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The Corridor’s Width as a Monetary Policy Tool

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The Corridor’s Width as a Monetary Policy Tool

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Abstract
Credit institutions borrow liquidity from the central bank’s lending facility and deposit (excess) reserves at its deposit facility. The central bank directly controls the corridor: the non-market interest rates of its lending and deposit facilities. Modifying the corridor changes the conditions on the interbank market and allows the central bank to set the short-term interest rate in the economy. This paper assesses the use of the corridor’s width as an additional tool for monetary policy. Results indicate that a symmetric widening of the corridor boosts output and welfare while addressing the central bank’s concerns over higher risk-taking in the economy.

Keywords: Monetary Policy; Interbank Market; Heterogeneous Interbank Frictions; the Corridor; Excess Reserves; Financial Intermediation

JEL Codes: E52, E58, E44

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1 Introduction

Monetary policy is implemented by setting the targeted short-term interest rate, the interest rate on the overnight interbank market. The main tool used by the central bank to set this market interest rate is the corridor: non-market interest rates of the lending and deposit facilities controlled directly by the central bank. Modifying the corridor changes the conditions on the interbank market and allows the central bank to set the overnight interest rate. Most macroeconomic models assume simply that the central bank directly controls the short-term interest rate in the economy and ignore the role of the corridor in monetary policy implementation. Yet ignoring the corridor prevents any analysis of how the operational framework of monetary policy impacts the real economy. This simplifying assumption is particularly problematic in the post-2008 world, where central banks experiment with new policy instruments to stabilize economic activity.

This paper proposes a model that contributes towards filling this gap. I introduce monetary policy implementation based on the central bank’s facilities and the corridor. Furthermore, I assess the use of the corridor’s width as an additional monetary policy tool allowing the central bank to boost output and increase welfare while simultaneously addressing concerns over the higher risk-taking involved in a low interest rate environment.

The core of the model is the financial sector, where financial intermediaries raise retail deposits from households, issue loans to non-financial firms and borrow or lend on the interbank market. They can also use the facilities offered by the central bank: they can borrow liquidity from the lending facility at a non-market interest rate, controlled directly by the central bank, and/or they can deposit reserves at the deposit facility and receive a non-market interest rate, controlled directly by the central bank, on these reserves. Moreover, intermediaries face heterogeneous frictions on the interbank market, and this heterogeneity plays a central role in their decisions to use each of the central bank’s facilities. Introducing heterogeneous frictions allows me to model a realistic use of these facilities, i.e. the fact that the financial intermediaries using the lending facility are of a different type from those using the deposit facility. Furthermore, the model includes price rigidities and conventional monetary policy.

I calibrate the model to match recent U.S. data. I focus on key interest rate spreads and banking variables such as the spread between the interest rate on excess reserves paid by the Fed and the interest rate paid by commercial banks on retail deposits, the ratio of retail deposits to commercial banks’ total assets, banks’ leverage, etc. Results from the quantitative analysis suggest that in an economy where the financial sector holds excess reserves at the central bank, the corridor’s width can be a valuable tool for monetary policy. More particularly, when excess reserves deposited by a financial intermediary at the central bank are excess reserves.

I do not include minimum reserve requirements in the model, therefore any reserves deposited by a financial intermediary at the central bank are excess reserves.

See the literature on monetary policy implementation, where the central bank’s facilities are used by banks of different types (see Hauck and Neyer, 2014, Ennis and Keister, 2008 and Colliard et al., 2016).
reserves deposited at the central bank are not held by financial intermediaries that lend to non-financial firms investing in new capital, then lowering the corridor (jointly lowering the interest rates of both the lending facility and the deposit facility) by 0.87% has the same positive impact on output as a symmetric 1% widening of the corridor (increasing the interest rate of the lending facility and jointly lowering the interest rate of the deposit facility). The symmetric widening of the corridor relieves the constraint on these intermediaries’ balance sheets, so they can increase their lending to the real economy. This in turn enhances output and welfare. Implementing this policy thus allows the central bank to boost welfare and the real economy, while simultaneously addressing concerns over higher risk-taking in a low interest rate environment (see Dell’Ariccia et al., 2017 and Lee et al., 2017): a symmetric widening of the corridor positively impacts both output and welfare while limiting the decrease in the short-term interest rate.\(^3\)

After the Great Contraction, several central banks decided to pay a negative interest rate on (excess) reserves to overcome the Zero Lower Bound on nominal interest rates. In these economies, credit institutions face a disconnection between the assets and liabilities sides of their balance sheets, receiving a negative interest rate on one component of their assets (excess reserves at the central bank) but paying a positive interest rate on retail deposits (one component of their liabilities).\(^4\) Policymakers stress the damaging effects that such a disconnection may have on the financial system (see McAndrews, 2015 and Borio and Zabia, 2016). This paper suggests an explanation for this continuing disconnection. It shows that when reserves are only an asset to financial intermediaries (not used to produce liquidity services), heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on excess reserves. When financial intermediaries are constrained in their ability to raise funds from households, they will deposit excess reserves at the central bank to relieve the constraint on their balance sheets even if the interest rate they receive on these reserves is lower than the interest rate they pay to households on their retail deposits.

This paper is based on the canonical model of Gertler and Kiyotaki (2011). Unlike their model, here financial intermediaries face heterogeneous instead of symmetric frictions on the interbank market and are able not only to borrow liquidity from the central bank but also to deposit excess reserves at its deposit facility. I also introduce price rigidities, which allows me to model the conventional monetary policy.

Since the Great Recession, there has been a renewed interest in assessing monetary policy implementation through the central bank’s facilities instead of just assuming that the central bank directly controls the short-term interest rate on the interbank market to be 47 basis points less than the decrease in this interest rate when the corridor is lowered by 0.87%.

\(^{3}\)To have the same positive impact on output, a symmetric widening of the corridor of 1% allows the decrease in the interest rate on the interbank market to be 47 basis points less than the decrease in this interest rate when the corridor is lowered by 0.87%.

\(^{4}\)The disconnection between the assets and liabilities sides of credit institutions’ balance sheets was present in the United States before the Great Contraction (the Fed did not pay interest on reserves before October 2008). However, the significant reserves since created by central banks have made this a crucial issue for the proper functioning of the financial system in the post-2008 era.
interest rate. Nonetheless, with the exception of Berentsen and Monnet (2008)\textsuperscript{5} and Bianchi and Bigio (2014)\textsuperscript{6}, to my knowledge all the papers considered the impact of at best only one of the two central bank’s facilities.

Berentsen and Monnet (2008) consider monetary policy under a corridor system. They show that a symmetric widening of the corridor is welfare decreasing and amounts to policy tightening by the central bank. They also show that it is optimal to have a positive spread between the interest rates of the central bank’s lending and deposit facilities if the opportunity cost of holding the collateral needed to borrow liquidity from the central bank is positive. In this paper I also focus on using the corridor’s width as a tool for monetary policy, but instead of using a Lagos and Wright (2005) framework I analyze its impact in a New Keynesian framework. Furthermore, unlike Berentsen and Monnet (2008), I show that a symmetric widening of the corridor is welfare increasing and amounts to policy loosening by the central bank. This difference is due to differences in the assumptions of the two models. Berentsen and Monnet (2008) base welfare on the utility of agents that borrow liquidity and/or deposit (excess) reserves at the central bank, thus implicitly considering the welfare of the banking sector. In contrast, the welfare I consider in this paper is based on the utility of households that are different agents from financial intermediaries. As a result, a symmetric widening of the corridor is welfare decreasing for the banking sector but welfare increasing for non-banking agents.

Bianchi and Bigio (2014) relax the standard assumption that the central bank directly controls the prevailing short-term interest rate and implements monetary policy through the banking sector. They focus on introducing classic insights from the liquidity management literature into a dynamic, general equilibrium model. My model also focuses on the implementation of monetary policy through financial intermediaries, but it focuses on how heterogeneous frictions on the interbank market affect the monetary policy transmission mechanism.

Several models consider the impact of credit institutions depositing reserves at the central bank. When reserves are considered simply as an asset for credit institutions, at equilibrium if credit institutions deposit excess reserves at the central bank the interest rate thereon must be strictly higher than the interest rate paid on retail deposits (see Ennis, 2014 and Martin et al., 2016). On the other hand, when reserves are part of the banking technology function, at equilibrium the interest rate paid on retail deposits can be higher than the interest rate paid on reserves. Cúrdia and Woodford (2011) show that when credit institutions’ consumption of real resources to produce loans decreases with an increase in reserve holdings, the interest rate paid on retail deposits is higher than the interest rate paid on reserves. Dressler and Kresting (2015) show that if banks face an idiosyncratic lending cost, when the

\textsuperscript{5}Several papers analyze some aspects of monetary policy under a corridor system from different standpoints: Martin and Monnet (2011) compared, in a variant of Berentsen and Monnet (2008), monetary policy with a Standing Facilities framework and an Open Market Operations framework. Whitesell (2006) compared implementing monetary policy mainly through reserve requirements and relying on standing facilities to implement the central bank’s policy.

critical cost such that a bank is indifferent between holding loans or excess reserves is high enough, some banks will deposit all their funds as excess reserves at the central bank. In this case, the banking system portfolio will include excess reserves even though the interest rate paid on reserves is lower than the interest rate paid to households on retail deposits. This paper complements this literature by showing that in a model where reserves are simply an asset for financial intermediaries, and without introducing any ad hoc operational cost, heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on reserves.

I describe the model in section 2. Section 3 presents the model analysis while section 4 concludes the paper.

2 The model

The core framework is based on the canonical Gertler and Kiyotaki (2011) model. To this, I introduce heterogeneous frictions on the interbank market and the central bank’s corridor through its lending and deposit facilities. There are four types of agents in the model: households, financial intermediaries, non-financial goods producers and capital producers. In addition, the central bank directly controls the corridor defined by the interest rates of its facilities: the non-market interest rate that financial intermediaries pay when they borrow liquidity from the central bank and the non-market interest rate that the central bank pays on the reserves deposited by the intermediaries.

Two versions of the model are suggested. The first version, hereinafter called the Standing Facilities version and referred to as SFs, assumes that the household member selected to be a banker manages a financial intermediary for one period. Each financial intermediary decides on its liquidity borrowing from the central bank at the end of the period (simultaneous with their intervention on the interbank market and their decision to deposit excess reserves at the central bank). Theoretical results are derived from this version of the model to simplify the computations. In the second version, hereinafter called the Open Market Operations version and referred to as OMOs, a household member selected to be a banker this period stays a banker next period with a probability $\sigma$ regardless of how long this household member has been a banker. The financial intermediary decides on its liquidity borrowing from the central bank at the beginning of the period (before its intervention on the interbank market and its decision to deposit excess reserves at the central bank). The quantitative assessment is based on this version of the model. The timing difference between the two versions closely mimics the monetary authorities’ operational framework in most developed economies, where the central bank conducts open market operations early in the morning at the opening of the interbank market and borrowing from the standing facility can only be conducted either when the
interbank market is closing or is already closed\textsuperscript{7}.

Without heterogeneous frictions on the interbank market, the model is very close to Gertler and Kiyotaki (2011). As I show, allowing for this heterogeneity is sufficient to explain the interest rate paid by commercial banks on retail deposits being higher than the interest rate paid by the central bank on reserves. When financial intermediaries are constrained in their ability to raise funds from households, they will deposit excess reserves at the central bank to relieve their constraint even if the interest rate they receive on these reserves is lower than the interest rate they pay to households on their retail deposits.

\section*{2.1 Households}

There is a continuum of identical households of measure unity. Each household contains two types of members: workers and bankers. Each period, a household member is a worker with i.i.d. probability $1 - f$ and a banker with probability $f$. A worker supplies labor and returns the wages earned to the household. In the first version of the model, a banker manages a financial intermediary for one period and transfers any earnings to the household. In contrast, in the second version of the model, a banker this period has probability $\sigma$ being a banker next period. Thus, the average survival time for a banker in any given period is $1/ (1 - \sigma)$. The financial intermediary managed by the banker is owned by the household, retained earnings are transferred by an exiting banker to the respective household. Within the family there is perfect consumption insurance. As well as consuming and supplying labor, each household saves by lending funds to financial intermediaries that it does not own.

Let $C_t$ be consumption, $L_t$ be the family labor supply, $W_t$ be the real wage, $R_t$ be the gross real return on deposits at financial intermediaries, $D_{t+1}$ be the total amount of funds the household lends to financial intermediaries, $\Pi_t$ be net payouts to the household from ownership of both non-financial and financial firms and $T_t$ be the lump sum transfers from the central bank.

The household chooses consumption, labor supply and deposits at financial intermediaries to maximize expected discounted utility subject to its budget constraint:

$$\max_{C_t, L_t, D_{t+1}} \left\{ E_t \sum_{m=0}^{+\infty} \beta^m \left[ \ln(C_{t+m} - \gamma C_{t+m-1}) - \frac{x}{1 + \epsilon} L_{t+m}^{1+\epsilon} \right] \right\}$$

s.t. $C_t + D_{t+1} = W_t L_t + R_t D_t + \Pi_t + T_t$

\textsuperscript{7}The European Central Bank conducts open market operations once a week (through its main refinancing operations) while the Federal Reserve does so once a day at the beginning of the day (through its interventions on the Fed Funds market), see Friedman and Kuttner (2011) for more details. The ECB’s standing facility (its marginal lending facility) is available every day; however restricting open market operations to once a week creates the timing difference noted above, see ECB (2014). The Fed grants banks’ requests for its standing facility (the main standing facility of the Fed is its primary discount window) assistance only very late in the day when the money markets are closing, see Clouse (1994). Other major central banks have relatively similar frameworks which lead to the same difference in timing between open market operations and lending through the standing facility.
where $0 < \beta < 1$ is the discount rate, $0 < \gamma < 1$ the habit parameter, $\chi > 0$ the relative utility weight of labor and $\epsilon > 0$ is the inverse Frisch elasticity of labor supply.

The household’s first order conditions are standard:

$$L_t: \quad U_{ct} W_t = \chi L_t^\epsilon$$

$$C_t / D_{t+1}: \quad \Lambda_{t,t+1} R_{t+1} = 1$$

Where:

$$U_{ct} = \frac{1}{C_{t-\gamma} C_{t-1}} - \frac{\beta \gamma}{C_{t+1-\gamma} C_t}$$

$$\Lambda_{t,t+1} = \beta \frac{U_{ct+1}}{U_{ct}}$$

### 2.2 Financial intermediaries

Intermediaries raise funds on retail (by collecting deposits from households) and wholesale (by borrowing from each other) financial markets, decide the amount they lend to non-financial firms, the amount of liquidity to borrow from the central bank and the amount of excess reserves to deposit at the central bank (any reserves deposited by a financial intermediary at the central bank are excess reserves, since there are no minimum reserve requirements in the model).

At the beginning of the period, each financial intermediary $j$ raises deposits $d_{j,t+1}$ from households on the national retail financial market at the deposit rate $R_{t+1}$. After the retail financial market closes, investment opportunity arrives randomly to a fraction $\pi^i$ of islands. This creates surpluses and deficits of funds across financial institutions, assuming that each intermediary can lend funds only to the non-financial firm on the same island, that capital is not mobile across islands and that only firms located on investing islands can acquire new capital. After learning about its lending opportunities, intermediary $j$ decides the volume of its loans $s_{j,t}^h$ to non-financial firms and the funds it borrows (or lends) on the interbank market $b_{j,t+1}^h > 0 \left(b_{j,t+1}^h < 0\right)$. The subscript $h \in \{i, n\}$ denotes the type of the island on which intermediary $j$ is located: $i$ for investing and $n$ for non-investing.

In the SFs version of the model, intermediary $j$ decides, after learning about its lending opportunities (simultaneously with its decision to intervene on the interbank market and its decision to deposit excess reserves at the central bank), the amount of liquidity $l_f^{h,j,t+1} \geq 0$ to borrow from the central bank and the amount of excess reserves $d_f^{h,j,t+1} \geq 0$ to deposit at the central bank. In the OMOs version of the model, the intermediary’s decision regarding the amount of liquidity $l_f^{h,j,t+1} \geq 0$ to borrow from the central bank is taken before learning about its
lending opportunities (before its decision to intervene on the interbank market and its decision to deposit excess reserves at the central bank), while the decision regarding the amount of excess reserves to deposit at the central bank $d_{f, t+1}^h \geq 0$ is always taken after learning about its lending opportunities. This closely mimics the difference in timing between open market operations and the standing facilities of monetary authorities in developed economies.\(^8\)

Intermediaries raise retail deposits at the deposit rate $R_{t+1}$, borrow from the central bank at the refinancing operations rate $R_{t+1}^{lf}$, deposit excess reserves at the central bank that pay the deposit facility interest rate $R_{t+1}^{df}$ and lend or borrow on the interbank market at the interest rate $R_{t+1}^{b}$. Furthermore, claims on non-financial firms located on islands of type $h$ have a market price $Q_{t}^h$.

I follow Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) and introduce an agency problem that constrains the ability of financial intermediaries to collect funds: at the end of the period, the banker can choose to divert a fraction $\theta$ of available funds from the intermediary and transfer it back to the household of which he is a member. The potential cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction $(1 - \theta)$ of assets.

I allow intermediary $j$ to be differently constrained from raising funds from households, from borrowing liquidity from the central bank, from intervening on the interbank market and from depositing excess reserves at the central bank. Divertable assets consist of total gross assets net of (plus) a fraction $\omega^h$ of interbank borrowing (lending), net of a fraction $\omega^{lf}$ of liquidity borrowed from the central bank and plus a fraction $\omega^{df}$ of reserves at the central bank. The general case where frictions on the interbank market are heterogeneous is considered below.

I assume that the central bank is better able to collect information about intermediaries and to enforce repayment than lenders on the interbank market, which implies that it is harder for an intermediary to divert liquidity borrowed from the central bank than to divert funds borrowed on the interbank market. Thus, if intermediaries on islands of type $h$ borrow on the interbank market and borrow liquidity from the central bank, the following strict inequality must hold: $\omega^{lf} > \omega^h$.

Furthermore, I assume that it is harder for households to recover funds lent by intermediaries on the interbank market than funds deposited at the central bank as excess reserves. This seems reasonable due both to the additional information and supervision the central bank has on intermediaries and to its capacity to perfectly track reserves if a banker decides to divert. Thus if intermediaries on islands of type $h'$ lend on the interbank market and deposit excess reserves at the central bank, the following strict inequality must hold: $\omega^{h'} > \omega^{df}$.

\(^8\)See footnote 7 for more details.
2.2.1 The Standing Facilities version

In this section I present the Standing Facilities version of the model, where a banker manages a financial intermediary for only one period and financial intermediary \( j \) decides on its liquidity borrowing from the central bank simultaneously with its entry to the interbank market. Theoretical results presented below are derived from this version for simplicity. Let \( \Pi_{j,t+1}^h \) be the profits of intermediary \( j \) collected between periods \( t \) and \( t + 1 \). Then in the SFs version, profits \( \Pi_{j,t+1}^h \) and the balance sheet of intermediary \( j \) during the period \( t \) are:

\[
\Pi_{j,t+1}^h = R_{t+1}^{k,h} Q_{j,t}^h + R_{t+1}^{d_j} d_{f,j,t+1}^h - R_{t+1}^b b_{j,t+1}^h - R_{t+1}^{lf} l_{f,j,t+1}^h - R_{t+1}^{d_j,t+1} d_{j,t+1}
\] (3)

\[
Q_{j,t}^h s_{j,t}^h + d_{f,j,t+1}^h = b_{j,t+1}^h + l_{f,j,t+1}^h + d_{j,t+1}
\] (4)

To ensure that lenders will be willing to supply funds to the banker, the following incentive constraint must be satisfied:

\[
\Pi_{j,t+1}^h \geq \theta \left( Q_{j,t}^h s_{j,t}^h + \omega d_{f,j,t+1}^h - \omega b_{j,t+1}^h - \omega l_{f,j,t+1}^h \right)
\] (5)

Intermediary \( j \)'s manager maximizes profits (3) under the incentive constraint (5), the intermediary’s balance sheet (4) and the non-negativity constraints from its liquidity borrowings from the central bank \( l_{f,j,t+1}^h \geq 0 \) and from its reserves deposits at the central bank \( d_{f,j,t+1}^h \geq 0 \).

Let \( \lambda_t^h \) be the Lagrangian multiplier for the incentive constraint (5). Then intermediary \( j \)'s first order conditions are the following:

\[
s_{j,t}^h : \quad R_{t+1}^{k,h} - R_{t+1}^b = \theta (1 - \omega^h) \frac{\lambda_t^h}{1 + \lambda_t^h}
\] (6)

\[d_{j,t+1} : \quad R_{t+1}^b - R_{t+1} = \theta \widetilde{\lambda}_t \frac{\lambda_t^h}{1 + \lambda_t^h}
\] (7)

\[l_{f,j,t+1} : \quad R_{t+1}^{lf} - R_{t+1}^b \geq \theta (\omega^f - \omega^h) \frac{\lambda_t^h}{1 + \lambda_t^h}
\] (8)

\[d_{f,j,t+1} : \quad R_{t+1}^b - R_{t+1}^{df} \geq \theta (\omega^h - \omega^{df}) \frac{\lambda_t^h}{1 + \lambda_t^h}
\] (9)

where \( \lambda_t^h = \sum_{h=i,n} \pi^h \omega^h \lambda_t^h \)

\[\widetilde{\lambda}_t = \sum_{h=i,n} \pi^h \lambda_t^h
\]

2.2.2 The Open Market Operations version

This section presents the OMOs version of the model, where a banker this period has probability \( \sigma \) of remaining a banker next period and financial intermediary \( j \) decides on its liquidity borrowing from the central bank before
entering the interbank market. The quantitative assessment discussed below is based on this version of the model.

The banker managing intermediary $j$ optimizes expected terminal wealth at the beginning of the period:

$$V_{j,t} = E_t \sum_{m=0}^{+\infty} \left[ (1 - \sigma) \sigma^m \Lambda_{t,t+1+m} \sum_{h=i,n} \pi^h N^h_{j,t+1+m} \right]$$

(10)

where $N^h_{j,t+1}$ is the net worth of a financial intermediary $j$ located on an island of type $h$ at the beginning of period $t + 1$ after the settlement of period $t$ contracts and before the opening of any market on period $t + 1$. Over time, intermediary $j$’s net worth evolves as the difference between earnings on assets and interest payments on liabilities:

$$N^h_{j,t+1} = R^h_{t+1} Q^h_{j,t} s^h_{j,t} + R^{df}_{t+1} d^f_{j,t+1} - R^b_{t+1} b^h_{j,t+1} - R^{lf}_{t+1} l^f_{j,t+1} - R_{t+1} d^f_{j,t+1}$$

(11)

I follow Gertler and Kiyotaki (2011) and assume a transfer of net worth between the financial intermediaries, at the beginning of each period after the settlement of the contracts of the previous period and before the opening of any market on period $t + 1$, such as to equalize the ratio of total intermediary net worth to total capital on all islands. This assumption frees me from having to keep track of the distribution of net worth across islands. As a consequence, the balance sheet of intermediary $j$ during period $t$ is:

$$Q^h_{t} s^h_{j,t} + d^f_{j,t+1} = N^h_{j,t} + b^h_{j,t+1} + l^f_{j,t+1} + d^f_{j,t+1}$$

(12)

To ensure that lenders will be willing to supply funds to the banker, the following incentive constraint must be satisfied:

$$V_{j,t} \geq \theta \left( Q^h_{t} s^h_{j,t} + \omega^d d^f_{j,t+1} - \omega^b b^h_{j,t+1} - \omega^l l^f_{j,t+1} \right)$$

(13)

To solve the model, I guess the following from for the value function and then verify this guess (see Appendix A.1 for more details):

$$V_{j,t} = \sum_{h=i,n} \pi^h \left[ \nu^h_{t} s^h_{j,t} + \nu^h_{df} d^f_{j,t+1} - \nu^h_{b} b^h_{j,t+1} - \nu^h_{lf} l^f_{j,t+1} - \nu^h_{d} d^f_{j,t+1} \right]$$

Intermediary $j$’s manager maximizes terminal wealth (10) under the incentive constraint (13), the intermediary’s balance sheet (12) and the non-negativity constraints from its liquidity borrowings from the central bank $l^f_{j,t+1} \geq 0$ and from its reserves deposits at the central bank $d^f_{j,t+1} \geq 0$.

Let $\lambda^h_t$ be the Lagrangian multiplier for the incentive constraint (13). Then intermediary $j$’s first order
conditions are the following:

\[ s_{j,t}^h : \frac{\nu_{b,t}^h}{Q_t^i} - \nu_{b,t}^h = \theta(1 - \omega^h) \frac{\lambda_t^h}{1 + \lambda_t} \]  

(14)

\[ d_{j,t+1} : \sum_{h=i,n} \pi^h (\nu_{b,t}^h - \nu_{b,t}^h) = \theta \frac{\bar{\lambda}_t}{1 + \Lambda_t} \]  

(15)

\[ l_{f,t+1} : \sum_{h=i,n} \pi^h (\nu_{f,t}^h - \nu_{b,t}^h) \geq \theta \sum_{h=i,n} \pi^h \frac{[\omega^{lf} - \omega^h] \lambda_t^h}{1 + \Lambda_t} \]  

(16)

\[ d_{f,t+1}^h : \nu_{b,t}^h - \nu_{d,f,t}^h \geq \theta (\omega^h - \omega^{df}) \frac{\lambda_t^h}{1 + \Lambda_t} \]  

(17)

where \( \bar{\lambda}_t = \sum_{h=i,n} \pi^h \omega^h \lambda_t^h \)

\[ \Lambda_t = \sum_{h=i,n} \pi^h \lambda_t^h \]

After verifying the form of the value function (see Appendix A.1 for more details), the marginal gains and costs have the following laws of motion:

\[ \nu_{b,t}^h = \Omega_{t+1}^h R_{t+1}^b \]  

(18)

\[ \nu_{s,t}^h = \nu_{b,t}^h R_{t+1}^h Q_t^i R_{t+1}^b \]  

(19)

\[ \nu_{l_{f,t}}^h = \nu_{b,t}^h R_{t+1}^{lf} R_{t+1}^b \]  

(20)

\[ \nu_{d_{f,t}}^h = \nu_{b,t}^h R_{t+1}^{df} R_{t+1}^b \]  

(21)

\[ \nu_t^h = \nu_{b,t}^h R_{t+1}^b \]  

(22)

where \( \Omega_{t+1}^h = \Lambda_{t+1} \left[ (1 - \sigma) + \sigma \left[ \nu_{b,t+1}^h (1 + \bar{\lambda}_{t+1}) - \theta \lambda_{t+1}^h \omega^h \right] \right] \)

2.3 Intermediate goods firms

There is a continuum of islands each of which has a goods-producing firm and a financial intermediary. I assume that a non-financial firm can borrow funds only from the financial intermediary on the same island. Each financial intermediary has perfect information about the non-financial firm on the same island and can enforce contractual obligations with this borrower. Therefore, given its supply of funds, a financial intermediary can lend frictionlessly to the non-financial firm on the same island against its future profits. The firm is thus able to offer the intermediary a perfectly state-contingent debt, which can be considered equity.

Firms at the beginning of each period face an idiosyncratic shock when investment opportunity randomly
reaches a fraction $\pi^i$ of islands, hereinafter referred to as investing islands. Only firms with investment opportunities can acquire new capital. The shock is i.i.d. across time and islands. As in Gertler and Kiyotaki (2011), it creates differences in liquidity needs across non-financial firms which will lead to differences in liquidity needs across financial intermediaries.

At the end of period $t$, goods-producing firm $m$ acquires capital $k_{m,t+1}^h$ for use in production in period $t + 1$. The firm finances its purchases of capital by borrowing funds from the financial intermediary on the same island. To acquire the funds, firm $m$ issues $s_{m,t}^h$ claims equal to the number of units of capital it holds at the price of a unit of capital on island $h$, $Q_t^h$:

$$Q_t^h k_{m,t+1}^h = Q_t^h s_{m,t}^h$$  \hspace{1cm} (23)

I assume that aggregate output $Y_t$ is a constant-returns-to-scale Cobb-Douglas production function with aggregate capital $K_t$ and aggregate labor $L_t$ as inputs: $Y_t = K_t^\alpha L_t^{1-\alpha}$. I follow Gertler and Karadi (2011) and assume that $P_{mt}$ is the price of intermediate goods output and that the replacement price of used capital is fixed at unity. Furthermore, I follow Gertler and Kiyotaki (2011) and assume that capital is not mobile, but that labor is perfectly mobile across firms and islands. Since labor is perfectly mobile, competitive goods producers choose labor in accordance with the national wage rate $W_t$ which determines the gross profit per unit of capital $Z_t$ as follows:

$$W_t = \frac{\partial Y_t}{\partial L_t} = P_{mt}(1-\alpha) \frac{Y_t}{L_t}$$
$$Z_t = \frac{Y_t - W_t L_t}{K_t} = P_{mt} \alpha \left( \frac{L_t}{K_t} \right)^{1-\alpha}$$

Given that non-financial firms earn zero profits state by state, the firm simply pays out the ex post return on capital, $R_{t+1}^{k,h}$, to the intermediary:

$$R_{t+1}^{k,h} = \frac{Z_{t+1} + Q_{t+1}^h - \delta}{Q_t^h}$$  \hspace{1cm} (24)

where $\delta$ is the capital depreciation rate.

### 2.4 Capital-producing firms

Capital-producing firms build new capital using input from final output and subject to adjustment costs. They sell new capital to goods-producing firms on investing islands at price $Q_t^i$ and redistribute any profits to households. The objective of a capital producer is to choose $I_t$ to solve:

$$\max_{I_t} \left\{ E_t \sum_{m=0}^{+\infty} \Lambda_{t,m} \left[ Q_m^i I_m - \left[ 1 + f \left( \frac{I_m}{I_{m-1}} \right) \right] I_m \right] \right\}$$
From profit maximization, the price of new capital is:

\[ Q_t^i = 1 + f \left( \frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f' \left( \frac{I_t}{I_{t-1}} \right) - E_t \Lambda_{t,t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 f' \left( \frac{I_{t+1}}{I_t} \right) \]  

(25)

### 2.5 Retail firms

Final output \( Y_t \) is a constant elasticity of substitution composite of a continuum of mass unity of differentiated retail firms that use intermediate output as input. The final output technology is given by

\[ Y_t = \left[ \int_0^1 \frac{Y_{ft}^{\epsilon_p-1}}{Y_{ft}^{\epsilon_p}} df \right]^{\epsilon_p} \]  

(26)

where \( Y_{ft} \) is output by retailer \( f \). From cost minimization of final output:

\[ Y_{ft} = \left( \frac{P_{ft}}{P_t} \right)^{-\epsilon_p} Y_t \]  

(27)

\[ P_t = \left[ \int_0^1 Y_{ft}^{1-\epsilon_p} df \right]^{\frac{1}{1-\epsilon_p}} \]  

(28)

I follow Gertler and Karadi (2011) and assume that retailers only re-package intermediate output. It takes one unit of intermediate output to produce one unit of retail output, therefore the marginal cost is the relative intermediate output price \( P_{mt} \). Like Gertler and Karadi (2011) I introduce price rigidities following Christiano et al. (2005): each period a firm can freely re-optimize its price with a probability \( 1 - \gamma_p \). In between these periods, the firm is able to index its price to the lagged rate of inflation. The retailers choose the optimal price \( P_t^* \) to solve:

\[
\max E_t \sum_{i=0}^{\infty} \gamma_p^i \beta^{i-1} \Lambda_{t,t+i} \left[ \frac{P_t^*}{P_{t+i}} X_t - P_{mt+i} \right] Y_{ft+i} 
\]

(29)

where \( X_t^i = \begin{cases} (1 + \pi_{t+i})^{\gamma_p} \cdots (1 + \pi_t)^{\gamma_p} & \text{if } i > 0 \\ 1 & \text{if } i = 0 \end{cases} \), \( \pi_t \) is the inflation rate from \( t-1 \) to \( t \). The first order condition is given by:

\[
E_t \sum_{i=0}^{\infty} \gamma_p^i \beta^{i-1} \Lambda_{t,t+i} \left[ \frac{P_t^*}{P_{t+i}} X_t - \frac{\epsilon_p}{\epsilon_p - 1} P_{mt+i} \right] Y_{ft+i} 
\]

(30)

From the law of large numbers, the evolution of the price level follows:

\[
P_t = \left[ (1 - \gamma_p) (P_t)^{1-\epsilon_p} + \gamma_p (1 + \pi_{t-1})^{\gamma_p} P_{t-1} (1-\epsilon_p) \right]^{\frac{1}{1-\epsilon_p}} 
\]

(31)
2.6 The central bank

The central bank in the model lends liquidity to financial intermediaries\(^9\) through its standing facility or its open market operations (depending on the version of the model) and offers them a deposit facility at which they can deposit excess reserves. The central bank controls both the lending and the deposit facilities’ nominal interest rates, \(R_{l+1}^{lf,nom}\) and \(R_{t+1}^{df,nom}\), following a simple Taylor rule. I also assume that it targets the nominal deposit interest rate \(R_{t+1}^{nom}\) following the same Taylor rule\(^10\):

\[
R_{l+1}^{lf,nom} = (1 - \rho_i) \left( R_{t}^{lf,nom} + \kappa_{\pi} \pi_t + \kappa_y \frac{Y_t - Y}{Y} \right) + \rho_i R_t^{lf,nom} + \epsilon_{l+1}^{lf} \tag{32}
\]

\[
R_{t+1}^{df,nom} = (1 - \rho_i) \left( R_{t}^{df,nom} + \kappa_{\pi} \pi_t + \kappa_y \frac{Y_t - Y}{Y} \right) + \rho_i R_t^{df,nom} + \epsilon_{t+1}^{df} \tag{33}
\]

\[
R_{t+1}^{nom} = (1 - \rho_i) \left( R_{t}^{nom} + \kappa_{\pi} \pi_t + \kappa_y \frac{Y_t - Y}{Y} \right) + \rho_i R_t^{nom} \tag{34}
\]

where the relation between the real and nominal interest rates is determined by the Fisher relation:

\[
R_{t+1}^{nom} = R_{t+1}^{\pi_{t+1}}
\]

\[
R_{t+1}^{lf,nom} = R_{t+1}^{l_{t+1}\pi_{t+1}}
\]

\[
R_{t+1}^{df,nom} = R_{t+1}^{df_{t+1}\pi_{t+1}}
\]

Any profits or deficits due to the central bank’s operations are transferred each period to the household:

\[
T_t = R_t^{lf} LF_t - R_t^{df} DF_t
\]

2.7 Aggregation and market clearing conditions

Each goods-producing firm \(m\) issues claims equal to the number of units of capital it holds following equation (23), where the price of a unit of capital depends only on the type of island where firm \(m\) is situated. Thus one can sum across firms situated on islands of the same type. Furthermore, the national investment is concentrated...
only on investing islands $i$ since only firms on these islands can invest in new capital which gives us:

\[
K_{i, t+1}^i = S_t^i \\
I_t + (1 - \delta) \pi^i K_t = S_t^i
\]

(35)

and

\[
K_{n, t+1}^n = S_t^n \\
(1 - \delta) \pi^n K_t = S_t^n
\]

(36)

This yields the standard capital accumulation equation:

\[
K_{t+1} = K_{i, t+1}^i + K_{n, t+1}^n \\
= I_t + (1 - \delta) \pi^i K_t + (1 - \delta) \pi^n K_t \\
= I_t + (1 - \delta) K_t
\]

(37)

On the financial side, after using the balance sheet constraint of financial intermediary $j$ to replace $b_{j, t+1}^h$, summing across intermediaries on islands of type $h$ yields the following incentive constraints in the Standing Facilities and the Open Market Operations versions of the model, respectively:

\[
\left[ \theta (1 - \omega^h) - (R_{t+1}^{k,h} - R_{t+1}^{b,h}) \right] Q_t^h S_t^h + \left[ \theta \omega^h - (R_{t+1}^{b} - R_{t+1}) \right] \pi^h D_{t+1} \\
\leq \theta (\omega^f - \omega^h) - (R_{t+1}^{f} - R_{t+1}^{b,h}) \right] l_f t_{t+1} + \left[ \theta (\omega^h - \omega^d f) - (R_{t+1}^{b} - R_{t+1}^{d,f}) \right] D F_{t+1}^h
\]

\[
\pi^h V_t \\
\leq \theta [\omega^h \pi^h (N_t^i + N_t^n) + (1 - \omega^h) Q_t^h S_t^h - (\omega^h - \omega^d f) D F_{t+1}^h - (\omega^f - \omega^h) \pi^h L F_{t+1}]
\]

(38)

(39)

where the aggregate function value for all intermediaries $V_t = \sum_{h=i, n} \pi^h O_{t+1}^h N_{t+1}^h$.

In addition, according to the balance sheet of the entire banking sector, retail deposits in the economy are equal to the difference between total assets (claims on the non-financial sector and excess reserves at the central
bank) and borrowed liquidity from the central bank:

**Standing Facilities version:**

\[ D_{t+1} = Q_t^i S_t^i + Q_t^n S_t^n + DF_{t+1}^i + DF_{t+1}^n - l f_{t+1}^i - l f_{t+1}^n \]  

\[ (40) \]

**Open Market Operations version:**

\[ D_{t+1} = Q_t^i S_t^i + Q_t^n S_t^n + DF_{t+1}^i + DF_{t+1}^n - (N_i^t + N_n^t) - LF_{t+1} \]  

\[ (41) \]

In the OMOs version, the aggregate net worth on islands of type \( h \) in the infinite life version is the sum of the net worth of existing bankers/intermediaries, \( N_{t+1}^{e,h} \), and the net worth of new bankers, \( N_{t+1}^{n,h} \):

\[ N_{t+1}^h = N_{t+1}^{e,h} + N_{t+1}^{n,h} \]

Since the fraction \( \sigma \) of bankers at \( t \) survive until \( t+1 \), \( N_{t+1}^{e,h} \) is given by:

\[ N_{t+1}^{e,h} = \sigma \left[ R_{t+1}^{b} \pi^h (N_i^t + N_n^t) + (R_{t+1}^{k,h} - R_{t+1}^{b}) Q_i^h S_i^h - (R_{t+1}^{b} - R_{t+1}^{df}) DF_{t+1}^h - (R_{t+1}^{lf} - R_{t+1}^{b}) \pi^h LF_{t+1} + (R_{t+1}^{b} - R_{t+1}) \pi^h D_{t+1} \right] \]

Like Gertler and Karadi (2011) and Gertler and Kiyotaki (2011), I assume that entering bankers receive “start up” funds from their respective households. I assume that the household transfers a small fraction of the net value of non-financial assets that exiting bankers had intermediated in their final operating period. Given that the probability is i.i.d., the total final period assets of exiting bankers at period \( t \) is \( (1 - \sigma) (S_t^i + S_t^n) \). Accordingly, each household transfers the fraction \( \xi / (1 - \sigma) \) of this value to its entering banker. In the aggregate, entering bankers on islands of type \( h \) receive:

\[ N_{t+1}^{n,h} = \pi^h \xi (S_t^i + S_t^n) \]

Finally, the law of motion of the aggregate net worth of intermediaries located on islands of type \( h \) is the following (see A.2 for the infinite life version aggregation details):

\[ N_{t+1}^h = \sigma \left[ R_{t+1}^{b} \pi^h (N_i^t + N_n^t) + (R_{t+1}^{k,h} - R_{t+1}^{b}) Q_i^h S_i^h - (R_{t+1}^{b} - R_{t+1}^{df}) DF_{t+1}^h - (R_{t+1}^{lf} - R_{t+1}^{b}) \pi^h LF_{t+1} + (R_{t+1}^{b} - R_{t+1}) \pi^h D_{t+1} \right] + \pi^h \xi (S_t^i + S_t^n) \]  

\[ (42) \]

The market clearing condition in the labor market is:

\[ (1 - \alpha) \frac{Y_t}{L_t} = \frac{\chi L_t^c}{U_{ct}} \]  

\[ (43) \]
Finally, the aggregate resource constraint in the economy is the following:

$$Y_t = C_t + \left[1 + f \left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$  \hspace{1cm} (44)

3 Model analysis

I consider in this section the implications of relaxing the standard assumption of the central bank’s direct control over the short-term interest rate and of introducing monetary policy implementation through the central bank’s facilities and the corridor. I focus on the case where intermediaries that borrow liquidity from the central bank and intermediaries that deposit excess reserves at the central bank are located on different island types. This is in line with the literature on monetary policy implementation where the central bank facilities are used by banks of different types (e.g. with different liquidity needs or of different sizes, see Hauck and Neyer, 2014, Ennis and Keister, 2008 and Colliard et al., 2016).

I start by considering the steady state of the SFs version of the model. I derive the determinants of activating each of the central bank’s facilities and show that heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on reserves. Then I calibrate the OMOs version of the model and illustrate how real and financial variables react to either a symmetric or an asymmetric widening of the corridor.

Results from the quantitative assessment suggest that the impact on intermediaries’ lending to the real economy depends on how the corridor is widened. More particularly, a symmetric 1% widening of the corridor has the same positive impact on output as lowering the corridor by 0.87%. When the financial intermediaries that lend to non-financial firms investing in new capital do not hold excess reserves at the central bank, a symmetric widening of the corridor, which will came all intermediaries on the interbank market to adjust their participation, relieves the constraint on the balance sheets of intermediaries that lend to non-financial firms investing in new capital\(^\text{11}\). Relieving the constraint on the balance sheets of intermediaries that lend to investing firms increases their lending to the real economy, which boosts output.

\(^{11}\text{The type of financial intermediaries that deposit excess reserves at the central bank is a key factor in the positive impact on the real economy of a symmetric widening of the corridor. However, how the two intermediaries act on the interbank market does not affect this impact. When financial intermediaries located on non-investing islands hold the excess reserves at the central bank: 1) if these intermediaries are the lenders on the interbank market they will react to a higher constraint on their balance sheets by increasing their lending on the interbank market which will relieve the constraint on the balance sheet of intermediaries located on investing islands }i\text{ which will allow these intermediaries to increase their lending to the real economy. 2) if intermediaries located on non-investing islands are the borrowers on the interbank market they will react to a higher constraint on their balance sheets by decreasing their borrowing on the interbank market, which will relieve the constraint on the balance sheets of intermediaries located on investing islands }i\text{ and increase their available funds to be invested in non-financial assets, thus increasing their lending to the real economy. See Appendix C for more details.}\)
3.1 Steady state analysis

I define here the conditions that determine when a financial intermediary will use each of the central bank’s standing facilities. Then I show that heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on reserves.

**Proposition 1** Financial intermediaries located on islands of type $h$ borrow liquidity from the central bank and intermediaries located on islands of type $h’$ deposit excess reserves at the central bank if and only if:

\[
\frac{\omega^{f} - \omega^{h’}}{1 - \omega^{h’}} < \frac{R^{k, h} - R^{b}}{R^{k, h'} - R^{b}} < \frac{\omega^{h'} - \omega^{df}}{1 - \omega^{h'}}
\]

**Proof.** See Appendix B.2 □

Proposition 1 shows that the characteristics and the real return from lending to the real economy of any intermediary compared to other intermediaries are the two determinants of its decision to use each facility of the central bank. It also shows that if the real returns from lending to the real economy are divergent enough

\[
\frac{(\omega^{f} - \omega^{h’})}{(1 - \omega^{h’})} > \frac{R^{k, h} - R^{b}}{R^{k, h'} - R^{b}} \text{ or } \frac{R^{k, h} - R^{b}}{R^{k, h'} - R^{b}} > \frac{\omega^{h'} - \omega^{df}}{1 - \omega^{h'}}
\]

then intermediaries of only one type will use both the lending and deposit facilities of the central bank.

**Proposition 2** If financial intermediaries of type $h$ borrow liquidity on the interbank market and from the central bank and financial intermediaries of type $h’$ lend on the interbank market and deposit excess reserves at the central bank then:

\[R^{df} < R\]

**Proof.** See Appendix B.3 □

As noted above, models assessing the impact of the central bank offering a deposit facility to credit institutions where reserves are considered simply as an asset show that, at equilibrium, if credit institutions deposit excess reserves at the central bank then the interest rate paid on reserves must be strictly higher than the interest rate paid on retail deposits $R^{df} > R$. However, several models show that introducing an ad hoc operational cost for financial intermediaries based on the amount of reserves deposited by these intermediaries can explain the banking sector holding excess reserves even if the central bank pays an interest rate on reserves lower than the interest
rate paid on retail deposits. Proposition 2 suggests another explanation: it shows that, in a model where reserves are simply an asset to financial intermediaries, heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on reserves. When financial intermediaries are constrained in their ability to raise funds from households, they will deposit excess reserves at the central bank to relieve their constraint, even if the interest rate they receive on these reserves is lower than the interest rate they pay to households on their retail deposits. Credit institutions choose to hold excess reserves rather than other assets (non-financial assets or interbank lending), even though the interest rate received on these excess reserves is lower than the interest rate paid on their main liability (retail deposits), because holding these excess reserves relieves the constraint on their balance sheets sufficiently to compensate for the difference in interest rates.

Finally, the model implies that frictions on the interbank market are at least of an intermediate level for the empirically observed case: (large) banks with liquidity deficit, represented in the model as intermediaries on investing islands $i$, borrow liquidity from the central bank and on the interbank market from (small) banks with excess funds, represented in the model as intermediaries on non-investing islands $n$. This result suggests that there is still room for policies aimed at reducing frictions on the wholesale financial market.

This is shown in Proposition 3, where $\omega^i$ and $x^n$ are the fractions of interbank borrowing and lending respectively that intermediaries on investing islands $i$ and non-investing islands $n$ cannot divert. $\omega^i + x^n = 0$ is the limit case of complete frictions on the interbank market and $\omega^i + x^n = 2$ is the limit case of a frictionless interbank market.

**Proposition 3** Under the assumption of a core-periphery interbank market, if intermediaries on investing islands $i$ borrow liquidity from the central bank and intermediaries on non-investing islands $n$ deposit excess reserves at the central bank then frictions on the interbank market are at least of an intermediate level:

$$\omega^i + x^n < 1$$

**Proof.** See Appendix B.4 ■

### 3.2 Quantitative assessment

#### 3.2.1 Calibration

The model is calibrated quarterly. With the exception of parameters specific to my model ($\theta, \omega^i, \omega^n, \omega^i f, \omega^d f, \sigma$ and $\xi$) and of those of the price rigidities block, all parameters are set as Gertler and Kiyotaki (2011). The discount factor $\beta = 0.99$, the depreciation rate $\delta = 0.025$, the capital share $\alpha = 0.33$, the habit
parameter $\gamma = 0.5$, the utility weight on labor $\xi = 5.584$ and the inverse elasticity of net investment to price of capital $I f''/f' = 1.5$. Like Gertler and Kiyotaki (2011), I choose a Frisch labor elasticity of ten, $\epsilon = 0.1$, to compensate partly for the absence of labor market rigidities. The nominal rigidity parameters are set as Gertler and Karadi (2011) following Primiceri et al. (2006) estimations. The elasticity of substitution between goods $\epsilon_p = 4.167$, the price rigidity parameter $\gamma_p = 0.779$ and the price indexing parameter $\gamma_{pr} = 0.241$.

My strategy is to calibrate the spread between the return on assets on investing islands $i$ and the interest rate on the interbank market, $R^{k,i} - R^b$, on the spread between Moody’s Baa and the Fed Funds rate, the spread between the return on assets on non-investing islands $n$ and the interest rate on the interbank market, $R^{k,n} - R^b$, on the spread between Moody’s Aaa and the Fed Funds rate, the spread between the interest rate paid on retail deposits and the interest rate paid on reserves, $R^{df} - R$, on the spread between the interest rate paid on excess reserves minus the weighted average of the rates received on the interest-bearing assets included in M2, the ratio of retail deposits to the financial sector total assets, $D/\left(Q^i S^i + Q^n S^n + DF\right)$, on the ratio of small-denomination time deposits$^{12}$ and savings deposits to commercial banks’ total assets, the ratio of excess reserves deposited at the central bank to the financial sector total assets, $DF/\left(Q^i S^i + Q^n S^n + DF\right)$, on the ratio of depositary institutions’ total borrowings from the Fed to commercial banks’ total assets, the ratio of financial intermediaries’ assets to total equity, $DF/\left(Q^i S^i + Q^n S^n + DF\right)$, on the inverse of the ratio of total equity to total assets for banks and the average survival time for a banker, $1/(1 - \sigma)$, to 10 years, as per Gertler and Kiyotaki (2011). I let the following parameters determined endogenously: the fraction of assets that can be diverted, $\theta$, the fraction of interbank borrowing that cannot be diverted, $\omega^i$, the fraction of interbank lending that can be diverted, $\omega^n$, the fraction of liquidity borrowed from the central bank that cannot be diverted, $\omega^{lf}$, the fraction of excess reserves deposited at the central bank that can be diverted, $\omega^{df}$, the survival probability of the bankers, $\sigma$, and the proportional transfer to the entering bankers, $\xi$. Table 1 in Appendix D summarizes the chosen moments and the matched data, Table 2 in Appendix D reports the parameter values of the model and Table 3 in Appendix D compares the steady state for the financial sector’s variables in the model with that for the first moments in the data.

Regarding the central bank’s policy rates, the steady state for the spread between the interest rate paid on borrowed liquidity from the central bank and the interest rate on the interbank market, $R^{lf} - R^b$, is calibrated on the spread between the Fed Funds Target rate and the Effective Fed Funds rate. The steady state for the spread between the interest rate on the interbank market and the interest rate paid on reserves, $R^b - R^{df}$, is calibrated on the Effective Fed Funds rate.

I consider the case where financial intermediaries on non-investing islands $n$ deposit excess reserves at the

$^{12}$The small-denomination time deposits component of M2 includes time deposits at banks and thrifts with balances less than $100,000$.  

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central bank. The model is calibrated based on U.S. data collected from the Federal Reserve Economic Data (FRED). Data is monthly and covers a period of one business cycle: from 2011M04 to 2016M12 (69 months)\(^\text{13}\). See Table 3 in Appendix D for a comparison between the model and the data.

### 3.2.2 Experiments

**An asymmetric widening of the corridor: \(\uparrow\) of \(R^{lf, nom}\).** As expected, increasing the higher bound of the corridor is a contractionary monetary shock: \(R^{lf, nom}\) is the nominal interest rate that financial intermediaries pay when they borrow from the central bank’s open market operations\(^\text{14}\). Impulse responses to a 1\% increase in the annualized \(R^{lf, nom}\) are plotted in Figure 1.

The initial increase in \(R^{lf, nom}\) impacts all financial intermediaries\(^\text{15}\). The higher cost of borrowing from the central bank decreases demand \(\downarrow LF\) and leads to weaker and more constrained balance sheets for all intermediaries \(\uparrow \lambda^i \& \uparrow \lambda^n\). Weaker balance sheets lead to reduced demand for non-financial assets, which decreases their prices on both types of islands \(\downarrow Q^i \& \downarrow Q^n\), with a simultaneous decrease in the return on these assets \(\downarrow R^{k,i} \& \downarrow R^{k,n}\). The reduced demand for non-financial assets on investing islands leads to a contraction in lending to non-financial firms that invest \(\downarrow S^i\), which decreases output \(\downarrow Y\) and welfare \(\downarrow Welfare\).

**An asymmetric widening of the corridor: \(\downarrow\) of \(R^{df, nom}\).** A decrease in \(R^{df, nom}\) impacts only those financial intermediaries that deposit excess reserves at the central bank: intermediaries located on non-investing islands \(n\). However, such a decision will also have an indirect effect on the real economy due to the interaction of financial intermediaries on the interbank market. Impulse responses to a 1\% decrease in the annualized \(R^{df, nom}\) are plotted in Figure 2.

The initial decrease in \(R^{df, nom}\) decreases the amount of excess reserves that intermediaries located on non-investing islands deposit at the central bank \(\downarrow DF\) and weakens the balance sheets of these intermediaries, which increases the constraint on their balance sheets \(\uparrow \lambda^n\). This reduces the demand from these intermediaries for non-financial assets, which pushes the price downwards \(\downarrow Q^n\) and reduces the return on these assets \(\downarrow R^{k,n}\) on non-investing islands \(n\). However, there is a second round effect on the economy due to the adjustment of participation on the interbank market of intermediaries located on non-investing islands, which relieves the constraint on the balance sheets of intermediaries on investing islands\(^\text{16}\) \(\downarrow \lambda^i\). This increases the demand for

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\textsuperscript{13}The National Bureau of Economic Research determines the average duration of business cycles between 1945 and 2009 to be \(\in [68.5, 69.5]\) months. See NBER (2017) for more details.

\textsuperscript{14}\(R^{lf, nom}\) plays the same role in the model as the Main Refinancing Operations rate in the euro area (post 2008) and, to a lesser extent, the Fed Funds target in the U.S..

\textsuperscript{15}Intermediaries on both investing \(i\) and non-investing \(n\) islands actually borrow liquidity from the central bank, since they decide on their borrowing from the central bank before learning about their investment opportunities.

\textsuperscript{16}See footnote 11.
non-financial assets by intermediaries on investing islands $i$ ($\uparrow S^i$), which pushes the asset price upwards on these islands ($\uparrow Q^i$) and boosts output ($\uparrow Y$) and welfare ($\uparrow Welfare$).

**A symmetric widening of the corridor:** $\uparrow$ of $R^{lf,nom}$ & $\downarrow$ of $R^{df,nom}$. The net effect on the real economy of a symmetric widening of the corridor will be whichever dominates: the negative impact of the monetary shock from increased $R^{lf,nom}$, of the positive impact from decreased $R^{df,nom}$ relieving the constraint on the balance sheets of intermediaries that lend to non-financial firms investing in new capital. Impulse responses to a 1% increase in the annualized nominal interest rate that financial intermediaries pay on their borrowings from the central bank, $R^{lf,nom}$, and a 1% decrease in the annualized nominal interest rate that the central bank pays on the excess reserves deposited by intermediaries are plotted in Figure 3.

The initial increase in $R^{lf,nom}$ and the initial decrease in $R^{df,nom}$ have the same impact on non-investing islands $n$: a higher cost of borrowing from the central bank and a decrease in return on excess reserves which results in a weaker and more constrained balance sheet ($\uparrow \lambda^n$). In reaction, both the price of non-financial assets located on non-investing islands $n$ ($\downarrow Q^n$) and the return on these non-financial assets ($\downarrow R^{k,m}$), fall due to a decrease in demand for these assets by intermediaries located on islands of type $n$. However, the impact on the real economy depends mainly on the net effect on intermediaries located on investing islands $i$: in this case the impact of increasing $R^{lf,nom}$ and the impact of decreasing $R^{df,nom}$ work against each other. Impulse responses in Figure 3 suggest that the positive effect of decreasing the interest rate on excess reserves dominates. The relief generated by the decrease in $R^{df,nom}$ for the constraint on the balance sheets of intermediaries located on investing islands $i$ dominates the effect of the increase in the cost of borrowed liquidity from the central bank, increasing $R^{lf,nom}$, ($\downarrow \lambda^i$). This increases these intermediaries’ demand for non-financial assets ($\uparrow S^i$) and their price ($\uparrow Q^i$), which positively impacts output ($\uparrow Y$) and welfare ($\uparrow Welfare$).

**Lowering the corridor:** $\downarrow$ of $R^{lf,nom}$ & $\downarrow$ of $R^{df,nom}$. As expected, lowering of the corridor has the effect of a standard expansionary monetary shock. Impulse responses to a 1% decrease in the annualized $R^{lf,nom}$ and $R^{df,nom}$ are plotted in Figure 3.

The initial decrease in the higher and lower bounds of the short term interest rate in the economy (the interbank rate), $R^{lf,nom}$ and $R^{df,nom}$, puts a downward pressure on it ($\downarrow R^b$), increases borrowed liquidity from the central bank ($\uparrow LF$) and decreases excess reserves deposited at the central bank ($\downarrow DF$). The decrease in the cost of borrowing from the central bank, $R^{lf,nom}$, relieves the constraint on the balance sheet of all financial intermediaries while the decrease in the interest rate paid on excess reserves $R^{df,nom}$ weakens the balance sheets of intermediaries located only on non-investing islands $n$. The negative impact on the balance sheet of intermediaries located on non-investing islands due to the decrease in $R^{df,nom}$ dominates the positive effect due to the decrease
in $R^{lf, nom}_l (\lambda^n)$ while the adjustment of participation on the interbank market of intermediaries located on non-investing islands together with the decrease in $R^{lf, nom}_l$ relieve the constraint on the balance sheets of intermediaries located on investing islands $i (\lambda^i)$. In reaction, both the price of non-financial assets located on investing islands $i (\lambda^i)$ and the return on these non-financial assets ($\lambda^i R^{k,i}$), rise due to an increase in demand for these assets by intermediaries located on islands of type $i (\lambda^i S^i)$, which positively impacts output ($\lambda^i Y$) and welfare ($\lambda^i Welfare$).

A symmetric widening of the corridor vs. lowering the corridor. Impulse responses to the central bank’s decision to widen the corridor and impulse responses to the central bank’s decision to lower the corridor plotted in Figure 3 suggest that lowering the corridor by 0.87% has the same positive impact on output as a symmetric 1% widening of the corridor. Since implementing these two different policies have a similar positive impact on the real economy, when should the central bank implement each of these policies?

Despite that these two policies affect the real economy in a similar way, their impact on the financial sector together with the transmission channels they activate differ significantly. The positive impact on output comes in both cases from the increase in lending to non-financial firms that invest in new capital. This increase is due to the relief in the constraint on the balance sheet of financial intermediaries located on investing islands $i$. However, this relief is achieved either by a sharp tightening of the constraint on the balance sheets of financial intermediaries that hold excess reserves at the central bank (intermediaries located on non-investing islands $n$) when the central bank decides to widen symmetrically the corridor or by a mild tightening of the constraint on the balance sheets of these intermediaries accompanied by a sharp decrease in the short term interest rate in the economy, the interbank rate. Widening symmetrically the corridor achieves the positive impact on the real economy through only a strong tightening of the constraint on the balance sheets of a part of financial intermediaries (intermediaries that hold excess reserves at the central bank) without impacting the whole economy: without significantly lowering the short term interest rate in the economy, the interbank rate, and lowering the corridor achieves also a similar positive impact on the real economy but accompanied with a strong impact on the whole economy: a sharp decrease in the short term interest rate in the economy, the interbank rate. This argues to symmetrically widening the corridor instead of lowering it when the central bank has concerns of higher risk taking due to a low interest rate in the economy (see Dell’Ariccia et al., 2017 and Lee et al., 2017).

4 Conclusion

This paper introduces monetary policy implementation based on the corridor and focuses on the role of frictions on the interbank market in the transmission of monetary policy to the real economy. Conventional monetary policy
is introduced into the model together with the interest rates of the central bank’s lending and deposit facilities (the corridor). Modifying the corridor affects the wholesale financial market and allows the central bank to impact the economy without decreasing its targeted interest rate.

The model shows that heterogeneous frictions on the interbank market are sufficient to explain the interest rate paid on retail deposits being higher than the interest rate paid on reserves. Furthermore, results from the quantitative assessment suggest that in an economy where the financial sector holds excess reserves at the central bank, the corridor’s width has significant potential as a tool for monetary policy. More particularly, when the financial intermediaries that lend to non-financial firms investing in new capital hold a negligible percentage of excess reserves at the central bank, a 1% symmetric widening of the corridor has the same positive impact on output as lowering the corridor by 0.87%. This symmetric widening of the corridor relieves the constraint on the balance sheets of these financial intermediaries, enhancing both output and welfare. Implementing this policy allows the central bank to boost welfare and the real economy and to simultaneously address concerns over higher risk-taking in a low interest rate environment (see Dell’Ariccia et al., 2017 and Lee et al., 2017). In other words, a symmetric widening of the corridor impacts output positively and increases welfare without decreasing the short-term interest rate.
Appendices

A The Open Market Operations version

A.1 Guess and verify

I start by guessing the following form of the value function:

\[ V_{j,t} = \sum_{h=i,n} \pi^h \left[ \nu_{s,t}^h s_{j,t}^h + \nu_{df,t}^h df_{j,t+1}^h - \nu_{b,t}^h b_{j,t+1}^h - \nu_{l,t}^h l_{j,t+1}^h - \nu_t^h d_{j,t+1}^h \right] \] (45)

Then, using intermediary \( j \)'s balance sheet (12), I substitute \( b_{j,t+1}^h \) into (45) and the function value reduces to:

\[ V_{j,t} = \sum_{h=i,n} \pi^h \left[ \nu_{b,t}^h N_{j,t}^h + \left( \frac{\nu_{s,t}^h}{Q_t^h} - \nu_{b,t}^h \right) Q_t^h s_{j,t}^h - \left( \nu_{b,t}^h - \nu_{df,t}^h \right) df_{j,t+1}^h - \left( \nu_{b,t}^h - \nu_t^h \right) l_{j,t+1}^h + \left( \nu_t^h - \nu_{l,t}^h \right) d_{j,t+1}^h \right] \] (46)

After substituting the F.O.C.s (14), (15), (16) and (17) into (46), the value function reduces to:

\[ V_{j,t} = \theta \frac{\bar{\lambda}_t}{1 + \lambda_t} d_{j,t+1} + \sum_{h=i,n} \pi^h \left[ \nu_{b,t}^h N_{j,t}^h + \frac{\lambda_t^h}{1 + \lambda_t} \left[ \theta (1 - \omega^h) Q_t^h s_{j,t}^h - \theta (\omega^h - \omega df) df_{j,t+1}^h - \theta (\omega l_f - \omega^h) l_{j,t+1}^h \right] \right] \] (47)

Replacing with the incentive constraint of intermediary \( j \) (13) in (47), the value function reduces to:

\[ V_{j,t} = \theta \frac{\bar{\lambda}_t}{1 + \lambda_t} d_{j,t+1} + \sum_{h=i,n} \pi^h \left[ \nu_{b,t}^h N_{j,t}^h + \frac{\lambda_t^h}{1 + \lambda_t} \left[ V_{j,t} - \theta \omega^h N_{j,t}^h - \theta \omega^h d_{j,t+1}^h \right] \right] \] (48)

After some algebraic manipulations, (48) reduces to:

\[ V_{j,t} (1 + \bar{\lambda}_t) = V_{j,t} \bar{\lambda}_t + \sum_{h=i,n} \pi^h \left[ \nu_{b,t}^h (1 + \bar{\lambda}_t) - \theta \omega^h \lambda_t^h \right] N_{j,t}^h \] (49)

Then the value function reduces to:

\[ V_{j,t} = \sum_{h=i,n} \pi^h \left[ \nu_{b,t}^h (1 + \bar{\lambda}_t) - \theta \omega^h \lambda_t^h \right] N_{j,t}^h \] (50)

From the function value of intermediary \( j \) (10):

\[ V_{j,t} = (1 - \sigma) \Lambda_{t,t+1} \sum_{h=i,n} \pi^h N_{j,t+1}^h + \sum_{m=1}^{\infty} \left[ (1 - \sigma)^m \Lambda_{t,t+1+m} \sum_{h=i,n} \pi^h N_{j,t+1+m}^h \right] = (1 - \sigma) \Lambda_{t,t+1} \sum_{h=i,n} \pi^h N_{j,t+1}^h + \sigma \Lambda_{t,t+1} V_{j,t+1} \] (51)
Then substituting \( V_{j,t+1} \) in (51) by (50), I obtain:

\[
V_{j,t} = (1 - \sigma) \Lambda_{t,t+1} \sum_{h=1}^{n} \pi^h N_{j,t}^h + \sigma \Lambda_{t,t+1} \sum_{h=1}^{n} \pi^h \left[ \nu^h_{b,t+1} (1 + \lambda_{t+1}) - \theta^h \omega \lambda_{t+1} \right] N_{j,t+1}^h
\]

\[
= \sum_{h=1}^{n} \pi^h \left[ (1 - \sigma) \Lambda_{t,t+1} N_{j,t+1}^h + \sigma \Lambda_{t,t+1} \left[ \nu^h_{b,t+1} (1 + \lambda_{t+1}) - \theta^h \omega \lambda_{t+1} \right] N_{j,t+1}^h \right]
\]  

(52)

After some algebraic manipulations, the function value reduces to:

\[
V_{j,t} = \sum_{h=1}^{n} \pi^h \left[ (1 - \sigma) \Lambda_{t,t+1} N_{j,t+1}^h + \sigma \Lambda_{t,t+1} \left[ \nu^h_{b,t+1} (1 + \lambda_{t+1}) - \theta^h \omega \lambda_{t+1} \right] N_{j,t+1}^h \right]
\]  

(53)

Then replacing \( N_{j,t+1}^h \) by its law of motion (11) yields the law of motion of the marginal gains and costs (18), (19), (20), (21) and (22).

**A.2 Aggregation**

The net worth:

From intermediary \( j \)'s balance sheet (12), I substitute \( b_{j,t+1}^h \) into the law of motion of intermediary \( j \)'s net worth \( N_{j,t+1}^h \) (11) and I find:

\[
N_{j,t+1}^h = R_{t+1}^b \left( \pi^i N_{j,t}^i + \pi^n N_{j,t}^n \right) + \left( R_{t+1}^b - R_{t+1}^b \right) Q_{t,s_{j,t}}^h - \left( R_{t+1}^b - R_{t+1}^d f \right) d_{f,j,t+1}
\]

\[
- \left( R_{t+1}^l f - R_{t+1}^b \right) l f_{j,t+1} + \left( R_{t+1}^b - R_{t+1} \right) d_{j,t+1}
\]  

(54)

Since the liquidity shock is i.i.d. across time and islands, any financial intermediary \( j \) (located on any type of island) during period \( t \) has a probability \( \pi^i \) of being on an island of type \( i \) during period \( (t-1) \) and a probability \( \pi^n = 1 - \pi^i \) of being on an island of type \( n \) during period \( (t-1) \). Then financial intermediary \( j \) located on an island of type \( h \) during period \( t \) has a probability \( \pi^i \) of inheriting \( N_{j,t}^i \) from period \( (t-1) \) and a probability \( \pi^n = 1 - \pi^i \) of inheriting \( N_{j,t}^n \) from period \( (t-1) \). Therefore the inheritance of net worth does not depend on the type of island intermediary \( j \) is located on during period \( t \). By consequence, summing for intermediaries on islands of type \( h \) yields the following aggregate inheritance: \( \pi^h \left( \pi^i N_t^i + \pi^n N_t^n \right) \). Since the fraction \( \sigma \) of bankers at \( t \) survive until \( t+1 \), then the aggregate law of motion of the net worth of existing intermediaries located on islands of type \( h \) is:

\[
N_{t+1}^c = \sigma \left[ \begin{array}{c} R_{t+1}^b \pi^h (\pi^i N_t^i + \pi^n N_t^n) + (R_{t+1}^b - R_{t+1}^b) Q_t^h S_t^h - (R_{t+1}^b - R_{t+1}^d f) D_{f,t+1}^h - (R_{t+1}^l f - R_{t+1}^b) L f_{t+1}^h + (R_{t+1}^b - R_{t+1}) \pi^h D_{t+1}^h \end{array} \right]
\]  

(55)

The value function:
Any intermediary \( j \) maximizes the following value function:

\[
V_{j,t} = \sum_{h=i,n} \pi^h \left[ (1 - \sigma) \Lambda_{t,t+1} + \sigma \Lambda_{t,t+1} \left[ \nu_{b,t+1}^h (1 + \lambda_{t+1}) - \theta \omega^h \lambda_{t+1}^h \right] \right] N_{j,t+1}^h
\]  

(56)

Then summing for all intermediaries on both types of islands \( h \in i, n \), yields the aggregate value function:

\[
V_t = \sum_{h=i,n} \pi^h \left[ (1 - \sigma) \Lambda_{t,t+1} + \sigma \Lambda_{t,t+1} \left[ \nu_{b,t+1}^h (1 + \lambda_{t+1}) - \theta \omega^h \lambda_{t+1}^h \right] \right] N_{t+1}^h
\]  

(57)

The incentive constraint:

The value function \( V_{j,t} \) of an intermediary \( j \) located on an island of type \( h \) does not depend on the type of island \( h \) (the intermediary maximizes the expected wealth at the beginning of the period). Thus summing the incentive constraint of intermediaries located on islands of type \( h \) yields the following aggregate incentive constraint:

\[
\pi^h V_t \leq \theta \left[ \omega^h \pi^h (N_t^i + N_t^n) + (1 - \omega^h) Q_t^h S_t^h - (\omega^h - \omega^d f) DF_{t+1}^h - (\omega^f - \omega^h) LF_{t+1}^h \right]
\]  

(58)

B Proofs

B.1 Lemma 1

I start by establishing a Lemma in which I show that when the central bank facilities are used by intermediaries of different types, the fraction \( \omega^{h'} \) that intermediaries who deposit excess reserves at the central bank can (cannot) divert from interbank lending (borrowing) is larger than the fraction \( \omega^h \) that intermediaries who borrow liquidity from the central bank can (cannot) divert from interbank lending (borrowing).

Lemma 1 If intermediaries located on islands of type \( h \) borrow liquidity from the central bank and intermediaries located on islands of type \( h' \) deposit excess reserves at the central bank then:

\[ \omega^{h'} > \omega^h \]

Proof. I assume that only intermediaries of type \( h \) borrow liquidity from the central bank and only intermediaries of type \( h' \) deposit excess reserves at the central bank and consider both cases:

1. If \( R_{k,h}^b - R_{b}^b > R_{k,h}^b - R_{b}^b \):
Then:
\[
\frac{\lambda^{h'}}{1 + \lambda^{h'}} (1 - \omega^{h'}) > \frac{\lambda^{h}}{1 + \lambda^{h}} (1 - \omega^{h})
\]  
(59)

From (59), if \(\omega^{h'} > \omega^{h}\) then \(\lambda^{h'} > \lambda^{h}\)

Since only intermediaries of type \(h\) borrow liquidity from the central bank, the following strict inequality holds:
\[
\frac{\lambda^{h}}{1 + \lambda^{h}} (\omega^{l} f - \omega^{h}) > \frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^{l} f - \omega^{h'})
\]  
(60)

From (60), if \(\lambda^{h'} > \lambda^{h}\) then \(\omega^{h'} > \omega^{h}\). This shows that:
\[
\lambda^{h'} > \lambda^{h} \iff \omega^{h'} > \omega^{h}
\]  
(61)

Since only intermediaries of type \(h'\) deposit excess reserves at the central bank, the following strict inequality holds:
\[
\frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^{h'} - \omega^{d} f) > \frac{\lambda^{h}}{1 + \lambda^{h}} (\omega^{h} - \omega^{d} f)
\]  
(62)

From (62), if \(\omega^{h} > \omega^{h'}\) then \(\lambda^{h'} > \lambda^{h}\). This cannot hold since (61) shows that \(\lambda^{h'} > \lambda^{h} \iff \omega^{h'} > \omega^{h}\). Therefore, \(\omega^{h'} > \omega^{h}\) and \(\lambda^{h'} > \lambda^{h}\).

2. If \(R^{k,h} - R^{b} > R^{k,h'} - R^{b}\):

Then:
\[
\frac{\lambda^{h}}{1 + \lambda^{h}} (1 - \omega^{h}) > \frac{\lambda^{h'}}{1 + \lambda^{h'}} (1 - \omega^{h'})
\]  
(63)

From (63), if \(\omega^{h} > \omega^{h'}\) then \(\lambda^{h} > \lambda^{h'}\).

Since only intermediaries of type \(h'\) deposit excess reserves at the central bank, (62) holds. From (62), if \(\lambda^{h} > \lambda^{h'}\) then \(\omega^{h'} > \omega^{h}\). As shown above, if \(\omega^{h} > \omega^{h'}\) then \(\lambda^{h} > \lambda^{h'}\).

Therefore, we reach a contradiction.

This shows that if intermediaries of type \(h\) borrow liquidity from the central bank and intermediaries of type \(h'\) deposit excess reserves at the central bank then \(\omega^{h'} > \omega^{h}\) ■
B.2 Proposition 1

**Proof.** From intermediary $j$’s F.O.C. for $s_{j,t}^h$ at the steady state one can obtain:

$$R^{k,h} - R^b = \lambda^h \left[ \theta (1 - \omega^h) - (R^{k,h} - R^b) \right] \tag{64}$$

In order for intermediaries implemented on islands of type $h$ to borrow from the central bank using its refinancing operations, the following strict inequality must hold:

$$\frac{\lambda^h}{1 + \lambda^h} (\omega^lf - \omega^h) > \frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^lf - \omega^{h'}) \tag{65}$$

From (64) I plug $\lambda^h$ and $\lambda^{h'}$ into (65) and after some algebraic manipulations, I obtain that the following strict inequality must hold:

$$\frac{\omega^lf - \omega^{h'}}{1 - \omega^{h'}} < \frac{R^{k,h} - R^b}{R^{k,h'} - R^b} \tag{66}$$

Furthermore, for intermediaries implemented on islands of type $h'$ to deposit excess reserves at the deposit facility of the central bank, the following strict inequality must hold:

$$\frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^{h'} - \omega^{df}) > \frac{\lambda^h}{1 + \lambda^h} (\omega^h - \omega^{df}) \tag{67}$$

As above, I get $\lambda^h$ and $\lambda^{h'}$ from (64) and plug them into (67). I obtain:

$$\frac{\omega^{h'} - \omega^{df}}{1 - \omega^{h'}} > \frac{R^{k,h} - R^b}{R^{k,h'} - R^b} \tag{68}$$

Furthermore, the following can be shown:

$$\frac{\partial}{\partial \omega^h} \left( \frac{\omega^h - \omega^{df}}{1 - \omega^h} \right) = \frac{1 - \omega^{df}}{(1 - \omega^h)^2} > 0 \tag{69}$$

And

$$\frac{\partial}{\partial \omega^h} \left( \frac{\omega^lf - \omega^h}{1 - \omega^h} \right) = -\frac{1 - \omega^lf}{(1 - \omega^h)^2} < 0 \tag{70}$$
From Lemma 1, \( \omega^{h'} > \omega^h \) therefore:

\[
\frac{\omega^f - \omega^{h'}}{1 - \omega^{h'}} < \frac{1 - \omega^h}{1 - \omega^{h'}} \tag{71}
\]

And

\[
\frac{\omega^{h'} - \omega^d_f}{1 - \omega^{h'}} > \frac{1 - \omega^h}{1 - \omega^{h'}} \tag{72}
\]

This shows that if intermediaries implemented on islands of type \( h \) borrow from the central bank and intermediaries implemented on islands of type \( h' \) deposit excess reserves at the central bank, then the strict inequalities in the proposition hold.

If I assume that the strict inequalities in the proposition hold then, using intermediary \( j \)'s F.O.C. for \( s^h_{j,t} \) and replacing \( R^k,h - R^b \), and using intermediary \( j \)'s F.O.C. for \( s^{h'}_{j,t} \) and replacing \( R^{k,h'} - R^b \), I obtain that

\[
\frac{\lambda^h}{1 + \lambda^h} (\omega^f - \omega^h) > \frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^f - \omega^{h'}). \tag{73}
\]

Doing the same I obtain that the \( df \) part of the inequality

\[
\frac{\lambda^{h'}}{1 + \lambda^{h'}} (\omega^{h'} - \omega^d_f) > \frac{\lambda^h}{1 + \lambda^h} (\omega^h - \omega^d_f) \tag{74}
\]

\[\blacksquare\]

**B.3 Proposition 2**

**Proof.** I assume that only intermediaries of type \( h \) borrow liquidity from the central bank and that only intermediaries of type \( h' \) deposit excess reserves at the central bank. I also assume that the incentive constraints of intermediaries of both types are binding. From (6), I find:

\[
\omega^h = 1 - \frac{(R^{k,h} - R^b) (1 + \lambda^h)}{\theta \lambda^h} \tag{73}
\]

And

\[
\omega^{h'} = 1 - \frac{(R^{k,h'} - R^b) (1 + \lambda^{h'})}{\theta \lambda^{h'}} \tag{74}
\]

From (8), I find:

\[
\omega^f = \omega^h + \frac{(R^f - R^b) (1 + \lambda^h)}{\theta \lambda^h} \tag{75}
\]

From (9), I find:

\[
\omega^d_f = \omega^{h'} - \frac{(R^b - R^d_f) (1 + \lambda^h)}{\theta \lambda^h} \tag{76}
\]
I substitute $\omega^h$ and $\omega^l_f$ obtained from (73) and (75) into the aggregate incentive constraint of intermediaries of type $h$ (38). After some manipulation, I obtain:

$$\frac{R^k,h - R^b}{\lambda^h} Q^h S^h + \left[ \theta - \frac{R^k,h - R^b}{\lambda^h} - (R^k,h - R) \right] \pi^h D = \frac{R^l_f - R^b}{\lambda^h} LF$$  \hspace{1cm} (77)

I substitute $\omega^{h'}$ and $\omega^{d_f}$ obtained from (74) and (76) into the aggregate incentive constraint of intermediaries of type $h'$ (38). After some manipulation, I obtain:

$$\frac{R^k,h' - R^b}{\lambda^{h'}} Q^{h'} S^{h'} + \left[ \theta - \frac{R^k,h' - R^b}{\lambda^{h'}} - (R^{k,h'} - R) \right] \pi^{h'} D = \frac{R^b - R^{d_f}}{\lambda^{h'}} DF$$  \hspace{1cm} (78)

From (40), I substitute $DF$ into (78).

$$\frac{R^k,h' - R^{d_f}}{\lambda^{h'}} Q^{h'} S^{h'} + \frac{R^b - R^{d_f}}{\lambda^{h'}} Q^h S^h = \left[ \frac{R^k,h' - R^{d_f}}{\lambda^{h'}} + (R^{k,h'} - R) - \theta \right] \pi^{h'} + \frac{R^b - R^{d_f}}{\lambda^{h'}} D + \frac{R^b - R^{d_f}}{\lambda^{h'}} LF$$  \hspace{1cm} (79)

From the assumption that banks of type $h$ are lenders on the interbank market, I substitute $LF = Q^h S^h - IB - \pi^h D$ into (79) and obtain:

$$\frac{R^k,h' - R^{d_f}}{\lambda^{h'}} Q^{h'} S^{h'} + \frac{R^b - R^{d_f}}{\lambda^{h'}} IB = \pi^{h'} \left[ \frac{R^k,h' - R^{d_f}}{\lambda^{h'}} + (R^{k,h'} - R) - \theta \right] D$$  \hspace{1cm} (80)

I also substitute $LF = Q^h S^h - IB - \pi^h D$ into (77) and obtain:

$$\frac{R^k,h - R^{l_f}}{\lambda^h} Q^h S^h + \frac{R^{l_f} - R^b}{\lambda^h} IB = \left[ \frac{R^k,h - R^b}{\lambda^h} + (R^{k,h} - R) - \theta \right] \pi^h D$$  \hspace{1cm} (81)

From (80) and (81) I substitute $\lambda^h$ and $\lambda^{h'}$ into (7). After some algebraic manipulations I obtain:

$$(R^k,h - R^{l_f}) Q^h S^h + (R^{k,h'} - R^{d_f}) + (R^{l_f} - R^{d_f}) IB = [(R - R^{d_f}) - \pi^h (R^{l_f} - R^{d_f})] D$$  \hspace{1cm} (82)

I conclude from (82) that the model exists only if:

$$(R - R^{d_f}) > \pi^h (R^{l_f} - R^{d_f})$$  \hspace{1cm} (83)

Which implies the following strict inequality: $R > R^{d_f}$

**B.4 Proposition 3**

**Proof.** From Lemma 1, if intermediaries on investing islands $i$ borrow liquidity from the central bank and inter-
mediaries on non-investing islands $n$ deposit excess reserves at the central bank, then: $\omega^i < \omega^n$.

I assume a core-periphery interbank market, where intermediaries on investing islands $i$ borrow on the interbank market from intermediaries on non-investing islands $n$. Therefore, $\omega^i$ is the fraction of interbank borrowing that intermediaries on investing islands $i$ cannot divert and $\omega^n$ is the fraction that intermediaries on non-investing islands $n$ can divert from interbank lending. Then $x^n = 1 - \omega^n$ is the fraction of interbank lending that intermediaries on non-investing islands $n$ cannot divert.

$\omega^i < \omega^n \Rightarrow \omega^i + x^n < 1$, but $0 < \omega^i + x^n < 2$ where $\omega^i + x^n = 0$ is when the interbank market presents total frictions and $\omega^i + x^n = 0$ is when the interbank market is frictionless.

C  Borrowers or lenders on the interbank market: is there an impact?

The type of financial intermediaries that deposit excess reserves at the central bank is crucial for the impact of lowering (symmetrically or asymmetrically) the interest rate paid on reserves. If intermediaries that lend to investing firms hold excess reserves at the central bank, decreasing the interest rate paid on reserves will increase the constraint on the balance sheets of these intermediaries. In reaction, these intermediaries will decrease their lending to the real economy, which will have a contractionary effect on output. The case where intermediaries that lend to non-investing firms are holding excess reserves is discussed at length in the paper above. However, how financial intermediaries act on the interbank market has no impact on relieving or not relieving the constraint on the balance sheets of intermediaries lending to investing firms. To illustrate this, I consider the case where the excess reserves at the central bank are held by intermediaries lending to non-financial firms that do not invest in new capital.

Case 1: intermediaries $i$ borrowers and intermediaries $n$ lenders

On the one hand, if the central bank lowers the interest rate paid on reserves, lending on the interbank market becomes more attractive to intermediaries located on non-investing islands, thereby increasing the offer on the interbank market. On the other hand, intermediaries lending to non-financial firms that invest in new capital are on the demand side of the interbank market, so decreasing the interest rate paid on reserves increases the offer on the interbank market, which relieves the incentive constraint of these intermediaries.

$$\Pi^i_{j,t+1} \geq \theta \left( Q^i_j s^i_{j,t} - \omega^i b^i_{j,t+1} - \omega^i\ell f^i_{j,t+1} \right)$$  \hspace{1cm} (84)

where

$$\Pi^i_{j,t+1} = R^{k,i}_{t+1} Q^i_j s^h_{j,t} - R^b_{t+1} b^i_{j,t+1} - R^{d,i}_{t+1} f^i_{j,t+1} - R^d_{t+1} d^i_{j,t+1}$$  \hspace{1cm} (85)

As discussed above, when the central bank decreases the interest rate paid on reserves, intermediaries located
on non-investing islands \( n \) increase their lending on the interbank market. Then the constraint on the balance sheets of intermediaries located on investing islands \( i \) is mainly affected by the shift from borrowing from the central bank towards borrowing on the interbank market. More particularly, a 1 dollar decrease in \( l_{j,t+1}^{f} \) and a 1 dollar increase in \( b_{j,t+1}^{i} \) increases the right-hand side of the balance sheet constraint \((84)\) by \((\omega^{f} - \omega^{i})\) and increases the left-hand side of the balance sheet constraint \((84)\) by \((R_{t+1}^{l} - R_{t+1}^{b})\). Since intermediaries located on non-investing islands are increasing their lending on the interbank market, the interbank rate faces downward pressure, so the increase in the left-hand side \((R_{t+1}^{l} - R_{t+1}^{b})\) will be greater than in the right-hand side \((\omega^{f} - \omega^{i})\). This relieves the constraint of intermediaries located on investing islands \( i \) (the Lagrangian multiplier \( \lambda^{i} \) must decrease to increase the right-hand side of the optimization equation of \( l_{j,t+1}^{f} \)).

**Case 2: intermediaries \( n \) borrowers and intermediaries \( i \) lenders**

If the central bank lowers the interest rate paid on reserves, then intermediaries on non-investing islands decrease their borrowing of funds, including from the interbank market, because the return on reserves decreases. Therefore, demand for loans on the interbank market decreases. Intermediaries located on investing islands react by shifting their investment from the interbank market towards non-financial assets, which relieves the constraint on their balance sheets. Therefore, decreasing the interest rate paid on reserves decreases lending on the interbank market and relieves the constraint on the balance sheets of intermediaries located on investing islands.

\[
\Pi_{j,t+1}^{i} = R_{t+1}^{k} Q_{t}^{i} s_{j,t}^{i} + R_{t+1}^{b} b_{j,t+1}^{i} - R_{t+1}^{l} l_{j,t+1}^{f} - R_{t+1}^{d} d_{j,t+1} \tag{87}
\]

As discussed above, when the central bank decreases the interest rate paid on reserves, intermediaries located on non-investing islands \( n \) decrease their borrowing on the interbank market. Then the constraint of the balance sheets of intermediaries located on investing islands \( i \) is mainly affected by the shift from investing on the interbank market towards investing in non-financial assets. More particularly, a 1 dollar decrease in \( b_{j,t+1}^{i} \) and a 1 dollar increase in \( Q_{t}^{i} s_{j,t+1}^{i} \) increases the right-hand side of the balance sheet constraint \((86)\) by \((1 - \omega^{i})\) and the left-hand side of the balance sheet constraint \((86)\) by \((R_{t+1}^{k} - R_{t+1}^{b})\). Since intermediaries located on non-investing islands are decreasing their borrowing on the interbank market, the interbank rate faces downward pressure, so the left-hand side increase \((R_{t+1}^{k} - R_{t+1}^{b})\) will be greater than the right-hand side increase \((1 - \omega^{i})\). This relieves the constraint of intermediaries located on investing islands \( i \) (the Lagrangian multiplier \( \lambda^{i} \) must decrease to increase the right-hand side of the optimization equation of \( s_{j,t+1}^{i} \)).

**Summary**
To summarize, when the excess reserves at the central bank are held by intermediaries that do not lend to non-financial firms that invest in new capital, decreasing the interest rate paid on reserves relieves the constraint on the balance sheets of intermediaries that lend to these non-financial firms, and how the intermediaries act on the interbank market (lending vs. borrowing) has no impact on the relief of this constraint.

D Calibration

<table>
<thead>
<tr>
<th>Table 1: Calibration: moments</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^{k,i} - R^b$: Return on capital on investing islands minus the interest rate on the interbank market</td>
<td>Moody’s Baa minus Fed Funds rate</td>
</tr>
<tr>
<td>$R^{k,n} - R^b$: Return on capital on non-investing islands minus the interest rate on the interbank market</td>
<td>Moody’s Aaa minus Fed Funds rate</td>
</tr>
<tr>
<td>$R^{df} - R$: Interest on Excess Reserves minus the interest rate paid on retail deposits</td>
<td>Interest on Excess Reserves minus M2 Own Rate</td>
</tr>
<tr>
<td>$\frac{D}{Q^i S^i + Q^n S^n + DF}$: The proportion of deposits in financial intermediaries’ total assets</td>
<td>Small-denomination time deposits + Savings deposits divided by Commercial banks’ total assets</td>
</tr>
<tr>
<td>$\frac{DF}{Q^i S^i + Q^n S^n + DF}$: The proportion of excess reserves at the central bank in financial intermediaries’ total assets</td>
<td>Excess reserves at the Fed divided by Commercial banks’ total assets</td>
</tr>
<tr>
<td>$\frac{Q^i S^i + Q^n S^n + DF}{N^i + N^n}$: Total assets to total equity</td>
<td>1 divided by Total equity to total assets for banks</td>
</tr>
<tr>
<td>$\frac{1}{1-\sigma}$: The average survival time for a banker</td>
<td>10 years: Gertler and Kiyotaki (2011)</td>
</tr>
</tbody>
</table>
### Table 2: Parameter Values

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>Habit parameter</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$\zeta$</td>
<td>Relative utility weight of labor</td>
<td>5.584</td>
</tr>
<tr>
<td></td>
<td>$\epsilon$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Financial intermediaries</strong></td>
<td>$\pi^i$</td>
<td>Probability of new investment opportunities</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$\theta$</td>
<td>Fraction of assets that can be diverted</td>
<td>0.8143</td>
</tr>
<tr>
<td></td>
<td>$\omega^i$</td>
<td>Fraction of interbank borrowing that cannot be diverted</td>
<td>0.4278</td>
</tr>
<tr>
<td></td>
<td>$\omega^n$</td>
<td>Fraction of interbank lending that can be diverted</td>
<td>0.1624</td>
</tr>
<tr>
<td></td>
<td>$\omega^{lf}$</td>
<td>Fraction of liquidity borrowed from the central bank that cannot be diverted</td>
<td>0.6086</td>
</tr>
<tr>
<td></td>
<td>$\omega^{df}$</td>
<td>Fraction of excess reserves deposited at the central bank that can be diverted</td>
<td>0.0281</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>Survival probability of the bankers</td>
<td>0.972</td>
</tr>
<tr>
<td></td>
<td>$\xi$</td>
<td>Proportional transfer to the entering bankers</td>
<td>0.0126</td>
</tr>
<tr>
<td><strong>Intermediate goods firms</strong></td>
<td>$\alpha$</td>
<td>Effective capital share</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td><strong>Capital producing firms</strong></td>
<td>$I f^{''}/f'$</td>
<td>Inverse elasticity of net investment to the price of capital</td>
<td>1.5</td>
</tr>
<tr>
<td><strong>Retail firms</strong></td>
<td>$\epsilon_p$</td>
<td>Elasticity of substitution</td>
<td>4.167</td>
</tr>
<tr>
<td></td>
<td>$\gamma_p$</td>
<td>Probability of keeping prices fixed</td>
<td>0.779</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{pr}$</td>
<td>Measure of price indexation</td>
<td>0.241</td>
</tr>
<tr>
<td><strong>The central bank</strong></td>
<td>$\kappa_\pi$</td>
<td>Inflation coefficient of the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$\kappa_y$</td>
<td>Output gap coefficient of the Taylor rule</td>
<td>0.5/4</td>
</tr>
<tr>
<td></td>
<td>$\rho_i$</td>
<td>Smoothing parameter of the Taylor rule</td>
<td>0.1</td>
</tr>
</tbody>
</table>
### Table 3: Steady State Properties, Model versus Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D$ [\frac{Q^i S^i + Q^n S^n + DF}{N^i + N^n}]</td>
<td>32.16%</td>
<td>54.81%</td>
</tr>
<tr>
<td>$LF$ [\frac{Q^i S^i + Q^n S^n + DF}{N^i + N^n}]</td>
<td>5.38%</td>
<td>0.02%</td>
</tr>
<tr>
<td>$DF$ [\frac{Q^i S^i + Q^n S^n + DF}{N^i + N^n}]</td>
<td>24.75%</td>
<td>14.57%</td>
</tr>
<tr>
<td>$\frac{S^i + S^n}{Y}$</td>
<td>5.57</td>
<td>1.65&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>$R^{k,i} - R^b$</td>
<td>3.36%</td>
<td>4.84%</td>
</tr>
<tr>
<td>$R^{k,n} - R^b$</td>
<td>1%</td>
<td>3.83%</td>
</tr>
<tr>
<td>$R^{lf} - R^b$</td>
<td>0.64%</td>
<td>0.64%</td>
</tr>
<tr>
<td>$R^b - R^{df}$</td>
<td>0.16%</td>
<td>0.16%</td>
</tr>
<tr>
<td>$R^{df} - R$</td>
<td>0.49%</td>
<td>0.17%</td>
</tr>
<tr>
<td><strong>Real variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K \frac{Y}{\bar{Y}}$</td>
<td>5.57</td>
<td>3.65</td>
</tr>
<tr>
<td>$I \frac{Y}{\bar{Y}}$</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Volatility (s.d.)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$log(Y)$</td>
<td>0.0092</td>
<td>0.02&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>$log(C)$</td>
<td>0.0048</td>
<td>0.02&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>$log(I)$</td>
<td>0.0722</td>
<td>0.08&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td><strong>Persistence (autocorrelation)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$log(Y)$</td>
<td>0.82</td>
<td>0.65&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>$log(C)$</td>
<td>0.97</td>
<td>0.71&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>$log(I)$</td>
<td>0.61</td>
<td>0.66&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Table 4: Steady State Properties: Model vs. Data. All data is collected by the author from the Federal Reserve Economic Data (FRED). 1) Data values of these moments are collected from Jorda et al. (2016).
E  Impulse responses
Figure 1: Impulse responses to an asymmetric widening of the corridor: 1% increase in the annualized $R^{lf,nom}$.
Figure 2: Impulse responses to an asymmetric widening of the corridor: 1% decrease in the annualized $R_{df,nom}$. 

Annualized $Δ$ from ss
Figure 3: Symmetric widening vs. lowering the corridor: 1% increase in the annualized $R^{lf, nom}$ and 1% decrease in the annualized $R^{df, nom}$ (blue line) vs. 1% decrease in the annualized $R^{lf, nom}$ and 1% decrease in the annualized $R^{df, nom}$ (red dashed line).
References


