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Environmental Tax Reform and Income Distribution with Imperfect Heterogeneous Labor Markets*

Diane AUBERT[†] and Mireille CHIROLEU-ASSOULINE[‡]

Abstract

This paper investigates the distributional and efficiency consequences of an environmental tax reform, when the revenue from the green tax is recycled by varying labor tax rates. We build a general equilibrium model with imperfect heterogeneous labor markets, pollution consumption externalities, and non-homothetic preferences (Stone-Geary utility). We show that in the case where the reform appears to be regressive, the gains from the double dividend can be made Pareto improving by using a redistributive non-linear income tax if redistribution is initially not too large. Moreover, the increase of progressivity acts on unemployment and can moderate the trade-off between equity and efficiency. We finally provide numerical illustrations for three European countries featuring different labor market behaviors. We show that a double dividend may be obtained without worsening the initial inequalities if the green tax revenues are redistributed with a progressivity index lower for France than for Germany and UK.

JEL classification: D62 - D63 - H23 - Q52 - Q58.

Keywords: Environmental tax reform - Heterogeneity - Unemployment - Welfare analysis - Tax progressivity

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1 Introduction

Can the use of carbon taxes still be advocated in a period characterized by secular stagnation, persistent unemployment and increased inequality? Environmental taxes are indeed often deemed strongly regressive, harming more the poor than the rich, which could easily be detrimental to growth and extend stagnation insofar as the propensity to consume decreases with income (Summers [2014]). Despite the growing - though unequally distributed - awareness of the need to reduce greenhouse gas emissions in order to limit the ongoing climate change, governments are most often reluctant to implement significant green tax shifts for fear of strong opposition from public opinion[1].

We take the opposite view of this wide-spread argument, arguing that seeking to implement green tax shifts, which do not widen inequalities may be an appropriate policy both to combat climate change, to reduce unemployment and to boost the economic growth and welfare.

Indeed, as with all indirect taxation, a green tax lower consumers' purchasing power, though this is somehow alleviated by the substitution effect that leads them to alter their consumption basket. This substitution effect is greater when the price elasticity of the demand for the polluting good is higher. As a consequence, the more efficient the green tax is in terms of environmental quality, the less purchasing power income is lowered ex post. Nevertheless, the combined effect on growth and employment is unambiguously negative when a green tax is implemented without any recycling of its tax revenues. But redistributing the revenues raised by the green taxes by reducing the rates of other distortionary taxes could yield a double dividend, namely an "environmental dividend" due to the improvement of the environment, and an "economic dividend", when the macroeconomic cost is equal to zero or is even negative; i.e. welfare increases (the *strong* double dividend case, as defined by Goulder [1995]).

The essential issue governments therefore face lies in finding a way for designing environmental fiscal reform with a distributive objective, without losing the efficiency advantage of environmental taxes. This paper intends to analyze this issue by investigating the distributional and efficiency consequences of a revenue-neutral environmental tax reform, when the revenue of a green tax is recycled via a variation of labor tax rates. We assess the usefulness of increasing the progressivity of labor taxes, in cases where intuitively there are doubts about the compatibility between environmental effectiveness, economic efficiency and intra-generational equity. Our work belongs to applied theory since we put forward explicit analytical conditions for the economic parameters for this compatibility. But we also calibrate our model and provide numerical illustrations assessing the empirical relevance of our results for three European countries.

To do so, we build a general equilibrium model with heterogeneous imperfect labor markets and

[1]Sweden is the only example of a country that implemented a significant green tax shift in the 1990s, as budget-neutral environmental tax reform.

pollution consumption externalities, in which we incorporate two assumptions that are supposedly detrimental to equity: firstly, heterogeneous households have a subsistence level of polluting goods and an elastic labor supply and secondly, there is unemployment. We remain in a second-best setting consistent with the literature on double dividend. To our knowledge, this paper is the first to address the issue of the fairness of environmental tax reforms by modeling heterogeneous labor markets. One of our key contributions is to combine unemployment and so the discrete labor supply of low-skilled workers, with the intensive supply of high-skilled workers (Keuschnigg and Ribi [2008], Saez [2002]). This links our paper to the income tax literature based on a discrete labor supply, without neglecting the detrimental impact of progressivity to incentives to work. This leads to a trade-off between equality and efficiency, as typically argued in public finance. Moreover, our specification of the labor market gives insight into the difference between high and low-skilled wage formations, which is crucial in the distributional properties of the green tax analysis. This distinction allows us to take into account a new channel through which, under conditions on the subsistence level of the dirty good, environmental taxes can be progressive: it acts through the wage formation and the difference of sensitivity between both employments. We assume that the production technology is a function of heterogeneous labor. Heterogeneous households only differ via their labor skill and earn wages corresponding to their skills. We assume structural unemployment in equilibrium caused by hiring costs, and we use a stationary search and matching model to formulate frictions on the labor market with individual worker-firm bargaining. By assumption, the risk of unemployment falls only on low-wage workers, whose labor supply is inelastic, whereas the high-skilled labor supply is endogenous. The tax system is initially composed of a progressive labor tax (tax on wages). As in Bovenberg and De Mooij [1994], pollution is due to the consumption of a polluting ("dirty") good. We characterize the necessary conditions for the obtainment of the environmental and welfare dividends when the revenue of the pollution tax is recycled by a change in the labor tax rates. The model is fully solved analytically as we have specified preferences and technologies in the simplest way.

Our first set of results points out the crucial role of the relative magnitudes of the elasticity of labor supply of high-skilled labor and of the sensitivity of low-skilled labor demand to the reservation wage. When the former is sufficiently high to ensure that the substitution effect between skills dominates the price effect on wages, an uncompensated increase in the green tax rate is unambiguously regressive, whatever the level of the subsistence consumption of thei polluting goods is. For lower values for the elasticity of the supply of high-skilled, the effect is ambiguous: the revenue effect (or "sources side effect"[2]) through wages and labor behavior, tends to be progressive, whereas the consumption effect (or the "uses side-effects") is always regressive. In countries where unemployment is above its optimal level, the regressivity of the tax is compounded. Even without any sectoral substitution, or any technological dif-

[2]We rely on the definitions of "the source/uses side-effect" given by Fullerton and Monti [2013]

ference between clean and dirty goods, the uncompensated tax may nevertheless appear as spontaneously progressive, once the high skilled labor supply is sufficiently low.

Our second set of analytical results concerns uniform revenue-neutral environmental tax reforms that redistribute the green tax revenues through the same decrease of the labor tax rate for both categories of workers. The existence of a subsistence level of consumption of polluting goods exacerbates the trade-off between the first and the second dividends, by decreasing the facility to substitute between clean and dirty goods. This compromises the first dividend but favors the second one, because the tax shift may be larger. As far as equity is concerned, this latter effect may reverse the distributive properties of the reform through the revenue effect. As a result, a progressive (resp. regressive) uncompensated environmental tax may lead to a regressive (resp. progressive) uniform revenue-neutral green tax shift out of. Contrary to previous studies, which argue that the double dividend hypothesis seems unrealistic, unless it is obtained through the worsening of inequalities, the main contribution of this paper is to show that, even in the case where a uniform revenue-neutral tax reform appears to be regressive, the use of a non-linear income tax affects unemployment and can moderate the trade-off between equity and efficiency. The gains from the double dividend can be made Pareto improving if redistribution is initially not too large, if and only if the wage elasticity of the high-skilled labor supply is not too high, such that the cost of progressivity in terms of high-skilled labor supply is more than compensated by the low-skilled employment effect. Using values of the core parameters given by the empirical literature, we show that inequalities will not be worsened, and that the double dividend is achieved, with a progressivity index of redistribution of the green tax revenues lower for France than for Germany or the United Kingdom.

The remainder of the paper is set out as follows. After a brief literature review in Section 2, we present our model in Section 3. In Section 4 we establish the conditions for regressivity of an uncompensated green tax, through some comparative statics, while in Section 5 we study two types of revenue-neutral environmental tax reforms and prove that it is possible to achieve simultaneously environmental, efficiency and distributional objectives. Section 6 provides some numerical illustrations of our results for the French case and comparisons with the German and British cases and Section 7 concludes.

2 Related Literature

Our work integrates the contributions of three different branches of economic literature: environmental economics, public finance and labor economics. The double dividend literature was initiated by Terkla [1984], Poterba [1993] and Parry [1995], who showed early on that redistributing the revenues raised by green taxes enables their overall macroeconomic cost to be reduced by lowering the loss of consumers' purchasing power and/or the costs to firms' production. The distribution channel depends on the targeted objectives.

According Baumol and Oates [1988], Pearce [1991] or Poterba [1993], such a reform aims at increasing the economic efficiency by reducing the distortions caused by some taxes. Pearce [1991] was thus the first one to suggest that recycling the green tax revenues by reducing the rates of other distortionary taxes could yield a double dividend, which was further precisely defined by Goulder [1995]. The welfare dividend (the second dividend) may correspond to an increase in growth, to an improvement of purchasing power, to a decrease of unemployment, etc. (Ekins [1997]). Parry [1995] establishes the general condition, which guarantees the existence of the second dividend: the revenue-recycling effect that reduces the existing tax distortions has to be greater than the tax-interaction effect, which increases the gross welfare cost of the green tax, through the erosion of the mutual tax base. But there is usually a trade-off between the first and the second dividend because the increase of output yields an increase in energy consumption, which counteracts the substitution effect induced by the green tax. Since Bovenberg and De Mooij [1994], a substantial theoretical and empirical literature has already emphasized the conditions favorable for the strong form of the double dividend: when there are several productive factors and/or several consumer groups, the double dividend can be obtained (Bovenberg and van der Ploeg [1996], Proost and van Regemorter [1995]). In particular, Goulder [1995] has shown that the existence of the double dividend essentially depends on the possibility of transferring the overall tax burden from the wage earners to some fixed production factors or to other consumers, thus emphasizing the role of heterogeneity. Heterogeneity of agents is above all a necessary but not sufficient condition for obtaining a double dividend. This highlights the existence of a trade-off between economic efficiency and equity.

Moreover, environmental taxes by themselves are usually considered as regressive taxes since they hurt the poorest households relatively harder, as these households are constrained to devote a larger share of their income to the consumption of polluting goods. These are often necessary goods, like energy products used for transportation or heating (Poterba [1991], Metcalf [1999], Wier et al. [2005], Ruiz and Trannoy [2008]). As these expenditures are constrained, and because poor households are not able to invest in less polluting vehicles or boilers, their substitution behavior is limited and they are unable to limit their loss of purchasing power.

This compatibility between equity and efficiency of an environmental tax reform was already addressed by Chiroleu-Assouline and Fodha [2011] and [2014], who were the first to take into account the intra-generational heterogeneity of agents, which previously had never been considered in depth in the double dividend literature. They use an overlapping-generations framework, that allows them to consider the inter-generational heterogeneity between workers and retired agents and they take into account different skills for workers, implying different levels of wage rates among households. Pollution is generated as a by-product of capital used in production (instead of being emitted by the consumption of a specific good or by the output, as in Bovenberg and De Mooij [1994]). They show that, even in very unfavorable circumstances, an adequately designed environmental tax reform can simultaneously lead to a decrease of pollution, an increase of the global welfare and a non-decreasing welfare for each class of household: whatever the degree of regressivity of the environmental tax alone, it is possible to re-design a recycling mechanism that renders the tax reform Pareto-improving. They propose to increase the progressivity of the labor tax together with a decrease of the lowest rate (and a lump-sum compensation to everybody who pays no income tax). However, they note that their results stem from the assumption of an inelastic labor supply and full employment.

In an optimal taxation framework (Mirrleesian-type partial equilibrium model), in which no strong double dividend can be obtained given its construction, Jacobs and de Mooij [2015] also acknowledge the need to focus on the income distributional consequences of environmental policies. By explicitly enabling the use of uniform lump sum transfers for distributional purposes, they show whether a *weak* double dividend[3] occurs, depending on the balance of the revenue-recycling and distributional effects. On the optimal tax system, the two exactly offset each other and finally the optimal environmental tax should not differ from the Pigouvian rate. Returning the revenues via income tax cuts would not be superior anymore as compared to lump-sum recycling, since further redistribution would be non-optimal. No weak double dividend occurs. Jacobs and van der Ploeg [2016] generalize this result in a model with non-homothetic preferences to capture the potential regressivity of green taxes (the poor spend a large part of their income on dirty goods). They find that the optimal carbon tax still amounts to the value of marginal climate damages (the Pigouvian tax) even though preferences are non-homothetic. Nevertheless, the optimal response could be a more progressive income tax to limit adverse income-distributional consequences. Both of these papers belong to an optimal taxation framework within the partial equilibrium model. But policy-makers may find it hard to implement an optimal income and pollution tax system. It is therefore relevant to investigate the question whether welfare-improving tax reforms exist which raise pollution taxes and reduce income taxes if the starting fiscal system is not necessarily optimized.

We intend to complement this limited stream of literature with a general equilibrium model, to obtain a bigger picture of the distributional and efficiency consequences of a revenue-neutral environmental tax reform. As in Jacobs and van der Ploeg [2016], we allow for non-homothetic (Stone-Geary) preferences that are compatible with the subsistence need for poor households to consume polluting goods (uses side effect). The general equilibrium framework also gives us the possibility to investigate the distributional impact of green tax reforms on the "income source side effect". Indeed, the standard analysis of environmental policies in partial equilibrium is questioned by some recent works, that prove that green taxes may be less regressive and even progressive when taking account of agents' entire life-cycle (Sterner [2012])

[3]As defined by Goulder [1995]: returning tax revenues through cuts in distortionary taxes leads to increase welfare relative to the case where tax revenues are distributed as lump sum transfers.

or of the general equilibrium effects on the factors prices (Dissou and Siddiqui [2014]). More specifically, our paper focuses on wage formation by allowing heterogeneous labor market and unemployment. As a result, our paper incorporates some features borrowed from the literature on labor market performance with progressivity tax, that uses either union-bargaining models (Koskela and Schöb [2009]) or search generated unemployment (Pissarides [1998], Hungerbühler et al. [2006] and Lehmann et al. [2013]). Hafstead and Williams [2016] incorporate a search model with friction as in Mortensen and Pissarides [1994], together with a simple two-sector general-equilibrium model of environmental policy, roughly calibrated to correspond to the effects of imposing a carbon policy in the United States. They adopt a dynamic framework that allows them to study long term effects, whereas we prefer to deal with a stationary model, which enables us to draw analytical results despite our assumption of heterogeneous labor market behavior. As typically argued in public finance, tax progressivity may be detrimental to incentives to work and so lead to an inevitable trade-off between equality and efficiency[4]. On the other hand, in a framework with a non-competitive labor market, the literature argues that a more progressive tax schedule remains beneficial for employment (Pissarides [1998], Hungerbühler et al. [2006], Lehmann et al. [2013], Sørensen [1999], Strand [2002], Piketty et al. [2014] and Cahuc-Carcillo-Le Barbanchon [2014]).

Lehmann et al. [2013], in a search and matching framework, support the unemployment-reducing effect of tax progressivity with two theoretical arguments. The first effect is a wage moderation effect, due to the bargaining between employers and workers and the local progressivity of the tax schedule. The second mechanism is a composition effect that occurs if the labor demand or the extensive margin of low-skilled labor is more sensitive to taxation as compared to high-skilled workers. In the latter case, progressive taxes contribute to the more efficient allocation of the total tax burden. The empirical literature suggests that the composition effect is key: the extensive margin elasticity is empirically higher for low-skilled (Røed et al. [1999] and Heckman [1993]) and the idea that low-paid employment is more responsive than high-paid employment is quite common in the literature (Kramarz and Phillipon [2001], Immervoll et al. [2007]).

As we combine unemployment and thus the discrete labor supply with the intensive supply of high-skilled workers (Keuschnigg and Ribi [2008], Saez [2002]), we refer to the income tax literature, and specifically to Saez [2002]. He has shown that the relative strength of the intensive and extensive responses is important in the design of optimal tax transfer schedules. If the extensive margin dominates at the low end of the income distribution, it can rationalize an earned income tax credit or wage subsidy.

Moreover, this specification of our labor market allows us to exhibit a new channel through which environmental taxes can be progressive: whereas for Dissou and Siddiqui [2013], the progressivity of the

[4]See Røed et al. [1999] and Diamond and Saez [2011] for a general discussion on this trade-off

environmental tax stems from the substitution of inputs, here it stems from the wage formation and the substitution of different categories of workers.

3 The Model

The model assumes a small open economy composed of firms, the government, and two different types of households, which differ with respect to their labor skills. There are a mass 1 of unskilled households (indexed with $i = L$) and a mass N of high-skilled households ($i = H$). We consider three types of commodities produced with the same technology: the polluting consumption commodity often called the "dirty" good (D), that harms the environment when consumed, the "clean" consumption commodity C and a fixed amount of clean public goods G. We assume their prices (before taxes) are fixed and normalized to unity because the rate of transformation is assumed to be constant[5] . Labor is assumed to be immobile internationally. Hence, wages are the only prices determined endogenously in the economy. We assume structural unemployment in equilibrium caused by hiring costs, and we use a search and matching model to formulate frictions on the labor market with individual worker-firm bargaining. In order to make the analysis as simple as possible, we adopt a stationary framework[6] . The government finances public goods G, provides fixed unemployment benefit payments to unemployed workers and imposes a tax t_i on the labor income of households of type i. Moreover we assume the government fights pollution externalities by imposing a green tax t_D on the consumption of the dirty good.

3.1 Households behavior

Our modeling of households is based on three main assumptions inspired by stylized facts: (i) low-skilled workers supply one unit of labor, whereas high-skilled labor supply is endogeneous (Roed et al. [1999]); (ii) the risk of unemployment falls on low-wage workers only (Keuschnigg and Ribi [2008], Koskela and Schöb [2009]); (iii) there is a subsistence level of consumption for the dirty good (like in Jacobs and van der Ploeg [2016]).

In this subsection, we describe and explain our specification choices.

[5] As observed by Bovenberg and De Mooij [1994], an alternative explanation of the fixed producer prices is possible: some commodities (for example the clean consumption commodity and the public consumption good) can be produced domestically while other goods (the dirty goods as fossil fuels) are imported at exogenous world-market prices.

[6] Diamond [1982] showed, that a static model still captures the major mechanisms of the dynamic version of the matching model and is able to describe "the essence of job search and recruiting externalities" . For other examples of stationary search and matching models, see Snower [1996], Sato [2004], Hungerbühler et al. [2006], Keuschnigg and Ribi [2008].

Consumption preferences

Although households differ with respect to their income and leisure time (which corresponds to their skill abilities and to their labor market activity), we assume that they are all identical in their consumer tastes. Agents are all risk neutral[7] and leisure is supposed to be weakly separable from consumption utility. This enables us to solve the model analytically. Moreover *"there is no strong empirical evidence suggesting that separability does not hold if we associate dirty consumption with the energy used by households*" (Bovenberg and de Mooij [1994]).

Clean goods and dirty goods are assumed to be imperfect substitutes in a composite commodity of quantity $Q = q(C,D)$. In contrast to the literature, we do not allow $q(C,D)$ to be linearly homogeneous in C and D. In fact, usual quasi-linear and homothetic preferences imply that the elasticity of substitution between clean and dirty goods is constant and thus independent of individual skill abilities. It results in constant expenditure shares of polluting goods. Hence, in most of the models, " *the income tax rate does not affect the allocation between clean and dirty goods, and the green tax on the dirty good is superfluous as a distributional device"* (Jacobs and van der Ploeg [2016]). However, poor people seem to devote a larger share of their consumption to dirty goods than do rich households (Ruiz and Trannoy [2008], Metcalf [1999]). An increase in carbon taxes is therefore likely more harmful for them. To render the trade-off between redistribution and the efficiency of green tax reforms more relevant, we follow Jacobs and van der Ploeg [2016], and assume Stone-Geary preferences captured by the following consumption utility function Q_i:

$$Q_i = q(C_i, D_i) = (C_i)^{1-\sigma}(D_i - \bar{D})^{\sigma} \tag{1}$$

where $\bar{D}$ denotes the subsistence level for the dirty good that is the same for all households. The Stone-Geary utility function makes it possible to model a share of consumption that is independant on price changes ($\bar{D}$) and another share that response instantaneously to price variations ($D_i - \bar{D}$). This specification allows us to represent dirty goods as necessities (their income elasticity is less than unity). Moreover, the dependence of these elasticities to skills captures the fact that the least able and poorest members of society are hurt most by a green tax (Deaton and Muellbauer [1980], Chung [1994], Jacobs and van der Ploeg [2016]).

As the environmental degradation acts as an externality, we assume households ignore the adverse effect of their demand for polluting goods on the quality of the environment. Consequently, households i choose C_i and D_i in order to maximize their utility subject to their budget constraint: $C_i + (1+t_D)D_i = I_i$ (with I_i denoting the income of households i). From the first order conditions of the maximization of (1), we

[7] The indirect utility function dependsthus positively and linearly on the income. This assumption places us in the worst case for the introduction of progressivity.

obtain the uncompensated demand for good D and that for good C, and the indirect utility of consumption:

$$D_i^* = \frac{\sigma}{1+t_D}\left[I_i - (1+t_D)\bar{D}\right] + \bar{D}; \qquad C = (1-\sigma)\left[I_i - (1+t_D)\bar{D}\right]; \qquad Q_i^* = \frac{\left[I_i - (1+t_D)\bar{D}\right]}{P_Q}$$

where $P_Q = \left[\frac{1}{1-\sigma}\right]^{1-\sigma}\left[\frac{(1+t_D)}{\sigma}\right]^{\sigma}$, and can be interpreted as the marginal price of consumption which is independant of individual abilities. Although, P_Q is equal to the inverse of the private marginal utility of income (i-e the Lagrange multiplier associated with the budget constraint of household), P_Q does not correspond to the implicit price of aggregated consumption . Because the Stone-Geary utility function is non homogeneous, the price index P_i depends on income and varies accross individuals ($P_i Q_i^* = I_i => P_i = \left(\frac{I_i}{I_i-(1+t_D)\bar{D}}\right) P_Q$). The marginal price of consumption P_Q is still constant as we constrain incomes to be sufficiently high to purchase the subsistence level of polluting good ($I_i > (1+t_D)\bar{D}$). [8]
Consumer first have to purchase the subsistence level of the polluting good that costs $(1+t_D)\bar{D}$. Then, they decide how to allocate their leftover income ($I_i - (1+t_D)\bar{D}$) between polluting and non polluting goods, according to their respective preference parameter (σ, $1-\sigma$) (similary to the case of classical Cobb-Douglas preferences). The assumption of households' risk neutrality implies that their indirect consumption utility (Q_i^*) is defined as the purchasing power of their leftover income.

Income and welfare

Concerning low-skilled workers, we consider a static framework of matching, so that the *ex ante* probability of being unemployed u, is equal to the *ex post* unemployment rate[9]. The indirect utility of the low-skilled workers, denoted V_L, is then defined as :

$$V_L = u * V_L^U + (1-u) * V_L^E$$

where V_L^U and V_L^E denote respectively the indirect utility of unemployed and employed low-skilled workers (hereafter, the relevant variables for the low-skilled workers are marked with the subscript E or U, depending on whether workers are employed or unemployed). Low-skilled workers supply one unit of labor at wage $w_L(1-t_L)$ net of tax, if employed. If unemployed, they receive a benefit B, given by some fixed nominal payment[10] and enjoy a utility of leisure Z (Pissarides [1998]). Assume further that the environmental externality enters the utility function linearly, the latter equation can be rewritten as :

8

9 For the dynamic framework, see Cahuc-Carcillo-Zylberberg [2014].

10 Since Koskela and Schöb [1999] already emphasized the impact of indexation of unemployment benefits on the efficiency of green tax shifts, we assume here that they are nominally fixed, which is their neutral case, with no employment effect of a revenue-neutral green tax shift.

$$V_L = u * [Q^*(B) + Z] + (1-u) * Q^*(w_L(1-t_L)) - \psi[D_{tot}] \tag{2}$$

where $-\psi[D_{tot}]$ denotes the disutility due to the environmental degradation caused by the aggregated consumption of the polluting good, $D_{tot} = D_L + D_H$.

On the other labor market, given an hourly wage net of tax $w_H(1-t_H)$, skilled workers supply variable labor H. High-skilled workers' welfare V_H is an increasing linear function of income I_H minus a given convex increasing effort cost $\varphi(H)$ and the disutility due to the environmental degradation:

$$V_H = \left(\max_H \left[Q^*(I_H) - \varphi(H)\right]\right) - \psi[D_{tot}] \qquad s.t.\, I_H = w_H(1-t_H) * H \tag{3}$$

Assuming an effort cost function of the form $\frac{H^{1+\frac{1}{\eta_H}}}{1+\frac{1}{\eta_H}}$ (where η_H reflects the Frish elasticity of labor supply to wage), we get a skilled labor supply of:

$$H^* = \left(\frac{w_H(1-t_H)}{P_Q}\right)^{\eta_H} \tag{4}$$

3.2 Firms' behavior

Technology

The production process applies to the production of the dirty good, and of the clean goods including the public one. As all goods are produced at constant and identical unit costs, we can normalize producer prices to unity. We assume a mass 1 of firms that produce output y with labor as the only variable input. Following Keuschnigg and Ribi [2008], the total technology is homothetic: a firm acquires labor services by high-skilled workers ($h = NH$) and low-skilled workers $l = L$, to produce raw value added:

$$y = f(l,h) = l^{\alpha} h^{1-\alpha} \quad with\, 0 < \alpha < 1. \tag{5}$$

f is linear homogeneous, the profit maximization $\pi(C) = \max_{l,h}[y - (lp_L + hp_H)]$ s.t. $y = f(l,h)$ gives $p_L l = \alpha y$ and $p_H h = (1-\alpha)y$: the Cobb-Douglas technology implies constant cost shares.
We use a stationary version of a simple search and matching model of labor market[11] to model unemployment among low-skilled workers. There are heterogeneities (or mismatches) in the labor market that make it costly for a low-skilled worker or a firm to find a partner with whom they can produce sufficiently

[11] See Diamond [1982], Snower [1996], Sato [2004], Hungerbühler et al. [2006], and Keuschnigg and Ribi [2008] for examples of stationary search and matching models.

high returns. "*Labor market heterogeneities are summarized in the matching function [...], that gives the rate at which good-quality matches are formed in the labor market*" (Pissarides [1998]). Given a mass 1 of job searchers, and the number of vacant jobs v, the matching function is defined as: $\mathscr{M} = m(v, 1)$, with positive first partial derivatives, negative second derivatives and constant returns to scale. According to the definition of Pissarides [1998], the matching function implies that a firm looking for a low-skilled worker finds one with probability less than one, equal to $\frac{\mathscr{M}}{v}$, even if there are enough jobs to satisfy all workers. Denoting $\theta = \frac{v}{1}$, the tightness ratio of the labor market, we can rewrite this probability as: $q(\theta) = \frac{\mathscr{M}}{v} = m(1, 1/\theta)$. It represents the Poisson matching probability of a vacant job: *i.e.* the rate at which vacant jobs are filled. Symmetrically, the rate at which an unemployed finds a job is given by $\theta q(\theta) = l = \mathscr{M}$. Then, for low-skilled workers, $\theta q(\theta)$ of workers are employed and $[1 - \theta q(\theta)]$ are unemployed. The standard Beveridge curve is defined as:

$$u = [1 - \theta q(\theta)] = 1 - l \tag{6}$$

Maintaining a vacancy costs c units of output. As in Piassaride [1998] and in Keuschnigg and Ribi [2008], we assume for simplicity "*one shot matching so that no other search opportunity is available*". The firm needs h units of skilled labor and l units of unskilled labor. Considering the wage bargaining, profits are given by:

$$\pi = \max_{h,v} [y - p_H h - w_L l - cv] \qquad s.t. \qquad l = v * q((\theta)) \ et \ y = f(l, h)$$

The traditional job creation and labor demand conditions are:

$$(f_L - w_L) * m = c \quad \Leftrightarrow \quad f_L = p_L = w_L + \frac{c}{m} \tag{7}$$

$$f_H = p_H = w_H \tag{8}$$

The skilled labor market is competitive: Equation (8) shows us that firms hire skilled labor until marginal productivity is equal to the wage. On the other hand, "*the total cost of an unskilled worker exceeds the wage by a recruitment cost equal to the search cost multiplied by the number of vacancies needed for a successful hire*" (Keuschnigg and Ribi [2008]). Equation (7) represents the traditional "job creation condition": the marginal cost of investing in a job vacancy (c) equal to the expected job rent $((f_L - w_L) * m)$. In fine the expected profit is zero in this model because of our assumption of constant return to scale.

Low-skilled wage determination

Once a suitably low-skilled worker is found, a job rent appears that corresponds to the sum of the expected search and hiring costs for the firm and the worker. The wage needs to share this economic (local-monopoly) rent, in addition to compensating both parties for its assets from forming the job. As in the original model of Pissarides [1998], we assume a decentralized Nash bargain, which imposes a particular splitting of the matching surplus between the two parties involved according to the relative bargaining power between them. For a low-skilled worker, the matching surplus is the difference between the expected utility when employed and that when unemployed: $Q^*((w_L(1-t_L)) - [Q^*(B)+Z]$. For a firm, the matching surplus is the difference between the profit when it fills a vacancy and when the firm remains with a job vacant: $(f_L - w_L - c) - c = f_L - w_L$.
w_L is determined as: $w_L = argmax\left\{\left(Q^*\left(I_L^E\right) - \left[Q^*(I_L^U)+Z\right]\right)^{\beta}(f_L - w_L)^{1-\beta}\right\}$ with β the low-skilled worker's bargaining power.
In the remain of the paper, we denote w_R the reservation wage, for which a household is indifferent between being employed or unemployed $\left(Q^E(w_R(1-t_L)) = Q^U(B)+Z\right)$, and M the wage mark-up, defined as the difference between the wage w_L and the reservation wage. Appendix A.I shows that the first-order condition for the maximization of the Nash product implies the following expression of the wage mark-up M:

$$M = \frac{P_Q\left(Q^*\left(I_L^E\right) - Q^*(I_L^U) - Z\right)}{(1-t_L)} = \frac{\beta}{1-\beta} * \left[\frac{c}{q(\theta)}\right] = \frac{\beta}{1-\beta}(p_L - w_L) \tag{9}$$

If hiring costs are zero ($c=0$), in equilibrium $M=0$. Thus, positive hiring costs increase the gap between the utility of employment and that of unemployment. Similarly, a drop in job vacancies (or θ) decreases the expected value of the firm's hiring costs ($\frac{c}{q(\theta)}$). This reduces the rents from the job match and also decreases the wage mark-up. Another formulation of equation (9) (see in Appendix A.I), can be given by:

$$w_L = w_R + \beta(p_L - w_R) \tag{10}$$

Workers receive their reservation wage w_R and a fraction β (union' s bargaining power) of the net surplus that they create by accepting the job (Pissarides [1998]) . In this model, $w_R = \frac{B+P_QZ}{1-t_L}$. If the bargaining power of the low-skilled worker equals one (*i.e.* $\beta = 1$), then the low-skilled wage equals the productivity of low-skilled labor (similarly to in a competitive labor market). Labor demand does not depend at all on hiring costs. By contrast, when the firm alone has the bargaining power ($\beta = 0$), the wage just equals the reservation wage.

3.3 The government budget constraint

To finance unemployment benefit payment and a fixed amount of some public goods G, the government levies a tax t_i on wages i and a green tax t_D on the consumption of the dirty good D. The burden of the labor income tax depends on the nominal wage rate w_i, and the income tax rate t_i, which is assumed to be progressive: $t_H \geqslant t_L$. The government budget constraint can then be written as:

$$G + (1-l)B = lw_Lt_L + NHw_Ht_H + t_DD_{tot} \qquad (11)$$

4 An uncompensated Green Tax: Comparative Statics

In this section, we analyze how wages, employment and finally welfare react to exogenous infinitesimal increases of taxes dt_D, dt_L and dt_H. For the moment, we do not constrain the government to balance its spending with its revenues. In fact, in this section everything is considered as if the additional revenue raised by the increase of green/labor taxes is saved, or sterilized, *i.e.* we do not take the government budget constraint (11) into account.

We thus define the partial equilibrium of the model as a tuple (l^*, H^*, θ^*, w_L^*, w_H^*, f_l^*, y^*) that satisfies the following conditions: the job creation condition (7), the wage mark up equation (10), the Beveridge curve (6), the high-skilled labor supply (4), firms' labor demands (7-8) and finally, the production function (5). Due to the properties of the matching function, it is not possible here to solve explicitly the equilibrium of this economy in levels. We thus examine the local behavior of a small open economy around the initial equilibrium: the model is log-linearized. Consequently, in the following, percentage changes relative to initial values are denoted with a tilde ($\sim$), *i.e.* $\widetilde{l} = \frac{dl}{l}$. Exceptions to this definition are separately indicated.

4.1 Revenue and employment effects

The low-skilled labor market

On the low-skilled labor market, the supply side links the employment rate l to market tightness θ. The log-linearization of the matching function and the Beveridge curve mentioned in (6) implies:

$$\tilde{l} = (1-\xi) * \tilde{\theta} \;\; (6bis), \qquad where\; 0 < \xi = -\frac{\partial q(\theta)}{q(\theta)} * \frac{\theta}{\partial \theta} < 1$$

A raise of the tightness θ increasess the probability for workers to find a job and reduces the rate $q(\theta)$ with which firms are able to fill vacancies (Keuschnigg and Ribi [2008]). Thus, employment l rises with labor market tightness whereas the unemployment rate u falls.

In this model, the wage-mark-up equation (9 or 10) can be seen as the replacement of the unskilled labor supply curve of the Walrasian model. The low-skilled labor supply is fixed and each low-skillled employee works a fixed number of hours. But the existence of a local monopoly power and the sharing rule used to solve for wage imply that even with fixed labor productivity and supply, there is an upward sloping relation between θ and w_L (Pissarides [1998]). Log linearizing the bargaining condition (10) and using (6 bis), leads to a wage response (Appendix A. II):

$$\widetilde{w_L} = \frac{M}{w_L}\xi\,\tilde{\theta} + \left[\frac{w_R}{w_L}\right]\tilde{w_R} \qquad (9\,bis)$$

where we denote $\tilde{w_R} = \left[\frac{dt_L}{1-t_L} + \frac{P_Q Z}{w_R(1-t_L)}\frac{\sigma}{1+t_D}dt_D\right]$, the variation of the reservation wage with the tax rate variations dt_D and dt_L. The increase of environmental taxes has a first direct effect on the marginal price of consumption : P_Q becomes higher. Thus, the utility of leisure in monetary terms $P_Q * Z$ increases. As a direct consequence, the reservation wage ($w_R = \frac{B+P_Q Z}{1-t_L}$) increases as well. Similarly to green taxes, labor taxes make the outside option more attractive for unemployed workers. "*Since workers bargaining strength assures a strictly positive job surplus, any policy raising the reservation wage shifts costs on to firms and inflates gross wages*" (Keuschnigg and Ribi [2008])[12].
On the low-skilled labor demand side, given a wage increase, the market tightness moves in order to satisfy the job creation condition:

$$\tilde{p_L} = \frac{w_L}{p_L} * \tilde{w_L} + \frac{c}{q(\theta)}\frac{\xi}{p_L} * \widetilde{\theta} \quad (7.1\,bis)$$

The job creation condition is a downward-sloping curve in (w_L, θ). The inverse relationship between wage costs and labor market tightness can be explained as follows. An increase in wage costs reduces the expected benefits of an occupied job, and by increasing hiring costs raises the cost of filling a vacant job. The market tightness is such that the expected profit from a new job is equal to the expected cost of hiring. Then, to re-establish arbitrage between the costs and benefits of hiring new workers, labor market tightness has to decrease in response to an increase of wage (Pissarides [1998]).

Abstracting from the substitution effect between low and high-skilled labor, partial equilibrium in the low-skilled labor market is determined by equations (6 bis), (9 bis) and (7.1 bis). We can therefore easily determine impacts of taxes on the partial equilibrium of the low-skilled labor market, through these three

[12] As noticed in Keuschnigg and Ribi [2008], tax shifting is weakened when benefits are indexed to net wages. Some tax shifting will occur as long as indexation of benefits is not complete.

equations. The uncompensated increase of environmental taxes leads to a *price effec*t: P_Q increases, which raises the reservation wage. Workers claim a higher wage, since the costs of unemployment are lower. The wage curve (9 bis) shifts upward. Hence, the low-skilled labor market becomes less tight, and firms create fewer jobs (the decrease of the job creation curve (7 bis). Unemployment increases. The same reasoning applies for low-skilled labor taxes.

Let us now recall that the matching model used here features two central market failures: congestion externalities and appropriability problems.The congestion externalities are as follows: workers do not internalize the fact that if they look for a job, they create new jobs at a lower rate than their own probability of finding a job. This externality leads to too much worker search: i.e. too much unemployment. The appropriability problems come from the process of wage bargaining, when workers and firms are engaged in a process to share the surplus of accepting a job. Workers only appropriate a fraction of the private value of the jobs they get. Hence the value of looking for a job (*i.e.* the opportunity cost of working) is underestimated. This appropriability problem leads to insufficient job search by workers and so too little unemployment. The Hosios Condition defines the 'optimality' condition of the low-skilled employment equilibrium, as the condition for which the appropriability and congestion problems exactly balance each other (Hosios [1990]). With our assumption of constant returns to scale on the matching function, the Hosios condition is satisfied if the workers' share in the surplus of a match (β) is equal to the elasticity of the matching function (ξ). If $\beta > \xi$, there is too much unemployment: the appropriability problem dominates the congestion externality on the firms' side, and conversely on the workers' side. Both effects push unemployment above the optimum. If $\xi > \beta$, the unemployment rate is below the optimum.

The high-skilled labor market

On the high-skilled labor market, that is assumed to be perfect, variations of taxes act through labor supply. The log-linearization of (4) gives us:

$$\tilde{h} = \eta_H * [\tilde{p_H} - \frac{\sigma}{1+t_D} * dt_D - \frac{1}{1+t_H} * dt_H] \quad (4\,bis)$$

The increase of P_Q due to a rise of green taxes decreases the real after-tax wage $\frac{w_H(1-t_H)}{P_Q}$, which discourages high-skilled workers to supply labor. Taxes on high-skilled labor have a similar effect. Thus, when abstracting the substitution effects between skills, labor and green taxes act through the supply side (the reservation wage for low-skilled labor and the labor supply for high-skilled workers), and lead both to an unambiguous decrease of labor.

Interaction between both labor markets

In order to understand the substitution effect, firms' labor aggregate demands have to be considered:

$$\tilde{l} = \tilde{y} - \widetilde{p_L}\,(7.2\,bis)\ , \quad \tilde{h} = \tilde{y} - \widetilde{p_H}\,(8\,bis)$$

Replacing them in the production function yields:

$$\widetilde{p_H} = -\frac{\alpha}{1-\alpha}\widetilde{p_L} \quad (5\,bis)$$

This can be seen as a *substitution effect*. Skilled and unskilled labor can be substitutes, and the actual substitution depends on the change in the relative prices, that is the ratio of labor productivities. If the high-skilled labor supply and outside option reactions to the environmental tax were exactly the same, then productivities would remain unchanged and labor would decrease with the same proportion for high and low-skilled workers. If it is not the case, firms have the option to substitute inputs, which impacts labor productivities, and then labor and wages.

Partial Equilibrium

By combining the Beveridge curve (6 bis), the wage mark-up equation (9 bis) and the demand for low-skilled labor (7 bis), we obtain low-skilled labor as a function of productivities and reservation wage.

$$\tilde{l} = \frac{1-\xi}{\xi}\left[\frac{p_L\widetilde{p_L} - w_R\widetilde{w_R}}{p_L - w_R}\right] = \eta_L\widetilde{p_L} - \eta_R\widetilde{w_R} = \tilde{y} - \widetilde{p_L}$$

with $\eta_L = \frac{1-\xi}{\xi}\left[\frac{p_L}{p_L - w_R}\right]$, and $\eta_R = \eta_L\frac{w_R}{p_L}$. Clearly, the labor productivity has to balance the labor "supply" and the labor demand. We can rewrite the previous equation in the following way: $\widetilde{p_L}\,(1+\eta_L) = \tilde{y} + \eta_R\widetilde{w_R}$.

Similarly, high-skilled labor equilibrium yields: $\widetilde{p_H}\,(1+\eta_H) - \eta_H d\tau_H = \tilde{y}$, where $d\tau_H = \frac{\sigma}{1+t_D} * dt_D + \frac{1}{1+t_H} * dt_H$ is the direct impact of taxes on real after tax wages. By replacing $\tilde{y}$ and $\tilde{h}$ with (8 bis) and (4 bis) and using (5 bis), we finally obtain:

$$\widetilde{p_L} = \frac{(1-\alpha)}{1+(1-\alpha)\,\eta_L + \alpha\eta_H}\left[\eta_R\widetilde{w_R} - \eta_H d\tau_H\right]$$

If the labor supply of high-skilled is more elastic than the low-skilled sensitivity to outside option ($\eta_H d\tau_H > \eta_R\widetilde{w_R}$), production becomes more intensive in low-skilled labor, and productivity of high-skilled labor increases compared to low-skilled labor. This *substitution effect* depends on the difference between high labor supply and outside option reactions to taxes. It impacts in an ambiguous way on labor productivities and then wages. Tables 1 and 2 in Appendix A.II contain respectively the log-linearized model, and the solutions of the system.

Proposition 1: *A higher uncompensated green tax raises unemployment and decreases high-skilled labor and production if and only if utilities depend on leisure. If this is not the case, green taxes leave employment unaffected.*

Proposition 2: *An uncompensated increase in the green tax rate has an ambiguous effect on wages, which depends on the magnitude of the elasticity of labor supply of high-skilled η_H compared to the sensitivity of low-skilled labor demand to reservation wage:*

- *If $\frac{\eta_H}{\eta_R} < p_1$, a higher uncompensated green tax increases p_L and w_L but decreases w_H*
- *If $\frac{\eta_H}{\eta_R} \in [p_1; \Gamma p_1]$, a higher uncompensated green tax decreases p_L, but increases w_H and w_L.*
- *If $\frac{\eta_H}{\eta_R} > \Gamma p_1$, a higher uncompensated green tax decreases w_L and p_L but increases w_H.*

where: $p_1 = \frac{P_Q Z}{P_Q Z + B}$ *and* $\Gamma = \frac{\eta_L(1-\alpha)+\beta(1-\beta)}{\eta_L(1-\alpha)} > 1$.

See the proof in Appendix A.III. The threshold p_1 increases with the utility of leisure Z and the overall price P_Q (hence with σ *and* τ_D) and decreases when B increases, while Γ is maximized for $\beta = 1/2$. Three kinds of situations may arise depending on the respective magnitudes of the price effect and the substitution effect: in the first one, when the ratio of the two elasticities η_H/η_R is low, the labor supply of high-skilled workers is so high compared to the sensitivity to the outside option that the productivity of the high-skilled labor decreases while the productivity of low-skilled labor increases (*substitution effect*). Low-skilled wage increases (*substitution and price effect*) whereas high-skilled wage decreases. The reverse situation is represented by the third case above, when η_H/η_R is very high: the high-skilled wage increases and low-skilled wage decreases. The intermediate case corresponds to a situation where the substitution effect induces a decrease of low-skilled productivity but not enough to overcome the price effect on the reservation wage. As a result both wages increase.

4.2 Welfare analysis

We measure the welfare effects of small tax changes with the marginal *excess burden*. It corresponds to the additional income that needs to be provided to the representative household in each category, in order to keep its utility at its initial level. This is the compensatory income variation, denoted by dR. It stands for the excess welfare loss of the consumers over and above the tax revenues collected by the government, and it can be interpreted as the hidden costs of financing public spending: a positive value for the marginal excess burden indicates a loss in welfare after the tax reform. Let us determine the compensatory income variation which, after the tax reform, would leave the welfare unchanged (dV_H, $dV_L = 0$).

$$dV_H = 0 \Leftrightarrow \frac{\partial V_H}{\partial C_H} dC_H + \frac{\partial V_H}{\partial D_H} dD_H + \frac{\partial V_H}{\partial H} dH + \frac{\partial V_H}{\partial D_{tot}} dD_{tot} = 0$$

Using the first order conditions of consumer's program, this leads to:

$$dV_H = 0 \Leftrightarrow \frac{dC_H}{P_Q} + \frac{(1+t_D)}{P_Q} dD_H + \frac{w_H(1-t_H)}{P_Q} dH + \frac{\partial V_H}{\partial D_{tot}} dD_{tot} = 0$$

By differentiating the budget constraint of high-killed households and using the definition of the compensatory income variation we obtain:

$$dC_H + (1+t_D)dD_H + D_H dt_D = (1-t_H)w_H dH + H(1-t_H)dw_H - Hw_H dt_H + dR_H$$

Combining the last two equations gives us:

$$dR_H = -\underbrace{\left[\underbrace{I_H \left(\widetilde{w_H} - \frac{dt_H}{1-t_H} \right)}_{\Delta_H^w} - \underbrace{D_H dt_D}_{\Delta_H^D} \right]}_{\Delta_H^{PP}} - \underbrace{\frac{\partial V_H}{\partial D_{tot}} P_Q dD_{tot}}_{\Delta_H^e} \tag{12}$$

The compensatory income variation is the variation of the revenue needed to overcome the purchasing power variation Δ_H^{PP}, which is equal to the sum of the variation of the after tax wage Δ_H^w and the increase of price of the polluting good Δ_H^D, and the deterioration of environmental quality Δ_H^e. Similarly, in Appendix A.IV we derive the compensatory variation for the low-skilled workers:

$$dR_L = -\underbrace{\left[\underbrace{l I_L^E \left(\widetilde{w_L} - \frac{dt_L}{1-t_L} \right)}_{\Delta_L^w} + \underbrace{P_Q \left(V_L^E - V_L^U \right) dl}_{\Delta_L^E} - \underbrace{D_L dt_D}_{\Delta_L^D} \right]}_{\Delta_L^{PP}} - \underbrace{\frac{\partial V_L}{\partial D_{tot}} P_Q dD_{tot}}_{\Delta_L^e} \tag{13}$$

where similar notations are used and Δ_L^{PP} is the purchasing power effect for the low-skilled (incorporating $I_L^E = w_L(1-t_L)$ the revenue of the unskilled worker when employed). Due to imperfections in the labor market of low-skilled workers, another term is added in the compensatory variation of the unskilled worker: the employment effect Δ_L^E, that is proportional to the difference between utility of employed (V_L^E) and unemployed worker (V_L^U). Another formulation of equation (9) (see in Appendix A.IV), can be given by:

$$dR_L = -\underbrace{\left[\underbrace{l\,I_L^E\left(\frac{\beta}{\xi}\frac{p_L}{w_L}\widetilde{p_L} - \frac{dt_L}{1-t_L}\right)}_{\Delta_L^w} + \underbrace{lI_L^E\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\widetilde{w_R}}_{\Delta_L^E} - \underbrace{D_L dt_D}_{\Delta_L^D}\right]}_{\Delta_L^{PP}} - \underbrace{\frac{\partial V_L}{\partial D_{tot}}P_Q dD_{tot}}_{\Delta_L^e} \tag{14}$$

This last equation gives us the opportunity to interpret the compensatory variation of low-skilled according to the Hosios condition.

In the case where $\beta = \xi$, then $dR_L = -\left[\underbrace{I_L^E\left(\frac{p_L}{w_L}\widetilde{p_L} - \frac{dt_L}{1-t_L}\right)}_{\Delta_L^w} - \underbrace{D_L dt_D}_{\Delta_L^D}\right] - \underbrace{\frac{\partial V_L}{\partial D_{tot}}P_Q dD_{tot}}_{\Delta_L^e}$. The employment effect disappears entirely. As for the high-skilled labor, the compensatory income variation would be only driven by the increase of the productivity p_L, the labor tax rate t_L, the increase of price of the polluting good Δ_L^D, and the deterioration of environmental quality Δ_L^e. Compensatory income variations of high- and low-skilled labor are really similar in this case, and the main difference between them is the sign of the variation of the productivity induced by the substitution effect.

If $\beta > \xi$, unemployment is higher than the optimal level. In this case, an increase of the reservation wage (due to an increase or environmental taxes or labor tax), pushes wages to increase and again creates more unemployment. The situation deteriorates and it increases the compensatory income variation of low-skilled workers. In the opposite case, if $\beta < \xi$, the unemployment rate lies below the optimum. An increase of the reservation wage would push wages to increase and create more unemployment. Regarding the externalities side, the labor market would get closer from the optimum. The compensatory income variation of low-skilled labor would decrease.

As in Bovenberg and de Mooij [1994], it is possible here to distinguish an environmental component and a non-environmental one: in expression (12) and (14), the marginal excess burden appears as the sum of an effect on the loss of purchasing power Δ^{PP} and an environmental effect Δ^e. Moreover, as we do not allow for heterogeneous valuations of damages from pollution in this paper, the regressivity of environmental taxes only appears through the loss of purchasing power, proportionally to the income of the agents. These specifications allow us to define the regressivity only in terms of purchasing power, abstracting from the environmental component, as is specified in the following Definition.

Definition: *A green tax is called regressive (resp. progressive), if and only if the relative loss of purchasing power caused by an increase of the tax rate is lower (higher) for high-skilled than for low-skilled workers.*

$$\frac{\Delta_H^{PP}}{I_H dt_D} = -\left[\underbrace{\frac{dw_H}{w_H dt_D}}_{\Delta_H^w} - \underbrace{\frac{D_H}{I_H}}_{\Delta_H^D}\right] \lesseqgtr \frac{\Delta_L^{PP}}{I_L dt_D} = -\left[\underbrace{\frac{I_L^E}{I_L}\left(\frac{\beta}{\xi}\frac{p_L}{w_L}\frac{dp_L}{p_L dt_D}\right)}_{\Delta_L^w} + \underbrace{\frac{I_L^E\left[\frac{\xi-\beta}{\xi}\right]\frac{1}{w_L}\frac{dw_R}{dt_D}}{I_L}}_{\Delta_L^E} - \underbrace{\frac{D_L}{I_L}}_{\Delta_L^D}\right]$$

Proposition 3: *An uncompensated increase in the green tax rate may be regressive or progressive, depending on the magnitude of the elasticity of labor supply of high-skilled η_H, and of the subsistence level of polluting consumption $\bar{D}$:*

- *If $\eta_H < p_1 f(\eta_R)$, a higher uncompensated green tax is unambiguously progressive when $\bar{D} = 0$ but becomes regressive for high $\bar{D} > 0$.*
- *If $\eta_H > p_1 f(\eta_R)$, a higher uncompensated green is regressive whatever $\bar{D}$.*

with $f(\eta_R) = \left[\frac{\left(\frac{I_L^E l}{I_L}\left((1-\alpha)\frac{\eta_R(1-\xi)\eta_R}{(1-\xi)\eta_R-(1-\beta)\xi}\right)+\frac{I_L^E l}{I_L}\left(\frac{(\xi-\beta)\eta_R}{\xi\eta_R+\beta(1-\xi)}\right)+\alpha\right)}{\left[\left(\alpha\left(1-\frac{I_L^E l}{I_L}\left(\frac{(\xi-\beta)\eta_R}{\xi\eta_R+\beta(1-\xi)}\right)\frac{P_Q Z}{P_Q Z+B}\right)+\frac{I_L^E}{I_L}\frac{\beta}{\xi}(1-\alpha)\frac{(1-\xi)\eta_R}{(1-\xi)\eta_R-(1-\beta)\xi}\right)\right]}\right]$*, increasing function of* η_R*.*

See Appendix A.V for the proof. We show moreover, that a sufficient condition for regressivity is given by $\frac{\eta_H}{\eta_R} > \Gamma\frac{P_Q Z}{P_Q Z+B}$, that is the same condition for a decrease of the wage of the low-skilled-workers, *i.e.* if the substitution effect between skills dominates the price effect of the green tax. If it is not the case, situation is ambiguous. The *revenue effect* (or the sources side effect) through wages and labor behavior, tends to be progressive: the reaction of labor-unions to environmental taxes is high compared to labor supply of high-skilled workers and leads to an increase of unskilled wages. But, due to the properties of the utility function (that is non homothetic), we have $\frac{D_L}{I_L} - \frac{D_H}{I_H} = (1-\sigma)\bar{D}\left(\frac{1}{I_L} - \frac{1}{I_H}\right) > 0$. The *consumption effect*, or the uses side effects, is therefore always regressive. Then, if $\bar{D}$ is really high, then consumption effect is always more important than the revenue effect: in this case, green taxes are always regressive.

Under the Hosios condition ($\beta = \xi$), one obtains $f(\eta_R) = 1$: the reform is unambiguously regressive if it increases the wage of the high-skilled workers. But, when $\beta > \xi$, the existence of too much unemployment and its increase because of the uncompensated increase in the green tax rate enhances the regressivity of the reform..

5 The revenue-neutral environmental tax reform

We can now analyze a revenue-neutral green tax reform that increases the tax on the dirty good and reduces the income tax correspondingly. In this section, we first characterize a uniform revenue-neutral reform and the necessary conditions for the obtainment of the environmental and welfare dividends. Distributional properties of the reform are then analyzed. We finally discuss the implication of a reform taking the redistributive objective into account, using the progressivity of labor tax schemes.

5.1 The specification of a uniform revenue-neutral tax reform

We focus on balanced tax reforms: the government keeps the amount of government spending ex post unchanged, and the tax policy has to maintain the amount of the tax revenues constant. We can describe a revenue-neutral green tax reform with the following expression:

$$\frac{dG}{GY} = G^*_{t_D} dt_D + G^*_{t_H} dt_H + G^*_{t_L} dt_L = 0$$

where $G^*_{t_D}$ denotes the marginal tax revenues from the tax on dirty good and $G^*_{t_H}$ and $G^*_{t_L}$ the marginal revenues from the low and high-skilled income taxes t_L and t_H.

Although we will relax this assumption later, we first assume that $dt_H = dt_L = dt$. This means that the increase of the environmental tax rate is accompanied by a homogeneous variation of all labor tax rates. In other words, for the moment, we do not allow the government to recycle the revenue with a distributive objective. Moreover, we assume that we are on the Laffer-efficient side of the three tax revenues, *i.e.* $G^*_{t_H} > 0, \quad G^*_{t_L} > 0 \quad G^*_{t_D} > 0$. In this case, we can express dt by: $dt = -\frac{G^*_{t_d}}{G^*_{t_H}+G^*_{t_L}} dt_D < 0$

Some algebra in Appendix A.VI shows that a uniform revenue-neutral tax reform is defined as follows:

$$-\frac{dt}{dt_D} = \left[\frac{\overbrace{D_{tot}}^{value\ effect} + \overbrace{B\frac{\partial L}{\partial t_D}}^{benefit\ effect} + \overbrace{\frac{t_D}{1+t_D}E_s}^{substitution\ effect} + \overbrace{\frac{\sigma t_D}{1+t_D}\frac{\partial I_{tot}}{\partial t_D} + \sum_{i=l,h} i w_i t_i \left(\frac{\partial w_i}{w_i \partial t_D} + \frac{\partial i}{i \partial t_D}\right)}^{revenue\ effect}}{\underbrace{(w_L l + w_H h)}_{value\ effect} + \underbrace{B\left(\frac{\partial L}{\partial t_L} + \frac{\partial L}{\partial t_H}\right)}_{benefit\ effect} + \underbrace{\sum_{j=l,h}\left(\frac{\sigma t_D}{1+t_D}\frac{\partial I_{tot}}{\partial t_j} + \sum_{i=l,h} i w_i t_i \left(\frac{\partial w_i}{w_i \partial t_j} + \frac{\partial i}{i \partial t_j}\right)\right)}_{revenue\ effect}} \right]$$

The numerator measures the effect of the change in the pollution tax rate on the share of its revenue which can be recycled (after the unemployment benefits have been paid). We can distinguish both *a value effect* (the tax revenue increases with the pollution tax rate, for unchanged consumption D_{tot}) and a *tax base effect* made up of three components. This tax base effect represents the possible erosion of the

tax base: the increase of the pollution tax induces agents to substitute between clean and polluting goods (*substitution effect*), and impacts also on wages and labor. Low-skilled labor decreases (Proposition 1) which leads to an increase in the amount of unemployment benefits provided by the government (*benefit effect*). Changes in wages and labor do not leave the total income of agents ($I_{tot} = I_L + NI_H$) unaffected. They lead to a decrease of the total consumption, and of D_{tot} according to its proportion into the total unconstrained consumption ($\frac{\sigma t_D}{1+t_D}$) (*revenue effect*).
The denominator measures the effect of the change in labor tax rates on their revenue. The *value effect* is proportional to labor revenues: the tax revenues increase with the tax rates, for unchanged labor incomes. The *tax base effect* works in the opposite way: the total revenue decreases (*revenue effect*) and a decrease of low-skilled labor implies an increase of unemployment benefits (*benefit effect*).
Note that the absolute variation of labor tax rate $\mid dt \mid$ is an increasing function with respect to the subsistence level of the polluting good $\bar{D}$. It acts through the numerator. Augmenting $\bar{D}$ increases the value effect by increasing D_{tot}, but also minimizes the substitution effect and so the erosion of the tax base.

5.2 Efficiency issue: the environmental and welfare dividends

We refer in this paper to Goulder's definition of the *strong* double dividend: an environmental dividend (*i.e.* a decrease in the consumption of the polluting good) together with a non environmental welfare dividend measured by an increase of the purchasing power as represented by a negative marginal excess burden. From equations for the marginal excess burden (12 and 13), and by considering an egalitarian criterion where the marginal excess burden of the economy is $dR_{tot} = dR_L + NdR_H$, conditions for btaining the double dividend are given by:

- $\Delta^{PP}_{tot} = -\left[\overbrace{\sum_{i=l,h}\left(iI_i[\tilde{w}_i - \frac{\frac{dt}{dt_D}}{1-t_i}]\right)}^{after-tax-wage-effect} + \overbrace{P_Q\left(V_L^E - V_L^U\right)\tilde{l}}^{employment\ effect} - \overbrace{D_{tot}}^{consumption}\right] < 0$
- $\Delta^{e}_{tot} = \left(-\frac{\partial\psi(D_{tot})}{\partial D_{tot}}\frac{dD_{tot}}{dt_D}\right) < 0$

Proposition 4: *A uniform revenue-neutral tax reform that increases the green tax rate t_D and decreases equally both the labor tax rates dt_i, can lead to a strong double dividend if and only if:*

$$D_{tot}dt_D + P_Q ZdL + \varphi'(H^*)NdH < dI_{tot} < (\frac{I_{tot}}{1+t_D})dt_D$$

where dI_{tot} is the variation of total revenue of the agents.

See proof in Appendix A.VII.
Proposition 4 is very intuitive: on the one hand, the positive impact on the total revenue of the agents has to be not too high in order for the polluting consumption D_i to remain decreasing (the right hand side of the inequality). On the other hand, the reform has to compensate agents for their loss on consumption D_{tot} and their disutility of work (left hand side of the inequality).
Our results here do not differ from those of the traditional literature on the double dividend conditions, except with the subsistence level in the dirty good. The introduction of $\bar{D}$ exacerbates the trade-off between the environmental and the non-environmental dividend. On the one hand, by decreasing the facility for substituting between clean and dirty goods, $\bar{D}$ makes the reform less efficient for achieving a better-quality environment. On the other hand, because of the low level of the substitution effect, the erosion of the tax base of the green tax is also low, which leads to a larger tax shift[13]. The tax system is then more efficient.

5.3 Equity issue: the distributive properties of the tax reform and the trade-off between efficiency and equity

Proposition 5: *A uniform revenue-neutral environmental reform has an ambiguous effect on wages, which depends on the magnitude of the sensitivity of employment to the labor tax and on the reform's efficiency.*
For $\left[\frac{\sigma}{(1+t_{D)}} - \frac{1}{(1-t_H)} \mid \frac{dt}{dt_D} \mid\right] > 0$

- *If* $\frac{\eta_H}{\eta_R} < p_2(\mid \frac{dt}{dt_D} \mid)$*, a uniform revenue-neutral green shift decreases* w_H *and increases* p_L *and* w_L*,*
- *If* $\frac{\eta_H}{\eta_R} \in \left[p_2(\mid \frac{dt}{dt_D} \mid); \Gamma p_2(\mid \frac{dt}{dt_D} \mid)\right]$*, a uniform revenue-neutral green shift increases* w_H *and* w_L *but decreases* p_L*,*
- *If* $\frac{\eta_H}{\eta_R} > \Gamma p_2(\mid \frac{dt}{dt_D} \mid)$*, a uniform revenue-neutral green shift decreases* w_L *and* p_L*but increases* w_H*where* $p_2(\mid \frac{dt}{dt_D} \mid) = \frac{\left[\frac{\sigma}{(1+t_{D)}}\frac{P_QZ}{B+P_QZ} - \frac{1}{(1-t_L)} \mid \frac{dt}{dt_D} \mid\right]}{\left[\frac{\sigma}{(1+t_{D)}} - \frac{1}{(1-t_H)} \mid \frac{dt}{dt_D} \mid\right]}$.

[13] Demand for dirty goods being less elastic, the Ramsey logic dictates that the optimal carbon tax must be set relatively high (Jacobs and van der Ploeg [2016]).

Inequalities must be reversed if $\left[\frac{\sigma}{(1+t_{D})}-\frac{1}{(1-t_H)}\mid\frac{dt}{dt_D}\mid\right]<0.$

See proof in Appendix A.VIII.
It should be noted that the double dividend can only be obtained if at least the denominator or the numerator of the ratio $p_2(\mid\frac{dt}{dt_D}\mid)$ is negative. It means that for a very low $\frac{\eta_H}{\eta_R}$ whereas the uncompensated environmental tax leads to an increase of w_L (see Proposition 2), the reform may lead to a decrease of w_L. If $\mid\frac{dt}{dt_D}\mid$is sufficiently large, the welfare improvement may reverse the distributive properties of the environmental tax reform through the *revenue effect.*

Corollary: *If the reform succeeds in providing a strong double dividend, the welfare improvement may reverse the distributive properties of the environmental tax reform, depending on the magnitude of the elasticity of labor to taxes, the initial progressivity of the tax system and the level of* $\bar{D}$.

Labor-unions and households react not only to the increase of environmental taxes but also to the decrease of labor taxes. This latter effect dominates in the case where the reform is efficient. If n_H is relatively low, *i.e.* in the case of a progressive uncompensated green tax (Proposition 4), the union's reaction would be high compared to labor supply of high-skilled workers. This leads to a decrease of low-skilled wages and an increase of high-skilled wages, and favors regressivity.

How to compensate the low-skilled workers?

In the cases where the environmental tax reform appears to be regressive, how is it possible to combine a redistributive objective with the double dividend conditions? As in Chiroleu-Assouline and Fodha [2014], we argue that the distributive properties of the labor tax policy could be one of the instruments of fair internalization of the environmental externalities. We propose in this paper to use the progressivity of labor taxes to increase the progressivity of the environmental tax reform. Consequently, we allow dt_L to differ from dt_H in the following way:

$$dt_L=-(1+\gamma)da=-(1+\gamma)\left(\frac{G^*_{t_D}}{(1-\gamma)G^*_{t_H}+(1+\gamma)G^*_{t_L}}\right)dt_D$$

$$dt_H=-(1-\gamma)da=-(1-\gamma)\left(\frac{G^*_{t_d}}{(1-\gamma)G^*_{t_H}+(1+\gamma)G^*_{t_L}}\right)dt_D$$

where γ represents the progressivity index. If $\gamma=0$, we redistribute proportionally to revenue of agents $dt_L=dt_H=dt$. If $\gamma=1$ (resp. $\gamma=-1$) we redistribute the whole tax revenue to the low-skilled workers (resp. to the high-skilled workers).

Lemma: *It is efficient to redistribute through $\gamma > 0$ if and only if:*

$$\triangle(\gamma) = \Delta_{tot}^{PP}(\gamma) - \Delta_{tot}^{PP}(\gamma = 0) < 0 \iff \frac{\partial \Delta_{tot}^{PP}}{\partial t_L} > \frac{G_{t_L}^*}{G_{t_H}^*} \frac{\partial \Delta_{tot}^{PP}}{\partial t_H}$$

See proof in Appendix A.IX. There is no trade-off between efficiency of the reform and the use of progressivity if and only if the ratio of the marginal indirect utilities in respect to labor taxes is larger than the ratio of marginal revenues from labor taxes. If the elasticity of labor supply of high-skilled is really high, it is not possible to avoid the trade-off between the efficiency and equality of the reform, but the reform may probably be progressive (if $\bar{D}$ is not really high).

Proposition 6: *In the particular case where the substitution effect between low- and high-skilled labor is absent, there is a threshold $\overline{\eta_H}$ defined as*

$$\overline{\eta_H} = \frac{1-t_H}{1-t_L} \frac{w_R}{w_L} \left[\left(\frac{\frac{t_D\sigma}{1+t_D}(1-t_L)+t_L}{\frac{t_D\sigma}{1+t_D}(1-t_H)+t_H} \right) \left(\frac{\beta-\xi}{\xi} \right) + \left(\frac{1-\frac{t_D\sigma}{1+t_D}}{\frac{t_D\sigma}{1+t_D}(1-t_H)+t_H} \right) \left(\frac{1-\xi}{\xi} \right) \beta \frac{B}{M} \right]$$

under (resp. above) which it is always (resp. never) efficient to recycle the revenue of environmental taxes by using a progressive fiscal index $\gamma > 0$.

This can be demonstrated from the proof of the previous lemma, abstracting from substitution effect and extracting η_H (see details in Appendix A.IX). The intuition of the last proposition is the following: the higher η_H, the more an increase of the progressivity index generates a decrease in the labor supply and in production, which erodes the fiscal base. But on the low-skilled labor market side, an increase of the initial progressive labor tax can act positively on employment depending on the Hosios condition. It should be remembered that under the Hosios condition ($\beta - \xi = 0$), the two externalities on the low-skilled labor market exactly offset each others. So the more $\beta > \xi$, the more the frictions on the low-skilled labor market are initially important and the more a decrease on the low-skilled labor tax rate reduces the unemployment rate. Then, the progressivity index can contribute to improve the efficiency of the tax system. $\overline{\eta_H}$ is such that the cost of progressivity (in terms of labor supply) will be equal to its potential beneficial impact on the employment of low-skilled labor. Accordingly, $\overline{\eta_H}$ increases with the difference in $\beta - \xi$. It is also worth noting that $\overline{\eta_H}$ is lower when the initial index of progressivity $\frac{1-t_L}{1-t_H}$ is higher: the efficiency of the tax reform is constrained by the progressivity of the initial tax system.

This result is in line with the literature on progressivity and efficiency (Sorensen [1999], Røed and Strøm [2002]). In order to illustrate some arguments of our model, the next section provides simulations to highlight the trade-off between first, second and "third" dividend (inequalities).

6 A numerical illustration

This section aims to provide some numerical illustrations of our theoretical model. We calibrate and simulate the economic and welfare consequences of an environmental tax reform, designed with an increase of the progressivity labor taxes rate. Since this paper mainly aims at providing theoretical contributions, our modeling choices may sometimes be restrictive. Therefore our simulations do not claim to be exhaustive and to give a complete and detailed picture of the impacts of an environmental tax reform. [14] But we do believe that it could help readers to get a better understanding of the economic effects underlined previously, namely the substitution and consumption effects. Through these numerical illustrations, we also aim at identifying the key parameters of the economy that could affect the equity and the performance of a green tax reform. For this purpose, we choose to calibrate and compare our model for three different developed countries: France, Germany and the UK. Then, we provide, for each of these countries, the interval of the plausible index of progressivity (γ) that allows a double dividend to be obtained, while leaving the initial distributive characteristics of the economy unaffected after the reform. We then compare and explain our different results for the three countries.

6.1 Calibration

To close the numerical model, we specify the functional form of the matching function. Empirically, a reasonable approximation to the matching function in equation $\mathscr{M}$, is a Cobb-Douglas function, where the index on each variable lies between 0 and 1. Following Pissarides [1986], we set the matching function as $\mathscr{M} = \omega v^{1-\xi} 1^{\xi}$, where ω is the scale parameter of the matching function and is always positive. It gives $q(\theta) = \frac{\mathscr{M}}{v} = \omega\theta^{-\xi}$. We also specify the unemployment benefit payment B as a percentage of the low-skilled wage: $B = \rho w_L$,, with ρ defined as the replacement rate. Finally, the set of all parameters is given by $\{\eta_H$,N, β, α,σ, ξ, ω,ρ, $\bar{D}$, B, Z , t_H ,t_L ,t_D,c, ω, $\bar{D}$, $Z\}$. As argued in the literature [15], η_H is traditionally estimated to be equal to 0,5 as well as ξ (Pissarides [1998]). Parameters linked to the labor market are taken from the Database on Institutional Characteristics of Trade Unions, Wage Setting, State

[14] In particular, the techonology is identical for the proudction of the clean and the dirty goods. Taking a sectoral approach in consideration would introduce supply side effects, which could lead to a reform initially less regressive compared to our results (see Dissou and Siddiqui [2014] for more details).

[15] See Chetty et al. [2011] and Appendix X for a discussion.

Intervention and Social Pacts (ICTWSS). We have :

	Parameters	France	Germany	UK
β:	bargaining power of the low-skilled workers	0,6	0,45	0,45
η_H:	elasticity of the high-skilled labor supply	0,5	0,5	0,5
ξ:	elasticity of the matching function	0,5	0,5	0,5
ρ:	replacement rate	0,35	0,22	0,15
N:	number of high-skilled/low-skilled workers	0,65	0,65	0,65
t_H:	high-skilled labor tax rate	0,2	0,15	0,35
t_L:	low-skilled labor tax rate	0,35	0,3	0,28
t_D	green tax rate	0,01	0,02	0,02

We choose to calibrate the remaining subset of parameters $\{c,\ \omega,\ \bar{D},\ Z,\ ,\ \sigma\ \}$ to fit the economic characteristics of the labor market of each countrie. Our calibration strategy and the pertinence of our parameter choice are explained in the Appendix A. X.

6.2 Design of an equity-neutral tax reform

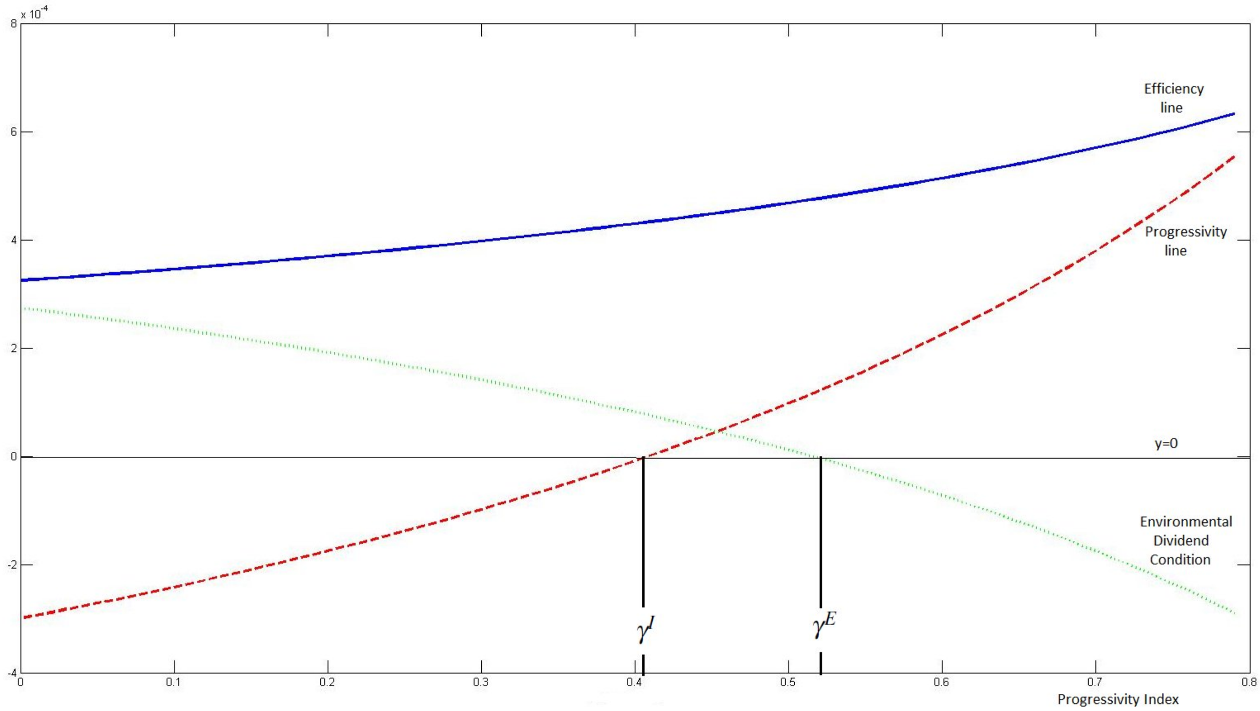
For each country, we try to define the corresponding index of progressivity γ necessary to obtain:

1. A reform that reduces the environmental damage and satisfies the first dividend hypothesis $\Longrightarrow$ γ^E is the upper bound.

2. A reform that increases the global non-environmental welfare (i-e the purchasing power), and thus satisfies the second dividend $\Longrightarrow \gamma^P$ is the lower bound.

3. A reform that leaves the initial inequalities unaffected $\Longrightarrow \gamma^I$ is the lower bound.

The next graph represents for the French case, the variation of the environmental damage, the variation of the global welfare, and the progressivity of the reform as a function of the progressivity index γ of the reform. The intersection of these curves with the horizontal line $y = 0$, gives the thresholds (γ^E, γ^P, γ^I).



We can observe that, under our calibration assumptions for France, an increase in the progressivity index is always better for the efficiency of the reform (the efficiency line is an increasing function). Typically progressivity increases the employment rate and the tax base of the reform, leading to a more efficient allocation. It should be noted that this result would not hold anymore if the labor supply elasticity of high-skilled labor (η_H) were much higher (see the Appendix the sensitivity test)[16] In this case the shape of the efficiency curve would change, which confirms our results of Proposition 6. There exists a threshold ($\overline{\eta_H}$) under which there is no trade-off anymore between efficiency and progressivity.

But we still need to implement $\gamma^I = 0,41$ in order to leave the inequalities unaffected. In this case the environmental tax reform will be neutral in term of redistribution. If γ is above this level, the reform will appear to be progressive. it should be noted that, here, even with an homothetic utility function ($\bar{D}$=0), a positive index of progressivity is always needed to leave the global economic welfare unaffected. This is due to the "income source side effect". The green tax reform associated with a uniform decrease of the labor tax rate is still regressive because of low-skilled labor market externalities.

Finally, the decreasing curve shows how the reduction of environmental damage varies with progressivity. The more the progressivity index increases, the less the reform is able to decrease pollution.

[16]This may also change in countries in the opposite case according the Hosios conditions ($\beta < \xi$). Typically,$\beta < \xi$ means that the economy lies under the optimal level of unemployment and then an increase of progressivity index by increasing employment may be counterproductive.

Basically, there is a trade-off between progressivity and obtaining the first dividend. This follows from two facts: here the overall economic welfare (or purchasing power) increases with the index of progressivity (the traditional trade-off between the first and second dividend). But by changing the distributive characteristics of the economy, it makes the poorer households better off, whereas the share of their dirty consumption in their total income is higher (Stones-Geary preferences).
Simulations show that there exists a range $\gamma \in \left[\gamma^I;\gamma^E\right]$, for which the environmental reform generates welfare gains that are Pareto improving. As shown in the graph, finally obtaining a neutral distributive goal is less compromised by the efficiency objective of the reform than by the environmental objective goal. This trade-off becomes more important when the minimum consumption of the dirty good is higher. The higher the minimum of dirty consumption ($\bar{D}$) is, the lower the magnitude of the interval $\left[\gamma^I;\gamma^E\right]$ is. Indeed, γ^I will increase in order to offset the increase of the regressivity of the green tax, due to the uses side effect. At the same time, γ^E will decrease: increasing the amount of the minimum of dirty good consumption will reduce the elasticity of substitution between the dirty and the clean good. It will become harder to reduce pollution. On the other hand, the efficiency of the reform will be high (due to the Ramsey rule).
Similar results for UK and Germany are summarized in the following table:

	France	Germany	UK
γ^E	0.52	0.55	0.88
γ^I	0,41	0,44	0,35

Surprisingly, the rate of progressivity needed to obtain a neutral reform in terms of equity, turns out to be smaller for France than for Germany. With an initial progressivity higher in France than in the two other countries, we may have expected the opposite result. How can this difference be explained? France presents several labor market features, which actually suggest a low γ^I is needed. Its labor distortions are usually much higher (above the Hosios condition) compared to the other countries as shown in the unemployment rate (10% in France, 4% and 5% respectively to Germany and the UK).[17] If institutional differences are often invoked as an explanation, the high level of unemployment benefit and the strong bargaining power of workers in France may also contribute to widening the gap between countries. As a result, recycling the revenue of green taxes through a variation of labor tax rates generates a higher double dividend in terms of employment. Moreover, this gain increases with progressivity index.This is no longer the case for Germany and the UK.

Then, the conflict between redistribution and efficiency is weaker for France. Whereas the success of the green tax reform for France can be challenged in terms of the trade-off between the redistribution

[17] Data form the Database on Institutional Characteristics of Trade-Unions, Wage Setting, State Intervention and Social Pacts (ICTWSS) for 2015

and the environmental goals, for Germany or UK the reform seems still to depend on the compatibility between efficiency and equality objectives (the efficiency line is greater than zero but decreasing). Another explanation comes from our assumption about the consumption of the polluting good. We consider the case in which the need of minimum dirty good is higher in Germany than in France: Germany uses carbon whereas France provides electricity through nuclear plants. It is therefore relevant to think that the elasticity between the clean and dirty good is higher in France. [18].

7 Conclusions

In this paper, we investigate the distributional and efficiency consequences of an environmental tax reform. Our model contains several features that contribute to identifying the key components of the regressivity/progressivity of the environmental tax. Our specification of utility, as a Stone-Geary function, allows us to represent dirty goods as necessities and emphasizes the importance of the magnitude of the subsistence level of consumption of polluting goods, like energy products. An increase of this subsistence level always leads to regressivity of the environmental tax through a *"uses side of income effect":* the share of dirty good in total consumption is higher for low-skilled, as compared to high-skilled labor, and so is the tax burden.

Moreover, the asymmetry between low- and high-skilled labor markets sheds light on the difference between high- and low-skilled wage formations, which is crucial in the distributional properties of the green tax analysis. We show that if the low-paid employment is more responsive than high-paid employment *(i.e.* if the labor supply of high-skilled is relatively low compared to low-skilled labor demand sensitivity) an uncompensated environmental tax can be progressive. This acts through a *"revenue effect"* that reflects the effect on the *"sources side of income".* However, the *revenue effect* of the revenue-neutral environmental reform - when the green tax is recycled by a variation of labor tax rates - is more ambiguous and depends clearly of the magnitude of the efficiency of the reform. We show that in the case where the reform appears to be regressive, the gains from the double dividend can be made Pareto improving by using a redistributive non-linear income tax if redistribution is initially not too large, and if the subsistence level of polluting consumption is not too high. Moreover, the use of a non-linear income tax acts on unemployment, which can moderate the trade-off between equity and efficiency. We finally provide numerical illustrations for the three European countries featuring different labor market behaviors. We show that a double dividend may be obtained without worsening the initial inequalities if the green tax revenues are redistributed with a progressivity index which is lower for France than for Germany and UK.

[18] The low initial rate of carbon tax in France compared to Germany can also contribute to generating this result.

Appendix

A.I Wage Bargaining

Wage of low-skilled workers is determined as: $w_L = argmax\left\{\left(Q^*\left(I_L^E\right) - \left[Q^*(I_L^U) + Z\right]\right)^{\beta}(f_L - w_L)^{1-\beta}\right\}$

First order condition gives: $\beta\left[\frac{(1-t_L)}{P_Q\left[Q_L^{E*} - Q_L^{U*} - Z\right]}\right] - (1-\beta)\left[\frac{1}{f_L - w_L}\right] = 0$. And with equation (7) we obtain:

$\frac{P_Q\left[Q_L^{E*} - \left(Q_L^{U*} + Z\right)\right]}{(1-t_L)} = \frac{\beta}{1-\beta} * [f_L - w_L] = \frac{\beta}{1-\beta} * \left[\frac{c}{q(\theta)}\right]$ (A.1)

It should be remembered that w_R is defined as w_L such that $Q^*\left(I_L^E\right) = \left[Q^*(I_L^U) + Z\right]$, which gives:

$w_R = \frac{B + P_Q Z}{1-t_L} = \frac{P_Q\left[Q^*(I_L^U) + Z\right]}{1-t_L}$. Then we can rewrite (A.1) as:

$\frac{P_Q\left[Q^*\left(I_L^E\right) - \left(Q_L^{U*} + Z\right)\right]}{(1-t_L)} = \frac{P_Q\left[Q^*\left(I_L^E\right)\right]}{(1-t_L)} - w_R = w_L - w_R = M$ (A.2)

Equation A.1 and A.2 finally lead to: $w_L = w_R + \beta(p_L - w_R)$.

A.II Comparative statics

Table 1 contains the log-linearized model. The tilde ($\sim$) denotes percentage changes relative to initial values, *i.e.* $\tilde{l} \equiv \frac{dl}{l}$. Exceptions to this definition are separately indicated.

Low-skilled labor	$\tilde{l} = (1-\xi) * \tilde{\theta} = \tilde{y} - \widetilde{p_L}$	(6-7)
Job creation condition	$\widetilde{p_L} = \frac{w_L}{p_L} * \widetilde{w_L} + \frac{c}{q(\theta)} \frac{\xi}{p_L} * \widetilde{\theta}$	(7)
Wage mark-up equation	$\widetilde{w_L} w_L = \beta \widetilde{p_L} p_L + [1-\beta] \widetilde{w_R} w_R$	(9)
High-skilled labor demand	$\tilde{H} = \xi * [\widetilde{p_H} - d\tau_H] = \tilde{y} - \widetilde{p_H}$	(4-8)
Production	$\tilde{y} = \alpha \tilde{l} + (1-\alpha)\tilde{h} \quad \Leftrightarrow \quad \widetilde{p_H} = -\frac{\alpha}{1-\alpha} \widetilde{p_L}$	(5)

where we denote $\widetilde{w_R} = \left[\frac{dt_L}{1-t_L} + \frac{P_Q Z}{w_R(1-t_L)} \frac{\sigma}{1+t_D} dt_D\right]$, the variation of the reservation wage with the tax rate variations dt_D and dt_L, and $d\tau_H = \frac{\sigma}{1+t_D} * dt_D + \frac{1}{1+t_H} * dt_H$. Solving the log-linearized model yields the solutions I, II, III, V, and VI in the following table. Notations are recalled at the end of the table.

Low-skilled labor	$\tilde{l} = -\mu_L \left[\widetilde{w_R} \frac{w_R}{p_L} + \eta_H \left(\alpha \widetilde{w_R} \frac{w_R}{p_L} + (1-\alpha) d\tau_H\right)\right]$	(I)
High-skilled labor	$\tilde{h} = \tilde{H} = -\mu_H \left[d\tau_H + \eta_L \left(\alpha \widetilde{w_R} \frac{w_R}{p_L} + (1-\alpha) d\tau_H\right)\right]$	(II)
Production	$\tilde{y} = -\left[\alpha \mu_L \widetilde{w_R} \frac{w_R}{p_L} (1+\eta_H) + (1-\alpha) \mu_H d\tau_H (1+\eta_L)\right]$	(III)
Low-skilled productivity	$\widetilde{p_L} = (1-\alpha) \frac{\mu_L}{\eta_L} [\eta_R \widetilde{w_R} - \eta_H d\tau_H]$	(IV)
High-skilled productivity	$\widetilde{p_H} = -(\alpha) \frac{\mu_H}{\eta_H} [\eta_R \widetilde{w_R} - \eta_H d\tau_H]$	(V)
Low-skilled wage	$\widetilde{w_L} = \frac{p_L}{w_L} \left[((1-\beta) + (1-\alpha)\beta \mu_L) \widetilde{w_R} \frac{w_R}{p_L} + -\beta ((1-\alpha)\mu_H) d\tau_H\right]$	(VI)

where $0 < \mu_H = \frac{\eta_H}{1+(1-\alpha)\eta_L + \alpha \eta_H} = \frac{\eta_H}{\eta_L} \mu_L < 1$, and $\eta_L = \frac{1-\xi}{\xi} \left[\frac{p_L}{p_L - w_R}\right]$ is the elasticity of the low-skilled labor to productivity p_L.

A.III Proposition 2

Sign of $\widetilde{p_H} = -\alpha \frac{\mu_H}{\eta_H}\left[\eta_L \widetilde{w_R}\frac{w_R}{p_L} - \eta_H d\tau_H\right]$? $\Rightarrow \widetilde{p_H} < 0 \Leftrightarrow \eta_L \widetilde{w_R}\frac{w_R}{p_L} > \eta_H d\tau_H$
Using the definitions of w_R, $\tilde{w_R}$, η_R and $dt_H = dt_L = 0$, we find:
$\eta_H < \eta_R \frac{P_Q Z}{w_R(1-t_L)} \Leftrightarrow \frac{\eta_H}{\eta_R} < \frac{P_Q Z}{P_Q Z + B} = p_1 (A.5)$

Sign of $\widetilde{w_L} = \frac{p_L}{w_L}\left[\left((1-\beta) + (1-\alpha)\beta\mu_L\right)\widetilde{w_R}\frac{w_R}{p_L} + -\beta\left((1-\alpha)\mu_H\right)d\tau_H\right]$?
Remember that $\mu_i = \frac{\eta_i}{1+(1-\alpha)\eta_L + \alpha\eta_H}$, and by multiplying everything by $1+(1-\alpha)\eta_L + \alpha\eta_H$, we obtain $\widetilde{w_L}$>0

$\Leftrightarrow \eta_R \widetilde{w_R}$> $\eta_H \left(\frac{(1-\alpha)\beta\eta_L}{(1-\beta)((1+\alpha)\eta_{HL}) + (1-\alpha)\eta_L}\right) d\tau_H$

Noting $\Gamma = \frac{(1-\alpha)\beta\eta_L}{(1-\beta)((1+\alpha)\eta_{HL}) + (1-\alpha)\eta_L}$ and using (A.5), we find: $\frac{\eta_R}{\eta_H} > p_1\Gamma$.

A.IV The compensatory income variation

Let us determine the compensatory income variation of low-skilled workers:

$$dV_L = 0 \Leftrightarrow (1-l)\,dV_L^U + l dV_L^E + \left(V_L^E - V_L^U\right) dl = 0$$

$$\Leftrightarrow (1-l)\frac{\partial V_L^U}{\partial C_L} dC_L^U + l\frac{\partial V_L^E}{\partial C_L} dC_L^E + (1-l)\frac{\partial V_L^U}{\partial D_L} dD_L^U + l\frac{\partial V_L^E}{\partial D_L} dD_L^E + \left(V_L^E - V_L^U\right) dl + \frac{\partial V_L}{\partial D_{tot}} dD_{tot} = 0$$

Using first order conditions of consumer's program, this leads to: $dV_L = 0$

$$\Leftrightarrow \frac{1}{P_Q}\left[(1-l)\,dC_L^U + l dC_L^E\right] + \frac{(1+t_D)}{P_Q}\left[(1-l)\,dD_L^U + l dD_L^E\right] + \left(V_L^E - V_L^U\right) dl + \frac{\partial V_L}{\partial D_{tot}} dD_{tot} = 0$$

By differentiating the budget constraint of low-skilled households and using the definition of the compensatory income variation we obtain:
$\left[I_L^E - B\right] dl + \left[(1+t_D)\left((1-l)\,dD_L^U + l dD_L^E\right) + (1-l)\,dC_L^U + l dC_L^E\right] = \left[w_L(1-t_L) - B\right] dl + I_L^E l\left(\widetilde{w_L} + \frac{dt_L}{1-t_L}\right) + dR_L$

$$\Leftrightarrow dR_L = -\left[I_L^E\left(\widetilde{w_L} + \frac{dt_L}{1-t_L}\right) + P_Q\left(V_L^E - V_L^U\right) dl - D_L dt_D - \frac{\partial V_L}{\partial D_{tot}} P_Q dD_{tot}\right] \text{ (eq. 13)}$$

Using the definition of $\widetilde{w_L}$, and dl we obtain:

$$dR_L = -\left[I_L^E\left(\left[\frac{\beta}{\xi}\frac{p_L}{w_L}\widetilde{p_L} + \left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\widetilde{w_R}\right] + \frac{dt_L}{1-t_L}\right) - D_L dt_D - \frac{\partial V_L}{\partial D_{tot}} P_Q dD_{tot}\right] \text{ (eq. 14)}$$

A.V Proposition 3: The regressivity of an uncompensated green tax

An increase of an uncompensated green tax appears regressive if and only if,when $dt_D > 0$ and $dt_L = dt_H = 0$
$\frac{\Delta_H^{PP}}{I_H dt_D} < \frac{\Delta_L^{PP}}{I_L dt_D}$. Combining with equations 12 and 14 leads to:

$$\left[\frac{dw_H}{w_H dt_D} - \frac{D_H}{I_H}\right] > \left[\frac{I_L^E l}{I_L}\left(\frac{\beta}{\xi}\frac{p_L}{w_L}\frac{dp_L}{p_l dt_D}\right) + \frac{I_L^E l}{I_L}\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\frac{dw_R}{w_R dt_D} - \frac{D_L}{I_L}\right]$$

Because of the functional form of the consumption utility, we have:

$$\frac{D_H}{I_H} - \frac{D_L}{I_L} = (1-\sigma)\bar{D}\left[\frac{1}{I_H} - \frac{1}{I_L}\right] \leq 0, \forall \bar{D} \geq 0$$

In the case where $\bar{D} = 0$, the definition of regressivity is equivalent to:

$$\left[\frac{dw_H}{w_H dt_D}\right] > \left[\frac{I_L^E l}{I_L}\left(\frac{\beta}{\xi}\frac{p_L}{w_L}\frac{dp_L}{p_l dt_D}\right) + \frac{I_L^E l}{I_L}\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\frac{dw_R}{w_R dt_D}\right]$$

$$\Leftrightarrow \left[\alpha\frac{\mu_H}{\eta_H}\left[\eta_H\frac{d\tau_H}{dt_D} - \eta_R\frac{dw_R}{w_R dt_D}\right]\right] > \left[\frac{I_L^E l}{I_L}\left(\frac{\beta}{\xi}\frac{p_L}{w_L}(1-\alpha)\frac{\mu_L}{\eta_L}\left[\eta_R\frac{dw_R}{w_R dt_D} - \eta_H\frac{d\tau_H}{dt_D}\right]\right) + \frac{I_L^E l}{I_L}\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\frac{dw_R}{w_R dt_D}\right]$$

By multiplying both sides by $(1+\alpha\eta_H + (1-\alpha)\eta_L)$, replacing $d\tau_H$ and $\frac{dw_R}{w_R dt_D}$ and dividing both members by $\frac{t_D\sigma}{1+t_{Df}}$ gives:

$$\eta_H > \frac{P_Q Z}{P_Q Z+B}\left[\frac{\left(\frac{I_L^E l}{I_L}\left((1-\alpha)\frac{(1-\xi)\eta_R}{(1-\xi)\eta_R-(1-\beta)\xi}\right)+\frac{I_L^E l}{I_L}\left(\frac{(\xi-\beta)\eta_R}{\xi\eta_R+\beta(1-\xi)}\right)+\alpha\right)}{\left[\left(\alpha\left(1-\frac{I_L^E l}{I_L}\left(\frac{(\xi-\beta)\eta_R}{\xi\eta_R+\beta(1-\xi)}\right)\frac{P_Q Z}{P_Q Z+B}\right)+\frac{I_L^E}{I_L}\frac{\beta}{\xi}(1-\alpha)\frac{(1-\xi)\eta_R}{(1-\xi)\eta_R-(1-\beta)\xi})\right)\right]}\right]$$

If and only if $\xi > \beta$, then

$$\frac{\left[\left(\frac{I_L^E l}{I_L}\frac{\beta}{\xi}\frac{p_L}{w_L}(1-\alpha)+\alpha\left[1-\left(\frac{P_Q Z}{P_Q Z+B}\right)\frac{I_L^E l}{I_L}\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}\right]\right)\right]\eta_H}{\left((1-\alpha)\frac{I_L^E l}{I_L}\left(\frac{\beta}{\xi}\frac{p_L}{w_L}+\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}*\frac{p_L}{w_R}\right)+\alpha\right)\eta_R+\frac{I_L^E l}{I_L}\left[\frac{\xi-\beta}{\xi}\right]\frac{w_R}{w_L}} > \left(\frac{P_Q Z}{P_Q Z+B}\right) = p_1$$

NOTE 1: If $\xi = \beta$, (*i.e.* the Hosios condition is verified), then the previous condition is given by: $\frac{\eta_H}{\eta_R} > \left(\frac{P_Q Z}{P_Q Z+B}\right) = p_1$

NOTE 2: According to Proposition 1, a higher uncompensated green tax raises unemployment $\frac{dl}{dt_D} < 0$. If $\frac{dw_L}{w_L dt_D} < 0$, (which implies necessarily $\frac{dw_H}{w_H dt_D} > 0$, see Proposition 2), we have:

$$\left[\underbrace{\frac{dw_H}{w_H dt_D}}_{>0}\right] > \left[\underbrace{\frac{lI_L^E}{I_L}\left(\frac{dw_L}{w_L dt_D}\right)}_{<0} + \underbrace{\frac{P_Q\left(V_L^E - V_L^U\right)\frac{dl}{dt_D}}{I_L}}_{<0}\right] + \left[\underbrace{\frac{D_H}{I_H} - \frac{D_L}{I_L}}_{<0}\right]$$

In this case, a higher uncompensated green tax is then unambiguously regressive.

A.VI A revenue-neutral tax reform

Remember the government budget constraint:

$G + (1-l)B = lw_L t_L + NHw_H t_H + t_D D_{tot} = w_L t_L + NHw_H t_H + \left(\sigma \frac{t_d}{1+t_d} I_{tot} + (N+1)(1-\sigma)\bar{D}\right)$

We want to identify $G^*_{t_d}$, $G^*_{t_L}$, and $G^*_{t_H}$. We have:

$$G^*_{t_D} = \frac{dG}{dt_D} = \frac{\partial G}{\partial t_D} + lw_L t_L \left(\frac{\partial w_L}{w_L \partial t_D} + \frac{\partial l}{l \partial t_D}\right) + NHw_H t_H \left(\frac{\partial w_H}{w_H \partial t_D} + \frac{\partial H}{H \partial t_D}\right) + B\frac{\partial l}{\partial t_D} + \sigma \frac{t_D}{1+t_D}\frac{\partial I_{tot}}{\partial t_D}$$

$$G^*_{t_L} = \frac{dG}{dt_L} = \frac{\partial G}{\partial t_L} + lw_L t_L \left(\frac{\partial w_L}{w_L \partial t_L} + \frac{\partial l}{l \partial t_L}\right) + NHw_H t_H \left(\frac{\partial w_H}{w_H \partial t_L} + \frac{\partial H}{H \partial t_L}\right) + B\frac{\partial l}{\partial t_L} + \sigma \frac{t_d}{1+t_d}\frac{\partial I_{tot}}{\partial t_L}$$

$$G^*_{t_H} = w_H H + lw_L t_L \left(\frac{\partial w_L}{w_L \partial t_H} + \frac{\partial l}{l \partial t_H}\right) + NHw_H t_H \left(\frac{\partial w_H}{w_H \partial t_H} + \frac{\partial H}{H \partial t_H}\right) + B\frac{\partial l}{\partial t_H} + \sigma \frac{t_d}{1+t_d}\frac{\partial I_{tot}}{\partial t_H}$$

Note that $\frac{\partial G}{\partial t_D} = \left(D_{tot} - \frac{\sigma t_D}{1+t_D}\frac{I_{tot}}{1+t_d}\right) = \left(D_{tot} - \frac{t_D}{1+t_D} \mid E_s \mid\right)$, with E_s, the elasticity of substitution. Finally we find:

$$-\frac{dt}{dt_D} = \left[\frac{\overbrace{D_{tot}}^{value\,effect} + \overbrace{B\frac{\partial L}{\partial t_D}}^{benefit} + \overbrace{\frac{t_D}{1+t_D}E_s}^{substitution\,effect} + \overbrace{\frac{\sigma t_D}{1+t_D}\frac{\partial I_{tot}}{\partial t_d} + \sum_{i=l,H} i w_i t_i \left(\frac{\partial w_i}{w_i \partial t_D} + \frac{\partial i}{i \partial t_D}\right)}^{the\,revenue\,effect}}{\underbrace{(w_L L + w_H H)}_{value\,effect} + \underbrace{B\left(\frac{\partial L}{\partial t_L} + \frac{\partial L}{\partial t_H}\right)}_{benefit} + \underbrace{\sum_{j=l,H}\left(\frac{\sigma t_D}{1+t_D}\frac{\partial I_{tot}}{\partial t_j} + \sum_{i=l,H} i w_i t_i \left(\frac{\partial w_i}{w_i \partial t_j} + \frac{\partial i}{i \partial t_j}\right)\right)}_{the\,revenue\,effect}}\right]$$

A.VII Proposition 4: Double dividend

Condition for obtaining the first dividend:

$$\widetilde{D_{tot}} < 0 \iff \left(-\left[\frac{\sigma}{(1+t_D)^2} * \frac{I_{tot}}{D_{tot}}\right] dt_D + \left[\frac{\sigma}{(1+t_D)} * \frac{dI_{tot}}{D_{tot}}\right]\right) < 0$$

Notice that $\mid E_s \mid$, the substitution elasticity between clean and polluting good is equal to: $\mid E_s \mid =$

$\frac{\sigma}{(1+t_D)} * \frac{I_{tot}}{D_{tot}}$. We obtain:$\widetilde{D_{tot}} < 0 \iff \frac{|E_s|}{(1+t_D)} * \left[(1+t_D)\widetilde{I_{tot}} - dt_D\right] < 0 \iff \left[\widetilde{I_{tot}} < \frac{dt_D}{(1+t_D)}\right]$

Condition for obtaining the second dividend:

$\Delta_{tot}^{PP} < 0 \iff P_Q(dV_L + NdV_H) > 0$. And we know that:

$$P_Q dV_L = lI_L^E\left(\widetilde{w_L} + \frac{dt_L}{1-t_L}\right) + P_Q\left(V_L^E - V_L^U\right)dl - D_L dt_D = dI_L - P_Q Zdl - D_L dt_D$$
$$P_Q dV_H = dI_H - \varphi'(H^*)dH - D_H dt_D$$

Combining the last two equations, the condition for the second dividend is given by:
$D_{tot}dt_D + P_Q ZdL + \varphi'(H^*)NdH < dI_{tot}$

A.VIII Proposition 5

$\widetilde{p_H} = -\alpha\frac{\mu_H}{\eta_H}[\eta_R\widetilde{w_R} - \eta_H d\tau_H]$<0 $\Leftrightarrow \eta_R\widetilde{w_R} > \eta_H d\tau_H$

Using the definitions of $\tilde{w}_R$, $d\tau_H$ and $dt_H = dt_L = dt$, we find :
$\eta_R\left(-\frac{|dt/dt_D|}{1-t_L} + \frac{P_Q Z}{w_R(1-t_L)}\frac{\sigma}{1+t_D}\right) > \eta_H\left(\frac{\sigma}{1+t_D} - \frac{|dt/dt_D|}{1+t_H}\right)$

Thus $\widetilde{p_H} < 0$ <=> $\eta_H < \frac{\eta_R\left[\frac{\sigma}{(1+t_D)}\frac{P_Q Z}{B+P_Q Z} - \frac{1}{(1-t_L)}|\frac{dt}{dt_D}|\right]}{\left[\frac{\sigma}{(1+t_D)} - \frac{1}{(1-t_H)}|\frac{dt}{dt_D}|\right]}$ if $\left[\frac{\sigma}{1+t_D} - \frac{|dt/dt_D|}{1+t_H}\right] > 0$

A.IX Proposition 6

1) Proof of Lemma:

$\Delta_{tot}^{PP}(\gamma) - \Delta_{tot}^{PP}(\gamma = 0) < 0$

$\Leftrightarrow \frac{\partial\Delta_{tot}^{PP}}{\partial t_L}(dt - dt_L) > \frac{\partial\Delta_{tot}^{PP}}{\partial t_H}(dt_H - dt)$

But $dt = -\frac{G_{t_d}^*}{G_{t_H}^* + G_{t_L}^*}dt_D$, $dt_L = -(1+\gamma)da$ and $dt_H = -(1-\gamma)da$ with:

$da = \left(\frac{G^*_{t_d}}{(1-\gamma)G^*_{t_H} + (1+\gamma)G^*_{t_L}} \right) dt_D$.

Using definitions of dt_L and dt_H, we find:

$\Rightarrow \left(\frac{dt_H - dt}{dt - dt_L} \right) = \frac{G^*_{t_L}}{G^*_{t_H}}$.

And then : $\Delta^{PP}_{tot}(\gamma) - \Delta^{PP}_{tot}(\gamma = 0) < 0 \iff \frac{\partial \Delta^{PP}_{tot}}{\partial t_L} > \frac{G^*_{t_L}}{G^*_{t_H}} \frac{\partial \Delta^{PP}_{tot}}{\partial t_H}$

2) Proof of Proposition 6:

Remember that:

Δ^{PP}_{tot}=$- \left[lI^E_L \left(\widetilde{w_L} - \frac{dt_L}{1-t_L} \right) + P_Q \left(V^E_L - V^U_L \right) dl + NHI_H \left(\widetilde{w_H} - \frac{dt_H}{1-t_L} \right) - D_{tot} dt_D \right]$

And then: $\frac{\partial \Delta^{PP}_{tot}}{\partial t_L} = - \left[lI^E_L \left(\frac{dw_L}{w_L dt_L} - \frac{1}{1-t_L} \right) + \left[lI^E_L \left(1 - \frac{w_R}{w_L} \right) \right] \frac{dl}{l dt_L} + NHI_H \left(\frac{dw_H}{w_H dt_L} \right) \right]$

Similarly: $\frac{\partial \Delta^{PP}_{tot}}{\partial t_H} = - \left[lI^E_L \left(\frac{dw_L}{w_L dt_H} \right) + \left[lI^E_L \left(1 - \frac{w_R}{w_L} \right) \right] \frac{dl}{l dt_H} + NHI_H \left(\frac{dw_H}{w_H dt_H} - \frac{1}{1-t_H} \right) \right]$

By developping and replacing the derivatives $G^*_{t_L}$ and $G^*_{t_H}$ with their expressions obtained in Appendix A.VI, we find the formula in Lemma is equivalent to:

$$
\begin{aligned}
&[NI_H * lI^E_L \left(1 - \frac{w_R}{w_L} \right) \left[\alpha + \gamma + \frac{(1-\alpha)}{1-t_H} \right] * \eta_R + \\
&- \left(NI_H * lI^E_L * \frac{p_L}{w_L} \left[(1-\beta)\, \alpha \left[\frac{t_L}{1-t_L} \right] \right] + (1-\alpha)\, I^E_L l \frac{p_L}{w_L} \left[(1-\gamma)\, lB + \frac{wr}{w_L} I^E_L l \right] \right) \eta_R \\
&- \beta\, (1-\alpha)\, lI^E_L \frac{p_L}{w_L} \left[\frac{t_L}{1-t_L} \right] + \left[\alpha \left(\frac{t_H}{1-t_H} \right) + \gamma_H \right] NI_H] * \eta_H \\
&\leq NI_H \frac{w_R}{w_L} \left[\eta \left[(1-\gamma) * (lB + \alpha NI_H) + lI^E_L \eta_R \left[\frac{p_L}{w_R} \beta \left[\frac{p_L - w_R}{p_L} \right] - (1-\alpha) \right] + \left(\alpha \gamma_H I_H N + \gamma_L I^E_L \right) \right] \right] \\
&\quad + NI_H \frac{w_R}{w_L} (1-\beta) \left[\left(\frac{t_L}{1-t_L} \right) \right] lI^E_L
\end{aligned}
$$

A.X: Calibration

The set of all parameters is given by $\{\eta_H, N, \beta, \alpha, \sigma\ \xi, \omega, \rho, c, \bar{D}, B, Z\ , t_H\ , t_L\ , t_D\}$.

Some of these parameters are known in the literature and common between countries.
The choice of a Frish form for the disutility of hours of work imposes a range of plausible values for the consumption-leisure substitution elasticity η_H between 0.35 and 0,75. These are typical estimates from the literature, and they are meant to represent the effects of changes in the real wage on average hours worked.
Our parameter η_H, the Frisch labor elasticity, is first set to a value of 0.5, a widely adopted value in the literature, in line with the estimates derived from microdata reported by, for example, Chetty et al. [2011].
The parameter ξ, which represents the elasticity of the matching function with respect to vacancies, is set to a value of 0.5 wich is an average of the range of possible values identified by Pissarides and Petrongolo [2001], and is widely adopted in literature.
The data on bargaining power and unemployment benefits are taken from the Database on Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts (ICTWSS). We choose $\beta = 0,6$, $\beta = 0.45$, $\beta = 0.3$, respectively for France, Germany and UK. The unemployment benefit payment B is defined as a percentage of the low-skilled wage: $B = \rho W_L$, with ρ defined as the replacement rate. We set $\rho = 0,35$, $\rho = 0.22$, $\rho = 0.15$, respectively for France, Germany and UK.
We assume a non equi-distribution of the workers among classes such that the high-skilled workers represent 40% of the mass of total workers (the 4 first deciles of workers). Thus N=0.65. Giving this assumption, we choose the initial labor tax rate accordingly and in line with the official labor taxes data. Thus, $t_L = 0.2;0.15;0.15$ and $t_H = 0.35; 0.3; 0.28$ respectively for France, Germany and UK.
The environmental tax rate corresponds for the French system closely to the budget of the French Environmental and Energy Management Agency (ADEME), which is entirely financed by environmental tax revenues. As in Chiroleu-Assouline and Fodha [2014], $t_D = 0.01$. We refer to Klenert and al [2015] for the environmental tax in Germany: $t_D = 0.02$.

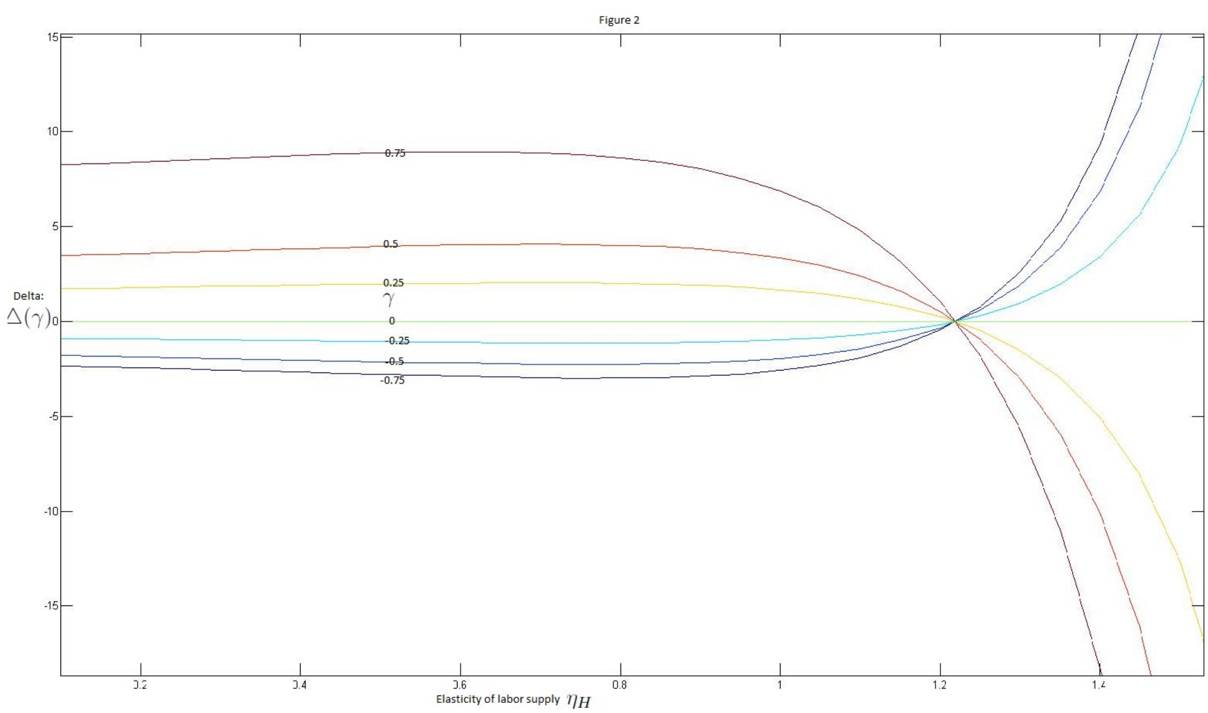
We choose to calibrate the remaining subset of parameters $\{c, \omega, \bar{D}, Z, , \sigma, \alpha\}$.
Following the empirical taxation and double dividend literature[19], we assume a share of the polluting good in the total output of 0.35, which gives $\sigma = 0.35$ if we assume $\bar{D} = 0$. But in the following, $\bar{D}$ varies between 0 and 50% of consumption of dirty good by unemployed people, which will induce also a variation of σ accordingly.

[19] See Cremer et al. [2003], Chiroleu-Assouline and Fodha [2014].

The constants ω, c Z and α , are free and chosen to yield a fixed value for the unemployment rate, u , a ratio of wages w_H/w_L that fits the characteristics of the specific country, and plausible gap between utility of unemployed and employed workers and a ratio of government expenditure in collective goods. Data for the ratio of government expenditure in collective goods are taken from the OECD. France, Germany and the UK feature collective public spending (in proportion to GDP) comparable to other OECD countries. This leads us to calibrate G/Y as a common parameter by assuming that these collective goods are chosen by the government in accordance to the households preferences.

A.XI: Robustness tests

In Figure 2, the range of the combinations (γ, η_H) for which the progressive reform leads to a higher efficiency is shaded. The curves stand for differents level of $\gamma \in]-1;1[$. On the y axis, we represent $\triangle(\gamma)$, the difference between$\Delta_{tot}^{PP}(\gamma)$ and $\Delta_{tot}^{PP}(\gamma=0)$.



We can see that, for low values of η_H, redistributing the whole revenue of the environmental tax to the low-skilled households ($\gamma = 1$) can be the best option. By contrast, when η_H becomes higher, progressivity leads in any case to inefficiency. Hence the range of critical elasticity of labor supply widens as the progressivity index increases. Based upon the two previous graphs, a conclusion may appear. If the elasticity of the high-skilled labor supply is really high, it is not possible to avoid the trade-off between the efficiency and equality of the reform. But, if $\bar{D}$ is not so high, the reform may initially be progressive for $\gamma = 0$.

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