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# Can we Identify the Fed's Preferences?\*

Jean-Bernard Chatelain<sup>†</sup> Kirsten Ralf<sup>‡</sup>

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## Abstract

Using US data, we estimate optimal policy with a probability below one that the Fed reneges on its commitment ("limited credibility") versus discretionary policy where the Fed reneges on its commitment at all periods with a probability equal to one ("zero credibility"). The transmission mechanism is the new-Keynesian Phillips curve with auto-correlated cost-push shock. It includes the labor cost channel or the working capital channel. Discretion with zero credibility of the Fed is rejected. The working capital channel fits the data before Volcker's mandate. The labor cost channel fits the data since Volcker's mandate.

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## 1 Introduction

Can we *test* if the Fed follows Ramsey optimal policy under the limited credibility of quasi-commitment with a *non-zero* probability to commit (Debortoli and Nunes (2014), Schaumburg and Tambalotti (2007), Roberts (1987)) versus the Fed follows a policy with a complete lack of credibility with a *zero* probability to commit (Oudiz and Sachs (1985))? Are the Fed's preferences facing the same *identification* problem as Taylor rule parameters for both types of policies? Cochrane (2011) found that the simple Taylor rule parameters are not identified in new-Keynesian models including forward-looking inflation, but only the auto-correlation parameters of non-observable shocks.

Beginning with Simon (1956) certainty equivalence property of the linear quadratic regulator, the quadratic loss function describing policy-maker's preferences is used for modelling stabilization policy (Duarte (2009)). It took more than two decades for the estimation of Fed's preferences to begin with Salemi (1995), using inverse control (Salemi (2010)). Salemi (1995) took into account two structural breaks (1970-1, 1979-10) for three monetary policy regime during the period 1947-1992 using monthly data. Salemi (1995) includes a careful investigation of identification issues when the monetary policy transmission mechanism is a backward-looking vector auto-regressive (VAR) model. These

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identification issues are the same for Ramsey optimal policy under quasi-commitment (Debortoli and Nunes (2014)) with forward-looking variables, besides the optimal initial jump anchoring forward-looking variables (Ljungqvist and Sargent (2012)).

Following Salemi (1995), there are more than twenty papers estimating Fed's preferences since the 1960's with a structural break since the 1980's (Volcker-Greenspan period) assuming inflation is a predetermined variable or a forward-looking variable, assuming the acceleration Phillips curve or new-Keynesian Phillips curve as a monetary policy transmission mechanism: e.g. Cechetti and Ehrmann (2002), Ozlale (2003), Favero and Rovelli (2003), Castelnuovo and Surico (2004), Castelnuovo (2006), Soderstrom et al. (2005), Juillard et al. (2006), Salemi (2006), Kara (2007), Adjemian and Devulder (2011), Adolfson et al. (2011), Ilbas (2012), Leveuge and Lucotte (2014), Paez-Farrell (2015), Debortoli and Lakdawala (2016) and many others.

Ramsey optimal policy including the probability to re-optimize each period is mentioned "stochastic replanning" (Roberds (1987)), "quasi commitment" (Schaumburg and Tambalotti (2007), Kara (2007)) or "loose commitment" (Debortoli and Nunes (2010)). This assumption is observationally equivalent to Chari and Kehoe (1990) optimal policy under sustainable plans facing a punishment threat at a given horizon  $T$  in case of deviation of an Ramsey optimal policy plan (Fujiwara, Kam, Sunakawa (2016)).

We estimate Debortoli and Nunes (2014) and Schaumburg and Tambalotti (2007) quasi commitment model, where the transmission mechanism is the new-Keynesian Phillips curve with an auto-regressive cost-push shock. These empirical results can be compared to estimations of the new-Keynesian Phillips curve (Mavroeidis et al. (2014)). We avoid the Lucas' critique taking into account the endogeneity of the policy instruments due to a feedback policy rule. We also estimate that case of the working capital channel with interest smoothing in the Fed's loss function. In this case, the model fits data as well as a non-structural VAR for the pre-Volcker period. We test Ramsey versus zero-credibility policy. Assuming the minimal number of lags for the transmission mechanism, this paper makes progress in these five directions:

- (1) We disclose the estimates of the reduced form stable eigenvalues of the estimated VAR for Ramsey versus zero-credibility and check Blanchard and Kahn (1980) determinacy condition. The estimations should not correspond to the same number of stable eigenvalues for both models. Söderlind (1999) algorithm provides no hints on checking the number of stable eigenvalues for Ramsey versus zero-credibility policy.

- (2) We disclose the conditions for the exact identification of parameters for each model.

- (3) We test the signs restrictions of reduced form policy rule parameters.

- (4) We test the misspecification of each models, using for example Durbin (1970) test.

- (5) We compare both estimations with the estimations of a non-structural vector auto-regressive model.

Section 2 presents our estimation methods of Ramsey optimal policy and zero-credibility policy. Section 3 details specification tests of zero-credibility versus Ramsey optimal policy. Section 4 presents estimations on US data during 1960-2006 for a labor cost channel and section 5 for a cost of working capital channel. The last section concludes.

## 2 Ramsey versus Zero-credibility Monetary Policy

### 2.1 Monetary Policy Transmission Mechanism Including or not the Cost of Working Capital

The reference new-Keynesian Phillips curve is the monetary policy transmission mechanism:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \text{ where } \kappa > 0, 0 < \beta < 1 \quad (1)$$

where  $x_t$  represents the welfare-relevant output gap, i.e. the deviation between (log) output and its efficient level.  $\pi_t$  denotes the rate of inflation between periods  $t - 1$  and  $t$ .  $u_t$  denotes a cost-push shock.  $\beta$  denotes the discount factor.  $E_t$  denotes the expectation operator. The cost push shock  $u_t$  includes an exogenous auto-regressive component:

$$u_t = \rho u_{t-1} + \varepsilon_{u,t} \text{ where } 0 < \rho < 1 \text{ and } \varepsilon_{u,t} \text{ i.i.d. normal } N(0, \sigma_u^2) \quad (2)$$

where  $\rho$  denotes the auto-correlation parameter and  $\varepsilon_t$  is identically and independently distributed (i.i.d.) according to a normal distribution with constant variance  $\sigma_u^2$ .

The reduced-form parameter (denoted  $\kappa$ ) of the slope of the new-Keynesian Phillips curve relates inflation to marginal cost or to the output gap. It depends on four structural parameters: the representative household discount factor  $\beta$ , the household's elasticity of substitution between each differentiated goods  $\varepsilon$ , the measure of decreasing returns to scale of labor in the production functions of the firms  $\eta$ , and the proportion of firms who do not reset their price each period  $\theta$  (Gali (2015), chapter 3):

$$\kappa(\beta, \varepsilon, \eta, \theta) = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \frac{(1 - \eta)}{(1 - \eta + \eta\varepsilon)}.$$

In the new-Keynesian model, labor is assumed to be the only input in the production function. The cost of capital channel of monetary policy is ruled out by assumption. Christiano, Trabandt and Walentin (2010) assume that labor cost is financed by working capital. The marginal cost in the new-Keynesian Phillips curve is then a linear combination of labor and capital cost (Christiano, Trabandt and Walentin (2010), equation 35):

$$\gamma(1 + \phi)x_t + \frac{\psi}{(1 - \psi)\beta + \psi} i_t \quad (3)$$

We estimate two polar cases. When material inputs are not used in production ( $\gamma = 1$ ) and when no labor cost is financed by working capital ( $\psi = 0$ ), the cost of labor is the only cost taken into account and the output gap is the policy instrument. When material inputs are all used in production ( $\gamma = 0$ ) and when all labor cost is financed by working capital ( $\psi = 1$ ), the cost of capital is the only cost taken into account with a slope of the new-Keynesian Phillips curve denoted  $\kappa_i$ . Then, the Federal funds rate is the policy instrument with an interest smoothing parameter  $\alpha_i$  for the Fed's preferences.

### 2.2 Monetary Policy Regimes and Central Bank Behavior

In a monetary policy regime indexed by  $j$ , a policy maker has a period loss function  $\frac{1}{2}(\pi_t^2 + \alpha_{x,j}x_t^2)$  for the labor cost model or the period loss function includes an interest

smoothing term  $\frac{1}{2}(\pi_t^2 + \alpha_{i,j}i_t^2)$  for the cost of working capital model. Fed's preferences are represented by the relative cost  $\alpha_{x,j} > 0$  or  $\alpha_{i,j} > 0$  of changing its policy instrument. A policy maker may re-optimize on each future period with exogenous probability  $1 - q$  strictly below one ("stochastic replanning" (Roberds, 1987), "quasi commitment" (Schaumburg and Tambalotti, 2007; Kara 2007) or "loose commitment" (Debortoli and Nunes, 2014)).

Following Schaumburg and Tambalotti (2007), we assume that the mandate to minimize the loss function is delegated to a sequence of policy makers with a commitment of random duration. The degree of credibility is modelled as if it is a change of policy-maker with a given probability of renegeing commitment and re-optimizing optimal plans. The length of their tenure or "regime" depends on a sequence of exogenous i.i.d. Bernoulli signals  $\{\eta_t\}_{t \geq 0}$  with  $E_t[\eta_t]_{t \geq 0} = 1 - q$ , with  $0 < q < 1$ . If  $\eta_t = 1$ , a new policy maker takes office at the beginning of time  $t$ . Otherwise, the incumbent stays on. A higher probability  $q$  can be interpreted as a higher credibility. As seen below, this leads to use a "credibility adjusted" discount factor  $\beta q$  in the policy maker's optimal behavior.

Secondly, in this new monetary policy regime indexed by  $k$ , there may be a switch of the transmission mechanism parameter (the slope of the Phillips curve)  $\kappa_k$  including in particular, a switch of the representative household's elasticity of substitution between each differentiated goods  $\varepsilon_k$ . Thirdly, the new policy maker may have new preferences  $\alpha_{x,k}$ . If the policy maker is maximizing welfare, its preferences are given by the new values of structural parameters of the transmission mechanism  $\alpha_{x,k} = \frac{\kappa_k}{\varepsilon_k}$  (Gali (2015)). More generally, and stepping beyond the welfare-theoretic justification for (1), one can interpret  $\alpha_x$  as the weight attached by the central bank to deviations of output from its efficient level (relative to price stability) in its own loss function, which does not necessarily have to coincide with the household's. Because structural parameters may change for a new regime  $k$ , long run equilibrium values may also change in the estimation.

Under regime  $j$ , policy plans solve the following problem (omitting subscript  $j$  for the central bank preferences  $\alpha_x$ , transmission mechanism parameter  $\kappa$ , the auto-correlation of the cost-push shock  $\rho$  and its variance of its disturbances  $\varepsilon_t$ ):

$$V^{jk}(u_0) = -E_0 \sum_{t=0}^{t=\infty} (\beta q)^t \left[ \frac{1}{2} (\pi_t^2 + \alpha_x x_t^2) + \beta (1 - q) V^{jk}(u_t) \right] \quad (4)$$

s.t.  $\pi_t = \kappa x_t + \beta q E_t \pi_{t+1} + \beta (1 - q) E_t \pi_{t+1}^k + u_t$  (Lagrange multiplier  $\gamma_{t+1}$ )  
 $u_t = \rho u_{t-1} + \varepsilon_t, \forall t \in \mathbb{N}, u_0$  given.

The utility the central bank obtains is next period objectives change is denoted  $V^{jk}$ . Since when objectives change, the central bank loses its commitment, this value function depends on the policies of the alternative regime. Inflation expectations are an average between two terms. The first term, with weight  $q$  is the inflation that would prevail under the current regime upon which there is commitment. The second term with weight  $1 - q$  is the inflation that would be implemented under the alternative regime, which is taken as given by the current central bank. The key change is that the narrow range of values for the discount factor  $\beta \in [0.99, 1]$  for quarterly data (4% discount rate at most) is much wider for the "credibility weighted discount factor" of the policy maker:  $\beta q \in ]0, 1]$ . In what follows, the notation  $\beta$  will correspond to  $\beta q$ , in order to simplify notations and because the theoretical model does not allow to identify separately  $\beta$  and

$q$  for the estimation.

### 2.3 VAR Representation of Ramsey optimal policy

Differentiating the Lagrangian with respect to the policy instrument (output gap  $x_t$ ) and to the policy target (inflation  $\pi_t$ ) yields the first order conditions (see appendix 2):

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial \pi_t} = 0 : \pi_t + \gamma_{t+1} - \gamma_t = 0 \\ \frac{\partial \mathcal{L}}{\partial x_t} = 0 : \alpha_x x_t - \kappa \gamma_{t+1} = 0 \end{cases} \Rightarrow \begin{cases} x_t = x_{t-1} - \frac{\kappa}{\alpha_x} \pi_t \\ x_t = \frac{\kappa}{\alpha_x} \gamma_{t+1} = \frac{\kappa}{\alpha_x} (\gamma_t - \pi_t) \end{cases}$$

that must hold for  $t = 1, 2, \dots$ . The central bank's Euler equation ( $\frac{\partial \mathcal{L}}{\partial \pi_t} = 0$ ) links recursively the future or current value of central bank's policy instrument  $x_t$  to its current or past value  $x_{t-1}$ , because of the central bank's relative cost of changing her policy instrument is strictly positive  $\alpha_x > 0$ . This non-stationary Euler equation adds an unstable eigenvalue in the central bank's Hamiltonian system including three laws of motion of one forward variable (inflation  $\pi_t$ ) and of two predetermined variables ( $u_t, x_t$ ) or ( $u_t, \gamma_t$ ).

The *natural boundary condition*  $\gamma_0 = 0$  minimizes the loss function with respect to inflation at the initial date:

$$\gamma_0 = 0 \Rightarrow x_{-1} = -\frac{\kappa}{\alpha_x} \gamma_0 = 0 \text{ and } \pi_0 = -\frac{\alpha_x}{\kappa} x_0 \neq 0$$

It predetermines the policy instrument which allows to anchor the forward-looking policy target (inflation). The inflation Euler equation corresponding to period 0 is not an effective constraint for the central bank choosing its optimal plan in period 0. The former commitment to the value of the policy instrument of the previous period  $x_{-1}$  is not an effective constraint. The policy instrument is predetermined at the value zero  $x_{-1} = 0$  at the period preceding the commitment. Combining the two first order conditions to eliminate the Lagrange multipliers yields the optimal initial anchor of forward inflation  $\pi_0$  on the predetermined policy instrument  $x_0$ .

Ljungqvist and Sargent (2012, chapter 19) seek the stationary equilibrium process using the augmented discounted linear quadratic regulator solution of the Hamiltonian system (Anderson, Hansen, McGrattan and Sargent (1996)) as an intermediate step (ChateLain and Ralf (2017) algorithm). The policy instrument is exactly correlated with private sectors variables:

$$x_t = F_\pi \pi_t + F_u u_t. \quad (5)$$

We estimate three observationally equivalent estimations of Ramsey optimal policy structural VAR of the two observable variables (inflation and the policy instrument), using feasible generalized non-linear least squares for a system of equations (see appendix 2).

The **first estimation** estimates the two stable eigenvalues: the exogenous auto-correlation of cost-push shock  $\rho$ , the reduced form eigenvalue  $\lambda$  and the reduced form policy rule parameter with respect to inflation  $F_\pi$ :

$$\begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} = \begin{pmatrix} \rho \lambda & (1 - \rho) \lambda \frac{1}{F_\pi} \\ \rho (\lambda - 1) F_\pi & \rho + \lambda - \rho \lambda \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} + \begin{pmatrix} \eta_{t+1} \\ \frac{\beta \lambda \rho - 1}{F_\pi} \varepsilon_{t+1} \end{pmatrix} \quad (6)$$

The auto-correlation coefficient  $\rho$  is *into* the matrix of the VAR of inflation and the

policy instrument. It does not appear into the disturbances, which are now *white noise*, a necessary condition for estimating a VAR. We *assume* an i.i.d. measurement error  $\eta_{t+1}$  in order to avoid the stochastic singularity of the perfect collinearity of the inflation equation predicted by the Ramsey optimal policy theory (appendix 2). A high  $R^2$  for the inflation equation would signal that the variance of this measurement error is sufficiently small to remain close to the theoretical model.

$$\begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} = \begin{pmatrix} \rho \frac{1-\kappa F_\pi}{\beta} & (1-\rho) \frac{1-\kappa F_\pi}{\beta} \frac{1}{F_\pi} \\ \rho \left( \frac{1-\kappa F_\pi}{\beta} - 1 \right) F_\pi & \rho + \frac{1-\kappa F_\pi}{\beta} - \rho \frac{1-\kappa F_\pi}{\beta} \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} + \begin{pmatrix} \eta_{t+1} \\ \frac{\beta \lambda \rho - 1}{F_\pi} \varepsilon_{t+1} \end{pmatrix} \quad (7)$$

The **second estimation** estimates the two stable eigenvalues: the exogenous  $\rho$ , the reduced form eigenvalue  $\lambda$  and the ratio of two structural parameters  $\frac{\alpha_x}{\kappa} = \frac{1}{\varepsilon}$  where  $\varepsilon$  is the household's elasticity of substitution between each differentiated goods, if the central bank maximizes the representative household welfare.

$$\begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} = \begin{pmatrix} \rho \lambda & (1-\rho)(1-\lambda) \frac{\alpha_x}{\kappa} \\ -\rho \lambda \frac{\kappa}{\alpha_x} & \rho + \lambda - \rho \lambda \end{pmatrix} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} + \begin{pmatrix} \eta_{t+1} \\ \frac{\beta \lambda \rho - 1}{F_\pi} \varepsilon_{t+1} \end{pmatrix} \quad (8)$$

The **third estimation** substitutes the reduced form stable eigenvalue  $\lambda$  by its *non-linear* function of the three structural parameters  $\beta, \alpha_x, \kappa$  in the above VAR of the second estimation.

$$\lambda = \frac{1 - \kappa F_\pi}{\beta} = \frac{1}{2} \left( 1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta \alpha_x} \right) - \sqrt{\frac{1}{4} \left( 1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta \alpha_x} \right)^2 - \frac{1}{\beta q}} = \delta \quad (9)$$

This requires an identification restriction that we set on the discount factor  $\beta$ . We estimate separately three structural parameters: the auto-correlation of the cost-push shock  $\rho$ , the preference of the Fed  $\alpha_x$ , and the slope of the new-Keynesian Phillips curve  $\kappa$ . Only three structural parameters can be identified:  $\rho, \lambda, F_\pi$  or  $\rho, \lambda, \frac{\alpha_x}{\kappa}$  or  $\rho, \alpha(\beta q), \kappa(\beta q)$  for a given value of the discount factor  $\beta q$  (see appendix 2):

$$\kappa(\beta) = \frac{1 - \lambda \beta q}{F_\pi} \Rightarrow \alpha_x(\beta) = \left( \frac{\lambda}{1 - \lambda} \right) \frac{1}{F_\pi} \kappa(\beta q). \quad (10)$$

If initial values of inflation and of the policy instrument (in deviation from their equilibrium values) were perfectly measured at the date of commitment, the ratio  $\frac{\alpha_x}{\kappa}$  would be over-identified by the optimal initial anchor of forward inflation on the predetermined policy instrument equation:

$$\frac{\alpha_x}{\kappa} = \frac{-\pi_0}{x_0}. \quad (11)$$

The semi-reduced form cost-push shock rule parameter  $F_u$  requires an identification restriction, for example, setting a value for  $\beta$  (see appendix 2):

$$F_u(\beta) = \frac{-1}{1 - \beta \rho \lambda} F_\pi < 0. \quad (12)$$

The standard error  $\sigma_u$  of cost-push shock is computed using the standard error of residuals  $\sigma_{\varepsilon, x}$  of the output gap rule equation in the VAR(1). It requires an identification



restriction, because it depends on  $F_u$ :

$$\sigma_u(\beta) = \frac{\sigma_{\varepsilon,x}}{F_u(\beta)} = \frac{\beta\rho\lambda - 1}{F_\pi} \sigma_{\varepsilon,x}. \quad (13)$$

The standard error of the measurement of the inflation equation  $\sigma_\pi$  (which is theoretically predicted to be zero) and its covariance with the cost push shock  $\sigma_{x\pi} = F_u\sigma_{xu}$  are also available.

One identifying equation is missing in order to identify the remaining four structural parameters  $(\alpha_x, \kappa, \beta, \sigma_u)$  and the negative feedback rule parameter  $F_u$ . We set an identification restriction on the credibility weighted discount factor to a given values in the estimations.

For each regime, we estimate the slope of the Phillips curve  $\kappa_j$ , the elasticity of substitution between each differentiated goods  $(\frac{1}{\varepsilon_j} = \frac{\alpha_{x,j}}{\kappa_j})$ , the Fed's preferences  $\alpha_{x,j}$ , the auto-correlation of the cost-push shock  $\rho_j$  and its variance  $\sigma_{u,j}^2$ . We calibrate the two parameters  $\beta q$ . The measure of decreasing returns to scale of labor in the production functions of the firms  $\eta$  and the proportion of firms who do not reset their price each period  $\theta$  face an under-identification problem detailed in the next section.

## 2.4 Infinite Horizon Zero-Credibility Policy

With quasi-commitment, the probability of not renegeing commitment could be infinitely small (near-zero credibility), but it remains strictly positive: for example,  $q = 10^{-7} > 0$  (Roberts (1987)) with  $q \in ]0, 1]$ . Before Roberts (1987) stochastic replanning article, Oudiz and Sachs (1985) invented an infinite horizon zero-credibility policy, which holds when the policy maker re-optimizes with certainty for all future periods. This forever complete lack of credibility corresponds to  $q = 0$ . This zero-credibility policy is mentioned as "discretion" by Gali (2015) with a very minor change with respect to Oudiz and Sachs (1985) detailed in the appendix. Testing Roberts (1987) against Oudiz and Sachs (1985) amounts to test the assumption of a non-zero probability of not replanning and renegeing commitment  $q \in ]0, 1]$  against a zero probability of not renegeing commitment  $q = 0$  for ever, so that Oudiz and Sachs (1985) equilibrium is unlikely.

The central bank minimizes its loss function subject to the new-Keynesian Phillips curve *and* such that private sector and the central bank policy instrument reacts only to the contemporary *predetermined variable*  $u_t$  at all periods  $t$  with a *perfect correlation*. These zero-credibility rules are determined by time-invariant rule parameters  $N_{ZC}$  and  $F_{u,ZC}$  (indexed by  $ZC$ ) which are optimally chosen for all periods:

$$\pi_t = N_{ZC}u_t \text{ and } x_t = F_{u,ZC}u_t \Rightarrow x_t = F_{\pi,ZC}\pi_t \text{ with } F_{\pi,ZC} = \frac{F_{u,ZC}}{N_{ZC}} \quad (14)$$

The subscript TC is for zero-credibility policy. The central bank zero-credibility rule responds only to inflation with a perfect correlation. Substituting the private sector's inflation rule and the policy rule in the loss function:

$$\max_{\{\pi,x_t\}} -\frac{1}{2}E_0 \sum_{t=0}^{+\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2) = \max_{\{F_{u,ZC}, N_{ZC}\}} -\frac{1}{2} (N_{ZC}^2 + \alpha_x F_{u,ZC}^2) \frac{u_0^2}{1 - \beta\rho^2}$$

The central bank first order condition is:

$$0 = N_{u,ZC} \frac{\partial N_{u,ZC}}{\partial F_{u,ZC}} + \alpha_x F_{u,ZC}$$

$$F_{\pi,ZC} = \frac{F_{u,ZC}}{N_{ZC}} = -\frac{1}{\alpha_x} \frac{\partial N_{u,ZC}}{\partial F_{u,ZC}}$$

Substituting the private sector's inflation rule and the policy rule in the inflation law of motion leads to the following relation between  $N_{ZC}$  on date  $t$ ,  $N_{ZC,t+1}$  and  $F_{u,ZC}$ :

$$\begin{aligned}\pi_t &= \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \Rightarrow \\ N_{ZC} u_t &= \beta N_{ZC,t+1} \rho u_t + \kappa F_{u,ZC} u_t + u_t \\ N_{ZC} &= \beta \rho N_{ZC,t+1} + \kappa F_{u,ZC} + 1\end{aligned}$$

The central bank foresees that  $N_{ZC,t+1} = N_{ZC}$  in its optimization (Oudiz and Sachs' (1985), appendix):

$$N_{ZC} = \frac{\kappa F_{u,ZC} + 1}{1 - \beta \rho} = \frac{\kappa F_{\pi,ZC} N_{ZC} + 1}{1 - \beta \rho} \Rightarrow \frac{\partial N_{u,ZC}}{\partial F_{u,ZC}} = \frac{\kappa}{1 - \beta \rho}$$

The endogenous rule parameters are increasing function of the central bank cost of changing the policy instrument  $\alpha_x$ . They are bounded by limit values of  $\alpha_x \in ]0, +\infty[$ :

$$\begin{aligned}0 &< \frac{\pi_{t,ZC}}{u_t} = N_{ZC}(\alpha_x) = \frac{\alpha_x (1 - \beta \rho)}{\alpha_x (1 - \beta \rho)^2 + \kappa^2} < N = \frac{1}{1 - \beta \rho} \\ -\frac{1}{\kappa} &< \frac{x_{t,ZC}}{u_t} = F_{u,ZC}(\alpha_x) = \frac{-\kappa}{\alpha_x (1 - \beta \rho)^2 + \kappa^2} < 0 \\ -\infty &< \frac{x_{t,ZC}}{\pi_{t,ZC}} = F_{\pi,ZC}(\alpha_x) = \frac{F_{u,ZC}}{N_{ZC}} = \frac{-\kappa}{\alpha_x (1 - \beta \rho)} < \frac{-\kappa}{\alpha_x} < 0\end{aligned}$$

The policy instrument (the output gap) is exactly *negatively* correlated ( $F_{\pi,ZC} < 0$ ) with the policy target, inflation.

## 3 Specification tests of Ramsey versus Zero-Credibility Policy

### 3.1 Sign restrictions of reduced form rule parameters

Oudiz and Sachs (1985) equilibrium implies a *bifurcation* (a sharp qualitative change) of the economy dynamic system with respect to the later paper of Roberts (1987) stochastic replanning equilibrium.

**Proposition 1. Opposite sign restrictions.** *The inflation policy rule parameter  $F_{\pi}$  is positive and bounded using Roberts (1987) stochastic replanning equilibrium. The inflation policy rule  $F_{\pi,ZC}$  is negative and unbounded using Oudiz and Sachs (1985) zero-credibility equilibrium. There is a saddle-node bifurcation on the inflation eigenvalue  $0 < \lambda < 1 < \lambda_{ZC}$  when shifting from negative-feedback of Ramsey optimal policy under quasi-*

commitment equilibrium (eigenvalue  $\lambda$ ) to the positive-feedback mechanism of infinite horizon zero-credibility equilibrium (eigenvalue  $\lambda_{ZC}$ ).

**Proof:** The reduced form inflation rule parameters  $F_{\pi,ZC}$  of zero-credibility policy (respectively  $F_{\pi}$  of Ramsey optimal policy) is an *affine negative function* of the inflation eigenvalue  $\lambda_{ZC}$  (respectively  $\lambda$ ). The inflation rule parameters satisfies the following inequalities:

$$F_{\pi,ZC} = -\frac{1}{1-\beta\rho} \frac{\kappa}{\alpha_x} < 0 < \frac{1-\beta}{\kappa} < F_{\pi} = \frac{1-\beta\lambda}{\kappa} < \frac{1}{\kappa} \quad (15)$$

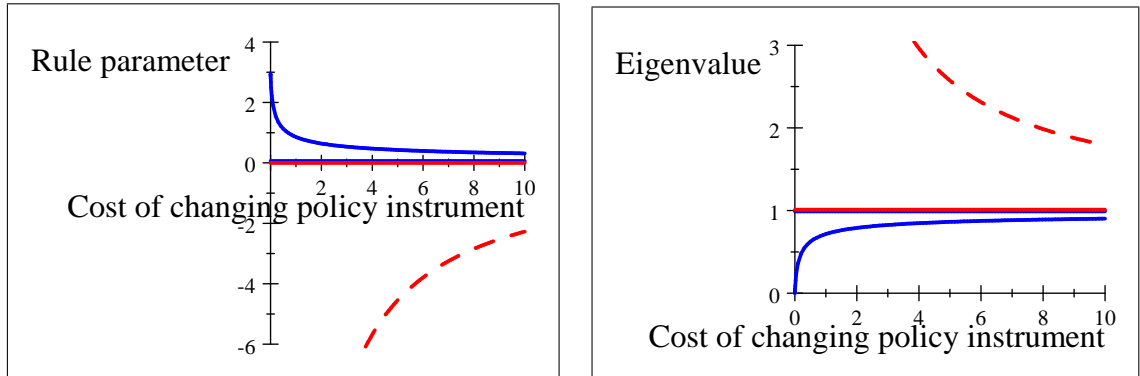
The inflation eigenvalues satisfies the following inequalities:

$$0 < \lambda = \frac{1}{2} \left( 1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right) - \sqrt{\frac{1}{4} \left( 1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} \right)^2 - \frac{1}{\beta}} < 1 \quad (16)$$

$$1 \leq \frac{1}{\beta} < \lambda_{ZC} = \frac{1-\kappa F_{\pi,ZC}}{\beta} = \frac{1}{\beta} + \frac{1}{\beta} \frac{1}{1-\beta\rho} \frac{\kappa^2}{\alpha_x} \quad (17)$$

The policy instrument (or the Lagrange multiplier on forward-looking inflation) is optimally predetermined for Ramsey optimal policy whereas it is forward-looking with infinite horizon zero-credibility policy. This implies an additional stable eigenvalue for Ramsey optimal policy with respect to zero-credibility policy, according to Blanchard and Kahn (1980) determinacy condition. QED.

Figures 1 and 2: Inflation rule parameter and inflation eigenvalues functions of  $\alpha_x$  for Ramsey optimal policy (solid line) and zero-credibility policy (dash line) for  $\rho = 0.995$ ,  $\kappa = 0.340$ ,  $\beta = 0.99$ .



The estimated parameters  $\rho = 0.995$ ,  $\kappa = 0.340$  for a given  $\beta = 0.99$  of the Ramsey optimal policy model are found during Volcker-Greenspan's Fed starting 1979q3-2006q2 in the next section. Figure 1 plots the positive-feedback inflation rule parameter  $F_{\pi,ZC} < 0$  of zero-credibility policy (dashed line) and respectively the negative-feedback inflation rule parameter  $F_{\pi} > 0$  of Ramsey optimal policy (continuous line) as non-linear increasing (respectively decreasing) function of the relative cost of changing the policy instrument  $\alpha_x$ . The estimated parameters correspond to the point:  $\alpha_x = 4.552$  and  $F_{\pi} = 0.447$ . Figure 2 plots the related eigenvalue  $\lambda_{ZC}$  of zero-credibility policy (and respectively the eigenvalue  $\lambda$  of Ramsey optimal policy) as non-linear decreasing (respectively increasing) function of the relative cost of changing the policy instrument  $\alpha_x$ . The estimated parameters correspond to the point  $\alpha_x = 4.552$  and  $\lambda = 0.856$ .

For an infinite cost of changing the policy instrument ( $\alpha_x \rightarrow +\infty$ , the Fed has maximal inertia): the negative-feedback rule parameter  $F_{\pi}$  decreases to its lowest value (zero). The

stable inflation eigenvalue  $\lambda$  tends to one for Ramsey optimal policy. For zero-credibility policy, the positive-feedback rule parameter  $F_\pi$  increases up to zero (with negative values). The unstable inflation eigenvalue  $\lambda_{ZC}$  decreases towards to  $1/\beta > 1$ .

For a near-zero cost of changing the policy instrument ( $\alpha_x \rightarrow 0$ , the Fed is an inflation nutter), the negative-feedback rule parameter  $F_\pi$  increases to its largest bounded positive value. The stable inflation eigenvalue  $\lambda$  tends to zero for Ramsey optimal policy. For zero-credibility policy, the positive-feedback rule parameter decreases to minus infinity (with negative values). The unstable inflation eigenvalue  $\lambda_{ZC}$  tends to plus plus infinity.

### 3.2 Specification tests of zero-credibility policy

Time-consistent policy is described by a permanent anchor of inflation on the output gap and by two AR(1) processes of inflation and of the output gap. Variables, such as inflation ( $\pi_t + \pi^*$ ) are not computed as deviations of equilibrium (already denoted  $\pi_t$ ). Estimates of equilibrium values ( $\pi^*, x^*$ ) are then sample mean values found in the estimates of intercepts. The reduced form zero-credibility policy rule to be tested (which corresponds to a permanent anchor of inflation on the output gap) allows to estimate the reduced form zero-credibility policy rule parameter  $F_{\pi,ZC}$ :

$$\begin{aligned} x_t + x^* &= F_{\pi,ZC} (\pi_t + \pi^*) + (x^* - F_{\pi,ZC}\pi^*) + \varepsilon_{x\pi,t} \text{ with} \\ \varepsilon_{x\pi,t} &= 0 \text{ for all dates, } R^2 = 1 \text{ and } F_{\pi,ZC} < 0 \end{aligned} \quad (18)$$

We assume i.i.d. measurement errors  $\eta_t$  for the inflation equation of Ramsey optimal policy. By the same token, for the infinite horizon zero-credibility policy, we also *assume* an i.i.d. measurement error  $\varepsilon_{x\pi,t}$ , for the policy rule equation, in order to avoid the stochastic singularity of the perfect negative correlation predicted by the theory. A high  $R^2$  for the inflation equation would signal this measurement error to be sufficiently small and close to the theoretical model. For example, one can perform a one-sided test of a composite null hypothesis of a simple correlation very close to minus one:  $H_{0,ZC,SS} : r_{x\pi} < -0.99$ . A first specification test of the negative sign restriction of zero-credibility policy is more informative.

$$\text{Negative sign restriction (NSR): } H_{0,ZC,NS} : r_{x\pi} < 0 \quad (19)$$

A positive sign of the zero-credibility rule suggests Ramsey optimal policy as an alternative, with an opposite sign on the policy rule parameter  $F_\pi$ . The simple correlation between the output gap and inflation provides another estimate of the zero-credibility policy rule parameter  $F_{\pi,ZC} = r_{x\pi}\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} < 0$ . It is equal to the one found using the ratio of standard errors of residuals of the AR(1) estimations for inflation and for the output gap :  $F_{\pi,ZC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi}$  only if the following condition is satisfied:  $r_{x\pi} = -1$  (stochastic singularity).

The rejection of stochastic singularity is expected to be stronger in the zero-credibility policy rule with a lower  $R^2$  (because it does not include a lagged dependent variable) than the one of the Ramsey inflation equation (which includes lagged inflation). *Hence, a second specification test of zero-credibility policy versus Ramsey optimal policy is a Durbin (1970) test of the auto-correlation of residuals of these measurement errors of the zero-credibility positive-feedback policy rule.* The measurement errors of the zero-credibility policy rule should not be auto-correlated according to the theory of zero-credibility policy:

White noise disturbances:  $H_{0,ZC,Durbin} : \rho_{\varepsilon,x\pi} = 0$  for  $\varepsilon_{x\pi,t} = \rho_{\varepsilon,x\pi}\varepsilon_{x\pi,t} + \eta_t$  with  $\eta_t$  i.i.d. (20)

If measurement errors are correlated, this alternative hypothesis suggests that at least one lagged policy instrument is missing in the regression of the policy rule. This is exactly a reduced form of the *Ramsey optimal policy rule* which also depends on inflation. Finally, we can perform another test. In the zero-credibility feedback rule, the policy instrument is a linear function of the policy target. The two AR(1) process for inflation and the output gap have common auto-correlation parameters:

$$\pi_t + \pi^* = \rho(\pi_{t-1} + \pi^*) + (1 - \rho)\pi^* + N_{ZC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \quad (21)$$

$$x_t + x^* = \rho(x_{t-1} + x^*) + (1 - \rho)x^* + F_{\pi,ZC}N_{ZC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \quad (22)$$

We can test the hypothesis of a common auto-correlation parameter for the policy target (inflation) and for the policy instrument (output gap):

$$\text{Common auto-correlation (CA): } H_{0,ZC,CA} : \rho_{\pi} = \rho_x \quad (23)$$

**Proposition 2:** *For zero-credibility policy, only the auto-correlation coefficient of the cost-push shock  $\rho$  can be identified. But the four other structural parameters are not identified: the Fed's preferences parameter  $\alpha_x$ , the slope of the new-Keynesian Phillips curve  $\kappa$ , the discount factor  $\beta$  and the variance of the cost-push shock  $\sigma_{\varepsilon,u}^2$ . If the Fed's preferences is a welfare loss function, where Fed's preferences parameter is endogenous ( $\alpha_x = \frac{\kappa}{\varepsilon}$ ), the representative household's elasticity of substitution between each differentiated goods  $\varepsilon$  is not identified.*

**Proof.** If common auto-correlation hypothesis is not rejected, the AR(1) estimates of the policy target and of the policy instrument identify the auto-correlation parameter of the non-observable cost-push shock:  $\rho$ . The ratio of the standard errors of residuals of each AR(1) estimations of inflation and output gap provides another estimate of the zero-credibility reduced form rule parameter  $F_{\pi,ZC}$ , (if  $r_{x\pi} = 1$ ), if the hypothesis of a negative sign is not rejected:

$$F_{\pi,ZC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} \quad (24)$$

The variance  $\sigma_{\varepsilon,\pi}^2$  of perturbations of the inflation AR(1) process is:

$$\sigma_{\varepsilon,\pi}^2 = N_{ZC}^2\sigma_{\varepsilon,u}^2 \Rightarrow N_{ZC}^2 = \frac{\sigma_{\varepsilon,\pi}^2}{\sigma_{\varepsilon,u}^2} \quad (25)$$

The cross equations covariance  $\sigma_{\varepsilon,\pi x}$  between the residuals of both AR(1) process of inflation and of the output gap does not allow to identify either the private sector reduced form parameter  $N_{ZC}$  anchoring inflation on the cost-push shock or the variance of the cost-push shock  $\sigma_{\varepsilon,u}^2$ . The simple correlation between the two residuals is predicted to be exactly negatively correlated ( $r_{\varepsilon,\pi x} = -1$ ):

$$\sigma_{\varepsilon,\pi x} = -\frac{\sigma_{\varepsilon,x}}{\sigma_{\varepsilon,\pi}}\frac{\sigma_{\varepsilon,\pi}^2}{\sigma_{\varepsilon,u}^2}\sigma_{\varepsilon,u}^2 = -\sigma_{\varepsilon,x}\sigma_{\varepsilon,\pi} < 0. \quad (26)$$

It is not possible to identify at least one of these four remaining structural parameters separately, because the identified parameter  $F_{\pi,ZC}$  does not depend only on one of these

four structural parameters:

$$F_{\pi,ZC} = \frac{-1}{1 - \beta\rho} \frac{\kappa}{\alpha_x} < 0. \quad (27)$$

Three identifying equations are missing in the case of zero-credibility policy. QED.

Finally, the tests of reduced form parameters of bivariate VAR(1) of zero-credibility policy versus Ramsey optimal policy are not feasible. The exact multicollinearity (exact correlation) between regressors (current output gap and inflation) imply a bivariate VAR(1) with infinite coefficients with denominator including the term  $1 - r_{x\pi}^2$  equal to zero:

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} +\infty & -\infty \\ -\infty & +\infty \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} N_{ZC} \\ F_{\pi,ZC} N_{ZC} \end{pmatrix} \varepsilon_t \quad (28)$$

The zero-credibility policy equilibrium predicts that out-of-equilibrium behavior corresponds to a non-stationary bivariate VAR including one unstable eigenvalue  $\lambda_{ZC}$  and one stable eigenvalue  $\rho$ , which cannot be estimated. By contrast, the stationary structural VAR(1) of output gap and inflation with Ramsey optimal policy allows to identify a larger number of structural parameters.

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} F_{u,R} \\ 0 \end{pmatrix} \varepsilon_t$$

Two additional reduced form parameters ( $b, c$ ) are available, because the stable subspace of the VAR process is of dimension two with Ramsey optimal policy instead of dimension one with zero-credibility policy.

## 4 Tests with Output Gap as Policy Instrument

### 4.1 Specification tests

The annualized quarter-on-quarter rate of inflation and the congressional budget office (CBO) measure of the output gap are taken from Mavroeidis' (2010) online appendix (detailed information at the end of this paper's appendix). The pre-Volcker sample covers the period 1960q1 to 1979q2 and the Volcker-Greenspan sample runs until 2006q2. The period of Paul Volcker's tenure is 1979q3 to 1987q2. The period of Alan Greenspan's tenure is 1987q3 to 2006q1.

According to Debortoli and Nunes (2015), a structural break corresponds to an new initial anchor of forward inflation on the output gap for Ramsey optimal policy with finite horizon. Clarida, Gali and Gertler (2000) and Mavroeidis (2010) consider the beginning of Paul Volcker's mandate 1979q3 as a structural break. Givens (2012) considers 1982q1 as a structural break, after 1981 fall of inflation and before the 1982 recession. Matthes (2015) estimation of the private sectors beliefs regarding central bank regimes also points to 1982q1 as a structural break. Table 2 presents summary statistics before and after the 1979q3 and 1982q1 structural breaks.

**Table 2: Summary statistics of inflation and output gap**

	dates	obs.	mean	min	max	after:	obs.	mean	min	max
$\pi_t$	< 79q3	78	4.39 (2.71)	0.59	11.79	$\geq$ 79q3	108	3.18 (2.03)	0.64	10.93
$x_t$	< 79q3	78	0.47 (2.59)	-4.97	6.10	$\geq$ 79q3	108	-1.11 (2.07)	-7.95	3.01
$\pi_t$	< 82q1	88	4.86 (2.90)	0.59	11.79	$\geq$ 82q1	98	2.64 (1.08)	0.64	5.61
$x_t$	< 82q1	88	0.20 (2.59)	-4.97	6.10	$\geq$ 79q3	98	-1.03 (2.12)	-7.95	3.01

Standard deviations are in parentheses.

The mean of inflation and of output gap are lower during Volcker-Greenspan than before Volcker. Excluding the period 1979q3 to 1981q4, in particular the sharp disinflation which occurred during 1981 (figure 3), the standard error of inflation decreases by half from 2.03 to 1.08 .

**Table 3: Pre-tests of zero-credibility policy rule**

Dates	obs	$r_{x\pi}$	Low 95% $r$	$p$	$R_{x\pi}^2$	$F_{\pi,ZC}$	$c$	$\rho_{\varepsilon,x\pi}$
< 79q3	78	-0.13	-0.21	< 0.001	0.02	-0.13 (0.11)	1.03 (0.56)	<b>0.92</b> (0.04)
< 82q1	108	-0.30	-0.42	< 0.001	0.09	-0.30 (0.09)	-0.14 (0.35)	<b>0.91</b> (0.04)
$\geq$ 79q3	88	-0.24	-0.31	< 0.001	0.06	<b>-0.22</b> (0.09)	1.25 (0.53)	<b>0.92</b> (0.05)
$\geq$ 82q1	98	-0.40	-0.53	< 0.001	0.16	-0.78 (0.18)	1.03 (0.52)	<b>0.89</b> (0.05)

The pre-tests of the null hypothesis of a quasi perfect negative correlation  $H_0 : r_{x\pi} < -0.99$  between observed inflation and observed output gap are rejected. The test uses Fisher's  $Z$  transformation using the procedure corr with the software SAS. The threshold of the composite null hypothesis  $-0.99$  is far away from the 95% single tail confidence interval, where the lowest 95% confidence limit reported in table 3 is at most equal to  $-0.53$  for the period beginning from 1982q1 (figure 4). The opposite null hypothesis  $H_0 : r(x_t, \pi_t) = 0$  is not rejected before 1979q3. zero-credibility policy predicts a perfect correlation for the anchor of inflation expectations with the output gap. If the zero-credibility policy equilibrium occurred before 1979q3, we do not reject the null hypothesis  $H_0 : r(E_{t-1}(\pi_t), \pi_t) = 0$  that the rational expectations of inflation are orthogonal to observed inflation.

The pre-tests of the null hypothesis of the auto-correlation of residuals  $H_0 : \rho_{\varepsilon,x\pi} = 0$  are strongly rejected, with a point estimate at least equal to 0.89 (figure 5). These tests gives a hint of model misspecification. They suggest an omitted lagged policy instrument in the policy rule. When it is included in Ramsey optimal policy rule, the  $R^2$  increases from 16% (table 3, last line) to 93% (table 6, last line) beginning in 1982q1.

**Table 4: Auto-correlation of inflation and output gap**

dates	obs.	var.	$r$	$R^2$	$\rho$	$c$	$\sigma_\varepsilon$	$\rho_\varepsilon$	$DF$	$PP$
< 79q3	78	$\pi_t$	0.86	0.74	0.88 (0.06)	0.62 (0.30)	1.38	-0.22 (0.12)	0.55	0.41
< 79q3	78	$x_t$	0.93	0.86	0.93 (0.04)	0.03 (0.11)	0.99	0.26 (0.11)	0.20	0.33
< 82q1	88	$\pi_t$	0.88	0.78	0.88 (0.05)	0.63 (0.28)	1.35	-0.19 (0.11)	0.34	0.23
< 82q1	88	$x_t$	0.92	0.85	0.93 (0.04)	-0.03 (0.11)	1.03	0.23 (0.11)	0.27	0.37
$\geq$ 79q3	108	$\pi_t$	0.89	0.79	0.85 (0.04)	0.42 (0.16)	0.93	-0.27 (0.09)	0.04	0.01
$\geq$ 79q3	108	$x_t$	0.94	0.88	0.94 (0.03)	-0.07 (0.08)	0.70	0.34 (0.09)	0.11	0.25
$\geq$ 82q1	98	$\pi_t$	<b>0.64</b>	0.41	<b>0.59</b> (0.07)	1.06 (0.21)	0.83	<b>-0.20</b> (0.10)	<b>0.00</b>	<b>0.00</b>
$\geq$ 82q1	98	$x_t$	<b>0.96</b>	0.92	<b>0.95</b> (0.03)	-0.02 (0.07)	0.60	<b>0.35</b> (0.09)	<b>0.07</b>	<b>0.35</b>

Table 4 investigates the auto-correlation and unit roots of inflation and output gap. The output gap and inflation are highly auto-correlated (respectively 0.93 and 0.86), except when inflation excludes the 1981 disinflation for the period after 1981q4. For the period 1982q1 to 2006q2, the inflation auto-correlation coefficient falls in the 95% confidence interval  $0.6 \pm 0.14$  and it is statistically different from the output gap auto-correlation coefficient in the 95% confidence interval  $0.95 \pm 0.06$  (figures 4 and 5). As the zero-credibility policy equilibrium predicts that the auto-correlation of the output gap and of inflation should be the same, this is an additional test against zero-credibility policy, which holds for the period 1982q1 to 2006q2.

There is a negative auto-correlation of residuals  $\rho_\varepsilon$  for inflation and a (statistically significant at the 5% level) positive auto-correlation of residuals for the output gap. The column DF reports the p-value of the Dickey-Fuller test of unit root with one lag without trend. The column PP reports the p-value of the Phillips-Perron test of unit root, which takes into account auto-correlation, with one lag without trend. The null hypothesis of a unit root is rejected for inflation after 1979q2 and after 1981q4.

## 4.2 Tests of Ramsey Optimal Policy

Table 5 presents estimates of structural parameters, Table 5 report estimates of three structural VAR estimations for  $(\rho, \lambda_R, F_{\pi,R})$ ,  $(\rho, \lambda_R, \frac{\alpha_x}{\kappa})$  and  $(\rho, \kappa(\beta), \alpha_x(\beta))$  with two given values for the discount factor  $\beta = 1$  or  $\beta = 0.99$  for the Volcker-Greenspan period. With these three estimations, delta method is not necessary to compute the standard errors of parameters in each case. Maximum likelihood did not converge for pre-Volcker period (unconstrained VAR corresponds to complex conjugate eigenvalues, which are excluded because of exogenous real auto-correlation of cost-push shock). Post 1982q1 estimations converged to unlikely estimates.

**Table 5: Ramsey optimal policy structural parameters**

Dates	$\rho$	$\lambda$	$F_\pi$	$\frac{\alpha_x}{\kappa} = \frac{1}{\varepsilon}$	$\beta$	$\kappa(\beta)$	$\alpha_x(\beta)$	$F_{u,R}(\beta)$	$\sigma_u(\beta)$
$\geq$ 79q3	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	1	0.321 (0.303)	4.296 (5.447)	-3.027	0.229
$\geq$ 79q3	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	0.99	0.340 (0.314)	4.552 (5.703)	-2.861	0.242

The cost-push shock faces is extremely persistent, close to a unit root, with  $\rho$  estimate close to one. The ratio  $\frac{\alpha_x}{\kappa}$  is statistically significant. If the Fed's preferences are identical to (welfare) household's preferences), then  $\frac{\alpha_x}{\kappa} = \frac{1}{\varepsilon}$  and  $\hat{\varepsilon} = 0.07$ . The new-Keynesian Phillips curve parameter  $\kappa$  is relatively large. The Fed's preference parameter  $\alpha_x$  is



relatively large (although not unheard of in previous estimations). for the period 1979-2006.

**Table 6: Inflation and output gap structural (S) versus unconstrained (U) VAR**

dates	obs.	var.	S/U	$\pi_{t-1}$	$x_{t-1}$	$c$	$\rho_\varepsilon$	$R^2$	$\lambda$
$\geq 79q3$	108	$\pi_t$	S	0.85	0.009	0.428 (0.16)	-0.25 (0.09)	0.793	0.857* (0.054)
$\geq 79q3$	108	$\pi_t$	U	0.85 (0.04)	0.009 (0.04)	0.43 (0.16)	-0.25 (0.09)	0.79	0.85
$\geq 79q3$	108	$x_t$	S	-0.064	<b>0.999</b>	0.198 (0.121)	0.29 (0.09)	0.888	<b>0.995*</b> (0.024)
$\geq 79q3$	108	$x_t$	U	-0.084 (0.03)	0.917 (0.03)	0.17 (0.11)	0.29 (0.09)	0.89	0.92

Table 6 compares the reduced form parameters of the VAR of Ramsey optimal policy with parameters of an unconstrained VAR. The inflation equation of the VAR are the same up to the third decimal of all statistics. For the output gap equation, *Ramsey optimal policy slightly over-estimates the persistence of the output gap: its VAR auto-correlation parameter shifts from 0.93 to 1*. For the output gap rule, the auto-correlation of residuals fell from 0.92 with zero-credibility policy to 0.29 with Ramsey optimal policy, but it remains statistically significant. As well, the inflation equation has a statistically significant auto-correlation (-0.25).

The reduced form Ramsey optimal policy rule of the structural VAR is observationally equivalent to the LQR first step representation including the non-observable cost-push shock, when taking into account the other equations of the Hamiltonian system:

$$\text{Ramsey : } x_t = 0.995x_{t-1} - 0.064\pi_{t-1} - 2.861\varepsilon_{u,t} \text{ or } x_t = 0.447\pi_t - 2.861u_t \text{ for } \beta = 0.99 \quad (29)$$

The policy instrument responds to two variables for Ramsey stable subspace of dimension two. The policy instrument responds to one variable in the zero-credibility policy stable subspace of dimension one. The reduced form policy rule for zero-credibility policy is:

$$\text{Time-consistent: } x_t = -0.22\pi_t \quad (30)$$

## 5 Tests with Federal Funds Rate Smoothing and Working Capital Channel

### 5.1 Specification tests

Assuming the polar case where all labor cost is financed by working capital instead than no labor cost at all, the federal funds rate is used here as a policy instrument. Its variance enters into the period loss function  $\pi_t^2 + \alpha_i i_t^2$  as an interest smoothing model. Figure 2 represents the time series of inflation and federal funds rate. Table 2 presents summary statistics before and after the 1979q3 and 1982q1 structural breaks.

**Table 2B: Summary statistics of inflation and federal funds rate**

	dates	obs.	mean	min	max	after:	obs.	mean	min	max
$\pi_t$	< 79q3	78	4.39 (2.71)	0.59	11.79	$\geq$ 79q3	108	3.18 (2.03)	0.64	10.93
$i_t$	< 79q3	78	5.47 (2.42)	1.68	12.09	$\geq$ 79q3	108	6.56 (3.76)	1.00	17.78
$\pi_t$	< 82q1	88	4.86 (2.90)	0.59	11.79	$\geq$ 82q1	98	2.64 (1.08)	0.64	5.61
$i_t$	< 82q1	88	6.84 (3.74)	1.68	17.78	$\geq$ 79q3	98	5.76 (2.84)	1.00	14.51

Standard deviations are in parentheses below the mean.

The means of inflation are lower after Volcker than before Volcker. Excluding the period 1979q3 to 1981q4, in particular the sharp disinflation which occurred during 1981 (figure 3), the standard error of inflation decreases by half from 2.03 to 1.08. The difference of means between the policy interest rate and inflation increased after Volcker.

**Table 3B: Pre-test of zero-credibility policy rule**

dates	obs	$r_{i\pi}$	$t$	$r_{i\pi} = 0 : p$	$R_{i\pi}^2$	$F_{\pi,D}$	$c$	$\rho_{\varepsilon,i\pi}$
< 79q3	78	0.83	12.85	< 0.001	0.68	0.74 (2.22)	2.22 (0.30)	0.61 (0.09)
< 82q1	88	0.79	11.83	< 0.001	0.62	1.01 (0.08)	1.55 (0.48)	0.73 (0.08)
$\geq$ 79q3	108	0.75	11.63	< 0.001	0.56	1.39 (0.12)	1.39 (0.11)	0.76 (0.06)
$\geq$ 82q1	98	0.53	6.08	< 0.001	0.28	1.39 (0.22)	2.09 (0.65)	0.80 (0.06)

The tests of the null hypothesis of a quasi perfect negative correlation  $H_0 : r_{i\pi} < -0.99$  between inflation and federal funds rate have been replaced by the usual tests of the null hypothesis:  $H_0 : r_{i\pi} = 0$ , because  $r_{i\pi} > 0$ . The negative sign of the zero-credibility policy rule is rejected, with large  $t$  statistics. The Durbin and Breusch-Godfrey tests strongly reject the lack of serial correlation (for one or two lags) with p-value below  $10^{-4}$ . Tests of the null hypothesis of the first order auto-correlation of residuals  $H_0 : \rho_{\varepsilon,i\pi} = 0$  are rejected, with a point estimate at least equal to 0.60 and at most 0.80. The infinite horizon zero-credibility positive-feedback mechanism *does not fit the data when the cost of capital is taken into account into the monetary transmission mechanism*. Finally, the Taylor principle (an inflation coefficient  $F_\pi$  larger than one) is not satisfied before Volcker and satisfied after Volcker.

Table 4B investigates the auto-correlation of inflation and federal funds rate. Inflation and Federal funds rate are highly auto-correlated (respectively 0.93 and 0.86), except for the period 1982q1 to 2006q2, the inflation auto-correlation coefficient falls in the 95% confidence interval  $0.6 \pm 0.14$  and it is statistically different from the federal funds rate auto-correlation coefficient in the 95% confidence interval  $0.95 \pm 0.02$ . Infinite horizon zero-credibility policy predict that the auto-correlation should be the same, which is not the case after 1982q1.

**Table 4B. Auto-correlation of inflation and federal funds rate**

dates	obs.	var.	$r$	$R^2$	$\rho$	$c$	$\sigma_\varepsilon$	$\rho_\varepsilon$	DF	PP
< 79q3	78	$\pi_t$	0.86	0.74	0.88 (0.06)	0.62 (0.30)	1.38	-0.22 (0.12)	0.55	0.41
< 79q3	78	$i_t$	0.93	0.87	0.95 (0.04)	0.33 (0.25)	0.89	0.43 (0.10)	0.17	0.54
< 82q1	88	$\pi_t$	0.88	0.78	0.88 (0.05)	0.63 (0.28)	1.35	-0.19 (0.11)	0.34	0.23
< 82q1	88	$i_t$	0.94	0.89	0.96 (0.04)	0.36 (0.27)	1.26	0.21 (0.11)	0.42	0.65
$\geq$ 79q3	108	$\pi_t$	0.89	0.79	0.85 (0.04)	0.42 (0.16)	0.93	-0.27 (0.09)	0.04	0.01
$\geq$ 79q3	108	$i_t$	0.94	0.92	0.96 (0.03)	0.23 (0.20)	1.05	0.15 (0.09)	0.38	0.44
$\geq$ 82q1	98	$\pi_t$	<b>0.64</b>	0.41	0.59 (0.07)	1.06 (0.21)	0.83	-0.20 (0.10)	0.00	0.00
$\geq$ 82q1	98	$i_t$	0.96	0.95	0.94 (0.02)	0.26 (0.14)	0.63	0.45 (0.09)	0.18	0.08

There is a negative auto-correlation of residuals  $\rho_\varepsilon$  for inflation and a positive auto-correlation of residuals for federal funds rate. The column DF reports the  $p$ -value of the Dickey-Fuller test of unit root with one lag without trend. The column PP reports the  $p$ -value of the Phillips-Perron test of unit root, which takes into account auto-correlation, with one lag without trend. The null hypothesis of a unit root is rejected at the 5% threshold for inflation after 1979q2 and after 1981q4. It is rejected for federal funds rate at the 10% level after 1981q4 only for the Phillips-Perron test.

## 5.2 Tests of Ramsey optimal policy

Table 5B reports estimates using four structural VAR estimations for  $(\rho, \lambda_C, F_{\pi,C})$ , for  $(\rho, \lambda_C, \frac{\alpha_i}{\kappa_i})$  and for  $(\rho, \kappa(\beta), \alpha_i(\beta))$  with the identification restrictions for the credibility adjusted discount factor. We check that these three estimations are observationally equivalent. In particular, each estimate satisfy the theoretical relations with the others implied by Ramsey optimal policy. When the federal funds rate is the policy instrument, the estimations only converged to plausible values for the pre-Volcker period, before 1979q3. The implicit rule parameter on the cost-push shock is computed using  $F_u(\beta) = \frac{-1}{1-\beta\rho\lambda}F_\pi$  and the variance of the cost push shock is  $\sigma_u(\beta) = \frac{\sigma_{\varepsilon,i}}{-F_u(\beta)}$  with the root mean square error of the residuals of the policy instrument equation equal to  $\sigma_{\varepsilon,i} = 0.876$ .

**Table 5B: Ramsey optimal policy structural parameters**

Dates	$\rho$	$\lambda$	$F_\pi$	$\frac{\alpha_i}{\kappa}$	$\beta$	$\kappa_i(\beta)$	$\alpha_i(\beta)$	$F_{u,C}(\beta)$	$\sigma_u(\beta)$
< 79q3	0.550* (0.092)	0.947* (0.047)	0.873* (0.131)	20.83 (18.26)	1	0.060 (0.06)	1.24* (0.34)	-1.822	0.481
< 79q3	0.550* (0.092)	-	-	-	0.99	0.071 (0.06)	1.47* (0.36)	-1.802	0.486
< 79q3	0.550* (0.092)	-	-	-	0.94	0.125* (0.063)	2.60* (0.16)	-1.710	0.512
< 79q3	0.550* (0.092)	-	-	-	0.93	0.136* (0.064)	2.83* (1.35)	-1.693	0.517
< 79q3	0.550* (0.092)	-	-	-	0.92	0.146* (0.065)	3.05* (1.54)	-1.676	0.522

Estimates of structural parameters are plausible values. The Fed's reduced form rule parameter  $F_\pi$  is significantly different from zero at the 5%. To reach the statistical significance at the 5% level of *both*  $\kappa_i(\beta)$  and  $\alpha_i(\beta)$  and not only one of the two parameters is obtained for identification restrictions of the credibility adjusted discount factor  $\beta \in [0.92, 0.94]$ . This corresponds to a Fed's probability to commit in this range:  $q \in [0.93, 0.95]$  for an identification restriction of the discount factor 0.99. For  $\beta > 0.94$ ,

only the Fed's preference  $\alpha_i(\beta)$  is statistically significant. For  $\beta < 0.92$ , only the slope of the new-Keynesian Phillips curve for working capital  $\kappa_i(\beta)$  is statistically significant. When the identification restriction on  $\beta$  increases, both estimates  $\kappa_i(\beta)$  and  $\alpha_i(\beta)$  increase, but not at the same pace.

Table 6B shows that the reduced form parameters of the structural VAR(1) (rows S) are *identical* to the unconstrained VAR(1) estimates (rows U) up to the third decimal.

**Table 6B: Inflation and federal funds rate structural (S) versus unconstrained (U) VAR**

dates	obs.	var.	S/U	$\pi_{t-1}$	$i_{t-1}$	$c$	$\rho_\varepsilon$	$R^2$	$\Delta R^2$	$\lambda$
< 79q3	77	$\pi_t$	S	0.521	0.488	-0.471 (0.340)	-0.12 (0.16)	0.804	0.06	0.550* (0.092)
< 79q3	77	$\pi_t$	U	0.52 (0.09)	0.49 (0.10)	-0.471 (0.343)	-0.12 (0.16)	0.804	0.06	0.550
< 79q3	77	$i_t$	S	-0.025	0.976	0.316 (0.226)	0.55 (0.12)	0.869	0.01	0.947* (0.047)
< 79q3	77	$i_t$	U	-0.025 (0.065)	0.976 (0.07)	0.317 (0.252)	0.55 (0.12)	0.869	0.01	0.947

(1) For the inflation equation, there is Granger causality from lagged federal funds rate to inflation. The exogenous cost-push shock auto-correlation is close to the auto-correlation of inflation in the VAR. The residuals are not auto-correlated controlling for endogenous lagged inflation and federal funds according to Durbin's test. The autocorrelation estimate is  $-0.12$ , It not statistically different from zero (Durbin's test:  $t = -0.77$ ,  $p = 0.44$ ).

(2) For the federal funds rate rule equation, the auto-correlation 0.976 is relatively close to a unit root. The endogenous stable eigenvalue corresponds to the auto-correlation of the federal funds rate.

One can compare our modest results with two recent papers using implicit functions of the parameters within a reduced form VAR including three US time-series (inflation, output gap and Federal funds rate). Givens (2012) finds out that Volcker-Greenspan period corresponds to zero-credibility policy whereas Matthes (2015) finds it corresponds to Ramsey optimal policy. The ratio of the estimated weight on output in the loss function in commitment versus discretion in Givens (2012):  $\hat{\lambda}_D/\hat{\lambda}_C = 0.0987/0.1351 = 0.7$  and 10 times larger in Matthes (2015)  $\hat{\lambda}_D/\hat{\lambda}_C = 0.49/0.07 = 7$ . For the transmission mechanism, Givens (2012) estimate of the intertemporal elasticity of substitution is 0.0089 (estimated standard error 0.0035, 95% confidence interval [0.0019, 0.0159]) for Ramsey optimal policy and 0.0002 for zero-credibility policy, with an estimated standard error 0.0001 so that zero belong to the 95% confidence interval. Matthes (2015) estimate is 70 times larger  $1/1.61 = 0.62$  for both policies. The estimated standard error of this parameter is not disclosed. Givens (2012) estimate of the slope of the new-Keynesian Phillips curve is  $\hat{\kappa} = 0.0045$ , with estimated standard error 0.0016 and 95% confidence interval [0.0019, 0.0159]), for Ramsey optimal policy and 0.0047 for discretion with estimated standard error 0.009 so that zero belongs to the 95% confidence interval ("*Estimates of  $\kappa$  are small but within the range typical of the literature.*" p.1043). In the discretion regime, both transmission mechanism parameters for the Federal funds rate not statistically different from zero. This result supports the hypothesis of monetary policy ineffectiveness during 1982-2006. Matthes (2015) estimate is 75 times larger:  $\hat{\kappa} = 0.7/(1 + 0.99) = 0.35$ : "*The estimate found here, while being at the other end, is not unheard of*". The estimated standard error of this parameter is not disclosed.

Givens (2012) assumes one lag of inflation, two lags of output and one lag for Federal funds rate. Matthes (2015) assumes one lag of inflation and two exogenous auto-regressive

forcing variables. The more lags of observable exogenous variables are added in the specification of the new-Keynesian Phillips curve, the more likely the weak identification of some parameters occurs (Mavroeidis (2005), Mavroeidis et al. (2014), Dees, Pesaran, Smith, Smith (2009), Canova and Sala (2009), Komunjer and Ng (2012) and Iskrev (2012)).

## 6 Conclusion

Using closed form solutions of Ramsey optimal policy and zero-credibility policy, we take *exactly* into account the identification restrictions related to the dimension of the stable subspace of each policy. Ramsey optimal policy with quasi-commitment has a comparative advantage with respect to infinite horizon zero-credibility policy for modelling persistence *with fewer parameters*. Further work may extend this approach to other transmission mechanisms than the new-Keynesian Phillips curve with labor cost channel or with working capital channel.

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## 7 Appendix 1: Definition of data variables

Mavroeidis data are running from 1960-Q1 to 2006-Q2.

Inflation is annualized quarter-on-quarter rate of inflation,  $400 * \text{LN}(\text{GDPDEF}/\text{GDPDEF}(-1))$  with GDPDEF: Gross Domestic Product Implicit Price Deflator, 2000=100, Seasonally Adjusted. Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis.

GAPCBO is the output gap measure:  $100 * \text{LN}(\text{GDPC1}/\text{GDPPOT})$  with GDPC1: Real Gross Domestic Product, Billions of Chained 2000 Dollars, Seasonally Adjusted Annual Rate, Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis and GDPPOT: Real Potential Gross Domestic Product, Billions of Chained 2000 Dollars. Source: U.S. Congress, Congressional Budget Office.

Federal Funds Rate : Averages of Daily Figures - Percent, Source: Board of Governors of the Federal Reserve System

### 7.1 Appendix 2: Augmented Discounted Linear Quadratic Regulator

The new-Keynesian Phillips curve can be written as a function of the Lagrange multiplier where  $\kappa > 0$ ,  $0 < \beta < 1$  and  $0 < q < 1$  (Debortoli and Nunes (2014, appendix A). We keep Gali (2015) chapter 5  $\gamma_{t+1}$  notation of the Lagrange multiplier with one step ahead subscript: it corresponds to Debortoli and Nunes (2014) notation  $\lambda_t$ . Our notation for the stable eigenvalue  $\lambda$  corresponds to Debortoli and Nunes (2014) notations " $\psi_y = 1/\gamma$ ".

$$E_t \pi_{t+1} + \frac{\kappa^2}{\beta q \alpha_x} \gamma_{t+1} = \frac{1}{\beta q} \pi_t - \frac{1}{\beta q} u_t - \frac{1-q}{q} E_t \pi_{t+1}^j$$

In what follows,  $\beta$  refers to  $\beta q$  to simplify notations. The solution of the Hamiltonian system are based on the demonstrations of the augmented discounted linear quadratic regulator in Anderson, Hansen, McGrattan and Sargent [1996], following the steps in Chatelain and Ralf (2017c):

$$\begin{pmatrix} 1 & \frac{\kappa^2}{\beta\alpha_x} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \pi_{t+1} \\ \gamma_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} & 0 & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_t \\ \gamma_t \\ u_t \end{pmatrix} + \begin{pmatrix} -\frac{1-q}{q} E_t \pi_{t+1}^j \\ 0 \\ 0 \end{pmatrix}$$

The Hamiltonian system is:

$$\begin{pmatrix} \pi_{t+1} \\ \gamma_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_t \\ \gamma_t \\ u_t \end{pmatrix} + \begin{pmatrix} -\frac{1-q}{q} E_t \pi_{t+1}^j \\ 0 \\ 0 \end{pmatrix}$$

The characteristic polynomial of this upper square matrix is:

$$\lambda^2 - \left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x}\right) \lambda + \frac{1}{\beta} = 0$$

The Hamiltonian matrix has two stable roots  $\rho$  and  $\lambda$  ( $\lambda$  is denoted  $\delta$  in Gali (2015)) and one unstable root  $\frac{1}{\beta\lambda}$ . The determinant of the matrix is  $\rho\lambda\frac{1}{\beta\lambda} = \rho\frac{1}{\beta}$ . Then  $\lambda < \sqrt{\frac{1}{\beta}} < \frac{1}{\beta\lambda}$ . The trace of the matrix is

$$\lambda = \frac{1}{2} \left( 1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - \sqrt{\left(1 + \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x}\right)^2 - \frac{4}{\beta}} \right)$$

**Identification of  $\frac{\kappa}{\alpha_x}$ :** The characteristic polynomial is equal to zero. The policy rule parameter with respect to inflation depends on  $\frac{\kappa}{\alpha_x}$ :

$$(1 - \lambda) \left(1 - \frac{1}{\beta\lambda}\right) = -\frac{\kappa^2}{\beta\alpha_x} \implies \left(\frac{1 - \lambda}{\beta\lambda}\right) \left(\frac{\beta\lambda - 1}{\kappa}\right) = -\frac{\kappa}{\beta\alpha_x} \implies F_\pi = \frac{1 - \beta\lambda}{\kappa} = \left(\frac{\lambda}{1 - \lambda}\right) \frac{\kappa}{\alpha_x}$$

**Hamiltonian system function of the stable eigenvalue  $\lambda$  (eliminating  $\frac{\kappa}{\alpha_x}$ ):**

$$\begin{pmatrix} \pi_{t+1} \\ \gamma_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \lambda + \frac{1}{\beta\lambda} - 1 & 1 + \frac{1}{\beta} - \lambda - \frac{1}{\beta\lambda} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_t \\ \gamma_t \\ u_t \end{pmatrix}$$

**Proposition 1: Solution of Riccati and Sylvester equation:** Rule parameters  $P_u$  and  $P_z$  of the response of the Lagrange multiplier on inflation to exogenous variables:

$$\gamma_t = P_\pi \pi_t + P_u u_t + P_z z_t \quad (31)$$

$$P_\pi = \frac{1}{1-\lambda}, P_u = \frac{1}{1-\lambda} \frac{\frac{1}{\beta}}{\rho - \frac{1}{\beta\lambda}} = \frac{1}{1-\lambda} \frac{\lambda}{\beta\lambda\rho - 1} \text{ and } P_z = \frac{1}{1-\lambda} \frac{\frac{1}{\beta}}{\mu - \frac{1}{\beta\lambda}} = \frac{1}{1-\lambda} \frac{\lambda}{\beta\lambda\mu - 1} \quad (32)$$

**Demonstration:** It uses the method of undetermined coefficients of Anderson, Hansen, McGrattan and Sargent's (1996), section 5. The solution is the one that stabilizes the state-costate vector for any initialization of inflation  $\pi_0$  and of the exogenous variables  $u_0$  in a stable subspace of dimension two within a space of dimension three  $(\pi_t, \gamma_t, u_t)$  of the Hamiltonian system. We seek a characterization of the Lagrange multiplier  $\gamma_t$  of the form:

$$\gamma_t = P_\pi \pi_t + P_u u_t + P_z z_t.$$

To deduce the control law associated with vector  $(P_\pi, P_u, P_z)$ , we substitute it into the Hamiltonian system:

$$\begin{pmatrix} \pi_{t+1} \\ P_\pi \pi_{t+1} + P_u u_{t+1} + P_z z_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta} - (1-\lambda) \left(1 - \frac{1}{\beta\lambda}\right) & (1-\lambda) \left(1 - \frac{1}{\beta\lambda}\right) & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_t \\ P_\pi \pi_t + P_u u_t + P_z z_t \\ u_t \end{pmatrix}$$

We write the last two equations in this system separately:

$$\begin{aligned} P_\pi \pi_{t+1} + P_u u_{t+1} + P_z z_{t+1} &= (P_\pi - 1) \pi_t + P_u u_t + P_z z_t \\ u_{t+1} &= \rho u_t \end{aligned}$$

It follows that:

$$\pi_{t+1} = \frac{P_\pi - 1}{P_\pi} \pi_t + \frac{(1-\rho) P_u}{P_\pi} u_t + \frac{(1-\mu) P_z}{P_\pi} z_t$$

The first equation is such that:

$$\pi_{t+1} = \left[ \frac{1}{\beta} - (1-\lambda) \left(1 - \frac{1}{\beta\lambda}\right) \right] \pi_t + (1-\lambda) \left(1 - \frac{1}{\beta\lambda}\right) (P_\pi \pi_t + P_u u_t + P_z z_t) - \frac{1}{\beta} u_t - \frac{1}{\beta} z_t$$

Factorizing:

$$\begin{aligned}\pi_{t+1} = & \left[ \frac{1}{\beta} - (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) + (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) P_\pi \right] \pi_t + \left[ (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) P_u - \frac{1}{\beta} \right] u_t \\ & + \left[ (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) P_z - \frac{1}{\beta} \right] z_t\end{aligned}$$

The method of undetermined coefficients implies for the first term:

$$\begin{aligned}\frac{P_\pi - 1}{P_\pi} &= \frac{1}{\beta} + (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) (P_\pi - 1) \\ P_\pi &= \frac{1}{1-\lambda}\end{aligned}$$

For the second term:

$$\begin{aligned}\frac{(1-\rho)P_u}{P_\pi} &= (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) P_u - \frac{1}{\beta} \Rightarrow \\ \frac{1}{\beta} &= \left( 1 - \frac{1}{\beta\lambda} - 1 + \rho \right) (1-\lambda) P_u \Rightarrow \\ P_u &= \frac{1}{1-\lambda} \frac{\frac{1}{\beta}}{\rho - \frac{1}{\beta\lambda}} \Rightarrow \frac{P_u}{P_\pi} = \frac{\frac{1}{\beta}}{\rho - \frac{1}{\beta\lambda}} = \frac{-\lambda}{1-\lambda\beta\rho}\end{aligned}$$

For the third term:

$$\frac{(1-\mu)P_z}{P_\pi} = (1-\lambda) \left( 1 - \frac{1}{\beta\lambda} \right) P_z - \frac{1}{\beta} \Rightarrow P_z = \frac{\frac{1}{\beta}}{(1-\lambda) \left( \mu - \frac{1}{\beta\lambda} \right)}$$

QED

**Proposition 2: Optimal policy rule parameters formulas:**

$$F_\pi = \frac{\kappa}{\alpha_x} (P_\pi - 1) = \lambda \frac{\kappa}{\alpha_x} P_\pi = \frac{\kappa}{\alpha_x} \frac{\lambda}{1-\lambda} = \frac{1-\beta\lambda}{\kappa} \quad (33)$$

$$F_u = \frac{\kappa}{\alpha_x} P_u = \frac{\kappa}{\alpha_x} P_\pi \frac{\lambda}{\beta\lambda\rho - 1} = \frac{\kappa}{\alpha_x} \frac{1}{1-\lambda} \frac{\lambda}{\beta\lambda\rho - 1} \quad (34)$$

$$F_z = \frac{\kappa}{\alpha_x} P_z = \frac{\kappa}{\alpha_x} P_\pi \frac{\lambda}{\beta\lambda\mu - 1} = \frac{\kappa}{\alpha_x} \frac{\lambda}{1-\lambda} \frac{1}{\beta\lambda\mu - 1} \quad (35)$$

$$\frac{F_u}{F_\pi} = A = \frac{1}{\lambda} \frac{P_u}{P_\pi} = \frac{1}{\beta\lambda\rho - 1} = \frac{P_u}{P_\pi - 1} = -1 + \beta\rho \frac{P_u}{P_\pi} \quad (36)$$

$$\frac{F_z}{F_\pi} = B = \frac{1}{\lambda} \frac{P_z}{P_\pi} = \frac{1}{\beta\lambda\mu - 1} \quad (37)$$

**Demonstration:**

The first order condition relates Lagrange multiplier to the policy instrument:

$$\begin{aligned}
x_t &= \frac{\kappa}{\alpha_x} \gamma_{t+1} = \frac{\kappa}{\alpha_x} (\gamma_t - \pi_t) \\
x_t &= F_\pi \pi_t + F_u u_t + F_z z_t = \frac{\kappa}{\alpha_x} (\gamma_t - \pi_t) = \frac{\kappa}{\alpha_x} (P_\pi \pi_t + P_u u_t + P_z z_t - \pi_t) \Rightarrow \\
F_\pi &= \frac{\kappa}{\alpha_x} (P_\pi - 1), F_u = \frac{\kappa}{\alpha_x} P_u \text{ and } F_z = \frac{\kappa}{\alpha_x} P_z
\end{aligned}$$

**Proposition 3: From LQR to Gali (2015) vector basis (replace policy target by policy instrument).**

One has:

$$\frac{-1 - \kappa F_u}{\beta} = \frac{-1 - \kappa A \frac{1-\beta\lambda}{\kappa}}{\beta} = \frac{1}{\beta\lambda\rho - 1} \lambda - \frac{1}{\beta} - \frac{1}{\beta\lambda\rho - 1} = \frac{(1-\rho)\lambda}{\beta\lambda\rho - 1} = (1-\rho)\lambda A$$

One has:

$$\begin{cases} \left( \begin{array}{c} u_{t+1} \\ \pi_{t+1} \end{array} \right) = \begin{pmatrix} \rho & 0 \\ (1-\rho)A\lambda & \lambda \end{pmatrix} \left( \begin{array}{c} u_t \\ \pi_t \end{array} \right) + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix} \\ \left( \begin{array}{c} u_t \\ x_t \end{array} \right) = \begin{pmatrix} 1 & 0 \\ AF_\pi & F_\pi \end{pmatrix} \left( \begin{array}{c} u_t \\ \pi_t \end{array} \right) = \mathbf{N} \left( \begin{array}{c} u_t \\ \pi_t \end{array} \right) \\ x_t = F_\pi \pi_t + AF_\pi u_t \end{cases} \\
\Leftrightarrow \begin{cases} \left( \begin{array}{c} u_{t+1} \\ x_{t+1} \end{array} \right) = \mathbf{N}^{-1} (\mathbf{A} + \mathbf{BF}) \mathbf{N} \left( \begin{array}{c} u_t \\ x_t \end{array} \right) + \mathbf{N}^{-1} \left( \begin{array}{c} \varepsilon_t \\ 0 \end{array} \right) \\ \left( \begin{array}{c} u_t \\ \pi_t \end{array} \right) = \mathbf{N}^{-1} \left( \begin{array}{c} u_t \\ x_t \end{array} \right) \\ \pi_t = \frac{1}{F_\pi} x_t - A\pi_t \end{cases}$$

One has:

$$\mathbf{N}^{-1} (\mathbf{A} + \mathbf{BF}) \mathbf{N} = \begin{pmatrix} \rho & 0 \\ (1-\lambda)F_\pi A\rho & \lambda \end{pmatrix}$$

Which is Gali (2015) representation of the solution:

$$x_t = \lambda x_{t-1} + (1-\lambda) F_\pi A \rho u_{t-1} = \lambda x_{t-1} + \frac{\kappa}{\alpha_x} \frac{\lambda}{\beta\lambda\rho - 1} \rho u_{t-1}$$

QED

Because the auto-correlation  $\lambda$  of the policy instrument  $x_t$  and the auto-correlation of the cost-push shock are competing to explain the persistence of the policy instrument  $x_t$ , this partial adjustment model with serially correlated shocks has a problem of identification and multiple equilibria (Griliches (1967), Blinder (1986), McManus et al. (1994), Fève, Matheron Poilly (2007)). Hence, we compute the representation of Ramsey optimal policy as a bivariate VAR of two observable variables (inflation and the policy instrument).

**Proposition 4: From LQR to a bivariate VAR with observable variables:**

$$\left\{ \begin{array}{l} \begin{pmatrix} \pi_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} \lambda & (1-\rho)A\lambda \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_t \end{pmatrix} \\ \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ F_\pi & AF_\pi \end{pmatrix}^{-1} \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} = \mathbf{M} \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} \\ i_t = F_\pi \pi_t + AF_\pi u_t \end{array} \right.$$

$$\Leftrightarrow \left\{ \begin{array}{l} \begin{pmatrix} \pi_{t+1} \\ x_{t+1} \end{pmatrix} = \mathbf{M}^{-1} (\mathbf{A} + \mathbf{B}\mathbf{F}) \mathbf{M} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} + \mathbf{M}^{-1} \begin{pmatrix} 0 \\ \varepsilon_t \end{pmatrix} \\ \begin{pmatrix} \pi_t \\ u_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{A} & \frac{1}{AF_\pi} \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} \\ u_t = -\frac{1}{A}\pi_t + \frac{1}{AF_\pi}x_t \end{array} \right.$$

Then:

$$\begin{pmatrix} 1 & 0 \\ F_\pi & AF_\pi \end{pmatrix} \begin{pmatrix} \lambda & (1-\rho)A\lambda \\ 0 & \rho \end{pmatrix} \begin{pmatrix} 1 & 0 \\ F_\pi & AF_\pi \end{pmatrix}^{-1} \\ = \begin{pmatrix} \lambda\rho & (1-\rho)\lambda\frac{1}{F_\pi} \\ \rho(\lambda-1)F_\pi & \lambda + \rho - \lambda\rho \end{pmatrix} = \begin{pmatrix} \lambda\rho & (1-\rho)(1-\lambda)\frac{\alpha_x}{\kappa} \\ -\rho\lambda\frac{\kappa}{\alpha_x} & \lambda + \rho - \lambda\rho \end{pmatrix}$$

QED.

## 8 NOT FOR PUBLICATION

### 8.1 Appendix 3: Identification issue for reduced form including a non-observable AR(1) shock.

Because the auto-correlation of the policy instrument  $x_t$  and the auto-correlation of the cost-push shock are competing to explain the persistence of the policy instrument  $x_t$ , this partial adjustment model with serially correlated shocks has a problem of identification and multiple equilibria (Griliches (1967), Blinder (1986), McManus et al. (1994), Fève, Matheron Poilly (2007)). This VAR(1) can be written as:

$$x_t = \lambda_R x_{t-1} + \eta_t \text{ and } \eta_t = \rho \eta_{t-1} + \varepsilon_{\eta,t}$$

where  $\eta_t = -\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1-\lambda_R\rho\beta)} u_t$ . It is an AR(2) model of the policy instrument rule:

$$\begin{aligned} x_t &= \lambda_R x_{t-1} + \rho(x_{t-1} - \lambda_R x_{t-2}) + \varepsilon_{\eta,t} \\ x_t &= b_1 x_{t-1} + b_2 x_{t-2} + \varepsilon_{\eta,t} \text{ with } b_1 = \lambda_R + \rho \text{ and } b_2 = -\lambda_R \rho. \end{aligned}$$

The structural parameter  $\rho$  and the semi-structural parameter  $\lambda_R$  are functions of reduced form parameters  $b_1$  and  $b_2$  solutions of:

$$X^2 - b_1 X - b_2 = 0$$

which are given by:

$$\lambda_R = \frac{b_1 \pm \sqrt{b_1^2 + 4b_2}}{2} \text{ and } \rho = \beta b - \lambda_R$$

where  $\Delta = b_1^2 + 4b_2 = (\rho - \lambda_R)^2$ . If  $\Delta \neq 0$  and  $\rho \neq \lambda_R$ , two sets of values for  $\lambda_R$  and  $\rho$  are observationally equivalent. The first solution is such that  $\lambda_R > \rho$  and the second solution is such that  $\lambda_R < \rho$ . The larger  $\Delta$ , the larger the identification issue, because it increases the gap between a more inertial monetary policy with lower correlation of monetary policy shocks and a less inertial monetary policy, that we cannot distinguish. The ADLQR representation and Gali (2015) representation of the stationary solution of the VAR(1) of optimal policy are not useful to identify parameters, because they include the cost-push shock  $u_t$  which is not observable.

The reduced form estimated variance  $\sigma_\eta$  provides another equation with a theoretical positive sign restriction  $\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1 - \lambda_R \rho \beta)} > 0$  for five unknowns structural parameters  $(\alpha_x, \kappa, \rho, \beta, \sigma_u)$ :

$$\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1 - \lambda_R \rho \beta)} \sigma_u = \sigma_\eta$$

## 8.2 Appendix 4: Oudiz and Sachs (1985) versus Gali (2015) time consistent policy

Substituting the private sector's inflation rule (8) and policy rule (9) in the inflation law of motion (1) and comparing it with the forcing variable law of motion (2) leads to the following relation between  $N_{ZC}$  on date  $t$ ,  $N_{ZC,t+1}$  and  $F_{u,ZC}$ :

$$\begin{aligned} \pi_t &= \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \Rightarrow \\ N_{ZC} u_t &= \beta N_{ZC,t+1} \rho u_t + \kappa F_{u,ZC} u_t + u_t \\ N_{ZC} &= \beta \rho N_{ZC,t+1} + \kappa F_{u,ZC} + 1 \end{aligned}$$

A myopic central bank does not notice that  $N_{ZC,t+1} = N_{ZC}$  (Gali (2015)) in its optimization:

$$\begin{aligned} N_{ZC,Gali} &= \beta \rho N_{ZC,t+1} + \kappa F_{u,ZC} + 1 \Rightarrow \frac{\partial N_{ZC,Gali}}{\partial F_{u,ZC}} = \kappa \\ F_{\pi,ZC} &= \frac{F_{u,ZC}}{N_{ZC}} = -\frac{\kappa}{\alpha_x} < 0 \end{aligned}$$

This first order condition of the central bank optimization is substituted into the new-Keynesian Phillips curve equation, where, only at this stage, players of the game discover that it is assumed  $N_{ZC,t+1} = N_{ZC,t} = N_{ZC}$ . Gali's (2015) solutions are:

$$\begin{aligned} F_{u,ZC,Gali} &= -\frac{\kappa}{\kappa^2 + \alpha_x (1 - \beta \rho)} = -\frac{\kappa}{\alpha_x} N_{ZC} \\ N_{ZC,Gali} &= \frac{\alpha_x}{\kappa^2 + \alpha_x (1 - \beta \rho)} \rightarrow \frac{1}{1 - \beta \rho} = N \text{ when } \alpha_x \rightarrow +\infty \end{aligned}$$

In zero-credibility equilibrium (Oudiz and Sachs (1985)), the central bank does foresee that  $N_{ZC,t+1} = N_{ZC}$  in its optimization, with the following solutions, that we consider for the remaining part of the paper:

$$\begin{aligned}
N_{ZC} &= \frac{\kappa F_{u,ZC} + 1}{1 - \beta\rho} = \frac{\kappa F_{\pi,ZC} N_{ZC} + 1}{1 - \beta\rho} \Rightarrow \frac{\partial N_{u,ZC}}{\partial F_{u,ZC}} = \frac{\kappa}{1 - \beta\rho} \\
F_{\pi,ZC} &= \frac{F_{u,ZC}}{N_{ZC}} = -\frac{\kappa}{\alpha_x} \frac{1}{1 - \beta\rho} = -\frac{\kappa}{\alpha_x} < 0 \\
F_{u,ZC} &= -\frac{\kappa}{\kappa^2 + \alpha_x (1 - \beta\rho)^2} \\
N_{ZC} &= \frac{\alpha_x (1 - \beta\rho)}{\kappa^2 + \alpha_x (1 - \beta\rho)^2} \rightarrow \frac{1}{1 - \beta\rho} = N \text{ when } \alpha_x \rightarrow +\infty
\end{aligned}$$

In Oudiz and Sachs' (1985) general solution, this is the condition after substitutions of the private sector's rule (matrix  $N_{ZC}$ ) and the policy maker's rule (matrix  $F_{u,ZC}$ ) for *both* dates  $t$  and  $t + 1$  into the law of motion of the private sector dynamics:

$$\begin{aligned}
N_{ZC,t} &= J - K F_{u,ZC} \\
J &= (A_{22} + N_{ZC,t+1} A_{12})^{-1} (N_{ZC,t+1} A_{11} + A_{21}) \\
K &= (A_{22} + N_{ZC,t+1} A_{12})^{-1} (N_{ZC,t+1} B_1 + B_2)
\end{aligned}$$

with general notations and equalities with Gali's (2015) transmission mechanism:

$$\begin{pmatrix} u_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} A_{11} = \rho & A_{12} = 0 \\ A_{21} = -\frac{1}{\beta} & A_{22} = \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} B_1 = 0 \\ B_2 = -\frac{\kappa}{\beta} \end{pmatrix} x_t$$

In Oudiz and Sachs (1985),  $N_{ZC,t+1} = N_{ZC,t}$  at all dates, whereas Gali (2015) assumes myopia (or  $N_{ZC,t+1} = 0$ ) for the policy maker. This assumption changes the initial jump of inflation, impulse response functions of inflation and the output gap and welfare. It does not change the identification problem of discretion raised in this paper, because the stable subspace of discretion have the same dimension (one) using the reference Oudiz and Sachs (1985) discretion equilibrium or Gali (2015) and Clarida, Gali, Gertler (1999) myopia assumption.

### 8.3 Appendix 5: Code for 3 estimations of Ramsey optimal policy

The **first estimation** estimates the two stable eigenvalues: the exogenous  $\rho$ , the reduced form  $\lambda$  and the reduced from policy rule parameter with respect to inflation  $F_{\pi}$ .

```

by id1: nlsur (inflation=({r}*{l})*linflation+({l}*(1-{r})/{F=1})*lfyff+{b3}) ///
(fyff=({r}*({l}-1)*{F})*linflation+({r}+{l}-{r}*{l})*lfyff+{b6}) ///
if tt>1

```

The **second estimation** estimates the two stable eigenvalues: the exogenous  $\rho$ , the reduced form  $\lambda$  and the ratio of two structural parameters  $\frac{\alpha_x}{\kappa}$  (the Fed's preference divided by the slope of the new-Keynesian Phillips curve). This estimate of  $\frac{\alpha_x}{\kappa}$  is an estimate of  $\frac{1}{\varepsilon}$ , the household's elasticity of substitution between each differentiated goods  $\varepsilon$ , if the



Fed's preferences corresponds to consumers' welfare.

```

by id1: nlsur (inflation={r=1}*{l=0.5}*linflation ///
+(1-{r})*(1-{l})*{G=1}*lfyff+{b3}) ///
(fyff=-{r}*{l}/{G=1}*linflation ////
+({r}+(1-{r})*{l})*lfyff+{b6}) ///
if tt>1

```

The **third estimation** requires an identification restriction that we set on the discount factor  $\beta$  ( $\beta = 1$  or  $\beta = 0.99$ ). We estimate separately three structural parameters: the auto-correlation of the cost-push shock  $\rho$ , the preference of the Fed  $\alpha_x$ , and the slope of the new-Keynesian Phillips curve  $\kappa$ .

```

by id1: nlsur (inflation={r=1}*(1+0.5*({k}^2/{a=1})-sqrt(-1+(1+0.5*({k}^2/{a}))^2)
)*linflation ///
+(1-{r})*(1-(1+0.5*({k}^2/{a})-sqrt(-1+(1+0.5*({k}^2/{a}))^2) ) ) *{a}/{k=1}*lfyff+{b3})
///
(fyff=-{r}*(1+0.5*({k}^2/{a})-sqrt(-1+(1+0.5*({k}^2/{a}))^2) ) *{k}/{a}*linflation
///
+({r}+(1-{r})*(1+0.5*({k}^2/{a})-sqrt(-1+(1+0.5*({k}^2/{a}))^2) ) ) *lfyff+{b6})
///
if tt>1
by id1: nlsur (inflation={r=1}*(1.005051+0.505051*({k}^2/{a=1})-sqrt(-1.010101+(1.005051+0.505051*({k}^2/{a}))^2)
)*linflation ///
+(1-{r})*(1-(1.005051+0.505051*({k}^2/{a=1})-sqrt(-1.010101+(1.005051+0.505051*({k}^2/{a}))^2) ) ) *{a}/{k=1}*lfyff+{b3}) ///
(fyff=-{r}*(1.005051+0.505051*({k}^2/{a=1})-sqrt(-1.010101+(1.005051+0.505051*({k}^2/{a}))^2) ) *{k}/{a}*linflation ///
+({r}+(1-{r})*(1.005051+0.505051*({k}^2/{a=1})-sqrt(-1.010101+(1.005051+0.505051*({k}^2/{a}))^2) ) ) *lfyff+{b6}) ///
if tt>1

```