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Abstract: This paper presents a small case of junction of two research fields that remained, until recently, relatively cut off from each other. Using concepts of the Assessment field, specified in a didactic approach, we develop and test a methodological tool on the analysis of students-teacher interactions in mathematics classroom. We discuss then its potentiality in the Assessment field. This illustrates a way of locally connect research areas, via a shared methodological tool.

Keywords: mathematics teaching practices, evaluation, interactions, formative regulation

Introduction

Initially inscribed in a particular research area of the didactics of mathematics field, the Double Approach (didactic and ergonomic) theory (Robert & Rogalski, 2005), our study aims at investigating the teacher's coherence in the processes of information-adaptation, when adjusting the situations, making decisions on the spot. Research on Assessment, a field traditionally cut from didactic ones, handles these questions with concepts as *formative* assessment or *regulations*. How can we describe these regulations and understand the way interactions guide them?

To study these questions, we thus went beyond the specific fields of didactics theories and carried out a literature review on formative interactions in Assessment, a research field in education, with its own models and conceptual frames[1]. Our review briefly summarized in section 1 shows needs for didactic references on learning and teacher activity, and led us to a theoretical and methodological development (reported in section 2) to support our analyses regarding the teacher-students interactions in mathematics as an adaptive dynamic process. The last part tests potentialities and limits of this tool on a short episode, before coming back to theoretical questions.

Literature in the field of Assessment: needs for a didactic approach

In Assessment research, the teacher-students interactions relate to three overlapping concepts: *feedback*, *formative assessment (*or *for learning*), and *regulation*.

Theoretical needs for a didactic approach

The teachers' feedback to learners' activities constitute an element of the process of formative evaluation, and research has been deepened to understand their variable effects according to their nature. Yet, Crahay (2007) notes that research on this concept is full of opposed results. He proposes to focus on the processual aspect of the link between teachers' reactions and students'

[1]This includes research on measurement in education, psychometric and edumetric approaches.

learning considered as an activity rather than a product, and taking into account the characteristics of the tasks, the school disciplines or the taught objects.

As for the concept of formative assessment, initially restrained to dedicated moments, it has been extended to "*all those activities undertaken by teachers, and/*or *by their students, which provide information to be used as feedback to modify the teaching and learning activities*" (Black and Wiliam (1998, p.10). Research points out a new concept, that of *regulation*, which can be interactive, retroactive or proactive. In this widened sense, the interactive regulations that may happen in didactic situations make a junction between evaluation and teaching-learning situations. Crahay's synthesis (2007) or those of Allal & Motier Lopez (2005) evoke a strong dependence between the formative evaluation and the very organization of teaching, encouraging research to integrate didactic issues (task, disciplinary contents…) in the analyses. More recently, Vantourout and Goasdoué (2014, p.142) claim the need for taking into account both didactical and psychological approaches by arguing that willing to foster learning requires understanding students' cognitive functioning with a task. Although some attempts exist in mathematics education, they stress the lack of research investing this joint perspective.

The theoretical needs addressed in this review join our didactic concerns, i.e. to pay particular attention to specificities of the taught and evaluated contents by analyzing the teacher-students interactions. In this perspective, the concept of interactive regulation described in research on formative assessment seems an interesting object regarding our didactical concern.

Regulation in formative assessment

Formative assessment research has been driven by two underpinning teaching and learning theories: originally the neo-behaviorism, then the cognitivist one with its strong necessity to consider classroom interactions, particularly real-time regulations. In Allal & Mottier Lopez's synthesis (2005), the concept of regulation is a process referring to the features of the concept of formative evaluation (collect, interpret, decide). Our didactic concerns relate to one of the five distinguished conceptual forms of regulations they bring out from literature: the online regulations resulting from interactions. Their model of regulations includes the situation; the teachers' interventions; and the interactions between students based on a Vygotskian approach using the ZPD concept. Aligned with this, recent research on assessment for learning (example Pryor & Crossouard, 2008) are referring to the Activity theory (Leontiev, 1975/84) in order to consider the formative assessment as a cultural historical activity shaped by teachers' and students' reciprocal acting, and co-determined by the subject and the situation, which should not be considered separately.

The Double Approach frame also borrows from the Activity theory, eploited in the field of ergonomy (Leplat, 1997). There, the term "activity", with a different unit of analysis, focuses on the activity of an individual subject. This didactic approach considers not only the mathematical knowledge to be taught, but also the procedures and the subject's activity to solve mathematical problems. Also considered as cultural-historical processes, the activity of a subject is here again co-determined by the subject herself and the situation carried out, which includes socio-cultural interactions with other subjects (Robert & Rogalski, 2005). In these interactions, we are interested in how the teacher's activity depends on that of the students; particularly while adapting, adjusting teaching to optimize learning. The Activity theory constitutes thus a theoretical link, enabling us to articulate the teaching practices and the students' learning. The next part clarifies these links and outlines our theoretical and methodological tool.

Connecting with a didactical approach of regulations of students learning

Towards tools for a didactic analysis of the formative regulations

We consider two activities reciprocally influencing each other: that of teacher, whose result *is* a situation for the students, impacting them; that of the student, who is co-determined by the student and the situation produced by the teacher. The teachers' activity therefore involves two levels: the double regulation, resulting from the activity they address to themselves (to teach) and that resulting from the activity they address to the students (to make learn).

As for Crahay, in the Double Approach, the learning is regarded as a process, not as a product and teachers' feedbacks are not studied independently of student's activity that creates them. From the didactic point of view, the analysis of the students' activity needs to consider the situation at stake, including the task and possible elements of the didactic contract; the students' knowledge and procedures (that are contextual elements), needed to solve the task; and the product (oral or written) of students' activity (answer and any element on how the task has been realized). Aligned with research on assessment, we consider that the teacher's activity in class interactions, is formative when he/she takes information on the students' activity in order to act on the learning. Our analyses of the teacher's regulations therefore need to specify the nature of information she collects and the actions she subsequently carries out to reduce the gap she can observe with her learning objectives.

Our research is therefore guided by the principle of identifying what is the collected information about: a product (the answer, the result), a procedure, or a piece of knowledge; and what is directly aimed at by the intervention (again a result, procedure or knowledge). Even if we do not have hierarchical assumptions on these types of intervention as for learning, not all interventions are equivalent. In case of an error for example, some interventions aim at correcting by giving the expected result (this lets the student the responsibility to find the underlying mathematical concept). Other interventions address the underlying procedure or knowledge (conform or not with what was aimed at). All constitute forms of accompaniment of the student's learning.

Results, procedures, knowledge

The Activity theory (Leplat, 1997) distinguishes the result of the activity from the procedure implemented and from the state of knowledge of the subject. This framework adapted to teaching, leads to associate to the student (the subject) a state of knowledge, which allows him to analyze and redefine the task prescribed by the teacher, to implement a procedure leading to an answer, guided by some knowledge (explicit or implicit). This student's activity (generally carried out in thought, but possibly verbal or written) can thus be observable in what it products: an answer/ a mathematical result... Among the possible feedback of the teacher, one can also distinguish those addressing the result only (indicating for example that the answer is false), or the procedure (indicating for example that the theorem used does not correspond to the hypotheses of the statement), or the student's state of knowledge (indicating for example confusion between a theorem and its reciprocal etc.). The choice of a type of feedback depends on various factors (time, prior explanations/examples, teacher's experience, etc.). In the same way, if the student says he ignores how to apply such theorem, the teacher perceives information on the procedure thought by the student. The possible feedback varies here again. It can remain on the procedure level, for example explain that the rule is not adapted; or change level by giving the solution (or begin the resolution and let the student finish), or it can approach the student's difficulty by clarifying the mathematical knowledge.

This way of analyzing interactive regulations leads to identify in each student-teacher interaction a couple information-feedback where the information, as well as the feedback, could be associated to a *result* (R), *procedure* (P) or *knowledge* ("*connaissance*", C), leading to 9 possible types (table 1). A qualitative didactic analysis of each interaction in a class session allows reporting the dynamics at play in these interactions by associating each of them to one of the nine possible types of regulation.

Action Information	Result	Procedure	Connaissance
Result	RR	RP	RC
Procedure	PR	PP	PC
Connaissance	CP	CP	CC

Table 1: Information-Action: 9 possible pairs

In some case, the "coding" could depend on what has been previously done, so that there is a need for interviews to support the coding.

The following section implements this tool to analyze an effective classroom episode.

A minute of verbal interactions in class of mathematics

We use a classroom video, collected from the French research project NEOPRAEVAL on evaluative practices in mathematics teaching to analyze the students-teacher interactions occurring there with the "RPC tool". The extract, situated at the beginning of a one-hour session with Grade 8 (14 y.o.) students, consists, as ritually in this class, of series of calculations intended to be treated quickly, called "Flash". A slide, titled "Mental calculus" is video-projected, and five calculations are successively displayed every 30sec, during which the students carry out their calculations and write answers. These writings are neither collected nor looked at by the teacher. After the fifth calculation, the teaching undertakes a collective correction, questioning students and writing the answers herself on the blackboard. Our analysis concerns the collective 1min dialogue-correction, of the very first calculation displayed: $\mathbf{3x10^{-2}}$. We aim at identifying verbal interactions that can be interpreted as moments of formative regulation, based on an *a priori* analysis of the task, of the knowledge concerned (presented in Appendix), of possible interventions; and an *a posteriori* analysis of the effective interactions (presented in the next section).

Analysis *a priori* of the teacher's interventions

In situations of interaction, the teacher faces various cases (answers are correct or not, wished or not, expected or surprising…), which could raise various types of regulation (explicitly agree or not, develop the procedure or not, indirectly disapprove by repeating an explanation/ not reacting/ questioning someone else…), depending, among others, on the objectives fixed by the teacher prior to the session and/ or on the spot. In all cases, the intervention is said formative if it enables the student to recognize whether a behavior, answer, is correct or not[2].

For the calculation $3x10^{-2}$, answers *a* or *b,* or even *f* (Annex) would be the acceptable correct ones (expected/not). Answer *a* could bring a simple approval or a development to clarify the procedure, for example by an oral explanation on the product by 10^{-2}, leading 3 at the hundredths digit by a technique on the rows of the decimal writing, or by using one of the other possible answers *b, f, g, h, i, j* or *k* as intermediate calculus. As for the answer *b*, it contains already a calculative step of procedure, but here again; the teacher could accept it or add an explanation via the answer *c*, even *d* or *e*. These *a priori* reactions not only depend on the mathematics aimed at but are to be adjusted with the context where the task takes place. As already underlined, neither the task alone, nor the students' answers (correct or not) suffice to explain the feedbacks. Here, the mental calculations context, in the "flash" ritual, requires not to spend too much time on the task; therefore answers *c, d, e,* or *j, k* (fractional answers) are less likely to be acceptable (even if the teacher may want to expose them during the correction). We assume that these answers will lead the teacher to some "negative" feedback, i.e. indicating, by a way or another, that it's not the expected answer in such context.

Analysis of the 1 minute interactions

Table 2 transcribes the turns to speak that occurred during the minute of this correction. The two last columns indicate whether the information (I) brought by the student's relate to result (R), procedure to obtain it (P), or subjacent knowledge ("connaissance", C); the same for the resulting teacher's action (A). The coding actually limits the researchers' inferences by addressing the facts: the nature of the feedback (from a mathematical point of view: a result, a procedure or knowledge). This feedback can be done by addressing the result level, others the procedure, still others the knowledge. When the teacher does not react verbally to an answer[3], we consider this silence an information too about the validity (or not) of the student's answer.

	Turns of speak	**I**	**A**
01	P : so / Camélia ?		
02	Camelia : three times one over ten times ten/	P	
03	P : what is ten to the negative two ?		P
04	Camelia : one over ten times ten	P	
05	P : and we know how to calculate it ?		P
06	E2 : one over three hundreds	R	
07	E3 : what's that ?		
08	P : you don't know how to calculate one over hundred??		R
09	Camelia : Ah yes ///		
10	E2 : it makes one over three hundreds. **P: --**	R	R
11	E3 : it's not possible	R	
12	P : Orlane		R
13	Orlane : one over three hundreds. **P: --**	R	R
14	E5 : three hundreds… one over three hundreds	R	
15	P : there is indeed a negative here [showing the sign of the exponent – 2 on the statement]		P
16	E5 : bah / negative one over three hundreds. **P: --**	R	R
17	E6 : zero point zero three	R	

[2] The exactness of an answer does not prejudge the teacher's reaction. A correct and awaited answer could still draw to repeat the solution, to develop a procedure, take the opportunity to review some concepts, etc.

[3] we consider only obvious cases (the student action is clearly audible and the teacher takes a time "not paying attention") and we count only utterances directly linked to the task as R, P or C, no other ones (as lines 7 or 21)

18	P : Yasmine		R
19	Yasmine : zero point zero three	R	
20	P : [writing on board 0,03 then $10^{-2} = \frac{1}{10 \times 10} = \frac{1}{100} = \frac{1}{10 \times 10} = \frac{1}{100} =$] So ten, negative two/ indeed Camélia / this is one / over ten times ten / so / one over hundred / one hundredth / you know how to write this?		P
21	Camélia : yes		
22	E8 : this is one percent !	R	
23	P : [she writes 0,01 after 1/100] It is 0,01 / so if one asks to make 3 times this/ 0,03		P

Table 2: A minute of verbal interactions

The pairs (I; A) then constitute basic units of formative evaluation and finally, the analysis of the interactions in this extract results in the following table:

Information/Action	Result	Procedure	Connaissance (knowledge)
Result	6	3	0
Procedure	0	2	0
Connaissance	0	0	0

Table 3: Synthesis of the interactions

Several observations can be made: 1) a majority of pairs remain on the same level, with a majority of RR among these. Only 3 feedbacks are changing the student's level of information making it pass from result R to procedure P; 2) Very few student intervention are situated at the P level in this particular case; 3) No interactions from students, nor from teacher, directly address knowledge. Many of these remarks could reasonably be explained by the specific nature of this episode: mental calculation, intended to be already mastered by students, the correction of which should be easy and short. Yet, we can notice a swift along teacher's interactions. After a time respecting the students' "result" level, she switches feedbacks towards procedural level. Indeed, the rare RP events occur all at the end of the exchange (line 15/20/23). This can reflect teacher's up-taking of information from the students' speeches. Seeing numerous mistakes instead of the expected answer, she adapts her feedback by entering into a procedural level. She does not always verbally react when the answer is wrong (see E2; repeated twice; or E3; E5); she possibly prolongs the interaction when the answer is acceptable even if not expected (it is the case for Camelia's answer, but not for the answer "1%" of E8) and writes on the board what is correct and acceptable (10^{-2}=0,01 and 0,03). Thus, when Camelia starts an answer that could lead to *d* (fractional expression) or *g* or *j* (decimal expression), she interrupts her to orient the discourse towards the meaning of the powers of 10 (here 10^{-2}) that lead to decimal notations (0,01). Referring to our *a priori* analysis, we assume that the teacher thus expects the answer *b* (direct decimal expression) not *f*, nor *i* that have intermediate fractional steps. Yet, the student continues with fractions. The expected answer seems to be long delayed. Students make the classical mistake which is not picked up by the teacher (E5), then an answer using percentage is expressed, not much taken into account by the teacher, who seems to get eager for the expected decimal. Observing the mistakes and students' difficulties, she reconsiders the change of the power of ten into a decimal notation via a fraction calculation, in accordance with the procedure suggested by Camelia, thus finally accepting to align with the cognitive path taken by her students.

Discussion and perspectives on didactic regulations

Aiming at analyzing, in a didactic approach, teachers' practices of formative regulation, we reviewed literature, which indicates that these practices are both very few and little diversified. We thus looked for a theoretical and methodological tool, which enables comparison between practices, even if the contents vary, in order to agglomerate results from various data and look for correlations. The analysis of the short episode above reveals many questions on the observed session as well as on the RPC tool. On the data, it suggests to test longer episodes, in other contexts, on other mathematical contents, and with other teachers. In the purpose of describing and understanding the teachers' adaptive activities to the students' one, it would be interesting to enrich analyses with various mathematical contents, and various didactical contexts of learning (application of former knowledge, problem solving, institutionalization…). The RPC tool makes possible the determination of trends in the regulating practices (between-variability of teachers or within-variability of teacher). On the theoretical tool, it results in prolonging the use of these RPC tables by devising tables being "average" of many same contextual tables, to characterize the regulation types for a given teacher. One could indeed calculate a table of the variations to the average of the tables corresponding to several sessions for the same teacher (intra-variability, to characterize the regulation practices for a given teacher), or examine the variations to the average for several teachers on a given content (inter-variability to characterize the profession, or how regulating practices depends on the specific knowledge).

Similarly, the other few analyses we have carried out with this tool look quite promising for research on teaching regulations. They confirm the variability of the practices observed in former research. Yet, they also reveal some tendencies: 1) the couples information-feedback observed are not of the same type but distributed among five or six of the nine possible types; 2) these regulations were not numerous in our data; 3) the couples RR are dominant for all the teachers, a result converging with research on evaluative practices quoted in our literature review; 4) the distribution of interactive regulations, for a given type of information (R, P or C) is higher along the diagonal, i.e. pairs RR are more frequent than RP or RC; pairs PP are more frequent than PR or PC. This suggests that teachers generally produce same level feedbacks as the students' answers. They rarely turn a R-answer onto a procedure level; or help a student, who is indicating a procedure, to formulate the underlying knowledge C. Therefore, are these specific cases of *regulation* (when the teacher's feedback and received information are not on the same level) fostering the students' learning? And are the ascending ones (RP, RC, PC) more supportive for learning?

In conclusion, the joint perspective taken here led us to a tool to analyze the mechanisms of formative regulation in the specific case of classroom interactions. If the tool appears fruitful in this case, then could it be more largely applied to other forms of formative assessment in mathematics education? Indeed, taking into account the "formative" character implies that the teacher has means to collect evidence or not of student learning. To turn towards the specificity of the *saviors*, identifying results, procedures or knowledge in interactions could *a priori* be means of taking into account the *savoirs* in other forms of formative assessment set-up by the teacher (questioning, discussions, peer/ self-assessments…). Such considerations lead to reflections about the nature of the beginning networking case here. The connection between didactics fields of research and the assessment one is realized here via the elaboration of a possible common methodological tool that possibly informs both research fields. This case of networking is rather original, although it is justified in Radford's remark: "*Using the semiosphere's spatial metaphor, theories Ti and Tj can be visualized as being "closer" or "further" depending on their own (Pi, Mi, Qi) and (Pj, Mj, Qj)* *structures. The connection c_k of Ti and Tj requires the identification of research questions Qij (tasks, problems, etc.) that guide the enterprise as well as the building of a new methodology Mij to answer the research questions under consideration*." (Radford, 2014, p.284).

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Annex: Excerpt of the a priori analysis of the task

Possible answers and procedures: Various categories of correct procedures and answers can be discerned following the knowledge used. **I)** Use of the scientific notation: directly move from this notation to the decimal one: **a.**$3\text{x}10^{-2}=0{,}03$ (note that the lack of explicit calculation can lead students to reject this procedure by didactic contract effect). **II)** Use of the decimal notation in calculation: **b.**3x0,01=0,03 **III)** Use of the fractional notation: different procedures according to the fractional form applied to ten powers: **c.** $3\times\frac{1}{10^2}=3\times\frac{1}{100}=\frac{3}{100}$ **d.** $3\times\frac{1}{10\times10}=3\times\frac{1}{100}=\frac{3}{100}$ **e.** $3\times\frac{1}{10^2}=3\times\frac{1}{10\times10}=3\times\frac{1}{100}=\frac{3}{100}$. **IV)** Mix fractional/ notation: various procedures depending on the forms applied to ten powers: **f.** $3\times\frac{1}{10^2}=3\times\frac{1}{100}=3\times0{,}01=\mathbf{0{,}03}$ **g.** $3\times\frac{1}{10\times10}=3\times\frac{1}{100}=3\times0{,}01=0{,}03$ **h.** $3\times\frac{1}{10^2}=3\times\frac{1}{10\times10}=3\times\frac{1}{100}=3\times0{,}01=0{,}03$ **i.** $3\times\frac{1}{10^2}=3\times\frac{1}{100}=\frac{3}{100}=\mathbf{0{,}03}$ **j.** $3\times\frac{1}{10\times10}=3\times\frac{1}{100}=\frac{3}{100}=0{,}03$ **k.** $3\times\frac{1}{10^2}=3\times\frac{1}{10\times10}=3\times\frac{1}{100}=\frac{3}{100}=0{,}03$.

Mathematical knowledge at stake: The task could be legitimately interpreted in multiple ways. Yet, in the context here (a series of quick calculus), short procedures (and short corrections), mentally easy, might be awaited by the teacher, so *a, b, c, f* or *i*. There, the passage from 10 powers to decimal or fractional form, do not explicitly rely on the definition of the exponent. This is important for interpreting her feedbacks taking into account professional constraints as the time factor.