ON DIALOGUES AND ONTOLOGY THE DIALOGICAL APPROACH TO FREE LOGIC

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THE DIALOGICAL APPROACH TO FREE LOGIC

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Abstract

Being a pragmatic and not a referential approach to semantics, dialogical logic does not understand semantics as mapping names, propositions and relationships into the real world to obtain an abstract counterpart of it, but as dealing (*handeln*) with them in a particular way. This allows a very simple formulation of free logic the core of which can be expressed in a nutshell, namely: in an argumentation, it sometimes makes sense to restrict the introduction of singular terms in the context of quantification to a formal use of them. That is, the proponent is allowed to use a constant iff this constant has been explicitly conceded by the opponent.

More technically, we show a new, dialogical way to build free logic systems for first-order logic with classical and intuitionistic features and present their corresponding tableaux.

1. *Introduction*

1.1. *Free logics*

The proposition "God does not exist" contains a paradox sometimes referred to as Plato's beard: if God does not exist and the proposition should be true, standard referential semantics for quantified logic fails to give meaning to the name "God". But, given compositionality, since the meaning of a sentence is combined from the meanings of its parts, "God does not exist" does not evaluate.

It is easy to see that related difficulties appear in every formula that contains singular terms. In standard logic, it is impossible to state that God is either good or evil without presupposing his existence, or that the round

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square is round, or even that the flying horse can fly. We always get caught by the lack of (referential) meaning of some of the parts of the sentences.

Several modifications to standard logic have been proposed to deal with formulae containing *referential gaps* while still defending reference (for instance, Russell's theory of descriptions [33] renders them false by translating "τ exists" to $\bigvee x.x = \tau$ and "τ is P" to $\bigvee x.x = \tau \wedge P(x)$). But they all suffer the same shortcoming: logic and ontology are conceptually distinct topics, but reference as a tool for giving meaning to formulae necessarily mixes them up. Only such names can be used to build meaningful formulae that denote some entity.[1]

An interesting class of more or less non-standard approaches to give meaning to singular terms is called *free logics*. The name is due to Lambert [13], [14] who first used it in 1960 for a system free of any existence assumptions about entities.[2] Bencivenga [1] gave one of the most fruitful definitions in his introduction to free logics:

> A free logic is a formal system of quantification theory [...] which allows for some singular terms in some circumstances to be thought of as denoting no existing object, and in which quantifiers are invariably thought of as having existential import.

Interestingly, Quine's [20] ontological criterion that "to be is to be the value of a bound variable" still holds; it even becomes the fundamental principle of most free logics. Existence is still part of our logic, expressed by the quantifiers: "everything" means "everything *that exists*", "anything" is "any *existing* thing". The crucial difference is that existence does not influence the truth or falsity of sentences containing singular terms that escape the scope of quantification in the same way as in Quine's point of view.

Many different free semantics have been presented, but basically they can be split into two main categories:

[1] *Cf.* Hintikka [12]. Héctor-Neri Castañeda [5] wrote about Plato-Meinong-like objects. Though he was mainly interested in ontology and not in logics, the formal nature of his style would be useful to develop non-free logical systems to argue about fictive or contradictory entities.

[2] Although free semantics had not yet been given birth to then and Lambert's concerns were purely syntactic this obviously was what he had in mind when designing his proof system. *Cf.* also Leonard [16].

1. In *outer domain* systems, reference still holds, but "God" may either denote an object from a domain of existents D_E or from a newly created domain of non-existents D_N. In some flavors of outer domain free logics, D_N is more complex and can itself contain more than one domain.
2. Systems based on modality are somewhat more sophisticated. Every singular term either denotes an existing entity or it does not denote at all; *there simply are no such things like non-existents*. To make a non-denoting singular term meaningful, models related to Kripke's possible worlds are used in one or the other way (*e.g.* by mapping formulae containing referential gaps to truth values using a convention like van Fraassen [8] did, or by directly evaluating the referential gaps in possible worlds like in Bencivenga [2]).

We will not go into the details of free logics based on reference nor will we discuss advantages and drawbacks (*cf.* Read [31], chapter 5). Instead, we will present a different approach that proposes a pragmatical view on the relations beween ontology and logic. We base this approach on *dialogues*.

1.2. *Dialogues*

Dialogical logic, suggested by Paul Lorenzen in 1958 and developed by Kuno Lorenz in several papers from 1961 onwards [18],[3] was introduced as a pragmatical semantics for both classical and intuitionistic logic.

The dialogical approach studies logic as an inherently pragmatic notion with help of an overtly externalised argumentation formulated as a *dialogue* between two parties taking up the roles of an *opponent* (*O* in the following) and a *proponent* (*P*) of the issue at stake, called the principal *thesis* of the dialogue.

P has to try to defend the thesis against all possible allowed criticism (*attacks*) of *O*, thereby being allowed to use statements that *O* may have made at the outset of the dialogue. The thesis *A* is logically valid if and only if *P* can succeed in defending *A* against all possible allowed criticism of the opponent. In the jargon of game theory: *P* has a *winning strategy* for *A*.

An interesting fact is that dialogues don't understand semantics as mapping names and relationships into the real world to obtain an abstract counterpart of it, but as acting upon them in a particular way.

[3] Further work has been done by Rahman [22] in his PhD thesis.

1.3. *Contents of this paper*

In this paper, we show how the ideas behind free logics can be captured with the dialogical approach to logic. In Section 2, we introduce an intuitionistic and a classical version of a very basic system called *DFL* that fits the definition of Bencivenga cited above. In Section 3, the notion of quantifiers of *DFL* is extended into several directions, some of them motivated by non-dialogical free semantics, others simply by the fact that dialogues open new perspectives on the topic.

2. *The core system*

This section introduces *DFL*, a simple system of dialogues that is *free* and *inclusive*. A logic is inclusive if it does not require any entity to exist at all (expressed in standard referential semantics, the domain of correspondence is allowed to be empty). But first, we need some formal definitions.

2.1. *Formulae*

A formula is a term generated according to the well-known standard conventions using constants (τ, σ,...), variables (x, y, z,...), predicates ($\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$,...), logical connectives ($\wedge, \vee, \rightarrow, \neg$) and quantifiers ($\bigwedge$ and $\bigvee$).

A formula is said to be atomic if it has the form $\mathcal{P}(\tau_1,...)$ for some predicate $\mathcal{P}$ and constants τ_i. Formulae that are not atomic are called complex. Atomic formulae are represented by small letters (a, b,...), formulae that might be complex by capitals (A, B,...).

2.2. *Dialogues again*

A dialogue is a sequence of labeled *formulae* that are stated by either *P* or *O*.[4] The label of a formula describes its role in the dialogue, whether it is an agressive or a defensive act. An *attack* is labeled with $?_{n/...}$, while $!_{n/...}$ tags a defense. (n is the number of the formula the attack or defense reacts on, the dots are sometimes completed with more information. The use of indices of labels will be made clear in the following.)

In dialogical logic the meaning in use of the logical particles is given by two types of rules which determine their *local (particle rules)* and their *global (structural rules)* meaning. The particle rules specify for each

[4]Sometimes, we use X and Y to denote P and O with $X \neq Y$.

particle a pair of moves consisting of an attack and (if possible) the corresponding defense. Each such pair is called a *round*. An attack *opens* a round, which in turn is *closed* by a defense if possible.

Before presenting a dialogical system *DFL* for free logics, we need the following definition.

- ► A constant τ is said to be *introduced by X* if (*i*) *X* states a formula $A[\tau/x]$ to defend $\bigvee(x)A$ or (*ii*) *X* attacks a formula $\bigwedge(x)A$ with $?_{n/\tau}$, and τ has not been used in either way before. Moreover, an atomic formula is said to be introduced by *X* if it is stated by *X* and has not been stated before.

2.3. *The system* DFL

DFL is closely related to Lorenz's standard dialogues for both intuitionistic and classical logic. The particle rules are identical, and the sets of structural rules differ in only one point, namely when fixing the way constants are dealt with.

Before we present the formal definition of *DFL*, we have a look at a simple propositional dialogue as an example for notational conventions:

O	*P*
	(0) $(a \wedge b) \rightarrow a$
(1) $?_0\ a \wedge b$	(4) $!_1\ a$
(3) $!_2\ a$	(2) $?_{1/\text{left}}$

P wins.

Formulae are labeled in (temporal) order of appearance. They are not listed in the order of utterance, but in a way that every defense appears on the same level as the corresponding attack.

Informally, the argument goes like this:

P: "If *a* and *b*, then *a*."

O: "Given *a* and *b*, show me that *a* holds."

P: "If you assume *a* and *b*, you should be able to show me that both hold. Thus show me that the left part holds."

O: "Ok, agreed: *a*."

P: "If you can say that *a* holds, so can I."

O runs out of arguments; *P* wins.

The particle rules are given in Fig. 1. The first row contains the form of the formula in question, the second one possible attacks to this formula, and the last one possible defenses to those attacks. (The symbol "⊗" indicates that no defense is possible.)

formula	attack	defense
$A \wedge B$	$?_{n/left}$	$!_m A$
	$?_{n/right}$	$!_m B$
$A \vee B$	$?_n$	$!_m A$
		$!_m B$
$\neg A$	$?_n A$	$\otimes$
$A \rightarrow B$	$?_n A$	$!_m B$
$\bigwedge(x)A$	$?_{n/\tau}$	$!_m A[x/\tau]$
$\bigvee(x)A$	$?_n$	$!_{m/\tau} A[x/\tau]$

Figure 1: Particle rules for *DFL* (orig. *Partikelregeln*).

Note that the symbol $?_{n/...}$ is a move —more precisely it is an attack— but not a formula. Thus if one partner in the dialogue states a conjunction, the other may initiate the attack by asking either for the left side of the conjunction ("show me that the left side of the conjunction holds", or $?_{n/left}$ for short) or the right one ("show me that the right side of the conjunction holds", or $?_{n/right}$). If one partner in the dialogue states a disjunction, the other may initiate the attack by requiring to be shown *any* side of the disjunction ($?_n$). As already mentioned, the number in the index denotes the formula the attack refers to. The notation of defenses is used in analogy to that of attacks. Rules for quantifiers work similarly.

Next, we fix the way formulae are sequenced to form dialogues with a set of structural rules (orig. *Rahmenregeln*).

(DFL0) Formulae are alternatingly uttered by *P* and *O*. The *initial formula* is uttered by *P*. It does not have a label, but provides the topic of argument. Every formula below the initial formula is either an attack or a defense to an earlier formula of the other player.

(DFL1) Both *P* and *O* may only make moves that change situation.[5]

[5]Fuhrmann [10] has used the same formulation. Intuitively, it replaces Lorenz's *Angriffsschranken*, but this point still remains to be made clear on a formal basis —for a precise formulation of this rule see Rahman/Rückert [28].

(DFL2) *(formal rule for atomic formulae)* P may not introduce atomic formulae: every atomic formula must be stated by O first.

(DFL3) *(formal rule for constants)* Only O may introduce constants.

(DFL4) *(winning rule)* X wins iff it is Y's turn but he cannot move (either attack or defend).

(DFL_I5) *(intuitionistic rule)* In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against *the last not already defended* attack.

DFL is an intuitionistic as well as a classical semantics. To obtain the classical version simply replace (DFL_I5) by the following rule:

(DFL_C5) *(classical rule)* In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against *any attack* (including already defended) of his partner.[6]

If we need to make explicit which system is meant, we write *DFLI* or *DFLC* instead of *DFL*.

A *DFL* dialogue is finite, since the particle rules satisfy the subterm property and (DFL1) ensures that no player may enter a loop iterating a finite sequence of arguments infinitely often.

The crucial rule that makes *DFL* behave like a free logic is (DFL3). To see the difference between standard and free dialogues (those with and those without (DFL3)), consider another example. Without (DFL3), we would obtain the following dialogue proving that if nothing is a vampire, Nosferatu is no vampire:

O	P
	(0) $\bigwedge(x)\neg\mathcal{P}(x) \rightarrow \neg\mathcal{P}(\tau)$
(1) $?_0 \bigwedge(x)\neg\mathcal{P}(x)$	(2) $!_1\ \neg\mathcal{P}(\tau)$
(3) $?_2\ \mathcal{P}(\tau)$	$\otimes$
(5) $!_4\ \neg\mathcal{P}(\tau)$	(4) $?_{1/\tau}$
$\otimes$	(6) $?_5\ \mathcal{P}(\tau)$

P wins.

[6]This rule is actually redundant, but we keep it to make the point explicit.

If we play the same dialogue again in *DFL*, things look different:

O	*P*
	(0) $\bigwedge(x)\neg\mathcal{P}(x) \rightarrow \neg\mathcal{P}(\tau)$
(1) $?_0 \bigwedge(x)\neg\mathcal{P}(x)$	(2) $!_1\ \neg\mathcal{P}(\tau)$
(3) $?_2\ \mathcal{P}(\tau)$	$\otimes$

O wins.

We observe that *P* runs out of arguments. He cannot attack (1) any more, because no single constant has been introduced so far, and he may not introduce one on its own. He also cannot defend himself against the atomic formula in (3) due to the particle rule for negation.

It is easy to see that *DFL* fits Bencivenga's definition. It is a formal system containing quantifiers, and existence can be defined according to Quine's dictum: "God exists" is formalized as $\bigvee x.x$ is God. Obviously, if we enter the denotational point of view at all, constants may be thought of as sometimes not denoting an existing entity.

To conclude this section, we would like to point out an interesting consequence of the inclusiveness of *DFL*. The rules do not allow closed formulae to be valid if they occur in an existential quantification (in the empty universe they would not be true for a single object):

O	*P*
(1) $?_0$	(0) $\bigvee(x)(\mathcal{P}(\tau) \rightarrow \mathcal{P}(\tau))$

O wins.

According to the rules, *P* has to use a constant to defend (0). Since no such constant has been introduced so far, he cannot move.

2.4. *Winning strategies and dialogical tableaux for* DFL

As already mentioned validity is defined in dialogical logic via winning strategies for *P*, *i.e.* the thesis *A* is logically valid iff *P* can succeed in defending *A* against all possible allowed criticism of *O*. In this case, *P* has a *winning strategy* for *A*.

A systematic description of the winning strategies available can be obtained from the following considerations:

- ► If *P* shall win against any choice of *O*, we will have to consider two main different situations, namely the dialogical situations in which O has stated a complex formula and those in which P has stated a complex formula. We call these main situations the O-cases and the P-cases, respectively.

In both of these situations another distinction has to be examined:

1. P wins by *choosing* an attack in the O-cases or a defense in the P-cases, iff he can win *at least one* of the dialogues he has chosen.
2. When O can *choose* a defense in the O-cases or an attack in the P-cases, P can win iff he can win *all of the* dialogues O can choose.

The closing rules for dialogical tableaux are the usual ones: a branch is closed iff it contains two copies of the same formula, one stated by O and the other one by P. A tree is closed iff each branch is closed. A closed tree for some formula A presents a winning strategy for A.

For the intuitionistic tableaux, the structural rule about the restriction on defenses has to be considered. The idea is quite simple: the tableaux system allows all the possible defenses (even the atomic ones) to be written down, but as soon as determinate formulae (negations, conditionals, universal quantifiers) of P are attacked all others will be deleted. Those formulae which compel the rest of P's formulae to be deleted will be indicated with the expression "$[O]_{[O]}$" (or "$[P]_{[O]}$") which reads *save O's formulae and delete all of P's formulae stated before*.

To obtain free tableaux from those described above, add the following restriction to the closing rules and recall the rule (DFL3) for constants in Subsection 2.3:

► *DFL*-restriction
Check that for every step in which P chooses a constant (*i.e.* for every P-attack on a universally quantified O-formula and for every P-defense of an existentially quantified P-formula) this constant has been already introduced by O (by means of an O-attack on a universally quantified P-formula or a defense of an existentially quantified O-formula).

This restriction can be technically implemented by a device which provides a label (namely a star) for each constant introduced by O. Thus, the *DFL*-restriction can be simplified in the following way:

► *DFL*-restriction with labels
Check that for every step in which P chooses a constant this constant has already been there labeled with a star.

All these considerations can be expressed by means of the tableaux systems for classical and intuitionistic *DFL* (see Fig. 2 for *DFLC*-T and Fig. 3 for *DFLI*-T).[7]

$[O]$-cases	$[P]$-cases
$\dfrac{[O]A \vee B}{\langle[P]?\rangle[O]A \;\Vert\; \langle[P]?\rangle[O]B}$	$\dfrac{[P]A \vee B}{\langle[O]?\rangle[P]A}$ $\langle[O]?\rangle[P]B$
$\dfrac{[O]A \wedge B}{\langle[P]?_{\text{left}}\rangle[O]A}$ $\langle[P]?_{\text{right}}\rangle[O]B$	$\dfrac{[P]A \wedge B}{\langle[O]?_{\text{left}}\rangle[P]A \;\Vert\; \langle[O]?_{\text{right}}\rangle[P]B}$
$\dfrac{[O]A \to B}{[P]A,\ldots \;\Vert\; \langle[P]?A\rangle[O]B}$	$\dfrac{[P]A \to B}{[O]A,}$ $[P]B$
$\dfrac{[O]\neg A}{[P]A,\otimes}$	$\dfrac{[P]\neg A}{[O]A,\otimes}$
$\dfrac{[O]\bigwedge(x)A}{\langle[P]?\tau\rangle[O]A[\tau/x]}$ τ has been labeled with a star before.	$\dfrac{[P]\bigwedge(x)A}{\langle[O]?\tau^{*}\rangle[P]A[\tau/x]}$ τ is a new constant.
$\dfrac{[O]\bigvee(x)A}{\langle[P]?\rangle[O]A[\tau^{*}/x]}$ τ is a new constant.	$\dfrac{[P]\bigvee(x)A}{\langle[O]?\rangle[P]A[\tau/x]}$ τ has been labeled with a star before.

Figure 2: Rules for classical *DFL*-tableaux. Observe that the formulae below the line represent pairs of attack-defense moves, *i.e.* they represent rounds. Also note that the expressions between the symbols ⟨ and ⟩, such as ⟨[**P**]?⟩ or ⟨[**O**]?A⟩ are moves —more precisely they are attacks— but not statements.

[7]See details on how to build the tableaux systems from the above considerations in Rahman [22] and Rahman and Rückert [27]. The use of these tableaux systems follows the very well known analytic trees of Raymund Smullyan [34]. *Cf.* also Felscher [7].

[O]-cases	[P]-cases
$\dfrac{[O]A \vee B}{\langle[P]?\rangle[O]A \;\;\|\;\; \langle[P]?\rangle[O]B}$	$\dfrac{[P]A \vee B}{\langle[O]?\rangle[P]A}$ $\langle[O]?\rangle[P]B$
$\dfrac{[O]A \wedge B}{\left\langle[P]?_{\text{left}}\right\rangle[O]A}$ $\left\langle[P]?_{\text{right}}\right\rangle[O]B$	$\dfrac{[P]A \wedge B}{\left\langle[O]?_{\text{left}}\right\rangle[P]A \;\;\|\;\; \left\langle[O]?_{\text{right}}\right\rangle[P]B}$
$\dfrac{[O]A \to B}{[P]A,\ldots \;\;\|\;\; \langle[P]?A\rangle[O]B}$	$\dfrac{[P]A \to B}{[O]_{[O]}A,}$ $[P]B$
$\dfrac{[O]\neg A}{[P]A,\otimes}$	$\dfrac{[P]\neg A}{[O]_{[O]}A,\otimes}$
$\dfrac{[O]\bigwedge(x)A}{\langle[P]?\tau\rangle[O]A[\tau/x]}$ τ has been labeled with a star before.	$\dfrac{[P]\bigwedge(x)A}{\left\langle[O]?\tau^{*}\right\rangle[P]_{[O]}A[\tau/x]}$ τ is a new constant.
$\dfrac{[O]\bigvee(x)A}{\langle[P]?\rangle[O]A[\tau^{*}/x]}$ τ is a new constant.	$\dfrac{[P]\bigvee(x)A}{\langle[O]?\rangle[P]A[\tau/x]}$ τ has been labeled with a star before.

Figure 3: Rules for intuitionistic *DFL*-tableaux.

Let us look at two examples, namely one for *DFLC*-T and one for *DFLI*-T, where we will use again the notation introduced in Subsection 2.3 for keeping track of the moves in a dialogue. First, we run the classical tableau for $\bigwedge(x)\neg\mathcal{P}(x) \to \neg\mathcal{P}(\tau)$.

$[P]$	$\bigwedge(x)\neg\mathcal{P}(x) \to \neg\mathcal{P}(\tau)$	(1)
$[O]$	$\bigwedge(x)\neg\mathcal{P}(x)$	(2)
$[P]$	$\neg\mathcal{P}(\tau)$	(3)
$[O]$	$\mathcal{P}(\tau)$	(4)

The tree remains open.

The tableau remains open for P cannot choose τ to attack the universal quantifier of O. The following *DFLI*-T strategy is slightly more complex.

$$
\begin{array}{lll}
[P] & \bigwedge(x)\mathcal{P}(x) \rightarrow \neg\bigvee(x)\neg\mathcal{P}(x) & (1) \\
[O]_{[O]} & \bigwedge(x)\mathcal{P}(x) & (2) \\
[P] & \neg\bigvee(x)\neg\mathcal{P}(x) & (3) \\
[O]_{[O]} & \bigvee(x)\neg\mathcal{P}(x) & (4) \\
[O] & \neg\mathcal{P}(\tau^{*}) & (5) \\
[O] & \mathcal{P}(\tau) & (6) \\
[P] & \mathcal{P}(\tau) & (7)
\end{array}
$$

The tree closes.

3. *Extensions*

In this section, we will analyze several extensions to *DFL* that target the modification of the set of quantifiers.

3.1. DFL *with four quantifiers* (DFL^4)

Church [6] proposed a semantics to prove free logics useless that, ironically, turned out to be one of the very first free semantics instead. It is based on the assumption that existence is nothing more than an ordinary predicate, say **E!**. The use of **E!** in combination with the standard (non-free) quantifiers yields a logic that looks very much like *DFL*.

Formally, take $\forall$ and $\exists$ as the quantifiers well-known from standard classical logic, and define two more quantifiers $\bigwedge$ and $\bigvee$ as follows.

$$
\begin{array}{lll}
\bigwedge(x)A & \equiv_{def} & \forall(x)(\mathbf{E!}x \rightarrow A) \quad (1) \\
\bigvee(x)A & \equiv_{def} & \exists(x)(\mathbf{E!}x \wedge A) \quad (2)
\end{array}
$$

As Leblanc and Thomason [15] (and others at the same time) have pointed out this leads directly to a semantics with two domains, one matching the scope of the existential predicate **E!** (the *inner domain*) and one containing all entities, existing or not (the *outer domain*).

Of course we cannot use definitions like (1) and (2) to describe a system of dialogues.[8] A new system DFL^4 with two pairs of quantifiers, one with

[8] Instead, in dialogical logics, semantics is always designed by a set of appropriate particle and structural rules.

existential import ($\bigwedge$ and $\bigvee$)[9] and one without ($\forall$ and $\exists$), is obtained from *DFL* by adding the following two particle rules:

formula	attack	defense
$\forall(x)A$	$?_{n/\tau}$	$!_m\, A[\tau/x]$
$\exists(x)A$	$?_n$	$!_{m/\tau} A[\tau/x]$

3.2. *Many quantifiers (*DFL^n *and* DFL$^{\langle n\rangle}$*)*

Consider the situation expressed by the following proposition:

The novel contains a passage in which Sherlock Holmes dreams that he shot Dr. Watson.

There is an underlying reality that the novel is part of, the outer reality of the story told in the novel, and an even outer reality of the dream of the protagonist. To formalize this situation in *DFL*^{4} we run out of quantification classes. To distinguish between the reality of Conan Doyle writing stories, Holmes's reality and the reality of Holmes's dream, we need three pairs of quantifiers expressing the three levels of reality.[10]

The solution is simple. Think of the pair of quantifiers of *DFL* as having upper index 0 and add new pairs of quantifiers with higher indices, as many as we need to express every level of reality (or fiction) that possibly could appear.

We call the so derived dialogical logic *DFL*^{n}. The new particle rules to be added to *DFL* are:

formula	attack	defense
$\bigwedge^i(x)A$	$?_{n/\tau}$	$!_m\, A[\tau/x]$
$\bigvee^i(x)A$	$?_n$	$!_{m/\tau} A[\tau/x]$

[9] Note that $\bigwedge$ and $\bigvee$ are used in *DFL*^{4} in exactly the same way as in *DFL*.

[10] Hugh MacColl (1837–1909), the father of formal non-classical logic, developed a system of logic which contains the seeds of a many sorted free logic that is somewhat related to the idea of outer domains, though expressed in different terms [19]. He distinguishes objects that have a meaning independent of discourse, the *real existents*, and the *non-existents* that are of symbolic nature and escape the ontological implications real entities carry with them. See Rahman [23], [24] and [25] for a detailed discussion about the subject.

The extended set of quantifiers requires a new notion of introduction:

- ► A constant τ is said to be *introduced at level i* iff it is used to attack a universal quantifier of level i or to defend an existential quantifier of level i and has not been used in the same way before.

We adapt (DFL3) to DFL^n:

(DFL^{n}3) *(first extended formal rule for constants)* At each level of quantification constants may only be introduced by O.

These formulations yield a logic containing a pair of standard quantifiers and arbitrary many more disjunct pairs of quantifiers dealing with different sorts of reality and fiction. In some contexts, it might be useful to have a logic where these different realities are ordered in a hierarchy. We call the system that establishes this ordering $DFL^{\langle n\rangle}$; it results from modifying (DFL3) again:

(DFL$^{\langle n\rangle}$3) *(second extended formal rule for constants)* P may introduce a constant τ on a level m iff O has introduced τ on some level n with $n < m$ before.

Consider two further examples. The first states that in DFL^n, whenever $\mathcal{P}$ has an instance in the scope of one or another $\bigvee$-quantifier, it has an instance in the scope of $\exists$; the second makes use of the ordering in $DFL^{\langle n\rangle}$.

O	P
	(0) $(\bigvee^1(x)\mathcal{P}(x) \vee \bigvee^2(x)\mathcal{P}(x)) \rightarrow \exists(x)\mathcal{P}(x)$
(1) $?_0\ \bigvee^1(x)\mathcal{P}(x) \vee \bigvee^2(x)\mathcal{P}(x)$	(6) $!_1\ \exists(x)\mathcal{P}(x)$
(3) $!_2 \bigvee^1(x)\mathcal{P}(x)$ $[?_2 \bigvee^2(x)\mathcal{P}(x)]$	(2) $?_1$
(5) $!_4\ \mathcal{P}(\tau)$	(4) $?_3$
(7) $?_6$	(8) $!_7\ \mathcal{P}(\tau)$

P wins.

	O		P
		(0)	$(\bigvee^1(x)\mathcal{P}(x) \wedge\bigwedge^2(x)(\mathcal{P}(x)\rightarrow\mathcal{Q}(x))) \rightarrow\bigvee^1(x)\mathcal{Q}(x)$
(1)	$?_0\ \bigvee^1(x)\mathcal{P}(x) \wedge\bigwedge^2(x)(\mathcal{P}(x)\rightarrow\mathcal{Q}(x))$	(2)	$!_1\ \bigvee^1(x)\mathcal{Q}(x)$
(3)	$?_2$	(14)	$!_3\ \mathcal{Q}(\tau)$
(5)	$!_4\ \bigwedge^2(x)(\mathcal{P}(x)\rightarrow\mathcal{Q}(x))$	(4)	$?_{1/\text{right}}$
(7)	$!_6\ \bigvee^1(x)\mathcal{P}(x)$	(6)	$?_{1/\text{left}}$
(9)	$!_8\ \mathcal{P}(\tau)$	(8)	$?_7$
(11)	$!_{10}\ \mathcal{P}(\tau)\rightarrow\mathcal{Q}(\tau)$	(10)	$!_{5/\tau}$
(13)	$!_{12}\ \mathcal{Q}(\tau)$	(12)	$?_{11}\ \mathcal{P}(\tau)$

P wins.

Obviously, there are formulae valid in $DFL^{\langle n\rangle}$ that are not valid in DFL^n. This example makes use of the fact that if some formula holds on a lower level of quantification, it may be used on a higher level as well.

4. *Related work*

Bencivenga [1] wrote a very illuminating introduction to referential free logics based on reference, including some involving modality or more exotic ideas. Many of his older articles contain the theories summarized there (*cf.* [3], [4]).

This article is one of a series based on the seminar "Erweiterungen der Dialogischen Logik" ("extensions to dialogical logic") held in Saarbrücken in summer 1998 by Shahid Rahman and Helge Rückert. The same seminar has motivated the publication of *On Frege's Nightmare* [24], and *Ways of Understanding Hugh MacColl's Concept of Symbolic Existence* [25], by Rahman, *The Dialogical Approach to Paraconsistency* by Rahman and Carnielli [26], *Dialogische Modallogik für T, B, S4 und S5* [28], *Dialogische Logik und Relevanz* [29], *Dialogical Connexive Logic* [30] by Rahman and Rückert, and *Why Dialogical Logic* [32] by H. Rückert.

5. *Conclusion and open problems*

In this paper several dialogical systems for free logics have been developed. These systems open a new approach to the problem of ontological presuppositions in logical argumentations.

A purely referential approach with the same goal has been proposed by van Fraassen [8], [9]. His idea is to ignore the fact that certain terms in first order quantified logics do not denote, and takes atomic formulae that lack a truth value to be either true or false, based on some arbitrary propositional convention. An important difference between this so called *supervaluational semantics* and *DFL* is that *DFL* is inclusive while supervaluations are exclusive. It would be interesting to analyze how *P* could be allowed to introduce constants in a controlled manner (making *DFL* exclusive) and prove an equivalence theorem that connects free *dialogical* logics with van Fraassen's ideas.

Definite descriptions have always been a major motivation for free logics. Whether the dialogical approach to free logics offers a new understanding of the problems involved remains to be examined.[11]

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