Preparing a (quantum) belief system

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Preparing a (quantum) belief system

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Abstract

In this paper we investigate the potential for persuasion linked to the quantum indeterminacy of beliefs. We first formulate the persuasion problem in the context of quantum-like beliefs. We provide an economic example of belief manipulation that illustrates the setting. We next establish a theoretical result showing that in the absence of constraints on measurements, any state can be obtained as the result of a suitable sequence of measurements. We finally discuss the practical significance of our result in the context of persuasion.

Keywords: belief, quantum-like, persuasion, measurement

1 Introduction

The theory of persuasion was initiated by Kamenica and Gentskow [7] and further developed in a variety of directions. The subject matter of the theory of persuasion is the use of an information structure (or measurement) that generate new information in order to modify a person's state of beliefs with the intent of making her act in a specific way. The question of interest is how much can a person, call him Sender, influence another one, call her Receiver, by a selecting a suitable measurement and revealing its outcome. A key feature is that Receiver is rational and knows the intent of Sender to manipulate her action.

Receiver's decision to act depends on her beliefs about the world. In [7] and related works the beliefs are given as a probability distribution over a set of states of the world. That is a central assumption [7] is that uncertainty is formulated in the standard classical framework. As a consequence the updating of Receiver's beliefs follows Bayes rule.

However as amply documented the functioning of the mind is more complex and often people do not follow Bayes rule. Cognitive sciences propose alternatives to Bayesianism. One avenue of research within cognitive sciences appeals to the formalism of quantum mechanics. A main reason is that QM has properties that reminds of the paradoxical phenomena exhibited in human cognition. Quantum cognition has shown successful in explaining a wide variety of behavioral phenomena (see for a survey

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Bruza and Busemeyer [2]). Moreover there exists by now a fully developed decision theory relying on the principle of quantum cognition (see [3, 4]). Therefore in the following we shall use the Hilbert space model to represent the belief of an individual and capture the impact of new information on those beliefs. Clearly, the mind is likely to be even more complex than a quantum system but our view is that the quantum cognitive approach already delivers interesting new insights with respect to persuasion.

In quantum cognition, the system of interest is the decision-maker's mental representation of the world is represented by a cognitive *state.* This representation of the world is modelled as a quantum-like system so the decision relevant uncertainty is of non-classical (quantum) nature. As argued elsewhere ([5]) this modeling approach allows capturing widespread cognitive difficulties that people exhibit when constructing a mental representation of a 'complex' alternative. The key quantum property that we use is that some characteristics (cf properties) of a complex mental object may be "Bohr complementary" that is incompatible in the decision-maker's mind: they cannot have definite value simultaneously. A central implication is that measurements (new information) modifies the cognitive state in a non-Bayesian well-defined manner.

It turns out that this has a close counter-part in Physics (see, for example, [6]). In Quantum Mechanics experiments one often needs to know the state of a particle before performing operations on it in order to be able to draw conclusions. In order to determine that state, physicists 'prepare' particles in a definite quantum state, e.g. by means of filtering: for instance they measure the spin of a set of particles and keep for further operation those that are in state + while throwing away the others. By doing so they have effectively created particles with the needed spin property. The term preparation is rather broad - it covers any kind of operations that affects the state of a single system or the composition of a set of systems. One such operation is measurement. A von Neumann direct measurement prepares a system in the (pure) state that obtains as the result of the measurement. Generally, a von Neumann measurement modifies a quantum state according to the von Neumann-Luders rule for updating.

In [4], we learned that dynamically consistent, that is rational, quantum-like decision-makers update their beliefs according to the von Neumann-Luder's postulate. We take this result as starting point to investigate the potential of manipulation of Sender when facing a rational quantum-like Receiver. Our central result in Theorem 1 is that, in the absence of any constraints on the choice of measurements, there always exists a sequence of measurements that secures obtaining any target state starting from any initial state. In terms of persuasion, Sender can always persuade Receiver to believe anything that favors him. This is in sharp contrast with the classical setting where the expected posterior must be equal to the prior. Theorem 1 is of course a theoretical result which does not mean that Sender can make Receiver do whatever he wants. First, Receiver may not be willing to take the desired action in any belief state. Second, even if there exists a belief state such that Receiver takes the desired action it may require measurements that are not practically feasible. They may be too costly or Receiver would get annoyed. Classical measurements face similar practical constraints. A distinction however is that classical measurements can always be performed as a single measurement while this is not true in a quantum situation.

Kamenica and Gentskow motivated their paper by the fact that attempts to manipulate command a sizable share of our resources in the economy. Persuasion is at the heart of advertising, courts hearings, lobbying, financial disclosures and political campaign among other activities. As suggested by Akerlof and Schiller [1], the power of manipulation seems however much larger and more determinant than what the classical approach suggests. Our results suggest that quantum cognition may provide a better model to explain the extent and power of manipulation in society. This research is a step in a broader project which aims at proposing an alternative to the foundations of the functioning of free markets in the spirit of Akerlof and Schiller.

2 The model

We shall formulate our model in terms of a communication game as we want to facilitate the comparison with Kamenica and Genskow (KG) [7]. There are two players R (Receiver) and S (Sender). We are interested in persuasion aimed at influencing Receiver's choice over uncertain alternatives which we model as quantum lotteries following Danilov et al [4].

In the most general context a belief of Receiver is a point B in some convex set $\mathcal{B}$ (the set of states of the Receiver beliefs). The Receiver chooses an action a from some finite set A. Any action can be considered as a linear function on $\mathcal{B}, a : \mathcal{B} \to \mathbb{R}$ where $a(B)$ is the expected utility of Receiver in state B that is when she holds belief B. Naturally she chooses an action $a^* = a^*_B$ that gives her the highest possible expected utility that is $a^*_B \in \arg\max(a(B), a \in A)$. So the set of states can be decomposed into convex regions $\mathcal{B}_a$ (a runs over A, $\mathcal{B} = \cup_a B_a$). In region $\mathcal{B}_a$, the optimal action of Receiver is a.

Sender's utility from actions is captured by his utility function $u : A \to \mathbb{R}$. Here we assume that Sender's utility only depends on the action chosen by Receiver. Sender's problem amounts to acting on Receiver's state of beliefs so as move it into a region that induces the most favorable decision to him, i.e. to persuade her to act in a particular manner. His means of persuasion are information (signal) structures, we shall also use the term "measurement".

In such a general framework, we do not know how to characterize a measurement and how Receiver's belief B changes in response to new information obtained from a measurement. We below provide a brief description of the classical setting and thereafter we develop our argument in the quantum context where we also do know how to proceed relying on the Hilbert space model.

2.1 The classical setting

The classical means of persuasion have been well described in KG. There is a set Ω of states of Nature. Receiver's belief is a probability distribution or measure β on Ω, $\beta(\omega) \geq 0$, $\sum \beta_\omega(\omega) = 1$ (above we denoted that β as B). For the sake of simplicity we shall assume that the set Ω is finite. So the set

$\mathcal{B}$ is the simplex $\Delta(\Omega)$ of probability distributions on Ω. An *information structure* (or *measurement device*) is a map $\varphi : \Omega \to \Delta(S)$, where S is a set of signals (outcomes) of our measurement device. In a state $\omega \in \Omega$ this device gives (randomized) signal $\varphi(\omega) \in \Delta(S)$. If we write this more carefully such a device is given by a family $(f_s, s \in S)$ of function f_s on Ω; $f_s(\omega)$ gives the probability of obtaining signal s in state $\omega \in \Omega$. Of course all the functions f_s must be nonnegative and their sum $(\sum_s f_s)$ must yield the unity function 1_Ω on Ω.

If Receiver holds a belief $\beta = (\beta(\omega), \omega \in \Omega)$ then she can easily compute the probability for the signals. The probability of receiving signal s given prior β, is equal to $p_s = \varphi(\beta)(s) = \sum_\omega \beta(\omega)\varphi(\omega)(s) = \sum_\omega \beta(\omega) f_s(\omega)$.

More interesting for us is that Receiver can use Bayes' rule when she receives signal s to update her beliefs, i.e. to form the posterior $\beta_s \in \Delta(\Omega)$, given as $\beta_s(\omega) = f_s(\omega)\beta(\omega)/p_s$. For Receiver (and for Sender) what is important is the change in the belief from β to β_s upon receiving signal s. Because as she receives signal s she will choose action $a^*(\beta_s)$ and Sender will receive utility $u(a^*(\beta_s))$. On average when Sender uses such an information (signal) structure he receives utility $\sum p_s u(a^*(\beta_s))$. And so we can ask which is the best measurement device for Sender? Kamenica and Genskow formulate the problem in the classical information context and characterize the optimal measurement.

2.2 The quantum setting

The description of a quantum system starts with the fixation of a Hilbert space (in our case a finite dimensional) over $\mathbb{R}$, that is a real vector space H equipped with a symmetric scalar product $(.,.)$.

We shall be interested not so much in the Hilbert space H as in operators (that is linear maps from H to H). For such an operator A we denote by A^* its conjugate operator which is defined by the following condition: $(v, Aw) = (A^*v, w)$ for all u, w in H. The self-conjugate operators (for which we have $A = A^*$) are called *Hermitian* will play most important role in our analysis.

The notion of trace will be a central instrument in what follows. The trace Tr of a matrix can be defined as the sum of its diagonal elements. It is known that the trace does not depend on the choice of basis. With the help of the trace one can introduce the notion of state of a quantum system. It is defined as a nonnegative Hermitian operator B (nonnegativity means that $(Bv, v) \geq 0$ for any $v \in H$) with trace equal to 1. This notion replaces the concept of probability distribution in the classical context. The nonnegativity of the operator is analogous to the nonnegativity of a probability measure, and the trace 1 to the sum of probabilities which equals 1.

General Hermitian operators play the role of random variables on Ω. In fact for any Hermitian operator A and a state B we can define the 'expected value' of A in state B as $Tr(AB)$. In our work [4] we showed that a quantum lottery can also be expressed as an Hermitian operator and that the expected utility of lottery A (in belief state B) is expressed as $Tr(AB)$.

In this way, Receiver's preferences over lotteries are determined by her belief state B. The set of (belief) states (that we earlier denoted by $\mathcal{B}$) we denote by $\mathbf{St}$. It is a compact convex set, the extreme points of this set are called *pure states* (they correspond to one-dimensional projectors). Lotteries or actions are understood as affine functions on **St**.

We still have to define what we understand as *measurement device* (or information structure, cf. KG). And here we must recognize that there exist three (more and more general) candidates to serve as measurement devices (see [8]).

1. *Direct* (or projection, or von Neumann) measurements.

Such a measurement is given by a family $(P_s, s \in S)$ of projectors with the property $\sum_s P_s = E$ where E is the identity operator on H. (A projector is an Hermitian operator such that $P^2 = P$.) The probability p_s to obtain a signal-outcome s (in a state $B \in \mathbf{St}$) is equal to $Tr\,(P_s B)$. And the posterior belief-state is $B_s = P_s B P_s / p_s$ (it is easy to check that it is a state). Here all is standard and simple especially if projector P_s is one-dimensional (that is a pure state); in this case the posterior B_s is equal to P_s. If we repeat the measurement we obtain the same outcome s and the state does not change. This type of measurement is *repeatable* or "*first kind*". The only thing that must be underlined is that the *expected posterior* $B' = \sum_s p_s B_s = \sum_s P_s B P_s$ is generally different from the prior B as is illustrated in the example below. This contrasts with the classical case.

2. *Indirect, or unsharp measurements* (or POVM).

Such a measurement is given by a family of Hermitian operators $(Q_s, s \in S)$ which are nonnegative and sum up to E ($\sum Q_s = E$). We have no problem defining probabilities $p_s = Tr(Q_s B)$ as before. But with the posterior the situation is more complicated. We can (as we did in our article [4]) take the positive square root $R_s = \sqrt{Q_s}$ (so that $R_s^2 = Q_s$). It always exists and define the posterior B_s as $R_s B R_s / p_s$. The expected posterior is given by the same formula: $B' = \sum_s p_s B_s = \sum_s R_s B R_s$.

This kind of measurements are not repeatable; the number of signals may be largely bigger than the dimension of H. They are called *indirect* because they can be understood as follows. We include our system in a larger one (which is described by a higher dimensional Hilbert space $\widetilde{H}$) and perform a direct measurement (see above) of this larger system.

3. *The most general measurements.*

Although we shall not make use of them much, a few words are in place. They generalize the measurements in 2) in terms of more general formation of the posterior. Above we used a canonical (or simplest) decomposition of Q_s into the product $R_s R_s$. However we could use a more general decomposition $Q_s = A_s A_s^*$ (where A_s is not necessarily a Hermitian operator; if A is Hermitian then $A = A^*$ and we return to the square roots). Now the most general measurement device is given by an arbitrary family of operators $(A_s,\ s \in S)$ with the sole condition that $\sum_s A_s A_s^* = E$. The probability for signal outcome s is defined as earlier $p_s = Tr(A_s^* B A_s) = Tr(A_s A_s^* B)$ but the posteriors are defined slightly differently as $B_s = A_s^* B A_s / p_s$.

This generalization may appear somehow artificial. But there are a highly meaningful considerations in defense of its use. Consider the case when we perform two (or more) measurements (even direct ones) in a row. First with device (P_s) and thereafter with device (Q_t). It is natural to consider that the outcome (s,t) is obtained with probability $Tr(Q_t\,(P_sBP_s)\,Q_t)$. And that the posterior state is (with precision up to a multiple) equal to $Q_t\,(P_sBP_s)\,Q_t$. However if the measurements are not compatible, then operators P_s and Q_t do not commute and their product $A_{st} = P_sQ_t$ is not Hermitian. And we have $(A_{st})^* = P_s^*Q_t^* = Q_tP_s$. So that a combination or a sequence of measurements is given by a collection (A_{st}).

On the other side this kind of most general measurements gives too much generality: they allow not only describing measurements but also a unitary evolution of a system. It is however not very clear how to realize an unitary evolution (true this concern may also be relevant for measurement even direct ones). And finally the expected posterior even in the state E/d can differ from E/d. In fact it is equal to $\sum_s A_s^*A_s/d$. This is a quite strange and unattractive property. Therefore we shall not use those most general measurements and instead we shall work (whenever needed) with repeated direct measurements.

Quantum measurements be they direct or more general are characterized by a few distinguishing features. Most importantly, the performance of a measurement impacts on the state of the system. As a first consequence and in contrast with the classical case, beliefs do not converge toward complete information about the 'true state' of the system. In the quantum context, the belief state may be pure, i.e. represent maximal (rather than complete) information and yet change upon the reception of new information. An expression of this is that measurements may be incompatible. As a consequence and in contrast with the classical case, a sequence of direct measurements cannot generally be merged into a single direct measurement. Therefore, in this paper we opted for the following approach. On the one hand we limit ourselves to direct measurements while on the other hand allow for sequences of direct measurements. Yet another variant amounts to performing mixed measurements. The procedure begin with playing a roulette lottery which determines which direct measurement will be performed. Then the selected measurement is carried out and a signal obtained. In the classical context, this procedure works very well (cf. [7]) but as we shall see we are able to obtain very powerful results even without mixing.

A second consequence of the impact of measurements on the state is the so called 'decoherence' which turns out to be of great value in our context. Assume that we perform a measurement (for instance a direct one given by the family of projectors P_s) but do not learn the result. Such a measurement can be simply ignored in the classical context. In the quantum context such a "blind' measurement induces nevertheless a change in the posterior $B' = \sum_s P_sBP_s$. As we shall see such blind measurements (clearly useless in a classical context) provide a powerful mean of changing the beliefs of Receiver. They also have a meaningful interpretation in the cognitive quantum context.

3 An illustrative example

Before introducing our central result we wish to provide an example of quantum persuasion in a very common context, i.e. when a seller wants to persuade a potential buyer to purchase an item. So in that situation we identify Sender with the seller and Receiver with the consumer.

More specifically, our consumer is considering the purchase of a second hand smartphone at price 30 euros of uncertain value to her, it depends on its technical quality which may be standard or excellent. She holds subjective beliefs about the probability that the smartphone is excellent. Based on those beliefs, she assigns an expected utility value to the smartphone which determines her decision to buy or not the item.

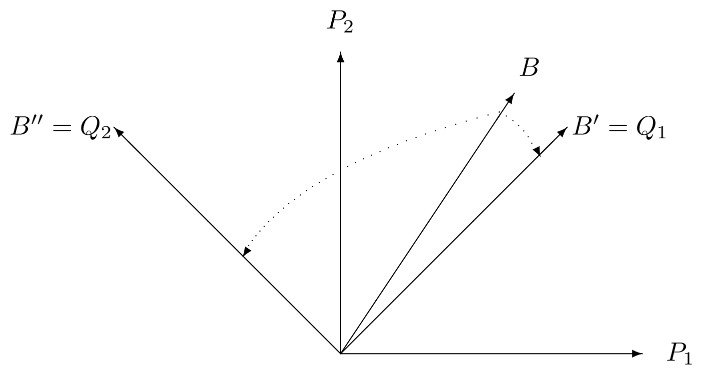
Let H be a two-dimensional Hilbert space with an orthonormal basis (e_1, e_2) and let (P_1, P_2) be the corresponding projectors, so that $P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Following [4], the smartphone is expressed as a quantum lottery A which gives a utility equal to 100 in state $|e_1\rangle$ (the smartphone is excellent) and 0 in $|e_2\rangle$ (the smartphone is standard); in matrix form the lottery writes as $A = \begin{pmatrix} 100 & 0 \\ 0 & 0 \end{pmatrix}$. Assume further that the utility when not buying the smartphone is $\underline{u} = 30$ (she keeps the money). The consumer's (Receiver) decision is $d \in \{Y, N\}$ accept or refuse to buy. The seller (Sender) receives utility 10 when selling the phone at price 30 whatever its quality and 0 otherwise.

Assume now that Receivers' belief (her cognitive state) is represented by the operator $B = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{pmatrix}$ in the basis (e_1, e_2). She assigns probability 1/5 to the state $|e_1\rangle$ (the smartphone is Excellent) and 4/5 to the state $|e_2\rangle$ (the smartphone is standard). Receiver's expected utility for the smartphone in the belief state B is represented by the trace of the product of operators A and B :

$$Eu\left(A; B\right) = \mathbf{Tr}\left(AB\right) = (1/5) * 100 + (4/5) * 0 = 20 < 30 = \underline{u}.$$

Given belief B Receiver does not want to buy the smartphone so the seller earns 0.

Can Sender persuade the Receiver by selecting an appropriate measurement? We next show that he indeed can induce her to buy with probability 1. Consider another property (characteristics) of the smartphone that we refer to as Glamour (i.e. whether celebrities have this brand or not). The Glamour property can measured with direct von Neumann measurement (Q_1, Q_2) with two possible outcomes Glamour $|G\rangle$ corresponding to projector $Q_1 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ and not Glamor $|NG\rangle$ corresponding to $Q_2 = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$. The Glamour property is represented by the basis $(|G\rangle, |NG\rangle)$ of the state space (of the mental representation of the smartphone) which is a 45° rotation of basis (e_1, e_2). This means that the $(|e_1\rangle, |e_2\rangle)$ and $(|G\rangle, |NG\rangle)$ are two properties (perspectives) that are incompatible in the mind of Receiver. Or equivalently (P_1, P_2) and (Q_1, Q_2) are measurements that do not commute with each other. Receiver can think in terms of either one of the two perspectives but she cannot synthesize (combine in a stable way) pieces of information from the two perspectives. This is illustrated in figure 1.



Assume that Sender brings the discussion to the Glamour perspective and performs the measurement so Receiver learns whether her preferred celebrity has this smartphone. With some probability p $(= 0.9)$ the new cognitive state is $B' = Q_1$ and with the complementary probability $1 - p = .1$ it is $B'' = Q_2$. We note that $Eu\,(a; B') = \mathbf{Tr}\,(AB') = .5 > .3$ and $Eu\,(a; B") = \mathbf{Tr}\,(AB") = .5 > .3$. In both cases Receiver is persuaded to buy and Sender gets utility 10.

Interestingly the example illustrates another non-classical phenomenon. Namely that the mere fact of measuring without knowing the outcome triggers a significant change in beliefs. Here the 'expected posterior' $EB = pB' + (1-p)B'' = \begin{pmatrix} 1/2 & 2/5 \\ 2/5 & 1/2 \end{pmatrix} \neq B$ in contrast with classical persuasion which is constrained by Bayesian plausibility meaning that the expected posterior must equal the prior.

In general, it is not possible to persuade Receiver with probability 1 by means of a single measurement. However as Theorem 1 below shows it is theoretically possible to manipulate beliefs with a sequence of appropriate measurements.

4 Our central result

In order to formulate our central result we return in more details to the description of direct measurements. A direct measurement device $\mathcal{M}$ is given by an orthonormal basis $(e_1, ..., e_n)$, where $n = \dim H$, and $(e_i, e_j) = \delta_{ij}$, and a collection of signals $(s_1, ..., s_n)$ (some s_i may coincide). The vector e_i should be understood as one-dimensional projectors $P_i; P_i\,(x) = (x, e_i)\,e_i$. How does such a measurement device operate? Upon the reception of signal s_i the system transits into state $|e_i\rangle$ (or P_i) with probability $p_i = Tr\,(BP_i)$. If we represent initial state B in matrix form in the basis $(e_1, ..., e_n)$, we have $p_i = b_{ii}$ where b_{ii} is the corresponding diagonal element of the matrix. This is true when the signal is fully disclosed that is distinct from all others. If all the s_i distinct but the result is not communicated (a *blind measurement*) the system transits into the mixed state $\sum p_i P_i$.

The existence of blind measurements is an interesting distinguishing feature of the quantum formalism. In the classical context a measurement without outcome has no impact whatsoever.

A second useful feature of the quantum situation is related to conditional measurements. Assume that we performed a measurement according to the above described device (with signal s_i). As we receive signal s we may thereafter perform a new measurement $\mathcal{M}_s$ (which can depend on signal s). In the classical context this kind of conditionality does not play any role because a compound measurement always is a measurement. But in the quantum context as we shall see further, such composition are generally not direct (von Neumann) measurements although they are measurements in the most general meaning. Of course we can iterate that procedure conditioning on the result from the second measurement and so on.

Our main result is that starting from any initial state it is possible to transit to any state by means of a suitable sequence of conditional measurements. In term of persuasion and beliefs, it means that Sender can always persuade Receiver to take any desirable for him decision independently of Receiver's prior. Of course, this is a theoretical result. In practice, it may not be possible to 'play' with Receiver so easily. Measurements are connected with costs, Receiver may be impatient or get tired of all information etc...

Theorem 1. *For any prior B, any target belief state T, and any $\varepsilon > 0$, there exists a sequence of direct conditional measurements such that with a probability larger than $1-\varepsilon$ the posterior is equal to T.*

We understand this result as follows: there exists in principle a rather simple and constructive strategy for Sender to persuade Receiver to believe anything Sender wants him to believe.

To start we show that the target state can be taken as pure. In fact the final state T can be represented (by force of the spectral theorem) as a mixture of orthogonal pure states, $T = \sum_i q_i P_i$, where $q_i \geq 0$, $\sum_i q_i = 1$, P_i are projectors on (unit) vector e_i. Let P be the projector on the following vector $e = \sum_i \sqrt{q_i} e_i$, that is $P(x) = (x, e)\, e$ for $x \in H$. In the basis $(e_1, ..., e_n)$ the projector P is given by the matrix $(\sqrt{q_i q_j})$.

We assert that if we perform a blind measurement with basis $(e_1, ..., e_n)$, state P transits into state T. In fact, the expected posterior is $P' = \sum_i P_i P P_i = \sum_i q_i P_i = T$.

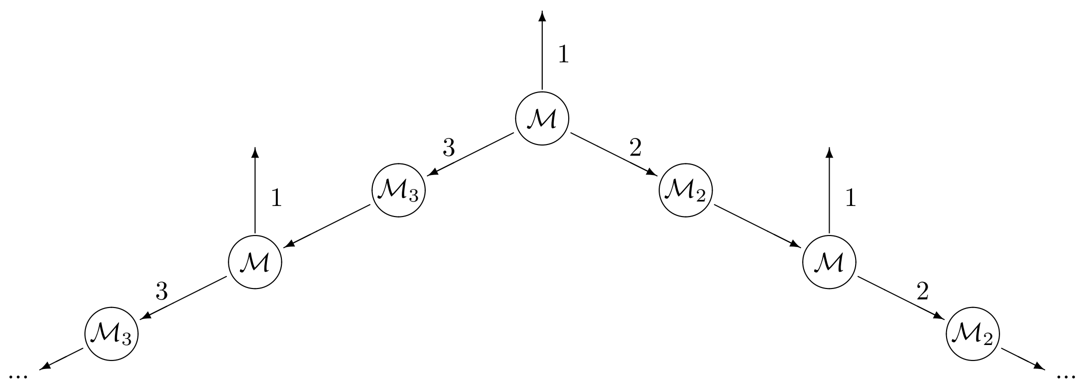
Hence, in order to arrive at state T it is sufficient to arrive at pure state P. We next show how to transit into an arbitrary pure state P.

Suppose that P is a projector on (unit) vector $|e_1\rangle$. We complete it to an orthonormal basis $(e_1, ..., e_n)$ and as our main measurement $\mathcal{M}$ we take a non-degenerated (complete) direct measurement in this basis. Whatever the initial state B **was**, after the performance of $\mathcal{M}$, the state transits into one of the pure states $|e_1\rangle, ..., |e_n\rangle$ and the signal informs us which one of them. Assume that the signal is s_2 so the system is now in state $|e_2\rangle$. In that case we construct an auxiliary direct measurement device $\mathcal{M}_2$ with the following orthonormal basis $(e_1 + e_2)/\sqrt{2}$, $(e_1 - e_2)/\sqrt{2}, e_3, ..., e_n$. If we perform

$\mathcal{M}_2$ (recalling that the system is now in $|e_2\rangle$) the system will with equal probability transit into $\left|(e_1+e_2)/\sqrt{2}\right\rangle$ or $\left|(e_1-e_2)/\sqrt{2}\right\rangle$ (we could let $\mathcal{M}_2$ be a blind measurement). Now we once more apply $\mathcal{M}$ and with probability 1/2 we obtain the desired state $|e_1\rangle$ (and the undesired state $|e_2\rangle$ with the same probability). If we obtain $|e_1\rangle$ we are done. If we obtain $|e_2\rangle$ we again apply $\mathcal{M}_2$ and thereafter $\mathcal{M}$. After N iterations the state will have transited into the target state $|e_1\rangle$ (or P) with probability $1-(1/2)^N$.

Above we consider the case when the first measurement gave outcome e_2. A similar procedure secures the desired state whenever the first outcome is $e_i,\ i\neq 1$. Instead of $\mathcal{M}_2$ we use $\mathcal{M}_i$ with basis $(e_1+e_i)/\sqrt{2},\ (e_1-e_i)/\sqrt{2}, e_3, ...e_{i-1}, e_{i+1}, ...e_n$. Which proves the Theorem 1.

In figure 2 we illustrate the conditional measurement scheme used in the proof of Theorem 1 for the case when $n=3$. $\mathcal{M}_{2(3)}$ represent blind measurements.



The strategy used in the proof of the theorem has two nice features. First it does not require knowing the prior of Receiver. Second it only appeals to a restricted number of measurements, $\mathcal{M}$ and the instrumental $\mathcal{M}_i$ ($i=2,...,n$). As earlier mentioned in practice the optimal strategy will have to take into account costs of measurements and constraints on the feasible number of iterations. Therefore, it can be interesting to consider the case when the number of measurement is limited.

5 Discussion

Manipulation is both an old and a very actual topic in social sciences. Recently an approach has received a lot of attention in the field of economics: Bayesian persuasion. The starting point is that you can manipulate rational people's behavior by acting upon their beliefs by choosing a suitable information structure or measurement that generate new information. This theory is developed in the classical uncertainty setting.

An alternative approach to decision-making under uncertainty uses the quantum formalism to describe uncertainty. Relying on the recent success of quantum cognition in explaining behavioral anomalies, we have investigated the scope of manipulation when a person's representation of the world, i.e. her belief are represented as a quantum-like system.

Quite remarkably, we establish that a person's beliefs are in theory fully manipulable. For any target state and any initial state there exists a sequence of direct measurements such that the target state is reached with a probability close to one.

In practice, this potential for manipulation is not expected to be realized because measurements are costly, people have limited patience or because constructing the appropriate measurements is not practically feasible. However, there are situations where it makes perfect sense. We provide a simple example showing that under some circumstances a single measurement in a very common situation can be sufficient to achieve desired behavior with probability one. The example also illustrates the impact and power of blind measurements. A blind measurement is a measurement that is performed but the signal is not communicated to Receiver. This capture the idea of changing the focus of a person's mind without bringing any new information. In our example simply to "divert" Receiver's attention to the Glamour perspective of the smartphone. As note by Akerlof and Schiller " just change people's focus and one can change the decisions they make." [1, p.173].

Our results suggest that the potential for manipulation of human behavior may indeed be much greater than what is proposed by main stream economic theory. This finding is in line with recent works in economics that emphasize the role of manipulation in the functioning of markets.

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