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October 24, 2017

Abstract

We propose a general model of oligopoly with firms relying on a two factor production function. In a first stage, firms choose a certain fixed factor level (capacity). In the second stage, firms compete on price, and adjust the variable factor to satisfy all the demand. When the factors are substitutable, the capacity constraint is "soft", implying a convex cost function in the second stage. We show that there is a unique equilibrium prediction in pure strategies, whatever the returns to scale, characterized by a price that increases with the number of firms up to a threshold. The main propositions are established under the general assumption that the production function is quasi-concave but the paper provides a general methodology allowing the model to be solved numerically for special parametrical forms.

Key words: price competition, tacit collusion, convex cost, capacity constraint, limit pricing strategy, returns to scale.

Code JEL: L13, D43

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1 Introduction.

This article investigates price competition between a variable number of capacity constrained firms producing a homogeneous good. In our model, firms rely on a production function with two substitutable factors that are chosen sequentially. The first factor, chosen in a first stage, remains "fixed" in the second during which firms compete on price and adjust the second "variable" factor to match their demand. In this setting, the fixed factor can be seen as a capacity. This means that firms in our model are capacity constrained but that this constraint is "soft" because they can always increase production beyond their optimal capacity, albeit at an increasing marginal production cost. Our results are general. The production function can be of any form and returns to scale do not have to be constant or decreasing. The results are as follows. 1) There is a continuum of subgame perfect Nash Equilibria in pure strategies. 2) In the second stage, the convexity of the short-run costs allows high prices to be sustained as an equilibrium, as reported by Dastidar (1995) and Cabon-Dhersin and Drouhin (2014). 3) Because the short-run cost function is convex whatever the returns to scale, the existence of an equilibrium for the whole game is disentangled from the nature of the returns to scale. 4) In the first stage, there is a threshold for the fixed factor below which competitors can adopt limit pricing strategies in the second stage. 5) Moreover, the equilibrium price appears to increase with the number of firms, a theoretical result that has seldom been reported (Rosenthal, 1980; Gabaix et al., 2016).

This paper bridges three lines of literature, the Bertrand-Dastidar convex cost approach to price competition, the Bertrand-Edgeworth constrained capacity approach to price competition, and the literature on capacities and limit pricing strategies.

In his seminal model of price competition, Joseph Bertrand (1883) considered interactions between two firms that have identical linear cost functions and simultaneously set their prices. According to this model, even if the number of competing firms is small, price competition leads to a perfectly competitive outcome in a market for a homogeneous good. The unique equilibrium price equals the firms' (constant and common) marginal cost and each firm's profit is equal to zero. This result is referred as the Bertrand Paradox. For a long time, following Edgeworth's 1925 initial insight, the belief was that there was a serious equilibrium existence problem (in pure strategies) when considering decreasing returns to scale and/or convex cost functions. However, Dastidar (1995) proved that a continuum of pure strategy Nash equilibria in price competition does exist when costs are strictly convex. As usual in price competition, a firm undercutting its rivals will attract all the demand but, because of the convexity of the cost function, this move may not necessarily be profitable. At equilibrium therefore, prices may be higher than the average cost and even higher than the marginal cost. Dastidar (2001) shows that, when the costs are sufficiently convex, the collusive outcome may even be an equilibrium. On the contrary, with strictly subadditive costs and symmetric firms, it can be shown that there is no equilibrium in price competition (Dastidar, 2011b)[1]. The source of subadditivity can be either increasing returns to scale or the existence of fixed costs when variable unitary costs are constant or not too convex (Hoernig, 2007; Baye and Kovenock, 2008; Saporiti and Coloma, 2010). In this Bertrand-Dastidar approach to price competition it is the convexity of the cost function that resolves the Bertrand Paradox. As mentioned in the introductory paragraph, in our model, the convexity of the short-run cost function in the second stage is due to the decreasing marginal productivity of the variable factor. Our model thus shares some of the properties of the Dastidarian framework, whatever the returns to scale.

As pointed out by Vives (1999), following Edgeworth (1925), there is a long tradition in Industrial Organization to solve the Bertrand Paradox by considering that firms are constrained by their production capacities when matching the incoming demand. In the modern literature, this argument has been put forward by Kreps and Scheinkman (1983), among others. In a two-stage game, they obtain that quantity pre-commitment,

[1] Dastidar (2011a) introduces asymmetric cost functions and proves that, in this case, when the monopoly break-even prices differ, an equilibrium can be found even if costs are strictly subadditive.

in the first stage, and price competition, in the second, sustain the Cournot outcome provided the constrained capacities are not "too high". As shown by Davidson and Deneckere (1986), this result is sensitive to the choice of a rationing rule for the residual demand (see Vives, 1999, p.124, for details). This result is built on "drastic" capacity constraints, that is, the marginal cost of production in excess of capacity is infinite. Our approach relies on the same type of two stage game with capacity chosen in the first. However, the softness of the capacity constraint induces a smoother cost function in the second stage. Less-rigid capacity constraints have been introduced previously in a number of studies(see Maggi, 1996; Boccard and Wauthy, 2000, 2004; Chowdhury, 2009, for example) directly in the cost function. Cabon-Dhersin and Drouhin (2014) have established a rigorous basis for such "soft" capacity constraints starting from the microeconomic production function and production factors chosen sequentially. Burguet and Sákovics (2017) follow the same approach, but with a very different model of price competition in the second stage, whereby firms can tailor their prices to each consumer.

Beyond the issue of price competition, strategic investment capacity decisions are also a very classical question in industrial organization with regard to entry deterrence (Spence, 1977; Dixit, 1980, among many others). In this kind of model, choosing an excess capacity in the first stage drives away potential competitors. In our model, all the competitors are already operating in the market. However, a firm that chooses too low a capacity in the first stage will be unable to match its competitors' prices profitably in the second. Firms must therefore choose a high enough capacity to avoid limiting their pricing strategies in the second stage.

In this context, we propose a general model of price competition with "soft" capacity constraints that allows the effects on the market price of the number of firms to be investigated. Notably, we find that when there are few firms in the market, the equilibrium price can increase when new firms enter whatever the returns to scale. While the concept of price-increasing competition is not new to the literature (Rosenthal, 1980;

Gabaix et al., 2016), our model highlights a simple explanation for this phenomenon: cost convexity combined with endogenous capacities induces a "capacity effect" that can offset the negative effect on prices of additional competing firms. When more firms operate in the market, the level of the fixed factor (capacity) tends to decrease and the stronger convexity of the cost function increases the equilibrium price.

The paper is organized as follows. Section 2 rigourously characterizes the notion of "soft" capacity constraints; the complete model is solved in Section 3; in Section 4 finally, a general method for numerical simulations is presented along with a "textbook example" to illustrate some interesting properties.

2 Characterization of soft capacity constraints

Firms produce a homogeneous good using the same technology represented by a two factor production function. The factors are chosen sequentially. We denote z the level of the factor chosen in the first stage (the fixed factor) and v, the level of the factor (the variable factor) chosen in the second stage. We denote y the level of production, and $f : \mathbb{R}_+^2 \to \mathbb{R}_+$, giving:

$$y = f(z, v) \tag{1}$$

The only assumptions are that f increases with z and v, shows decreasing marginal factor productivity, and is quasi-concave. Thus: $f_z > 0, f_v > 0, f_{zz} < 0$, $f_{vv} < 0$ and $-f_{zz}f_v^2 + 2f_{zv}f_vf_z - f_{vv}f_z^2 > 0$.

It is important to emphasize that we make no general assumptions about the nature of the returns to scale or the level of substitutability between the two production factors.

When z is fixed, Equation (1) defines the variable factor as an implicit function of z and y, $\hat{v}(y, z)$.

Lemma 1. *1) The function $\hat{v}$ is quasi-convex and fulfils:*

$$\hat{v}_y(y,z) = \frac{1}{f_v(z,v)} > 0 \tag{2}$$

$$\hat{v}_z(y,z) = -\frac{f_z(z,v)}{f_v(z,v)} < 0 \tag{3}$$

$$\hat{v}_{yy}(y,z) = -\frac{f_{vv}(z,v)\hat{v}_y(y,z)}{f_v(z,v)^2} > 0 \tag{4}$$

$$\hat{v}_{zz}(y,z) = \frac{-f_{zz}f_v^2 + 2f_{zv}f_vf_z - f_{vv}f_z^2}{f_v(z,v)^3} > 0 \tag{5}$$

$$\hat{v}_{yz}(y,z) = \hat{v}_{zy}(y,z) = -\frac{f_{vz}(z,v) + \hat{v}_z(y,z)f_{vv}(z,v)}{f_v(z,v)^2} < 0 \tag{6}$$

2) Moreover, if f is (strictly) concave then $\hat{v}$ is (strictly) convex.

Proof: Implicit differentiation of $\hat{v}$ yields Equations (2) to (6).

The quasi-concavity of f means that:

$$-\hat{v}_{zz}\hat{v}_y^2 + 2\hat{v}_{zy}\hat{v}_z\hat{v}_y - \hat{v}_{yy}\hat{v}_z^2 = f_{zz}f_v < 0 \tag{7}$$

This proves that v is quasi-convex.

Moreover, it is easy to check that:

$$\begin{vmatrix} \hat{v}_{yy} & \hat{v}_{yz} \\ \hat{v}_{zy} & \hat{v}_{zz} \end{vmatrix} = \frac{1}{f_v^4}\begin{vmatrix} f_{zz} & f_{zv} \\ f_{vz} & f_{vv} \end{vmatrix}$$

If f is concave then this determinant is necessarily positive. The second order pure derivatives of $\hat{v}$ are also positive (cf. (4) and (5)), proving part 2) of the Lemma. □

We can therefore define the cost as a function of (y,z). With w_1,the price of factor z and w_2, the price of the factor v, we have:

$$C(y,z) = \underbrace{w_1 z}_{FC(z)} + \underbrace{w_2\hat{v}(y,z)}_{VC(y,z)} \tag{8}$$

Setting the level of the fixed factor corresponds to choosing a capacity. In this model, it is possible to match any incoming demand but at an increasing marginal cost. It is in this way that the capacity constraint is"soft". The sequential choice of production factors implies that the cost function is convex, whatever the returns to scale. Thus, when firms compete on price in the second stage, our model inherits the general properties of the Dastidarian framework.

It is noteworthy that as always, the fixed cost depends on the level of the fixed factor, but so does the variable cost. The level chosen for the capacity will have qualitative implications for the shape of the variable cost function. Models that start from an arbitrary cost function usually miss this effect.

Finally, it is important to notice that the softness of the capacity constraint comes from the substitutability of the production factor. As pointed out by Cabon-Dhersin and Drouhin (2014, p. 428) and Burguet and Sákovics (2017), if the production factors are fully complementary (Leontief technology), our approach is equivalent to the usual "drastic" capacity constraint.

3 Equilibrium of the game

Firms first set their fixed factor to a certain level then compete on price in a second stage. The demand of the whole market is continuous, twice differentiable and decreasing.

$D : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ with $D(p_{max}) = 0, D(0) = Q_{max}$.

The strategic variable for the firms in stage 2 is price. We denote p_i the price of firm i and $\vec{p} = (p_1, .., p_n)$, the vector of prices of all the n firms in the market. We denote $p_L = \text{Min}\{p_1, .., p_n\}$ and we define the set $M = \{j \in \{1, .., n\} | p_j = p_L\}$. We denote $m = \text{Card}(M)$ the number of firms quoting the lowest price. Firms have to supply all the demand they face in stage 2 at price p_i. The demand function of firm i is defined as follows:

$$D_i(\vec{p}) = \begin{cases} 0 & \text{if } p_i > p_L \\ \frac{D(p_i)}{m} & \text{if } p_i = p_L \end{cases}$$

We can now express the profit π_i of each firm i, when m firms operate in the market (set the lowest price).

$$\pi_i(\vec{p}, z_i) = pD_i(\vec{p}) - w_1 z_i - w_2 \hat{v}\left(D_i(\vec{p}), z_i\right)$$

$$\pi_i(\vec{p}, z_i) = \begin{cases} -w_1 z_i & \text{if } p_i > p_L \\ p\frac{D(p)}{m} - w_1 z_i - w_2 \hat{v}\left(\frac{D(p)}{m}, z_i\right) \stackrel{\text{def}}{=} \hat{\pi}(p, z_i, m) & \text{if } p_i = p_L = p \end{cases}$$

The function $\hat{\pi}(p, z_i, m)$ represents the profit of firm i when m firms (including firm i) quote the lowest price, p. This function depends on the level of the fixed factor set in the first stage of the game. The assumptions described above for the production function are sufficient to ensure that $\hat{\pi}_{zz} < 0$. However, even if the profit is necessarily concave in y, this does not guarantee that $\hat{\pi}$ is strictly concave in p. This only occurs if the demand function is not too concave or too convex:

$$-\frac{D'(p)^2}{m}\frac{\hat{v}_{yy}}{\hat{v}_y}\left(\frac{D(p)}{m}, z\right) < D''(p) < -2\frac{D'(p)}{p}$$

The left-hand side of the inequality corresponds to the sufficient condition for the short run cost to be convex in p and the right-hand side is the sufficient condition for the revenue function to be concave in p.[2] Moreover, if we want this condition to hold whatever the number of firms, in particular when this number grows to infinity, the left-hand side should tend to zero and the demand function has to be convex (non-strictly).

To solve the equilibrium of the game in stage 2, the first step is to test whether firm

[2]This is a standard assumption to make, even if when starting directly from a cost function, it is hidden within the general assumption that the profit function is concave in p.

i can deviate profitably from an outcome in which m firms (including firm i) quote the same price. The following two thresholds are defined with this purpose in mind. The first, $\bar{p}$, is the maximum price for which firms cannot increase their profits by lowering their prices. The second, $\hat{p}$, is the minimum price for which firms cannot increase their profits by increasing their prices. In the traditional Bertrand competition model with constant average/marginal costs, these two thresholds are equal and correspond to the unique equilibrium of the game, implying marginal cost pricing. In our more general setting with a convex short-run cost function, as in Dastidar (1995), the two thresholds are never equal and define a price interval for each firm for which there is no profitable deviation.

Let us start by studying a firm's ability to increase its profit by undercutting its competitors. For that purpose we define, for $m \geqslant 2$, the function $\Omega(p,z,m) \stackrel{\text{def}}{=} \hat{\pi}(p,z,1) - \hat{\pi}(p,z,m)$. Ω can be interpreted as the incentive for a firm to lower its price when the market price is p. Thus when $\Omega \leq 0$, it is not profitable for the firm to lower its price while when $\Omega > 0$, it is profitable for the firm to do so.

Lemma 2. *For a given z and m, there is a unique threshold $\bar{p}(z,m) \in (0, p_{max})$ that solves $\Omega(p,z,m) = 0$*
When $p \leq \bar{p}$, $\Omega(p,z,m) \leq 0$ and when $p > \bar{p}$, $\Omega(p,z,m) > 0$

Proof: The first step is to expand Ω and Ω_p

$$\Omega(p,z,m) = \left(\frac{m-1}{m}\right) D(p)p - w_2 \left(\hat{v}(D(p),z) - \hat{v}\left(\frac{D(p)}{m}, z\right)\right)$$

Differentiating gives:

$$\begin{aligned} &\Omega_p(p,z,m) \\ &= \left(\frac{m-1}{m}\right) D(p) + D'(p) \left[p - w_2 \hat{v}_y(D(p),z) - \frac{1}{m}\left(p - w_2 \hat{v}_y\left(\frac{D(p)}{m}, z\right)\right)\right] \end{aligned} \tag{9}$$

We are now going to prove existence.

For a given z and m, $\Omega(0, z, m) = -w_2 \left(\hat{v}(Q_{max}, z) - \hat{v}\left(\frac{Q_{max}}{m}, z\right)\right) < 0$ (because $\hat{v}_y > 0$ and $Q_{max} > Q_{max}/m$). We also have $\Omega(p_{max}, z, m) = 0$ with $\Omega_{p^-}(p_{max}, z, m) = D'^{-}(p_{max})p < 0$ (with D'^{-} being the left derivative of the demand function). Ω is continuous in p over the interval $[0, p_{max}]$, initially negative and finally converging to zero from above. This implies that there is necessarily a $\bar{p}(z, m) \in (0, p_{max})$ that solves $\Omega(p, z, m) = 0$

We now prove the uniqueness of $\bar{p}(z, m)$ in $(0, p_{max})$. Over this interval, we have $D(p) > D(p)/m > 0$. Moreover, the strict convexity of $\hat{v}$ implies that:

$$\hat{v}_y\left(\frac{D(p)}{m}, z\right) < \frac{\hat{v}(D(p), z) - \hat{v}\left(\frac{D(p)}{m}, z\right)}{D(p) - \frac{D(p)}{m}} < \hat{v}_y(D(p), z)$$

From the definition of $\bar{p}$, $\frac{m-1}{m}D(\bar{p})\bar{p} = w_2\left(\hat{v}(D(\bar{p}), z) - \hat{v}\left(\frac{D(\bar{p})}{m}, z\right)\right)$ and thus

$$w_2\hat{v}_y\left(\frac{D(\bar{p})}{m}, z\right) < \bar{p} < w_2\hat{v}_y(D(\bar{p}), z) \tag{10}$$

Finally, considering Equation (9), it is now obvious that $\Omega_p(\bar{p}, z, m) > 0$. This means that, in the interval $(0, p_{max})$, Ω can only intercept the x-axis from below. And since Ω is a continuous functions, this can only happen once. $\square$

$\bar{p}(z, m)$ is the highest price with no incentive to deviate when m firms operate the market. From Inequality (10), we can see that it corresponds to a strictly positive *markup*.

We can now study the possibility for a firm to increase its profit by increasing its price. This case is much simpler because in the second stage, the fixed cost, $w_1 z$, is sunk, and the firm it only motivated to produce if the variable part of the profit is positive. If this is not the case at the current price, p, increasing the price will induce zero demand for the firm and thus zero production and will reduce its losses.

Lemma 3. *For a given z and $m \geq 1$, there is a unique $\hat{p}(z, m)$ in the interval $(0, p_{max})$ for which: $\hat{\pi}(\hat{p}, z, m) = -w_1 z$*

Moreover, $\hat{\pi}(\hat{p}, z, m)$ decreases with m and $\hat{p}(z, m) < \bar{p}(z, m)$.

Proof: It is easy to check that $\hat{\pi}(0, z, m) < -w_1 z$, $\hat{\pi}(p_{max}, z, m) = -w_1 z$ and $\hat{\pi}_{p^-}(p_{max}, z, m) < 0$. Then, the strict concavity of $\hat{\pi}$ in p implies that $\hat{p}$ exists and is unique. Implicit differentiation of $\hat{\pi}$ for a given z yields

$$\left.\frac{\mathrm{d}\hat{p}}{\mathrm{d}m}\right|_{dz=0} = \frac{1}{m} \frac{D(\hat{p})\left(\hat{p} - w_2 \hat{v}_y\left(\frac{D(\hat{p})}{m}, z\right)\right)}{D(\hat{p}) + D'(\hat{p})(\hat{p} - w_2 \hat{v}_y(\frac{D(\hat{p})}{m}, z))} < 0 \tag{11}$$

For $p < p_{max}$, we have $\frac{D(p)}{m} > D(p_{max}) = 0$. The strict convexity of $\hat{v}$ then implies that $\hat{v}\left(\frac{D(p)}{m}, z\right) - 0 < \left(\frac{D(p)}{m} - 0\right)\hat{v}_y\left(\frac{D(p)}{m}, z\right)$. By definition, $\hat{p}$ is such that $\hat{p}\frac{D(\hat{p})}{m} = w_2 \hat{v}\left(\frac{D(\hat{p})}{m}, z\right)$ and then $\hat{p} < w_2 \hat{v}_y\left(\frac{D(\hat{p})}{m}, z\right)$, which gives the sign of the implicit derivative and proves that $\hat{p}$ decreases with m. Thus, for $m \geq 2$, $\hat{p}(z, m) < \hat{p}(z, 1)$ and $\hat{\pi}(\hat{p}(z, m), z, 1) < -w_1 z$. It follows that for $m \geq 2$, $\Omega(\hat{p}(z, m), z, m) < 0$, implying $\hat{p}(z, m) < \bar{p}(z, m)$. $\square$

For a given z, $\hat{p}(z, m)$ is the minimum price that motivates production in the second stage when m firms operate in the market.

The price interval $[\hat{p}(z, m), \bar{p}(z, m)]$ is crucial to solving for the equilibrium of the game in stage 2. These prices will have to be compared with the *purely collusive* price, p^*, when m firms operate in the market.

Lemma 4. *For a given z and $m \geq 1$, there is a unique $p^*(z, m)$ in the interval $(0, p_{max})$ for which $p^*(z, m) \overset{def}{=} \arg\max_p \{\hat{\pi}(p, z, m)\}$*
Moreover, $p^*(z, m) > \hat{p}(z, m)$.

Proof: It is easy to verify that $\hat{\pi}_p(0, z, m) > 0$ and $\hat{\pi}_{p^-}(p_{max}, z, m) < 0$. $\hat{\pi}_p$ is continuous, ensuring that the program has an interior maximum. The strict concavity of $\hat{\pi}$ with p ensures that the maximum is unique. Because $\hat{\pi}_p(\hat{p}, z, m) > 0$, $p^*(z, m) > \hat{p}(z, m)$ $\square$

A simple interpretation is that this is the collusive price when all firms chose to set their fixed factor to the same level in the first stage (when $m = 1$, it is the monopoly price). We will see that p^* can fall within $[\hat{p}(z,m), \bar{p}(z,m)]$, but not necessarily.

Lemma 5. *$\forall m \in [1,n]$, $\hat{p}(z,m), \bar{p}(z,m)$ and $p^*(z,m)$ are strictly decreasing in z and m over their respective domains.*

Proof: At $\hat{p}$, $\frac{D(\hat{p})}{m}\hat{p} - w_2\hat{v}(\frac{D(\hat{p})}{m}, z) = 0$.

The derivative of the above expression with respect to z is:

$$\left.\frac{\mathrm{d}\hat{p}}{\mathrm{d}z}\right|_{dm=0} = \frac{w_2\hat{v}_z(\frac{D(\hat{p})}{m}, z)}{\frac{D'(\hat{p})}{m}\hat{p} + \frac{D(\hat{p})}{m} - w_2\frac{D'(\hat{p})}{m}\hat{v}_y(\frac{D(\hat{p})}{m}, z)} < 0$$

From Equation (11), we have

$$\left.\frac{\mathrm{d}\hat{p}}{\mathrm{d}m}\right|_{dz=0} < 0$$

At $\bar{p}$, we have $\Omega(\bar{p}, z, m) = 0$. The derivatives of the above equality with respect to z and m are:

$$\left.\frac{\mathrm{d}\bar{p}}{\mathrm{d}z}\right|_{dm=0} = -\frac{\Omega_z(\bar{p}, z, m)}{\Omega_p(\bar{p}, z, m)} = \frac{w_2\left(\hat{v}_z(D(\bar{p}), z) - \hat{v}_z(\frac{D(\bar{p})}{m}, z)\right)}{\Omega_p(\bar{p}, z, m)}$$

which is < 0 since $\Omega_p(\bar{p}, z, m) > 0$ and $\hat{v}_z < 0, \hat{v}_{yz} < 0$.

$$\left.\frac{\mathrm{d}\bar{p}}{\mathrm{d}m}\right|_{dz=0} = -\frac{\Omega_m(\bar{p}, z, m)}{\Omega_p(\bar{p}, z, m)} = -\frac{\frac{D(\bar{p})}{m^2}(\bar{p} - w_2\hat{v}_y(\frac{D(\bar{p})}{m}, z))}{\Omega_p(\bar{p}, z, m)}$$

which is < 0 since $\Omega_p(\bar{p}, z, m) > 0$ and from Equation (10), $\bar{p} > w_2\hat{v}_y(\frac{D(\bar{p})}{m}, z)$.

Finally, we obtain,

$$\left.\frac{\mathrm{d}p^*}{\mathrm{d}z}\right|_{dm=0} = -\frac{\hat{\pi}_{pz}(p^*, z, m)}{\hat{\pi}_{pp}(p^*, z, m)} = w_2\frac{D'(p^*)}{m}\frac{\hat{v}_{yz}(\frac{D(p^*)}{m}, z)}{\hat{\pi}_{pp}(p^*, z, m)} < 0$$

and

$$\left.\frac{\mathrm{d}p^*}{\mathrm{d}m}\right|_{dz=0} = -\frac{\hat{\pi}_{pm}(p^*, z, m)}{\hat{\pi}_{pp}(p^*, z, m)} = -w_2\frac{D'(p^*)}{m^3}\frac{D(p^*)\hat{v}_{yy}(\frac{D(p^*)}{m}, z)}{\hat{\pi}_{pp}(p^*, z, m)} < 0$$

□

We have now gathered all the elements required to characterize the equilibrium prediction for the whole game.

Proposition 1. *An outcome of the game* $(\vec{p}, \vec{z})$ *in which n firms operate in the market at the same price* p^N *is a* Subgame Perfect Nash Equilibrium *(SPNE) if and only if the three following conditions are verified simultaneously:*

1. ***Efficiency:*** *All n firms set their fixed factor to the same level,* $z^N = z^*(p^N, n)$, *with* $z^*(p, n)$ *being a solution of the program:*

$$(\mathcal{P}_1) \begin{cases} \max\limits_{z} \hat{\pi}(p, z, n) \\ s.t.\ \ p \leq \bar{p}(z, n) \end{cases}$$

2. ***Profitability:*** $\hat{\pi}(p^N, z^N, n) \geq 0$

3. ***Non-existence of limit pricing strategies:***
$\hat{\pi}(p^N, z^N, n) \geq \hat{\pi}(\hat{p}(z^N, n), \underset{z}{\operatorname{argmax}}\, \hat{\pi}(\hat{p}(z^N, n), z, 1), 1)$

Proof: Assuming that we are in a *SPNE* in which all n firms (indexed by i) operate in the market at price p^N and have fixed factors that can range from z_L, the lowest level, to z_H, the highest. Because p^N is a Nash equilibrium in the second stage, the firms have no incentive to deviate in the second stage: $p^N \in \bigcap\limits_i [\hat{p}(z_i, n), \bar{p}(z_i, n)] = [\hat{p}(z_L, n), \bar{p}(z_H, n)] \neq \emptyset$. There are three cases to consider depending on the location of p^N in this interval.

The case in which $p^N = \hat{p}(z_L, n)$ can be discarded because by definition $\hat{\pi}(\hat{p}(z_L, n), z_L, n) = -w_1 z_L < 0$. z_L cannot be an equilibrium strategy in the first stage (the firm will earn a strictly higher profit by playing $z = 0$).

Another possibility is that p^N falls within the interval, $p^N \in (\hat{p}(z_L, n), \bar{p}(z_H, n))$. The derivative $\hat{\pi}_z(p^N, z_i, n)$ can be calculated for each firm i. If this derivative is negative, there is an incentive for firm i to slightly decrease its fixed factor. Thus z_i cannot be an equilibrium strategy in the first stage. Symmetrically, when the derivative is positive, the firm has an incentive to slightly increase its fixed factor and z_i is therefore not an equilibrium strategy in the first stage either. A *SPNE* is thus only obtained in the second stage if for all i, $\hat{\pi}_z(p^N, z_i, n) = 0$. Because $\hat{\pi}$ is strictly concave in z, $\hat{\pi}_z$ is strictly decreasing. This implies that $z_L = z_H$, i.e. that in a *SPNE* in which all the firms operate in the market at the same price, p^N, all the firms also have their fixed factor set to the same level.

The third (and last) possibility is that $p^N = \bar{p}(z_H, n)$. As before, we can compute the derivative $\hat{\pi}_z(p^N, z_H, n)$. When the derivative is zero, there is no incentive to deviate for the firm with the highest fixed factor. When this derivative is negative, firm H has an incentive to reduce its fixed factor in the first stage, which is incompatible with a SPNE. What happens when the derivative is strictly positive? In this case, Lemma 5 shows that for any $\epsilon > 0$, $\bar{p}(z_H + \epsilon, n) < \bar{p}(z_H, n) = p^N$. By increasing its fixed factor in the first stage, the firm whose fixed factor is the highest prevents itself from sustaining p^N as a Nash equilibrium in the second stage. In this case, a necessary condition to have a *SPNE* is thus $\hat{\pi}_z(p^N, z_H, n) \geq 0$. Finally, is it possible in this case to have $z_L < z_H$? Because $\hat{\pi}$ is strictly concave in z, $\hat{\pi}_z(p^N, z_H, n) \geq 0$ and $z_L < z_H$ imply that $\hat{\pi}_z(p^N, z_L, n) > 0$. From Lemma 5 furthermore, $\bar{p}(z_L, n) > \bar{p}(z_H, n)$, the firm whose z is strictly lower initially can increase its profit by slightly increasing z without destabilizing the equilibrium at price p^N in the second stage. In this case therefore, the inequality $z_L < z_H$ cannot be verified in the *SPNE*.

Cases 2 and 3 are mutually exclusive and cover all the possible *SPNE*. Thus, any *SPNE* necessary fulfils $z_i = z_H = z_L = z^N$ for all i and:

$$\left\{\begin{array}{c} p^N < \bar{p}(z^N, n) \\ \text{and} \\ \hat{\pi}_z(p^N, z^N, n) = 0 \end{array}\right. \quad \text{OR} \quad \left\{\begin{array}{c} p^N = \bar{p}(z^N, n) \\ \text{and} \\ \hat{\pi}_z(p^N, z^N, n) \geq 0 \end{array}\right.$$

It is easy to check that this last logical necessary condition is the same as the one required for z^N to be a solution of program $(\mathcal{P}1)$, completing the proof of Part *1.* of

Proposition 1. This efficiency condition is required to obtain a *SPNE* but it is not sufficient, because it only tests for the profitability of slight adjustments of the fixed factor in the first stage. We must also rule out the profitability of substantial deviations in z in the first stage.

The first substantial deviation to consider is setting $z = 0$ in the first stage and a price $p > p^N$ in the second. This move will induce zero profit for the firm. (It is not profitable as long as $\hat{\pi}(p^N, z^N, n) \geq 0$.) This proves part 2. of the proposition.

The second substantial variation to consider is when a firm sets z high enough to sustain a price in the second stage that is low enough to "exclude" its competitors from the market. This is a limit pricing strategy. If all the other firms play z^N in the first stage, their interest will be to sustain any $p^N \geq \hat{p}(z^N, n)$ (remember that in the second stage the fixed cost is sunk and the decision criterion is the positivity of the variable profit). The limit price is thus $p^L = \hat{p}(z^N, n) - \epsilon$ with $\epsilon > 0$ and as low as possible. The most profitable value of z to sustain such a price is $\operatorname*{argmax}_z \hat{\pi}(\hat{p}(z^N, n), z, 1)$. If a firm deviates using this "limit pricing strategy", it will operate in the market alone. This move is profitable only when part 3. of the proposition is not fulfilled. □

Proposition 2. *The outcome in which all n firms choose the same fixed factor level, z^C, in the first stage and quote the same price, p^C, in the second, with p^C being a solution of the program,*

$$(\mathcal{P}_2) \left\{ \begin{array}{ll} & \max_p \hat{\pi}(p, z, n) \\ s.t. & z = \operatorname{argmax}\{\mathcal{P}_1(p)\} \\ & \hat{\pi}(p, z, n) \geq 0 \\ & \hat{\pi}(p, z, n) \geq \hat{\pi}(\hat{p}(z, n), \operatorname*{argmax}_{\tilde{z}} \hat{\pi}(\hat{p}(z, n), \tilde{z}, 1), 1) \end{array} \right.$$

and $z^C = \operatorname{argmax}\{\mathcal{P}_1(p^C)\}$, is a Subgame Perfect Nash Equilibrium of the game. Moreover, $\hat{\pi}(z^C, p^C)$ is the Payoff Dominant Subgame Perfect Nash Equilibrium of the game.

Proof: Because all firms have the same technology, it is obvious that any (p, z)

that fulfil the constraints of Program $\mathcal{P}_2$ are *SPNE*. It follows that (p^C, z^C) is the *Payoff Dominant SPNE*.$\square$

In the remainder of the article, we will consider (p^C, z^C) to be the unique and symmetric predictable outcome of the price competition game with soft capacity constraints. As pointed out by Cabon-Dhersin and Drouhin (2014), the solution of program $(\mathcal{P}_2)$ is collusive by nature (i.e. it corresponds to a joint profit maximisation program). When neither of the constraints on programs $(\mathcal{P}_1)$ and $(\mathcal{P}_2)$ are binding, the predictable outcome will verify $p^C = p^*(z^C, n)$, with p^* defined as in Lemma 4. In this case the predictable outcome is fully collusive. When the constraint on Program $(\mathcal{P}_1)$ is binding (i.e. no profitable deviation in stage 2, $p \leq \bar{p}(z, n)$), then $p^C = \bar{p}(z^C, n)$ and the predictable outcome is "weakly collusive". However, as proved in Proposition 2, this "collusive" solution is a Subgame Perfect Nash equilibrium, a result that is very unusual in a non-repeated game.

4 A textbook example

The model in this article is built on very general assumptions: two substitutable factors chosen sequentially, a quasi-concave production function, decreasing marginal factor productivity. Propositions (1) and (2) show that the equilibrium prediction for the whole game can be seen as a solution to a maximisation program subject to three different inequality constraints. We can exploit this unusual and remarkable property to compute equilibrium predictions of parametrical versions of price competition models with soft capacity constraints.

4.1 General procedure and parametrization

In this section, we provide a general procedure to compute equilibria and apply it to solve a simple parametrical example numerically as a "textbook case", assuming a Cobb-Douglas production function and linear demand.

The production function is:

$$f(z, v) = A\left(z^{1-\alpha} v^{\alpha}\right)^{\rho} \tag{12}$$

with $\rho > 0$, the scale elasticity of production, and, $\rho\,\alpha < 1$ and $\rho\,(1-\alpha) < 1$ because of the decreasing marginal factor productivity. In general of course, the Cobb-Douglas production function is quasi-concave. It will be concave when $\rho = 1$ (constant returns to scale) and strictly concave when $\rho < 1$ (decreasing returns to scale). Taking $y = f(z, v)$, the function $\hat{v}$ is easily obtained by direct calculation:

$$\hat{v}(y, z) = \frac{y^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}} z^{\frac{1-\alpha}{\alpha}}} \tag{13}$$

Thus, when n firms operate in the market, the function $\hat{\pi}$ can be written:

$$\hat{\pi}(p, z, n) = p\frac{D(p)}{n} - w_1 z - w_2 \frac{y^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}} z^{\frac{1-\alpha}{\alpha}}} \tag{14}$$

The demand function is assumed to be linear:

$$D(p) = b(p_{max} - p) \tag{15}$$

with $b > 0$. We will show that although the assumptions are simple, they are sufficient to demonstrate the full richness of our theoretical framework. We take the Payoff dominant subgame perfect Nash Equilibrium of Proposition (2) - the solution of programme $(\mathcal{P}_2)$- as the predictable outcome of our general model of price competition with soft capacity constraints. But because the "non-existence of limit pricing strategies" condition can be tricky to deal with directly, we will proceed sequentially.

Step 1. We solve program $(\mathcal{P}_1)$ for a given number of firms, n, a given price $p \in (0, p_{max})$, and a given vector of parameters $(\alpha, \rho, A, b, w_1, w_2)$, and obtain $z^*(p, n)$ the level of the fixed factor that efficiently sustains price p. We are thus able to calculate $\Pi(p, n) = \hat{\pi}(p, z^*(p, n), n)$.

Step 2. For a given n, the process in step 1 can be repeated for any $p \in (0, p_{max})$. So we are able to draw $\Pi(p, n)$ as a function of p point by point.

Step 3. The **profitability** condition $\Pi(p, n)) \geq 0$ can be tested for each point $(p, \Pi(p, n))$ calculated in Step 2.

Step 4. For each point $(p, \Pi(p, n))$ calculated in Step 2, we can calculate $\hat{p}(z^*(p, n), n)$ and then test for the **non-existence of limit pricing strategies**: $\hat{\pi}(p, z^*(p, n), n) \geq \hat{\pi}(\hat{p}(z^*(p, n), n), \underset{\tilde{z}}{\operatorname{argmax}}\, \hat{\pi}(\hat{p}(z^*(p, n), n), \tilde{z}, 1), 1)$.

Step 5. For each point $(p, \Pi(p, n))$ calculated in Step 2, we can check if the **non profitable deviation in stage 2** condition is binding or not.

Step 6. Among all the $(p, \Pi(p, n))$ that satisfy the **profitability** test in Step 3 and the **non-existence of limit pricing strategies** test in Step 4, we search for the price that provides the highest profit, which is p^C, the equilibrium price solution of $(\mathcal{P}_2)$. If in Step 5 the **non profitable deviation in stage 2** condition is binding, $p^C = \bar{p}(z^*(p^C, n), n)$, otherwise, $p^C = p^*(z^*(p^C, n), n)$.

For a given number of firms and all prices, the constraints of Program $(\mathcal{P}_2)$ can all be binding or slack. In each case, there will be a threshold price delimiting the subdomain in which each constraint is binding. We will denote: $\tilde{p}(n) = \bar{p}(z^*(p, n), n)$, the threshold above which profitable deviations are excluded in the second stage; $p^0(n)$, the threshold above which firms earn a positive profit; and $p^L(n)$, the threshold above which a fixed factor can be chosen in the first stage that can make limit pricing strategies profitable in the second.

Consider the following numerical example.[3] Figure 1 shows the behavior predicted for a duopoly (n=2), with constant returns to scale (ρ=1), $\alpha = .7$, $p_{max} = 10$, and all the other parameters normalized to 1.

[3]Numerical simulations were performed using Wolfram Research Mathemathica 11. The optimization programs were solved numerically using the NMaximize function and the value of $\hat{p}(z^*(p, n), n)$ was obtained using the Findroot function.

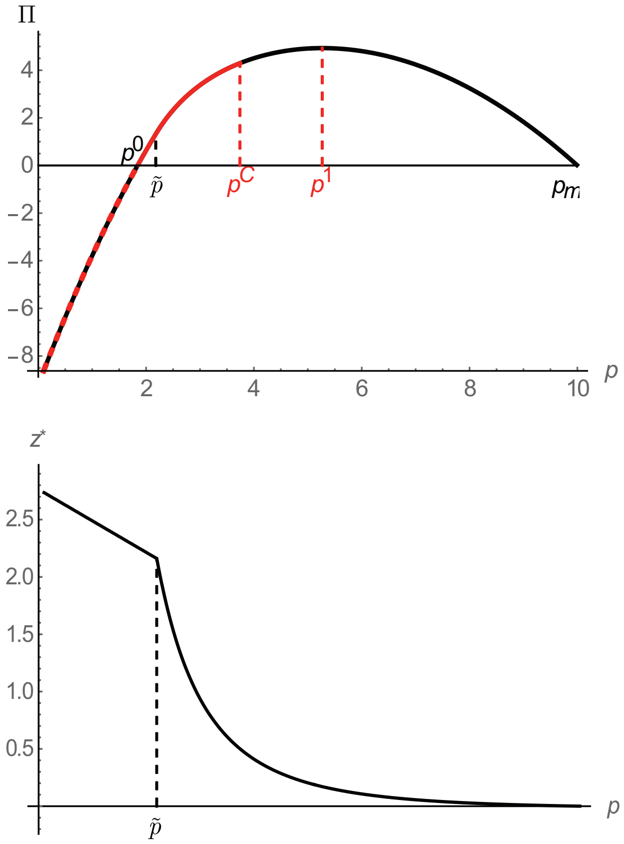

Figure 1: The *SPNE* interval for price (plain red) and associated fixed factor levels when $\rho = 1$, $\alpha = .7$, $n = 2$, w_1=1, w_2=1.

The lower graphic represents $z^*(p, n)$ the solution of the Program $(\mathcal{P}_1)$ *i.e.* the efficient level of the fixed factor taking into account the **non profitable deviation in stage 2** constraint. For $p \leq \tilde{p}$, the constraint is slack. Of course, z^* decreases as p increases, converging to zero as p tends to p_{max}. Conversely, for $p \geq \tilde{p}$, the constraint is binding ($p = \bar{p}(z^*, n)$). It will become apparent that a binding constraint implies a much lower level of z for a given price.

The upper graph in Figure 1 shows the whole function $\Pi(p, n) = \hat{\pi}(p, z^*(p, n), n)$ for $p \in (0, p_{max})$. The left dotted part, for $p \in (0, p^0)$, corresponds to negative profits. This price interval can therefore not be a *SPNE* of the two-stage game. The right part of the curve (in black) does not fulfil the **non-existence of limit pricing strategies** condition (verified in Step 4 of our procedure), and cannot correspond either to a *SPNE*.

Consequently, the remaining (red) part of the curve corresponds to values of p for which both the **profitability** and **non-existence of limit pricing strategies** conditions are fulfilled. This means that all (p, z) pairs for which p belongs to $[p^0, p^C]$ and $z = z^*(p, n)$ are *SPNE* of the two-stage game. It is easy to check that p^C corresponds to the "Payoff dominant" *SPNE* of the whole game (the solution of program ($\mathcal{P}_2$)). With this vector of parameters, we can see that the price p^1 that maximizes $\Pi(p, n)$ does not correspond to a *SPNE* (note that the Π function is defined in step 1 of our general procedure).

4.2 Effect of the concavity of the variable cost and the fixed factor price

The convexity of the cost function in the second stage is a crucial feature of our model. With a Cobb-Douglas production function, this convexity in the second stage (with z fixed) is determined by the product of α and ρ. When $\alpha\rho$ tends to one, the variable cost function becomes linear. For a given level of scale elasticity, ρ, a lower α corresponds to a "more convex" production function.

Figure 2 shows the effect of different levels of convexity on the equilibrium prediction of the whole game when five firms operate in the market.

In the upper graphic, $\alpha = .45$, meaning that the variable cost function is highly convex. In this case, $\tilde{p}$ is higher. The more convex variable cost function implies that price deviation in the second stage is more costly (here, with five firms, the deviating firm will have to produce approximatively five times more.) Thus, p^C and p^L differ from $\bar{p}$, and the corresponding z^* will be higher. This is why $p^L > p^1 = p^C$. The maximum of $\Pi(p, n)$ corresponds to the solution of Program ($\mathcal{P}2$).

In the middle graphic, with $\alpha = .6$, $\tilde{p}$ is now lower than p^1 and $p^L = p^C$. p^1 and $p^L = p^C$ are thus $\bar{p}$, meaning that the non profitable deviation condition in the second stage is binding (implying that z^C is much lower). At p^1, limit pricing strategies are profitable ($p^1 > p^L$); p^1 is not a *SPNE*. This is why $p^C = p^L$ is the highest possible profit in the absence of of limit pricing strategies.

The lower graph shows that the behavior is similar with $\alpha = .75$, with p^C being much more lower than p^1. The **non-existence of limit pricing strategies** constraint excludes more than half of the prices between p^0 and p^1 from being *SPNE*.

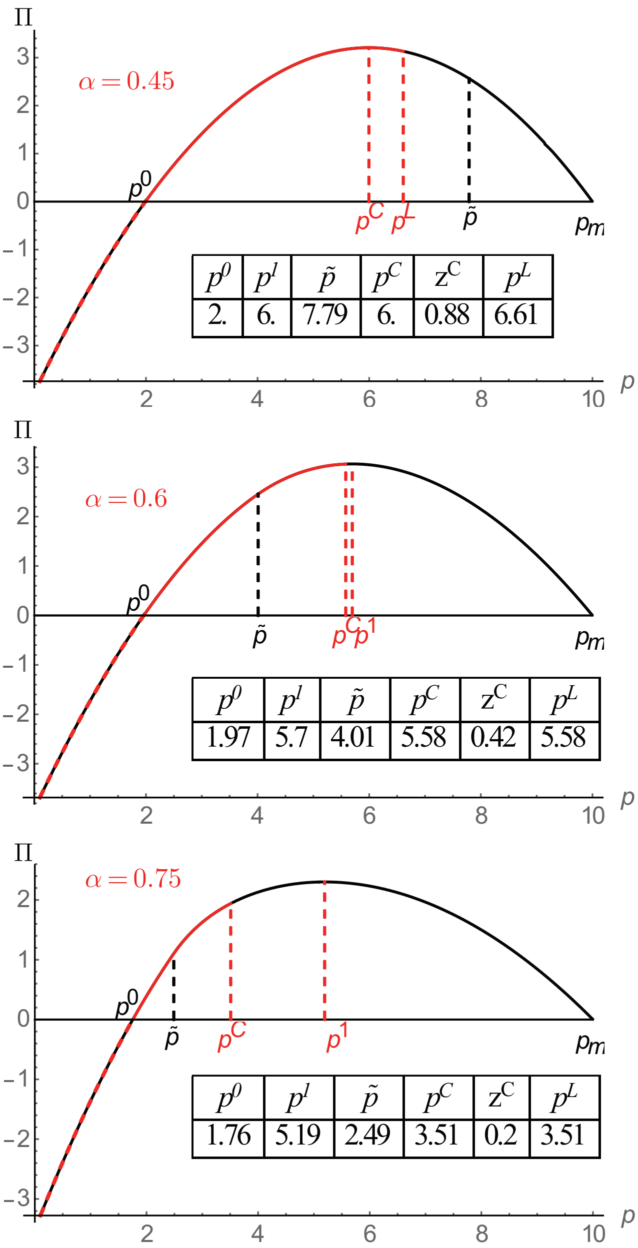

Figure 2: Effect of α on the *SPNE* interval for price (plain red) ($\rho = 1$, $n = 5$, w_1=1, w_2=1).

Finally, in this example, when $\alpha \to 0.45 \to 0.6 \to 0.75$, $p^C \to 6 \to 5.58 \to 3.51$ and $z^C \to 0.88 \to 0.42 \to 0.20$: the lower the convexity is, the lower the equilibrium price is. The effect on the level of the fixed factor is more complex to analyze. A lower price implies a higher demand and thus, all things being equal, optimally requires a higher fixed factor to produce. However, a higher α implies that the production process uses the variable factor more intensively, and thus that z is lower. Moreover, when the "non profitable deviation in the second stage" condition is binding, the optimal level of z (the solution of program $\mathcal{P}_1$) is much lower (cf. Figure 1).

The same method can be used to study the effect of all parameters other than the number of firms.

4.3 Effect of the number of firms and of returns to scale

We will now study how the price varies with the number of firms. We will show that the nature of the returns to scale has a qualitative impact on this relation.

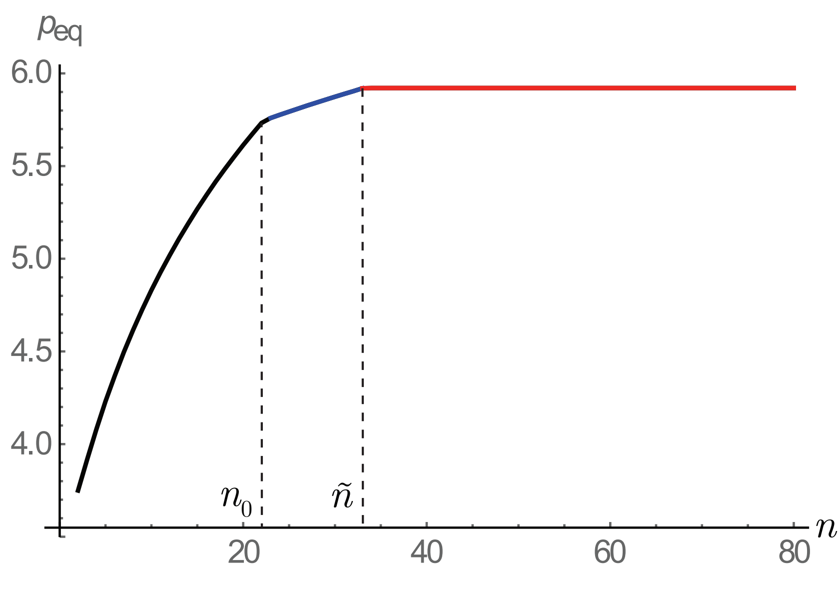

Figure 3: Effect of the number of firms on equilibrium prices ($\rho = 1$, $\alpha = 0.7$, w_1=1, w_2=1).

Considering constant returns to scale first, Figure 3 shows the equilibrium prediction of p^C as a function of the number of firms when $\rho = 1$, $\alpha = 0.7$, w_1=1, and w_2=1. Two important thresholds are apparent. When the number of firms is low (between 2 and n_0), the **non-existence of limit pricing strategies** constraint is binding. The lower n is, the more effective this constraint is, meaning that the $p^L(n) = p^C$ threshold increases with n. From $n = n_0$ onwards, the **non-existence of limit pricing strategies** constraint is no longer binding. Between n_0 and $\tilde{n}$, the non-profitable deviation constraint is binding. The corresponding prices are $\bar{p}$. The higher n is, the less effective this constraint is. Prices continue to increase with n. Beyond $\tilde{n}$, none of the constraints are binding at equilibrium. p^C corresponds to a purely collusive outcome. Because of the constant returns to scale, the market price p^C is independent of the size of the firm and thus of the number of firms sharing the market.

This result is very unusual! When the number of firms is lower than $\tilde{n}$, the price increases with the number of firms. This is still because the cost function is convex. In our model, any firm that deviates (either in the second stage by lowering its price or in the first stage by following a limit pricing strategy in z) will capture the entire market (i.e. operate in the market alone). The increase in production is proportional to the number of firms. Because of the convexity of the variable cost, the higher the increase in production is, the lower the incentive to deviate is. However, a convex short-run cost function does not guarantee that the price increases with the number of firms. As shown in Lemma 5, and also by Dastidar (2001), for a given z, $\hat{p}$, $\bar{p}$ and p^* decrease with n. It is the endogeneity of z, a specificity of our model, that adds a "capacity effect" that overcomes the direct effect of n on p for a given z. At equilibrium, z will tend to decrease with n and will, according to Lemma 5, have an indirect positive effect on p. As shown in the lower part of figure 1 this effect of z on p dominates when the constraint $p \leq \bar{p}$ is binding.

Let us now consider the limit case of free entry, in which firms enter the market as long as it is profitable to do so. With constant returns to scale, the endogenous number of firms will tend to infinity and each firm's market share and profit to zero. However, prices will remain at the collusive level, significantly higher than the constant long-run average/marginal cost. The markup will remain strictly positive and constant (i.e. never tending to zero)! In a sense, the convexity of the short-run cost in the second stage is stronger than the optional free entry condition. In our framework, free entry does not imply average cost pricing.

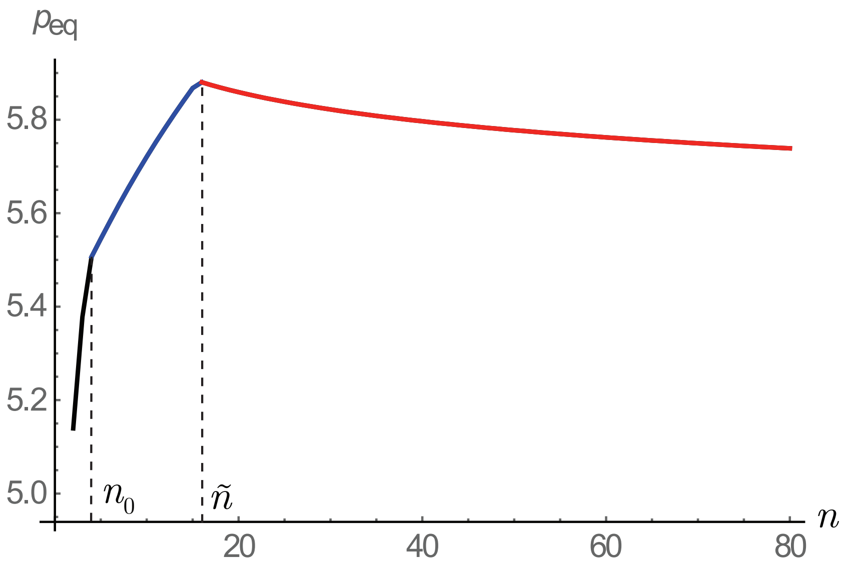

Figure 4: Effect of the number of firms on equilibrium prices ($\rho = 0.9$, $\alpha = 0.7$,w_1=1 , w_2=1).

Considering now decreasing returns to scale ($\rho = 0.9$) with all the other parameters left unchanged, the properties represented in Figure 4 are similar to those shown in Figure 3. The only difference is that in Figure 4 the right part of the curve (in red) decreases as the number of firms increases. This is a direct effect of the decreasing returns to scale. Smaller firms will be more efficient and will have an interest, when the outcome is purely collusive, in sustaining slightly lower prices.

Figure 5 illustrates the case of increasing returns to scale ($\rho = 1.02$). The lower envelop of the three curves on the left-hand side of the figure (up to n_1) follows the same trend as described above for the corresponding parts of Figures 3 and 4, with the purely collusive part (in red, between $\tilde{n}$ and n_1) being slightly increasing because of the increasing returns to scale (a greater number of smaller firms sharing the market is less efficient). The novelty is that beyond the n_1 threshold, the **non-existence of limit pricing strategies** constraint becomes binding again. Limit pricing strategies are more efficient because of the increasing returns to scale. Beyond n_1, this gain is sufficient to cancel the effect of the convexity of the variable cost function described previously.

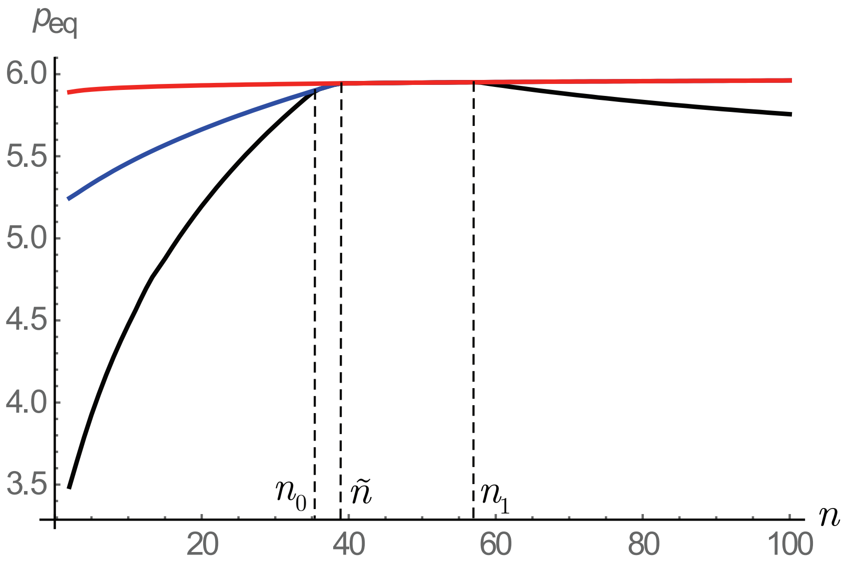

Figure 5: Effect of the number of firms on equilibrium prices($\rho = 1.02$, $\alpha = 0.7$,w_1=1 , w_2=1).

This last figure illustrates a very original property of our general model of price competition: there is an equilibrium even when returns to scale are increasing. As already pointed out in the introduction, this result stems from the sequential choice of production factors which makes the short-run cost convex in the second stage whatever the returns to scale. With a homogeneous single product, the long-run cost is clearly subadditive when returns are increasing. But in our model, in contrast with Dastidar's 2011b, this does not entail the existence of an equilibrium because the production factors are chosen sequentially and because the fixed factor z is set endogenously. It is noteworthy that the case of increasing returns to scale (or more generally, of cost subadditivity) is usually associated with the notion of a *natural monopoly*. In the *contestable market* theory (Baumol and Willig, 1981; Baumol et al., 1988), the threat of entry is assumed to be sufficient to drive the price down to the average cost with a single firm operating in the market. As pointed out by Tirole (1988) (p. 310) *contestable market* theory has been seen as a "generalisation of Bertrand competition to markets with increasing returns to scale". Our general model of price competition with soft capacity constraints clearly refutes this claim. Not only can more than one firm operate in the market when returns are increasing, but they do so with positive profits and potentially high markups. Thus, in presence of increasing returns to scale, a market driven by price competition with soft capacity constraints suffers simultaneously from market inefficiency (existence of a markup) and production inefficiency (the average cost is not minimum).

The same general pattern occurs in all the different cases of returns to scale that has already been analyzed for constant returns to scale. Because of a "capacity effect", before a threshold number of firms, $\tilde{n}$, is reached, the price increases with the number of firms. Beyond this threshold, prices can decrease both when the returns to scale are decreasing (because of the collusive nature of the outcome) and when they are increasing (because of the combined effect of the non-existence of limit pricing strategies and technological inefficiencies). The general model of price competition that we propose accounts for high markups that can increase with the number of firms. Gabaix et al. (2016) obtained the same pattern for a homogeneous-good market but based on a very different model with random utility. These authors surveyed the empirical literature on this phenomenon.

5 Conclusion

The general model of price competition with soft capacity constraints we propose is a simple and natural extension of existing studies of price competition that bridges three lines of literature: capacity constraints, cost convexity and limit pricing strategies. We show that an equilibrium prediction in pure strategies exists whatever the number of firms and the nature of the returns to scale. This equilibrium prediction is characterized by high markups and prices that can increase with the number of firms in the market. It is the balance between the "natural" feature of this extension and its paradoxical outcome that is the principle contribution of our work.

As economists, we have been taught that price competition is stronger than quantity competition and also that tacit collusion can only result from threats and retaliation in a dynamic setting when firms are not too numerous. Our model clearly undermines these ideas in a very general framework, starting from a production function that is only required to be quasi-concave with decreasing factor marginal productivity.

The core property of the model is summarized in Propositions 1 and 2 in section 3. The predictable non-cooperative outcome of a non-repeated game of price competition with soft capacity constraints is equivalent to the solution of a joint profit maximization program.

This result is very unusual and it is important to understand the special mechanism that operates behind the scene. In the second stage, for a given soft capacity (i.e. a given level of the fixed factor), because of the Dastidarian property of the model stemming from a convex short-run cost function, there is a continuum of Nash Equilibria in prices. A direct reverse implication is that there is a continuum of levels of the fixed factor (chosen in the first stage) that lead to the same price equilibrium in the second. Thus, for a given price in the second stage, it can be profitable for firms to deviate in the first from any fixed factor level that is not the one that maximizes the profit for that price, as long as this price remains sustainable as a Nash equilibrium in the second stage. It is thus the combination of a sequential choice of production factors with the continuum of Nash equilibria in the second stage that is at the origin of the joint profit maximisation property described in Proposition 2.

This has two important consequences. First, from a positive point of view, this property can be used to compute the equilibrium of the game. In general, studies of price competition are cursed by discontinuities, making it impossible to use standard reasoning based on continuous reaction functions to compute the equilibrium. The general methodology provided in section 4 offers a much more tractable method to compute the predictable outcome of the price competition game. Second, from a normative point of view, this outcome, which can be termed "collusive", is obtained in a non-cooperative framework, generalizing the claim of Cabon-Dhersin and Drouhin (2014) that the model offers an alternative mechanism for tacit collusion.

However, as general it is, our model relies on a number of assumptions that are debatable. Of course, as is always the case for Bertrand-Dastidar competition, the assumption that firms are committed to satisfying all incoming demand is a limit that has been commented upon at length. More interesting is the question of the robustness of our results with respect to some of the simplifying assumptions we made. What happens when firms use different technologies. What happens when a new firm arrives on an existing market? What happens if uncertainty (about demand, costs, etc.) is introduced into the model? What happens if dynamic effects are included? The textbook example that we provide here is sufficiently striking to prove that it is worth pursuing.

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