Remarks on the Mathematics and Philosophy of Space-time in Early Imperial China

Daniel Patrick Morgan

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There are things you learn early in your academic training that stick with you, and that you hear repeated, but that you are longer sure how you know—Truths, until otherwise proven, that even then are difficult to unlearn. Two of the Truths that I picked up from somewhere, and heard repeated since, are that, first, Chinese thought is somehow unique in recognizing the interconnectedness of space and time, and that, second, Chinese astronomy is a "calendar science" dealing exclusively with the latter.[1] These Truths, it took me some time to realize, cannot both be *true* if for the simple reason that the one, *a priori*, excludes the other. I forget how I had come to believe these things, and, to reassure myself that I had not made them up, I went back through the literature to familiarize myself with their genesis. Both go back to the nineteenth century, it turns out, and both have been problematized since before I learned to read, yet *both of them*, for whatever reason, flutter still like forgotten prayer flags generating merit in the wind.[2] The goal of this paper is to take them down.

What do I propose we say of Chinese thought as concerns space, time, cosmology, and the calendar if not this? Big picture, what I am proposing is very simple. First, I should like that we stop speaking

[1] Both at once, Marc Kalinowski tells us for example that "Le système astronomique chinois n'était pas conçu dans l'espace mais dans le temps, sur la base du calendrier," then, three pages later, that "Dans sa *Pensée chinoise* (1934), Marcel Granet consacrait un chapitre aux conceptions de l'espace et du temps en Chine. Nul mieux que lui n'a mis en évidence l'indissociabilité de ces notions, leur interchangeabilité et leur dimension concrète" ("Astrologie calendaire et calcul de position dans la Chine ancienne: les mutations de l'hémérologie sexagésimale entre le IV^{e} et le II^{e} siècles avant notre ère," *Extrême-orient, Extrême-occident* 18 [1996]: 71–113 [pp. 71, 74]). One should find this puzzling.

[2] This paper began with a list compiled of statements similar to Kalinowski's, in Note 1, and reactions thereto spanning the eighteenth to the twenty-first century and organized by filiation. If I have unduly singled out Kalinowski in the previous note it is for no other reason than that *he*, exceptionally, attributes these truisms to a historical progenitor. It is the roots of these platitudes, rather than their branches, so to speak, that are the focus of this paper.

about "the Chinese" and "Chinese thought" as if either were a single thing, let alone one informed exclusively by Marcel Granet's (1884–1940) *La pensée chinoise* of 1934.[3] Second, in allowing for a plurality of *pensées*, I propose that one of the voices that sinologists should try to accommodate in a discussion of space, time, cosmology, and the calendar is that of the *astronomer* and of *expert literature* on those subjects. Based primarily on the peculiarities of the word *yuzhou* 宇宙 "space-time" for "cosmos" as it appears (no more than ten times) in early political and ethical philosophy, Derk Bodde asserts an "emphasis on space over time in Chinese thinking."[4] And maybe that's true *of early political and ethical philosophy*,[5] it is important to specify, but who would a *historian of mathematics* be fooling with a commensurate assertion about *politics and ethics* based solely on his own corpus?

The interest in granting the astronomer a voice in a discussion of space and time is not simply *fairness*; it is that the expert voice, more often than not, has more interesting things to say, and that "the astronomer," more often than not, is but a role played by someone we otherwise know by a different label. More interesting than an outsider's musings about whether "Chinese thought" holds time over space or space over time, I hope to show, are the physics behind their interdependence, the plasticity with which they yield their forms, and the thought[s] of th[os]e Chinese who knew how to ply them to know the future, to move forwards and backwards in space and time, and to collapse the one into the other as effortlessly as a

[3] This is not to deny that Granet's study is beautifully written—it is—or the value of the early light it shed on the metaphysics and metrosophy one finds in certain forms of divination and elite ritual, occult, political, and medical writing; it is rather to insist that what Granet is describing is neither *unique* to China, *timeless*, nor *exclusive* of other "thought" therein. I am not the first to say this. For important critiques and alternatives, see Joseph Needham, *Science and Civilisation in China, vol.2: History of Scientific Thought* (Cambridge: Cambridge UP, 1956), esp. 288–9; John B. Henderson, *The Development and Decline of Chinese Cosmology* (New York: Columbia UP, 1984); A.C. Graham, *Yin-Yang and the Nature of Correlative Thinking* (Singapore: The Institute of East Asian Philosophies, National University of Singapore, 1986), esp. 8–11; Derk Bodde, *Chinese Thought, Society, and Science: The Intellectual and Social Background of Science and Technology in Pre-Modern China* (Honolulu: University of Hawaii Press, 1991).

[4] *Chinese Thought, Society, and Science*, 119.

[5] It is not. Also, note that the counterpart in Bodde's obligatory East-versus-West binary—"whereas Westerners, in speaking of 'time and space', clearly give priority to the former" (ibid., 106)—is easily disproven via Google search.

child might one string figure into another. Unlike mystics, our sources are only more than happy to share how they go about this, and the goal of this chapter is to distil that literature into metaphors, images, stories, and explanations by which the non-expert reader may visualise how the early Chinese astronomer experienced space and time from the big picture down to the fine grain.

Remembering how time works

The main problem with the opening propositions about space versus time, as any historian of astronomy would tell you, is that space *is* time when space is turning at a constant rate against a static point of reference. For short, *space is time when space is turning*.[6]

That, whether the reader is aware of it or not, is what is happening outside at this very moment: the sky is turning on its axis, east to west, carrying the sun from dawn to noon to dusk and back again in twenty-four hours just as reliably as the hour-hand of an analogue clock returns to twelve. The sun too is moving, albeit slower and in the opposite direction, charting its annual course through the ecliptic. In Cancer—where it was when I was born—it stands some twenty degrees north of the celestial equator, rising earlier, further north, and mounting higher in the sky over the course of a day; in Scorpio—where it was when one Vincent Leung was born—it has dropped just as far below the equator, rising later, towards the south, and culminating right where you can see it in your window. *All of these*—twelve o'clock, dawn and dusk, Cancer and Scorpio, summer and winter—*all of these* are *positions* (*fang* 方) as much as they are *times* (*shi* 時). Space is time when space is turning.

This simple fact is at the core of ancient correlative matrices of *yin* and *yang*, the five agents, and so on, and so too is at the core of

[6] I offer this reductionist formula not because it is *true* from a *modern perspective* but because it is *helpful to think with* as reflective of a *pre-modern perspective* that it is the historian's duty to understand. In reality, it is not *space* that is turning on its axis but *the earth*, and both the earth's diurnal and annual motion experience minute fluctuations and secular change over time. For an introduction to the basics of astronomy through the perspective of ancient practices and ideas, see Christopher Cullen, *Heavenly Numbers: Astronomy and Authority in Early Imperial China* (Oxford: Oxford UP, 2017); James Evans, *The History and Practice of Ancient Astronomy* (New York: Oxford UP, 1998); Anthony F. Aveni, *Skywatchers of Ancient Mexico* (Austin: University of Texas Press, 1980).

modern discussions of "Chinese thought," but there is nothing particularly *Chinese* about it. The Chinese divided the "circumference of heaven" (*zhoutian* 周天) into 365¼, one *du* 度—"span"—for each *day* of the solar year. The Mesopotamians divided *theirs* into 360 UŠ for much the same reason, approximating an "ideal year" to suit their sexagesimal numbers.[7] And so it was in Greece, in Rome, in India and in the caliphates and empires in between that we, to this day, count sixty minutes to an hour, sixty seconds to a minute, and 360 degrees to a circle—because it is in sixties that the Mesopotamians counted, and because space is time when space is turning.

We uses *hours*, of course, but so too did the Chinese, and what is an "hour," anyways, except fifteen degrees of arc?[8] When the sun stands due south, on the median, at its highest point in the sky, that is what we call noon; an hour later—"post meridian" (P.M.)—it has moved fifteen degrees towards the west as measured in the plane of the celestial equator. In modern times, we call these "hour angles": an *angle* measured in *hours* that, if added to 12:00 P.M., gives you *the time of day*.[9] In Greece, they called them χρόνοι ἰσημερινοί ("equatorial times") or simply χρόνοι ("times"); there were twenty-four and they counted them in letters.[10] In China, they called them *jiashi* 加時 ("added times") or simply *chen* 辰 ("times"); there were twelve, or twenty-four, and they counted them (primarily) in earthly branches (fig. 1). Still today, *zhongwu* 中午 is noon, and it is time to eat *wufan* 午飯 (lit. "wu_{η} rice," thus "lunch"), but it is also the position of the sun (lit. "[sun] centered at wu_{η}"), opposite where it is at midnight (zi_{α}). Space is time when space is turning.

Sure, our units, coordinates, and usages are different, but *what we are counting is the same*, and *the way we do it is mathematically*

[7] See David Brown, "The Cuneiform Conception of Celestial Space and Time," *Cambridge Archaeological Journal* 10, no. 1 (2000): 103–22. On sexagesimal place value notation, see Georges Ifrah and David Bellos, *The Universal History of Numbers: From Prehistory to the Invention of the Computer* (New York: Wiley, 2000), esp. 23–46, 91–95, 121-161.

[8] That 1h = 15° and 4m = 1° is a matter of reflex to astronomers, engineers, etc., used to working with these units. Consider fig. 1: we have a circle of, by definition, 360° that is divided into 24 hours: 360° ÷ 24 hours = 15°/hour, and 15° ÷ 60 minutes = 0.25°/minute.

[9] See W.M. Smart, *Textbook on Spherical Astronomy*, ed. Robin M. Green (Cambridge: Cambridge UP, 1977), 25–32.

[10] On χρόνοι "times' as measures of arc, see Toomer *Ptolemy's Almagest*, 2d ed. (Princeton: Princeton UP, 1998), 23. On Greek alphabetic numbers, see Ifrah and Bellos, *The Universal History of Numbers*, 182–262.

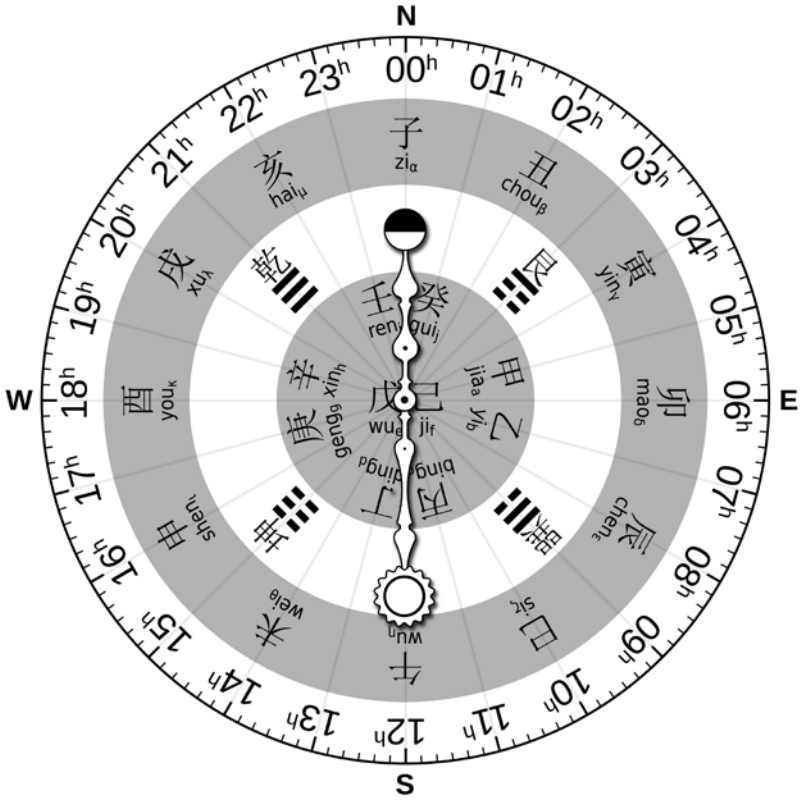

Fig. 1 Early medieval twenty-four *chen* or *jiashi* typical of an armillary equatorial ring, as per Qu Anjing 曲安京. "Zhongguo gudai lifa zhong de jishi zhidu" 中國古代曆法中的計時制度. Hanxue yanjiu 漢學研究 12, no. 2 (1994): 157–72. The twenty-four *chen*/*jiashi* are an extension of the twelve "earthly-branch" (*dizhi* 地支) "double-hours," zi_{α}, $chou_{\beta}$, yin_{γ}, mao_{δ}… hai_{μ}, between which are inserted the eight non-medial "heavenly stems" (*tiangan* 天干), jia_{a}, yi_{b}, $bing_{c}$, $ding_{d}$, $geng_{g}$, xin_{h}, ren_{i} gui_{j}, and, in the "four corners" (*siwei* 四維), the *Book of Changes* trigrams Qian 乾 ☰, Kun 坤 ☷, Gen 艮 ☶, and Xun 巽 ☴. In later geomancy, or *fengshui*, this configuration is known as the "twenty-four mountains" (*ershisi shan* 二十四山). The clock hands represent the position of the sun and moon at noon (local apparent time), full moon day. Note that I have flipped this diagram upside-down so that the modern reader might feel more comfortable reading this with north/midnight at the top.

commensurable.[11] The same goes for bees. Bees navigate by a combination of methods, one of which, for dead reckoning in the ab-

[11] There are, as always, interesting exceptions to such absolutist statements. The *du* and UŠ, for example, are not measures of *angle*, like the degree, but linear

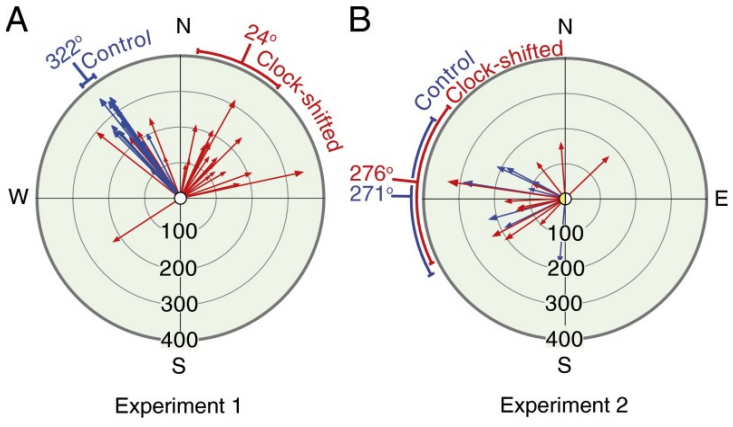

Fig. 2 Initial vector flight components of captured and released bees from James F. Cheeseman et al., "Way-Finding in Displaced Clock-Shifted Bees Proves Bees Use a Cognitive Map," *Proceedings of the National Academy of Sciences* 111, no. 24 (2014): 8949–8954 (p. 8951, fig. 3). Clock-shift = 6 hours, clock-shifted bees in red and control bees in blue. (*A*) Experiment 1: bees released in an open field; clock-shifted mean vector angle 24°, (95% CI 7°, 41°); control mean vector angle 322°, (95% CI 319°, 325°). (*B*) Experiment 2: bees released next to a familiar hedge; clock-shifted mean angle 276°, (95% CI 244°, 308°); control mean vector angle 271°, (95% CI 240°, 301°). Scale in meters.

sence of salient landmarks, scientists call a "sun compass." In short, to get from a given location back to the hive, bees allocate familiar places with a *direction* and a *distance* home, the direction being relative to the sun. Distance, of course, they measure in time, and the same goes for direction, because while bees do not have rulers, they are installed with clocks.[12] The sky is always turning, and the same circadian clock by which they measure a distance flown they use to adjust the remembered vector in compensation for the sun's displacement. Because the sun is south at noon, because between south and north is east, and because bees are not afraid of a little astronomy, bees can always find their way right back home.

They can, that is, unless you anaesthetize them for several hours so as to disable those clocks while the heavens go on turning. Then,

measures of perimeter. For some of the more interesting ramifications of this difference, see Huang Yi-long 黃一農, "Jixing yu gudu kao" 極星與古度考, *Tsing Hua Journal of Chinese Studies* 22, no. 2 (1992): 93–117.

[12] K. Tomioka and A. Matsumoto, "A Comparative View of Insect Circadian Clock Systems," *Cellular and Molecular Life Sciences* 67, no. 9 (2010): 1397–1406.

as studies like Cheeseman et al. show, funny things begin to happen: they wake up, and they fly in the wrong direction, roughly as many degrees in error as the amount of time they were unconscious (fig. 2). Space is time when space is turning, and "Chinese thought" is hardly unique for recognizing this as the foundation of our cosmos and the key to finding our way therein.

This is all no doubt somewhat mysterious to someone who's never studied these sorts of things, but it is nowhere near as mysterious as the idea that Chinese astronomy could somehow function independently of space. Saying this, as we were taught, is the equivalent of saying that this people could only (or, indeed, *could*) build one-dimensional clocks—an unmoving needle without a face. This might make for an excellent riddle or meditational aide, like the sound of one hand clapping, but bees know better, and so too should we. There is a reason behind this riddle, of course, but it is not one that has anything to do with physics or entomology.

Li*: not your father's calendar*

The study of these things, which I just described, was called *li* 曆, and it was called that starting from at least the second century BCE.[13] It was called that when Buddhists brought *jyotiṣa* into the mix, the Nestorians *iṣṭrunumiia*, the Muslims *ilm al-nujuum*, and the Catholics, finally, *astronomia*. Each time, some Chinese expert took a look at this foreign thing and said, "I know that: that is *li*."[14] We

[13] For a more nuanced explanation of the meaning of *li* 曆 than that offered here, see Jean-Claude Martzloff, *Le calendrier chinois: structure et calculs, 104 av. JC–1644* (Paris: Champion, 2009), esp. 367–72; Nathan Sivin, "Mathematical Astronomy and the Chinese Calendar," in *Calendars and Years II: Astronomy and Time in the Ancient and Medieval World*, ed. John M. Steele (Oxford: Oxbow Books, 2011), 39–51; Christopher Cullen, *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems* (New York: Routledge, 2017), esp. 7–25; Daniel P. Morgan, *Astral Sciences in Early Imperial China: Observation, Sagehood and the Individual* (Cambridge: Cambridge UP, 2017), chap. 1.

[14] For recent studies on these various transmissions, see Bill Mak, "The Transmission of Buddhist Astral Science from India to East Asia: The Central Asian Connection," *Historia Scientiarum* 24, no. 2 (2015): 59–75; "Astral Science of the East Syriac Christians in China during the Late First Millennium AD," *Mediterranean Archaeology and Archaeometry* 16, no. 4 (2016): 87–92; Dror Weil, "Islamicated China: China's Participation in the Islamicate Book Culture during the Seventeenth and Eighteenth Centuries," *Intellectual History of the Islamicate World* 4,

have been less kind: *li* is the one among these that does not go back to the Greeks, and in as much as Greekness was a necessary condition for "science," "theory," "proof," and "abstraction" in nineteenth-century Europe, it was there decided that *li* is nothing other (i.e. more) than the "practical," "concrete" art of "calendar-making."[15] This was decided, ironically, on a continent where the transmission of Arabic-language astronomy had, some centuries earlier, undermined and marginalized the indigenous tradition of computus on the self-same charge.[16] Regardless, *this* and the momentum of convention is the only reason we learn that *li* is "calendars" and that it is independent of space.

Li, however, are *not* calendars—not in any normal sense of the word—and anyone who doubts this is free to read one or to look for a single quote in Chinese history to the effect of "Lunch tomorrow? Let me check my *li*."[17] *That thing*—the table of dates, and months, with holidays and the sort—*that thing* is called many things, and none of them is *li*: it is called *ri* 日 ("day/s"), *zhiri* 質日 ("duty day/s"?), and, yes, even *liri* 曆日 ("*li* day/s"; see fig. 3), but a *liri* is no more a *li* than a *matong* 馬桶 is a *ma* 馬.[18] To a Jean-Baptiste Biot (1774–1862), of course, a *matong* is precisely where *li* belongs:

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no. 1–2 (2016): 36–60; Catherine Jami, *The Emperor's New Mathematics: Western Learning and Imperial Authority during the Kangxi Reign (1662–1722)* (Oxford: Oxford UP, 2012). I thank Y. Isahaya for helping me sort out some of these actors' categories.

[15] Ironically, similar charges have been brought against the Mesopotamians, from which the Greeks learned much of their astronomy. For a critique of the discourse *Occident : theoretical : science :: Orient : practical : non-science* in this context, see Francesca Rochberg, *The Heavenly Writing: Divination, Horoscopy, and Astronomy in Mesopotamian Culture* (Cambridge: Cambridge UP, 2004), esp. 14–43.

[16] See Charles Homer Haskins, *Studies in the History of Mediaeval Science* (Cambridge, MA: Harvard UP, 1924), chap. 5. I thank Philipp Nothaft for directing me to relevant sources on this topic (personal communication, March 6, 2017).

[17] If you are curious to learn from first-hand experience about the nature of primary sources that self-identify as *li*, I suggest starting with the expertly annotated translations of the *Santong li* 三統曆 (*c.*5 CE), *Sifen li* 四分曆 (85 CE) and *Qianxiang li* 乾象曆 (*c.*206 CE) in Cullen, *Foundations of Celestial Reckoning* or that of the *Shoushi li* 授時曆 (1280 CE) in Nathan Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records* (New York: Springer, 2009).

[18] On "calendars" in the sense of "a table showing the division of a given year into its months and days…" (*Oxford English Dictionary*), see Yoshimura Masayuki 吉村昌之, "Shutsudo kandoku shiryō ni mirareru rekihu no shūsei" 出土

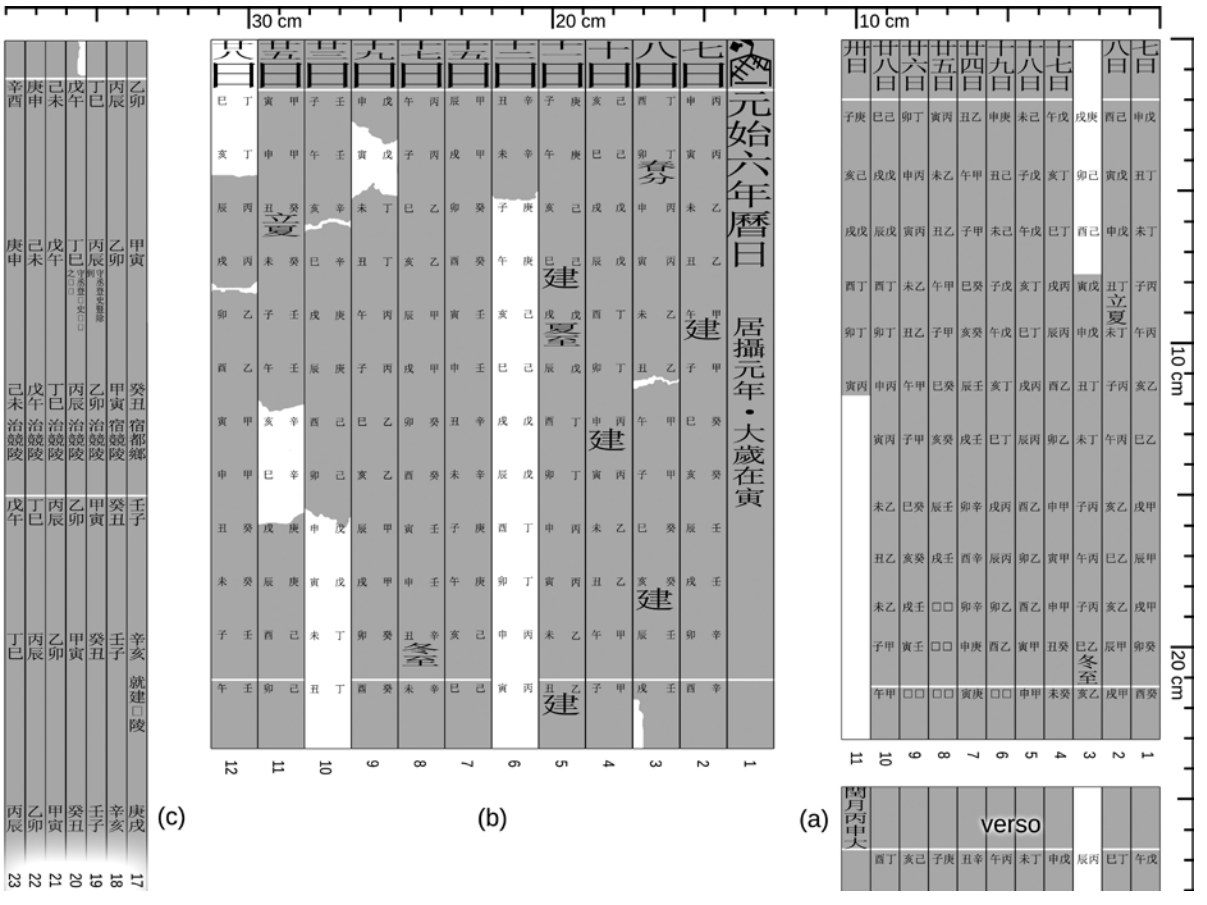

Fig. 3 Wood bookmat *liri* 曆日 -style day calendars. Right (a): Dunhuang 敦煌 T6b Shenjue year 3 (59 BCE) *liri*. Center (b): Jinguan 金關 T23 'Yuanshi liu nian liri' 元始六年曆日 (5 CE). Left (c): slips from the Zhoujiatai 周家臺 M30 Shihuang year 34 (213 BCE) *zhiri* 質日 for comparison. Note that the two *liri* are incomplete.

> On s'est plu souvent à remarquer que les Chinois, dans leurs idées, dans leurs usages, leurs préjugés même, offrent un singulier et perpétuel contraste avec les peuples européens […] Leur astronomie ne fait pas exception à cette règle. Elle n'a jamais constitué, chez eux, une **science spéculative**, apanage spécial d'un petit nombre d'esprits. Dans tous les siècles, elle a été une œuvre de gouvernement. Son principal office consiste à préparer chaque année, plusieurs mois à l'avance, le calendrier impérial. […] Elle est chargée, en outre, de l'avertir personnellement des phénomènes extraordinaires qui arrivent dans le ciel, pour en tirer les présages, favorables ou défavorables, qui concernent son gouvernement. Aussi, mus par ces deux intérêts **purement pratiques**, a-t-on vu, de tout temps…[19]

What is at stake in translating *li* any differently is admitting that "Chinese thought" was capable of *science* or *spéculation* on its own, and that, in 1862, was a little much for "Western thought" to handle.[20] *All of this*, however, is a problem of observer's categories, and observer's categories are beside the point.

Li, befitting the bibliographic classification that it heads in the *Han shu* 漢書 "Yiwen zhi" 藝文志, is comprised of "numbers and

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簡牘資料にみれる暦譜の集成, in *Henkyō shutsudo mokkan no kenkyū* 邊疆出土木簡の研究, ed. Tomiya Itaru 冨谷至 (Kyōto: Hōyū shoten, 2003), 459–516; Morgan, *Astral Sciences*, chap. 3. For a detailed look at how one produces a *liri* from a *li*, see Martzloff, *Le calendrier chinois*.

[19] *Etudes sur l'astronomie indienne et sur l'astronomie chinoise* (Paris: M. Lévy frères, 1862), 268; emphasis added. Note that, in the opening pages of his "Précis de l'histoire de l'astronomie chinoise," the only source that Biot cites as concerns this history is the *Rites of Zhou* as translated by his late son eleven years earlier. Coincidentally, his description of Chinese astronomy as constituting a "singulier et perpétuel contraste" to Europe, "dans tous les siècles," "de tout temps," etc., comes nearly word for word from the "Grand annaliste (*Ta-ssé*)" 大史 and "Officier chargé de préserver et d'éclaircir (*pao-tchang-chi*)" 保章氏 headings in Édouard Biot, *Le Tcheou-li, ou Rites des Tcheou, traduit pour la première fois du chinois par feu Édouard Biot*, ed. Jean-Baptiste Biot, 3 vols. (Paris: Imprimerie Nationale, 1851), vol. 2, 104–5, 113–14. The history of astronomy, needless to say, does not begin an end with the *Rites of Zhou*.

[20] Note that the formula *Europe : astronomy : theoretical :: China : calendars : practical* is a historical construct propagated and perpetuated only in the last two hundred years. In *Observations mathématiques, astronomiques, geographiques, chronologiques et physiques…*, 3 vols. (Paris: Chez Rollin libraire, 1729–1732), for example, Jesuit astronomer Antoine Gaubil (1689–1759) instinctively translates the word *li* as *l'Aftronomie* or, in the case of specific procedure texts like the *Kaihuang li* 開皇曆 of 584 CE, *une Aftronomie*.

procedures" (*shushu* 數術).[21] A *li*, like the Han *Sifen li* 四分曆 (85 CE), more specifically, is a chain of numbers, tables and algorithms; it reads fortuitously like computer code,[22] and *translating it into computer code* is, fortuitously, one of the best ways to learn it.[23] This no doubt sounds very abstruse, so allow me to show you what this looks like.

The *Sifen li*, authored by *li* workers (*zhili* 治曆) Bian Xin 編訢 and Li Fan 李梵 and preserved in *Hou Han shu* 後漢書, *zhi* 3, opens with the following numbers (for the moment, don't worry yourself with "why?", just pretend you're watching a command-line process run):

> 當漢高皇帝受命四十有五歲[…]冬十有一月甲子夜半朔旦冬至，日月閏積之數皆自此始[…]
> In the forty-fifth year after Han Emperor Gao[zu] 高祖 (r. 206–196 BCE) received the mandate […], in winter, month XI, day *jiazi*$_{a\alpha}$ 甲子, at midnight, [at the coincidence of] new moon and winter solstice (00:00, December 25, 162 BCE)—all the numbers (*shu*) of solar and lunar intercalation and accumulation start from this point. …
>
> 又上兩元，而月食五星之元，並發端焉。[…]

[21] On the six-part bibliographic category "Shushu" 數術 and its historical evolution, see Marc Kalinowski, ed., *Divination et société dans la Chine médiévale: étude des manuscrits de Dunhuang de la Bibliothèque nationale de France et de la British Library* (Paris: Bibliothèque nationale de France, 2003), 11–17. This actor's category has been appropriated by post-1980s scholars of excavated divinatory literature so as to place the study of subaltern, "superstitious" sources under the aegis of a sort of alternative history of science that at once lays claim to and excludes *li* and *suan* 算 mathematics. Whatever its schizophrenic relationship with math, this field does tend to recognize the "numbers and procedures" of *li* and *suan* as being as being the origin/inspiration of those *shushu* that deal neither with numbers or calculation; see Li Ling 李零, *Jianbo gushu yu xueshu yuanliu* 簡帛古書與學術源流, revised edn. (Beijing: Sanlian shudian, 2008), 403–4; Chao Fulin 晁福林, "Cong 'shushu' dao 'xueshu': shanggu jingshen wenming yanjin de yige xiansuo" 從「數術」到「學術」：上古精神文明演進的一個線索, *Gudai wenming* 古代文明 4, no. 4 (2010): 40–49 (p. 44).

[22] On thinking about the algorithms comprising *suan* procedure texts through the lens of computer language, see Karine Chemla, "Should They Read FORTRAN as If It Were English?," *Bulletin of Chinese Studies* 1, no. 2 (1987): 301–16.

[23] Christopher Cullen, "Translating Ancient Chinese Astronomical Systems with EXCEL: How Not to Stew the Strawberries?," *Journal for the History of Astronomy* 36, no. 3 (2005): 336–8.

Two origins (2 × 4,560 years) further up (00:00, December 25, 9282 BCE), and that is the origin for lunar eclipses and the five stars (planets), which all start from this point. […]

元法 Origin divisor 4,560
紀法 Era divisor 1,520
紀月 Era months 18,800
蔀法 Obscuration divisor 76
蔀月 Obscuration months 940
章法 Rule divisor 19
章月 Rule months 235
周天 Circuits of heaven 1,461
日法 Day divisor 4
蔀日 Obscuration days 27,759
…
日餘 Day remainder 168
中法 Medial [*qi*] divisor 32
大周 Big circuits 343,335
…[24]

The only datum the user need enter for things to begin is "the [number of years from] high origin to the year sought," which, for 2020, would be 2020 − −9,281 = 11,301 years.[25] Let's plug this into procedure 2:

```
Enter years from high origin: 11,301.
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推入蔀術曰：
[2.] Procedure for calculating [the sexagenary year number of] the obscuration entered:

以元法除去上元，
Eliminate (*modulo*) the origin divisor (4,560) from the distance from (i.e., years passed since) high origin.[26]

[24] *Hou Han shu* (Zhonghua shuju edn., 1962; hereafter HHS), *zhi* 3, 3057–60. Note that here and what follows are my own translations and explanations, but that for more serious purposes one is advised to refer to those in Cullen, *Foundations of Celestial Reckoning*, chap. 3, whose numbering I have adopted

[25] There is no 0 BCE/CE, so for the sake of calculation 9282 BCE is −9281.

[26] In the *Sifen li*, a ***yuan* 元** is the coincidence of: (a) new moon, month XI, (b) winter solstice, (c) midnight, (d) day $jiazi_{aa}$, and (e) year $gengchen_{ge}$, where 1 "origin" = 4,560 years = 56,400 months = 1,665,540 days. ***Modulo***, or *mod*, means dividing for the remainder.

11,301 ÷ 4,560 → 2; **rem. 2,181**

其餘以紀法除之，所得數，從天紀，筭外則所入紀也。
With the remainder, one eliminates (divides) it by the era divisor (1,520), and the number obtained, counting exclusively from "heaven origin," [leads you to] the era entered.[27]

2,181 ÷ 1,520 → 1; rem. 661,
i.e., filled 1 era, 658 years into era no. 2:
[1] heaven, **[2] earth,** [3] man.

不滿紀法者，入紀年數也。
Any [remainder] that does not fill an era divisor (1,520) is the number of years entered into the [current] era.

Retrieve: **661 years** into the earth era.

以蔀法除之，所得數，從甲子蔀起，筭外，所入【蔀】歲名命之，[…]
Eliminate (divide) this by the obscuration divisor (76), and by the number obtained, counting exclusively from the obscuration *jiazi*$_{\alpha\alpha}$, one names off the [the obscuration heads to find] the year name [of the obscuration] entered. […][28]

661 years ÷ 76 → 8; rem. 53,
i.e., filled 8 obs., 53 years into obs. 9:
[1] *jiazi*$_{\alpha\alpha}$, [2] *guimao*$_{j\delta}$, … → [**9**] ***bingzi***$_{c\alpha}$.

We now know that the last "obscuration head" (*bushou* 蔀首), 54 years prior, started with a coincidence of new moon, month

[27] In the *Sifen li*, a ***ji* 紀** is the coincidence of: (a) new moon, month XI, (b) winter solstice, (c) midnight, (d) day *jiazi*$_{\alpha\alpha}$, ~~and (e) year *gengchen*$_{g\varepsilon}$~~, where 1 "era" = 1,520 years = 18,800 months = 555,180 days. Each "era," the sexagenary year of this coincidence shifts forward twenty places, from "the era of heaven," starting year *gengchen*$_{g\varepsilon}$ 庚辰, to "the era of earth," starting year *gengzi*$_{g\alpha}$ 庚子, to "the era of man," starting year *gengshen*$_{g\iota}$ 庚申, and back to the "era of heaven." Cullen, *Foundations of Celestial Reckoning*, 162–3, provides a look-up table omitted here.

[28] In the *Sifen li*, a ***bu* 蔀** is the coincidence of: (a) new moon, month XI, (b) winter solstice, (c) midnight, ~~(d) day *jiazi*$_{\alpha\alpha}$~~, ~~and (e) year *gengchen*$_{g\varepsilon}$~~, where 1 "obscuration" = 76 years = 940 months = 27,759 days. Each "obscuration," the sexagenary date of this coincidence shifts forward thirty-nine places from *jiazi*$_{\alpha\alpha}$. Ibid. provides a look-up table omitted here.

month XI and winter solstice at midnight, day *bingzi*$_{c\alpha}$ 丙子 (January 6, 1967).[29] Now, to procedures 5 and 6 for 2020:[30]

推天正術：
[5.] Procedure for calculating astronomical month I:

置入蔀年減一，
Set out the years entered into the [current] obscuration and diminish (subtract) by 1.[31]

Retrieve: **53 years** into obs. no. 9, *bingzi*$_{c\alpha}$,
53 − 1 → **52 ac. years**.

以章月乘之，滿章法得一，名為積月，不滿為閏餘，
Multiply this by the rule months (235), and get 1 [for each time it] fills the rule divisor (19)—this is called the "months accumulated [into current obscuration]," and that which does not fill [the divisor] is the "intercalary remainder." […][32]

52 ac. years × 235/19 → **643 ac. months**; rem. 3.

推天正朔日，
[6. Procedure for] calculating new moon day, astronomical month I:

置入蔀積月，
Set out the [integer number of] months accumulated since entry into [the current] obscuration.

Retrieve: **643 ac. months**; rem. 3.

---

[29] Note that the true winter solstice fell on December 22, 1966, and the true new moon on December 12, 1966. The discrepancy in the *Sifen li*'s predictions here and in the following pages is due in part to the omission of ten days from the Christian calendar following the Gregorian reform of 1582 and, more importantly, to accumulated errors to be expected when using the *Sifen li* centuries beyond its intended lifespan. In 85 CE, the best of *li* were not expected to function beyond 300 years; see Morgan, *Astral Sciences*, 179–88.

[30] Procedure 3 has to do with eclipses and can be skipped for our purposes here.

[31] "Astronomical month I" = civil month XI = the month containing winter solstice, so for 2020 we need to count from the winter solstice of the year prior, 2019, thus the need to subtract 1.

[32] In the *Sifen li*, a ***zhang* 章** is the coincidence of: (a) new moon, month XI, (b) winter solstice, ~~(c) midnight~~, ~~(d) day *jiazi*$_{a\alpha}$~~, ~~and (e) year *gengchen*$_{g\varepsilon}$~~, where 1 "rule" = 19 years = 235 months = 6,939¾ days. Each "rule," the hour of this coincidence shifts forward ¾ day. To convert from years to months, one multiplies by $^{235}/_{19}$ (i.e. 12 $^{7}/_{19}$ months/year).

以蔀日乘之，滿蔀月得一，名為積日，不滿為小餘，
Multiply this by the obscuration days (27,759), and get 1 [for each time it] fills the obscuration months (940)—this is called the "days accumulated [into current obscuration]," and that which does not fill [the divisor] makes the "little remainder."[33]

643 ac. months × 27,759/940 → **18,988 ac. days**; rem. 317.

積日以六十除去之，其餘為大餘。以所入蔀名命之，筭盡之外，則前年天正十一月朔日也。[…]
With the accumulated days, remove [all] 60[s] by elimination (*modulo*) therefrom—the remainder makes the "big remainder." [Counting down from this number], name off [the sexagenary days starting] from the name of [the sexagenary date heading] the obscuration entered, and that which lies after the counting rods are exhausted (i.e., counting exclusively) is the new moon day of astronomical month I—[civil] month XI—of the previous year. […][34]

18,988 ac. days ÷ 60 → 316; **rem. 28**,
i.e., 28 days past sex. date at obs. head.

Retrieve sex. date at obs. head: ***bingzi***$_{c\alpha}$.

Name off 28 sex. days from *bingzi*$_{c\alpha}$, exclusively:
[1] *bingzi*$_{c\alpha}$, [2] *dingchou*$_{d\beta}$, … → [**29**] ***jiachen***$_{a\varepsilon}$.

For 2020, new moon, month XI, thus falls (317/940 day past midnight) on day *jiachen*$_{a\varepsilon}$ 甲辰 (January 2, 2020).[35] On to procedure 8 to find the winter solstice:

推二十四氣術曰：
[8.] Procedure for calculating the twenty-four *qi*:

置入蔀年減一，
Set out the [number of] years entered into the [current] obscuration and diminish (subtract) by 1.[36]

Retrieve: 53 years into obs. 9,
53 − 1 → **52 ac. years**.

[33] As 1 "obscuration" = 76 years = 940 months = 27,759 days, to convert from months to days, one multiplies by 27,759/940 (i.e. $29\,^{499}/_{940} \approx 29.53085$ days/month).
[34] HHS, *zhi* 3, 3062.
[35] Note that said new moon actually fell on November 26, 2019.
[36] As to why one subtracts 1, see Note 31.

以日餘乘之，滿中法得一，名曰大餘，不滿為小餘。大餘滿六十除去之，其餘以蔀名命之，筭盡之外，則前年冬至之日也。[…]

Multiply this by the day remainder (168), and get 1 [for each time it] fills the medial divisor (32)—this is called the "big remainder," and that which does not fill [the divisor] makes the "little remainder." With the big remainder, remove remove [all] full 60[s] by elimination (*modulo*) therefrom, and [counting down] from the remainder, name off the names of the [sexagenary days starting from] obscuration [head], and that which lies after the counting rods are exhausted (i.e., counting exclusively) is the day of the winter solstice of the previous year. […][37]

52 ac. years × 168/32 → **273 ac. days**, rem. 0,
273 ÷ 60 → 4; **rem. 33.**

Retrieve sex date at obs. head: ***bingzi***$_{c\alpha}$.

Name off 33 sex. days from *bingzi*$_{c\alpha}$, exclusively:
[1] *bingzi*$_{c\alpha}$, [2] *dingchou*$_{d\beta}$, … → **[34]** ***jiyou***$_{f\kappa}$.

For 2020, winter solstice thus falls (0/32 day past midnight) on day *jiyou*$_{f\kappa}$ 己酉 (7 January 2020), five days after new moon.[38] For subsequent new moons and *qi*, the elided text of procedures 5 and 6 instructs the user to cumulatively add 29 $^{499}/_{940}$ and 15 $^{7}/_{32}$ days, respectively, up to the end of said year. And at that, we have produced the luni-solar framework of a [*li/zhi*]*ri* calendar like those in fig. 3.[39]

How did this happen? Nathan Sivin long ago invited us to think of the numbers with which we have been operating as intricately connected gears. Describing the (different) values used by the *Santong li* 三統曆 (*c.*5 CE), he tells us that…

> We can look at this set of constants as a complex of circles turning upon each other (Figure [4]). The Epoch Cycle [元] simply specifies what motion of the integral system is needed to return all cycles

[37] HHS, *zhi* 3, 3063. In the *Sifen li*, 168/32 (= 5¼) is the number of days by which one solar year (= 365¼ days) exceeds six sexagenary cycles (= 360 days). As such, multiplying this by the years accumulated, from obscuration head gives us the days in the sexagenary cycle advanced therefrom.

[38] Note that said winter solstice actually fell on December 22, 2019.

[39] In the above presentation, I have elided matters of intercalation, otherwise treated in procedure 5 ("Procedure for calculating astronomical month I") and procedure 9 ("Calculating the position of the intercalary month" 推閏月所在), because the *Sifen li* does not give 2020 an intercalary month.

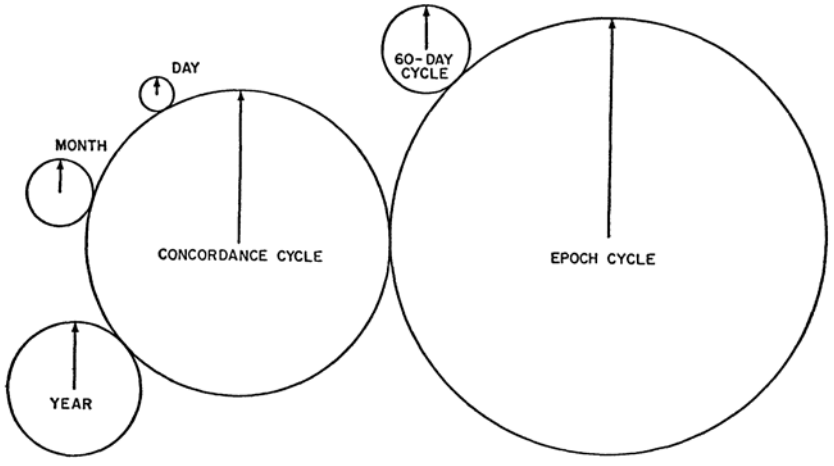

Fig. 4 Sivin's gear train ("Cosmos and Computation," 13, fig. 1). Original caption: System of calendrical constants in the Triple Concordance treatise [*Santong li*]. In a scale model, circumference would be proportional to length of cycle. The rotating arrows all point upward at the same time only once every 4617 years.

> simultaneously to their original orientations. In such a system, if we know the original orientation and the number of revolutions any one circle has passed through at a given moment, we can predict the orientation of any other circle. […] an astronomical system [i.e. *li*] was meant to be like the gear train of a well-functioning machine, requiring no human intervention.[40]

It is an elegant metaphor, but it is one—contrary to Sivin's intentions—that subsequent scholars cite as evidence that "the history of astronomy in ancient China was largely a history of calendar making."[41] Think about it: what other kind of gear train could this be

[40] "Cosmos and Computation in Early Chinese Mathematical Astronomy," *T'oung Pao* 2nd ser., 55, no. 1/3 (1969): 1–73 (pp. 13, 58). Note that the gear train metaphor goes back to Joseph Needham, who describes the various cycles surrounding the civil calendar and the *Santong li* as "cogwheels" (*Science and Civilisation in China, vol.3: Mathematics and the Sciences of the Heavens and the Earth* [Cambridge: Cambridge UP, 1959], 390–408). The metaphor persists; see for example Marc Kalinowski, "Fonctionnalité calendaire dans les cosmogonies anciennes de la Chine," *Études chinoises* XXIII (2004): 169–91 (pp. 88–9); Sun Xiaochun, "Chinese Calendar and Mathematical Astronomy," in *Handbook of Archaeoastronomy and Ethnoastronomy*, ed. C.L.N. Ruggles (New York: Springer, 2015), 2059–68 (p. 2062).

[41] Sun Xiaochun, "Chinese Calendar and Mathematical Astronomy," 2059. Note that this precise equation dates back to Nakayama Shigeru: "The history of Chinese astronomy is, for the most part, the history of calendar-calculation" ("Charac-

other than that you find in a clock? And is not a *clock*, like a "calendar," (just) for telling time?

One could bring in *astronomical clocks* like the Antikythera mechanism to buttress Sivin's metaphor, but instead I would like to propose another—that we *return*, to be more precise, to that with which contemporary actors conceived of what they were doing. Nothing is wrong with a little anachronism if it helps us grasp a difficult subject, but there are three points where Sivin's gear train metaphor falls short.

First, *li* are *not* written in computer code, nor is their operation nearly as "intervention"-less as either metaphor implies. *People* did these calculations, and they did them *by hand*. In the Eastern Han (25–220 CE) astronomical office—the Clerk's Office (*shiguan* 史官), office of the [Prefect] Grand Clerk (*taishi* [*ling*] 太史令)—this was the job of six "*li* workers" (*zhili*), like Bian Xin and Li Fan, who were *daizhao* 待詔 "expectant appointees" specially appointed for their skills.[42] The Northern Dynasties (386–581 CE) and Sui (518–618 CE) furthermore saw the same office appoint erudites (*boshi* 博士) and students (*sheng* 生) to begin training their own talent in-house.[43] Why go to the trouble? Probably because *li*-calculation is anything but automatic. Converting between cardinal and ordinal numbers can be confusing, especially in a language that does not distinguish between them. Far more confusing, however, is a word like *chu* 除 "eliminate," which can refer to one of four mathematical operations (subtraction, division, *modulo*, or sequence subtraction). Get confused *just once* in this sort of text and the mistake propagates, leaving you a garbled mess for all your efforts.[44] Yes, *li*

---

teristics of Chinese Calendrical Science," *Japanese Studies in the History of Science*, no. 4 [1965]: 124–31 [p. 125]).

[42] On the role of the *daizhao* in the state astronomical office, see Lai Swee Fo 賴瑞和, "Tangdai de Hanlin daizhao he Sitiantai" 唐代的翰林待詔和司天臺, *Tang yanjiu* 唐研究 9 (2003): 315–42; Morgan, *Astral Sciences*, chap. 1.

[43] On the Clerk's Office, see Thatcher Elliott Deane, "The Chinese Imperial Astronomical Bureau: Form and Function of the Ming Dynasty *Qintianjian* from 1365 to 1627" (Ph.D. diss., University of Washington, 1989); Chen Xiaozhong 陳曉中 and Zhang Shuli 張淑莉, *Zhongguo gudai tianwen jigou yu tianwen jiaoyu* 中國古代天文機構與天文教育 (Beijing: Zhongguo kexue jishu chubanshe, 2008).

[44] This is precisely why Cullen, "Translating," proposes the use of spreadsheet software to perform these texts—they are too hard, time-consuming, and prone to cascading errors for the modern scholar to do by hand. On the problem of *chu*, see Karine Chemla, "Shedding Some Light on a Possible Origin of a Concept of Frac-

workers are "computers," but in an older sense of the word, and a *li* is a lot less like a machine or a computer program than it is a liturgy—a liturgy to be *performed*.

Second, "origins," "eras," "obscurations," and "rules" are not just matters of *time*, like you might expect of clockwork, they are matters of *space-time*. The gear-train metaphor gets a little strained as we move, for example, to procedure 13:

> 推合朔所在度：
> [13. Procedure for] calculating the position of syzygy in *du*:
>
> 置入蔀積月，
> Set out the [integer number of] months accumulated [since] entry into the [current] obscuration.
>
> Retrieve: **643 ac. months**; rem. 3.
>
> 以蔀日乘之，滿大周除去之，其餘滿蔀月得一，名為積度，不盡為餘分。
> Multiply this by obscuration days (27,759), remove full big circuits (343,335) by elimination (*modulo*) therefrom, and, with the remainder, get 1 [for each time it] fills the obscuration months (940)—this is called the "*du* accumulated [into current obscuration]," and that which is not exhausted makes the "remainder parts."
>
> 643 ac. months × 27,759 → 17,849,037,
> 17,849,037 ÷ 343,335 → 51; rem. 338,952,
> 338,952 ÷ 940 → **360; rem. 552**.
>
> 積度加斗二十一度，加二百三十五分，
> With the *du* accumulated, add Dipper$_{L08}$ 21 *du* and add 235 parts.[45]
>
> 360;552 + 21;235 → **381;787**,
> or Dipper$_{L08}$ 381 *du* and 787[/940] parts.

---

tions in China: Division as a Link between the Newly Discovered Manuscripts and 'The Gnomon of the Zhou [Dynasty]'," *Sudhoffs Archiv* 97, no. 2 (2013): 174–98; "Observing Mathematical Practices as a Key to Mining Our Sources and Conducting Conceptual History: Division in Ancient China as a Case Study," in *Science after the Practice Turn in the Philosophy, History, and Social Studies of Science*, ed. Léna Soler et al. (New York: Routledge, 2014), 238–68; cf. Morgan, *Astral Sciences*, 127–31.

[45] In the *Sifen li*, **Dipper$_{L08}$ 21 $^{234}/_{940}$ *du*** (= Dipper$_{L08}$ 21¼ *du*) is the right ascension—i.e. the *equatorial position*—of winter solstice, the zero-point of the solar year and celestial circuit.

以宿次除之，不滿宿，則日月合朔所在星度也。
Sequentially eliminate (subtract) the [twenty-eight] lodges therefrom, and that which does not fill the [last] lodge is the star-*du* position of sun and moon at syzygy.[46]

381;787 – 8 (Ox$_{L09}$) – 12 (Maid$_{L10}$) … – 11 (Basket$_{L07}$) →
Dipper$_{L08}$ 16 *du* and 552[/940] parts.

What just happened? We started with a measure of *time*, we've plucked two "day" and "month" gears from the clockwork, looped them with a third, and suddenly we have *a position* in lodges and *du* (fig. 5). This isn't a calendar! This isn't how a gear train or a clock is supposed to work! And this starts, *after the calendar portion is done*, only thirteen of fifty-three procedures in!

Third, and far more importantly, none of our "*li* numbers" (*li shu* 曆數) bear measuring units.

Lü*: water into wine*

This last point may sound anti-climactic, but it is the most important of the three, because where similar lists of operable numbers without units are given in *suan* 算 mathematics, they are called *lü* 率, and there is a whole theoretical and philosophical apparatus surrounding them in early literature.[47] *Lü* is not a word that translates, because it is not a mathematical construct that *exists* in other traditions, but rest assured that it is one that is nonetheless easy enough to grasp.

You know *lü*; you have seen them and you have used them in their modern form. You have seen them, namely, on reader boards and mobile applications like that in fig. 6: *huilü* 匯率 "exchange rates." You have used them in this form to go from one currency to another in your head—to calculate, as per fig. 6, that if a dollar gets you 30.48 TWD, 100 get you 304.85. Such a table is useful when changing money, but there are limits to what it can do. You can't

[46] HHS, *zhi* 3, 3063–4.

[47] The treatment of *lü* in this and the following section draws extensively from Chemla's insights as presented in "Mathematics, Nature and Cosmological Inquiry in Traditional China," in *Concepts of Nature: A Chinese-European Cross-Cultural Perspective*, ed. Günter Dux, Hans Ulrich Vogel, and Mark Elvin (Leiden: Brill, 2010), 255–84, and as communicated to me in person over the years of our work together. My principal contribution in what follows is simply the application of her insights in *suan* to *li*.

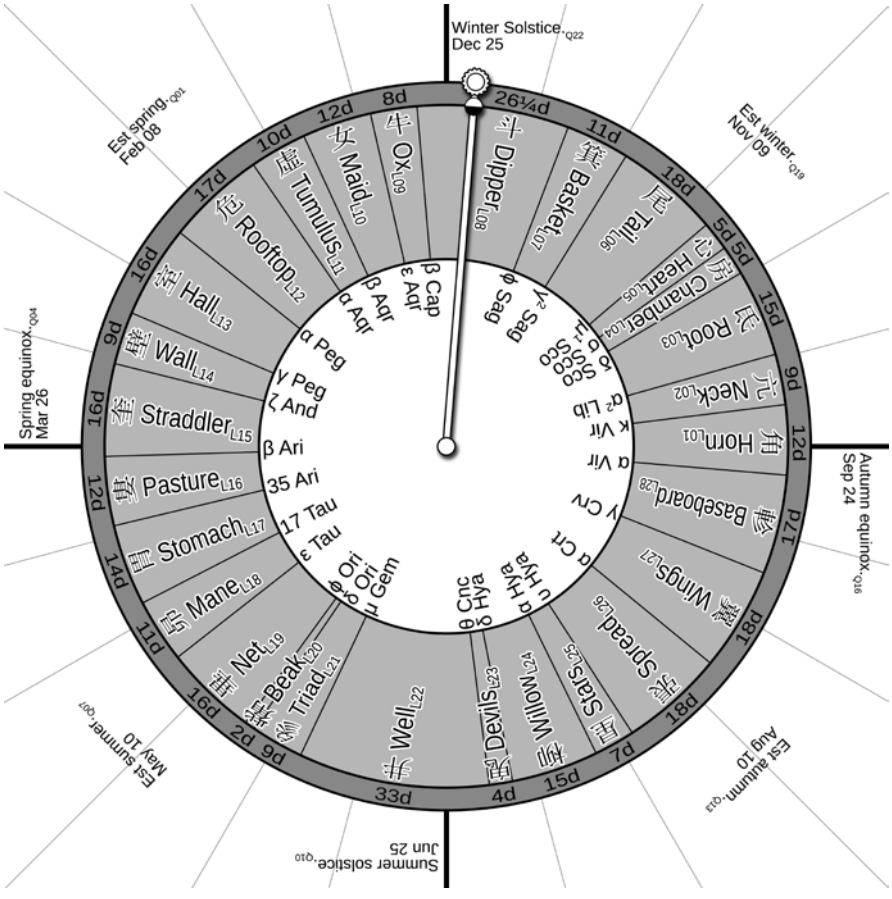

Fig. 5 The twenty-eight equatorial lodges as per the *Sifen li* (85 CE). The sun and moon progress *counterclockwise* through the ring of lodges (L01–L28), and, unbeknownst to the *Sifen li*'s authors, the ring of lodges is effectively turning in the same direction vis-à-vis the twenty-four *qi* (the winter solstice$_{Q22}$, establishment of spring$_{Q01}$, spring equinox$_{Q04}$, etc.), albeit *very slowly*, in what we call the precession of the equinoxes. The letter *d* represents equatorial lodge-width, where one *du* 度 equals the displacement of the mean sun through the stars in one day, for an annual "circuit" of 365¼ *du* in 365¼ days. Due to their interrelation, *d* can be read either "*du*" *or* "days" and thought of in terms of time *or* space. As per our result for procedure 13, the sun and moon are represented here in conjunction—i.e. the point of "syzygy," at "new moon," where the two bodies fall on the same line in a given reference plane (here, the equator)—at "Dipper$_{L08}$ 16 *du* and 552 parts," which is to say $16\frac{552}{940}$ *du* (equivalent to 16.35°, but as a measure of perimeter rather than angle) into [Southern] Dipper$_{L08}$ as counted from its 'guide star' (*juxing* 距星), φ Sagittarii. Note that the time/position of winter solstice and the other *qi* has since moved from where they are in this diagram, and that "Dipper$_{L08}$ 16 *du* and 552 parts" is not a particularly accurate prediction for the right ascension of this particular syzygy (see Note xx).

(easily) go *backwards*, for example, or from euros to yen, but there's no one table by which you can convert *any two things* as simply as that, right? Wrong; and welcome to the world of *lü*.

Lü, in their *pre-modern form*, allowed exactly that, and to that end let us turn to the "classic" (*jing* 經) *Jiuzhang suan-shu* 九章算術 (TAQ first century CE), which devotes one of its "nine chapters" to the topic.[48] Like the *Sifen li*, the chapter "Su mi" 粟米 (Unhulled and Hulled) begins with a table of unit-less numbers:[49]

粟率 [Unhulled] millet *lü* 50
糲米 hulled grain 30
粺米 milled grain 27
鑿米 milled grain, fine 24
御米 milled grain, superior 21
小麥 small oats 13½
大麥 big oats 54
糲飯 hulled grain (cooked) 75
粺飯 milled grain (cooked) 54
鑿飯 milled grain, fine (cooked) 48
御飯 milled grain, superior (cooked) 42
菽荅麻麥 soy, adzuki, hemp or wheat 45
稻 Paddy rice 60
豉 fermented soy 63
飧 diluted rice (cooked) 90
熟菽 soy (cooked) 130½
蘖 fermented grain 175

After these numbers (*shu*) comes a procedure (*shu*):

> 今有術曰：以所有數乘所求率爲實。以所有率為法。實如法而一。
>
> The "Suppose-you-have" procedure: Multiply the quantity of what you have by the *lü* of that which you seek to make dividend; take the *lü* of what you have as the divisor; and [divide] the dividend by the divisor.[50]

[48] For a complete critical edition, translation, and study of the *Jiuzhang suanshu*, see Karine Chemla and Guo Shuchun, *Les neuf chapitres: le classique mathématique de la Chine ancienne et ses commentaires* (Paris: Dunod, 2004).

[49] The following translation is modified from (hereafter "mod.") ibid., 222–3.

[50] Tr. ibid., 222–5 (mod.).

Fig. 6 Visualizing *lü* 率, ancient and modern. Left: *Huilü* 匯率 "exchange rates," drawn after the real-world application *Huilü* 匯率 available on Android™ everywhere on Google Play. Right: the list of *lü* 率 under the heading "Norms for Grain Conversion" 粟米之法 at the opening of *The Nine Chapters*, chapter 2, as represented in the form the same sort smartphone app. On the left, the quantity entered is 91.150 TWD, and on the right it is 7 *dou* 8 *sheng* (= 78 *sheng*) of unhulled millet, as per *The Nine Chapters*, problem 2.16. The modern *lü*—a "rate"—comprises the juxtaposition of two quantities, the one of which is normally pegged at 1, and the other expressed as a decimal fraction (i.e. "1 USD : 30.485 TWD"). To find the equivalent of the starting value of 91.150 TWD in USD, one converts via the rule of three thus: $91.150\ \text{TWD} \times 1\ \text{USD}/30.485\ \text{TWD} = 2.9900\ \text{USD}$. The pre-modern *lü*, by contrast is a single integer value assigned to an element in a larger list (i.e. "unhulled millet *lü*: fifty" 粟率五十). To find the equivalent of the starting value of 78 *sheng* unhulled millet in fermented soy, one also converts via the rule of three—the " 'suppose you have' procedure" 今有術—but one does so using the simpler integer values of the appropriate *lü*: $78 \times 63/50 = 98\ ^{14}/_{50}$ *sheng* fermented soy

Using "Suppose-you-have"—i.e., the rule of three—with an appropriate list of *lü*, one can transform *any one thing* into *any other* in *whichever direction one desires*. Take for instance problem 2.16:

> 今有粟七斗八升，欲爲豉。問得幾何？
> Suppose you have 7 *dou* 8 *sheng* (≈ 15.6 liters) of [unhulled] millet, and you desire to make it into fermented soy. How much do you get?

荅曰：爲豉九斗八升二十五分升之七。
Answer: It makes 9 *dou* 8 *sheng* and 7/25 *sheng* (≈ 19.7 liters) of fermented soy.

術曰：以粟求豉，六十三之，五十而一。
The procedure: seeking fermented soy from [unhulled] millet, multiply by [the fermented soy *lü*] 63 and divide by [the millet *lü*] 50.[51]

```
7 dou 8 sheng = 78 sheng
78 sheng × 63 → 4914 sheng
4914 sheng ÷ 50 → 98 7/25 sheng
```

One can go backwards just as easily, of course, by flipping the *lü* in this ratio from 63/50 to 50/63, and either can be swapped with any other to get you something else.

This multiply–divide combo—"Suppose-you-have"—is precisely the same operation we see in *li* mathematical astronomy, and it is performed there using numbers that are identical in form, function and presentation to those in *suan*. It is probably safe to call these *lü*, particularly since that is what many of them call themselves: "circuit *lü*" 周率, "day *lü*" 日率, "coincidence *lü*" 會率, "discrepancy *lü*" 差率, "decrease–increase *lü*" 損益率, etc.[52] *Lü,* moreover, is exactly how contemporaries active in *suan* speak of these numbers in philosophizing upon the mathematics of *li* mathematical astronomy and *lü* 律 tono-metrology. Consider, for example, Li Chunfeng 李淳風 (602–70 CE). Li rose through the Tang (618–907 CE) Clerk's Office by merit of his talent in *tianwen* 天文 and *li* from an auxiliary appointment (*zhi* 直) in 627/629 CE to its directorship in 649 CE; in 656 CE, he also oversaw a project to commentate and canonize works like the *Jiuzhang suanshu* as part of the *Suanjing shishu* 算經十書—the mathematical counterpart to the *Wujing zhengyi* 五經正

[51] Tr. ibid., 234–5 (mod.).

[52] These examples are taken from the *li* translated in Cullen, *Foundations of Celestial Reckoning* (Cullen, unfortunately, translates *lü* throughout as "rate"). On *lü* in mathematical astronomy, see Morgan, *Astral Sciences*, 21–3, Daniel P. Morgan and Howard L. Goodman, "Numbers with Histories: Li Chunfeng on Harmonics and Astronomy," in *Monographs in Tang Official History: Perspectives from the Technical Treatises of the* History of Sui *(*Sui Shu*),* ed. Daniel P. Morgan and Damien Chaussende (forthcoming); Karine Chemla, "Conjunctions between the Sun and the Moon, and Pursuit Problems: Mathematical Reasoning in Chinese Writings on Astral Sciences," in *Mathematical Practices in Relation to Astral Sciences*, ed. Matthieu Husson et al. (forthcoming).

義 of 653 CE.[53] Li knows his stuff, and this is how he presents the underpinnings of these pursuits in the *Sui shu* 隋書 "Lü-li zhi" 律曆志:

> 探賾索隱，鉤深致遠，莫不用焉。一、十、百、千、萬，所同由也。律、度、量、衡、歷、率，其別用也。故體有長短，檢之以度，則不失毫釐[…]三光運行，紀以曆數，則不差晷刻。事物糅見，御之以率，則不乖其本。故幽隱之情，精微之變，可得而綜也。
>
> In exploring the recondite and searching the hidden, in snaring what is deep and eliciting what is distant, [counting rods] can never be done without. Ones, tens, hundreds, thousands, and myriads derive alike from them, and pitches, lengths, capacities (≈ volumes), weights, *li* 曆, and *lü* 率 are [simply] their distinct applications. Thus it is that bodies can be long or short, but that if one examines them with a ruler, then one will not miss by [a single] hair; […] that the three luminaries (i.e., the sun, moon, and planets) travel in revolutions, but that if one marks them with *li* numbers, then one will not err in gnomon and waterclock [timing]; and that matters and things can appear jumbled together, but that if one takes charge of them with *lü* 率, then one will not pervert their bases. Thus it is[—with counting rods—]that even dark and hidden natures (*qing* 情) and fine and subtle transformations (*bian* 變) can be fully grasped and synthesized.[54]

It is clear that these are the same *lü* 率 we see in *suan*, because Li Chunfeng next defines his terms via the chapters of the *Jiuzhang suanshu* in words borrowed from Liu Hui's 劉徽 263 CE commentary thereto:

> 夫所謂率者，有九流焉：一曰方田，以御田疇界域。二曰粟米，以御交質變易。[…]皆乘以散之，除以聚之，齊同以通之，今有以貫之。則算數之方，盡於斯矣。
>
> Now, as for that which we refer to as "*lü*," there are nine [subjects] that flow from it: [chap.] 1, "Rectangular Field" 方田, for dealing with the boundaries and areas of cultivated fields; [chap.] 2, "Un-

[53] On Li Chunfeng and his oeuvre, particularly his historiography, see chapters 2–7 of Morgan and Chaussende, eds., *Monographs in Tang Official History*.

[54] *Sui shu* (Zhonghua shuju edn., 1973), 16.387. Note that Li Chunfeng is appropriating a parallel passage in *Han shu* 漢書 (Zhonghua shuju edn., 1962), 21A.956., and HHS, *zhi* 1, 2999, which he modifies to emphasize the centrality of *lü*. On this point, see chapters 3 and 4 in Morgan and Chaussende, eds., *Monographs in Tang Official History*.

> hulled and Hulled" 粟米, for dealing with transformations (*bian* 變) and changes (*yi* 易) of the exchange of goods; […] All of these [subjects/chapters] multiply to disaggregate them and simplify to assemble them; [they] homogenize and equalize to make them communicate and "Suppose you have" to link them together—and so it is that the methods of calculating numbers all come down to this (i.e., to *lü* 率).[55]

Going from a number of months to the position of the sun in *li*, as we did above, is a "transformation" (*bian*) that Li Chunfeng, for one, likens to that between millet and fermented soy in *suan*—a transformation mediated by *lü* and performed by "Suppose-you-have."[56] Yes, the mechanical metaphor is helpful for thinking about how "*li* numbers" (*li shu* 曆數) are *structured*, but a gear train can no better explain the procedures (*shu* 術)—the numbers' *use*—than it can the transformation of millet into soy, adzuki beans, or wheat. Luckily, we need not go searching for a better metaphor, because that with which our historical subjects have left us is rather fitting.

Of milfoil, counting rods, and transformation

When actors philosophize about such "transformations" in mathematics, as Karine Chemla has written about at length, they tend to turn to the *Book of Changes*.[57] This makes sense, and it makes sense on several levels.

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[55] *Sui shu*, 16.387, tr. Chemla, "Mathematics, Nature and Cosmological Inquiry," 278 (mod.). Li Chunfeng is citing Liu Hui's *Jiuzhang suanshu zhu* 九章算術注 as concerns the six-character chapter summaries and paraphrasing him in the conclusion: "Multiply to disaggregate them, simplify to assemble them, homogenize and equalize to make them communicate, how could those not be the key-points of *suan* (computations/mathematics)?" 乘以散之，約以聚之，齊同以通之，此其筭之綱紀乎 (tr. Chemla and Guo, *Les neuf chapitres*, 158–9 [mod.]).

[56] On Li Chunfeng's treatment of *lü* 率 in the *Sui shu* "Lü-li zhi," see Chemla, "Mathematics, Nature and Cosmological Inquiry," 278–9, and chapters 3–4 in Morgan and Chaussende, eds., *Monographs in Tang Official History*.

[57] Chemla, "What Is at Stake in Mathematical Proofs from Third Century China?," *Science in Context* 10, no. 2 (1997): 227–51; Chemla, "Philosophical Reflections in Chinese Ancient Mathematical Texts: Liu Hui's Reference to the *Yijing*," in *Current Perspectives in the History of Science in East Asia*, ed. Kim Yung Sik and Francesca Bray (Seoul: Seoul National UP, 1999), 89–100; Chemla, "Mathematics, Nature and Cosmological Inquiry."

Mythologically, both *li* and *suan* go back to the demiurge Fuxi's 伏羲 awakening to the incipient, civilizing order in heaven, earth, and the "myriad creatures" in between. The version of the story that *you* know, from the "Xici zhuan" 系辭傳, may end with the trigrams, but the version that mathematicians told went a little different. Here, for example, is Liu Hui on the origins of math:

> 昔在庖犧氏始畫八卦，以通神明之德，以類萬物之情，作九九之術，以合六爻之變。暨于黃帝神而化之，引而伸之，於是建曆紀，協律吕，用稽道原。
> In the past, [Fu]xi first drew the eight trigrams to enter into communication (*tong* 通) with the virtue of the spirits-illuminant and to classify (*lei* 類) the inner tendencies of the myriad creatures; he [also] created the "nine-nine" procedure (i.e., the multiplication table) to accord with the transformations (*bian* 變) of the six lines (of the hexagrams). The Yellow Thearch, in his time, transformed (*hua* 化) them through spiritualization and expanded them through extension, thereupon establishing the rules of *li* and harmonizing the pitch-pipes (*lülü* 律吕), which [he] used to investigate the source of the *dao*.[58]

Going even further back, the *Zhoubi suanjing* 周髀算經 has the Duke of Zhou 周公 (r. 1042–1036 BCE) ask the following of Shang Gao 商高:

> 竊聞乎大夫善數也，請問古者包犧立周天曆度：夫天不可階而升，地不可將尺寸而度，請問數從安出？
> I have heard, sir, that you excel in numbers. May I ask how [Fu]xi laid out the *li du* of the circumference of heaven in ancient times? Heaven cannot be scaled like a staircase, and earth cannot be measured out with a footrule, [so I] would like to know where is it that these numbers come from.[59]

The answer, the Duke of Zhou learns, is that Fuxi derived them from the *lü* of "the circle and the square" 圓方, pertaining, respectively, to heaven and earth, and that "it was thus that Yu [the Great] was able

[58] Tr. Chemla and Guo, *Les neuf chapitres*, 127 (mod.). Note that, more typically, "it is said that Lishou 隸首 invented numbers" 云隸首作數 in conjunction with/service of the Yellow Emperor (*Sui shu*, 16.395).

[59] *Zhoubi suanjing* (Sibu congkan 四部叢刊 edn.; rpt. Shangwu yinshuguan, 1919–36), 1.1a–2a; tr. Christopher Cullen, *Astronomy and Mathematics in Ancient China: The* Zhou Bi Suan Jing (Cambridge: Cambridge UP, 1996), 174 (mod.).

to bring order to the sub-celestial realm—this [triumph] was brought about by numbers" 故禹之所以治天下者，此數之所生也.[60]

Operationally, such *shushu* "numbers and procedures" literature does not speak of "calculating the corresponding number of" so much as "*making*" 為 *a* into *b*, the algorithm-based approach typical of *li* and *suan* being more a process of "transformation" (→) than of equation (=).[61] With the *Sifen li*, we began with a single datum—a year, 2020—which was transformed before our eyes into the date of the solstice (*jiyou*$_{fκ}$), that of the new moon (*jiachen*$_{aε}$), and the position of the sun and moon at syzygy (Dipper$_{L08}$ 16 $^{552}/_{940}$ *du*). More to the point, people of this time did not calculate as we do, let alone in the faux computer code offered above; they did so with counting rods (*suan* 筭), on a physical surface, using their own set of conventions. Within those conventions, the multiplication and division of the "Suppose-you-have" procedure present, in Chemla's words, "opposed but complementary operations" whose interplay vis-à-vis the object of change—the operand—is suggestive of the transformations of *yin* and *yang*.[62] Namely, on the calculating surface, "the digits of the number by which one multiplies all vanish" as they are carried into the product that they "mount" (*cheng* 乘); and so too is the product physically "eliminated" (*chu*) from the counting surface, step by step, in building the quotient (fig. 7).[63] In this sense, achillemancy is not the only stick-based magic act in town.[64]

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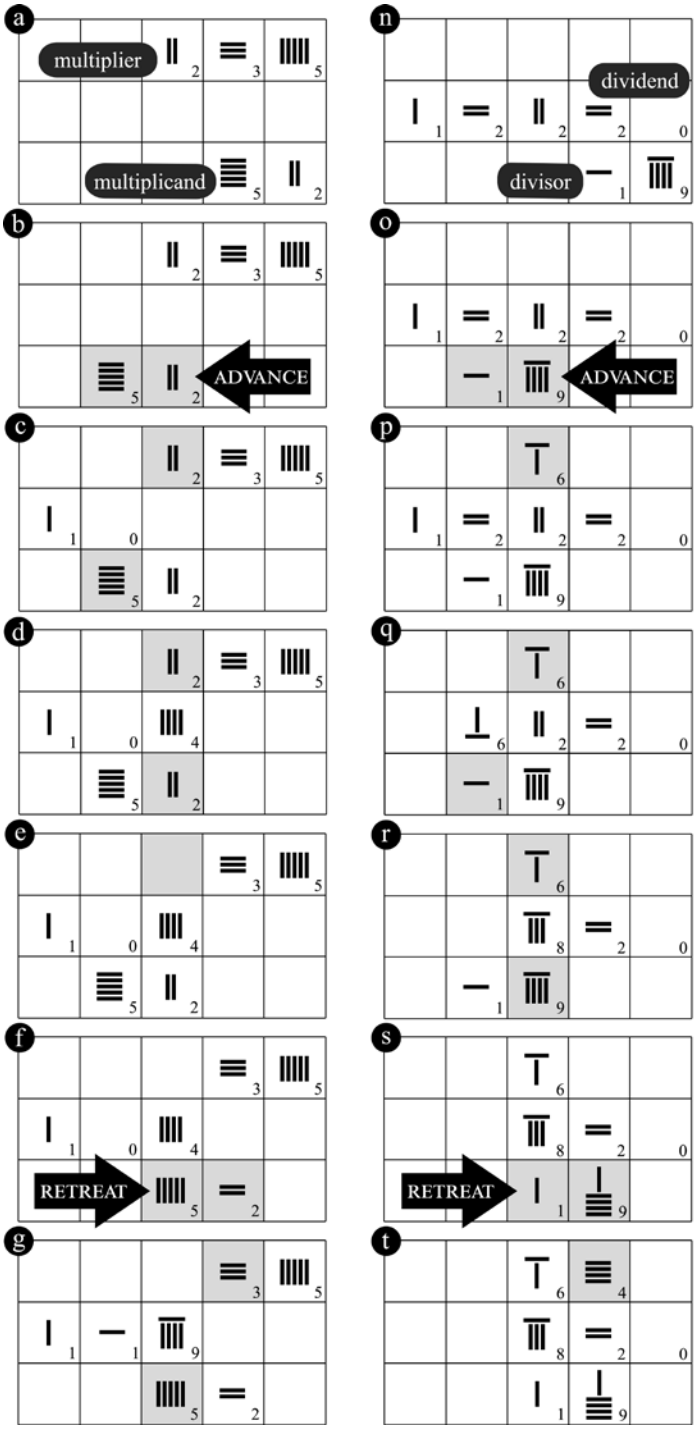
[60] *Zhoubi suanjing*, 1.4a; tr. Cullen, *Astronomy and Mathematics*, 174 (mod.). On the date and authenticity of the *Zhoubi*, see ibid., 138–56, and note that the two were in question early on (Daniel P. Morgan, "Heavenly Patterns," in Morgan and Chaussende, op. cit.). Elsewhere, the credit for *zuo* 作 "inventing' *li* often goes to a variety of figures surrounding the Five Thearchs, particularly Zhuanxu 顓頊.

[61] Chemla, "Philosophical Reflections," 90.

[62] "Mathematics, Nature and Cosmological Inquiry," 257.

[63] Ibid., 261.

[64] The ritual/symbolic parallel in physical support was not lost on contemporary actors. On *Changes* symbolism in philosophical treatments of counting rods, see Li Yan 李儼, "Suanchou zhidu kao" 算籌制度考, in *Zhongsuan shi luncong* 中算史論叢, by adem. (Beijing: Kexue chubanshe, 1955), vol. 4, pp. 1–8; Zhu Yiwen 朱一文, "Shu: suan yu shu—yi jiu shu zhi fangcheng weili" 數：筭與術——以九數之方程為例, *Hanxue yanjiu* 漢學研究 28, no. 4 (2010): 73–105. In this vein, it may prove interesting to compare the relevant liturgy for milfoil counting in, say, Zhu Xi's 朱熹 (1130–1200) *Zhouyi benyi* 周易本義, *j*. 14, with that for counting rods in mathematics (e.g. Karine Chemla, "Positions et changements en mathématiques à partir de textes chinois des dynasties Han à Song-Yuan. Quelques remarques," *Extrême-Orient, Extrême-Occident* 18, no. 18 [1996]: 115–47;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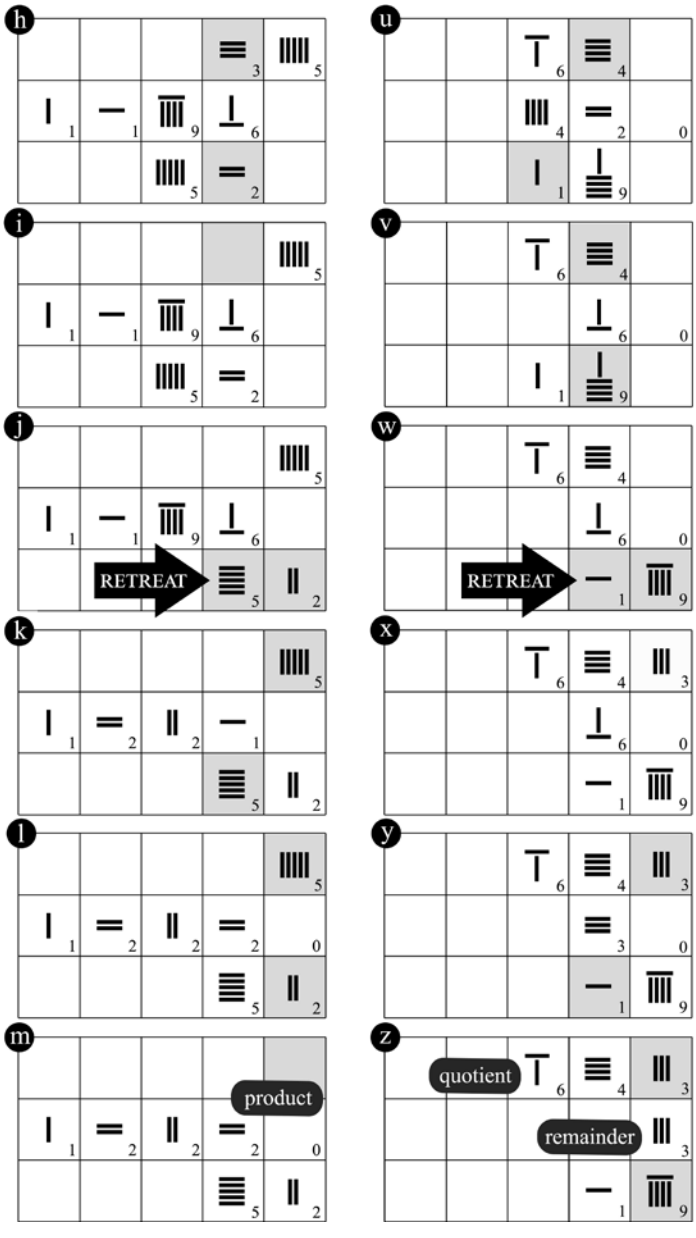

Fig. 7 The "Suppose-you-have" procedure behind the *Sifen li*'s "[5.] Procedure for calculating astronomical month I" as performed with counting rods on a physical calculating surface for the year 2020, i.e. $52 \times 235/19$ (reconstruction after Chemla, "Mathematics, Nature and Cosmological Inquiry"). Note, on the left, how the multiplier in step *a* gradually disappears in "making" 為 the product in step *m*, on the right, how the dividend in step *n* likewise disappears but for the "little remainder" in "making" the quotient in step *z*, and, lastly, how the steps of multiplication and division, left and right, mirror one another at each step.

Terminologically, algorithms present a meditation upon "the capacity of realities to be transformed without being destroyed in the process,"[65] and philosophical commentators, like Liu Hui and Li Chunfeng, bring a similar vocabulary to bear upon the subject. Whether it be milfoil or counting rods, one speaks equally of "transformation" (*bian* / *hua*), "[ex]change" (*yi* 易), and "placing in communication" (*tong* 通). When speaking of mathematics, moreover, commentators tend to frame these transformations in parallel prose around *yin–yang* operational pairs: "homogenize" (*qi* 齊) and "equalize" (*tong* 同), "disaggregate" (*san* 散) and "assemble" (*ju* 聚), "gain" (*de* 得) and "loss" (*shi* 失), "advance" (*jin* 進) and "retreat" (*tui* 退).[66] We are, of course, talking about "advancing" place values, "disaggregating" fractions, and "homogenizing" those with different denominators, but such is the magic that turns time to space and rice to beans.

Coming back to space, time, and their representation, therefore, we may think about *li* mathematical astronomy thus, from the perspective of the *Changes*. Time and space are the warp and woof of the self-same fabric—tug at one end, and the other comes mysteriously with.[67] This fabric is constant (*chang* 常), but only in so much as it is in constant flux—time is space when space is turning, but only *against a fixed background* and *at a constant rate*. Were you to stop that somehow, time and space would cease to be, because the two are only possible in concert, in opposition, and in flux—without the sun by day and stars by night, one couldn't tell north (zi_{α}) from south (wu_{η}), and without a fixed horizon by which to "clock" them, one couldn't tell midnight (zi_{α}) from noon (wu_{η}). Round and square, heaven and earth—space and time are *in communication* (*tong* 通), which means that you can *pass freely* (*tong* 通) from one to the other and back. You can, that is, if you know the trick, because long ago, before our day, "[Yao 堯] ordered Chong and Li to sever the *tong* of

"Mathematics, Nature and Cosmological Inquiry") and athletic score-keeping (e.g. Morgan, *Astral Sciences*, 163–74).

[65] Chemla, "What Is at Stake," 243.

[66] Ibid., 240–1; Chemla, "Mathematics, Nature and Cosmological Inquiry," 276–8. Compare these terms as they appear in the glossaries of Chemla and Guo, *Les neuf chapitres*, 897–1035, and Bent Nielsen, *A Companion to* Yi Jing *Numerology and Cosmology: Chinese Studies of Images and Numbers from Han (202 BCE–220 CE) to Song (960–1279 CE)* (London: RoutledgeCurzon, 2003).

[67] In this vein, see David Pankenier, "Weaving Metaphors and Cosmo-Political Thought in Early China," *T'oung Pao* 2nd ser., 101, no. 1–3 (2015): 1–34.

earth and heaven that there be no further descent and ascent [between]" 乃命重黎絕地天通，罔有降格.[68] By what sorcery, then, are we to tame the waters, conquer the world, achieve a destiny denied us by the gods, and move in and out of time and space?

You can do this with counting rods, and "if one takes charge of them with *lü* 率," Li Chunfeng promises, then "even dark and hidden natures and fine and subtle transformations can be fully grasped and synthesized" on the calculating surface. *Lü*, to reiterate, are unitless integer numbers that, like time and space, *yin* and *yang*, bear meaning only in concert, in opposition and in flux—i.e., in proportional relationship to one another and in the act of "transformation." To insert a modern metaphor in place of another, *lü* are not so much cogs in a machine as they are a reader board of exchange rates that one (manually) consults when wanting to "change" (*yi*) a fistful of money, commodities, time, or space into something else—everything has a price, modern economics has taught us, and so too, in the ancient world, did everything have its *lü*. You cannot just *"make"* 為 water into wine, of course, but with a little imagination you might well "suppose you have," because in the sortilege on the calculating surface, at least, *the numbers* are physically transformed step by step via opposed but complementary operations until from "what you have" 所有 nothing is left but "what you seek" 所求.

Cosmos, empire, and colonialism

Echoing Jean-Baptiste Biot and later anthropologists of what would until the mid-twentieth century be unblushingly called "primitive" or "savage thought," Marcel Granet would argue that, as with astronomy, "Chinese thought" allowed no place for what we would call mathematics:

> L'idée de quantité ne joue autant dire aucun rôle dans les spéculations philosophiques des Chinois. Les Nombres, cependant, **intéressent passionnément** les Sages de l'ancienne Chine. Mais, – quelles qu'aient pu être les connaissances arithmétiques ou géométriques de certaines corporations (arpenteurs, charpentiers, architectes, charrons, musiciens...), – nul Sage n'a accepté de les utiliser, si ce n'est

[68] *Shangshu zhushu* 尚書注疏 (*Chongkan Shisanjing zhushu* 重刊宋本十三經注疏 edn., 1815; rpt. Taibei: Yiwen yinshuguan, 1965), 19.297b; cf. Anne Birrell, *Chinese Mythology: An Introduction* (Baltimore: Johns Hopkins UP, 1993), 91–95.

> dans la mesure où, sans jamais contraindre à des opérations dont le résultat ne se pût commander, ce savoir facilitait des jeux numériques. […] Un symbole numérique *commande* à tout un lot de réalités et d'emblèmes ; mais, à ce même mot, peuvent être attachés divers nombres, que l'on considère, *en l'espèce*, comme *équivalents*. À côté d'une valeur quantitative qui les distingue, mais **qu'on tend à négliger**, les Nombres possèdent une valeur symbolique **beaucoup plus intéressante**, car, n'offrant aucune résistance au génie opératoire, elle les laisse se prêter à une sorte d'alchimie. Les Nombres sont susceptibles de *mutations*. Ils le sont en raison de l'efficience multiple dont ils paraissent dotés et qui dérive de leur fonction principale ; ils servent et valent en tant que *Rubriques emblématiques*.[69]

[69] *La pensée chinoise* (Paris: La Renaissance du livre, 1934), 127–8; italics are original, bold is added to enforce the following point about the *négligance* of "uninteresting" sources. One sees an echo of Granet's interest in numbers as *emblèmes*, *symboles*, *aspects*, *ensembles*, and *groupements* ***concrets*** in E.B. Tylor's (1832–1917) "examination of the methods of numeration in use among the lower races": that, compared to "the philosopher" and "our advanced system of numeration," "the savage" counts in small numbers and concrete terms, and "the still-used Roman and Chinese numeration are indeed founded on savage picture-writing, while the abacus and the swan-pan [算盤], the one still a valuable school-instrument, and the other in full **practical use**, have their germ in the savage counting by groups of objects, as when South Sea Islanders count with coco-nut stalks, putting a little one aside every time they come to 10, […] or when African negroes reckon with pebbles or nuts, and every time they come to 5 put them aside in a little heap" (*Primitive Culture: Researches into the Development of Mythology, Philosophy, Religion, Art, and Custom*, 2 vols. [London: John Murray, 1871], vol. 1, pp. 219, 244–5). As to Granet's insistence on the exclusively "emblematic" (vs. quantitative) function of *les Nombres* as concerns time and space, one likewise hears an echo of Lévy-Bruhl (1857–1939): "These minds will not picture space as a uniform and immaterial quantum. On the contrary, to them it will appear burdened with qualities; its regions will have virtues peculiar to themselves; they will share in the mystic powers which are revealed therein. Space will not be so much imagined, as *felt*, and its various directions and positions will be qualitatively differentiated from one another. […] To the primitive time is not, as it is to us, a kind of intellectualized intuition, an 'order of succession'. Still less is it a homogeneous quantity. It is felt as a quality, rather than represented" (*Primitive Mentality*, tr. Lilian A. Clare [London: George Allen & Unwin, 1923], 95, 124). On Granet's intellectual influences, see Miranda Brown, "Neither 'primitives' nor 'others,' but Somehow Not Quite Like 'us': The Fortunes of Psychic Unity and Essentialism in Chinese Studies," *Journal of the Economic and Social History of the Orient* 49, no. 2 (2006): 219–52.

Granet is simply wrong about *l'idée de quantité*, and he got there by conflating several things. He got there, first of all, by conflating what interests *les Chinois* and what interests Marcel Granet.[70] Of the eminent philosophers and classicists (*ru* 儒) of which you may have heard, here, for example, are a few from early and early imperial times that were involved in numbers in a quantitative sense: Mozi 墨子 (*c*.468–*c*.391 BCE), Sima Qian 司馬遷 (*c*.145–*c*.86 BCE), Yang Xiong 揚雄 (53 BCE–18 CE), Liu Xin 劉歆 (*c*.50 BCE–23 CE), Zheng Zhong 鄭眾 (d. 83 CE), Ban Zhao 班昭 (44/49–118/121 CE), Ma Rong 馬融 (79–166 CE), Zheng Xuan 鄭玄 (127–200 CE), Du Yu 杜預 (222–85 CE), Jia Gongyan 賈公彥 (*fl*. 637 CE), and Kong Yingda 孔穎達 (574–648 CE).[71]

He also got there by conflating imagery and substance, and in this he is not the first. Commentaries and introductions to mathematical works appeal to the *Changes* and its affiliated "cosmology/ies" as a *metaphor*, but magic numbers do indeed find their way into the *practice* of *li* mathematical astronomy.[72] Indeed, when *li* emerges on the historical stage, it does so wrapped in the metrosophy of tono-

[70] I thank K. Chemla bring for revealing to me this pattern in Granet's writing: that wherever he mentions what "interests" or "is interesting" to "the Chinese," it is usually to justify the omission of a swath of primary and secondary sources that disinterests *Granet* in that it does not fit his own *pensée*; cf. the critique in Needham, *Science and Civilisation in China*, vol. 2, p. 217.

[71] On Mozi, see Graham, *Yin-Yang*, 8–11; *Later Mohist Logic, Ethics, and Science*, 2d ed. (Hong Kong: Chinese UP, 2003). On the Han-era figures listed here, see Christopher Cullen, "People and Numbers in Early Imperial China: Locating 'mathematics' and 'mathematicians' in Chinese Space," in *Oxford Handbook of the History of Mathematics*, ed. Eleanor Robson and Jacqueline A. Stedall (Oxford: Oxford UP, 2009), 591–618. On the commentators, see Daniel P. Morgan, "Calling out Zheng Xuan (127–200 CE) at the Crossroads of Ritual, Maths, Sport and Classical Commentary," in *Mathematical Commentaries in the Ancient World*, ed. Karine Chemla, Mark Geller, and Glenn Most (forthcoming).

[72] I object to the use of the observer's category "cosmology" in post-1980s sinology to present what actors called "*yin–yang*" and "five-agents" (*wuxing* 五行) correlative, "sympathetic" (*ganying* 感應) thinking as an *exclusive* and *monolithic* world-view. For a reminder of what we used to call "cosmology"/宇宙論 in Chinese studies, see Michael Loewe, "The Cosmology of Early China," in *Ancient Cosmologies*, ed. Carmen Blacker, Michael Loewe, and J. Martin Plumley (London: Allen and Unwin, 1975), 87–109; Xi Zezong 席澤宗 and Zheng Wenguang 鄭文光, *Zhongguo lishi shang de yuzhou lilun* 中國歷史上的宇宙理論 (Beijing: Renmin chubanshe, 1975). On the nineteenth- and twentieth-century origins of the new "ancient Chinese cosmology," see Haun Saussy, "Correlative Cosmology and Its Histories," *Bulletin of the Museum of Far Eastern Antiquities* 72 (2000): 13–28.

metrics and the *Changes*. Liu Xin, whose *li* of *circa* 5 CE is the earliest to survive, cites the *Changes* throughout as the supposed origin of his *lü*,[73] and Liu Hong 劉洪 (fl. 167–206 CE), some two centuries later, would push this about as far as it could go. Here is Li Chunfeng on the latter:

> 其為之也，依易立數，遁行相號，潛處相求，名為乾象曆。又創制日行遲速，兼考月行，陰陽交錯於黃道表裏，日行黃道，於赤道宿度復有進退。方於前法，轉為精密矣。
> What [Liu Hong] did was establish numbers based on the *Changes* [such that] they called out to one another in hidden motion and sought each other out from secret parts—and [at this he] named it the *Qianxiang li* (after the hexagram Qian 乾 ䷀). Also, [he] created the solar/daily motion slow–fast while at once investigating lunar motion, [concluding that] *yin* and *yang* cross inside and outside the yellow road, and that the sun travels on the yellow road, experiencing advance and retreat in red-road lodge *du*—only with this was there a turn towards the fine and tight relative to prior methods.[74]

Compare this to Jia Kui's 賈逵 (30–101 CE) somewhat less enthusiastic appraisal of one Zhang Long 張隆:

> 永平中，詔書令故太史待詔張隆以四分法署弦、望、月食加時。隆言能用易九、六、七、八爻知月行多少。今案隆所署多失。臣使隆逆推前手所署，不應，或異日，不中天乃益遠，至十餘度。
> In [57/75 CE], there was an edict ordering Zhang Long, former expectant appointee to the Grand Clerk, to (predictively) note the added hour of [lunar phases] and eclipses according to the *Sifen* method. Long said that he was able to use the nine, six, seven and eight lines from the *Changes* to know the extent of lunar motion. [We] now know Long's [predictive] notes to have missed the mark in most cases. [I,] Your servant, made Long retrodict [added hours] noted by former hands, and they did not correspond, sometimes [even] falling on different days; he was even further off in failing to hit the mark in heaven, [erring a matter of] up to more than ten *du*.[75]

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[73] On Liu Xin's historically-situated application of *Book of Changes* numerology to the synthesis of *lü* 律 tono-metrology and *li* mathematical astronomy as well as the limited historical shelf-life of said synthesis, see Morgan and Goodman, "Numbers with Histories," and the sources cited therein.

[74] *Jin shu* 晉書 (Zhonghua shuju edn., 1974), 17.498.

[75] Cited in HHS, *zhi* 2, 3030. On the hexagram line numbers, see the entry "Da yan zhi shu 大衍之數" in Nielsen, *A Companion to* Yi Jing *Numerology*.

One of these men *no one* has heard of, the other was later enshrined at temple among those "renowned from the days of yore for calculating numbers" 自昔著名算數者, and you can probably guess which is which.[76] As it turns out, Liu Hong's numerology was only impressive in so far as it capped off "revolutionary" mathematical models for lunar anomaly (*chiji* 遲疾), latitude (*yinyang* 陰陽), eclipse "crossing" (*jiao* 交), and solar reduction to the equator (*jintui* 進退)—models thanks to which Liu Hong was winning predictive competitions against the living several decades after his death.[77] Liu Hong, moreover, was one of only two or three to actually bother; mostly, *li* men didn't dress up their *lü* as something else—as something, no less, that *didn't work*.[78]

Suan, for its part, is likewise devoid of similar contents: one does not perform a cubic root extraction with "fire," nor can one operate upon "metal," *yin*, Mingyi 𝌆, stems, or branches.[79] There too, the question of "what one seeks" 所求 is not formulated in terms of the "bane" 凶 or "auspice" 吉 of human events, let alone colors or directions to "avoid" 避, but in the sort of 1–2–3 numbers of "what one

[76] *Song shi* 宋史 (Zhonghua shuju edn., 1977), 105.2552. The correct answer is Liu Hong, who was posthumously promoted to Viscount of Mengyin 蒙陰子 and whose sacrificial icon was repainted according in King Wenxuan Temple 文宣王廟, Bianjing, in 1109; see Morgan, *Astral Sciences*, 177–78.

[77] On Liu Hong's technical innovations and posthumous victories in live-trial testing and debate, see ibid., 140–76, 199; Cullen, *Heavenly Numbers*, 325–92.

[78] Martzloff, *Le calendrier chinois*, 38–44; Morgan and Goodman, "Numbers with Histories."

[79] Excluding expressions like *jin* 金 "cash" and *zhi wuxing* 置五行 "set out five columns," the very words *yin*, *yang*, *wuxing*, "wood," "fire," etc., do not appear once in the base-text of the *Jiuzhang suanshu*. The one place where they do appear in the commentary, moreover, it is to explicitly deny them a place in such matters as calculating the volume of the sphere: "La théorie de Zhang Heng 張衡 (78–139 CE) veut naturellement s'accorder avec la théorie du pair et de l'impair, du *Yin* et du *Yang*, et ne prend pas en considération la precision" 衡說之自然，欲恊其陰陽奇耦之說，而不顧踈密矣 (Chemla and Guo, *Les neuf chapitres*, 383). As to the *operability* of the heavenly branches and earthly stems in fig. 1, one notes that while Granet speaks of them as "les nombres des séries dénaire et duodénaire" (op. cit., 129), the elements these ordinal series are no more *numbers* than are "Sunday," "Monday," or "Tuesday." Like the days of the week, one cannot add, subtract, multiply, or divide *geng*$_{g}$ 庚 from hai$_{\mu}$ 亥, and where they appear in the context of *li* coordinates one notes they are "named off" (*ming* 命) rather than "counted" (*shu* 數), and that this comes only after all the operations on the *real numbers* are finished (see *Sifen li*, procedures 6 and 8 above).

has” 所有.[80] Where mathematicians *do* bring the vocabulary of the *Changes* into actual calculations, Chemla notes, it is “linked to the practice of proof in the context of exegesis, and, more precisely, to a specific function assigned to it: identifying the ‘fundamental transformations’ at play in all procedures.”[81] It is deployed in establishing the correctness of an algorithm, in drawing parallels with others, and in unveiling the mathematical strategies, hidden operations and “reason”/“internal constitution” (*li* 理) behind the problems of the Classic. It is heuristic framework, in other words, towards “a mathematical research on the rationality of change.” [82]

And that’s the thing about “numbers,” and why *they* enjoyed a place among the “six arts” 六藝 of the gentleman and in the normal, imperial curriculum ever after: with numbers, “transformations” can be reliably reversed, repeated and explained, and with numbers, more importantly, there is *proof*. Proof is what a paperwork empire demands—proof of travel, proof of payment, proof of proficiency, and the list goes on—and there is a reason that that empire entrusted such things as taxation, censuses, and market payments to *accountants*: because *quantities*, in this regard, are far more crucial to the work of sagecraft than are *Rubriques emblématiques*. And when it went about “observing the signs and granting the season” 觀象授時, so too is there a reason why the Empire appointed relevant experts to the task—why they recruited talent, why they hosted debates, why they ran competitive trials, and why Zhang Long, above, is introduced as a “former appointee.” Time and space are commutable, and by numbers you can take command of them to literally see the future and to *prove* it, to all under heaven, all at once, in the minute the midday sun goes black.[83]

They may address different questions, but the “numbers and procedures” (*shushu*) of *li* and *suan* are by no means a lesser magic than the *Changes*. They are, let us recall, sister sciences revealed by the demiurge at the beginning of human time, and they are, in the

[80] “Bane,” “auspice,” “avoidance,” etc., are problematics that come up in the other five subdivisions of the *Han shu* “Yiwen zhi” category “Shushu,” notably the sort of “Five Agents” hemerology studied in Donald Harper and Marc Kalinowski, eds., *Books of Fate and Popular Culture in Early China : The Daybook Manuscripts of the Warring States, Qin, and Han* (Leiden: Brill, 2017).

[81] “Mathematics, Nature and Cosmological Inquiry,” 280.

[82] Ibid., 281.

[83] On the subject of recruitment, debate, and testing in *li*, see Morgan, *Astral Sciences*, chaps. 1, 4; Cullen, *Heavenly Numbers*, chap. 7.

Documents, the very first thing to which Yao 堯 and Shun 舜 attended upon the throne.[84] And where the *Han shu* "Lü-li zhi" opens on numbers citing a lost *Document* to the effect that "one must prioritize one's *suanming*" 先其算命 [85] it is telling how Yan Shigu 顏師古 (581–645 CE) chooses to read this. Normally "fate divination," Yan, by virtue of its context, understands *suanming* to mean something altogether greater for the fate of man:

> 言王者統業，先立算數以命百事也。
> This means to say that, in consolidating his patrimony, he who is to be king must first establish the calculation of numbers so as to take command of the hundred affairs.[86]

On that note—on the "mathematical mandate"—I want to end on a question concerning further Truths that we might want to rethink in Chinese studies: Why is it, for starters, that the *shushu* of *li* and *suan* are systematically excluded from *shushu* studies? How is it also that that the *quantitative* study of space (*yu* 宇) and time (*zhou* 宙) is generally excluded from the discussion of "cosmology" (*yuzhou lun* 宇宙論)? Would not the story of "Chinese thought" be more *interesting* (let alone more *historically accurate*) if it were to include *all* of what the Chinese thought to do with numbers?

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[84] The "Yao dian" 堯典 chapter opens with Sage King Yao "ordering the Xi and He [brothers], in reverent accordance with prodigious heaven, to *li* and *xiang* the sun, moon and stars and respectfully grant the seasons of man" 乃命羲和，欽若昊天，歷象日月星辰，敬授人時 (*Shangshu zhushu*, 2.21a). When Yao abdicates to Shun, furthermore, he does so by declaring that "the *li* numbers of heaven rest in thy person; ascend thou at last [to the throne] of the great sovereign" 天之歷數在汝躬，汝終陟元后 (ibid., 4.55b; cf. *Analects* XX.1). Immediately following his ascension, lastly, Shun "attended to the Rotating Device and Jade Traverse (i.e., Beidou 北斗; UMa) so as to order the seven governors/government affairs" 在璿璣玉衡，以齊七政 (*Shangshu zhushu*, 3.35b).

[85] *Han shu*, 21A.956.

[86] Ibid. (comm.). Cf. Archytas (428–347 BCE) (B 3): "The invention of calculation put an end to discord and increased concord […] A standard and a barrier to the unjust, it averts those who can calculate from injustice, persuading them that they would not be able to stay unexposed when they resort to calculation, and prevents those who cannot calculate from doing injustice by showing through calculation their deceit" (tr. Leonid Zhmud, *The Origin of the History of Science in Classical Antiquity*, tr. Alexander Chernoglazov [Berlin: de Gruyter, 2006], 71).