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## Working Papers / Documents de travail

## Arranged Marriages under Transferable Utilities

Pauline Morault

# Arranged Marriages under Transferable Utilities 

Pauline Morault*

June 2017

Abstract: In many societies, marriage is a decision taken at the familial level. Arranged marriages are documented from Renaissance Europe to contemporary rural Kenya, and are still prevalent in many parts of the developing world. However, this family dimension has essentially been neglected by the existing matching literature on marriages. The objective of this paper is to introduce family considerations into the assignment game. We explore how shifting decision-making to the family level affects matching on the marriage market. We introduce a new concept of familial stability and find that it is weaker than individual stability. The introduction of families into the marriage market generates coordination problems, so the central result of the transferable utility framework no longer holds: a matching can be family-stable even if it does not maximize the sum of total marital surpluses. Interestingly, even when the stable matching is efficient, family decision-making drastically modifies how the surplus is shared-out. These results may have fundamental implications for pre-marital investments. We find that stable matchings depend on the type of family partitioning. Notably, when each family contains one son and one daughter, familial and individual stability are equivalent.

Keywords: marriage, family, matching, transferable utility

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## I. Introduction

In many societies, marriage is a decision taken at the family level. Examples range from Renaissance Europ $\mathbb{1}^{1}$ to contemporary rural Kenya ${ }^{2}$ In fact, arranged marriages are still prevalent in many parts of the developing world ${ }^{3}$. In the survey conducted in India in 2003 by Luke and Munshi (2011), $89.5 \%$ of the 4000 respondents reported that their marriage was "arranged" by their parents, and $88.7 \%$ of their children's marriages were also arranged. Even in Western countries, where arranged marriages are considered to have disappeared, parents still heavily influence the choice of the spouse ${ }_{4}^{4}$. The upper classes in particular exert this influence through private schooling and the organization of expensive and selective social events (e.g. "rallies", in France) More alarmingly, UNICEF (2014) revealed that 700 million women alive in 2014 worldwide had been forced into child marriages, more than a third of them under 15 years old.

Yet this family dimension is basically neglected by the existing matching literature on marriage. Surprisingly, even papers studying phenomena related to arranged marriages, such as premarital transfers or marital payments, do not take family structure into account. We seek to fill this gap here by introducing family considerations into the assignment game of Shapley and Shubik (1971). Our objective is to explore how shifting decision-making from individuals to families affects matching on the marriage market. In this paper, we study an extension of the transferable utility matching model by introducing families and considering the marriage decision to be taken at the family level. We extend the concept of stability to families and explore how the shift from individual to familial decision-making changes stable matchings. We show that stability at the family level is weaker than for individuals. In a transferable utility framework,

[^1]individual stability implies aggregate surplus maximization. Moreover, this framework allows utility to be shared with family members. Consequently, an individual-stable matching must be family-stable. By contrast, family-stable matchings are not always stable for individuals. We find two main configurations in which this happens. First, family-stable matchings may be inefficient due to coordination problems between families. In this case, the loss generated by potential deviations for some members of the family is too large to be compensated for by any benefits this deviation might provide for other members. Second, even efficient matchings may not be stable for individuals. This is because families loosen constraints on the shares of the surplus: they agree on some sharings-out of surplus their children would never accept individually, because they are taking into account the family as a whole. Thus we find that the set of the shares of surplus that support efficient matchings as family-stable includes the set of the shares of surplus that support them as individual-stable. As a result, our model predicts that we should observe more stable outcomes when marriages are arranged by parents rather than by individuals. In this sense, our extension with families is less predictive than the classical matching models on marriage. However, it can help explain why, in certain contexts, we do not observe the marriage structure and the sharing of marital surplus predicted by existing matching models.

We find that family-stable matchings strongly depend on the structure and composition of families. In particular, we find that when families are heterogenous in terms of size and when gender is not distributed uniformly across families, inefficient stable matchings may emerge. We also show through examples that the set of shares of surplus that support efficient matchings as stable tends to shrink as competition increases. In particular, for a family partition such that each family is composed of one son and one daughter, the set of shares is minimal.

Our analysis builds on the literature of matching theory applied to the marriage market and the economics of the family, in particular Becker (1973, 1991), and recently reviewed by Browning et al. (2014). The main novelty of our model lies in shifting the decision-making process from individuals to families. To our knowledge, we are the
first to introduce families into the assignment game (Shapley and Shubik 1971) ${ }^{6}$.
There is an extensive literature on the economics of marriage examining situations related to arranged marriages under very restrictive assumptions on family structure. Peters and Siow (2002), when they consider parents choosing a premarital transfer to their children and study equilibria in which children use these investments to compete for spouses, use a two-sided market setting, with families composed of one female facing families composed of one male. Actually, a family here can be modeled as an individual making an investment decision prior to the matching decision. Anderson (2003) analyzes the importance of the caste in the evolution of dowry payments with modernization, Anderson and Bidner (2015) formalize the dual role of dowry as both a premortem bequest from parents to daughters and a market clearing price, and Do et al. (2013) analyze the consequences of marital payments on consanguineous marriages when commitments are not credible. In all these papers, however, each family is composed of one child only. By contrast, we model families as arbitrary subsets in a population of males and females.

Only a few papers deal with family structure in the matching literature related to marriage. Laitner (1991) explores premarital transfers from parents to their two children, one son and one daughter, to induce their marriages in a non-transferable utility framework. He restricts attention to symmetric equilibria and focuses on the impact of assortative mating on neutrality results, but he provides a very interesting model of spouse selection by families which would be worth extending ${ }^{7}$. By contrast, we consider a transferable utility framework with arbitrary family structure and study the impact of family decision-making on stable matchings.

Our analysis contributes to an expanding literature on the impact of family composi-

[^2]tion on outcomes related to marriage. Botticini and Siow (2003) study how parents decide to allocate their capital between their son and their daughter, and show that in a virilocal environment, dowry endogenously emerges. Their paper differs from ours in that they focus on one family and do not study a matching problem. Fafchamps and Quisumbing (2008) study how parents allocate their wealth among a given number of sons and daughters through transfers of assets at the time of marriage and levels of human capital. They find that children receive more when their parents are wealthier or when they have fewer siblings. They do not put any restriction on family composition and find that siblings compete for limited resources. By contrast, we show that the constraints due to being part of the same family are different when the family chooses the spouse. Vogl (2013) uses an optimal stopping model to explore how daughter competition affects the quality of the spouse and human capital outcomes in South Asia, where the norm is to marry the first-born before the younger children. Our model also stresses the constraint connected with same-gender siblings on the marriage market, but without restrictive assumptions on family structure and cultural norms. The impact of family composition is also studied for other social and economic outcomes such as education (Lafortune and Lee 2014), labor (Baland et al. 2016), migration (Bratti et al. 2016), or health (Black et al. 2017). However, all of these studies neglect the equilibrium effects of family structure. By contrast, we show that type of family partition deeply affects stable matchings.

Some papers compare the effects of parental consent versus individual consent on the marriage market. Edlund and Lagerlöf (2006) argue that a shift from parental to individual consent redistributes resources from old to young and from men to women. They show with an overlapping-generation model that such redistribution may have further consequences on growth. Huang et al. (2012) use data on urban couples in China in the early 1990s and find that parental matchmaking may distort children's spouse choice, parents being more willing to substitute money for lov $8^{8}$ In this case, the parents' preferences differ from those of the children, and should be modeled

[^3]with non-transferable utilities. We also compare and contrast stable matchings when marriages are arranged by families and when they are decided at the individual level in a transferable utility framework. Our results help identify which matching framework should be used to address arranged marriages in different applied contexts.

Finally, our paper establishes a new connection between the literatures on matching and on network formation. In our model, families are composed of a given number of individuals, each linked to a member of a different family through a marital relationship. In this setting, the assignment game generates a network among the families. Jackson (2010) begins his textbook on social and economic networks by discussing the example of the Renaissance Florentine marriage network. Relying on Padgett and Ansell (1993), he suggests that the central position of the Medici family in the marriage network may have allowed them to dominate the Florentine oligarchy. Our model provides a theoretical framework which may shed light on which type of marriage network emerges in different social and economic contexts.

The remainder of the paper is organized as follows. We present the model and the new concept of familial stability in Section 2. We explore the properties and structures of family-stable matchings in Section 3. Section 4 concludes.

## II. The model

We consider an economy composed of marriageable sons and daughters. We assume that the population is partitioned into families by a family partition $\mathcal{F}$. A family $f$ is a subset of agents, which can a priori contain any number of sons and daughters. Figure 1 illustrates different family partitions for a population of two sons $i_{1}, i_{2}$ and two daughters $j_{1}, j_{2}$.

Family partitioning generates a coalition structure which is critical for the characterization of stable matchings. In Section 3, we study the particular case of family partitioning such that each family is composed of one son and one daughter. Parents


Figure 1: Family Partitions
seek to marry off their children on the marriage market in order to maximize the utility of the family $u_{f}$, which is equal to the sum of the utilities of the children. We consider a transferable utility framework, so a marriage between a son and a daughter from two different families generates a marital surplus $\pi_{i j} \geq 0$ endogenously allocated between the groom and the bride, who receive respectively $u_{i} \geq 0$ and $u_{j} \geq 0$, with $\pi_{i j}=u_{i}+u_{j}$. We assume siblings cannot marry, which is equivalent to setting $\pi_{i j}<0$ if $i$ and $j \in f$. Finally, we assume the normalization that being single provides no payoff, i.e. $\pi_{i 0}=\pi_{0 j}=0$ for all $i, j$. Therefore we can state the definition of a matching ( $\mu, u$ ) on individuals with families formally. We consider a unique output matrix with entries $\pi_{i j}$ that specify the total surplus from possible marriages. Because we assume transferable utilities, this marital surplus can be divided between the husband and the wife. Thus, by definition, if $i$ and $j$ from two different families form a match, i.e. if $\mu_{i j}=1$, we have $u_{i}+u_{j}=\pi_{i j}$. Thus, a matching on individuals with families induces family utilities $u_{f}=\sum_{i \in f} u_{i}+\sum_{j \in f} u_{j}$.

For instance, when each family is composed of one child only, we have the classical matching model with individuals. Introducing families shifts decision-making on the marriage market from individuals to parents. Parents consider the utility of the family, which generates some interdependence in the utilities of its members, who would otherwise act individually. They choose partners for their children in such a way as to maximize the utility of the whole family, which may mean arranging a worse marriage
for one child if it enables the other children to marry better. We show in Section 3 that this setting changes stable matchings. It is also noteworthy that, in our framework, a matching generates a network of families. In a network analysis perspective, each node or family can be linked to one or more families through marital connections. Two families could be united through several links, as several of their children could be matched. In fact, when families are taken into account, matching can also be considered a model of strategic network formation. This is in sharp contrast with the classical one-to-one matching models on marriage. We do not specifically study the network structure that emerges from this setting, but we discuss in the Conclusion the broader economic and social implications of family links through marriage, based on this network structure.

To solve our matching problem, we introduce a new concept of familial stability. Classical matching models on marriage only considering individuals define a matching as stable if there are no two persons, married or unmarried, who would like to form a new union. In other words, if there are no blocking pairs. As a direct extension of this notion, we consider that a matching is stable if there are no two families who would like to form one or several new unions for some of their children. Thus, we say that a matching is family-stable if there are no blocking pairs of families. This definition is consistent with empirical evidence that families negotiate their children's mariages bilaterally. In their study on the Luo in Kenya, Luke and Munshi (2006) explain that arranged marriages are organized by a matchmaker, or jagam, who is usually one of the man's sisters, sisters-in-law or other extended relatives. Molho (1994) provides evidence of this practice in detailed descriptions of some arranged marriages in medieval Florence. Literally, we say that a matching is family-stable if there are no two families who would like to sever their existing links for one or several of their children to create new ones with the other family, such that the utilities of both families increase, one of which increasing strictly. To state the definition formally, we introduce the notation $C_{f}$, which represents a subset of children in $f$.

Definition 1 A matching $(\mu, u)$ is not family-stable with respect to the family partition
$\mathcal{F}$ if $\exists\left(f, f^{\prime}\right) \in \mathcal{F}^{2}, \exists\left(C_{f}, C_{f^{\prime}}\right)$ of the same size, $\exists\left(\mu^{\prime}, u^{\prime}\right)$ such that
(1) $\forall i \in C_{f} \exists j^{\prime} \in C_{f^{\prime}}$ such that $\mu_{i j^{\prime}}^{\prime}=1$. .
(2) $\mu_{i j^{\prime}}=\mu_{i j^{\prime}}^{\prime}$ if $i \notin C_{f}$ and $j^{\prime} \notin C_{f^{\prime}}$.
(3) $u_{f}^{\prime} \geq u_{f}$ and $u_{f^{\prime}}^{\prime} \geq u_{f^{\prime}}$ with at least one strict inequality.

Condition (1) says that the alternative matching $\left(\mu^{\prime}, u^{\prime}\right)$ is such that some of the children of families $f$ and $f^{\prime}$ are married to each other, formally children in $C_{f}$ and $C_{f^{\prime}}$. Families $f$ and $f^{\prime}$ may already be matched through some of their children in the initial matching $(\mu, u)$ and may decide to sever some of their existing links to create new ones. They can sever some of their links with other families to create new links between themselves, and/or swap existing marriages among their children ${ }^{10}$. Condition (2) states that the alternative matching only differs from the initial one for members of $C_{f}$ and $C_{f^{\prime}}$ and their partners in the initial matching. Condition (3) requires that the two families $f$ and $f^{\prime}$ gain from the new matching, with at least one family gaining strictly. It is worth noting that when each family is composed of one child only, our concept of familial stability is equivalent to the classical notion of stability.

Our definition of familial stability considers only deviations by pairs of families. In Section 3, we show that this concept of familial stability generates some coordination problems which may lead to inefficient social outcomes. As a consequence, we find that the set of family-stable matchings exceeds the core, as defined in Shapley and Shubik (1971). We also discuss an alternative definition of familial stability that considers families as able to deviate in triples or more.

[^4]
## III. Stable Matchings with Families

In this section, we explore the properties and structures of family-stable matchings, and we compare them with individual-stable matchings. We call individual stability the usual concept of stability used in the Becker-Shapley-Shubik mode ${ }^{11}$. We find in particular that familial stability is weaker than individual stability: while individual stability implies familial stability, a family-stable matching may be not stable for individuals. We find that there are two main configurations in which a matching may be stable for families but not for individuals. First, inefficient matchings, i.e. matchings that do not maximize the sum of total marital surplus, may be family-stable. This is in sharp contrast with individual-stable matchings, as the central result of the transferable utility framework is that individual stability implies aggregate surplus maximization. Second, even efficient matchings may be stable for families but not for individuals. In this case, the difference lies in the shares of surplus, not in the assignment itself: there are some shares of surplus that support efficient matchings as family-stable, but not as individual-stable. Finally, we find that family partitioning has a direct impact on the characterization of family-stable matchings. In particular, for the family partition such that each family is composed of one son and one daughter, familial stability implies individual stability.

Our first result is that individual stability implies familial stability. This result may seem counter-intuitive, as we usually place arranged marriages and love marriages in opposition. The intuition for this result is that, as we are in a transferable utility framework, the utility generated by a marriage between a son and a daughter from different families can be transferred entirely and without friction to their respective families. It is as if the benefits the two individuals experience from a love marriage could be perfectly shared with their respective parents. Indeed, when children individually maximize their own utility on the marriage market, these utility maximizations directly benefit the family as a whole. However, in a non-transferable utility frame-

[^5]work, the utility of the parents could be misaligned with the utilities of their children. For instance, parents could care only about the wealth or the education of their children's partners, while grown-up children could care about shared interests or affinity, as documented in urban China in Huang et al. (2012). In this case, individual stability would not imply familial stability. In such a framework, we could observe sharp differences in terms of outcomes on the marriage market depending on whether the decision-maker is the family or the individual. Our results should help determine which assumptions on utility will be most relevant to different arranged marriages settings.

We now state this result formally in Theorem 1.
Theorem 1 An individual-stable matching is always family-stable.
Before proceeding to the proof, we introduce the notations $u_{C_{f}}$ and $\bar{C}_{f}$. Let $u_{C_{f}}$ be the sum of the utilities of the members in $C_{f}$, and $\bar{C}_{f}$ be such that $C_{f} \cup \bar{C}_{f}=f$.
Proof. Consider a matching $(\mu, u)$ which is stable for individuals. A matching $(\mu, u)$ is individual-stable if $u_{i}+u_{j}>\pi_{i j}$ if $i$ and $j$ are not married, and $u_{i}+u_{j}=\pi_{i j}$ if $i$ and $j$ are married ${ }^{12}$. Assume this matching is not family-stable. Therefore $\exists\left(f, f^{\prime}\right), \exists\left(C_{f}, C_{f^{\prime}}\right)$ of the same size, $\exists\left(\mu^{\prime}, u^{\prime}\right)$, which satisfy conditions 1,2 and 3 of Definition 1. Indeed we have that $u_{f}^{\prime}+u_{f^{\prime}}^{\prime}>u_{f}+u_{f^{\prime}} \Leftrightarrow u_{C_{f}}^{\prime}+u_{\bar{C}_{f}}^{\prime}+u_{C_{f^{\prime}}}^{\prime}+u_{\bar{C}_{f^{\prime}}}^{\prime}>u_{C_{f}}+u_{\bar{C}_{f}}+u_{C_{f^{\prime}}}+u_{\bar{C}_{f^{\prime}}}$. But we know that $u_{\bar{C}_{f}}^{\prime}+u_{\bar{C}_{f^{\prime}}}^{\prime} \leq u_{\bar{C}_{f}}+u_{\bar{C}_{f^{\prime}}}$, because children in $\bar{C}_{f}$ or $\bar{C}_{f^{\prime}}$ are either unaffected by the deviation or have their link severed. Therefore this implies that $u_{C_{f}}^{\prime}+u_{C_{f^{\prime}}}^{\prime}>u_{C_{f}}+u_{C_{f^{\prime}}}$ and hence $\sum_{i \in C_{f} \text { and } j \in C_{f^{\prime}}} \pi_{i j}>\sum_{i \in C_{f}} u_{i}+\sum_{j \in C_{f^{\prime}}} u_{j}$.
But because the matching $(\mu, u)$ is individual-stable, we have a contradiction.
This first result on the relationship between individual stability and familial stability enables us to derive interesting properties of family-stable matchings. From the literature on matching, we know that individual-stable matchings always exist and that they always maximize the sum of total marital surplus $\mathbb{S}^{13}$. This implies that Theorem

[^6]1 suffices to prove the existence of family-stable matchings. Moreover, thanks to the equivalence result of the transferable utility framework, we know that there always exists a set of shares of marital surplus that satisfy familial stability for assignments that maximize aggregate surplus. So our model predicts that if parents allowed their children to choose their own partners, the ensuing matching would be stable for families. This would argue for promoting individual choice of spouse instead of parental matchmaking in societies where arranged marriage is still prevalent, especially since individual choice should always lead to efficient social outcomes.

By contrast, we next show that parental matchmaking may lead to inefficient matchings.

Proposition 1 A matching can be family-stable and inefficient.
Consider the two-men-two-women case and the family partition $\mathcal{F}_{1}$ illustrated in Figure 2. Family $f_{1}$ is composed of two sons $i_{1}$ and $i_{2}$, while families $f_{2}$ and $f_{3}$ are composed of one daughter each, respectively $j_{1}$ and $j_{2}$. Note that with this family partition, assignments $\left(\mu_{1}\right) i_{1}-j_{1}, i_{2}-j_{2}$, represented by dashed lines, and $\left(\mu_{2}\right) i_{1}-j_{2}, i_{2}-j_{1}$, represented by thick lines, are feasible. Let us assume that matching $\mu_{1}$ is inefficient, while matching $\mu_{2}$ is efficient.


Figure 2: Families versus individuals

In this configuration, if individuals chose their spouse, the outcome would be efficient matching $\mu_{2}$, as theory predicts. However, when families decide who their children will marry, they may end up stuck with inefficient matching $\mu_{1}$. The intuition for this is that even if both son $i_{1}$ and daughter $j_{2}$ as individuals have an incentive to sever
their respective links so as to marry, family $f_{1}$ would prevent a marriage between its son $i_{1}$ and $j_{2}$ if the loss generated thereby in terms of utility for its second son $i_{2}$ is too large. In this case, inefficient matching $\mu_{1}$ is family-stable but not stable for individuals. If there were no families, individuals would be able to sever their links and remarry in order to reach the efficient assignment. But families forbid such deviations. Inefficient matchings emerge when potential deviations for a family are such that some of its members end up single or worse off, and the benefits its other members obtain from the new matching are not sufficient compensation. This happens when families are composed of several children of the same gender ${ }^{14}$ and are facing smaller families or families with few children of the complementary gender. In these cases, the bigger families could oppose potential deviations, as they would be more likely to involve one of their children ending up single. In this configuration, stable matchings differ in the assignment itself, depending on whether the decision-maker is the family or the individual. We may actually observe matchings that are not predicted by the classical theory on matching, but which can be explained if we take families into account. For instance, if we assume that each son is characterized by a single characteristic $x$, that each daughter is characterized by a single characteristic $y$ and that there is complementarity (substitution) in traits, i.e. that the marital surplus is a supermodular (submodular) function of the attributes of the two partners, the classical matching model predicts positive (negative) assortative mating. By contrast, with these same assumptions, matchings with no positive (negative) assortative mating can be family-stable.

Relying on our example, we now prove the existence of such inefficient matchings, showing that there exists a set of shares of surplus that support the inefficient matching $\mu_{1}$ as family-stable. By assumption, $\mu_{1}$ is the inefficient matching, and $\mu_{2}$ is the efficient one, which means that $\pi_{12}+\pi_{21}>\pi_{11}+\pi_{22}$. We consider possible deviations

[^7]of pairs of families from the inefficient assignment. We first note that families $f_{2}$ and $f_{3}$ cannot deviate together, both being composed of one single daughter. The only two possible family deviations from the inefficient assignment are (1) the deviation involving $f_{1}$ and $f_{2}$, in which case they would form $i_{2}-j_{1}$; and (2) the deviation involving $f_{1}$ and $f_{3}$, in which case they would form $i_{1}-j_{2}$. Let us consider the first family deviation: $f_{1}$ and $f_{2}$ could decide to sever their existing links to marry $i_{2}$ and $j_{1}$. In particular, family $f_{1}$ would sever its link with $f_{3}$ to marry its son $i_{2}$ to $j_{1}$ from family $f_{2}$ instead of $j_{2}$ from family $f_{3}$. This threat generates an upper bound on the share $u_{j_{2}}$ that $f_{3}$ can expect from $f_{1}$ in the marriage $i_{2}-j_{2}$. $f_{1}$ and $f_{2}$ would have an incentive to deviate if $u_{f_{1}}+u_{f_{2}}<u_{f_{1}}^{\prime}+u_{f_{2}}^{\prime} \Leftrightarrow u_{i_{1}}+u_{i_{2}}+u_{j_{1}}<u_{i_{2}}^{\prime}+u_{j_{1}}^{\prime}$. By definition, $u_{i_{2}}=\pi_{22}-u_{j_{2}}$, therefore, the highest share that $f_{3}$ could expect from $f_{1}$ in the marriage $i_{2}-j_{2}$ is $u_{j_{2}}$ such that $f_{1}$ and $f_{2}$ are indifferent between the inefficient assignment and deviation, formally $u_{j_{2}}$ such that $u_{i_{1}}+u_{i_{2}}+u_{j_{1}}=u_{i_{2}}^{\prime}+u_{j_{1}}^{\prime}$. We replace $u_{i_{2}}$ by its expression in terms of $u_{j_{2}}$ and find $u_{j_{2}} \leq \pi_{11}+\pi_{22}-\pi_{21}{ }^{15}$. We follow the same reasoning for the second deviation involving $f_{1}$ and $f_{3}$ and find that the upper bound on $u_{j_{1}}$ is: $u_{j_{1}} \leq \pi_{11}+\pi_{22}-\pi_{12}$. We find that all pairs ( $u_{j_{1}}, u_{j_{2}}$ ) satisfying inequalities $u_{j_{1}} \leq \pi_{22}+\pi_{11}-\pi_{12}$ and $u_{j_{2}} \leq \pi_{22}+\pi_{11}-\pi_{21}$ with $0 \leq u_{j_{1}} \leq \pi_{11}$ and $0 \leq u_{j_{2}} \leq \pi_{22}$ yield imputations $u_{j_{1}}, u_{j_{2}}, u_{i_{1}}=\pi_{11}-u_{j_{1}}$ and $u_{i_{2}}=\pi_{22}-u_{j_{2}}$ that support the inefficient assignment as family-stable. To represent this set graphically, assume $\pi_{22}>\pi_{21}, \pi_{12}>\pi_{11}$, and $\pi_{12}=\pi_{21}{ }^{16}$,

This example shows that considering families generates some coordination problems which may translate into inefficient outcomes. The coordination problem emerges here because deviations are only allowed for pairs of families. It is interesting to note that, in our example, if we allowed families to deviate in triples, the three families could coordinate their deviations to reach the efficient assignment. This means that

[^8]

Figure 3: Inefficient family-stable matching
the three families could obtain a higher aggregate surplus to share, and could find a sharing mode that would benefit all three. This is in sharp contrast with classical matching models on marriage in which individual-stable matchings are located in the core, defined in Shapley and Shubik (1971) as "the set of outcomes that no coalition can improve upon". In our setting, the core is the set of the shares of surplus that support efficient matchings as family-stable, as no coalition of families would be able to improve upon it without making other families worse off. In other words, either the coalition would reach an inefficient matching which would destroy the surplus for other families, or the coalition would modify their shares of surplus within the efficient matching, reducing the surplus for other families. Our result here is that familystable matchings exceed the core, as some inefficient family-stable outcomes could be improved upon by a coalition of a subset of families. This result comes from our definition of familial stability, which considers deviations by pairs of families. However, we could also choose an alternative definition which considers deviations of a subset of families. We could assume that families negotiate the marriage of their children multilaterally and commit through betrothal contracts. This alternative definition would resolve some situations where families are stuck in an inefficient matching. In reality, however, deviations by $k>2$ families should generate coordination costs that
may offset this positive result. In any case, as long as the number of families who can deviate together is bounded, inefficient outcomes are still likely to arise.

Interestingly, we also find that a matching can be efficient and family-stable but not individual-stable.

Proposition 2 A matching can be efficient and family-stable but not stable for individuals.

This means that the assignment itself might be the same for families and individuals, while the shares of surplus that support it as stable differ. We find that the set of shares of surplus that support efficient assignments as family-stable includes the set of shares of surplus that support them as individual-stable. The intuition here is that when we consider families instead of individuals, constraints are less binding, and therefore families may accept a wider range of sharings-out of surplus than individuals.

Consider again the two-men-two-women case presented previously. We now study the efficient matching. For individuals, we follow Browning et al. (2014), who characterize the shares of surplus that support the efficient matching $\mu_{2}$ as individual-stable ${ }^{17}$. The authors show that all pairs $\left(u_{j_{1}}, u_{j_{2}}\right)$ satisfying the inequalities $\pi_{12}-\pi_{11} \geq u_{j_{2}}-$ $u_{j_{1}} \geq \pi_{22}-\pi_{21}$ with $\pi_{21} \geq u_{j_{1}} \geq 0$ and $\pi_{12} \geq u_{j_{2}} \geq 0$ yield imputations $u_{j_{1}}, u_{j_{2}}$, $u_{i_{1}}=\pi_{12}-u_{j_{2}}$, and $u_{i_{2}}=\pi_{21}-u_{j_{1}}$, which support $\mu_{2}$ as stable for individuals. Indeed we observe that when the decision-maker is the individual, the share of surplus that woman $j_{2}$ can expect to obtain is bounded and depends on the share of surplus that woman $j_{1}$ obtains ${ }^{18}$. For families, we first consider the family partition $\mathcal{F}_{1}$, already described. We characterize formally the set of the shares of surplus that support the efficient assignment $\mu_{2}$ as family-stable with $\mathcal{F}_{1}$, and compare it with the set of surplus that supports it as individual-stable. Note that for this purpose, it is important to

[^9]choose a family partition for which assignments $\mu_{1}$ and $\mu_{2}$ are both possible ${ }^{19}$. This is so that we can isolate the impact of family decision-making on the set of the shares of surplus which support the stable matching, from the impact of siblings of different sex, who cannot marry. We follow the same reasoning as before and consider possible deviations by pairs of families from the efficient assignment. We find that all pairs $\left(u_{j_{1}}, u_{j_{2}}\right)$ satisfying inequalities $u_{j_{1}} \leq \pi_{12}+\pi_{21}-\pi_{22}$ and $u_{j_{2}} \leq \pi_{12}+\pi_{21}-\pi_{11}$ with $\pi_{21} \geq u_{j_{1}} \geq 0$ and $\pi_{12} \geq u_{j_{2}} \geq 0$ yield imputations $u_{j_{1}}, u_{j_{2}}, u_{i_{1}}=\pi_{12}-u_{j_{2}}$ and $u_{i_{2}}=\pi_{21}-u_{j_{1}}$ that support the efficient assignment as family-stable. It is worth noting that, unlike when the marriage decision is taken by individuals, there is no lower bound on $u_{j_{1}}$ and $u_{j_{2}}$ other than 0 , and $u_{j_{1}}$ and $u_{j_{2}}$ are independent of each other. The reason for this is that family partition $\mathcal{F}_{1}$ is such that alternative husbands for $j_{1}$ and $j_{2}$ are part of the same family $f_{1}$, which makes the threat of the wife leaving her current husband for the other potential husband obsolete. The only constraint on the shares of surplus is that $u_{j_{1}}$ (resp. $u_{j_{2}}$ ) should be such that $u_{f_{1}}+u_{f_{3}} \leq \pi_{22}$ (resp. $\left.u_{f_{1}}+u_{f_{2}} \leq \pi_{11}\right)$ 20. Otherwise families $f_{1}$ and $f_{3}$ (resp. $f_{1}$ and $f_{2}$ ) would both have an incentive to deviate, even if this means son $i_{1}$ (resp. son $i_{2}$ ) ending up single, because they would have more surplus to share with $\pi_{22}$ (resp. $\pi_{11}$ ).

We represent these two sets graphically in Figure 4, assuming as before that $\pi_{22}>\pi_{21}$, $\pi_{12}>\pi_{11}$, and $\pi_{12}=\pi_{21}$. On the left, the shaded area represents all the pairs that satisfy the requirements for individual stability. On the right, the shaded area represents all the pairs that satisfy the requirements for familial stability with $\mathcal{F}_{1}{ }^{21}$.

We observe that the set of the shares of surplus supporting the efficient assignment as family-stable with family partition $\mathcal{F}_{1}$ includes the set of the shares of surplus that support it as individual-stable ${ }^{22}$, which is consistent with Theorem 1.

[^10]

Figure 4: Shares of surplus

Now let us consider the family partition $\mathcal{F}_{2}$, such that family $f_{1}$ is composed of men $i_{1}$ and $i_{2}$ and family $f_{2}$ is composed of women $j_{1}$ and $j_{2}$. This family partition corresponds to configuration (a) in Figure 1 with two families, and here again, is chosen to ensure that assignments $\mu_{1}$ and $\mu_{2}$ are feasible. We note that in this configuration, the two possible husbands for each woman are part of the same family $f_{1}$, and the two possible wives for each man are part of the same family $f_{2}$. This familial configuration eliminates the threat of switching wives or husbands, which determines the upper and lower bounds on the shares that men and women can expect individually. It becomes straightforward that any sharing-out of the aggregate surplus will support the efficient assignment as family-stable. If this set were represented in Figure 4, the whole square would be shaded. Moreover, we note that with this family configuration, no inefficient matching is family-stable: the two families will obviously choose the assignment under which they would have the most to share. These results imply that we should end up with more stable configurations when two families marry off several of their children, as they have more leeway to rearrange the aggregate surplus among them. This helps explain why the practice of watta-satta, a bride exchange involving the simultaneous marriage of a brother-sister pair from two households, is common in some developing

[^11]countries (Jacoby and Mansuri 2010).
Thus, we find that for a given efficient assignment, families would accept some sharingsout of surplus its members would never accept if they were acting alone. As a consequence, our model predicts that arranged marriage could leave the assignment itself unaffected while greatly changing the surplus-sharing accepted by the married children. This result may have drastic implications ex ante in terms of premarital investment, in particular for the education of daughters. Moreover, it implies that in some societies, we may observe assignments that are predicted by one-to-one matching models but for which the sharing of surplus within the household is unexplained by classical theory. Our extension helps explain these situations.

Overall, we find that family partitioning determines the properties of family-stable matchings. For some family partitions, we may observe inefficient outcomes. As explained above, this seems to be the case when the distribution of sons and daughters is not uniform across families or when there is heterogeneity in families' size. For certain other family partitions, we may observe only efficient outcomes but drastic differences in terms of the size of the set of shares supporting them. For some family partitions, any sharing-out of aggregate surplus across families supports the efficient assignment as family-stable, as we illustrated with $\mathcal{F}_{2}$ and more broadly for all partitions that divide the population into two families. For others, the set is smaller and even the same as that obtained under individual decision-making: this is straightforward for the family partition that partitions the population into single individuals, for instance. Less trivially, this is also the case when each family is composed of one son and one daughter. It seems that the more competition between families, the smaller the set of shares supporting the efficient outcomes, as shown in Figure 5. For a population of three men and three women, we observe how the family partition affects the set of shares of surplus that support the efficient assignment $\left(\mu^{*}\right) i_{1}-j_{2}, i_{2}-j_{3}, i_{3}-j_{1}$. We thus assume that $\pi_{12}+\pi_{23}+\pi_{31}$ maximizes the sum of total marital surplus over all possible assignments $\sqrt[23]{23}$ Shaded volumes represent the set of shares of surplus that

[^12]support $\mu^{*}$ as family-stable. When volumes are in several colors, the set of shares of surplus supporting $\mu^{*}$ as family-stable is the intersection of these volumes.


Figure 5: Sets of surplus and family partitions

We observe that in family partitions for which all alternative husbands for the daughters are in the same family, as in (a), (b) and (c), the shares of surplus are independent of each other. By contrast, when alternative husbands are scattered among different families (as in (d), (e) and (f)), we observe not only lower bounds for women's shares, but also a functional relationship between the shares of surplus. Moreover, we observe that the more competition (i.e. the more families for the same number of males and partitions (b) and (d). To graphically represent the sets in Figure 5, we choose $\pi_{12}=\pi_{23}=\pi_{31}$.
females), the smaller the set of shares: the set shrinks when we go from (b) to (c), and when we go from (d) to (e). Finally, the family partitions for which inefficient matchings can be family-stable, (b), (c), (d) and (e), are characterized by families having same-gender children and are also heterogenous in terms of family size, as opposed to (a) and (f).

In particular, we find that for the family partition such that each family is composed of one son and one daughter, familial stability implies individual stability. Therefore for this family partition, the only family-stable assignments are the efficient ones and the sets of the shares of surplus that support the efficient assignments as stable are the same for individuals and families. We state our result formally in Theorem 2.

Theorem 2 For the family partition such that each family is composed of one son and one daughter, a family-stable matching must be stable for individuals.

Proof. Consider the family partition such that each family is composed of one son and one daughter. Consider a matching $\left(\mu_{i j}^{*}, u_{i j}^{*}\right)$. This matching is family-stable if there is no pair of families who would like to deviate from it together (see Definition 1). We need to consider all possible deviations from this matching, which should cover families that are linked and families that are not linked.
First consider any pair of linked families, $f_{k}$ and $f_{k^{\prime}}$. If these two families are already linked in terms of all four of their children, then they cannot deviate together. This is because this family partition is such that each family is composed of one son and one daughter, so two families already linked through two marriages cannot deviate by swapping the marriages of their children. If these two families are linked only in terms of their children $i_{k}$ and $j_{k^{\prime}}$, they could deviate together if they chose a marriage between their two other children, $j_{k}$ and $i_{k^{\prime}}$. Conditions on the sharing of surplus of linked families for $\left(\mu_{i j}^{*}, u_{i j}^{*}\right)$ to be a family-stable matching are: $u_{f_{k}}^{*}+u_{f_{k^{\prime}}}^{*}>u_{f_{k}}+u_{f_{k^{\prime}}} \Leftrightarrow$ $\pi_{k, k^{\prime}}+u_{i_{k^{\prime}}}^{*}+u_{j_{k}}^{*}>\pi_{k, k^{\prime}}+\pi_{k^{\prime}, k} \Leftrightarrow u_{i_{k^{\prime}}}^{*}+u_{j_{k}}^{*}>\pi_{k^{\prime}, k}$, which is a condition for individual stability.

Now consider any pair of unlinked families, $f_{k}$ and $f_{k^{\prime}}$. These two families are not
linked, so they have three options for deviation: marrying $i_{k}$ to $j_{k^{\prime}}$; marrying $j_{k}$ to $i_{k^{\prime}}$ or both these marriages. Considering only the two first deviations, we derive the conditions on surplus-sharing with unlinked families for ( $\mu_{i j}^{*}, u_{i j}^{*}$ ) to be a family-stable matching ${ }^{24}$, as follows:
$u_{f_{k}}^{*}+u_{f_{k^{\prime}}}^{*}>u_{f_{k}}+u_{f_{k^{\prime}}} \Leftrightarrow u_{i_{k}}^{*}+u_{j_{k}}^{*}+u_{i_{k^{\prime}}}^{*}+u_{j_{k^{\prime}}}^{*}>\pi_{k, k^{\prime}}+u_{i_{k^{\prime}}}^{*}+u_{j_{k}}^{*} \Leftrightarrow u_{i_{k}}^{*}+u_{j_{k^{\prime}}}^{*}>\pi_{k, k^{\prime}} ;$ $u_{f_{k}}^{*}+u_{f_{k^{\prime}}}^{*}>u_{f_{k}}+u_{f_{k^{\prime}}} \Leftrightarrow u_{i_{k}}^{*}+u_{j_{k}}^{*}+u_{i_{k^{\prime}}}^{*}+u_{j_{k^{\prime}}}^{*}>\pi_{k^{\prime}, k}+u_{i_{k}}^{*}+u_{j_{k^{\prime}}}^{*} \Leftrightarrow u_{i_{k^{\prime}}}^{*}+u_{j_{k}}^{*}>\pi_{k^{\prime}, k} ;$ which are conditions for individual stability.
Conditions for $\left(\mu_{i j}^{*}, u_{i j}^{*}\right)$ to be family-stable imply that $\pi_{k, k^{\prime}}=u_{i_{k}}^{*}+u_{j_{k^{\prime}}}^{*}$ if $i_{k}$ and $j_{k^{\prime}}$ are married, and $\pi_{k, k^{\prime}}<u_{i_{k}}^{*}+u_{j_{k^{\prime}}}^{*}$ if they are not, which is exactly the definition of a stable matching for individuals. Indeed, for the family partition such that each family is composed of one son and one daughter, familial stability implies individual stability.

This result is consistent with our previous observations: this family partition is such that there is a uniform distribution of sons and daughters across families, homogeneity of family size, and competition between families.

## IV. Conclusion

Our paper introduces families into the assignment game and extends the notion of stability to families in order to study arranged marriages. We explore how the shift from individuals to families in the decision-making process changes stable matchings in the marriage market. We find that individual-stable matchings are always familystable. By contrast, family-stable matchings may be not stable for individuals. A matching can be both family-stable and inefficient, due to coordination problems. Moreover, a matching can be family-stable and efficient but not stable for individuals. This arises from the fact that constraints are less rigid for families, as they can accept a poorer match for one of their children if this will benefit the whole family. As a consequence, our model predicts that there are more stable configurations when

[^13]marriages are arranged by parents rather than by individuals. We also find that the family partition impacts family-stable matchings. It seems that when families are heterogenous in size and when gender is not distributed uniformly across families, inefficient matchings are likely to appear. Finally, for efficient matchings we show through examples that the set of shares of surplus tends to shrink as competition increases. In particular, when families are composed of one child only or when they are composed of one son and one daughter, the set of shares is minimal. Thus, the theoretical framework we provide to capture consequences of family decision-making on stable matchings should be an aid to understanding outcomes in societies where arranged marriage is still prevalent.

In our model we consider transferable utilities, but it should be noted that we do not assume that parents can use the share of the surplus obtained from the marriage of one of their children to secure the marriage of another child. We could capture this dimension by introducing some dynamics into the model and assuming that each family marries off one child at each period. We could also see this emerging if we assumed credit-constrained families and explicit marriage payments. This would be a nice extension of our model for future research, which would enable us to capture some interesting features of arranged marriages in societies where marriage payment prevails. It has been documented that in such societies, the marriage of a child (e.g. a daughter) entails a marital transfer (e.g. a brideprice) to the wife-giving family, who can use it to finance the marriage payment of another child (e.g. the brideprice for a brother ${ }^{25}$.

In our paper, we find that different family partitions lead to different family-stable matchings, which restricts parents' range of decision-making. As a consequence, we support the idea that at the micro level, family composition has an impact on the way parents decide to marry off their children. For instance, Nassiet (2000) shows that in

[^14]the French nobility of the Ancien Régime, good marriages for first-born sons were more important than for younger children, due to male primogeniture. Moreover, in this historical context, women without brothers were very valuable partners as they would be the only heiresses of the family, while in other social contexts, such as rural South India (Kapadia 1995), women without brothers are less valuable mates. Vogl (2013) also provides evidence that in South Asia the quality of older daughters' marriages decreases as the number of their sisters increases. In future research it would be interesting to study this issue more deeply, by introducing more assumptions into our model. In particular, we could introduce birth order and asymmetry between sons and daughters, in order to more thoroughly capture the effect of family composition on marriage decisions.

In our model, family size and sex ratio are given, but we could also imagine an extension in which these two dimensions are endogenous. This would contribute to the growing literature on parents' decisions in terms of family size and sex selection in a marriage perspective (Edlund 1999, Bhaskar 2011).

Moreover, we could study the broader economic and social implications of family marriages. Marriages between families create a network of families whose structure determines the degree of segmentation of the society, which in turn has direct consequences in terms of redistribution, inequality and social mobility. As we show in our paper, the structure of families, described by the family partition, has direct impacts on family-stable matchings, and in turn on observed networks of families linked through marriage. It would be interesting to explore how family partitioning impacts this network formation.

Our intuition is that the impacts on stable matchings would be even sharper if we considered a non-transferable utility framework, in which the utility of the parents and the utilities of the children are misaligned. This would also be an interesting avenue for future research.

Finally, our model introduces pre-existing coalitions into the assignment game of Shap-
ley and Shubik (1971). The matching problem we explore here could therefore have relevance for a wider range of topics than simply marriage.

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[^1]:    ${ }^{1}$ Goody (1983), Nassiet (2000).
    ${ }^{2}$ Hakansson (1990), Luke and Munshi (2006).
    ${ }^{3}$ Hamon and Ingoldsby (2003), Anukriti and Dasgupta (2017).
    ${ }^{4}$ Kalmijn (1998) explains p. 401 that "although in Western societies parental control over children's marriage decisions is limited, there are still ways in which parents can interfere. They set up meetings with potential spouses, they play the role of matchmaker, they give advice and opinions about the candidates, and they may withdraw support in the early years of the child's marriage."
    ${ }^{5}$ Arrondel and Grange (1993), Pinçon and Pinçon-Charlot (1998).

[^2]:    ${ }^{6}$ No theoretical paper in the matching literature explores the matching problem we address. Some papers study many-to-one markets and many-to-many markets (Sotomayor 1999) applied to the marriage market. For instance, Baiou and Balinski (2000) study a matching model in which every man may have several wives and every woman several husbands and Bansal et al. (2007) study stable assignments with multiple partners. However, these models are different from ours, as we consider individuals to have one partner only.
    ${ }^{7}$ Zhang (2001) extends Laitner (1991) by introducing gender asymmetry.

[^3]:    ${ }^{8}$ Hortaçsu (2007) uses data on the urban Turkish family and finds that in comparison to familyinitiated marriages, couple-initiated marriages are more emotionally involving.

[^4]:    ${ }^{9}$ In the remainder of the paper, we implicitly assume that all definitions and proofs consider the respective case of a daughter $j \in f$ being married to a son $i^{\prime} \in f^{\prime}$, in order to avoid heavy notations.
    ${ }^{10}$ For instance, consider families $f_{1}$ and $f_{2}$, and assume that $i_{1}$ and $i_{2}$ are part of $f_{1}$ and $j_{1}$ and $j_{2}$ are part of $f_{2}$. Assume that $i_{1}$ and $j_{1}$ are matched together, and $i_{2}$ and $j_{2}$ are matched to other families in the initial matching. $f_{1}$ and $f_{2}$ could decide to deviate together by rearranging their marriages to have $i_{1}$ married to $j_{2}$ and $i_{2}$ married to $j_{1}$.

[^5]:    ${ }^{11}$ See Shapley and Shubik (1971), Becker (1973), Browning et al. (2014).

[^6]:    ${ }^{12}$ See Becker (1973).
    ${ }^{13}$ Shapley and Shubik (1971), Becker (1973), Browning et al. (2014).

[^7]:    ${ }^{14}$ This issue is also addressed by Vogl 2013 : "For instance, siblings of the same gender participate in the same marriage market, sharing a pool of potential spouses. In some ways, they are like any other participants on the same side of the market, but their membership in the same family introduces special constraints on their marriages." (p.1018).

[^8]:    ${ }^{15} \pi_{22}-u_{j_{2}}+u_{i_{1}}+u_{j_{1}}=u_{i_{2}}^{\prime}+u_{j_{1}}^{\prime} \Leftrightarrow \pi_{22}-u_{j_{2}}+\pi_{11}=\pi_{21} \Leftrightarrow u_{j_{2}}=\pi_{22}+\pi_{11}-\pi_{12}$, which is the higher bound on $u_{j_{2}}$.
    ${ }^{16}$ This is the same assumption as that made by Browning et al. (2014) in Chapter 8. We can use a numerical example to explore this result. For instance with $\pi_{22}=8, \pi_{21}=\pi_{12}=6, \pi_{11}=2$ we have that the shares $u_{j_{1}}=2, u_{j_{2}}=3, u_{i_{1}}=0, u_{i_{2}}=5$ support the inefficient assignment as family-stable, but obviously not as individual-stable.

[^9]:    ${ }^{17}$ See Example 1 in Section 8.1 of Browning et al. (2014).
    ${ }^{18}$ Browning et al. (ibid.) explain p. 318 "Woman $j_{2}$, who is matched with man $i_{1}$, cannot receive in that marriage more than $\pi_{12}-\pi_{11}+u_{j_{1}}$ because then her husband would gain from replacing her by woman $j_{1}$. She would not accept less than $u_{j_{1}}+\pi_{22}-\pi_{21}$ because then she can replace her husband with man $i_{2}$, offering to replace his present wife." Notations are adapted.

[^10]:    ${ }^{19}$ This would not be the case for the family partition (b) in Figure 1 with two families, as $i_{1}$ would be the brother of $j_{1}$, and $i_{2}$ the brother of $j_{2}$.
    ${ }^{20}$ Which is equivalent to $\left(\pi_{12}-u_{j_{2}}\right)+\left(\pi_{21}-u_{j_{1}}\right)+u_{j_{2}} \leq \pi_{22} \Leftrightarrow u_{j_{1}} \leq \pi_{12}+\pi_{21}-\pi_{22}\left(\left(\pi_{12}-\right.\right.$ $\left.\left.u_{j_{2}}\right)+\left(\pi_{21}-u_{j_{1}}\right)+u_{j_{1}} \leq \pi_{11} \Leftrightarrow u_{j_{2}} \leq \pi_{12}+\pi_{21}-\pi_{11}\right)$.
    ${ }^{21}$ The left hand side of Figure 4 is the same as Figure 8.1 in Browning et al. (2014). On the right hand side, the upper bound of $u_{j_{2}}$ is $\pi_{12}$, as $\pi_{12}+\pi_{21}-\pi_{11}>\pi_{12}$ with the assumptions made on the marital surpluses for this graphical representation.
    ${ }^{22}$ We can use the same numerical example as before to verify that the shares $u_{j_{1}}=0, u_{j_{2}}=0$,

[^11]:    $u_{i_{1}}=6, u_{i_{2}}=6$ support the efficient assignment as family-stable, but not as individual-stable.

[^12]:    ${ }^{23}$ We also assume $\pi_{12}+\pi_{31} \geq \pi_{11}+\pi_{32}$, otherwise $\mu^{*}$ would not hold as family-stable in family

[^13]:    ${ }^{24}$ Considering the third deviation would give weaker restrictions on the shares of surplus.

[^14]:    ${ }^{25}$ The 2015 documentary Sonita presents an Afghan family trying to marry one of its daughters to obtain a brideprice so that her elder brother could purchase a bride. Nassiet (2000) points out that the in-coming dowry of the bride was used to compensate for the out-going dowries of the sisters of her husband in the French nobility of the Ancien Régime.

