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A note on risk sharing versus instability in international financial integration: When Obstfeld meets Stiglitz *

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Abstract

International risk sharing is one of the main arguments in favor of financial liberalization. The pure risk sharing mechanism highlighted by Obstfeld (1994) implies that liberalization is growth enhancing for all countries as it allows the world portfolio to shift from safe low-yield capital to riskier high yield capital. This result is obtained under the assumption that **the volatility figures for risky assets prevailing under autarky are not altered after liberalization**. This note relaxes this assumption within the standard two-country model with intertemporal portfolio choices, formally incorporating the instability effect invoked by Stiglitz (2000). We show that putting together the pure risk sharing and instability effects in the latter set-up enriches the analysis and delivers predictions more consistent with the contrasted related empirical literature.

Keywords: Optimal growth, financial liberalization,risk sharing, volatility.
JEL classification: F21, G15, O16, O41.

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1 Introduction

The growth impact of financial liberalization is still at the heart of many ongoing research programs. While the arguments in favor of international financial integration are pretty clear (essentially, access to larger savings and international risk sharing), the findings of the related empirical studies are much more ambiguous. Indeed, a common view in the literature is that financial liberalization may be beneficial or not depending on whether the countries fundamentals are above certain threshold levels (see for example Kose et al., 2011). In particular, it is nowadays broadly argued that financial and institutional development, in particular the soundness of the national financial systems, should be above a certain level in order to reduce the risks associated with financial openness.

In an important review paper, Stiglitz (2000) took a remarkable stance against the potential inconveniences of financial liberalization, going well beyond the "threshold" literature mentioned just above. In particular, Stiglitz wrote: "...As the crisis spread from East Asia to Russia, and then to Latin America, it became clear that even countries with good economic policies and relatively sound financial institutions (at least as conventionally defined) were adversely affected, and seriously so. Indeed, this was consistent with earlier research that had shown that changes in capital flows, and even crises, were predominantly precipitated by events outside the country, such as changes in interest rates in the more developed countries" (page 1075). That's to say, even if, in the spirit of Kose et al., the threshold conditions for a beneficial financial liberalization are met, the scope for potential instability is so big that the intended advantages (access to larger savings, risk sharing) can be perfect offset.

A perfect illustration of Stiglitz claim is the case of emerging economies. Intuitively, capital inflow to the emerging markets should lower the cost of capital (Bekaert and Harvey, 2000; Chari and Henry, 2004; Harvey 1995) and thus stimulates economic growth in these economies (Bekaert et al, 2005, 2006; Monshirian, 2008; Levine, 2001). At the same time, there are also evidence showing that the effect from financial market liberalization is unclear or mixed (Prasad et al., 2003; Eichengreen, 2001; Rodrik, 1998). Arguably, the opposite views lie on different concerns. One of these concerns are that

financial market liberalization may *increase the volatility* of the emerging markets (Bae et al., 2004; Li et al., 2004; Umutlu et al., 2010; Harvey, 1995; Newbery, 1987) and volatility is negatively related to economic growth (Ramey and Ramey, 1995). The latter argument is at the heart of Stiglitz stance for the regulation of (notably short-term) capital flows.

This paper is a theoretical contribution to this debate. In particular, we shall focus on the international risk sharing argument. In a seminal contribution, Obstfeld (1994) explored the **pure** international risk sharing mechanism and its impact on growth. The main conclusion of Obstfeld is that after opening the asset markets to trade, **the expected growth rate must rise in all countries** thanks to the **world portfolio shift from safe low-yield capital to riskier high yield capital**. Obviously, there is no mistake (neither technical nor conceptual) in Obstfeld's argument, the analysis singles out the pure risk diversification mechanism and then extracts the implication in terms of growth. In this sense, the empirical studies showing negative correlation between financial liberalization and growth cannot be opposed to Obstfeld's conclusion because it is only concerned with the pure international risk sharing and deliberately disregards the many other mechanisms that can matter in this story. In particular, Obstfeld draws his conclusions based on the explicit assumption that **liberalization does not change the volatility of the risky assets**, that's the volatility figures prevailing under autarky are not altered after opening the asset markets.

In this paper, we relax the latter assumption and allow these volatilities to change as a result of financial liberalization. We argue that by proceeding so, we extend Obstfeld's analysis to allow for the instability argument highlighted by Stiglitz. Needless to say, putting the pure risk sharing and instability mechanisms together will markedly enrich the analysis, which in turns permits to cope much more easily with the contrasted related empirical literature. Two remarks are worth doing here. First of all, we could have indeed taken an agnostic view of how financial liberalization alters the volatilities of risky assets, which would have significantly increased the set of possible outcomes. To unburden the presentation and to make it focused enough, we concentrate on the emerging economies case for which, as mentioned above, there is a compelling evidence that liberalization has indeed raised volatility. Second, we do not **endogenize** the volatility variations induced

by liberalization, and only consider exogenous changes in these figures. Of course, there are several mechanisms and events which can cause such variations, some may be indeed exogenous due to contagion as outlined by Stiglitz (see above) and others may derive from the countries' deep characteristics. Boucekkine et al. (2016) (see also Boucekkine and Pintus, 2012) have for example highlighted the role of external debts in driving macroeconomic and financial instability.

The rest of paper is organized as follows. We restate the closed and open economy of Obstfeld (1994) in Section 2 and 3. In particular, the long-run average growth rates are obtained in both cases. In Section 4, we study in detail the case of emerging economies. Section 5 concludes.

2 Autarkic economy

Suppose that in one closed economy, all individuals are identical and live forever. At each moment of time, individuals face the choice between consumption and investment. Suppose furthermore, the utility of each individual is $U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$ where c_t reads time t consumption, $\gamma \in (0,1)$ represents relative-risk aversion parameter and then $\frac{1}{\gamma}$ is intertemporal substitution elasticity.

Each individual has initial wealth W_0 as endowment, which can be consumed or invested. Thus, the time t per capital wealth holding is $W_t = B_t + K_t$. Here, B_t is per capita risk free asset holding whose dynamics are simply

$$dB_t = rB_t dt$$

with r the return from risk free asset, assumed to be a fixed constant. K_t is the per capita risky asset holding, it is driven by the following stochastic law of motion

$$dK_t = \alpha K_t dt + \sigma K_t dz_t,$$

where $z = (z_t)_{t\geq 0}$ is a standard Wiener process, α is risky asset drift and σ is risky asset volatility, respectively. For simplicity, we assume both are fixed constants. Following

Obstfeld (1994), we assume also that there is no nondiversifiable income (such as labor income), thus the asset markets in this autarkic economy are complete.

Combining the above two dynamic processes together and taking into account consumption behaviour, the dynamics of wealth is governed by

$$dW_t = (rB_t + \alpha K_t - c_t)dt + \sigma K_t dz_t.$$

Denote by $\omega_t = \frac{K_t}{W_t}$ the share of risky assets in total wealth holding, we also have $\frac{B_t}{W_t} = 1 - \omega_t$. The law of motion of wealth is then

$$dW_t = (rW_t + (\alpha - r)W_t\omega_t - c_t)dt + \sigma W_t\, \omega_t\, dz_t. \tag{1}$$

To obtain explicit solution to the related optimal intertemporal choices problem, we follow Merton (1969) and suppose that individuals have CRRA (Constant Relative Risk Aversion) preferences, implying the optimization problem:

$$\max_{c_t,\omega_t} \mathbb{E}\int_0^\infty e^{-\rho t}U(c_t)dt = \mathbb{E}\int_0^\infty e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt,$$

where parameter $\rho(>0)$ is time preference and $\mathbb{E}$ is mathematical expectation, subject to the budget constraint (1).

Denote by $v(W)$ the maximum feasible level of lifetime utility with wealth level at W. Then the standard Hamilton-Jacobi-Bellman equation is

$$\rho v = rv + \sup_{c\geq 0}\left\{-cv' + \frac{c^{1-\gamma}}{1-\gamma}\right\} + \sup_{\omega\in\mathbb{R}}\left\{(\alpha - r)\omega W v' + \frac{1}{2}\sigma^2\omega^2 W^2 v''\right\}$$

with v', v'' the first and second derivatives of the function v respectively.

Following the standard solution process proposed by Merton to solve stochastic lifetime portfolio choice model (see Merton, 1971), one can easily express maximum consumption and the share of risky assets as

$$c_t^* = [v_0(1-\gamma)]^{-\frac{1}{\gamma}}W_t^*, \qquad \omega_t^* \equiv \frac{1}{\gamma}\frac{\alpha - r}{\sigma^2}. \tag{2}$$

with

$$v(W) = v_0 W^{1-\gamma} \text{ and } v_0 = \frac{1}{1-\gamma}\left(\frac{\rho - r}{\gamma} - \frac{(1-\gamma)}{2\gamma^2}(\alpha - r)^2\sigma^2\right)^{-\gamma}.$$

Substituting the optimal choice into the dynamic equation of wealth accumulation, it yields

$$\frac{dW_t}{W_t} = \mu dt + \nu dz_t,$$

where the constants μ and ν are defined as

$$\mu := r + \frac{(\alpha - r)^2}{\sigma^2\gamma} - [v_0(1+\gamma)]^{-1/\gamma}, \qquad \nu := \frac{1}{\gamma}\frac{\alpha - r}{\sigma}. \tag{3}$$

Thus, the optimal wealth is

$$W_t^* = W_0 \exp\left[\left(\mu - \frac{\nu^2}{2}\right)t + \nu z_t\right].$$

Therefore, the expected almost sure growth rate (growth rate of the almost surely trajectory) is

$$g_a := \mu - \nu^2/2 = r - \frac{\rho - r}{\gamma} + \frac{1}{2\gamma}\frac{(\alpha - r)^2}{\sigma^2} \tag{4}$$

which is constant over the probability space.[1]

Expression (2) and (3) imply that the share of risky assets in total wealth is decreasing in the risky asset's volatility ($\frac{\partial \omega_t^*}{\partial \sigma} < 0$), which in turns implies that the volatility in wealth is also decreasing in the latter volatility ($\frac{\partial \nu}{\partial \sigma} < 0$).

Rewrite (3) as $\gamma\nu = \frac{\alpha - r}{\sigma}$ and recall that γ measures relative risk aversion. Thus, expression $\frac{\alpha - r}{\sigma}$ can be renamed as relative-risk-averse-wealth volatility. The above study can be concluded with the following important statement.

Proposition 1. *Both share of risky asset holding and the relative-risk-averse-wealth volatility are decreasing in term of volatility of risky asset.*

[1]Notice that the economic system is *exponentially stable* if and only if $\mu - \nu^2/2 < 0$. However, it is easy to see that if $\sigma \to 0$, we have $r - \frac{\rho - r}{\gamma} + \frac{1}{2\gamma}\frac{(\alpha - r)^2}{\sigma^2} \to +\infty$. In other words, stability is almost impossible in the deterministic case.

In the next section, we shall open the economy to international asset trade. In particular, we shall move to a two-country framework. To single out the pure risk sharing effect, Obstfeld (1994) assumed that the volatilities of the risky assets (here σ for the closed economy case) remains unchanged after liberalization. We shall depart from this assumption. Accordingly, we shall denote the volatility of risky assets before opening for each of the countries, say $i = 1, 2$, σ_{ia}, while these volatilities turn to σ_{io} after liberalization.

3 Open economy – the two-country case

Consider now two countries which open their asset markets to each other with the same risk free asset yielding the same world interest rate, and two risky assets. Denote then by $W = (W_t^i)_{t\geq 0}$ the wealth process of country i (i=1, 2), B_t^i being the amount of riskless asset held at time t by country i, and $K_t^{i,j}$ being the amount of risky asset j (j=1, 2) held by country i at time t. We set $\mathbf{K}^i = \begin{pmatrix} K^{i,1} \\ K^{i,2} \end{pmatrix}$, which now follows a stochastic differential equation in a two dimensional space. Furthermore, denote by $z^i = (z_t^i)_{t\geq 0}$, $i = 1, 2$, the two Brownian motions involved, and denote by $\xi \in (-1, 1)$, the correlation between these two motions. Set $\mathbf{z} = \begin{pmatrix} z^1 \\ z^2 \end{pmatrix}$. We can write it as $\mathbf{z} = \mathbf{\Xi y}$ with $\mathbf{y} = (\mathbf{y}_t)_{t\geq 0} := \begin{pmatrix} (y_t^1)_{t\geq 0} \\ (y_t^2)_{t\geq 0} \end{pmatrix}$ two-dimensional Brownian motion and $\mathbf{\Xi} = \begin{pmatrix} 1 & 0 \\ \xi & \sqrt{1-\xi^2} \end{pmatrix}$. Let r be the common riskless asset rate of return and α_{io} the drift of the risky asset of country i. σ_{io} is the volatility of the risky asset of country i after opening as already explained. In matrix form, we set $\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{1o} \\ \alpha_{2o} \end{pmatrix}$, $\mathbf{\Sigma_o} = \begin{pmatrix} \sigma_{1o} & 0 \\ 0 & \sigma_{2o} \end{pmatrix}$, $\mathbf{1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \in \mathbb{R}^2$.

Similar to the autarkic case, the budget equation for country i at time t is

$$W_t^i = B_t^i + K_t^{i,1} + K_t^{i,2}$$

and the dynamics of wealth accumulation for country i is

$$dW_t^i = (rB_t^i + \langle \boldsymbol{\alpha}, \mathbf{K}_t^i \rangle - c_t^i)dt + \langle \mathbf{K}_t^i, \mathbf{\Sigma_o \Xi} d\mathbf{y}_t \rangle,$$

with

$$\mathbf{M} := \mathbf{\Sigma_0 \Xi} = \begin{pmatrix} \sigma_{1o} & 0 \\ \xi\sigma_{2o} & \sqrt{1-\xi^2}\sigma_{2o} \end{pmatrix}.$$

Define now the capital share vector in total wealth as

$$\boldsymbol{\omega}_t^i = \begin{pmatrix} \omega_t^{i,1} \\ \omega_t^{i,2} \end{pmatrix} := \mathbf{K}_t^i / W_t^i,$$

we have $B_t^i/W_t^i = 1 - \langle \boldsymbol{\omega}_t^i, \mathbf{1} \rangle$ and wealth accumulation equation becomes[2]

$$dW_t^i = (rW_t^i + \langle \boldsymbol{\alpha} - r\mathbf{1}, \boldsymbol{\omega}_t \rangle W_t^i - c_t^i)dt + W_t^i \langle \boldsymbol{\omega}_t^i, \mathbf{M} d\mathbf{y}_t \rangle. \tag{5}$$

As in the previous section, the country i's optimization problem is

$$\max_{c_t^i, \omega_t^i} \mathbb{E} \int_0^\infty e^{-\rho_i t} \frac{(c_t^i)^{1-\gamma_i}}{1-\gamma_i} dt,$$

subject to the budget constraint (5).

If we define $v_i(W^i)$ as the maximum feasible level of lifetime utility of country i at wealth level W^i, then one can use a similar strategy as in the previous section,[3] and obtain country i's optimal consumption as

$$c_t^{i,*} = [v_0^i(1-\gamma_i)]^{-\frac{1}{\gamma_i}} W_t^{i,*},$$

[2]Here $\langle \cdot, \cdot \rangle$ denotes scalar product in $\mathbb{R}^2$, i.e.,

$$\langle \mathbf{x}, \mathbf{x}' \rangle = x_1 x_1' + x_2 x_2', \quad \forall \mathbf{x} = (x_1, x_2),\ \forall \mathbf{x}' = (x_1', x_2').$$

[3]In the two country case, the Hamilton-Jacob-Bellman equation for each country is given by

$$\rho v_i = rv_i + \sup_{c_i \geq 0} \left\{ -c^i v_i' + \frac{(c^i)^{1-\gamma_i}}{1-\gamma_i} \right\} + \sup_{\boldsymbol{\omega} \in \mathbb{R}^2} \left\{ (\boldsymbol{\alpha} - r\mathbf{1}) \cdot \omega W^i v_i' + \frac{1}{2} \langle \mathbf{\Omega}\boldsymbol{\omega}, \boldsymbol{\omega} \rangle (W^i)^2 v_i'' \right\}$$

with

$$\mathbf{\Omega} := \mathbf{M}\mathbf{M}^\mathbf{T} = \begin{pmatrix} \sigma_{1o}^2 & \sigma_{1o}\sigma_{2o}\xi \\ \sigma_{1o}\sigma_{2o}\xi & \sigma_{2o}^2 \end{pmatrix}.$$

The current value function is then given by

$$v_i(W^i) = v_0^i (W^i)^{1-\gamma_i} \ \text{ with } \ v_0^i = \frac{1}{1-\gamma_i} \left[\frac{\rho - r}{\gamma_i} - \frac{(1-\gamma_i)\Gamma}{2\gamma_i^2} \right]^{-\gamma_i}$$

and the optimal share of risky asset vector as

$$\boldsymbol{\omega}_t^{i,*} \equiv \frac{1}{\gamma_i}\boldsymbol{\Omega}^{-1}(\boldsymbol{\alpha} - r\mathbf{1}) = \frac{1}{\gamma_i}\begin{pmatrix} \frac{\alpha_{1o}-r}{(1-\xi^2)\,\sigma_{1o}^2} + \frac{\xi(\alpha_{2o}-r)}{(\xi^2-1)\,\sigma_{2o}\sigma_{1o}} \\ \frac{\xi(\alpha_{2o}-r)}{(\xi^2-1)\,\sigma_{2o}\sigma_{1o}} + \frac{\alpha_{2o}-r}{(1-\xi^2)\,\sigma_{2o}^2} \end{pmatrix}. \tag{6}$$

Substituting these optimal choices into the law of motion of wealth accumulation equation (5), we find that wealth accumulation of country i follows

$$\frac{dW_t^i}{W_t} = \mu_i dt + \langle \boldsymbol{\nu}_i, d\mathbf{y}_t \rangle,$$

where the constants μ_i and $\boldsymbol{\nu}_i = (\nu_i^1, \nu_i^2)$ are [4]

$$\mu_i := r + \frac{1}{\gamma_i}\langle \boldsymbol{\Omega}^{-1}(\boldsymbol{\alpha} - r\mathbf{1}), \boldsymbol{\alpha} - r\mathbf{1}) \rangle - [v_0^i(1-\gamma_i)]^{-1/\gamma_i},$$

$$\nu_i := \frac{1}{\gamma_i}\mathbf{M}^{-1}(\boldsymbol{\alpha} - r\mathbf{1}).$$

Therefore, the explicit form of optimal wealth for country i in this open economy case is

$$W_t^{i,*} = W_0^i \exp\left[\left(\mu_i - \frac{|\boldsymbol{\nu}_i|^2}{2}\right)t + \langle \boldsymbol{\nu}_i, \mathbf{y}_t \rangle\right].$$

Hence, the expected almost surely growth rate of country i in this open economy is defined as

$$g_{io} := \mu_i - \frac{1}{2}|\boldsymbol{\nu}_i|^2 = r - \frac{\rho - r}{\gamma_i} + \frac{\Gamma}{2\gamma_i}. \tag{7}$$

and

$$\Gamma = < (\boldsymbol{\alpha} - r\mathbf{1}), \boldsymbol{\Omega}^{-1}(\boldsymbol{\alpha} - r\mathbf{1}) > = \frac{1}{1-\xi^2}\left[\frac{(\alpha_{1o}-r)^2}{\sigma_{1o}^2} + \frac{(\alpha_{2o}-r)^2}{\sigma_{2o}^2} - \frac{2\xi(\alpha_{1o}-r)(\alpha_{2o}-r)}{\sigma_{1o}\sigma_{2o}}\right].$$

[4] Where

$$\Omega^{-1} = \begin{pmatrix} \frac{1}{(1-\xi^2)\,\sigma_{1o}^2} & \frac{\xi}{(\xi^2-1)\,\sigma_{2o}\sigma_{1o}} \\ \frac{\xi}{(\xi^2-1)\,\sigma_{2o}\sigma_{1o}} & \frac{1}{(1-\xi^2)\,\sigma_{2o}^2} \end{pmatrix}, \quad M^{-1} = \begin{pmatrix} \frac{1}{\sigma_{1o}} & 0 \\ -\frac{\xi}{\sqrt{1-\xi}\,\sigma_{1o}} & \frac{1}{\sqrt{1-\xi^2}\,\sigma_{2o}} \end{pmatrix}.$$

4 The gains and losses from financial integration: the case of emerging economies

As explained before, we depart from Obstfeld (2014) by assuming that the volatility of country i under autarky, σ_{ia} may be different from the counterpart after opening, σ_{io}. For simplicity, we keep all other parameters, such as γ_i, α_i and so on, unaltered after liberalization, though they may differ between the two countries.

Combining equation (4) and (7), it yields

$$g_{io} - g_{ia} = \frac{1}{2\gamma_i}\left[\Gamma - \frac{(\alpha_i - r)^2}{\sigma_{ia}^2}\right]. \tag{8}$$

It is easy to see that if $\sigma_{ia} = \sigma_{io}$, then for both countries, $i = 1, 2$, we have

$$g_{io} - g_{ia} \geq 0.$$

That is the case studied by Obstfeld (1994) and one can safely re-state his main result.

Proposition 2. *Financial market liberalization always promotes both countries long-run economic growth if there is no volatility change from autarky economy to open economy.*

However, as documented in the related empirical literature (see Bae et al., 2004; Li et al., 2004; Umutlu et al., 2010; Harvey, 1995), at least some emerging markets did experience increases in volatility after financial market liberalization. More precisely, it has been documented that in some cases the volatility of the risky asset did rise. In other words, the equality $\sigma_{ia} = \sigma_{io}$ may be violated in practice. We prove hereafter that that if $\sigma_{ia} < \sigma_{io}$, the unambiguous growth gain from financial integration pointed out by Obstfeld may be offset by this induced volatility increase (Stiglitz instability effect) under some appropriate conditions.

Actually, two different cases may appear when scrutinizing the set of emerging countries opening, say, to a single developed country.

1. The emerging market is sufficient small compared to the developed world financial market, thus only the emerging economy's volatility change and the other economy's volatility is unchanged.

2. The emerging economy is large enough, such as China and India. When these countries join the world financial market, it may affect the rest of the world as well. Thus, both the developing and emerging economy volatilities may change after financial market liberalization.

However, to make our point, it is sufficient to study only the first case, which is indeed the best case for financial liberalization. In the second case, both the emerging economy and the originally developed economy may face a decrease in economic growth, provided the emerging economy is sufficiently large and, hence, influential (contagion effect).

Denote by $i = 1$ the emerging economy, $i = 2$ being the developed one. Suppose that $\sigma_{2o} = \sigma_{2a} = \sigma_2$ is unchanged. In the Appendix, we demonstrate the following results.

Proposition 3. *Suppose that after financial market liberalization, there is no volatility change in country* 2*, but the emerging economy,* $i = 1$*, faces volatility increase to some extent, such that,*

$$\sigma_{1o} > \frac{\alpha_1 - r}{y_2}, \tag{9}$$

with

$$y_2 = \xi \left(\frac{\alpha_2 - r}{\sigma_2} \right) + \sqrt{(1 - \xi^2) \left[\left(\frac{\alpha_1 - r}{\sigma_{1a}} \right)^2 - \left(\frac{\alpha_2 - r}{\sigma_2} \right)^2 \right]}.$$

Furthermore, if the two risky assets are negatively correlated, i.e. $\forall \xi \in (-1, 0]$*, the emerging economy experiences* $g_{1o} < g_{1a}$ *if and only if*

$$\frac{\alpha_2 - r}{\sigma_2} < \sqrt{1 - \xi^2} \left(\frac{\alpha_1 - r}{\sigma_{1a}} \right) \quad . \tag{10}$$

Proposition 4. *Under the same assumptions as in the previous proposition, including (9), and provided the two risky assets are positively correlated, that is ,* $\forall \xi \in (0, 1)$*, then* $g_{1o} < g_{1a}$ *if and only if*

$$\frac{\alpha_2 - r}{\sigma_2} < \frac{\alpha_1 - r}{\sigma_{1a}}. \tag{11}$$

The proofs are in the Appendix. A few remarks are in order here. First notice that Propositions 3 and 4 state conditions for growth detrimental financial liberalization under

negative and positive correlation ξ respectively. As it transpires, the case of negative correlation is more demanding as fulfilling condition (10) is harder than fulfilling (11). In other words, (10) implies (11) . Moreover, the larger the absolute value of ξ the more demanding condition (10). This should not be surprising: the pure risk sharing effect works when risk diversification makes more sense, and this happens under negative correlation of the national risky assets' returns. Obviously, the "more negative" the correlation is, the harder for the Stiglitz instability effect to offset the Obstfeld risk sharing effect.

Second, in each Proposition, 2 conditions are indeed invoked. The first one, which is common to the 2 propositions, that is: $\sigma_{1o} > \frac{\alpha_1 - r}{y_2}$, gives the size of the instability effect induced by the liberalization, required to offset the Obstfeld effect. The second condition ((10) in Proposition 3 and (11) in Proposition 4) concerns the autarky phase. Recall that we define $\frac{\alpha_i - r}{\sigma_i}$ as relative-risk-averse-wealth volatility. Thus, the latter type of condition states that, for given developed world financial markets, if under autarky the emerging economy's wealth volatility is (at least) higher than the wealth volatility of the developed economy (the negative correlation case being the more demanding), there could be the case that after financial liberalization its long-run growth rate is lower than under autarky.

Summing up, as it transpires from the interpretation of the two types of conditions involved in the Propositions just above, our analysis allows to bridge the gap between the threshold literature à la Kose et al. (2011) and the instability literature inspired by Stiglitz: the outcomes of financial liberalization do not only depend on the fundamentals of the economies, that's on their characteristics before opening (as in the stream of literature led by Kose et al., 2011) but also on the size of the instability conveyed by liberalization. We do identify in our framework how big should be instability for the virtuous pure diversification effect to be dominated.

We now state some of the residual implications of the Propositions above. In particular, not surprisingly, it's shown that the developed country, for which there is no instability effect assumed, always benefits from financial integration.

Corollary 1. *Suppose that after financial market liberalization, there is no volatility change in country* 2*, the developed economy, then*

- *country* 2 *always benefits from financial market liberalization of the emerging economy, that is,* $g_{2o} > g_{2a}$ *is always true;*
- *if condition (9) fails, or one of the two conditions (10) or (11) fail, then in the long run, both countries benefit from financial market liberalization.*

5 Conclusion

In this note, we have entended Obstfled's international risk sharing model to account for the instability effect rightly pointed out by Stiglitz. We do so by relaxing one crucial assumption in the original framework, at a very reasonable algebraic cost. We have identified the conditions under which an emerging economy joining the international financial markets may experience a drop in growth. Interestingly, these conditions combine the typical threshold conditions one can find in the related literature, which concern the deep characteristics of the economies under autarky, and size conditions on the instability effects induced by liberalization (consistently with Stiglitz).

Needless to say, our paper is limited by the fact that we do not model the instability effect as an endogenous mechanism but as an exogenous shock. Clearly part of the instability comes exogenously from external crises as outlined by Stiglitz. But it is also obvious that instability is favored by countries' own characteristics related to financial fragility and external vulnerability.

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A Proof of Proposition 3, 4 and their corollary

It is easy to check that $g_{1o} < g_{1a}$ if and only if

$$\Gamma - \frac{(\alpha_1 - r)^2}{\sigma_{1a}^2} < 0,$$

that is,

$$\frac{(\alpha_1 - r)^2}{\sigma_{1o}^2} + \frac{(\alpha_2 - r)^2}{\sigma_2^2} - \frac{2\xi(\alpha_1 - r)(\alpha_2 - r)}{\sigma_{1o}\sigma_2} < (1 - \xi^2)\frac{(\alpha_1 - r)^2}{\sigma_{1a}^2}.$$

To short notation, denote

$$x = \frac{\alpha_1 - r}{\sigma_{1a}}, \;\; y = \frac{\alpha_1 - r}{\sigma_{1o}}, \;\; A = \frac{\alpha_2 - r}{\sigma_2}.$$

Then the above inequality can be rewritten in form of second degree polynomial:

$$F(y) = y^2 - 2\xi A y + A^2 - (1 - \xi^2)x^2 < 0, \tag{12}$$

which have two roots:

$$y_1 = \xi A - \sqrt{(1 - \xi^2)(x^2 - A^2)}, \;\; y_2 = \xi A + \sqrt{(1 - \xi^2)(x^2 - A^2)}. \tag{13}$$

Thus, $F(y) < 0$ if and only if $y_1 < y < y_2$ with $y_{1,2}$ real. Otherwise, it yields $g_{1o} > g_{1a}$.

The two roots $y_{1,2}$ may be complex or real, if only real roots, there may be negative one(s). For $F(y) < 0$ making sense if and only if $y > 0$. In the following, we study case by case.

Case I. Existence of complex roots.

Since $\xi \in (-1, 1)$, the above roots, $y_{1,2}$, are real if and only if $x^2 - A^2 > 0$, that is,

$$\frac{\alpha_1 - r}{\sigma_{1a}} > \frac{\alpha_2 - r}{\sigma_2},$$

given our assumption that $\alpha_i > r$ (risky assets have higher return than risk-free asset).

So the first conclusion is that if initially

$$\frac{\alpha_1 - r}{\sigma_{1a}} < \frac{\alpha_2 - r}{\sigma_2},$$

we will always have $g_{1a} < g_{1o}$.

Case II. $y_{1,2}$ are real and $y_1 < y_2 < 0$.

If both roots of (12) are negative, we must have $\xi < 0$ and

$$\sqrt{1-\xi^2}\left(\frac{\alpha_1 - r}{\sigma_{1a}}\right) < \left(\frac{\alpha_2 - r}{\sigma_2}\right).$$

If so, for any $y > 0$, we would have $F(y) > 0$. Thus, we always have $g_{1o} > g_{1a}$.

Case III. $y_{1,2}$ are real and $y_1 < 0 < y_2$.

Then inequality (12) is true, that is $g_{1o} < g_{1a}$, if and only $0 < y < y_2$, given assumption $\alpha_i > r$. In other words, $y_1 < 0 < y_2$ if and only if

$$A^2 < (1-\xi^2)x^2.$$

Case IV. $y_{1,2}$ are real and $0 < y_1 < y_2$.

In this case, we must have $\xi \in (0,1)$ (otherwise, $y_1 < 0$) and, $y_1 > 0$ if and only if

$$\sqrt{1-\xi^2}\left(\frac{\alpha_1 - r}{\sigma_{1a}}\right) < \left(\frac{\alpha_2 - r}{\sigma_2}\right).$$

Then, with the above condition under autarky, $g_{1o} < g_{1a}$ if and only if,

$$y_1 < y < y_2.$$

However, it is straightforward to see that as long as $\sigma_{1o} > \sigma_{1a}$, we we have

$$y_1 < y.$$

Regrouping the case III and IV depending on $\xi \in (-1, 0]$ and $\xi \in (0, 1)$, we have the following: (1) $\forall \xi \in (-1, 0]$, $g_{1o} < g_{1a}$ if and only if

$$\frac{\alpha_2 - r}{\sigma_2} < \sqrt{1-\xi^2}\left(\frac{\alpha_1 - r}{\sigma_{1a}}\right) \quad \left(< \frac{\alpha_1 - r}{\sigma_{1a}}\right);$$

(2) $\forall \xi \in (0, 1)$, $g_{1o} < g_{1a}$ if and only if

$$\frac{\alpha_2 - r}{\sigma_2} < \frac{\alpha_1 - r}{\sigma_{1a}}.$$

The rest cases, where $F(y) > 0$ are the conditions in the corollary.

That finishes the proof.