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JEL Codes: E62, F34, H12, H6, O11, P16
Keywords: Political Economy, Fiscal Crises, Favoritism, Entitlements, Public Debt, Inequality, State Capacity
Engineering Crises: Favoritism and Strategic Fiscal Indiscipline*

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Abstract

If people understand that some macroeconomic policies are unsustainable, why would they vote for them in the first place? We develop a political economy theory of the endogenous emergence of fiscal crises, based on the idea that the adjustment mechanism to a crisis favors some social groups, that may be induced ex-ante to vote in favor of policies that are more likely to lead to a crisis. People are entitled to a certain level of a publicly provided good, which may be rationed in times of crises. After voting on that level, society votes on the extend to which it will be financed by debt. Under bad enough macro shocks, a crisis arises: taxes are set at their maximum but despite that some agents do not get their entitlement. Some social groups do better in this rationing process than others. We show that public debt – which makes crises more likely – is higher, as is the probability of a crisis, the greater the level of favoritism. If the favored group is important enough to be pivotal when society votes on the entitlement level, favoritism also leads to greater public expenditure. We show that the favored group may strategically favor a weaker state in order to make crises more frequent. Finally, the decisive voter when choosing expenditure may be different from the one when voting on debt. In such a case, constitutional limits on debt may raise the utility of all the poor, relative to the equilibrium outcome absent such limits.

Keywords: Political Economy, Fiscal Crises, Favoritism, Entitlements, Public Debt, Inequality, State Capacity.

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1 Introduction

Crises that follow a period of protracted macroeconomic imbalances are a well documented phenomenon. In many cases, such crises arise from unsustainable fiscal and monetary policies, which are themselves associated with generous electoral promises. This scenario has been dubbed “Macroeconomic populism” by Dornbusch and Edwards (1991), who analyze a number of episodes, particularly in the context of Latin American countries. Yet an obvious question remains: if people understand that some macroeconomic policies are unsustainable, why would they vote for them in the first place?

This paper addresses this issue. We develop a political economy theory of the endogenous emergence of fiscal crises, broadly defined as situations where the government is unable to finance its overall expenditures, including both the cost of a publicly provided good, and the cost of servicing the public debt.

We use a simple three period model where, in the first period, society votes on the (constant) level of a good to be provided by the public sector in the second and in the third period of the game. This level defines a quantity of that good that each citizen is entitled to consume, and it may be financed by debt or by taxes. In the second period, society votes on how to finance the level of expenditure which was decided in the first period: taxes vs. debt. The government can borrow on the international financial market, and debt has to be repaid in the third period of the game. In the third period the economy is subject to a random shock. A fiscal crisis occurs when a negative aggregate income shock makes maximum potential government revenues fall short of government expenditures, calling for some correcting action.

We assume that in the event of a fiscal crisis, the government is forced to cut down on its provision of the entitlement good promised ex ante, which is reduced through some rationing scheme. Such a rationing scheme is nonconvex, implying some individuals get their entitlement level of the publicly provided good, while others are not served. Also, rationing entails a deadweight loss, which is larger, the larger the number of people who are rationed and the larger the entitlement level.

A key assumption of our theory is that people differ in two dimensions: their pre-tax income, some people are rich and some people are poor (and the poor are the majority), and their degree of “connection”. That is, some people are better connected than others to the public sector.\(^1\) Better connected people are better treated than others in case of a fiscal crisis,

\(^1\)This notion of connection stands several different interpretations. Favoured people may be either public sector employees, or citizens living in a particular region of the country, or belonging to a particular etc.
in the sense that they are less exposed to the rationing of the entitlement good, which makes them naturally less worried of the potential occurrence of a fiscal crisis.

As a consequence, better connected people are more likely to vote for a high level of public spending, and, given that level, they are also more likely to favor debt financing over tax financing. This, in spite of the fact that both policies raise the likelihood of a fiscal crisis, and that agents correctly internalize that effect.

By voting for higher debt, better connected people trade a tax cut now in exchange for a lower probability of getting one’s entitlement in the case of a crisis – but this lower probability disproportionately affects less connected people. Hence, better connected people gain from debt financing of public expenditure, and these gains are themselves associated with the occurrence of crises – in the absence of crisis, Ricardian equivalence would hold, and people would be indifferent between debt finance and tax finance, regardless of how well they are connected.\(^2\)

For this reason, when voting on the entitlement level in period 1, a voter will also favor a higher expenditure level, the better he is connected, and the more important connections are in the crisis rationing scheme. This is because in equilibrium part of this expenditure will be financed by debt, and issuing debt in period 2 guarantees that an increase in expenditure will be disproportionately financed by less connected people, through their lower access to the publicly provided good in case of a fiscal crisis.

Another implication of our model is that, depending on parameter values, different coalitions may arise when society votes on the level of expenditure as opposed to its financing. We assume there are two income levels: rich and poor, and that the poor are split in two groups, connected and unconnected. We focus on the interesting case where neither the rich nor the unconnected poor may be a political majority on their own. By assumption, the rich do not benefit from the publicly provided good, perhaps because they can get a higher quality from the private sector. As a result, they are immune from the consequences of fiscal crises, and always support debt financing over tax financing. Consequently, in the vote at date 2, the connected poor are the decisive voters: the unconnected want to issue less debt, and the rich want to issue more debt. At date 1, however, the rich want the lowest level of expenditure, while the connected vote for the highest level; therefore, the unconnected poor are the decisive

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\(^2\)In our model, as long as crises occur, Ricardian equivalence fails for two reasons. First, reneging on the state’s preagreed provision of public goods has, by assumption, a resource cost. Second, adjustment to a fiscal crisis is through a reduction in expenditure, so that expenditures are not independent of the way they are financed.
voters, unless the connected ones are numerous enough to achieve a political majority of their own. If not, then the rich are in some sense the swing voters, in that they support different groups in the first and in the second election.

Summarizing, uneven connections, and the related favoritism enjoyed by a particular social group, potentially concur to explain in our model the occurrence of fiscal crises. At the root of a crisis is the relative insulation from it of politically connected people, which makes them both desire (and impose to society when they are pivotal in the political process) a relatively high level of provision of the entitlement good (a source of crisis), and of public debt (still an additional source of fiscal crisis).

The literature related to our paper includes first, and foremost, the influential work of Alesina and Drazen (1991), which argues that fiscal stabilizations are delayed due to inability of rival social groups, which are imperfectly informed about the opponent’s cost of postponing reforms, to agree on how to share the cost of a stabilization policy. As a result, in a fiscal crisis, a war of attrition causing deadweight losses occurs before a fiscal stabilization eventually takes place.\(^3\) Our paper differs from Alesina and Drazen (1991) essentially as we address the specular question of why a fiscal crisis emerges endogenously, a question which has so far received little attention in the literature to the best of our knowledge. Our setup does not allow for wars of attrition. However, it may be interpreted in a way consistent with Alesina and Drazen: the connected group may be viewed as the one which rationally expects to win a war of attrition in case of a crisis.

Also closely related to our paper is the literature on the political economy of budget deficits, including Persson and Svensson (1989), Aghion and Bolton (1990), Alesina and Tabellini (1990) and Tabellini and Alesina (1990).\(^4\) These papers typically explain why fiscal policy can deviate from the standard prescriptions of the normative theory of public debt, by making two assumptions: the existence of heterogeneity within the society, which translates into heterogeneous preferences regarding government spending in different public goods, and the existence of political uncertainty on the identity of the party in office in the future.\(^5\) In our model, excess debt also arises for strategic reasons, but this is due to heterogeneity in the distribution of the

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\(^3\) Drazen and Grilli (1993) generalize Alesina and Drazen (1991) main result, by showing that a crisis can actually be beneficial for reform. This is because in their model a crises increases the fiscal distortions present in the status quo regime, and therefore raises the cost of delaying a fiscal stabilization, which, as a result, comes sooner, given the extent of the distributional conflict associated with the sharing of the burden of the reform. See Drazen (1996) for a comprehensive survey of the literature on the political economy of delayed reform.

\(^4\) See also Alesina and Perotti (1995) for a comprehensive discussion of this literature.

\(^5\) Specifically, the government in power today knows that in the future a different government, with different policy preferences, may be in power and in control of fiscal policy. Hence, the present incumbent government may attempt to “tie” the hands of its successor by leaving it a large public debt.
burden of the crisis, not in preferences for different public goods. Sill an additional rationale for the existence of budget deficits in absence of tax smoothing concerns is offered by Velasco (1999, 2000), who proposes a dynamic domestic common pool model of public finance. In this model, a society fragmented in a number of social groups accumulates public debt since each social group does not internalize the true social cost of financing a group-specific transfer (part of which is paid by the rival groups). As a result, society as a whole finds itself accumulating inefficiently high public debt, and a “stabilization” occurs when debt eventually reaches a critical threshold. This mechanism is clearly quite different from ours: in Velasco’s paper, agents are symmetrical and the common pool problem leads to a distorted discount factor; our model relies in asymmetries among the poor in the allocation of the burden of a crisis. Despite the differences in their specific setups, we remark that, as the discussion above should have made clear, all political economy theories of budget deficits discussed here (including of course our own) share the common denominator of relying on the inability of the group in power of fully internalizing the social cost of its decisions.

As noted above, the by now classic work on the macroeconomics of populism of Dornbusch and Edwards (1991), is also related to our work. Our model represents an attempt to formalize some of the key insights offered by Dornbusch and Edwards, since it demonstrates that, when the pivotal voter is, through his connections, relatively insulated from fiscal crises, he will tends to support a “macroeconomic populist” i.e. a leader who rationally implements policies

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6 More generally, our paper is related to the large literature explaining inefficient policies, which usually emphasizes either commitment problems (e.g. Acemoglu and Robinson, 2001), or informational asymmetries (e.g. Coate and Morris, 1995). Within this literature, our paper is most closely related to the contribution of Robinson and Verdier (2013), who provide a theory of clientelism, which is a special form of favoritism in that it involves a form of selective and inefficient redistribution of resources.

7 Common pool situations leading to “tragedy of commons,” outcomes, have been studied in a variety of setups. For instance, a static common pool model, explaining inefficient overspending is provided by the influential paper of Weingast et al. (1981). A dynamic common pool logic similar to Velasco’s (1999, 2000) explains the “voracity effect” studied by Tornell and Lane (1999), whereby a capital windfall reduces the rate of endogenous growth.

8 The importance of a political economy approach to explaining budget deficits is highlighted by many empirical studies which find that, while the tax smoothing model is reasonably successful in explaining the dynamics of public debt in the U.S. (e.g. Barro, 1986), it is much less successful at explaining the same dynamic pattern in OECD countries (e.g. Roubini and Sachs, 1989), and in developing countries (e.g. Edwards and Tabellini, 1991; Roubini, 1991).

9 We remark that the conceptualization of the multi-faced phenomenon of populism suggested by our paper is rather different from Acemoglu, Egorov and Sonin’s (2013). These authors define political populism essentially as the implantation of policies which are to the left of the median voter’s ideal point, an outcome which takes place for signaling reasons in a world of asymmetric information regarding the true nature of politicians (honest or corrupt). Instead ours is a world with symmetric information and perfect political agency; also, in our setup otherwise identical voters are treated differently by politicians due to uneven connections; lastly, the distinctive concern of our paper is to explain the occurrence of a specific macroeconomic outcome, i.e. potential fiscal crises of the state (and the related outcome of the potential failure of the principle of Ricardian Equivalence).
characterized by a relative lack of fiscal discipline and a high probability of crisis. Consistent with our result that the high debt policy in period 2 is supported by the rich and the connected poor, this literature has also shown that populist regimes are often supported by a cross-cutting coalition, often involving part of the economic elite and part of the lower classes (e.g. Drake, 1982; Kaufman and Stallings, ch. 2, pp. 15-16, in Dornbush and Edwards, 1991).

Finally, the empirical findings of Acemoglu, Robinson, and Thaicharoen (2003) are relevant for our theory. These authors have highlighted the role of “weak” institutions as a cause for bad macroeconomic policies. In our model, democratic institutions are weak in the sense that equal treatment is upheld absent a crisis, but not in a crisis since the rationing scheme favors some people over others.

The paper is organized as follows. Section 2 describes the basic setup. Section 3 characterizes the equilibrium choice of public debt conditional on the predetermined entitlement level of the public good. Section 4 studies the determination of the public good level in the first stage of our game. Section 5 discusses two extensions. First, it shows that if the connected poor are decisive voters in the first stage, they would tend to support a reduction in the state’s fiscal capacity, in order to strategically engineer more frequent crises, so as to shift the burden of financing the public good upon the unconnected. Second, it shows that if the unconnected poor are decisive in the first stage, the welfare of all the poor may be improved by a constitutional limit on the ability to issue debt. This limit provides a commitment device which allows the connected poor, who decide on debt, to trade a reduction in debt against a higher entitlement level, which is set ex-ante by the unconnected poor, and from which all the poor benefit. Section 6 concludes.

2 The Model

2.1 Basic Environment

We consider an economy in three periods, \( t = 0, 1, 2 \), populated by a continuum of agents of measure one. This endowment economy has a single final good produced and consumed at dates 1 and 2.

Agents differ in two dimensions: a fraction of them are rich and the remaining are poor; the proportion of rich people in the population is equal to \( \theta < 1/2 \). For any realization of the aggregate income \( y \) (which may vary across time as explained below), a poor person’s income
is equal to $\beta y$, while a rich person’s income is equal to $\gamma y$. We assume that $\beta < 1$ and that
\[
\gamma = \frac{1 - (1 - \theta) \beta}{\theta} > 1,
\]
which guarantees that average income is equal to $y$. Also, $\beta$ is clearly a direct measure of income equality, and a lower value of $\beta$ corresponds to a mean-preserving spread in inequality.

All the poor have the same utility function, given by
\[
U_P(c_1, c_2, \mu_1, \mu_2, G_1, G_2) = \sum_{t=1}^{2} u_P(c_t, G_t, \mu_t)
= \sum_{t=1}^{2} c_t + G_t - \mu_t,
\]
where $c_t$ is consumption of the final good at date $t$, $G_t$ is consumption of a publicly provided good, and $\mu_t$ denotes the utility cost of rationing in the allocation of the public good, to be discussed below. As for the rich, for simplicity we assume that they do not benefit from consuming the publicly provided good. Their utility is then given by
\[
U_R(c_1, c_2, \mu_1, \mu_2) = \sum_{t=1}^{2} u_R(c_t, \mu_t)
= \sum_{t=1}^{2} c_t - \mu_t.
\]

Empirically, this would be the case, for example, if the publicly provided good is health or education, for which the rich can get higher quality suppliers on the private market. Nevertheless, our results do not hinge on that particular assumption.

In addition, a fraction $\lambda$ of the people, which we refer to as type $H$, are potentially favored, in a sense made more precise below, in the provision of a publicly provided good $G$, in times of fiscal crisis, i.e. in a situation where the government is unable to finance its total expenditures. The remaining will be denoted as type $L$ citizens. Note that $\lambda$ represents an inverse measure of favoritism, which disappears in the limit case of $\lambda = 1$ where all people are equally treated.

The level of provision of the publicly provided good is decided by majority voting and once-and-for-all at the beginning of period 0. Typically, the poor will favor different values of $G$, depending on which group ($H$ or $L$) they belong to. This is not the case for the rich who, since they do not value $G$, will vote for the lowest possible value of $G$, regardless of the group they belong to.\footnote{Also for simplicity, we assume that the rich do consume their entitlement of the publicly provided good whenever they can, despite that this delivers no utility to them.} Hence, we will lump together the rich of both groups, so that there are essentially three groups in society, denoted by $H$, $L$, and $R$, where $R$ stands for the rich.
We assume that the publicly provided good has an entitlement dimension. That is, at date \( t = 0 \) society has to decide on the amount of good \( G \) that each citizen is entitled to consume, and this choice involves some irreversibility: A level of entitlement \( G \) has a once-and-for-all administrative cost, which is assumed to be increasing and convex in \( G \). For tractability, we assume that this cost is equal to \( (k/2)G^2 \).

Furthermore, in case of a fiscal crisis, the government has to renege on its commitment to citizens and to deny some citizens their entitlement. This adjustment process is non convex: some citizens get their full entitlement, and others don’t get it at all, although this occurs with some probability. Furthermore, reneging on entitlements involves some cost, which we specify as a non transferable utility cost to each citizen. This cost is equal to

\[
\mu = \varepsilon (1 - \phi) G,
\]

where \( \varepsilon \in (0, \beta) \) is a parameter and \( \phi \) is the proportion of citizens that get their entitlement. This cost may be interpreted as an administrative cost or, in the tradition of the rent-seeking literature, as the resources spent by individuals in competing to get their entitlement. Note that fiscal crises are an inefficient outcome in our model because they entail such loss of utility. We assume that it is small enough, in particular that \( \varepsilon \) is smaller than \( \beta \), the poor’s income relative to average. We denote by

\[
z = \beta - \varepsilon
\]

the difference between the two. This quantity is the net gain to a poor citizen of financing one unit of his entitlement by rationing another citizen, as compared to paying taxes for it.

The entitlement good \( G \) is provided at dates 1 and 2, and may be financed by two means. First, the government may levy proportional income taxes at date \( t = 1, 2 \) at rate \( \tau_t \). Second, at date \( t = 1 \) the government may issue a stock of public debt denoted by \( D \), to be paid back at \( t = 2 \). Taxation generates no distortion but has an upper bound, in the sense that \( \tau_t \leq \tau \), where \( \tau \in (0, 1) \) reflects the fiscal capability of the state.\(^{11}\) For simplicity, there is no private saving or borrowing. Consequently, consumption of the final good at date \( t \) is given by

\[
c_{Pt} = \beta y_t (1 - \tau_t)
\]

for the poor and by

\[
c_{Rt} = \gamma y_t (1 - \tau_t)
\]

for the rich.

\(^{11}\)Section 5 partly analyzes the consequences of voting on fiscal capacity.
If the government issues debt at time $1$ it borrows from the international financial market, in terms of the final good, at a fixed interest rate which is normalized to $1$. We assume that debt is always paid back, implying it is senior relative to the citizens’ entitlements, that may be defaulted upon in a fiscal crisis.$^{12}$ The equilibrium level of borrowing $D$ is decided by majority voting at $t = 1$. That is, people first vote on an entitlement level of the publicly provided good and then on how it should be financed. Intuitively, choosing greater entitlements and/or to finance them by borrowing raises the probability of a fiscal crisis, which captures the definition of “macroeconomic populism” by Dornbusch and Edwards.

The timing of income shocks realizations is as follows: No income is produced at time $0$; aggregate income at time $1$, $y_1$, is fixed and known at time $0$; aggregate income at time $2$, $y_2$, is random and drawn from a uniform distribution with support $[\bar{y}, \bar{y}]$. Its realization is publicly known at the beginning of time $2$ only. Note that $\sigma = (\bar{y} - \bar{y})$ can be interpreted as an index of macroeconomic volatility. We also assume that $y_1$ is in the same range as $y_2$, i.e. $\bar{y} \leq y_1 \leq \bar{y}$.

We make the following technical assumptions, for the sake of tractability and limiting the number of regimes to be discussed.

**Assumption 1** We assume that $D$ has the following bounds

$$\bar{y} G \geq D \geq \tau y_1.$$ (2)

Assumption 1 guarantees that (i) creditors can always be repaid in full,$^{13}$ leaving the burden of adjustment to citizens, regardless of the realization of the shock at date $2$, and (ii) there is never a fiscal crisis at period $1$, since public debt is always set by the government at a level that is large enough to allow it to finance $G$ by taxing at capacity or less.$^{14}$ Consequently, in particular, $\mu_1 = 0$ and the tax rate date date $1$ is given by

$$\tau_1 = \frac{G - D}{y_1}.$$ (3)

The randomness of $y_2$ will introduce aggregate uncertainty in the last period of the game. Because the entitlement $G$ is chosen at time $0$ and the fiscal capacity of the state is given,

---

$^{12}$This no default assumption is consistent with our assumption of an exogenous interest rate, since the risk premium is always equal to zero.

$^{13}$Absent assumption 1, for low realizations of the income shock $y$ no citizen would get his entitlement and creditors would get a haircut. The interest rate would endogenously adjust to reflect those potential haircuts. Preliminary calculations suggest that these effects would greatly complicate our algebraic derivations.

$^{14}$The inexistence of a fiscal crisis at period 1 is, as already remarked, by assumption. However, none of the main results of the paper would change if we allowed for fiscal crises to take place in both period 1 and period 2. Assumption 1 greatly simplifies the model by forcing the timing of a fiscal crisis to be exogenous. Otherwise, the model would have to include a mechanism for determining whether, in equilibrium, a fiscal crisis occurs at $t = 1$, as opposed to entitlements being financed by debt.
such uncertainty may lead to a fiscal crisis at period 2, whereby the government is unable to provide the level of entitlement originally voted (called “notional”) and at the same time to pay the outstanding public debt, even if taxes are set at their maximum level.

We assume an upper bound on $G$ which guarantees that the range of values of $D$ defined by Assumption 1 is nonempty.\textsuperscript{15}

**Assumption 2** We assume that $G$ has the following upper bound

$$G \leq G_{\text{max}} = \bar{\tau} (y_1 + \bar{y}).$$  \hfill (4)

We also assume the upper bound of the distribution of shocks is large enough for a no crisis situation to always occur with positive probability.

**Assumption 3** $\bar{y} > y_1 + 2y$.

Finally, we assume that the administrative cost $k$ of setting up the entitlement $G$ is large enough, which will guarantee concavity of the poor’s preferences in $G$ in the first stage of the game.

**Assumption 4** We assume that $k$ has the following lower bound

$$k > \frac{z(1 - \lambda)^2}{\sigma \bar{\tau}(1 - \lambda z)}.$$  

A fiscal crisis occurs at time $t = 2$ when the maximum level of tax revenues the government can collect falls short of “notional” government spending, defined as the level obtained if all citizens get their entitlement. This will be the case if $\tau y_2 < G + D \iff y_2 < (G + D) / \tau$. In a crisis, taxes are set at their maximum value (meaning that users of the public service are considered as senior relative to taxpayers) and a fraction $\phi$ of the people get their entitlement level of $G$, where $\phi$ is the highest possible level consistent with government receipts, i.e.

$$\tau y_2 = \phi G + D.$$  \hfill (5)

Expression (5) allows us to compute the average service probability $\phi$:

$$\phi = \frac{\tau y_2 - D}{G}.$$  \hfill (6)

Note that $\phi$ generally differs from the individual rationing probability, which may be higher or smaller than $\phi$ depending on the extent of the favoritism enjoyed by a particular individual in

\textsuperscript{15}Clearly, for $G$ too high, a fiscal crisis at date 1 could not be avoided, given fiscal capacity.
the provision of the entitlement good in time of crisis. We assume that the $H$ group is always served before the $L$ group. That is, no member of the $L$ group can access the public good unless all $H$ people are served. Consequently, we need to distinguish between a mild crisis, where only the type $L$ poor are rationed, and a supercrisis, where the $H$ group is rationed and the $L$ group has no access to the public good. Clearly, a mild crisis obtains when the average service rate $\phi$ is greater or equal than the mass $\lambda$ of favored people, whereas a supercrisis obtains in the opposite configuration. The following result then follows straightforwardly.

**Lemma 1** At date $t = 2$,

(i) there is no crisis if and only if

$$y_2 \geq \frac{G + D}{\bar{\tau}},$$

(ii) there is a mild crisis if and only if

$$\frac{\lambda G + D}{\bar{\tau}} \leq y_2 < \frac{G + D}{\bar{\tau}},$$

(iii) there is a supercrisis if and only if

$$y_2 < \frac{\lambda G + D}{\bar{\tau}}.$$

The next lemma, which is also derived from straightforward arithmetics, characterizes the tax rate in a no crisis scenario and the probability of being served of both groups if there is a crisis.

**Lemma 2** (i) If there is no crisis the equilibrium tax rate is

$$\tau_2 = \frac{D + G}{y_2} \leq \bar{\tau}. \tag{7}$$

(ii) If there is a mild crisis all type $H$ people get their entitlement and a type $L$ person gets his entitlement with probability

$$\phi_L = \frac{\phi - \lambda}{1 - \lambda}, \tag{8}$$

where $\phi$ is given by (6).

(iii) If there is a supercrisis no type $L$ person is served and type $H$ people are served with probability

$$\phi_H = \frac{\phi}{\lambda}. \tag{9}$$
Finally, a straightforward corollary of Lemma 1 characterizes the probability of being in one of the three states.\(^{16}\)

**Lemma 3** Let the probability of being in the supercrisis, mild crisis and no crisis states be denoted by \(A\), \(B\), and \(C\) respectively. Then, \(A\), \(B\), and \(C\) are given by the following table:

<table>
<thead>
<tr>
<th>Range for (D)</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (G - \bar{\gamma} y_1 \leq D \leq \bar{\gamma} y - G)</td>
<td>(0)</td>
<td>(0)</td>
<td>(1)</td>
</tr>
<tr>
<td>(2) (\bar{\gamma} y - G &lt; D \leq \bar{\gamma} y - \lambda G)</td>
<td>(0)</td>
<td>(\frac{1}{\alpha} \left( \frac{G + D}{\bar{\gamma}} - y \right))</td>
<td>(\frac{1}{\alpha} \left[ \bar{y} - \frac{G + D}{\bar{\gamma}} \right])</td>
</tr>
<tr>
<td>(3) (\bar{\gamma} y - \lambda G &lt; D \leq \bar{\gamma} y)</td>
<td>(\frac{1}{\alpha} \left( \frac{\lambda G + D}{\bar{\gamma}} - y \right))</td>
<td>(\frac{1}{\alpha} \left( 1 - \lambda G \bar{\gamma} \right))</td>
<td>(\frac{1}{\alpha} \left( \bar{y} - \frac{G + D}{\bar{\gamma}} \right))</td>
</tr>
</tbody>
</table>

*Table I* – Probability of the three regimes depending on \(D\)

Clearly, the supercrisis probability \(A\) and the mild crisis probability \(B\) are continuous, nondecreasing functions of \(G\) and \(D\), while the no crisis probability \(C\) is nonincreasing and continuous in \(G\) and \(D\).\(^{17}\)

At this junction, it is useful to summarize the timing of the baseline version of the dynamic political game, which is as follows.

1. At time \(t = 0\), citizens vote for the entitlement good \(G\), which is provided at time \(t = 1, 2\).
2. At time \(t = 1\), citizens produce their income, equal respectively for poor and rich to \(\beta y_1\) and \(\gamma y_1\), and vote for the public debt \(D\). Everybody consumes his or her entitlement \(G\) and pays a tax rate determined residually by the government’s budget constraint, \(\tau_1 = \frac{G - D}{y_1}\).
3. At time \(t = 2\), the income shock \(y_2\) is realized. Its allocation between rich and poor is the same as in date 1. Taxes and consumption of the entitlement good are as described in Lemma 2.

Since we have a finite sequential game with perfect information, it is natural to compute its subgame perfect equilibrium (SPE), proceeding by backward induction.

### 3 Choice of the Public Debt Conditional on Entitlement

We now characterize the equilibrium of the game at \(t = 1\), conditional on the choice that was made for \(G\) at date \(t = 0\). We denote by \(V_i(D, G)\) the reduced form total expected utility of group \(i\), \(i \in \{H, L, R\}\), as a function of \(D\) and \(G\).

\(^{16}\)Because of Assumptions 1, 2, and 3, we have that \(G + D \leq G_{\text{max}} + \bar{\gamma} y = 2\bar{\gamma} y + \bar{\gamma} y_1 < \bar{\gamma} y\), so that it is not feasible to select a value of \(D\) so high that the probability of no crisis \(C\) is equal to zero.

\(^{17}\)Depending on the initial choice for \(G\), some ranges may be empty. It is always the case, however, that \(G - \bar{\gamma} y_1 \leq \min(G, \bar{\gamma} y)\), due to Assumption 2. Therefore, at least one of the three ranges is nonempty.
We first characterize the marginal utility of increasing debt for the poor of group $H$, and then compare it to the marginal utility of increasing debt for the poor of group $L$ and for the rich. We then establish a single-crossing condition and, as a corollary, the fact that the poor of group $H$ will be pivotal, because they benefit from debt more than the poor of group $L$ but less than the rich. Finally, we perform comparative statics on the equilibrium value of debt, conditional on the value of $G$ inherited from date $t = 0$.

### 3.1 The preferences of the poor of type $H$

The following proposition characterizes the marginal utility of debt for the poor of type $H$.

**Proposition 1** The marginal utility of debt for the poor of type $H$ is given by:

$$\frac{\partial V_H}{\partial D} = -A \left( \frac{1}{\lambda + \epsilon} \right) - B\epsilon - C\beta + \beta,$$

(10)

where $A$ is the probability of a supercrisis, $B$ is the probability of a mild crisis, and $C$ is the probability of no crisis, as defined in Lemma 3.

**Proof.** See Appendix. ■

Absent a crisis, an increase in debt by one dollar reduces the poor’s utility at date 2 by $\beta$ dollars, since average taxes have to increase by one dollar, and the poor pay $\beta$ of taxes per dollar of average tax. This cost of debt is captured in Equation (10) by the $\epsilon$ term, while the benefit to the poor of issuing debt at $t = 1$ is, by the same token, equal to $\beta$—hence the last term on the RHS of (10). In a mild crisis, group $H$ is served its entitlement of the publicly provided good anyway, and the only cost to them of raising debt is that the level of rationing in society goes up, and so does its distortionary cost. This effect is captured by the term $-B\epsilon$ on the RHS of (10). Finally, in a supercrisis, the marginal cost of debt is twofold: First, the distortionary cost of rationing still applies; second, contrary to the mild crisis regime, the preferred group have a lower probability of being served, the lower the proportion $\phi$ of individuals who are served, i.e. the greater the level of debt. From Equation (9) we see that there is a **multiplier effect**: since the preferred group only accounts for a fraction $\lambda$ of society and since the adjustment potential of the $L$ group has been exhausted in a supercrisis, a reduction in the aggregate probability of being served $\phi$ by one unit has to be matched by a reduction in the individual probability of being served for a member of group $H$ by $1/\lambda > 1$ units. Altogether, these two dimensions of the cost of debt to group $H$ are summarized by the first term in the RHS of (10), $-A \left( \frac{1}{\lambda + \epsilon} \right)$.  

12
Remark 1 We note that in range (1) of Table I, that is, if crises never occur, \( \frac{\partial V_H}{\partial D} = 0 \). This is because Ricardian Equivalence holds locally as long as there is no crisis, since the distortions associated with public debt only materialize in a fiscal crisis.

The last result of this section shows that the utility function of the poor of group \( H \) is well behaved as \( D \) varies.

Lemma 4 The poor’s utility function \( V_H(D, G) \) is continuously differentiable, single-peaked in \( D \) and reaches its maximum for some \( D \geq \tau y - \lambda G \).

Proof. See Appendix. ■

Hence, as long as that is feasible, i.e. range (3) is nonempty, the poor of group \( H \) would always prefer a level of \( D \) such that there is a positive probability of both a mild crisis and a supercrisis. Supercrisis is the only regime where this group loses on net from raising \( D \). In a mild crisis the only cost for them of raising \( D \) is the small distortion \( \varepsilon \), which is smaller than the benefit \( \beta \) of paying lower taxes at date 1 while shifting the burden of adjustment upon group \( L \). Consequently, as long as \( A = 0 \), the poor of group \( H \) gain from raising \( D \).

We now turn to the analysis of the preferences of group \( L \) and of the rich, which will allow us to establish single-crossing and that the poor of group \( H \) are pivotal.

3.2 The preferences of the poor of type \( L \)

Proposition 2 The marginal utility of debt for the poor of type \( L \) is given by:

\[
\frac{\partial V_L}{\partial D} = -A \varepsilon - B \left( \frac{1}{1 - \lambda} + \varepsilon \right) - C \beta + \beta.
\]  

(11)

Proof. See Appendix. ■

Compared to type \( H \), type \( L \) loses most from an increase in debt in mild crisis states, since their probability of being served their entitlement falls. In contrast, in supercrises, they only lose the marginal resource cost of rationing, since they do not get their entitlement at all anyway.

The key aspect of interest of group \( L \)’s preferences for our purposes is that the marginal value of debt is lower than for group \( H \).

Lemma 5 \( V_L \) is continuously differentiable with respect to \( D \). Furthermore, \( \frac{\partial V}{\partial D} \leq \frac{\partial V_H}{\partial D} \).

Proof. See Appendix. ■

This single crossing property lies at the root of the existence of a political equilibrium at \( t = 1 \). A similar result can be proved for the rich.
3.3 The preferences of the rich

Proposition 3 The marginal utility of debt for the rich of both types is given by:

\[
\frac{\partial V_R}{\partial D} = -(A + B)\varepsilon - C\gamma + \gamma.
\] (12)

Proof. See Appendix. ■

Since \( \gamma > 1 > \varepsilon \) and \( A + B + C = 1 \), the rich always gain from an increase in public debt at \( t = 1 \). This is because they trade a certain tax cut today against a tax increase of the same amount tomorrow, which only occurs with some probability, since in a crisis adjustment takes place through a reduction in \( \phi \), which does not affect the rich who do not value the public good. Furthermore, it is possible to show that the marginal value of debt is always larger for the rich than for the favored poor. This is intuitive: in mild crises, an additional unit of debt has the same effect on the welfare of the rich and on the poor of group \( H \): the former pay the same taxes, the latter pay the same taxes and get the same access to \( G \). Both suffer only through the marginal distortionary cost of rationing \( \varepsilon \). In contrast, in a supercrisis, the marginal cost of debt is higher for the poor of group \( H \) than for the rich, since the former suffer from a reduced access to the public good, while the latter do not value it. Finally, absent a crisis, the rich suffer more from the tax hikes than the poor, but this is more than compensated by their gains from the tax cut at \( t = 1 \), since the tax cut is certain while the tax hike only occurs with some probability. Therefore, we have that

Lemma 6 \( V_R \) is continuously differentiable with respect to \( D_2 \). Furthermore, \( \frac{\partial V_R}{\partial D} \geq 0 \) and \( \frac{\partial V_R}{\partial D} \geq \frac{\partial V_H}{\partial D} \), with strict inequalities absent Ricardian Equivalence, i.e. in ranges (2) and (3) of Table I.

Proof. See Appendix. ■

3.4 Equilibrium determination of \( D \)

Putting together Lemmas 5 and 6, we have the following result.

Proposition 4 For any given \( G \) satisfying Assumption 2, there exists a unique equilibrium for the voting game at \( t = 1 \). The equilibrium value of \( D \) is

\[
D^*(G) = \arg \max V_H(D, G),
\]

where maximization takes place over \( D \in [G - \bar{\tau}y_1, \bar{\tau}y] \).
We now characterize the equilibrium $D^*$ and perform some comparative statics conditional on $G$.

**Proposition 5** Let

$$G_{\text{min}} = \frac{1 - \lambda z}{1 + \lambda - 2\lambda z} \bar{\tau}(y_1 + y).$$  

(13)

If $G < G_{\text{min}}$, (region I), then

$$D^* = \frac{\lambda(1 - z)}{1 - \lambda z} G.$$  

(14)

If $G_{\text{min}} \leq G \leq G_{\text{max}}$, (region II), then

$$D^* = G - \bar{\tau} y_1.$$  

Proof. See Appendix.

We note that there are two possible regimes.

If $G$ is not too large (region I), society votes for a level of debt which is a decreasing function of $G$. An increase in the entitlement level $G$ is matched by a reduction in borrowing, so that the total commitment of the state $D + G = \bar{\tau} y + \frac{1-\lambda}{1-\lambda z} G$, rises less than one for one with $G$. People compensate the greater risk of fiscal crisis induced by $G$ by a reduction in debt which is larger, the larger $\lambda$, i.e. the lower the degree of favoritism. The following proposition summarizes the effect on debt and on the probability of crises of inequality, favoritism, fiscal capacity and entitlements.

**Proposition 6** Let $P = 1 - C$ be the probability of a crisis at $t = 2$. Assume $G < G_{\text{min}}$. Then

1. $P = \frac{G}{\bar{\tau}} \left( \frac{1 - \lambda}{1 - \lambda z} \right)$.  
2. $\frac{\partial P}{\partial G} > 0, \frac{\partial D}{\partial G} < 0$.  
3. $\frac{\partial P}{\partial z} > 0, \frac{\partial D}{\partial z} > 0$.  
4. $\frac{\partial P}{\partial \lambda} < 0, \frac{\partial D}{\partial \lambda} < 0$.  
5. $\frac{\partial P}{\partial \bar{\tau}} < 0, \frac{\partial D}{\partial \bar{\tau}} > 0$.

Proof. Straightforward computations.

We see that a reduction in inequality, i.e. an increase in $z$, tends to raise indebtedness and therefore the probability of a crisis. Intuitively, if the poor of group $H$ are richer, they value the publicly provided good less, given that they have to contribute more to it, and therefore

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*This compensating effect disappears if there is an infinite level of favoritism ($\lambda = 0$) or if there is no inequality and no distortion from rationing ($z = 1$). If there is no favoritism at all ($\lambda = 1$), there is full crowding out between public debt and entitlements.*
they are more willing to trade a reduction in taxes at date 1 against a lower probability of
being served at date 2.

A reduction in $\lambda$, i.e. an increase in favoritism, also raises the equilibrium level of debt
and the probability of a crisis, given $G$. A lower $\lambda$ raises $B$ and reduces $A$, i.e. it makes it less
likely that the favored group has to bear the burden of adjustment of a crisis, making it more
valuable for these pivotal voters to reduces taxes at date $t = 1$.

An increase in state capacity tends to reduce the probability of a crisis, all else equal. This
reduces the marginal cost of debt for the favored group, which selects a higher level of debt.\footnote{Also, the economy is more likely to be in region I, the stronger are connections, the greater is state fiscal
capacity, the more equal is the society, and the smaller are the distortions caused by a crisis.}

If $G$ is larger than $G_{\text{min}}$ (region II), debt is constrained by the fact that a crisis is ruled
out at $t = 1$. Taxes at date 1 are set at their maximum and debt is set at the minimal level
necessary to finance entitlements at date 1. As a result debt goes up with $G$, and neither
depends on inequality nor on favoritism. The response of $D$ to a raise in $G$ further raises the
crisis probability at $t = 2$ instead of dampening it.

We now proceed and analyze the equilibrium determination of $G$ from the vote at date
t $t = 0$.

4 Equilibrium determination of the entitlement level $G$

4.1 The structure of preferences

The following proposition allows us to rank the preferences of the three groups of interest when
voting over $G$ at date $t = 0$. This in turn determines who the decisive voters are.

**Proposition 7** (i) $V_H(D^*(G), G)$ is $C^1$ and concave in $G$.

(ii) $V_L(D^*(G), G)$ is continuous, differentiable everywhere except at $G = G_{\text{min}}$, and concave
in $G$. Furthermore, \( \frac{dV_L}{dG} < \frac{dV_H}{dG} \) for any $G \in [0, G_{\text{max}}] - \{G_{\text{min}}\}$.

(iii) $V_R(D^*(G), G)$ is continuous, differentiable everywhere except at $G = G_{\text{min}}$. Furth-
more, \( \frac{dV_R}{dG} < 0 \) for any $G \in [0, G_{\text{max}}] - \{G_{\text{min}}\}$.

This proposition implies that the preferred group of poor wants a higher level of $G$ than
the non preferred one, while the rich wants the lowest possible one. This is intuitive since the
favored group gets a better deal in times of crises, while the rich do not value the publicly
provided good.
4.2 Equilibrium

From there it follows that there exists a voting equilibrium at $t = 0$.

**Proposition 8** (i) There exists a unique voting equilibrium at date $t = 0$ such that $G = G^*$.

(ii) If $\lambda(1 - \theta) > 1/2$, then

$$G^* = \arg \max_G V_H(D^*(G), G).$$

(iii) If $\lambda(1 - \theta) < 1/2$, then

$$G^* = \arg \max_G V_L(D^*(G), G).$$

**Proof.** – (i) and (ii) are straightforward if $\lambda(1 - \theta) > 1/2$. Assume $\lambda(1 - \theta) < 1/2$. We can replace $V_R$ by $\tilde{V}_R = AV_R$, for any $A > 0$. Since $dV_R/dG < 0$, we can pick $A$ large enough so that $d\tilde{V}_R/dG < dV_L/dG$ for any $G \in [0, G_{\max}] - \{G_{\min}\}$. With this new cardinal representation of the preferences, given Proposition 7, they are single crossed, and standard results apply, implying that (i) and (iii) hold.\(^{20}\)

QED

We note from this proposition that unless the favored group has an absolute majority, the pivotal group when voting on $G$ is group $L$. This is because while the rich favor the highest possible level of debt when voting on financing a given level of expenditure, they favor the lowest possible level of expenditure. That is, they side with the favored group of poor in opposing higher taxes at date 1, but side with the unfavored group in opposing higher expenditures at date 0.

Consequently, the determinants of public expenditure will differ depending on which of the poor, the $L$ group or the $H$ group, is pivotal at date $t = 0$. We now address that issue.

4.3 Determinants of public spending at $t = 0$

We now discuss the determinants of public spending at $t = 0$. We distinguish between the case where $\lambda(1 - \theta) \geq 1/2$, i.e. group $H$ is pivotal, and the case where $\lambda(1 - \theta) < 1/2$, i.e. group $L$ is pivotal.

\(^{20}\)Alternatively, we can show that the preferred $G$ of the poor of group $L$ is an equilibrium directly. Since $dV_R/dG < 0$, any higher value of $G$ is opposed by a coalition of the rich and of the poor of group $L$. Since $dV_L/dG > dV_L/dG$, any lower value of $G$ is opposed by a coalition of all the poor.
4.3.1 Group $H$ is pivotal

In the case where group $H$ is pivotal, the main properties of the equilibrium value of $G$ are summarized by the following proposition. Note that if equilibrium is in region II, we get unambiguous comparative statics results only with respect to fiscal capacity $\bar{\tau}$.

**Proposition 9** There exists $\hat{k}$ such that

(i) The preferred entitlement level of the poor of group $H$, $G^*_H$, is in region I: $G^*_H \leq G_{\text{min}}$, if and only if $k > \hat{k}$.

(ii) If $k > \hat{k}$ then the following comparative statics results apply:

- $\partial G^*_H / \partial \tau < 0$, $\partial G^*_H / \partial \varepsilon < 0$, $\partial G^*_H / \partial \sigma < 0$, $\partial G^*_H / \partial \lambda < 0$, and $\partial G^*_H / \partial \beta < 0$.

- $\partial B / \partial \tau < 0$, $\partial B / \partial \varepsilon < 0$, $\partial B / \partial \sigma < 0$, $\partial B / \partial \lambda < 0$, and $\partial B / \partial \beta < 0$, where $B$ is the probability of a mild crisis.

- $\partial A / \partial \tau < 0$, $\partial A / \partial \varepsilon < 0$, $\partial A / \partial \sigma < 0$, where $A$ is the probability of a supercrisis. Furthermore, $\partial A / \partial \lambda < 0$ if $A$ is large and $> 0$ if $A$ is small, while $\partial A / \partial \beta > 0$ if $\beta$ is small.

- The probability of a crisis, $P = A + B$ is such that $\partial P / \partial \beta < 0$ and $\partial P / \partial \lambda < 0$.

(iii) If $k_{\text{min}} \leq k < \hat{k}$ then $\partial G^H / \partial \tau > 0$.

**Proof.** See Appendix. ■

The most salient results are obtained if the equilibrium $G$ is in region I, which will be the case if the convex resource cost of setting up the entitlement is large enough. In particular, we have that

- A reduction in favoritism, i.e. an increase in $\lambda$, makes it more likely that the favored group experiences a lower probability of getting its entitlement in a fiscal crisis. That is, it reduces the probability of a mild crisis $B$, for any given $G$. This raises the marginal cost of $G$ to the poor of group $H$, thus reducing the equilibrium entitlement level. While an increase in $\lambda$ makes fiscal policy more sound, at the same time it mechanically has a positive effect on the probability of a supercrisis, since a supercrisis is defined as an outcome where the most favored group is rationed. This explains the nonmonotonic response of $A$ to $\lambda$ in equilibrium, and in particular the fact that for low values of $\lambda$ it has to go up with $\lambda$. 

18
• An increase in fiscal capacity $\tau$ reduces the probability of a crisis and makes it more likely that $G$ is financed by taxation as opposed to a reduction of the probability of being served for group $L$. Again, the cost of $G$ goes up for group $H$, which favors a lower level of expenditures. This effect stands in contrast to the standard one, by which greater fiscal capacity, i.e. lower distortions from taxation, raises public spending. Here greater fiscal capacity reduces public spending because, by making crises less frequent, it raises the average contribution of the decisive group $H$ to financing entitlements. In contrast, in region II where taxes are at their maximum level at $t = 1$, $G$ is constrained by fiscal capacity, and the standard effect prevails: A greater value of $\tau$ raises $G$.

• An increase in $\beta$ has two effects. First, as in Meltzer and Richard (1981), the poor have to contribute more to public spending, and therefore prefer a lower level of $G$. Second, however, the poor benefit more from debt financing, because, as they have to pay higher taxes, they save more in mild crises when entitlements are financed by reducing access to the public good for group $L$. Through that effect, a reduction in inequality reduces the cost of $G$ for group $H$. Our computations, however, imply that the first effect dominates. However, the fact that lower inequality favors debt financing implies that for low levels of $\beta$, on net the probability of a supercrisis goes up when inequality falls.

These results reflect the fact that, contrary to the standard model, group $H$ does not have to finance its own entitlement in a mild fiscal crisis. In that respect, group $L$ is in an opposite situation, so that we do not expect the same comparative statics whenever the poor of group $L$ are pivotal. We now turn to this case.

4.3.2 Group $L$ is pivotal

If Group $L$ is pivotal, we know from Proposition 7 that it will always choose a value of $G$, $G_L^*$, lower than the one preferred by group $H$. Besides that, it is not possible to establish unambiguous comparative statics results regarding the optimal $G$, with the sole exception that if $k$ is large enough, the optimal $G$ chosen by the poor of group $L$ is an increasing function of fiscal capacity $\tau$, contrary to $G_H^*$ (See Appendix for derivations).

Consider, for example, the effect of greater favoritism, i.e. a lower $\lambda$. There are now two conflicting effects on the choice of $G$ by group $L$. First, the economy is more likely to end in a mild crisis where any increase in $G$ is met by a reduction in the probability of being served by group $L$. This tends to reduce the marginal utility of $G$ to this group. This effect is captured by the expression for $B$ in the third line of Table I. Second, the size of the unfavored group
is larger, which generates a *dilution effect*: as rationing is allocated over more people, the incremental probability of being rationed as \( G \) goes up falls when \( \lambda \) is lower. This is effect is captured by the expression for \( \phi_L \) in Lemma 2, (ii).

5 Extensions

5.1 Endogenous Fiscal Capacity

As noted in our introduction, it is often observed that stabilizations programs are made difficult for lack of fiscal capacity. Yet one may wonder why fiscal capacity is low in some countries, but not others. Our model predicts that under favoritism, the preferred group will favor a reduction in fiscal capacity as long as it is in power and the economy is in regime I, i.e. as long as the level of debt is not constrained by fiscal capacity.

**Proposition 10** Assume that (i) \( \lambda(1 - \theta) > 1/2 \), and (ii) \( k > \hat{k} \), where \( \hat{k} \) is defined in Proposition 9. Then \( \frac{d}{dG} V_H(D*(G^*), G^*) < 0 \).

**Proof.** See Appendix. ■

Intuitively, the probability of a crisis increases as the state becomes weaker (see Proposition 9), but a situation of crisis is precisely the state of the world where the type \( H \) poor benefit from favoritism. Therefore, a weak state tends to induce some selective redistribution in favor of the type \( H \) poor, through the channel of making a fiscal crisis more likely to occur.\(^{21}\)

5.2 Consequences of a Debt Ceiling

We have seen that if \( \lambda(1 - \theta) < 1/2 \), the decisive voter is not the same when society sets the entitlement level vs. when it decides on the debt level. This opens up the scope for coordination between the poor of group \( L \) and those of group \( H \) for improving outcomes, provided they could commit to vote for different fiscal policies than the one they choose in our sequential equilibrium. In this section, we show that a constitutional debt ceiling contingent on \( G \) can provide such a commitment device and deliver Pareto-improving outcomes, from the viewpoint of the poor, compared to the equilibrium ones that we have characterized so far.

Assume that group \( L \) is initially pivotal, and that the equilibrium is in region I, which, as shown in the Appendix, is always the case if \( k \) is large enough. Then the equilibrium policy

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\(^{21}\)\( \hat{k} \) depends on \( \tau \) and is larger, the smaller \( \tau \). If group \( H \) can choose \( \tau \) freely, it will pick a low enough value such that \( k < \hat{k} \), i.e. the economy is in regime II where the debt level issued at date 1 is constrained by the fact that taxes are set at their maximum at this date.
(G^*, D^*) is such that
\[ \frac{\partial V_L}{\partial G} + D'(G) \frac{\partial V_L}{\partial D} = 0, \]  
(15)

where the D(G) function is defined by (14), so that we also have that
\[ \frac{\partial V_H}{\partial D} = 0. \]

From Lemma 5 it follows that \( \frac{\partial V_L}{\partial D} < 0 \) and therefore, from (15) and the fact that \( D'(G) = -\frac{\lambda(1-z)}{1-\lambda z} < 0 \), that \( \frac{\partial V_L}{\partial G} < 0 \).

Now consider a small perturbation of this equilibrium such that \( G \) is replaced by \( G + \Delta G \) and \( D \) by \( D + \Delta D \). Assume that \( \Delta G > 0 \). The incremental change in the welfare of the poor of group \( L \) is
\[
\Delta V_L = \frac{\partial V_L}{\partial G} \Delta G + \frac{\partial V_L}{\partial D} \Delta D = \Delta G \frac{\partial V_L}{\partial D} \left( \frac{\Delta D}{\Delta G} - D'(G) \right).
\]

This is positive as long as \( \frac{\Delta D}{\Delta G} < D'(G) \), i.e. if the reduction in debt associated with a given increase in \( G \) is stronger than the one that group \( H \) would implement when voting on \( D \).

Consider now the welfare effects on the poor of group \( H \). They are equal to
\[
\Delta V_H = \frac{\partial V_H}{\partial G} \Delta G + \frac{\partial V_H}{\partial D} \Delta D = \frac{\partial V_H}{\partial G} \Delta G.
\]

From Proposition 7 we know that \( dV_H/dG > dV_L/dG = 0 \), and furthermore \( \frac{\partial V_H}{\partial D} = \frac{\partial V_H}{\partial G} + \frac{\partial V_H}{\partial D} D'(G) = \frac{\partial V_H}{\partial G} \). Therefore \( \frac{\partial V_H}{\partial D} > 0 \), implying that type \( H \)'s welfare goes up as long as \( \Delta G > 0 \).

Putting these conclusions together, we see that the welfare of the poor of both types goes up, relative to the initial equilibrium, if one could impose an increase in public expenditures matched by a reduction in debt larger than the one group \( H \) would elect when voting on \( D \).

The following Proposition gives formal content to this claim.

**Proposition 11** Assume (i) \( \lambda(1-\theta) < 1/2 \), so that group \( L \) is pivotal when voting on public expenditures and (ii) \( k \) is large enough so that \( G^*_L < G_{\text{min}} \). Let \( (G^*, D^*) \) be the equilibrium values of \( G \) and \( D \) and \( V^*_H, V^*_L \) the corresponding utility levels of the poor of groups \( H \) and \( L \), respectively.

Consider the debt ceiling
\[ D \leq D_{\text{max}}(G) = D^* - \omega(G - G^*). \]
Then there exists $\omega > \frac{\lambda(1-z)}{1-z^2}$ such that the equilibrium if the constraint $D \leq D_{\max}(G)$ is imposed, is such that $V_H > V_H^*$ and $V_L > V_L^*$.

Proof. See Appendix. ■

The ceiling on debt considered here forces group $H$ to reduce debt by a larger amount in response to an increase in $G$ decided by group $L$. As a result, the probability of a crisis, where group $L$ does not get its full entitlement, goes up by less for any increase in $G$, which induces group $L$ to choose a larger $G$ while at the same time making it better-off. As for group $H$, its loss from being able to issue less debt is only of second order as long as the new equilibrium is not too different from the original one, while its gain from a higher level of entitlement is first order. Therefore, the poor of group $H$ are better-off too.

While this constitutional constraint makes all the poor better-off, the rich are worse-off. Indeed, the new equilibrium has a higher expenditure level but a lower debt level than the original one, and, as seen above, the utility of the rich falls with expenditure and goes up with debt. Therefore, they are necessarily worse off.

6 Conclusions

This paper has provided a novel political economy theory of the emergence of fiscal crises, based on a simple dynamic model of public finance. Our key contribution is to relate the macroeconomic determinants of fiscal policies and fiscal crises to microeconomic features of the adjustment mechanism.

Fiscal crises occur because they are associated with rationing in accessing publicly provided goods, and some people are in a better situation to get their entitlement, through mechanisms such as information networks, political connections or corruption. Such favored agents are in favor of implementing policies which create fiscal stress in bad economic times, including relatively high levels public spending in entitlement goods and high levels of public deficits and debt. While such policies are likely to lead to a fiscal crisis, the favored people are not so worried about them – indeed, since they are favored in the rationing process and since rationing only occurs during a crisis, the benefits of their connections accrue only in times of troubles. This explains their relative support for expansionary fiscal policies, even though they rationally perceive that such policies are socially inefficient. Indeed, as we have seen, the incentives of connected people to create crises are so strong that they would also benefit from a fiscally weaker state.
Since uneven connections can be regarded as a form of weak institutionalization, i.e. departure from anonymity in accessing one’s entitlement to publicly provided goods, our model can shed new light on why crises are especially frequent in developing countries. As shown, these effects are amplified by high income inequality which leads to greater public spending and, in presence of favoritism, to accumulate more public debt.
7 Appendix

7.1 Proof of Proposition 1

From Lemma 2 and Equation (6) it is straightforward to fill the following table.\textsuperscript{22}

<table>
<thead>
<tr>
<th>State</th>
<th>(c_{H2} )</th>
<th>(E(G_{H2}) )</th>
<th>(\mu_2 )</th>
<th>(E(u_P(c_{H2}, G_{H2}, \mu_2) \mid y_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No crisis</td>
<td>(\beta(y_2 - D - G)\beta)</td>
<td>(G\beta)</td>
<td>(0\beta)</td>
<td>(\beta y_2 - \beta D + (1 - \beta)G\beta)</td>
</tr>
<tr>
<td>Mild crisis</td>
<td>(\beta(1 - \bar{\tau})y_2\beta)</td>
<td>(G\beta)</td>
<td>(\varepsilon(G + D - \bar{\tau}y_2)\beta)</td>
<td>([\beta(1 - \bar{\tau}) + \varepsilon\bar{\tau}]y_2 + G(1 - \varepsilon) - \varepsilon D\beta)</td>
</tr>
<tr>
<td>Supercrisis</td>
<td>(\beta(1 - \bar{\tau})y_2\beta)</td>
<td>(\frac{\tau y_2 - D}{\lambda}\beta)</td>
<td>(\varepsilon(G + D - \tau y_2)\beta)</td>
<td>(\beta(1 - \bar{\tau}) + \bar{\tau}(\frac{1}{\lambda} + \varepsilon)y_2 - \varepsilon G - (\frac{1}{\lambda} + \varepsilon)D\beta)</td>
</tr>
</tbody>
</table>

Table II – Consumption of private, publicly provided good, rationing cost, and utility at date \(t = 2\) for the poor of group \(H\).

Therefore, the marginal utility of debt in period 2 in the supercrisis, mild crisis and no crisis regimes are equal to \(-\left(\frac{1}{\lambda} + \varepsilon\right), -\varepsilon,\) and \(-\beta,\) respectively.

To complete the proof, two remarks remain to be made.

First, at the frontier between the mild crisis regime and the no crisis regime, we have that \(\bar{\tau}y_2 = G + D\). Substituting the implied value of \(y_2, y_m = (G + D)/\bar{\tau}\), into the last column of Table II, we get the same expression for \(E(u_P(c_{H2}, G_{H2}, \mu_2) \mid y_2)\) in the no crisis and mild crisis regimes. Therefore, utility is continuous at \(y_m\). Similarly, the frontier between the supercrisis regime and the mild crisis regime is such that \(\bar{\tau}y_2 = \lambda G + D\). Substituting the implied value of \(y_2, y_s = \frac{\lambda G + D}{\bar{\tau}}\), into the last column of Table II, we get the same expression for \(E(u_P(c_{H2}, G_{H2}, \mu_2) \mid y_2)\) in the supercrisis and mild crisis regimes. Therefore, utility is again continuous at \(y_s\). It follows that whenever one or both of these thresholds are interior, the effects on expected utility at date 2 of the marginal changes in \(A, B,\) or \(C\) implied by changes in \(y_m\) and \(y_s\) cancel out. Therefore, the contribution of date 2 to the marginal utility of raising debt is equal to \(-A\left(\frac{1}{\lambda} + \varepsilon\right) - \varepsilon B - \beta C\).

Second, from our assumption of no fiscal crisis at date 1, we see that the utility of a poor of any group at date 1 is equal to

\[u_{1P} = \beta y_1(1 - \tau_1) + G.\]

From Equation (3) we have that \(\tau_1 = \frac{G - D}{y_1}\). Substituting into the preceding formula, we get

\[u_{1P} = \beta y_1 + (1 - \beta)G + \beta D.\]

Therefore, the marginal utility of debt at \(t = 1\) to the poor of either group is equal to \(\beta\).

Putting these two remarks together, we get the expression in (10).

QED

\textsuperscript{22}Expectations only refer to the fact that access to \(G\) is random.
7.2 Proof of Lemma 4

Starting from (10) and noting that $A + B + C = 1$, we can rewrite marginal utility as

$$\frac{\partial V_H(D, G)}{\partial D} = -\frac{A}{\lambda} + z(1 - C),$$

where $z = \beta - \varepsilon > 0$. From there we clearly see that $\frac{\partial V_H(D, G)}{\partial D^2} = 0$ in range (1) of Table I, $\frac{\partial V_H(D, G)}{\partial D^2} > 0$ in range (2). This proves the last part of the claim. Since $A$ and $C$ are continuous functions of $D$, so is $\frac{\partial V_H(D, G)}{\partial D^2}$, implying that $V_H$ is C1. Next, note that in range (3), we have that

$$\frac{\partial^2 V_H(D, G)}{\partial D^2} = -\frac{1}{\lambda} \frac{\partial A}{\partial D} - z \frac{\partial C}{\partial D} = \frac{1}{\sigma} \left( -\frac{1}{\lambda} + z \right) < 0,$$

where the second line comes from Table I and the last inequality from the fact that $z < \beta < 1 < 1/\lambda$.

Putting these observations together with the continuity of $\frac{\partial V_H(D, G)}{\partial D}$, it follows that $V_H(D, G)$ is concave in $D$ over range (3) and therefore single peaked over ranges (1)-(3), which completes the proof of the first part of the claim.

NB – Clearly, if range (3) is empty, the maximum of $V_H$ is reached at the upper limit of range (2), i.e. $D = \bar{y} - \lambda G$.

QED

7.3 Proof of Proposition 2

The proof is the same as that of Proposition 1, with Table II replaced by the following:

<table>
<thead>
<tr>
<th>State</th>
<th>$c_{L2}$</th>
<th>$E(G_{L2})$</th>
<th>$E(u_F(c_{L2}, G_{L2}, \mu_2) \mid y_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No crisis</td>
<td>$\beta(y_2 - D - G)$</td>
<td>$G$</td>
<td>$\beta y_2 - \beta D + (1 - \beta) G$</td>
</tr>
<tr>
<td>Mild crisis</td>
<td>$\beta(1 - \tau)y_2$</td>
<td>$\frac{y_2 - D - \lambda G}{1 - \lambda}$</td>
<td>$\left[ \beta (1 - \tau) + \tau \left( \varepsilon + \frac{1}{1 - \lambda} \right) \right] y_2 - \left( \varepsilon + \frac{\lambda}{1 - \lambda} \right) G - \left( \varepsilon + \frac{1}{1 - \lambda} \right) D$</td>
</tr>
<tr>
<td>Supercrisis</td>
<td>$\beta(1 - \tau)y_2$</td>
<td>$0$</td>
<td>$\left[ \beta (1 - \tau) \varepsilon \tau \right] y_2 - \varepsilon G - \varepsilon D$</td>
</tr>
</tbody>
</table>

Table III – Consumption of private, publicly provided good, and utility at date $t = 2$ for the poor of group $L$.

QED.

23 Rationing costs are the same as in Table II.
7.4 Proof of Lemma 5

That $V_L$ is continuously differentiable can be proved in the same way as in the proof of Lemma 4. Comparing the LHS of (11) with that of (10), we see that

$$\frac{\partial V_L}{\partial D} \leq \frac{\partial V_H}{\partial D} \iff \frac{B}{1-\lambda} \geq \frac{A}{\lambda}.$$ 

This obviously holds over ranges (1) and (2) of Table I, since $A = 0$ over those ranges. Over range (3), the last inequality is equivalent to

$$D \leq \pi y,$$

which holds by Assumption 1.

QED.

7.5 Proof of Proposition 3

The proof is the same as that of Proposition 1, with Table II replaced by the following:

<table>
<thead>
<tr>
<th>State</th>
<th>$c_{R2}$</th>
<th>$u_R(c_{L2}, G_{L2}, \mu_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No crisis</td>
<td>$\gamma(y_2 - D - G)$</td>
<td>$\gamma(y_2 - D - G)$</td>
</tr>
<tr>
<td>Crisis</td>
<td>$\gamma(1 - \tau) y_2$</td>
<td>$[\gamma (1 - \tau) + \varepsilon \pi] y_2 - \varepsilon G - \varepsilon D$</td>
</tr>
</tbody>
</table>

Table IV – Consumption of the private good and utility at date $t = 2$ for the rich.

One also has to note that utility at date $t = 1$ is

$$u^i_{1,R} = \gamma (1 - \tau_1) y_1 = \gamma(y_1 + D - G).$$

QED

7.6 Proof of Lemma 6

From (12) we have that $\frac{\partial V_R(D,G)}{\partial D} = -(1 - C)\varepsilon - C\gamma + \gamma = (1 - C)(\gamma - \varepsilon) > 0$. This proves the first part of the claim. Furthermore, subtracting the RHS of (10) from that of (12), we have that

$$\frac{\partial V_R}{\partial D} - \frac{\partial V_H}{\partial D} = A/\lambda + (1 - C)(\gamma - \beta) \geq 0.$$ 

This proves the second part of the claim. Clearly, the above inequalities are strict as long as $C < 1$, i.e. in ranges (2) and (3) of Table I.

QED

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24 Rationing costs are the same as in Table II.
7.7 Proof of Proposition 5

Assume that \( G \geq G_{\min} \) (implying in particular that one must have \( G_{\min} \leq G_{\max} \)). Note that
\[
G_{\min} = \frac{1 - \lambda \sigma}{1 + 2 \lambda \sigma} \tau(y_1 + y) > \frac{\tau(y_1 + y)}{1 + \lambda}.\]
Therefore, \( G > \frac{\tau(y_1 + y)}{1 + \lambda} \), or equivalently \( G - \tau y_1 > \tau y - \lambda G \). Consequently, only range (3) is nonempty. In that range, \( V_H \) is concave, hence if \( \partial V_H / \partial D \leq 0 \) at \( D = G - \bar{\tau} y_1 \) then that is the optimal choice. Recall that \( \partial V_H / \partial D = -A/\lambda + z(1 - C) \).

Computing that expression at \( D = G - \bar{\tau} y_1 \) using the formulas of Table I, we get that
\[
\frac{\partial V_H}{\partial D}(G - \bar{\tau} y_1, G) \leq 0 \iff -\frac{1}{\lambda}((1 + \lambda)G - \bar{\tau}(y_1 + y)) + z(\sigma \bar{\tau} - \bar{\tau}(y + y_1) + 2G + D) \leq 0,
\]
\[\iff G \geq G_{\min}.\]

Therefore, \( D = G - \bar{\tau} y_1 \) is indeed the optimal choice.

Now assume that \( G < G_{\min} \). Since \( G_{\min} < \bar{\tau}(y_1 + y) \), it follows that \( G - \bar{\tau} y_1 < \bar{\tau} y \). Consequently, range (3) is non empty. It follows that the optimal \( D \) must necessarily lie in range (3), given that \( \partial V_H / \partial D = 0 \) in range (1) and \( \partial V_H / \partial D > 0 \) in range (2). Since \( G < G_{\min} \), \( \partial V_H / \partial D(G - \bar{\tau} y_1, G) > 0 \). Furthermore, computing \(-A/\lambda + z(1 - C)\) at \( D = \bar{\tau} y \), using the expressions in Table I, yields
\[
\frac{\partial V_H}{\partial D}(\bar{\tau} y, G) = -(1 - z)\frac{G}{\sigma \bar{\tau}} < 0.
\]

Therefore, the optimal choice of \( D \) is in the interior of range (3) and such that \(-A/\lambda + z(1 - C) = 0\). Substituting again the expressions for \( A \) and \( C \) from Table I, we get the expression on the RHS of (14).

QED

7.8 Proof of Proposition 7

7.8.1 Proof of claim (i)

Using Table II, we can write their full utility as
\[
V_H(D, G) = \int_{y}^{y_1 + y} \left[ a_0 y - \varepsilon G - \left( \frac{1}{\lambda} + \varepsilon \right) D \right] \frac{dy}{\sigma} + \int_{y}^{y_1 + y} \left[ a_1 y + (1 - \varepsilon) G - \varepsilon D \right] \frac{dy}{\sigma}
\]
\[+ \int_{y_1 + y}^{G + D} [\beta y - \beta D - \beta G + G] \frac{dy}{\sigma} + \beta [y_1 + D] + (1 - \beta)G - k \frac{G^2}{2}, \tag{16}
\]
where
\[a_0 \equiv \beta (1 - \bar{\tau}) + \bar{\tau} \left( \frac{1}{\lambda} + \varepsilon \right),
\]
and
\[a_1 \equiv [\beta (1 - \bar{\tau}) + \varepsilon \bar{\tau}].
\]
In region I where $G \leq G_{\text{min}}$, the value of $D$ chosen by the pivotal group $H$ of poor at date $t = 1$ is interior. Consequently the envelope theorem applies, i.e. $\frac{dV_H}{dG} (D^*(G), G) = \frac{\partial V_H}{\partial D} (D^*(G), G)$. Differentiating (16) with respect to $G$, we then get that

$$\frac{dV_H}{dG} = -\varepsilon A + (1 - \varepsilon)B + (1 - \beta)C + 1 - \beta - kG. \quad (17)$$

Using (14) and the expressions in Table I, we get that in region I,

$$\frac{dV_H}{dG} = G \left[ -k + \frac{z(1 - \lambda)^2}{\sigma \bar{\tau}(1 - \lambda z)} \right] + 2(1 - \beta). \quad (18)$$

In region II where $G > G_{\text{min}}$, we have that $D^* = G - \bar{\tau}y_1$, implying that $\frac{dD^*}{dG} = 1$. Differentiating (16) we now get that

$$\frac{dV_H}{dG} = \left[ \varepsilon + \left( \varepsilon + \frac{1}{\lambda} \right) \frac{dD^*}{dG} \right] A + \left[ (1 - \varepsilon) - \varepsilon \frac{dD^*}{dG} \right] B \quad (19)$$

$$+ \left[ (1 - \beta) - \beta \frac{dD^*}{dG} \right] C + 1 - \beta + \beta \frac{dD^*}{dG} - kG.$$

Using again the expressions in Table I, the fact that $\frac{dD^*}{dG} = 1$, and the fact that $D^* = G - \bar{\tau}y_1$, we can reexpress this as

$$\frac{dV_H}{dG} = G \left[ -k - \frac{1}{\sigma \bar{\tau}} (2 + \lambda + 1/\lambda - 4z) \right] + 2(1 - \beta) + \frac{1}{\sigma} (1 + 1/\lambda - 2z)(y + y_1). \quad (20)$$

Next, we prove the continuity of $\frac{dV_H}{dG}$ at $G = G_{\text{min}}$. For $G \leq G_{\text{min}}$, we have that $\frac{dV_H}{dG} = \frac{\partial V_H}{\partial G}$. For $G > G_{\text{min}}$, we have that $\frac{dV_H}{dG} = \frac{\partial V_H}{\partial D} + \frac{\partial V_H}{\partial G}$. All these derivatives are computed at $D = D^*(G)$. But $G_{\text{min}}$, by definition, is such that $\frac{\partial V_H}{\partial D} (G_{\text{min}} - \bar{\tau}y_1, G_{\text{min}}) = \frac{\partial V_H}{\partial D} (D^*(G_{\text{min}}), G_{\text{min}}) = 0$. Therefore, $\lim_{G \to G_{\text{min}}^+} \frac{dV_H}{dG} = \lim_{G \to G_{\text{min}}^-} \frac{dV_H}{dG}$. Since $V_H (D^*(G), G)$ is regular elsewhere in the $G \leq G_{\text{max}}$ region, this proves that $V_H$ is $C^1$ throughout.

Finally, we note from Assumption 4 that the first term in brackets in (18) is negative, and that is obviously so in (20). This proves that $V_H (D^*(G), G)$ is concave in both regions. Since $\frac{dV_H}{dG}$ is continuous at the frontier $G = G_{\text{min}}$ between the two regions, $V_H$ is clearly globally concave.
7.8.2 Proof of claim (ii)

From Table III we get that

\[ V_L(D, G) = \int_y^{G+D} \left[ a_1 y - \varepsilon G - \varepsilon D \right] \frac{dy}{\sigma} \]

\[ + \int_y^{G+D} \left[ a_2 y - \left( \varepsilon + \frac{\lambda}{1-\lambda} \right) G - \left( \varepsilon + \frac{1}{1-\lambda} \right) D \right] \frac{dy}{\sigma} \]

\[ + \int_y^{G+D} \left[ \beta y - \beta D - \beta G + G \right] \frac{dy}{\sigma} + \beta \left[ y_1 + D \right] + (1 - \beta)G - k \frac{G^2}{2} , \]

where

\[ a_2 = \beta (1 - \tau) + \tau \left( \varepsilon + \frac{1}{1-\lambda} \right) . \]

Differentiating the RHS of (21) and making use of (11) we get that

\[ \frac{dV_L}{dG} = A \left[ -\varepsilon - \varepsilon \frac{dD^*}{dG} \right] + B \left[ - \left( \varepsilon + \frac{\lambda}{1-\lambda} \right) - \left( \varepsilon + \frac{1}{1-\lambda} \right) \frac{dD^*}{dG} \right] \]

\[ + C \left[ 1 - \beta - \beta \frac{dD^*}{dG} \right] + (1 - \beta) + \beta \frac{dD^*}{dG} - kG. \]

Observe that (19) also holds in region I.\(^{25}\) Therefore, taking differences, over \( G \in [0, G_{\text{max}}] \) we have that

\[ \frac{dV_H}{dG} - \frac{dV_L}{dG} = A \left[ -\frac{1}{\lambda} \frac{dD^*}{dG} \right] + B \left[ \frac{1}{1-\lambda} \left( 1 + \frac{dD^*}{dG} \right) \right] . \]

In region I, we have from equation (14) that \( \frac{dD^*}{dG} = -\lambda(1 - z)/ (1 - \lambda z) \in (-1, 0) \). Clearly, then \( \frac{dV_H}{dG} - \frac{dV_L}{dG} > 0 \). In region II, we have that \( \frac{dD^*}{dG} = 1 \). The condition \( \frac{dV_H}{dG} - \frac{dV_I}{dG} > 0 \) is then equivalent to

\[ \frac{2B}{1-\lambda} > \frac{A}{\lambda} . \]

Substituting the expressions in Table I we get that this inequality holds iff \( G < \tau(y_1 + y)/ (1 - \lambda) \), which is always true since this expression is greater than \( G_{\text{max}} \). This proves single-crossing between the preferences of group \( H \) and those of group \( L \).

To prove concavity, we proceed as follows:

1. In region I, we use the equilibrium value of \( D^* \) and the fact that \( \frac{dD^*}{dG} = -\lambda(1 - z)/ (1 - \lambda z) \). Using Table I we see that the RHS of (22) is linear in \( G \) and we can gather the terms in \( G \). We get the following expression

\[ \frac{1 - \lambda}{(1-\lambda z)\sigma} \left\{ -\varepsilon \frac{\lambda(1-\lambda)z}{1-\lambda z} - \varepsilon(1-\lambda) - \lambda z - (1 - \beta \frac{1-\lambda}{1-\lambda z}) \right\} - k, \]

\(^{25}\)The envelope theorem implies that in this zone the contribution of all terms in \( dD/dG \) sum up to zero.
which, given that \( \beta < 1 \) and that \( \frac{1-\lambda}{1-\lambda z} < 1 \), is clearly negative. Therefore \( \frac{d^2V_L}{dG^2} < 0 \) over region I.

2. In region II, we do the same, and we get
\[
\frac{1+\lambda}{\sigma^2}(-2\varepsilon) + \frac{1-\lambda}{\sigma^2}((-2\varepsilon - \frac{1+\lambda}{1-\lambda}) - \frac{2}{\sigma^2}(1-2\beta) - k.
\]

Recalling that \( z = \beta - \varepsilon \) and rearranging, this expression is equal to
\[
\frac{4z - (3 + \lambda)}{\sigma^2} - k.
\]

It is easy to prove that \( 4z - (3 + \lambda) < \frac{z(1-\lambda)^2}{1-\lambda z} \). To see this, note that at \( z = 1 \), these two expressions would be equal. Then differentiate the expression \( z \left[ 4 - \frac{(1-\lambda)^2}{1-\lambda z} \right] \) with respect to \( z \) and note that its derivative is equal to \( 4 - \frac{(1-\lambda)^2}{(1-\lambda z)^2} > 3 \). Therefore, this expression is increasing with \( z \), implying that \( 4z - (3 + \lambda) < \frac{z(1-\lambda)^2}{1-\lambda z} \) for \( z \in (0, 1) \). Since by assumption \( k > \frac{z(1-\lambda)^2}{1-\lambda z} \), we have that \( \frac{4z - (3 + \lambda)}{\sigma^2} - k < 0 \).

3. Finally, we compute the difference between the right and left derivatives of \( V_L \) at \( G = G_{\text{min}} \), equal to, from (22),
\[
\frac{dV_L^+}{dG} - \frac{dV_L^-}{dG} = \Delta \left[ -A \varepsilon - B \left( \varepsilon + \frac{1}{1-\lambda} \right) + \beta(1-C) \right],
\]

where \( \Delta = \frac{dD^\ast}{dG} (G_{\text{min}})^+ - \frac{dD^\ast}{dG} (G_{\text{min}})^- = 1 + \frac{\lambda (1-z)}{1-\lambda z} > 0 \). Clearly, \( \frac{dV_L^+}{dG} - \frac{dV_L^-}{dG} \) has the same sign as \( (1-C)z - \frac{B}{1-\lambda} \). Using the values in Table I, and the fact that \( D^\ast (G_{\text{min}}) = G_{\text{min}} - \tilde{\tau} y_1 \), that is equal to \( \frac{\sigma}{\tilde{\tau}} (2G_{\text{min}} - \tilde{\tau}(y + y_1)) - \frac{G_{\text{min}}}{\sigma} \), which, given the expression for \( G_{\text{min}} \), can be shown to be always negative.

These three facts, altogether, imply that \( V_L(D^\ast (G), G) \) is globally concave in \( G \).

7.8.3 Proof of claim (iii)

The utility of the rich is
\[
V_R(D, G) = \int_y^{G+D \bar{y}} \left[ a_3 y - \varepsilon y - \varepsilon D \right] \frac{dy}{\sigma} + \int_{G+D \bar{y}}^{y} \gamma \left[ y - D - G \right] \frac{dy}{\sigma} + \gamma \left[ y_1 + D - G \right] - k \frac{G^2}{2},
\]

where
\[
a_3 = \gamma (1 - \bar{\tau}) + \varepsilon \bar{\tau}.
\]

From there we get that
\[
\frac{dV_R}{dG} = -(A + B)\varepsilon (1 + \frac{dD^\ast}{dG}) - C\gamma (1 + \frac{dD^\ast}{dG}) - kG + \gamma (\frac{dD^\ast}{dG} - 1).
\]
Since \(-1 < \frac{dD^p}{d\theta} \leq 1\) in both regions, it is immediate that \(\frac{dV_H}{dG} < 0\).

QED

7.9 Proof of Proposition 9

We know that \(V_H\) is C1 and concave in \(G\). Let \(V'_H(G)\) denote the LHS of (18) and \(G^*_H = V'_H^{-1}(0)\). Clearly, the optimal \(G\) for group \(H\) is equal to \(G^*_H\) iff \(G^*_H \leq G_{\min}\).

Assume that this is the case. Then, from (18),

\[
G^*_H = G^*_H = \frac{2 (1 - \beta ) (1 - z \lambda ) \sigma \tau}{(1 - \lambda z) \sigma \tau k - z (1 - \lambda)^2}.
\]

Clearly, then,

\[
G^*_H \leq G_{\min} \iff k \geq \tilde{k} = \frac{2(1 - \beta ) (1 + \lambda - 2 \lambda z) + z (1 - \lambda)^2}{\tau (1 - \lambda z) (y_1 + y)}.
\]

It is then straightforward from (25) that \(\frac{\partial G^*_H}{\partial \tau} < 0\), \(\frac{\partial G^*_H}{\partial \alpha} < 0\), \(\frac{\partial G^*_H}{\partial \lambda} < 0\), and \(\frac{\partial G^*_H}{\partial \beta} > 0\), implying, since \(z = \beta - \varepsilon\), that \(\frac{\partial G^*_H}{\partial \beta} < 0\).

Now, differentiating the RHS of (25) with respect to \(\beta\), and rearranging, we get an expression which is proportional to and has the same sign as

\[
-(k - \frac{z (1 - \lambda)^2}{\tau (1 - \lambda z)}) + (1 - \beta) \frac{(1 - \lambda)^2}{\tau (1 - \lambda z)^2}.
\]

We now prove that this expression is negative. To see this, compute the following:

\[
\tilde{k} = \left[ (1 - \beta ) \frac{(1 - \lambda)^2}{\tau (1 - \lambda z)^2} + \frac{z (1 - \lambda)^2}{\tau (1 - \lambda z)} \right] = \frac{1 - \beta}{\tau (1 - \lambda z)} \left( \frac{2 (1 + \lambda - 2 \lambda z)}{y_1 + y} - \frac{(1 - \lambda)^2}{\sigma} \right).
\]

Now note that (i) \(y_1 + y < \sigma\) by Assumption 3, and (ii) \(2 (1 + \lambda - 2 \lambda z) > (1 - \lambda)^2\). Consequently, the RHS of (28) is \(> 0\), implying from (26) that the expression in (27) is \(< 0\). Therefore, \(\frac{\partial G^*_H}{\partial \beta} < 0\).

Since \(B = \frac{(1 - \lambda)G}{\sigma \tau}\), it follows that \(\frac{\partial B}{\partial \sigma} < 0\), \(\frac{\partial B}{\partial \alpha} < 0\), \(\frac{\partial B}{\partial \lambda} < 0\), and \(\frac{\partial B}{\partial \beta} < 0\).

From Table I and (14) we get that \(A = \frac{z \lambda (1 - \lambda) G}{\sigma \tau (1 - \lambda z)}\). It follows that \(\frac{\partial A}{\partial \tau} < 0\), \(\frac{\partial A}{\partial \alpha} < 0\). Substituting in the equilibrium value of \(G\) from (25), we get that

\[
A = \frac{2 (1 - \beta ) \lambda (1 - \lambda)}{(1 - \lambda z) \sigma \tau k / z - (1 - \lambda)^2},
\]

from which it follows that \(\partial A / \partial \varepsilon > 0\) and therefore \(\partial A / \partial \varepsilon < 0\). Finally,

\[
\frac{\partial A}{\partial \beta} \propto (1 - \lambda)^2 + \frac{\sigma \tau k}{\varepsilon^2} (1 - 2 z - \varepsilon + \lambda z^2) \gg 0,
\]

31
which is furthermore clearly positive for \( z \) small. Finally, it is clear from the expression in (29) that \( A \) must be increasing with \( \lambda \) for small values of \( \lambda \) and decreasing for large values of \( \lambda \).

Now note that \( P = 1 - C = A + B = \frac{(1-\lambda)G}{\sigma \tau (1-\lambda \bar{z})} \), which clearly is decreasing in \( \beta \) and \( \lambda \).

Assume now that \( G_{H1}^* > G_{\min} \). Then the optimal \( G \) is in region II and equal to \( G_{H2}^* \), where \( G_{H2}^* \) equates the RHS of (20) to zero. Clearly, then, from (20),

\[
G_H^* = G_{H2}^* = \frac{2(1 - \beta) + \frac{1}{\sigma \tau}(1 + 1\lambda - 2\lambda z)(y + y_1)}{k + \frac{1}{\sigma \tau}(2 + \lambda + 1/1\lambda - 4\lambda z)},
\]

which is clearly increasing in \( \bar{\tau} \).

QED

7.10 Equilibrium value of \( G \) if group \( L \) is pivotal

In the zone where \( G < G_{\min} \), we have that \( D^* = \bar{\tau}y - \frac{\lambda(1-z)}{1-\lambda z} G \), implying from (22) and the expressions in Table I that

\[
\frac{dV_L}{dG} = \frac{z(1 - \lambda)}{1 - \lambda z} \frac{G}{\sigma \bar{\tau}} (\bar{\tau}z) + \frac{(1 - \lambda)G}{\sigma \bar{\tau}} \left[ \left( (\lambda + 1\lambda - 2\lambda z)(1 + \lambda z) + \varepsilon(1 - \lambda)^3 \right) \right] + \frac{1 - \lambda z}{\sigma (1 - \lambda z)} \left( (1 - z)(1 + \lambda z) + \varepsilon z(1 - \lambda)^2 \right) - kG.
\]

This expression can be rearranged as

\[
\frac{dV_L}{dG} = G \left( -(k - \frac{(1 - \lambda)(1 - z)(1 + \lambda z) + \varepsilon z(1 - \lambda)^3}{\sigma \bar{\tau}(1 - \lambda z)^2}) \right) + 2(1 - \beta).
\]

From this and the fact that \( V_L \) is concave, we see that the optimal \( G \) is such that \( G \leq G_{\min} \) if and only if \( \frac{dV_L}{dG}(G_{\min}) \leq 0 \), or equivalently, given that \( G_{\min} = \frac{1 - \lambda z}{1 + \lambda - 2\lambda z} (y_1 + y) \),

\[
k \geq \frac{1}{\tau(1 - \lambda z)} \left\{ \frac{2(1 - \beta)(1 + \lambda z)}{y_1 + y} + \frac{1 - \lambda z}{\sigma (1 - \lambda z)} \left( (1 - z)(1 + \lambda z) + \varepsilon z(1 - \lambda)^2 \right) \right\} = k_{L \max}.
\]

In this case, the optimal \( G \) is given by \( \frac{dV_L}{dG} = 0 \), that is

\[
G_L^* = G_{L1}^* = \frac{2(1 - \beta)}{k + \frac{(1 - \lambda)(1 - z)(1 + \lambda z) + \varepsilon z(1 - \lambda)^3}{\sigma \bar{\tau}(1 - \lambda z)^2}}.
\]

It is immediate that \( \frac{dG_{L1}^*}{d\bar{\tau}} > 0 \).

In the zone where \( G > G_{\min} \), we have that \( D^* = G - \bar{\tau}y_1 \), implying from (22) and the expressions in Table I that

\[
\frac{dV_L}{dG} = \frac{1}{\sigma \bar{\tau}} (-2\varepsilon) \left( (1 + \lambda)G - \bar{\tau}(y_1 + y) \right) + \frac{(1 - \lambda)G}{\sigma \bar{\tau}} \left[ -2\varepsilon - \frac{1 + \lambda}{1 - \lambda} \right]
+ \frac{1}{\sigma \bar{\tau}} \left( \bar{\tau}(y_1 + y) - 2G \right) (1 - 2\beta) + 1 - kG.
\]
This expression can be rearranged as
\[
d\frac{V_L}{dG} = G \left( \frac{4z - (3 + \lambda)}{\sigma \tau} - k \right) + \frac{y_1 + y}{\sigma} (1 - 2z) + 2(1 - \beta).
\]

The equilibrium is such that \( G > G_{\text{min}} \) if and only if this expression is strictly positive at \( G = G_{\text{min}} \), that is
\[
k < \frac{4z - (3 + \lambda)}{\sigma \tau} + \frac{1 + \lambda - 2\lambda z}{\tau(1 - \lambda z)} \left[ \frac{1 - 2z}{\sigma} + \frac{2(1 - \beta)}{y_1 + y} \right] = k_{L_{\text{min}}} < k_{L_{\text{max}}},
\]
where the inequality comes from point 3 in the proof of claim (ii) in Proposition 7.

In this case we have that
\[
G_{L}^{*} = G_{L2}^{*} = \frac{y_1 + y}{k - \frac{4z - (3 + \lambda)}{\sigma \tau}}.
\]

Finally, if
\[
k_{L_{\text{min}}} \leq k < k_{L_{\text{max}}},
\]
then \( \frac{dV_L}{dG} (G_{\text{min}}) \leq 0 < \frac{dV_L}{dG} (G_{\text{min}}) \) and the optimal value of \( G \) is\(^{26}\)
\[
G_{L}^{*} = G_{\text{min}}.
\]

### 7.11 Proof of Proposition 10

From (16) and the expressions in Table I, we get that
\[
\frac{\partial V_H}{\partial \tau} = \left( \frac{1}{\lambda} - \beta \right) \frac{1}{2\sigma} \left[ \left( \frac{\lambda G + D}{\tau} \right)^2 - y_2^2 \right] - \frac{z}{2\sigma} \left[ \left( \frac{G + D}{\tau} \right)^2 - \left( \frac{\lambda G + D}{\tau} \right)^2 \right].
\]

In the regime which we consider we have that
\[
D = \frac{\tau y}{1 - \lambda z} G + \frac{\lambda(1 - z)}{1 - \lambda z} G.
\]

Substituting into the preceding expression, we have that
\[
\frac{\partial V_H}{\partial \tau} \propto E \equiv \frac{1 - \lambda \beta}{1 - \lambda z} \left[ 2y + \frac{\lambda(1 - \lambda)z}{1 - \lambda z} G \right] - \left[ 2y + \frac{(1 - \lambda)(1 + \lambda z)}{1 - \lambda z} G \right]
= -2y + \frac{\lambda z}{\tau(1 - \lambda z)} \left[ (1 - \lambda \beta) \frac{\lambda z}{1 - \lambda z} - (1 + \lambda z) \right] G < 0,
\]
since the last term in squared brackets is negative. Furthermore, in this regime, both \( G \) and \( D \) are set optimally by group \( H \). By the envelope theorem, we have that
\[
\frac{dV_H}{d\tau} = \frac{\partial V_H}{\partial \tau} < 0.
\]

QED.

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\(^{26}\)If \( k = k_{L_{\text{max}}} \), then \( G_{L1}^{*} = G_{\text{min}} \).
7.12 Proof of Proposition 11

The proof follows as long as the debt ceiling delivers a perturbation of the initial equilibrium

\[ G = G^* + \Delta G, \quad D = D^* + \Delta D, \]

such that \( \Delta G > 0 \) and \( \Delta D / \Delta G < D'(G) = -\frac{\lambda(1-z)}{1-\lambda z} \). First, observe that as long as \( \omega > \frac{\lambda(1-z)}{1-\lambda z} \), the debt ceiling will be binding in region I if and only if \( G > G^* \). In this zone group \( H \) will choose \( D = D_{\text{max}}(G) \). Second, as long as \( \omega \) is close enough to \( \frac{\lambda(1-z)}{1-\lambda z} \), the proof of Proposition 7 can be replicated, implying that there exists a voting equilibrium over \( G \) whose outcome maximizes the preferences of the poor of group \( L \). Third, note that in our regime where \( D \) is constrained by \( D_{\text{max}} \),

\[
\frac{dV_L}{dG}(G^*, D^*)^+ = \frac{\partial V_L}{\partial G} - \omega \frac{\partial V_L}{\partial D} = -\frac{\partial V_L}{\partial D} \left( \omega - \frac{\lambda(1-z)}{1-\lambda z} \right) > 0, \tag{31}
\]

where the derivatives are computed at \((G^*, D^*)\). It follows that for \( \omega \) greater than \( \frac{\lambda(1-z)}{1-\lambda z} \) but arbitrarily close to it, \( G \) is greater than \( G^* \) but arbitrarily close to it. Furthermore, as the debt ceiling is binding, \( D - D^* = \Delta D = -\omega(G - G^*) = -\omega \Delta G < D'(G) \Delta G \). Since \( \Delta G > 0 \) and \( \Delta D / \Delta G < D'(G) \), the welfare of the poor of both groups increases.
8 References


