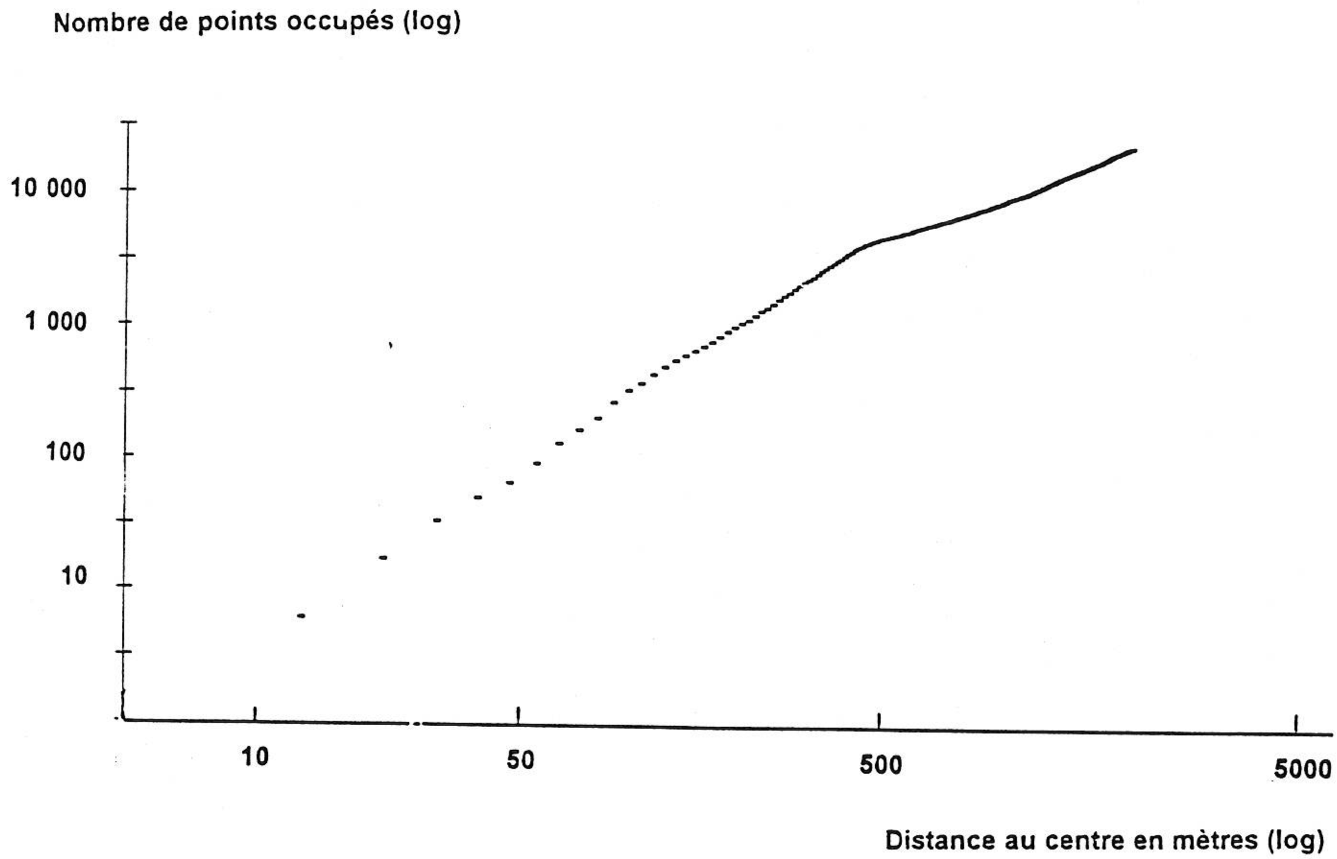
Figure 4 : Smoothed curve of the slopes of a radial analysis of Moscow

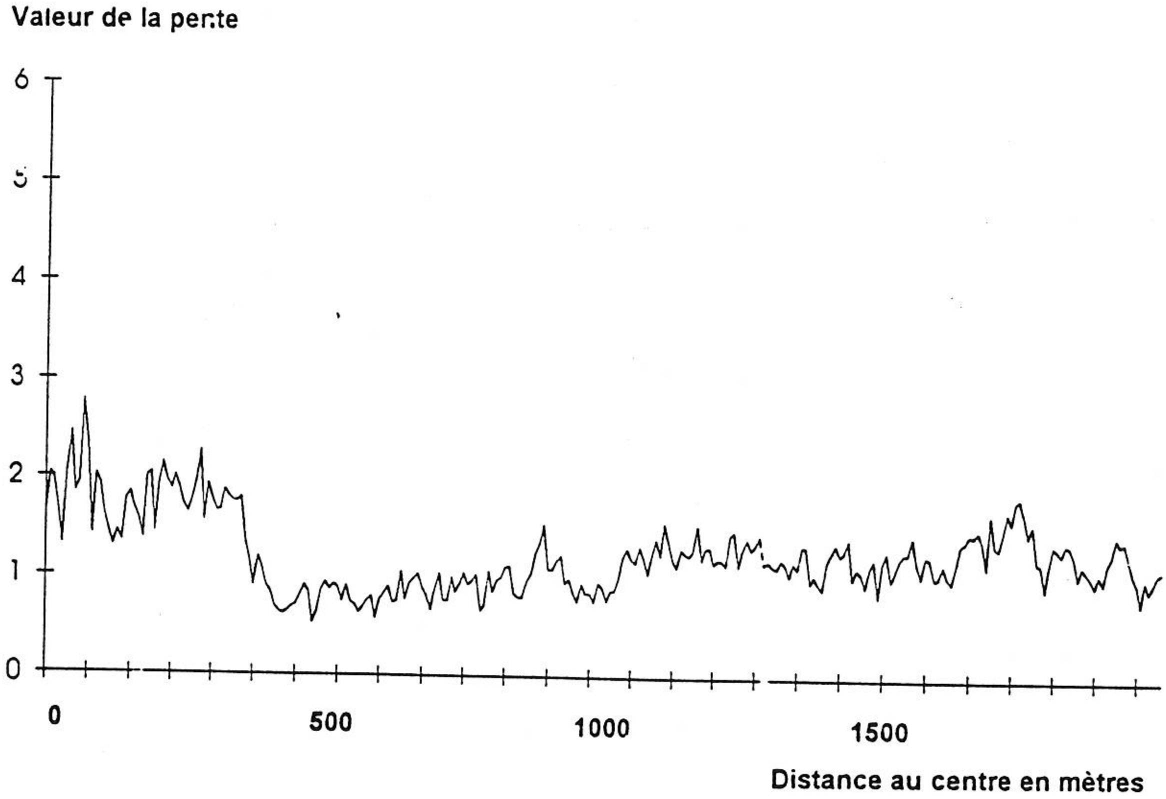
(Y axis) slope
(X axis) distance to the center in kilometers

Figure 5 : The spatial pattern of the French urban system
(urban agglomerations over 10 000 inhabitants)

Figure 6 : Spatial pattern weighted by urban populations


0 250 m


Nombre de points occupés (log)
10 000
1 000
100
10
10
50
500
5000
Distance au centre en mètres (log)


Valeur de la pente
6
5
4
3
2
1
0
0
500
1000
1500
Distance au centre en mètres

Moscou centre: courbe des pentes, lissée