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Jérôme Pouyet, Thomas Trégouët

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PARIS-JOURDAN SCIENCES ECONOMIQUES
48, BD JOURDAN – E.N.S. – 75014 PARIS
TÉL. : 33(0) 1 43 13 63 00 – FAX : 33 (0) 1 43 13 63 10
www.pse.ens.fr

# Vertical Mergers in Platform Markets*

Jérôme Pouyet† Thomas Trégouët‡

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**Abstract**

We analyze the competitive impact of vertical integration between a platform and a manufacturer when platforms provide operating systems for devices sold by manufacturers to customers, and, customers care about the applications developed for the operating systems. Two-sided network effects between customers and developers create strategic substitutability between manufacturers' prices.

When it brings efficiency gains, vertical integration increases consumer surplus, is not profitable when network effects are strong, and, benefits the non-integrated manufacturer. When developers bear a cost to make their applications available on a platform, manufacturers boost the participation of developers by affiliating with the same platform. This creates some market power for the integrated firm and vertical integration then harms consumers, is always profitable, and, leads to foreclosure. Introducing developer fees highlights that not only the level, but also the structure of indirect network effects matter for the competitive analysis.

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## 1. Introduction

Motivation. Software platform industries have recently witnessed many sudden changes in the nature of the relationship between software and hardware producers. A prime example is Google's venture in the smartphone markets, which started six years ago. Google initially maintained arm's length relationships with several smartphone hardware producers to build the Nexus range, even after the acquisition of Motorola. Perhaps not surprisingly, that acquisition has had the side-effect of making relationship with the

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†Paris School of Economics (CNRS & ENS). E-mail: jerome.pouyet@ens.fr. Address: PSE, 48 boulevard Jourdan, 75014 Paris, France.

‡Université de Cergy-Pontoise, THEMA. E-mail: thomas.tregouet@u-cergy.fr. Address: Thema – Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy Pontoise Cedex, France.

main licensees of its Android mobile operating system increasingly strained.[1] It certainly came as a greater surprise that Motorola was sold back only three years afterwards, in 2014, and many observers believed that more integration with hardware makers was not such a smart strategy for software platforms. As of October 2016, praising the synergies associated with the so-called full-stack (that is, integrated) model, Google announced the launch of its Pixel, the first smartphone entirely conceptualized and engineered in-house, which will benefit from exclusive Google's technologies and be the first to boast the new Android operating system. Rumors are now rampant that Samsung, subjugated to Google for the use of its Android platform but delivering Google substantial money through services installed on its phones, may move all of its devices with its own Tizen operating system, despite the problem of getting enough traction from application developers.

The issues we are interested in this paper can be stated as follows: In platform markets, what are the competitive effects of vertical integration and the risks of foreclosure? And, how direct or indirect network effects, which are endemic in these industries, ought to be considered in the analysis? In short, we show that indirect network effects affect the competitive analysis of vertical integration by changing the nature of the strategic interaction at the downstream level. As a result, vertical integration is likely to enhance consumer surplus, is not privately profitable when these effects are strong, and, the issue of foreclosure is often moot in this context, except when there are gains for manufacturers to coordinate on the same platform (a situation which arises when developers bear a cost to port their application on each platform or when there are direct network effects on one side of the market) or when product market competition dominates network effects (in a sense defined more precisely later on).

Before detailing our analysis, it is worth reminding some of the key elements of the standard (that is, absent network effects) competitive analysis of vertical integration.[2] Vertical integration has pro-competitive effects, through the removal of a double marginalization or the creation of synergies between the merging parties. But vertical integration also affects competition on the product market as well as the pricing on the upstream market and may soften downstream competition, raise wholesale prices and lead to foreclosure of the non-integrated competitors. The overall effect on prices, and thus on consumer surplus, is then a priori ambiguous. Key to these results is the fact that downstream competitors' prices are strategic complements, so that when the integrated firm raises its downstream price or when it increases its upstream price, competition on the downstream market ends up being softened.

Network Effects and Downstream Strategic Interaction. Our first contribution is to show that network effects make the manufacturers' downstream prices strategic substitutes. To highlight this mechanism in the simplest way, we start with the

[1] At the time of Motorola's acquisition, some experts argued that Google's primary objective was to strengthen its patents portfolio; many now retrospectively think that this also was a test of the feasibility of a more integrated business model.

[2] We focus on the literature that determines circumstances under which vertical integration creates some market power, and, thus, may lead both to soften downstream competition at the expense of final customers and to harmful foreclosure of non-integrated competitors. Another strand, following Hart and Tirole (1990), shows that vertical integration may be used as a way not to create but to restore the upstream market power, which was eroded by a lack of commitment. See Rey and Tirole (2007) and Riordan (2008) for surveys.

following base model. Two platforms compete to offer operating systems to two downstream manufacturers, which then sell devices to final customers. A device gives buyers access to the applications developed for that platform. Accordingly consumers care not only about the devices' prices but also about the number of developers joining a platform. Two assumptions are made: application developers can join a platform for free and care only about the total number of customers; manufacturers are local monopolists in the downstream market, that is, there is no direct product market competition between manufacturers.

In that context, a manufacturer's demand increases following a decrease in the price of the other manufacturer: intuitively, when one manufacturer reduces its price, more customers buy that device and the platform becomes more attractive for developers; since developers can join any platform at no cost, this boosts the demand faced by the other manufacturer, which then increases its price. Two-sided network externalities generate a form of demand complementarity between manufacturers, which makes manufacturers' prices strategic substitutes.[3]

IMPACT OF VERTICAL INTEGRATION. We start with our base model, in which one platform is more efficient than the other, or, alternatively, vertical integration creates merger-specific synergies. The competitive effect of a vertical merger between the efficient platform and a manufacturer can be decomposed by means of three channels. First, an 'efficiency effect': a vertical merger eliminates a double marginalization between the platform and the manufacturer. Second, an 'accommodation effect': the integrated firm reduces its price so as to boost the profit earned from selling its operating system to the non-integrated firm. Last, an 'upstream market power effect': because the integrated firm is more accommodating on the downstream market, the non-integrated firm is more willing to buy its operating system; this allows in turn the integrated firm to raise the royalty for its operating system. Overall, thanks to the strategic substitutability between downstream prices, a vertical merger leads unambiguously to a decrease of the integrated manufacturer's price and an increase of the non-integrated manufacturer's price.

We show then that vertical integration benefits the non-integrated manufacturer. Indeed, a manufacturer always gains when the other manufacturer becomes more efficient, for the latter then sets a lower retail price, which boosts the number of applications, and, allows the former to enjoy a higher demand from its buyers. The royalty paid by the non-integrated manufacturer increases, however, but that increase is constrained by the outside option of buying the less efficient platform's operating system. Overall, our analysis shows that vertical integration leads to an increase in the non-integrated manufacturer's profit. In that sense, foreclosure concerns are rather moot: while vertical integration may well lead to a higher royalty, it does not adversely impact the non-integrated manufacturer with respect to the pre-merger situation.

Vertical integration is however not always privately profitable, even when it creates synergies. For a given price set by the non-integrated manufacturer, vertical integration always benefits the merging parties, for it removes a double marginalization and allow

[3] In a broad sense, that the "two-sidedness" nature of a market can change the nature of strategic interactions has appeared in other contexts. For instance, in media markets, Reisinger et al. (2009) show that advertising levels can be either strategic complements or substitutes. Amelio and Jullien (2012) make a similar observation in the context of tying in two-sided markets.

a better alignment of incentives along the value chain. Strategic substitutability, however, generates an adverse strategic response by the non-integrated firm, which raises its price following the merger. When network effects are sufficiently strong, that strategic response makes the merger unprofitable in the first place. From a managerial perspective, our model predicts that in platform markets with strong indirect network externalities, platforms may well prefer to remain at arm's length relationship with their downstream manufacturers.

Fragmentation of Operating Systems. Once an application is developed, a developer has to bear some costs to make it available on a specific platform or on a specific operating system. These costs lead to the phenomenon of 'fragmentation', according to which the sometimes-prohibitive costs to port an application on different operating systems lead to the scattering of developers across competing platforms. These 'costs to port' imply a first departure of our base model, for by coordinating on the same platform manufacturers implicitly reduce the developers' total cost and increase the number of available applications.[4] Our analysis proceeds then by assuming that platforms are symmetric but that manufacturers strictly gain from coordinating on the same platform.

Intuitively, such motives for coordination between manufacturers reinforce a platform's market power, for some gains are lost if manufacturers do not affiliate with the same platform. When platforms are symmetric, and thus compete fiercely to license their operating systems, these gains end up being fully pocketed by the manufacturers. Integration, however, forces coordination on the integrated platform, thereby creating some upstream market power. Exactly as in the base model, the royalty paid by the non-integrated manufacturer increases above the pre-merger level, as a way for the integrated firm to extract some of the coordination gains from the non-integrated manufacturer. The crucial difference is, however, that the non-integrated manufacturer is now harmfully foreclosed, for its outside option of not joining the integrated platform lies below its pre-merger profit level because of the positive motive for coordination. Therefore, with motives for coordination the usual foreclosure concerns are reinstated.

Our analysis has argued so far that, as far as foreclosure is concerned, antitrust authorities should pay attention to the source of the integrated firm's market power. Interestingly, whether buyers gain or lose from vertical integration follows a similar pattern.

Consumer Surplus. The analysis of consumer surplus is, however, made complicated for two reasons. First, vertical integration generates a price decrease in the market where the merger occurs and a price increase in the other market. The impact on total consumer surplus will thus depend on the relative variation of downstream prices. Second, prices affect the participation of developers and, thus, the surplus of consumers through the indirect network effects. We first find a sufficient condition on the variation of prices consecutive to the merger under which developers' participation and total surplus increase. That condition is then used in a linear specification of our model to show that, at equilibrium, consumer surplus and developers' participation indeed increase following

[4]The base model neutralizes these motives for coordination since the choice of platform by a manufacturer did change neither the participation of developers (who could freely join the platforms and thus access to all buyers whatever the manufacturers' choice of operating system) nor the competition on downstream markets (because manufacturers are local monopolists).

vertical integration in our base model with efficiency gains; with motives for coordination between manufacturers and no efficiency effect, this result is reversed, however.

Product Market Competition. Product market competition between manufacturers on top of network effects is then added to our base model. As is usual in markets without network effects, price competition with differentiated products makes prices strategic complements; but, as we have shown in our base model, two-sided network effects push towards strategic substitutability. Hence, when product market competition is weak relative to network effects, downstream prices remain strategic substitutes and our analysis carries over; when product market competition is strong relative to network effects, prices become strategic complements and we find the results of the extant literature. From an antitrust perspective, everything happens as if indirect network effects 'scale down' the intensity of product market competition at the retail level. This suggests to adopt a more lenient stance vis-à-vis vertical integration than in standard markets, and all the more so that indirect network effects are strong.

A slightly different setting, which fits the motivating example given at the beginning of the introduction, is then analyzed: one platform has a dominant position on the market for operating systems and decides to launch its own smartphone, which will compete directly with the device of the manufacturer using the platform's operating system. The key difference is that the introduction of the new product is also a source of demand creation. Whether such a downstream expansion by the dominant platform generates harmful foreclosure or decreases consumer surplus depends, again, on the nature of the strategic interaction between downstream prices.

Fees on Applications Developers. Our last extension considers introducing fees that developers have to pay to join the platforms. When manufacturers join different platforms, developers have to pay each platform's fee to access all the buyer. By contrast, when manufacturers join the same platform, developers have to pay that platform's fee only to access all buyers. Put differently, developer fees create a horizontal double marginalization on the developer side, which is alleviated when manufacturers affiliate with the same platform: an endogenous motive for coordination, which may be a source of upstream market power.

With no efficiency gains and symmetric platforms, that market power can be used by the integrated firm to raise the developer fee above its pre-merger level, but not the royalty. The impact of integration on the non-integrated manufacturer and on consumer surplus is ambiguous for two reasons: setting a positive developer fee deprives the buyers' demand and the non-integrated firm's profit; however, the integrated firm then adopts an accommodating behavior on its downstream market to protect the revenues it earns from developers. We show that the impact of vertical integration depends on the structure of the indirect network effects (that is, roughly speaking, whether buyers value more the participation of developers than the reverse), rather than on their level.

Related literature. To the best of our knowledge, our paper is the first to link, on the one hand, the literature on two-sided markets, and, on the other hand, the literature on vertical relations in the specific context of platforms-manufacturers relationships.

From the literature on two-sided markets, we borrow the general insight that indirect network effects are key to understanding the platform pricing and competition (Armstrong, 2006, Rochet and Tirole, 2006, Armstrong and Wright, 2007, Weyl, 2010). That literature has considered more recently the effect of exclusive dealing between a platform and content providers (that is, developers in our model): Evans (2013) discusses the antitrust of such vertical relations in platform industries; Doganoglu and Wright (2010) and Hagiu and Lee (2011) provide a rationale for why platforms sign exclusive contract with content providers; Church and Gandal (2000) describe the incentives of a manufacturer that is integrated with a developer to make its applications compatible with the hardware of a rival manufacturer; Hagiu and Spulber (2013) show that investment in first-party content (that is, vertical integration with one side of the market) depends on whether a platform faces a "chicken-and-egg" coordination problem; In the videogame industry, Lee (2013) finds that exclusivity tends to be pro-competitive, in that it benefits more to an entrant platform than to an incumbent platform. While we share with these papers the issue of the competitive impact of vertical restraints in a two-sided market, our work also differs substantially, for we are interested in the vertical relations between platforms/operating systems and manufacturers when devices are an essential link to connect buyers and developers.

Our analysis also belongs to the strategic approach to foreclosure theory initiated by Ordover et al. (1990). A message conveyed by that literature is that vertical integration can lead to input foreclosure and be detrimental to consumer surplus. Analyses that feature trade-offs between the pro- and the anti-competitive effects of vertical integration include: Ordover et al. (1990) and Reiffen (1992) when integration generates an extra commitment power; Riordan (2008) and Loertscher and Reisinger (2014) when the integrated firm is dominant; Chen (2001) when manufacturers have switching costs; Choi and Yi (2000) when upstream suppliers can choose the specification of their inputs; Chen and Riordan (2007) when exclusive dealing can be used in combination with integration; Nocke and White (2007) and Normann (2009) when upstream suppliers tacitely collude; Hombert et al. (2016) when there are more manufacturers than upstream suppliers.[5] In the context of platform markets, we bring several new insights: first, indirect network effects change the nature of the strategic interaction in the downstream market and are pro-competitive; second, foreclosure may nevertheless emerge if manufacturers have incentives to coordinate on the same platform; third, direct network effects and indirect ones have different implications in terms of foreclosure; fourth, the structure of indirect network effects play a critical role with developer fee.

Organization of the Paper. Section 2 gives a quick overview of the smartphone industry, putting the emphasis on the relations between operating systems and manufacturers of devices. Section 3 describes the model. Section 4 characterizes the impact of vertical integration on manufacturers' prices and profits. Section 5 shows that, in platform markets, several characteristic features provide manufacturers with the incentives to join the same platform and derives the implications of such a motive for coordination. Section 6 studies the impact of vertical integration on consumer surplus. Section 7 discusses the role of product market competition and how it should be combined with network effects in competition analyses. Section 8 analyzes the impact of vertical merger when platforms charge fees to developers. Section 9 discusses briefly several other extensions. Section 10 concludes. All proofs are relegated to the Appendix.

[5]For empirical analyses, see, e.g., Lafontaine and Slade (2007) and Crawford et al. (2016) and the references therein.

## 2. A Quick Background on the Smartphone Industry

As of 2016, the smartphone industry is dominated by two software platforms: market shares are 84% for Google Android, 15% for Apple iOS and 1% for the other platforms (Windows Phone, Blackberry, etc.). Google and Apple have different business models: Android is an open-source platform that can be installed by any manufacturers willing to do so; Apple is fully integrated and does not license its operating system. Google makes profits through ads displayed on Android phones and from its mobile applications store (Google Play). It also collects user data from a set of applications: Google Search, Google Maps, etc. Though Android is an open-source platform, manufacturers can use the Android brand and the core applications developed by Google (Google Play, Google Search, Google Maps, etc.) only if they sign a Mobile Application Distribution Agreement (MADA). A MADA requires that a manufacturer makes its device "compatible" with Android (that is, a device must satisfies minimum requirements established by Google) and that all Google applications are installed and placed prominently (that is, not far from the default home screen). This makes the manufacturers' applications and application stores less visible to the end users. Accordingly, a MADA determines how the stream of revenue from the purchasing of applications and the monetization of users data (through advertising for instance) is shared between Google and the manufacturer.[6,7]

The top Android devices' manufacturers are, in order of market share, Samsung, Huawei, Oppo and LG. These manufacturers produce a number of new devices each year, ranging from high-end expensive "flagship" smartphones to low-end cheap smartphones. Though they all use Android, manufacturers often add an in-house user interface (Touchwiz for Samsung, Emotion UI for Huawei, etc.). This allows a manufacturer to differentiate its products from its competitors and to promote its own services and applications. Until recently, Google did not engineered smartphones for Android.

It is worth noting that the major manufacturers have developed or are in the process of developing their own mobile operation systems: Samsung's Tizen is already installed on a handful of devices (TV, watches, etc.); rumors indicate that Huawei is currently working on its own operating system with former developers from Nokia; in 2013, LG bought a license of WebOS from HP; Amazon developed a non-compatible "fork" of Android named FireOS; etc. In addition, there are alternative licensable mobile operating systems: Microsoft sells licenses of Windows 10 Mobile; Firefox OS (whose development has been abandoned recently); various forks of Android (Aliyun OS, Cyanogen, etc.). Manufacturers are however reluctant to launch smartphones with operating systems non compatible with Android, for they will lose the benefits of the thousands of applications available on the Google Play Store. In addition, manufacturers that offer devices on which Google applications are installed sign an anti-fragmentation agreement which prevents them for selling devices non compatible with Android.[8]

[6] Choi and Jeon (2016) study how a platform can leverage some market power through such tying agreement. See also Edelman (2015).

[7] The MADAs are confidential. However, the agreements between HTC and Samsung have been made available to the public during a litigation in 2014. See "Secret Ties in Google's "Open" Android," http://www.benedelman.org/news/021314-1.html

[8] In 2012, Acer attempted to launch a smartphone with Aliyun OS, a mobile operating system developed by Alibaba. Aliyun was partially compatible with Android applications and, in particular, with Google applications. Despite the claims of Alibaba, Google argued that Aliyun OS was based on An-

Concerns about foreclosure of Android manufacturers are currently investigated by the EU commission.[9] Similar concerns were also raised in two recent merger cases. In the Google-Motorola merger: "The Commission considered whether Google would be likely to prevent Motorola's competitors from using Google's Android operating system."[10] Similarly, in the Microsoft-Nokia merger, the Commission "investigated the vertical relationships between the merged entity's activities in the downstream market for smart mobile devices and Microsoft's upstream activities in mobile operating systems."[11]

## 3. Model

We consider a two-sided market where buyers and developers of applications may interact through competing software platforms offering operating systems to manufacturers. Manufacturers must choose which operating system to install on their devices. Interactions between buyers and developers require the former to buy a device from a manufacturer and the latter to affiliate with the platforms.

### 3.1. Main Assumptions

There are two platforms, denoted by $I$ and $E$. The incumbent $I$ has a competitive advantage over the entrant $E$: $I$'s marginal cost to provide its operating system is nil whereas it is equal to $\delta \geq 0$ for $E$, with $\delta$ not too large to ensure that platform $E$ puts an effective competitive pressure on the more efficient platform $I$. Platforms levy royalties from manufacturers. Let $w_{jk}$ denote the royalty paid to platform $j$ ($j = I, E$) by manufacturer $k$ ($k = 1, 2$) for each device using the former's operating system and sold to buyers by the latter.

A few comments are in order with respect to the interpretation of these assumptions. First, the competitive advantage of platform $I$ may come alternatively from an existing base of developers, a better software, etc. Accordingly, the assumption that $I$ has a lower marginal cost is a convenient shortcut to capture all these scenarios. Second, the least efficient platform $E$ can be seen as an alternative mobile operating system to $I$ that is developed in-house by a manufacturer (e.g., Tizen for Samsung). Third, the royalty can also be interpreted as the share of the revenue generated from user data that accrues to the platform. Contracts between a manufacturer and a platform, such as the MADA discussed in Section 2, typically specify which party owns the data and, accordingly, who can monetize it.

For developers, we make the following assumption.

ASSUMPTION 1. *Developers (i) pay no fees to publish their application with either platform, and, (ii) only care about the total number of buyers they can reach.*

The first item in Assumption 1 is a restriction on platforms' pricing instruments. The second item is related to the developers' technology: once an application is developed, it can be made available on both platforms to reach all the buyers at no additional cost.

droid and, accordingly, had to have a license to use the Google applications. Acer promptly renounced to launch an Aliyun smartphone when Google threatened it to cancel its Android license.

[9] http://europa.eu/rapid/press-release_IP-16-1492_en.htm

[10] http://europa.eu/rapid/press-release_IP-12-129_en.htm

[11] http://europa.eu/rapid/press-release_IP-13-1210_en.htm

Assumption 1 allows to have a clear benchmark; it is relaxed in Section 5, where we introduce cost to port applications, and in Section 8, where platforms charge developers a fee to publish their applications.

Hence, the number of applications $n_S$ can be written as a function of the total number of buyers only, or

$$n_S = Q_S(n_{B1} + n_{B2}), \tag{3.1}$$

where $n_{Bk}$ denotes the number of buyers using manufacturer $M_k$'s device. $Q_S(\cdot)$ is assumed to be increasing.

Manufacturers are symmetric and produce at the same constant marginal cost normalized to nil. In our two-sided framework, buyers' demand for manufacturer $M_k$'s device depends both on the number of applications developed on the operating system elected by $M_k$ (which coincides with the total number of developers $n_S$ under Assumption 1) and on the prices, denoted by $p_1$ and $p_2$, charged by manufacturers to buyers.

To highlight the specificity of vertical mergers in platform industries, we make the following assumption on the demand faced by each manufacturer.

ASSUMPTION 2. *Manufacturers are local monopolies: buyers' demand for product $k$ depends only on manufacturer $M_k$'s price $p_k$ and on the number of applications available on the operating system chosen by $M_k$.*

Assumption 2 in conjunction with Assumption 1 implies that the number of buyers who purchase the device produced by manufacturer $M_k$ can be written as

$$n_{Bk} = Q_B(p_k, n_S), \tag{3.2}$$

where the quasi-demand $Q_B(\cdot,\cdot)$ is assumed to be decreasing in $p_k$ and increasing in $n_S$. Under Assumption 2, there is no product-market competition between manufacturers: the demand faced by one manufacturer does not directly depend on the price set by the other manufacturer. While admittedly extreme, this assumption allows both to differentiate our analysis from the extant literature on the competitive effects of vertical integration and to highlight in a clear-cut way how indirect network effects nevertheless generate a specific form of strategic interaction between manufacturers. Furthermore, Assumption 2 is relaxed in Section 7.

The timing is as follows. In stage 1, platforms $I$ and $E$ set royalties for manufacturers $M_1$ and $M_2$. Then, in stage 2, $M_1$ and $M_2$ decide which platform to affiliate with.[12] Once operating systems are chosen, manufacturers set the prices of their devices. Last, in stage 3, buyers decide whether to buy a device, and, simultaneously, developers decide whether to develop an application. All prices and affiliation decisions are public. We look for subgame-perfect equilibria of the game.

[12]Since platforms are not differentiated, manufacturers have incentives to make their devices available on one platform only.

*3.2. Strategic Interaction between Manufacturers Induced by Indirect Network Effects*

Even though manufacturers are not competing on the product market, the demand faced by a manufacturer depends on the price set by the rival manufacturer through the developers' participation decision, which itself depends on the manufacturers' demands through the buyers' decision. Formally, given prices $(p_1, p_2)$, the demand $D_k(p_k, p_l)$ faced by manufacturer $k \neq l$ $(k, l = 1, 2)$ and the developers' demand for affiliation $D_S(p_1, p_2)$ solve the following system[13]

$$\begin{cases} D_1(p_1, p_2) &= Q_B(p_1, D_S(p_1, p_2)), \\ D_2(p_2, p_1) &= Q_B(p_2, D_S(p_1, p_2)), \\ D_S(p_1, p_2) &= Q_S(D_1(p_1, p_2) + D_2(p_2, p_1)). \end{cases} \tag{3.3}$$

As is usual in the two-sided markets literature, we assume that indirect network effects are not too strong to ensure that, for all relevant prices, a solution of system (3.3) exists, is unique, and, is such that manufacturers face a demand that is locally elastic with respect to prices.[14] The products sold by manufacturers exhibit then a form of demand complementarity as stated in the next lemma.

LEMMA 1. *The demand faced by a manufacturer for its product and the number of developers are decreasing in both $p_1$ and $p_2$.*

*Proof.* See Appendix A.1. □

That a manufacturer's demand is decreasing in its own price is straightforward: following an increase in, say, $p_1$, fewer buyers purchase device 1, and, therefore, fewer developers also join that platform, which further adversely impacts manufacturer $M_1$'s demand. That a manufacturer's demand is decreasing in the rival manufacturer's price can be explained intuitively as follows: if, say, $p_1$ increases, the number of buyers of product 1 decreases, fewer developers propose applications, and, therefore, fewer buyers are also willing to purchase device 2. Since manufacturers are local monopolies on the buyers market, $p_1$ has no direct effect on the number of buyers of device 2, only an indirect effect through the participation of developers. Therefore, as soon as indirect network effects are non nil, $\partial D_k(p_k, p_l)/\partial p_l < 0$, or, equivalently, the products sold by manufacturers exhibit a form of demand complementarity even though manufacturers are local monopolies.

This implies in turn that manufacturers' prices tend to be strategic substitutes. Indeed, given a royalty $w_k$ that it pays, $M_k$'s profit writes as $\pi_k(p_k, p_l) = (p_k - w_k)D_k(p_k, p_l)$, $k \neq l$. Assuming that the best response in price of manufacturer $M_k$, denoted by $R_k(p_l)$, can be uniquely characterized by the first-order condition $\frac{\partial \pi_k}{\partial p_k}(R_k(p_l), p_l) = 0$, the nature of the strategic interaction between manufacturers, that is, the sign of $R_k'(\cdot)$, is given by the sign of

$$\frac{\partial^2 \pi_k}{\partial p_k \partial p_l}(p_k, p_l) = (p_k - w_k)\frac{\partial^2 D_k}{\partial p_k \partial p_l}(p_k, p_l) + \frac{\partial D_k}{\partial p_l}(p_k, p_l).$$

The second term in the right-hand side is negative according to Lemma 1. The sign of the first term depends on the cross-derivative of the demand with respect to manufacturers'

[13] As manufacturers face the same quasi-demand, their demand functions are symmetric, or $D_1(p_1, p_2) = D_2(p_2, p_1)$ for all $p_1$ and $p_2$.

[14] A precise statement of that assumption can be found in Appendix A.1.

prices. As is usual, we assume that the first term is negligible compared to the second one for all prices, which leads to our third main assumption.

Assumption 3. *Manufacturers' prices are strategic substitutes, that is, the manufacturers' best-responses in downstream prices are downward-sloping.*

Finally, several standard technical assumptions are needed to ensure that the manufacturers' problem is indeed well-behaved. First, the slope of manufacturers' best-responses is smaller than 1 in absolute value. Accordingly, there exists a unique pair of prices $(p_1^*, p_2^*)$ which forms the Nash equilibrium of stage 2. Second, equilibrium prices are increasing in marginal cost, and, the cost pass-through is smaller than 1: $0 < \frac{\partial p_k^*}{\partial w_k}(w_k, w_l) < 1,\ k \neq l$. Third, the demand for a device is more responsive to its own price than to the price of the other device: $\frac{\partial D_k}{\partial p_k}(p_k, p_l) < \frac{\partial D_k}{\partial p_l}(p_k, p_l) \leq 0,\ k \neq l$.

### *3.3. Main Example*

For illustration, we use the following Main Example.

- A unit mass of heterogeneous buyers have utility $U_B = v + u_B n_S - p_k - \tilde{\varepsilon}$, where $v$ is the stand-alone benefit of buying product $k = 1, 2$ at price $p_k$, $u_B > 0$ measures the strength of network effects on the buyers' side of the market, and $\tilde{\varepsilon}$ is distributed on $[0, \overline{\varepsilon}]$ according to a cdf $G(\cdot)$ with (strictly positive) density $g(\cdot)$. Buyers' quasi-demand is then given by $Q_B(p_k, n_S) = 1 - G\left(v + u_B n_S - p_k\right)$. To ensure that buyers indeed purchase in equilibrium, $v$ is large enough with respect to $\delta$.
- A unit mass of heterogeneous developers have utility $U_S = u_S(n_{B1} + n_{B2}) - \tilde{f}$, where $u_S > 0$ measures the strength of network effects on the developers' side of the market, and $\tilde{f}$, the cost to develop an application, is distributed on $[0, \overline{f}]$ according to a cdf $F(\cdot)$ with (strictly positive) density $f(\cdot)$. Developers' quasi-demand is then given by $Q_S(n_B) = 1 - F\left(u_S n_B\right)$.
- When $\tilde{\varepsilon}$ and $\tilde{f}$ are uniformly distributed on $[0, 1]$, provided that $\mu = u_B u_S < 1/2$, a unique solution of (3.3) exists and is given by

$$\begin{cases} D_1(p_1, p_2) &= \frac{1}{1-2\mu}(v - (1-\mu)p_1 - \mu p_2), \\ D_2(p_1, p_2) &= \frac{1}{1-2\mu}(v - (1-\mu)p_2 - \mu p_1), \\ D_S(p_1, p_2) &= \frac{1}{1-2\mu}u_S(2v - p_1 - p_2). \end{cases}$$

Prices are thus strategic substitutes and all the technical assumptions are satisfied.

## 4. Competitive Impact of Vertical Integration

This section is devoted to the analysis of the impact of vertical integration on, first, royalties and manufacturers' prices, and, second, profits.

### *4.1. Benchmark: Separation*

Absent any merger between a platform and a manufacturer, price competition between platforms leads to the following outcome.

PROPOSITION 1. *With no merger, the most efficient platform $I$ supplies both manufacturers at a royalty equal to the least efficient platform's marginal cost, i.e.* $w_I = \delta$.

*Proof.* Immediate. □

This is the logic of Bertrand competition: the most efficient platform $I$ cannot set a price above platform $E$'s marginal cost $\delta$, for otherwise manufacturers would prefer to choose $E$'s operating system; the least efficient platform $E$ cannot set a royalty below its marginal cost, for otherwise it would make losses.

For future reference, the best response of manufacturer $M_k$ is solution of

$$D_k(p_k, p_l) + (p_k - \delta)\frac{\partial D_k}{\partial p_k}(p_k, p_l) = 0. \tag{4.1}$$

Denote the equilibrium prices, manufacturers' profit and platform $I$'s profit under separation by $\hat{p}_1 = \hat{p}_2$, $\hat{\pi}_1 = \hat{\pi}_2$ and $\hat{\pi}_I$ respectively.[15]

### *4.2. Royalty*

We consider from now on that manufacturer $M_1$ is integrated with platform $I$ and denote the integrated firm by $I1$. Its profit is given by $\pi_{I1}^{(I1)}(p_1, p_2, w_I) = p_1 D_1(p_1, p_2) + w_I D_2(p_2, p_1)$ if $M_2$ affiliates with $I1$ at some royalty $w_I$, and, $\pi_{I1}^{(E)}(p_1, p_2, w_E) = p_1 D_1(p_1, p_2)$ if $M_2$ affiliates with $E$ at some royalty $w_E$. When supplied by platform $j$ at a royalty $w$, $M_2$'s profit is $\pi_2^{(j)}(p_1, p_2, w) = (p_2 - w) D_2(p_2, p_1)$. As it will play an important role later on, let us call $w_I D_2(p_2, p_1)$ the upstream profit of the integrated firm, that is, the profit earned from selling the operating system to the non-integrated manufacturer.

We start by comparing the manufacturers' prices depending on the choice of operating system made by the non-integrated manufacturer $M_2$ for a given level of royalty. To this end, consider the subgame starting at stage 2, and, denote by $p_1^{(j)}(w)$ and $p_2^{(j)}(w)$ the manufacturers' equilibrium prices when $M_2$ affiliates with platform $j$ at a royalty $w$;[16] profits are thus defined accordingly.

LEMMA 2 (Accommodation Effect). *Suppose that both platforms offer the same royalty $w > 0$. The integrated (respectively, the non-integrated) manufacturer sets a lower (respectively, a higher) price when the non-integrated manufacturer affiliates with the integrated platform than when the non-integrated manufacturer affiliates with the non-integrated platform, that is, for all $w > 0$*

$$p_1^{(I1)}(w) < p_{I1}^{(E)}(w) \text{ and } p_2^{(I1)}(w) > p_2^{(E)}(w).$$

*Proof.* See Appendix A.2. □

This result hinges only on the indirect network effects between buyers and developers. By lowering the price of its device, the integrated firm attracts additional customers for device 1, which increases developers' participation in both platforms; this then leads to a higher demand for device 2, and, thus, to a larger upstream profit for $I1$.

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[15] $M_k$'s profit when it faces a royalty $\delta$ is $(p_k - \delta)D(p_k, p_l)$. Equilibrium profits under no integration are thus given by $\hat{\pi}_1 = (\hat{p}_1 - \delta)D_1(\hat{p}_1, \hat{p}_2)$ and $\hat{\pi}_I = \delta(D_1(\hat{p}_1, \hat{p}_2) + D_2(\hat{p}_2, \hat{p}_1))$.

[16] Those prices solve the two first-order conditions $\frac{\partial}{\partial p_1}\pi_{I1}^{(j)}(p_1, p_2, w) = 0$ and $\frac{\partial}{\partial p_2}\pi_2^{(j)}(p_1, p_2, w) = 0$.

Formally, given a royalty $w$, the best response in downstream price of the integrated firm when it supplies $M_2$ is characterized by

$$D_1(p_1,p_2)+p_1\frac{\partial D_1}{\partial p_1}(p_1,p_2)+w\frac{\partial D_2}{\partial p_1}(p_2,p_1)=0, \tag{4.2}$$

whereas it is given by Equation (4.1) (with $\delta=0$) when it does not supply $M_2$. Since the last term in the left-hand side of Equation (4.2) is negative, the integrated firm's best response moves downward when it supplies the non-integrated manufacturer. Because downstream prices are strategic substitutes, if $M_2$ chooses the $I1$'s operating system, this leads to an increase of the $M_2$'s price and a decrease of $I1$'s.

Next, we study $M_2$'s incentives to affiliate with $I1$ or with $E$, as well as $I1$'s incentives to supply $M_2$.

LEMMA 3 (Upstream Market Power Effect).

*(i) If the integrated platform $I1$ and the non-integrated platform $E$ offer the same royalty $w>0$, then the non-integrated manufacturer $M_2$ is better-off buying from $I1$ than from $E$, that is, for all $w>0$*

$$\pi_2^{(I1)}(w)>\pi_2^{(E)}(w).$$

*(ii) If the non-integrated platform $E$ offers a royalty $w>0$, then the integrated platform $I1$ is better-off supplying the non-integrated manufacturer $M_2$ even if it makes no upstream profits, that is, for all $w>0$*

$$\pi_{I1}^{(I1)}(0)>\pi_{I1}^{(E)}(w).$$

*(iii) The highest royalty $\overline{w}$ that the integrated firm $I1$ can charge while still supplying the non-integrated manufacturer $M_2$ is therefore such that $\pi_2^{(I1)}(\overline{w})=\pi_2^{(E)}(\delta)$.*

*Proof.* See Appendix A.3. □

If $M_2$ affiliates with $I1$, the integrated manufacturer reacts by lowering its price according to Lemma 2. That decision increases the number of applications available on the integrated platform, and, in turn, allows $M_2$ to extract more profits from its customers. This explains the first result in Lemma 3.

Next, the second part in Lemma 3 stems from the following observation: thanks to the demand complementarity between products, $I1$ is better off the lower $M_2$'s marginal cost is. Indeed, if $M_2$ becomes more efficient, it sets a lower price, attracts more buyers and thus more developers, for the benefit to the integrated firm. It follows that $I1$ prefers to sell its operating system to $M_2$ at a nil royalty rather than to let $M_2$ affiliate with the $E$ at any strictly positive royalty.

The third result in Lemma 3 is an immediate consequence of items (i) and (ii): since $I_1$'s accommodating behavior makes $M_2$ willing to buy the input from that firm, the integrated firm is willing to leverage its royalty to capture part of the extra profit earned by the non-integrated manufacturer. The royalty cannot be too high, however, for otherwise $M_2$ would rather take the option of buying $E$'s operating system.

We summarize the findings of this section in the next proposition.

Lemma 4. *In equilibrium, the vertically integrated platform $I1$ supplies the non-integrated manufacturer $M_2$ at a royalty $w^* > 0$ solution of*

$$\max_{w} \pi_{I1}^{(I1)}(w) \text{ subject to } w \in [0, \overline{w}].$$

*Proof.* See Appendix A.4. □

Observe that $w^*$ can, a priori, be either above or below the pre-merger upstream price $\delta$: Since the integrated firm benefits from having a more efficient manufacturer $M_2$, it may be willing to lower the input price below its pre-merger level $\delta$, thus sacrificing some upstream profits but benefiting from stronger network effects. To focus on the more interesting cases, the next assumption restricts attention to situations where vertical integration increases the royalty levied from the non-integrated manufacturer and is thus likely to raise some suspicion from antitrust authorities.[17]

Assumption 4. *Vertical integration leads to an increase in the royalty with respect to the separation benchmark, or $w^* > \delta$.*

We show in Appendix A.11 that Assumption 4 holds provided that $\delta$ is sufficiently small and for any relevant value of $\delta$ in the Main Example with uniform distributions.

### *4.3. Manufacturers' Prices*

The next result describes the impact of vertical integration on downstream prices with respect to the benchmark scenario.

Proposition 2 (Impact on Manufacturers' Prices). *With respect to the separation benchmark, a vertical merger between platform $I$ and manufacturer $M_1$ leads to a decrease in the integrated manufacturer's price and an increase in the non-integrated manufacturer's price, that is, for all $w^* \in (\delta, \overline{w}]$*

$$p_1^{(I1)}(w^*) < \hat{p}_1 \text{ and } p_2^{(I1)}(w^*) > \hat{p}_2.$$

*Proof.* See Appendix A.5. □

The intuition underlying the first result in Proposition 2 is perhaps best understood analyzing the impact of vertical integration on manufacturers' best responses, as illustrated in Figure 1. Points $S$ and $I$ in Figure 1 represent the equilibrium prices under separation and under integration when $M_2$ buys from $I1$ at royalty $w^* > \delta$ respectively. In order to understand the various forces at work, the move from $S$ to $I$ is decomposed in three steps: from $S$ to $a$; from $a$ to $b$; from $b$ to $I$.

- *Efficiency effect* (from $S$ to $a$). Point $a$ corresponds to a situation where $I$ and $M_1$ are integrated and $M_2$ buys from $E$ at royalty $\delta$. With respect to $S$, the only difference with $a$ is that $M_1$ faces a lower marginal cost: vertical integration eliminates a double marginalization; in that sense, $\delta$ can also be interpreted as the

[17]For the sake of conciseness, the case $w^* \leq \delta$ is nevertheless studied in Appendix A.11, which shows that either both prices decrease and the analysis is straightforward or $p_1$ decreases more than $p_2$ increases and the impact of vertical integration follows from Sections 4 and 6.

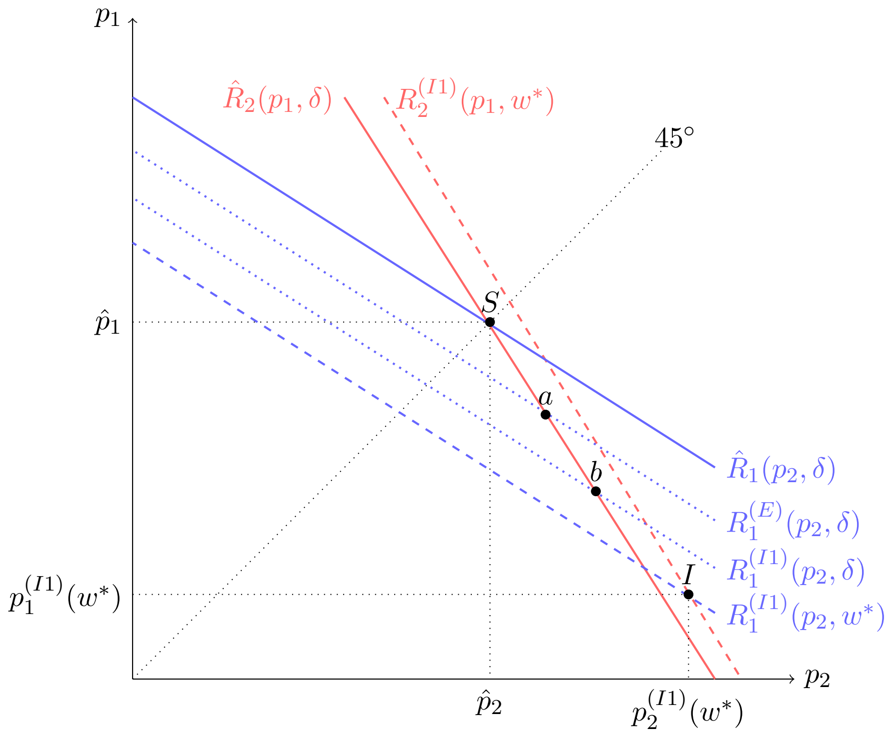


Figure 1 – The impact of vertical integration on equilibrium downstream prices is decomposed in three steps. $\hat{R}_k(p_l,\delta)$ (respectively, $R_k^{(j)}(p_l,w)$) denotes $M_k$'s best responses to $M_l$'s price $p_l$ under separation when both manufacturers buy at $\delta$ (respectively, under integration when $M_2$ is affiliated with platform $j$ at a royalty $w$).

efficiency gains associated to the merger between $I$ and $M_1$. As a result, $M_1$'s best response shifts downward (from $\hat{R}_1(p_2,\delta)$ to $R_1^{(E)}(p_2,\delta)$) but $M_2$'s best response is left unchanged ($R_2^{(I1)}(p_1,\delta)=\hat{R}_2(p_1,\delta)$).

- *Accommodation effect* (from $a$ to $b$). Moving from $a$ to $b$, the only difference is that $M_2$ buys at the same royalty $\delta$ but, now, from the integrated firm rather than from $E$. The integrated firm now (partially) internalizes the effect of $p_1$ on $M_2$'s demand through the impact on its upstream profit. Since an increase in $p_1$ has an adverse effect on the demand faced by $M_2$, $M_1$'s best response shifts downward (from $R_1^{(E)}(p_2,\delta)$ to $R_1^{(I1)}(p_2,\delta)$) but $M_2$'s best-response is still left unchanged

- *Upstream market power effect* (from $b$ to $I$). Moving finally from $b$ to $I$, the only difference is that $M_2$ buys from the integrated firm at price $w^*>\delta$ instead of $\delta$. A first consequence is that $M_2$ faces a higher marginal cost, implying that its best response shifts upward (from $\hat{R}_2(p_1,\delta)$ to $R_2^{(I1)}(p_1,w^*)$). A second consequence is that the integrated firm puts more weight on its upstream profits and thus internalizes more strongly its effect on the number of buyers in market 2, implying that $M_1$'s best response shifts downward (from $R_1^{(I1)}(p_2,\delta)$ to $R_1^{(I1)}(p_2,w^*)$).

It is remarkable that all these effects go in the same direction: with respect to the separation benchmark, vertical integration leads to a lower price for the integrated manufacturer and a higher price for the non-integrated one.

### 4.4. *Industry Profits*

The next result highlights a first consequence of vertical integration in platform markets.

PROPOSITION 3. *A vertical merger between platform $I$ and manufacturer $M_1$ always benefits the non-integrated manufacturer $M_2$.*

*Proof.* See Appendix A.6. □

Proposition 3 may be explained as follows. Remind that $M_2$ must earn in equilibrium at least its outside option, namely $\pi_2^{(E)}(\delta)$, which corresponds to its profit if it buys from $E$ at a royalty $\delta$ and faces a manufacturer $M_1$ with a perceived marginal cost of 0. Since, in our base model, a manufacturer always benefits from facing a more efficient rival (for a more efficient manufacturer sets a lower price, which boosts the demand for both products), $M_2$'s outside option is strictly larger than its profit under separation where both manufacturers are supplied at royalty $\delta$. Vertical integration in our two-sided context does not hurt the non-integrated manufacturer with respect to the separation benchmark, even if the royalty does increase. In a nutshell, the foreclosure argument becomes irrelevant, and royalty is a poor guide to assess the competitive impact of integration.

To conclude this section, consider the profitability of the vertical merger. A vertical merger is said to be profitable if and only if the joint profit of manufacturer $M_1$ and platform $I$ is higher under integration than under no integration, that is, when

$$\pi_{I1}^{(I1)}(w^*) \geq \hat{\pi}_1 + \hat{\pi}_I. \tag{4.3}$$

To investigate whether Condition (4.3) holds, we focus on the MAIN EXAMPLE with uniform distributions.

PROPOSITION 4. *Consider the* MAIN EXAMPLE *with uniform distributions. A vertical merger is profitable if and only if network effects are small enough, that is, there exists $\hat{\mu} \in (0, 1/2)$ such that*

$$\pi_{I1}^{(I)}(w^*) \geq \hat{\pi}_1 + \hat{\pi}_I \Leftrightarrow \mu \leq \hat{\mu},$$

*Proof.* See Appendix A.7. □

This proposition may appear surprising at a first glance. After all, the vertical merger enables both to get rid of a double marginalization and to align the manufacturer's price in the interest of the joint profit. But the merger also triggers a negative strategic response from the non-integrated manufacturer, which increases its price. When indirect network effects are strong, the price increase by the non-integrated manufacturer has a strong negative impact on developers' participation, which makes the merger between $I$ and $M_1$ not profitable.[18]

[18] In the MAIN EXAMPLE with uniform distributions, we can also show that the value of integration $\pi_{I1}^{(I1)}(w^*) - (\hat{\pi}_1 + \hat{\pi}_I)$ is first increasing and then decreasing in $\mu$. For $\mu = 0$, $p_1$ is lower and there are more developers under integration. Slightly increasing $\mu$ has a stronger impact on the joint profit associated to product 1 (that is, $p_1 D_1$) under integration, mostly because $p_1$ maximizes that joint profit under integration but not under separation. This also has a stronger impact on the upstream profit associated to product 2 (that is, $w^* D_2$) under integration, because there are more developers so that when indirect network effects start to kick in, they have a stronger effect on the buyers' demand than under separation. Computations are available from the authors upon request.

A somewhat similar phenomenon occurs in the case of a merger between Cournot competitors and strategic substitutability between quantities. There, as shown by Salant et al. (1983), the merger is profitable if and only if it involves a sufficiently large number of firms, or, equivalently, if and only if the strategic response of the non merging parties (which increase their production consecutive to the merger) is not too strong. The size of the network effects is related to the intensity of the strategic response by the non merging party in our model.

## 5. FRAGMENTATION OF OPERATING SYSTEMS

Assumption 1 has the following implication: if, for instance, both platforms set the same royalty, then the total numbers of developers and of buyers do not depend on whether manufacturers choose the same operating system or not. While that assumption has allowed us to obtain clear-cut results, it may not always be satisfactory for the following reasons:

- *Cost to port applications.* In practice, the cost of porting an application to a new platform, while usually less than the cost of writing it from scratch, is non-negligible. Such costs provide manufacturers with incentives to coordinate on the same platform in order to limit the fragmentation of developers across different systems.
- *Direct network effects.* The larger the community of developers using the same programming langage is, the easier it becomes to find help to overcome coding issues. Similarly, platforms often promote in-house services that encourages interaction among its users (for instance, messaging apps or games). Finally, as highlighted by Katz and Shapiro (1985), more users on a platform may lead to a higher quality of postpurchase services, a relevant dimension for electronic devices that require both hardware maintenance and software update on a regular basis. In short, intra-group network externalities make manufacturers willing to affiliate with the same platform.

The very nature of platform industries therefore makes situations where manufacturers have incentives to coordinate on the same platform the norm rather than the exception. This section analyzes how these motives for coordination across manufacturers affect the competitive analysis of vertical integration. Section 5.1 presents the general argument: vertical integration forces now the coordination of manufacturers and creates some market power. Section 5.2 then discusses several applications directly related to the examples mentioned above.

To emphasize the differences with the previous analysis, we assume from now on that platform $I$ has no cost advantage, that is, $\delta = 0$. In the base model, this implies that a vertical merger has no impact with respect to the separation benchmark.

### *5.1. Main Argument*

The base model is modified to account for the fact that, all else equal, the numbers of buyers or of developers are larger when both manufacturers choose the same platform.

Denote respectively by $D_k^{(i,j)}(p_k, p_l)$ and $D_{Si}^{(i,j)}(p_1, p_2)$ the buyers' demand for device $k$ and the developers' participation on platform $i$ when manufacturers $M_1$ affiliates with platform $i$ and $M_2$ with $j$, with $i \neq j \in \{I, E\}$.[19] Notations for profits are adapted accordingly: $\hat{\pi}_k^{(i,j)}(w_1, w_2)$ denotes $M_k$'s stage 2 equilibrium profit under separation when $M_1$ and $M_2$ affiliate with platforms $i$ and $j$ and pay royalties $w_1$ and $w_2$ respectively; $\pi_2^{(I1,j)}(0, w_2)$ denotes $M_2$'s profit under integration between $I$ and $M_1$ when $M_2$ affiliates with platform $j \in \{I1, E\}$ and pays royalty $w_2$. The next assumption traduces the existence of coordination gains between manufacturers.

Assumption 5 (Gains from Coordination). *When manufacturers affiliate with the same platform rather than with different ones, the total numbers of buyers and of developers increase, and, all else equal, the profit of a non-integrated manufacturer increases. Formally, for all $k \in \{1, 2\}$, $(p_1, p_2)$, $(w_1, w_2)$, and $i \neq j \in \{I, E\}$ under separation or $i \neq j \in \{I1, E\}$ under integration*

*(i)* $D_k^{(i,i)}(p_k, p_l) > D_k^{(i,j)}(p_k, p_l)$ *and* $D_{Si}^{(i,i)}(p_1, p_2) > D_{Si}^{(i,j)}(p_1, p_2) + D_{Sj}^{(i,j)}(p_1, p_2)$ *;*

*(ii)* $\hat{\pi}_k^{(i,i)}(w_1, w_2) > \hat{\pi}_k^{(i,j)}(w_1, w_2)$ *and* $\pi_2^{(I1,I1)}(0, w_2) > \pi_2^{(I1,E)}(0, w_2)$.

Part (i) traduces the fact that there are more buyers and more developers when manufacturers join the same platform. Part (ii) considers that this increase in participation on both sides of the market benefits the non-integrated manufacturer both under separation and integration.[20]

In the separation benchmark, Bertrand competition in the upstream market drives royalties down to 0.[21] Competition in the upstream market prevents platforms from capturing the gains associated to the coordination of manufacturers. Those gains are fully pocketed by manufacturers.

Consider now that manufacturer $M_1$ is integrated with platform $I$. The main change brought by Assumption 5 concerns the non-integrated manufacturer's outside option: If it affiliates with the non-integrated platform, $M_2$ looses the benefit of coordination between manufacturers. In other words, vertical integration somewhat forces coordination on the integrated platform and allows the integrated platform to capture part of the non-integrated manufacturer's extra gain from coordination through a higher royalty: a motive for coordination between manufacturers creates some upstream market power for the integrated firm, which becomes then accommodating in the downstream market. The next proposition describes some implications of this mechanism.

Proposition 5 (Impacts of Integration with Coordination Motives). *Assume no efficiency gains (that is, $\delta = 0$) and there exists a motive for coordination across manufacturers. Then:*

[19] Note that the number of applications may now vary across platforms, hence the subscript '$Si$'.

[20] In imperfectly competitive industries, it could be that a common positive shock on demand ends up decreasing the firms' profit. In a Cournot framework, Seade (1980) finds conditions under which a common increase in the firms' marginal cost increases or decreases equilibrium profits and Cowan (2004) extends the analysis to demand shocks. Given the focus of our analysis, we directly consider that (given royalties) a positive demand shock increases the manufacturers' profit.

[21] To be more precise, manufacturers $M_1$ and $M_2$ play a coordination game in stage 2. Accordingly, for given royalties, there can be multiple equilibria in the choice of platforms by manufacturers. We ignore this possible coordination problem and assume instead that, for all relevant royalties, manufacturers coordinate on the cheapest platform. See also the discussion in Section 9.4.

*(i) The non-integrated manufacturer affiliates with the integrated platform and pays a royalty $w^{**}$ strictly above the pre-merger level.*

*(ii) The non-integrated manufacturer is made worse-off by the merger.*

*(iii) Vertical integration is always strictly profitable.*

*Proof.* See Appendix A.8. □

While we argued that, in the base model, the increase in the royalty paid by the non-integrated manufacturer was not synonymous of foreclosure, things are different now. At best, the non-integrated manufacturer could earn $\pi_2^{(I1,I1)}(0,0)$ if the integrated platform would charge a nil royalty. But, absent any efficiency effect when $\delta = 0$, that profit coincides with $M_2$'s profit under separation, namely $\hat{\pi}_2^{(I,I)}(0,0)$. Because the integrated firm leverages the motive for coordination into a higher royalty, the non-integrated manufacturer's profit is necessarily reduced with respect to its pre-merger level. In that sense, motives for coordination between manufacturers give rise to harmful foreclosure.

Observe that by offering a royalty equal to the pre-merger level (that is, 0) the integrated firm secures a profit equal to the joint profit of $I$ and $M_1$ under separation. Vertical integration is now always strictly profitable.

From an antitrust perspective, efficiency gains and motives for coordination give rise to strikingly opposite views on the risk of foreclosure. The royalty charged by the integrated platform increases in both cases because vertical integration creates an accommodation and an upstream market power effects. The impact on the non-integrated manufacturer's profit is, however, positive with efficiency gains and negative with motives for coordination. In a nutshell: efficiency gains commit the integrated firm to lower its downstream price, which benefits the non-integrated manufacturer through indirect network effects;[22] coordination motives lower the non-integrated manufacturer's outside option, which allows the integrated firm to capture some of the non-integrated manufacturer's gain from the motives for coordination. Before comparing consumer surplus and welfare in both cases (Section 6), we detail two applications, particularly relevant for platform markets, in which motives for coordination emerge naturally.

### 5.2. *Cost to Port Applications and Intra-Group Network Effects*

COST TO PORT APPLICATIONS. Let us amend the base model as follows. Suppose developers are heterogeneous in the cost to make their application available on a platform: a share $\alpha \in [0,1]$ bear the development cost each time they want to publish their application on a platform; the remaining $1-\alpha$ share of developers incur only the development cost before making their application available on both platforms. Parameter $\alpha$ captures the idea that porting an application on several operating systems can be costly and is thus an inverse measure of scale economy in application development: when $\alpha = 1$, developers care only about the number of customers on the platform when deciding whether to

[22] When $\delta > 0$, the integrated manufacturer is committed, through the efficient effect, to lower its downstream price whatever the level of royalty. The upstream market power effect also leads the integrated firm to lower its price (through the accommodation effect) and thus the non-integrated firm to increase its price. However, from the viewpoint of the integrated firm, the price variations associated to the upstream market power effect can be fully controlled through the royalty.

develop for that platform; when $\alpha = 0$ (which corresponds to Assumption 1), developers do not care about which platform customers are affiliated with, but only about the total number of customers brought by the manufacturers.

Hence, under separation, when $M_1$ and $M_2$ affiliate with $I$ and $E$ respectively, developers' quasi-demands are given by[23]

$$\left\{ \begin{array}{lcl} n_{SI} & = & \alpha Q_S(n_{B1}) + (1-\alpha) Q_S(n_{B1} + n_{B2}), \\ n_{SE} & = & \alpha Q_S(n_{B2}) + (1-\alpha) Q_S(n_{B1} + n_{B2}). \end{array} \right. \tag{5.1}$$

If, instead, manufacturers coordinate on, say, platform $I$, developers' quasi-demands are given by

$$\left\{ \begin{array}{lcl} n_{SI} & = & Q_S(n_{B1} + n_{B2}), \\ n_{SE} & = & 0. \end{array} \right. \tag{5.2}$$

For $\alpha < 1$, $Q_S(n_{B1} + n_{B2}) > \alpha Q_S(n_{Bk}) + (1-\alpha) Q_S(n_{B1} + n_{B2})$ $(\forall k \in \{1, 2\})$: manufacturers have incentives to coordinate their affiliation decisions and choose the same operating system so as to benefit from a larger number of applications. This specification of the developers' quasi-demands leads to demand functions that satisfy Assumption 5.

Next, we specify the model to obtain the following proposition.

Proposition 6. *(Cost to Port Application) Consider the* Main Example *with uniform distributions and a cost to port application. Then, Proposition 5 applies. Moreover, the profitability of vertical integration and the extent of foreclosure decrease with the degree of scale economy in application development, that is,* $\pi_{I1}^{(I1,I1)}(0, w^{**}) - (\hat{\pi}_2^{(I,I)}(0,0) + 0)$ *is positive and increasing in* $\alpha$,[24] *and,* $\pi_2^{(I1,I1)}(0, w^{**}) - \hat{\pi}_2^{(I,I)}(0,0)$ *is negative and decreasing in* $\alpha$.

*Proof.* See the Online Appendix. □

When scale economies in application development are small, that is, when $\alpha$ is large, the manufacturers benefit highly from affiliating with the same platform. Put differently, these are the situations where the non-integrated manufacturer's outside option is the worse and, accordingly, the integrated platform captures a large part of the non-integrated manufacturer's gain from coordination.

Intra-Group Network Effects. Last, we briefly consider the possibility of intra-group network effects. To account for network effects among, say, buyers, the quasi-demand of our base model is modified as follows. If device $k$ is equipped with the operating system of platform $j$, then the number of buyers of device $k$ is given by

$$n_{Bk} = Q_B(p_k, n_S, n_{Bj}),$$

where $n_{Bj}$ is the total number of users on platform $j$. Positive direct effects among buyers arise when $Q_B(\cdot,\cdot,\cdot)$ is increasing in $n_{Bj}$. If device $k$ is the sole running platform

[23] $n_{Si}$ stands for the number of developers that affiliate with platform $i$ and $n_{Bk}$ denotes the number of customers that $M_k$ faces.

[24] Observe that, at equilibrium under separation, the joint profit of $I$ and $M_1$ depends neither on $\alpha$ nor on the platform chosen by the manufacturers.

$j$'s operating system, then $n_{Bj} = n_{Bk}$. If, however, both devices run that operating system, then $n_{Bj} = n_{B1} + n_{B2}$. It is immediate to check that demand functions derived from these quasi-demands satisfy Assumption 5. Hence, similar results obtain.

## 6. CONSUMER SURPLUS AND WELFARE

This section deals with the impact of vertical integration on consumer surplus. Since consumer surplus depends both on prices and on developers' participation, we need to address two issues: First, given a level of developers' participation, is the price decrease more or less pronounced than the price increase? Second, how do these price variations impact the number of applications available at equilibrium?

To progress in that direction, we consider the MAIN EXAMPLE. The surplus of consumers who purchase from $M_k$ is thus given by

$$S_k(p_1, p_2) = \int_0^{v+u_B D_S(p_1,p_2)-p_k} (v + u_B D_S(p_1, p_2) - p_k - \varepsilon)\, dG(\varepsilon),$$

which rewrites, after an integration by parts, as follows

$$S_k(p_1, p_2) = \int_0^{v+u_B D_S(p_1,p_2)-p_k} G(\varepsilon) d\varepsilon. \tag{6.1}$$

Since $G(\cdot)$ is increasing, Equation (6.1) shows that the surplus of buyers in market $k$ is increasing in the developers' participation $D_S(p_1, p_2)$, and decreasing in $p_1$ and $p_2$.

### *6.1. Relative Variations of Prices*

We first express the impact of vertical integration on consumer surplus in terms of the relative variation of downstream prices, taking into account the participation of developers.

LEMMA 5 (Impact on Developers' Participation and Consumer Surplus). *Consider the* MAIN EXAMPLE. *If $G(\cdot)$ is concave (respectively, convex) and the price variation in the market where vertical integration takes places is stronger (respectively, smaller) than in the other market, then vertical integration increases (respectively, decreases) developers' participation and total consumers surplus.*

*Proof.* See Appendix A.9. □

Figure 2 provides a graphical illustration of the main intuition for Proposition 5 in the case where $G(\cdot)$ is concave. We represent there the best responses and the equilibrium prices under separation (point $S$) and under integration (point $I$). We also represent the developers' iso-participation curve, that is, the locus of prices $(p_1, p_2)$ such that the number of developers is constant ($dD_S(p_1, p_2) = 0$) and equal to the number of developers at the equilibrium under separation.[25] The assumption that $G(\cdot)$ is concave implies that this iso-participation curve is convex. Next, we represent the iso-total price curve, that

[25]The developers' iso-participation curve is clearly a decreasing curve in the plane $(p_1, p_2)$ as, say, a decrease in the price $p_1$ must be compensated by an increase in the price $p_2$ to maintain the number of developers constant.

is, the locus of prices $(p_1, p_2)$ such that the total price is constant ($d(p_1 + p_2) = 0$ with slope $-1$) and equal to the equilibrium total price under separation. When the price decrease in market 1 is stronger than the price increase in market 2, the equilibrium prices under integration must be somewhere in the shaded area represented in Figure 2. But this implies in turn that the developers' iso-participation curve moves in the south-west quadrant, or equivalently that the total number of developers increases following the merger.[26]

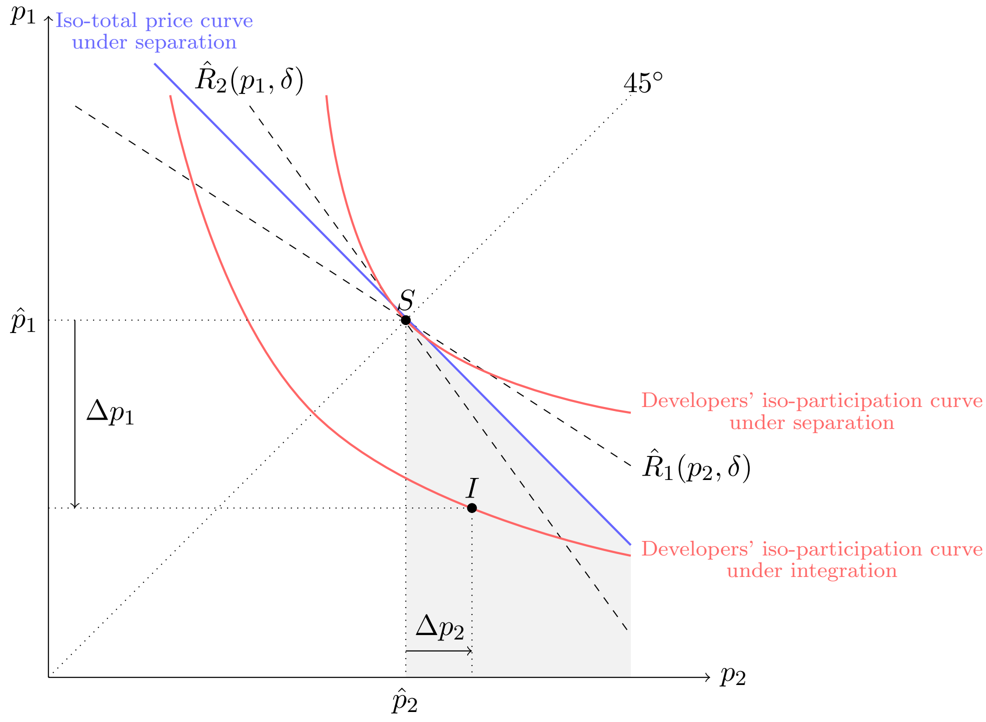


Figure 2 – When the price variation in market 1 is stronger than in market 2 and $G(\cdot)$ is concave, the number of developers increases following vertical integration.

It then comes immediately that consumer surplus in market 1 increases following the merger, for both the price $p_1$ paid by those buyers decreases and the number of applications they benefit from increases. The impact of the merger on buyers from market 2 is a priori ambiguous, for $p_2$ increases. However, given the symmetry of the downstream markets, the total buyers surplus increases as stated in Proposition 5.[27]

Observe, finally, that in the case of uniform distributions, $G(\cdot)$ is linear. A necessary

[26]Observe that $G(\cdot)$ concave is grossly sufficient to obtain that the number of developers increases following the merger. Even with, say, concave developers' iso-participation curves, it could well be that the integration leads to more applications.

[27]$G(\cdot)$ concave implies that the (quasi-) demand $Q_B(p, n_S)$ is convex in $p$ for all $n_S$, which has the following implication. First, for an exogenously fixed number of developers, if the price in, say, market 1 decreases by some amount, and, simultaneously, the price in market 2 increases by the same amount, then there are more additional buyers in market 1 than there are lost customers in market 2. Total consumers surplus therefore increases. Second, since developers care about the total number of buyers, there must be an increase of developers following such variations of downstream prices, which further enhances consumers surplus.

and sufficient condition for vertical integration to increase developers' participation is then $-\Delta p_1 \geq \Delta p_2$.

### 6.2. *Equilibrium Impact on Consumer Surplus and Welfare*

While the previous analysis has considered some exogenous price variations, it remains to understand how equilibrium prices actually vary to assess the impact of vertical integration on consumer surplus and welfare. Referring now to Figure 1, we note that both the efficiency and the accommodation effects lead to a price decrease in market 1 that is larger (in absolute value) than the price increase in market 2.[28] The upstream market power effect goes, however, in the opposite direction. The next result gives the net effect in the two scenarios studied so far.

PROPOSITION 7. *Consider the* MAIN EXAMPLE *with uniform distributions.*

*(i) With efficiency gains and no cost to port applications (that is, $\delta > 0$ and $\alpha = 0$), vertical integration leads to a price decrease in the market where the merger takes place stronger than the price increase in the other market. Developers' participation and total consumer surplus thus increase.*

*(ii) With no efficiency gains and a cost to port applications (that is, $\delta = 0$ and $\alpha > 0$), vertical integration leads to a price decrease in the market where the merger takes place smaller than the price increase in the other market. Developers' participation and total consumer surplus thus decrease.*

*Proof.* See the Online Appendix. □

An intuitive way to understand whether the upstream market power effect is dominated by the efficiency and the accommodation effects consists in looking at the impact of vertical integration of the best responses in downstream prices. The best response of manufacturer $M_1$ moves from (4.1) (under separation) to (4.2) (under integration), which amounts to, roughly speaking, a downward move of a magnitude given by

$$(6.2) \qquad -\underbrace{w\frac{\partial D_2}{\partial p_1}(p_2,p_1)}_{\text{Accommodation effect}}\ -\underbrace{\delta\frac{\partial D_1}{\partial p_1}(p_1,p_2)}_{\text{Efficiency effect}}.$$

For the non-integrated manufacturer, vertical integration moves $M_2$'s best response upwards by a magnitude given by

$$(6.3) \qquad -\underbrace{(w-\delta)\frac{\partial D_2}{\partial p_2}(p_2,p_1)}_{\text{Upstream market power effect}}.$$

Comparing (6.2) and (6.3), vertical integration has a stronger impact on $p_1$ than on $p_2$ if the impact is stronger on $M_1$'s best response than on $M_2$'s, that is, if

$$(6.4) \qquad w \leq \delta\,\frac{\frac{\partial D_1}{\partial p_1}(p_1,p_2)+\frac{\partial D_2}{\partial p_2}(p_2,p_1)}{\frac{\partial D_2}{\partial p_2}(p_2,p_1)-\frac{\partial D_2}{\partial p_1}(p_2,p_1)}.$$

[28]This comes from our assumptions on the slope of the downstream manufacturers' best responses.

With positive motives for coordination and no efficiency gains, Section 5 has shown that the merged entity raises the royalty above the pre-merger level: Condition (6.4) is thus never satisfied and the price increase is stronger than the price decrease. As an illustration, with a cost to port applications (that is, $\alpha > 0$) and no efficiency gains (that is, $\delta = 0$), there are fewer applications, consumer surplus in market 1 increases, consumer surplus in market 2 decreases, and, total consumer surplus decreases.

Absent positive motives for coordination but with efficiency gains, as in Section 4, the extent to which the integrated firm increases the royalty depends on $\delta$. With no cost to port applications (that is, $\alpha = 0$) and efficiency gains (that is, $\delta > 0$), it turns out that Condition (6.4) is always satisfied and the price decrease is stronger than the price increase: total consumer surplus and developers' participation both increase. Moreover, computations unveil that even the surplus of customers in market 2 increases following vertical integration.

Finally, we conclude with the impact on welfare.

Proposition 8. *Consider the* Main Example *with uniform distributions.*

*(i) With efficiency gains and a cost to port applications (that is, $\delta > 0$ and $\alpha = 0$), vertical integration always increases welfare.*

*(ii) With no efficiency gains and a cost to port applications (that is, $\delta = 0$ and $\alpha > 0$), vertical integration always decreases welfare.*

*Proof.* See the Online Appendix. □

## 7. Product Market Competition

This section has two objectives. First, we relax Assumption 2 and discuss how product market competition affects our results. Second, we discuss entry on the downstream market by a dominant platform through the development of a new product (rather than through the acquisition of an existing manufacturer).

To this end, assume that the devices offered by manufacturers are imperfectly differentiated from the perspective of buyers. One way to introduce such a differentiation is to use Shubik and Levitan (1980)'s linear demands system, to which we append network effects additively. The gross utility function of the representative buyer is then given by

$$q_0 + (v + u_B n_S) \sum_{k=1,2} q_k - \frac{1}{2} \frac{1}{2(1+\gamma)} \left( 2 \sum_{k=1,2} q_k^2 + \gamma \left( \sum_{k=1,2} q_k \right)^2 \right),$$

where $q_0$ is the numéraire and $q_k$ is the consumption of product $k$. Provided that both products are supplied at equilibrium, quasi-demands write as follows

$$Q_{Bk}(p_k, p_l, n_S) = v - p_k - \gamma \left( p_k - \frac{p_1 + p_2}{2} \right) + u_B n_S, \tag{7.1}$$

$$Q_S(n_B) = u_S n_B, \tag{7.2}$$

where, now, $\gamma \geq 0$ measures the extent of substitutability between products (or of product market competition).

### *7.1. Product Market Competition between Manufacturers*

Consider the vertical integration between platform $I$ and manufacturer $M_1$.

PROPOSITION 9. *When manufacturers compete on the product market and quasi-demands are given by Equations (7.1)-(7.2), manufacturers' prices are strategic substitutes if network effects are strong relative to the degree of product market differentiation (that is, $2\mu > \gamma/(1+\gamma)$) and strategic complements otherwise (that is, $2\mu < \gamma/(1+\gamma)$).*

*Moreover, assuming prices remain strategic complements, the non-integrated firm is hurt by the merger, but the extent of foreclosure decreases with the strength of network effects.*

*Proof.* See Appendix A.10. □

With the specification of quasi-demands given by (7.1)-(7.2), the compounding of the two forces shaping the strategic interaction between manufacturers, namely indirect network effects and product market interaction, leads to a simple result: given some product substitutability, manufacturers' prices are strategic substitutes if indirect network effects are large enough, and strategic complements otherwise.

As far as foreclosure is concerned, the role of the strategic interaction may be simply explained as follows. With the specification (7.1)-(7.2), prices are strategic complements (respectively, strategic substitutes) when the products sold by the manufacturers are demand substitutes (respectively, demand complements). Remind now that, at the equilibrium under integration, the royalty is increased up to the level where the non-integrated manufacturer becomes indifferent with buying from the less efficient platform $E$ at a royalty $\delta$. The extent of foreclosure is thus given by the difference between $\pi_2^{(E)}(\delta)$ and $\hat{\pi}_2$. With demand substitutes (respectively, demand complements) the former is always lower (respectively, larger) than the latter, for $M_2$ suffers (respectively, benefits) from facing a more efficient competitor. Hence, since they tend to reduce the strategic complementarity between prices, stronger network effects also lessen the extent of foreclosure when product market competition makes downstream prices strategic complements.

### *7.2. Downstream Expansion by a Platform*

A platform may also expand vertically by creating a downstream division ex nihilo, as Google did for its smartphone brand Pixel. An important question is whether nonintegrated manufacturers, such as Samsung, are hurt by this downstream expansion.

To answer this question, we assume that, first, in the separation benchmark there is only manufacturer $M_2$ active in the downstream market, and, second, platform $I$ opens a downstream division $M_1$ under integration.

If there is no integration, only device 2 is available. $M_2$'s quasi-demand is derived from consumer's preferences accordingly, which leads to

$$Q_{B2}(p_2, n_S) = \frac{2(1+\gamma)}{(2+\gamma)} (v - p_2 - u_B n_S) .$$

Otherwise, when platform $I$ is integrated, the manufacturers' quasi-demands are given by Equation (7.1).

When $I$ is active in the downstream market, there is an expansion of total demand (because a new product is available for the consumer) and an increase of competition for $M_2$. It turns out that a mechanism similar to Proposition 9 is at work: with strategic substitutability, $M_2$ suffers from facing a more efficient competitor; the opposite holds with strategic complementarity.

PROPOSITION 10. *Vertical expansion into the downstream market by platform I benefits to the non-integrated manufacturer if and only if manufacturers' prices are strategic substitutes (that is,* $2\mu > \gamma/(1+\gamma)$*).*

*Proof.* See the Online Appendix. □

## 8. Developer Fees

Software platforms often charge developers on participation (for instance, Google charges developers $25 for each application published on the Play Store) or on transaction each time an application is sold on the platform (for instance, both Apple and Google charges a 30% royalty on each transaction on their respective applications stores). Fees paid by developers to platforms are now introduced in our base model. Developer fees and manufacturer royalties may now be used to balance the network externalities across both sides of the market.

Denote by $a_j$ the fee charged by platform $j \in \{I, E\}$ when a developer wants to publish its application. The marginal cost of publishing an application is normalized to 0 and is assumed to be identical across platforms. For sake of simplicity, we also assume that royalties and developer fees are non-negative, or, for all $(j,k) \in \{I,E\} \times \{1,2\}$, $w_{jk} \geq 0$ and $a_j \geq 0$.[29,30] Last, platforms set their developer fees and royalties simultaneously in stage 1; the game then proceeds as before.

To emphasize the difference with the previous analysis, we assume hereafter that platform $I$ has no cost advantage, that is, $\delta = 0$. To streamline the analysis, we also assume that, when indifferent, a manufacturer affiliates with $I$.

### *8.1. Developer Fee Creates an Endogenous Motive for Coordination*

The developers' quasi-demand depends now both on the number of buyers of each device and on fees charged by platforms. More importantly, that quasi-demand also depends on whether manufacturers are affiliated to the same platform. If manufacturers affiliate with, say, platform $I$, the developers' quasi-demand may be written as[31]

$$n_{SI} = Q_S(a_I, n_{B1} + n_{B2}). \tag{8.1}$$

[29]To prevent opportunistic behavior from developers, platforms typically implement fees based on realized transactions between buyers and developers; this translates into non-negative developer fees in our model.

[30]That royalties and developer fees must both be non-negative rules out situations where a platform recoups losses on one side of the market with profits made on the other. The analysis of such situations is surprisingly intricate, for it bears resemblance with issues (related to equilibrium existence and characterization) encountered in models of competition in two-part tariffs in vertically related markets; see Schutz (2012) for a detailed exposition.

[31]$Q_S(\cdot,\cdot)$ is decreasing in the first argument and increasing in the second one.

By contrast, when manufacturers affiliate with different platforms, a developer has to pay $a_I + a_E$ to reach all consumers. The developers' quasi-demands are then given by[32]

$$n_{SI} = n_{SE} = Q_S(a_I + a_E, n_{B1} + n_{B2}). \tag{8.2}$$

Equation (8.2) reveals a key implication of adding developer fees to the base model: the total membership fee is higher when manufacturers affiliate with different platforms than when they coordinate on the same platform, or, $a_I + a_E \geq \max\{a_I, a_E\}$. Accordingly, all else equal, manufacturers are willing to coordinate on the same platform to eliminate this double marginalization on the developers side of the market: Developer fees create a positive motive for coordination between manufacturers. The difference with Section 5 is that this motive for coordination is now endogenous, for it depends on the fees charged by platforms to developers.[33]

### *8.2. Separation*

Consider the separation benchmark. If, say, platform $E$ offers $(w_E > 0, a_E > 0)$, then platform $I$ can undercut that offer (by offering, for instance, $w_I = w_E - \epsilon$ and $a_I = a_E - \epsilon$, $\epsilon$ positive and sufficiently small) so that both manufacturers are willing to join in order to reduce their costs and boost developers' participation. Bertrand competition in the upstream market drives royalties and developer fees to zero. Observe that, in equilibrium, the motive for coordination has disappeared since platforms set nil royalties.

For future references, observe that since manufacturers pay no royalties and developers pay no fees, the manufacturers' best responses in prices under separation are characterized by Equation (4.1) (with $\delta = 0$).

### *8.3. Integration: Equilibrium Royalties and Developer Fees*

Assume now that platform $I$ is integrated with manufacturer $M_1$. In the competition to attract the non-integrated manufacturer $M_2$, the integrated platform $I1$ has, a priori, several strategic advantages over platform $E$: $M_2$ benefits from being affiliated with $I1$, for this triggers an accommodation effect if $w_I > 0$; furthermore, if $a_E > 0$, there is a motive for coordination on the integrated platform, for this eliminates a double marginalization on the developers side of the market.

Accordingly, in equilibrium, the integrated platform is able to leverage these competitive advantages and the best offer the non-integrated platform can make consists in a nil royalty and a nil developer fee, that is, $w_E = a_E = 0$. We have now to determine whether the integrated firm's market power translates into a higher royalty (as suggested by the presence of an upstream market power effect), a higher developer fee (as suggested by the presence of a positive motive for coordination), or both.

[32] We are interested in situations where all developers multihome and, therefore, all pay $a_I + a_E$. This amounts to assuming that $a_I$ and $a_E$ are not too large.

[33] More precisely, that the number of application developers increases when both manufacturers affiliate with the same platform depends only on developer fees. Whether manufacturers gain from that increase depends on the level of royalties too. Since we show later on that equilibrium royalties are nil both under separation and integration, we take the shortcut that there is a positive motive for coordination in the sense of Assumption 5.

It turns out that the integrated platform cannot leverage its competitive advantage into a higher royalty. Indeed, if $w_I > 0$, then $M_2$ is better off affiliating with $E$ because, first, the accommodation effect does not compensate for the nil royalty offered by $E$, and, second, the affiliation with $E$ has no adverse effect on developers' participation since $a_E = 0$. This result directly echoes the analysis undertaken in Sections 4 and 5: absent efficiency gains (because $\delta = 0$) and without motives for coordination (because $I1$ faces $a_E = 0$ in equilibrium), there are no upstream and accommodation effects and competition forces the integrated platform to license its operating system at a nil royalty in equilibrium. Slightly abusing notations, we consider from now on that $w_I = 0$.

The integrated platform can, however, increase its membership fee $a_I$, for this has no impact on manufacturer $M_2$'s affiliation decision: in order to reach consumers in market 1, developers have to pay $a_I$ irrespective of whether $M_2$ affiliates with $I1$ or not. Intuitively, being integrated with a manufacturer provides a platform with a market power over that manufacturer's customers, which enables to raise the fee developers have to pay to reach those buyers. Accordingly, vertical integration is strictly profitable as soon as the integrated platform is willing to charge $a_I > 0$.

It is actually a priori unclear whether $I1$ is willing to charge developers a strictly positive fee, for this decreases developers' participation and, accordingly, the integrated platform's downstream profit. Formally, the integrated platform's profit writes now as

$$\pi_{I1}^{(I1)}(a_I, p_1, p_2) = p_1 D_1(a_I, p_1, p_2) + a_I D_S(a_I, p_1, p_2),$$

and standard computations show that (accounting for the fact that this profit is maximized in $p_1$ at stage 2 of the game)

$$\text{(8.3)} \qquad \frac{d}{da_I}\pi_{I1}^{(I1)}(a_I, p_1^{(I1)}(a_I), p_2^{(I1)}(a_I)) = D_S + a_I \frac{\partial D_S}{\partial a_I} + p_1^{(I1)} \left( \frac{\partial D_1}{\partial a_I} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2^{(I1)}}{\partial a_I} \right).$$

The fee $a_I$ allows to earn revenues from developers (this corresponds to the first two terms in Equation (8.3)) but impacts negatively the profit earned from selling devices to consumers in market 1 (this corresponds to the last term in Equation (8.3)). Whether the positive effect dominates the negative one depends, intuitively, on the structure of indirect network effects, that is, on magnitude of $u_S$ relative to $u_B$.

It is noteworthy that, up to this point, our analysis has relied mostly on the role of the intensity of network effects (that is, $\mu$ in the MAIN EXAMPLE with uniform distributions), but not on their structure. When platforms can charge both sides of the markets, they are in a position to make each side internalize the effect of its decision on the other side. Following the insight of the two-sided market literature, we expect that, given an intensity of the network effects, charging developers a strictly positive fee is profitable when developers are more responsive to buyers' participation than buyers are to developers'.

### *8.4. Integration: Equilibrium Manufacturers' Prices*

That an integrated platform charges developers a positive fee has two important consequences on the equilibrium prices set by manufacturers.

- First, with respect to the pre-merger situation, charging $a_I > 0$ on developers impacts negatively the demands for both devices: accordingly, the manufacturers' best responses move downwards.

- Second, this also creates an accommodation effect, much in the spirit of Section 4. With respect to the pre-merger situation, the integrated firm chooses the downstream price with an eye on its impact on the revenues earned from the developers. Since $\partial D_S/\partial p_1 < 0$, the integrated firm's best response moves further downwards with respect to the pre-merger situation.

Figure 3 illustrates these two effects. Point $S$ corresponds to the separation benchmark

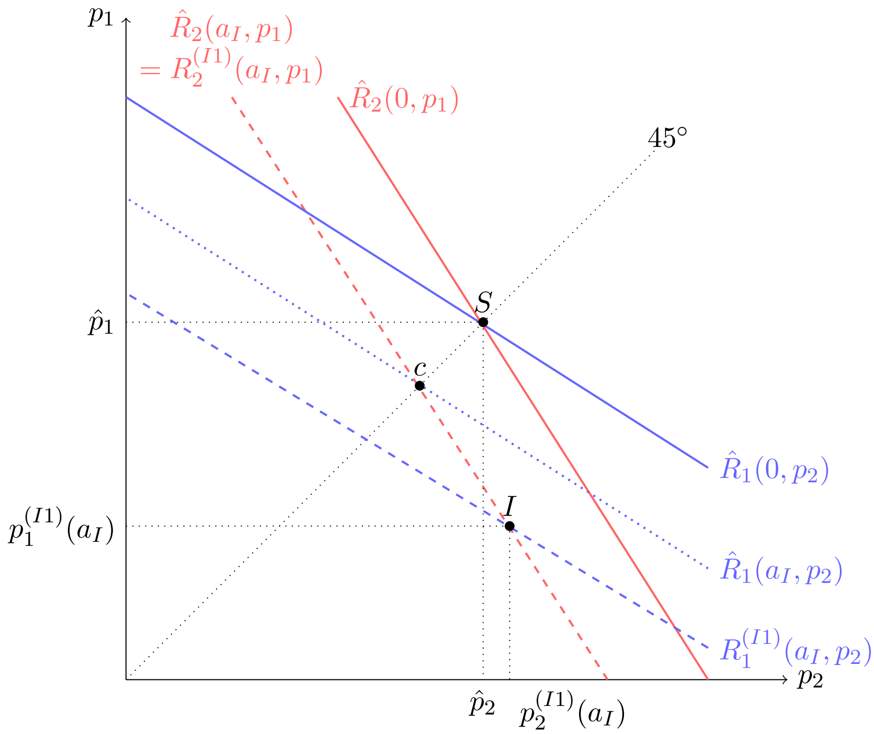


Figure 3 – The impact of vertical integration on downstream prices given a developers fee $a_I > 0$ charged by the integrated platform. $\hat{R}_k(0, p_l)$ denotes $M_k$'s best response under separation with nil royalty and developers fee. $\hat{R}_k(a_I, p_l)$ denotes $M_k$'s best response under separation when both manufacturers affiliate with $I$ but pay no royalty and a fee $a_I > 0$ is charged on developers. $R_1^{(I1)}(a_I, p_2)$ denotes $M_1$'s best response under integration when $M_2$ is affiliated with platform $I1$ at a royalty 0 and a fee $a_I > 0$ is charged on developers.

where manufacturers affiliate with, say, platform $I$ and both the royalty and the developers fee are nil. Point $c$ corresponds to a hypothetical situation in which, under separation, platform $I$ is elected by manufacturers and it is able to charge developers a strictly positive fee; demands for the devices would then be depreciated, leading manufacturers to lower their prices. Point $I$ corresponds to the integration outcome, in which manufacturers charge prices $p_1^{(I1)}(a_I)$ and $p_2^{(I1)}(a_I)$; the sole difference with respect to $c$ is that the integrated manufacturer's best response moves downwards thanks to the accommodation effect. While both effects lead to a lower price of the integrated manufacturer, the impact on the non-integrated manufacturer's price is a priori ambiguous and depends on the level of developers fee charged by the integrated platform.

In order to understand how these considerations combine at equilibrium, we specify the model and obtain the following proposition, which emphasizes the new role played by the structure of indirect network effects.

PROPOSITION 11. *Assume platforms are symmetric ($\delta = 0$) and can charge developers a fee for publishing an application. Consider the* MAIN EXAMPLE *with uniform distributions. For any intensity of indirect network effects $\mu = u_B u_S \in [0, 1/2)$, there exist thresholds $\underline{u}_S(\mu)$, $\tilde{u}_S(\mu)$, $\overline{u}_S(\mu)$, with $\overline{u}_S(\mu) \geq \tilde{u}_S(\mu) \geq \underline{u}_S(\mu)$, such that:*

*(i) Vertical integration is strictly profitable (and entails a strictly positive developer fee and a nil royalty) if and only if $u_S > \underline{u}_S(\mu)$.*

*(ii) When vertical integration is profitable, consumers in market 1 are better off if and only if $u_S > \tilde{u}_S(\mu)$; the non-integrated manufacturer is foreclosed and consumers in market 2 are better off if and only if $u_S > \overline{u}_S(\mu)$; the impact on total consumer surplus is ambiguous.*

*Proof.* See the Online Appendix. □

Figure 4 summarizes the results of Proposition 11. The relevant zone of parameters is the light grey one, which is the intersection of two conditions: network effects must not be too strong (that is, $\mu \leq 1/2$) and vertical integration is strictly profitable (which boils down to $a_I > 0$). Notice that a sufficient condition for the integrated platform to charge a strictly positive developer fee is $u_S > u_B$, which confirms the intuition developed previously.

To understand why vertical integration has an ambiguous impact on the non-integrated manufacturer's profit, consider the situation where network effects are stronger on the developer side than on the buyer side of the market (that is, $u_S > u_B$). Following a two-sided market logic, maximizing the industry profit requires to implement a low price on buyers and a high fee on developers in order to internalize the indirect network externalities. Under separation, platform competition prevents such an internalization of network effects. Since it leads to a lower price of the integrated manufacturer and a higher developer fee, vertical integration may bring the industry closer to the optimal price structure. While, roughly speaking, most of this increase of industry profit is captured by the integrated firm, the non-integrated manufacturer may still benefit substantially from a lower price set by the integrated manufacturer. In the MAIN EXAMPLE with uniform distributions, the non-integrated firm is harmfully foreclosed when $u_S \geq \overline{u}_S(\mu)$, but benefits from the merger otherwise.

The literature on two-sided markets has also repeatedly shown that the price structure implemented by a monopoly platform is similar to that chosen by a benevolent planner, though the price levels obviously differ. A similar argument can be made in our context: by bringing prices closer to an efficient structure, vertical integration may well enhance consumer surplus. In the MAIN EXAMPLE with uniform distributions, consumer surplus in market 1 (respectively, market 2) increases if and only if $u_S \geq \tilde{u}_S(\mu)$ (respectively, $u_S \geq \overline{u}_S(\mu)$). In particular, total consumer surplus decreases when $u_S \leq \tilde{u}_S(\mu)$.

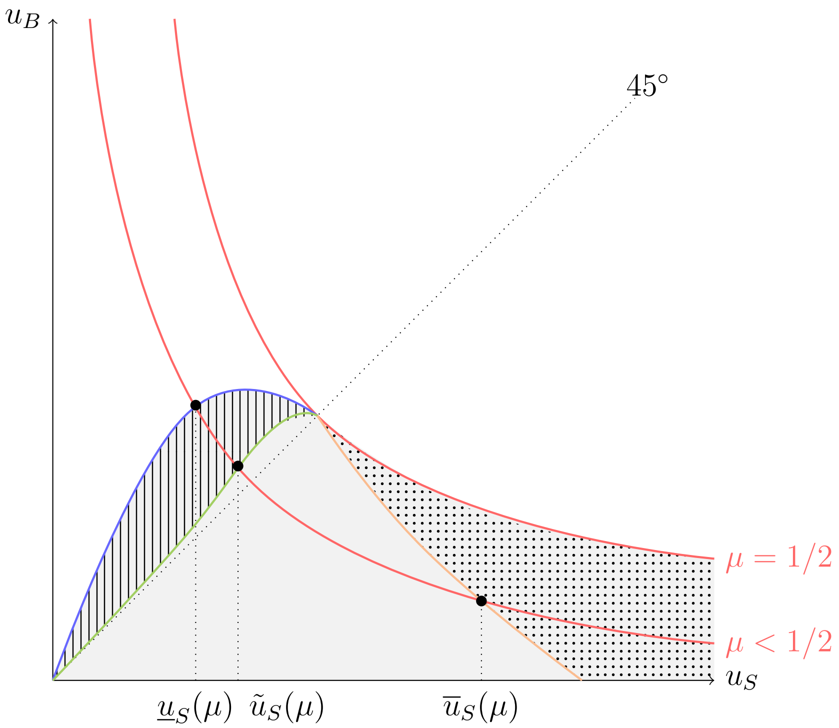


Figure 4 – Impact of a vertical merger when platforms can charge developers fees: vertical integration is strictly profitable in the light-grey area; $M_2$ is foreclosed and consumers in market 2 are better off in the dotted area; consumers in market 1 are worse off in the vertically-dashed area.

Observe that total consumer surplus and the non-integrated manufacturer's profit no longer vary in the same direction following the merger. Section 6 has shown that vertical integration leads to higher post-merger consumer surplus and non-integrated manufacturer's profit, whereas both are lower with motives for coordination. The logic at work with developer fee appears thus to be somewhat different, and is linked to the structure, rather than the level, of indirect network effects, and requires to study more closely how prices and participation levels are affected by vertical integration.

As far as prices and demands are concerned, computations reveal several interesting facts. When $u_S \leq \tilde{u}_S(\mu)$ (respectively, $u_S \leq \overline{u}_S(\mu)$), the price in market 1 (respectively, 2) decreases, the number of buyers in market 1 (respectively, 2) increases and the participation of developers decreases.

The reason why $u_S$ must be sufficiently greater than $u_B$ for consumer surplus to increase may now be explained as follows. Consider the impact of the fee $a_I$ on the participation of developers. There is a direct effect, which is negative because $\partial D_S / \partial a_I < 0$. There is also an indirect effect, related to the fact that developers care about the total number of buyers, which is itself linked to the total price $p_1 + p_2$; in the Main Example with uniform distributions, that total price paid decreases with the fee paid by developers (because of the accommodation effect created by a positive developer fee), so that the indirect effect is positive. Overall, the direct effect offsets the indirect one when $u_S \leq \tilde{u}_S(\mu)$ (respectively, $u_S \leq \overline{u}_S(\mu)$) in market 1 (respectively, market 2), for, in that case, the number of new buyers brought by the reduction of the total price does not compensate the loss of developers associated to the increase in the developer fee. This explains why when $u_S \leq \tilde{u}_S(\mu)$ (respectively, $u_S \leq \overline{u}_S(\mu)$) the surplus of buyers in market 1 (respectively, 2) decreases, even though the number of buyers in that market increases: while the price paid by buyers on that market decreases, there are less developers. Finally $\tilde{u}(\mu) < \overline{u}(\mu)$ because consumers in market 1 always benefit from a price decrease in market 1, unlike those in market 2.

To conclude, observe that total consumer surplus decreases following vertical integration when $u < \tilde{u}(\mu)$ and increases when $u > \overline{u}(\mu)$.

## 9. Extensions

This section discusses briefly some extensions of our base model.

### *9.1. Endogenous Vertical Integration*

Following Ordover et al. (1990) and Chen (2001), $M_2$ may counter the merger between $I$ and $M_1$ by integrating with $E$. To allow for this strategy, consider our base model and append to the timing a stage 0 which runs as follows: Manufacturers bid to acquire platform $I$. If there is no integration, then stage 0 ends and the game continues at stage 1 of our initial game. If a merger occurs between, say, $I$ and $M_1$, then $E$ and $M_2$ decide whether to vertically integrate in order to counter the first merger. After the counter-merger has occurred or not, the game continues at stage 1 of our initial game.

Then, the following result can be shown.[34]

In the base model of Section 4, where the incumbent has an efficiency advantage and there are no motives for coordination between manufacturers, one vertical merger occurs if the joint profit of $I$ and $M_1$ increases, that is, if and only if Condition (4.3) holds; otherwise, no merger occurs at equilibrium. Therefore, the condition stated in Proposition 4 is relevant whether the merger arises exogenously or endogenously.

With a cost to port applications and no efficiency gains, as in Section 5, a different result emerges: there is one vertical merger if and only if $\pi_{I1}^{(I1,I1)}(0, w^{**}) \geq \pi_2^{(I1,I1)}(0, w^{**}) + \hat{\pi}_I$ and that condition is always satisfied.[35]

The reason why the two models lead to different conditions for a vertical merger to occur endogenously at equilibrium stems from the fact that, in the second case, the non-integrated manufacturer is made worse-off by the merger so that manufacturers compete for not being non-integrated, whereas, in the first case, a vertical merger always benefits the non-integrated manufacturer and competition for being integrated is accordingly less intense.

[34] See the Online Appendix.

[35] With no efficiency gains, platform $I$'s profits under separation are nil, that is $\hat{\pi}_I = 0$. Then, the condition obtains since $\pi_{I1}^{(I1,I1)}(0, w^{**}) \geq \pi_{I1}^{(I1,I1)}(0, 0) = \pi_2^{(I1,I1)}(0, 0) \geq \pi_2^{(I1,I1)}(0, w^{**})$.

### 9.2. Diseconomies of Scope in Operating System Dissemination

In practice, diseconomies of scope that make manufacturers worse off when they choose the same operating system may exist: for instance, their devices might be perceived less differentiated by buyers, which intensifies downstream competition; or, the quality of the operating system might be lower because it must be compatible with different hardware configurations.

To account for this possibility, consider the extreme where manufacturers receive no demand if they affiliate with the same platform, that is, for all $k \in 1,2$ and $i \in \{I, E\}$, $D_k^{(i,i)}(p_k, p_l) = 0$. The immediate consequence is that manufacturers affiliate with different platforms, irrespective of whether a vertical merger has taken place.

Consider the separation benchmark and suppose, for instance, that $M_1$ and $M_2$ affiliate with $I$ and $E$ respectively at some royalties $w_I$ and $w_E$. The resulting downstream prices are denoted by $p_k^*(w_I, w_E)$, $k \in \{1,2\}$ and platforms' profits write $\pi_I(w_I, w_E) = w_I D_1(p_1^*(w_I, w_E), p_2^*(w_I, w_E))$ and $\pi_E(w_I, w_E) = w_E D_2(p_1^*(w_I, w_E), p_2^*(w_I, w_E))$. A platform has no incentives to undercut its rival, for manufacturers never affiliate with the same platform. Accordingly, equilibrium royalties $\hat{w}_I = \hat{w}_E = \hat{w}$ are positive and solve the first-order conditions $\frac{\partial}{\partial w_i}\pi_i(\hat{w}, \hat{w}) = 0$, $i \in \{I, E\}$.

Assume now that platform $I$ is integrated with manufacturer $M_1$. Vertical integration impacts downstream and upstream prices. First, there is an efficiency effect, that is, vertical integration eliminates a double marginalization between $I$ and $M_1$, and, accordingly, $p_1$ decreases and $p_2$ increases by strategic substitutability. Second, there is an upstream market power for platform $E$: through a higher royalty $w_E^* > \hat{w}$, platform $E$ captures part of $M_2$'s gains from facing a more efficient rival. Accordingly, $p_2$ increases, and, by strategic substitutability, $p_1$ decreases. Therefore, with diseconomies of scope, a vertical merger triggers a negative reaction from both the non-integrated manufacturer ($p_2$ increases) and the non-integrated platform ($w_E$ increases).

We provide two results for the case of the MAIN EXAMPLE with uniform distributions in the Online Appendix. First, vertical integration between $I$ and $M_1$ is profitable only when indirect network effects are not too strong: diseconomies of scope create some upstream market power for the non-integrated platform, which further increases the price of the non-integrated manufacturer. Second, total consumer surplus increases because the removal of one double marginalization more than compensates the price increase in market 2.

### 9.3. Compatibility between Platforms

Section 5 focused on the effect of scale economy in application development. A related question arises when a platform provides developers with tools to help them port their applications (an "adapter" to use the terminology of Katz and Shapiro, 1985). For instance, Microsoft Project Islandwood helps developers to port their iOS application in the Windows 10 mobile system.[36]

[36]Microsoft Project Astoria, aimed to build a bridge for developers between Android and Windows 10 mobile, has been recently discarded, because of technical reasons and the redundancy between both bridges.

If, say, platform $I$ is compatible with platform $E$ whereas the reverse is not true (platform $E$ is not compatible with $I$), then, all else equal, $I$ receives a higher demand from developers.[37] Hence, a unilaterally compatible platform has a competitive advantage to sell its operating system since it is more attractive for manufacturers.[38] Accordingly, our analysis still applies if there is no cost advantage ($\delta = 0$) but if platform $I$ is unilaterally compatible with platform $E$.

### *9.4. Coordination Failure and Incumbency Advantage*

When manufacturers had motives of coordination (Section 5), we assumed that they were able to coordinate on the most profitable platform for them. The literature sometimes refers to an incumbency advantage, according to which a lack of coordination in one side of the market empowers the incumbent platform with some market power.[39]

If manufacturers fail to coordinate on the entrant platform when it is the cheapest platform, then the incumbent platform can charge a supra-competitive royalty while securing the affiliation of both manufacturers. Hence, under separation, $I$ makes a strictly positive profit even if it has no cost advantage.

It is a priori unclear how such an incumbency advantage impacts the incentives to integrate vertically, for what matters is the joint profit of the incumbent platform $I$ and manufacturer $M_1$. To illustrate, consider the model with motives for coordination and suppose that, under separation, $I$ can attract both manufacturers with a royalty $w > 0$, and denote by $\hat{p}_k(w)$, $\hat{\pi}_k(w)$ and $\hat{\pi}_I(w)$ the $M_k$'s price, $M_k$'s profit and $I$'s profit respectively. The joint profit of $I$ and $M_1$ is then given by

$$\begin{aligned}\hat{\pi}_I(w) + \hat{\pi}_1(w) &= (\hat{p}_1(w) - w)\, D_1(\hat{p}_1(w), \hat{p}_2(w)) + w\,(D_1(\hat{p}_1(w), \hat{p}_2(w)) + D_2(\hat{p}_2(w), \hat{p}_1(w)))\,,\\ &= \hat{p}_1(w) D_1(\hat{p}_1(w), \hat{p}_2(w)) + w D_2(\hat{p}_2(w), \hat{p}_1(w)).\end{aligned}$$

Simple computations show that[40]

$$\left.\frac{d}{dw}(\hat{\pi}_I(w) + \hat{\pi}_1(w))\right|_{w=0} > 0,$$

[37]Consider the following model of developers' demand. Developers are differentiated solely by their costs $f_I$ and $f_E$ to develop applications for $I$ and $E$ respectively. A developer with cost $f_j$ develops an application for platform $j$ if $u_S n_{Bj} - f_j \geq 0$. Suppose $f_I$ and $f_E$ are identically and independently distributed and denote by $F(\cdot)$ the probability distribution. The developers' quasi-demand for platform $j$ is defined as follow: $Q_{Sj}(n_{Bj}) = \Pr(u_S n_{Bj} \geq f_j) = 1 - F(u_B n_{Bj})$. If $I$ is compatible with $E$, we assume that a developer with costs $(f_I, f_E)$ has a cost to develop an application for $I$ equal to $\min\{f_I, f_E\}$. Accordingly the developers' quasi demand for $I$ is given by: $Q_{SI} = 1 - \hat{F}(u_S n_B)$, where $\hat{F}(f) = F(f)(2 - F(f))$ is the distribution of $\min\{f_I, f_E\}$. Clearly, for all $n_B$, $Q_{SI}(n_B) \geq Q_{SE}(n_B)$.

[38]Note that this competitive advantage cancels out if the other platform also decides to be compatible.

[39]See, e.g., Caillaud and Jullien (2003), Hagiu (2006) and Jullien (2011).

[40]Omitting arguments

$$\left.\frac{d(\hat{\pi}_I + \hat{\pi}_1)}{dw}\right|_{w=0} = D_2^{(I,I)} + \hat{p}_1 \frac{\partial D_1^{(I,I)}}{\partial p_2}\frac{\partial \hat{p}_2}{\partial w} \geq D_2^{(I,I)} + \hat{p}_1 \frac{\partial D_1^{(I,I)}}{\partial p_2} \geq D_2^{(I,I)} + \hat{p}_2 \frac{\partial D_2^{(I,I)}}{\partial p_2} = 0,$$

where: the first inequality stems from the fact that pass-throughs are positive and smaller than 1; the second inequality comes from the fact that $\hat{p}_1(0) = \hat{p}_2(0)$ and that the demand for a device is more responsive to its own price; the last equality stems from the first-order condition associated to $\hat{p}_2(0)$.

or, starting from a nil royalty (which is the equilibrium royalty under separation absent any incumbency advantage), a small increase enhances the joint profit of platform $I$ and manufacturer $M_1$. The intuition is that the increase in upstream profits more than makes up for the loss from the increase in $p_2$, an insight already present in Ordover et al. (1990). It follows that, if the royalty set by the incumbent platform is positive but not too large – which is the case if coordination problems are not too severe–, the joint profit of $I$ and $M_1$ under separation is higher when there is a coordination problem between manufacturers. Accordingly, the incentives to integrate are weaker.

## 10. Conclusion

We develop a model of a platform market, in which platforms interact with device manufacturers and there are indirect network effects between buyers of devices and application developers. While our prime example is the smartphone market, our analysis is relevant, more generally, to the market of connected devices also called 'the Internet of Things.'

The main messages conveyed by our analysis are twofold: first, indirect network externalties change the nature of the strategic interaction between manufacturers, and, therefore, the competitive assessment of a vertical merger; second, the sources of upstream market power, and their consequences in terms of foreclosure or consumer surplus, are different from those unveiled in the extant literature. In doing so, we warn policy-makers against a blind application of the traditional view about foreclosure in platform markets.

As in standard markets, antitrust authorities may want to limit the anti-competitive effects of vertical integration by constraining the pricing of the royalty. Our analysis somewhat supports indeed that idea: with coordination motives giving rise to harmful foreclosure, limiting the royalty paid by manufacturers prevents the non-integrated manufacturer from being hurt by the merger. In the context of platform markets, this remedy raises, however, several issues. First, capping the royalty reduces the price decrease in the market where the merger takes place, which may thus be detrimental to consumer surplus. Second, capping the royalty paid by manufacturers may prove ineffective if the relationship between platforms and manufacturers is ruled by secret contracts specifying other non observable variables. Third, a cap on the royalty is likely to impact the pricing on the developer side of the market. We leave for further research the analysis of such a behavioral remedy when a vertical merger between a platform and a manufacturer is deemed anticompetitive.

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# A. APPENDIX

## A.1. *Proof of Lemma 1*

In order to avoid 'cornered-market' solutions, where all consumers and all developers participate in equilibrium, we make the usual assumption that network effects are not too strong so that each manufacturer faces a demand that is locally elastic with respect to prices in the relevant range.

ASSUMPTION A.1 (Indirect Network Effects Are Not Too Strong). *For all relevant downstream prices $(p_1, p_2)$, total number of buyers $n_{B1} + n_{B2}$ and number of developers $n_S$,*

$$Q_S'(n_{B1} + n_{B2}) \left( \frac{\partial Q_B}{\partial n_S}(p_1, n_S) + \frac{\partial Q_B}{\partial n_S}(p_2, n_S) \right) < 1.$$

We can then show the following result.

LEMMA A.1. *For all relevant prices $(p_1, p_2)$, system (3.3) has a unique solution.*

*Proof.* Let $D(p_1, p_2) = D_1(p_1, p_2) + D_2(p_2, p_1)$. From system (3.3), we have

$$D(p_1, p_2) = Q_B(p_1, Q_S(D(p_1, p_2)) + Q(p_2, Q_S(D(p_1, p_2)). \tag{A.1}$$

Therefore, for a given pair $(p_1, p_2)$, $D(p_1, p_2)$ is a fixed point of function $\psi(x) = Q_B(p_1, Q_S(x)) + Q_B(p_2, Q_S(x))$. Notice then that

$$\psi'(x) = Q_S'(x) \left( \frac{\partial Q_B}{\partial n_S}(p_1, Q_S(x)) + \frac{\partial Q_B}{\partial n_S}(p_2, Q_S(x)) \right).$$

Assumption A.1 then implies that $|\psi'(\cdot)| < 1$: $\psi(\cdot)$ is a contraction mapping and Equation (A.1) has a unique solution. □

We can now prove Lemma 1. By the implicit function theorem, $D(p_1, p_2)$ is continuously differentiable. Differentiating wrt $p_1$ in Equation (A.1) and rearranging terms, we find

$$\frac{\partial D}{\partial p_1}(p_1, p_2) \left\{ 1 - Q_S'(D(p_1, p_2)) \left( \frac{\partial Q_B}{\partial n_S}(p_1, D(p_1, p_2)) + \frac{\partial Q_B}{\partial n_S}(p_2, D(p_1, p_2)) \right) \right\} = \frac{\partial Q_B}{\partial p_1}(p_1, D(p_1, p_2)).$$

By Assumption A.1 the term in curly brackets is positive, and, therefore, $\frac{\partial D}{\partial p_1}(p_1, p_2)$ is negative. Similarly, $\frac{\partial D}{\partial p_2}(p_1, p_2) < 0$. Using a similar argument, it follows that $D_1(\cdot, \cdot)$, $D_2(\cdot, \cdot)$ and $D_S(\cdot, \cdot)$ are all decreasing in $p_1$ and $p_2$.

## A.2. *Proof of Lemma 2*

By definition, $(p_1^{(E)}(w), p_2^{(E)}(w))$ and $(p_1^{(I1)}(w), p_2^{(I1)}(w))$ solve respectively

$$\begin{cases} D_1\left(p_1^{(E)}(w), p_2^{(E)}(w)\right) + p_1^{(E)}(w) \dfrac{\partial D_1}{\partial p_1}\left(p_1^{(E)}(w), p_2^{(E)}(w)\right) = 0 \\ D_2\left(p_2^{(E)}(w), p_1^{(E)}(w)\right) + \left(p_2^{(E)}(w) - w\right) \dfrac{\partial D_2}{\partial p_2}\left(p_2^{(E)}(w), p_1^{(E)}(w)\right) = 0 \end{cases} \tag{A.2}$$

and
(A.3)

$$\begin{cases} D_1\left(p_1^{(I1)}(w), p_2^{(I1)}(w)\right) + p_1^{(I1)}(w)\dfrac{\partial D_1}{\partial p_1}\left(p_1^{(I1)}(w), p_2^{(I1)}(w)\right) + w\dfrac{\partial D_2}{\partial p_1}\left(p_2^{(I1)}(w), p_1^{(I1)}(w)\right) = 0 \\ D_2\left(p_2^{(I1)}(w), p_1^{(I1)}(w)\right) + \left(p_2^{(I1)}(w) - w\right)\dfrac{\partial D_2}{\partial p_2}\left(p_2^{(I1)}(w), p_1^{(I1)}(w)\right) = 0 \end{cases}$$

Let $R_k^{(j)}(p_l, w)$ denote manufacturer $M_k$'s best response to manufacturer $M_l$ $(l \neq k)$ when $M_2$ affiliates with platform $j \in \{I, E\}$ at some royalty $w$. From Equations (A.2) and (A.3), $R_2^{(E)}(p_1, w) = R_2^{(I1)}(p_1, w)$ and $R_1^{(E)}(p_2, w) = R_1^{(I1)}(p_2, 0)$. By definition $\frac{\partial \pi_{I1}^{(I1)}}{\partial p_1}(R_1^{(I)}(p_2, w), p_2, w) = 0$. Therefore (omitting some notations to ease the exposition)

$$\frac{\partial R_1^{(I1)}}{\partial w} = -\frac{\frac{\partial^2 \pi_{I1}^{(I1)}}{\partial p_1 \partial w}}{\frac{\partial^2 \pi_{I1}^{(I1)}}{\partial^2 p_1}} = -\frac{\frac{\partial D_2}{\partial p_1}}{\frac{\partial^2 \pi_{I1}^{(I1)}}{\partial^2 p_1}},$$

which is strictly negative since the technical assumptions require that the second-order condition is satisfied. Therefore, $M_1$'s best response shifts downward when $w$ increases. Hence, in particular, $R_1^{(I1)}(p_2, w) < R_1^{(I1)}(p_2, 0) = R_1^{(E)}(p_2, w)$. It follows that

$$\text{(A.4)} \qquad p_1^{(I1)}(w) = R_1^{(I1)}\left(R_2^{(I1)}(p_1^{(I1)}(w), w), w\right) < R_1^{(E)}\left(R_2^{(E)}(p_1^{(I1)}(w), w), w\right).$$

Define now $\Phi(p) = R_1^{(E)}(R_2^{(E)}(p, w), w) - p$, and notice that $\Phi(p_1^{(E)}(w)) = 0$. $\Phi(\cdot)$ is continuously differentiable and strictly decreasing, since the slopes of the best responses are strictly smaller than 1 from the technical assumptions. Therefore, $\Phi(p) > 0$ if and only if $p < p_1^{(E)}(w)$. Together with inequality (A.4), this implies that $p_1^{(I1)}(w) < p_1^{(E)}(w)$.

The second part of Lemma 2 is immediate from the strategic substitutability between downstream prices.

### *A.3. Proof of Lemma 3*

*(i)* Let $w > 0$. We have

$$\begin{aligned} \pi_2^{(I1)}(w) &= (p_2^{(I)}(w) - w)D_2\left(p_2^{(I1)}(w), p_1^{(I1)}(w)\right) \\ &> (p_2^{(E)}(w) - w)D_2\left(p_2^{(E)}(w), p_1^{(I1)}(w)\right) \qquad \text{(by revealed preferences)} \\ &> (p_2^{(E)}(w) - w)D_2\left(p_2^{(E)}(w), p_1^{(E)}(w)\right) = \pi_2^{(E)}(w), \end{aligned}$$

where the last inequality comes from the fact that $p_1^{(I1)}(w) < p_1^{(E)}(w)$ (by Lemma 2) and $D_2(p_2, p_1)$ is decreasing in $p_1$.

*(ii)* Similarly, we have

$$\begin{aligned} \pi_{I1}^{(I1)}(0) &= p_1^{(I1)}(0)D_1\left(p_1^{(I1)}(0), p_2^{(I1)}(0)\right) \\ &> p_1^{(E)}(w)D_1\left(p_1^{(E)}(w), p_2^{(I1)}(0)\right) \qquad \text{(by revealed preferences)} \\ &> p_1^{(E)}(w)D_1(p_1^{(E)}(w), p_2^{(E)}(w)) = \pi_{I1}^{(E)}(w), \end{aligned}$$

where the last inequality comes from the fact that $p_2^{(I1)}(0) = p_2^{(E)}(0) < p_2^{(E)}(w)$ and $D_1(p_1, p_2)$ is decreasing in $p_2$.

*(iii)* Let us show that $\pi_2^{(I1)}(w)$ is decreasing in $w$: (omitting some notations to ease the exposition)

$$\begin{aligned}\left(\pi_2^{(I1)}\right)'(w) &= -D_2 + (p_2^{(I1)}(w) - w)\tfrac{\partial D_2}{\partial p_1}\tfrac{\partial p_1^{(I1)}}{\partial w} && \text{(by the envelope theorem)}\\ &= (p_2^{(I1)}(w) - w)\tfrac{\partial D_2}{\partial p_1}\left(1 + \tfrac{\partial p_1^{(I1)}}{\partial w}\right) && \text{(using } M_2\text{'s first-order condition)}\\ &< 0,\end{aligned}$$

where the last inequality is obtained using the facts that the pass-throughs are smaller than 1 under the technical assumptions, so that $1 + \frac{\partial p_1^{(I1)}}{\partial w} > 0$.

Then, since $\pi_2^{(I1)}(\delta) > \pi_2^{(E)}(\delta) > 0$ by Lemma 3 and $\pi_2^{(I1)}(w) = 0$ when $w$ is large enough, there exists a unique royalty $\overline{w} > \delta$ such that $\pi_2^{(I)}(w) > \pi_2^{(E)}(\delta)$ iff $w < \overline{w}$. By definition of $\overline{w}$, $M_2$ affiliates with platform $I$ iff $w < \overline{w}$.

### *A.4. Proof of Lemma 4*

We only have to prove that the optimal royalty $w^*$ is positive. To this end, let us show that $\pi_{I1}^{(I)}(w)$ is increasing in $w$ in the neighborhood of 0. We have

$$\begin{aligned}\left(\pi_{I1}^{(I1)}\right)'(0) &= \frac{\partial p_2^{(I1)}}{\partial w}(0)\frac{\partial D_1}{\partial p_2}\left(p_1^{(I1)}(0), p_2^{(I1)}(0)\right)p_1^{I1}(0) + D_2\left(p_2^{(I1)}(0), p_1^{(I1)}(0)\right),\\ &> \frac{\partial D_1}{\partial p_2}\left(p_1^{(I1)}(0), p_2^{(I1)}(0)\right)p_1^{(I1)}(0) + D_2\left(p_2^{(I1)}(0), p_1^{(I1)}(0)\right),\\ &> \frac{\partial D_2}{\partial p_2}\left(p_2^{(I1)}(0), p_1^{(I1)}(0)\right)p_2^{(I1)}(0) + D_2\left(p_2^{(I1)}(0), p_1^{(I1)}(0)\right) = 0,\end{aligned}$$

where the first inequality stems from the fact that pass-throughs are positive and smaller than 1, $0 < \frac{\partial p_2^{(I1)}}{\partial w}(0) < 1$, and $\frac{\partial D_1}{\partial p_2} \le 0$; the second inequality stems from the fact that $p_1^{(I1)}(0) = p_2^{(I1)}(0)$ and $|\frac{\partial D_2}{\partial p_2}(p_1, p_2)| > |\frac{\partial D_1}{\partial p_2}(p_1, p_2)|$; the last equality stems from the first-order condition associated to $p_2^{(I1)}(0)$.

### *A.5. Proof of Proposition 2*

Let $\hat{p}_k(w)$ manufacturer $M_k$'s price under no integration when it pays a royalty $w$. We start by proving the following lemma.

LEMMA A.2. *For all $w$, $p_1^{(I1)}(w) < \hat{p}_1(w) = \hat{p}_2(w) < p_2^{(I1)}(w)$.*

*Proof.* It is immediate that $\hat{p}_1(w) = \hat{p}_2(w) \equiv \hat{p}(w)$ since $M_1$ and $M_2$ are symmetric under separation.

By definition, prices $\hat{p}(w)$ and $(p_1^{(I1)}(w), p_2^{(I1)}(w))$ solve respectively (we omit some arguments to ease exposition)

$$D_1(\hat{p}, \hat{p}) + (\hat{p} - w)\frac{\partial D_1}{\partial p_1}(\hat{p}, \hat{p}) = 0, \tag{A.5}$$

and

$$\begin{cases} D_1\left(p_1^{(I1)}, p_2^{(I1)}\right) + p_1^{(I1)}\dfrac{\partial D_1}{\partial p_1}\left(p_1^{(I1)}, p_2^{(I1)}\right) + w\dfrac{\partial D_2}{\partial p_1}\left(p_2^{(I1)}, p_1^{(I1)}\right) = 0,\\ D_2\left(p_2^{(I1)}, p_1^{(I1)}\right) + \left(p_2^{(I1)} - w\right)\dfrac{\partial D_2}{\partial p_2}\left(p_2^{(I1)}, p_1^{(I1)}\right) = 0.\end{cases} \tag{A.6}$$

Let $\hat{R}_k(p_l, w)$ (respectively, $R_k^{(I1)}(p_l, w)$) denote $M_k$'s best response to $p_l$ ($l \neq k$) when both $M_1$ and $M_2$ pay a royalty $w$ under separation (respectively, under integration when $M_2$ affiliates with $I1$). Notice that, for all $p_1$, $\hat{R}_2(p_1, w) = R_2^{(I1)}(p_1, w)$.

By definition, $\frac{\partial \pi_1}{\partial p_1}\left(\hat{R}_1, p_2, w\right) = \frac{\partial \pi_{I1}^{(I1)}}{\partial p_1}\left(R_1^{(I1)}, p_2, w\right) = 0$, where $\pi_1(p_1, p_2, w)$ is $M_1$'s profit under separation: $\pi_1(p_1, p_2, w) = (p_1 - w)D_1(p_1, p_2)$. Notice that $\pi_{I1}^{(I1)}(p_1, p_2, w) = \pi_1(p_1, p_2, w) + w\,(D_1(p_1, p_2) + D_2(p_1, p_2))$. Therefore we have

$$\frac{\partial \pi_1}{\partial p_1}\left(\hat{R}_1, p_2, w\right) - \frac{\partial \pi_1}{\partial p_1}\left(R_1^{(I1)}, p_2, w\right) = 0 - w\left(\frac{\partial D_1}{\partial p_1}(R_1^{(I1)}, p_2) + \frac{\partial D_2}{\partial p_1}(p_2, R_1^{(I1)})\right).$$

Since the right-hand side is positive when $w > 0$ and $\frac{\partial^2 \pi_1}{\partial p_1^2} < 0$ from the technical assumptions, it follows that, for all $p_2$, $R_1^{(I1)}(p_2, w) < \hat{R}_1(p_2, w)$. In particular

$$\text{(A.7)}\qquad p_1^{(I1)}(w) = R_1^{(I1)}\left(R_2^{(I1)}\left(p_1^{(I1)}(w), w\right), w\right) < \hat{R}_1\left(\hat{R}_2\left(p_1^{(I1)}(w), w\right), w\right),$$

since, for all $p_1$, $\hat{R}_2(p_1, w) = R_2^{(I1)}(p_1, w)$.

Define now $\Phi(p) = \hat{R}_1\left(\hat{R}_2(p, w), w\right) - p$, and notice that $\Phi(\hat{p}(w)) = 0$. $\Phi(.)$ is continuously differentiable and strictly decreasing, since the slopes of the best-response functions are strictly smaller than 1. Therefore, $\Phi(p) > 0$ if and only if $p < \hat{p}(w)$. Together with inequality (A.7), this implies that $p_1^{(I1)}(w) < \hat{p}(w)$.

To conclude, notice that

$$p_2^{(I1)}(w) = R_2^{(I1)}\left(p_1^{(I1)}(w), w\right) = \hat{R}_2\left(p_1^{(I1)}(w), w\right) > \hat{R}_2\left(\hat{p}(w), w\right) = \hat{p}(w),$$

where the inequality comes from the fact that prices are strategic substitutes. □

The first item in Proposition 2 then obtains immediately. Since $M_1$ and $M_2$'s best responses shift downward and upward respectively when $w$ increases, we have that $p_1^{(I1)}(w)$ is decreasing in $w$ and $p_2^{(I1)}(w)$ is increasing in $w$. Then, since $w^* > \delta$,

$$p_1^{(I1)}(w^*) < p_1^{(I1)}(\delta) < \hat{p}_1(\delta) = \hat{p}_1,$$
$$p_2^{(I1)}(w^*) > p_2^{(I1)}(\delta) > \hat{p}_2(\delta) = \hat{p}_2.$$

### *A.6. Proof of Proposition 3*

We have

$$\begin{aligned}
\pi_2^{(I1)}(\overline{w}) &= \pi_2\left(p_1^{(I1)}(\overline{w}), p_2^{(I1)}(\overline{w}), \overline{w}\right) \\
&= \pi_2\left(p_1^{(E)}(\delta), p_2^{(E)}(\delta), \delta\right) && \text{(by definition of } \overline{w}) \\
&\geq \pi_2\left(p_1^{(E)}(\delta), \hat{p}_2, \delta\right) && \text{(by revealed preferences)} \\
&\geq \pi_2(\hat{p}_1, \hat{p}_2, \delta) = \hat{\pi}_2 && \text{(since } \hat{p}_1 > p_1^{(E)}(\delta) \text{ and } \tfrac{\partial \pi_2}{\partial p_1} < 0)
\end{aligned}$$

To conclude notice that, for all $w^* \in [\delta, \overline{w}]$, $\pi_2^{(I1)}(w^*) \geq \pi_2^{(I1)}(\overline{w})$.

### A.7. *Proof of Proposition 4*

After calculations, we find

$$\begin{aligned}\pi_1^{(I)}(w^*) - (\hat{\pi}_1 + \hat{\pi}_I) = \; & 2\mu^2(1-2\mu)(2-3\mu)(2-3(2-\mu)\mu) \\ & +(1-\mu)(16-\mu(80+\mu(-104+\mu(-48+\mu(172+\mu(-88+9\mu))))))\tfrac{\delta}{v},\end{aligned}$$

Notice that

- $2\mu^2(1-2\mu)(2-3\mu)(2-3(2-\mu)\mu)$ has the sign of $2-3(2-\mu)\mu$ on $[0,1]$. Therefore, it is positive iff $\mu < \hat{\mu}_1 = 1-1/\sqrt{3} \simeq 0.42$.
- Numerical simulations shows that $(1-\mu)(16-\mu(80+\mu(-104+\mu(-48+\mu(172+\mu(-88+9\mu))))))$ has only one root $\hat{\mu}_2 \simeq 0.43$ in $(0,1)$ and that it is positive iff $\mu < \hat{\mu}_2$.

It follows that integration is profitable if $\mu < \min\{\hat{\mu}_1, \hat{\mu}_2\} = \hat{\mu}_1$ and is not profitable if $\mu > \max\{\hat{\mu}_1, \hat{\mu}_2\} = \hat{\mu}_2$. Therefore, there exists $\hat{\mu} \in (\hat{\mu}_1, \hat{\mu}_2)$ such that a vertical merger is profitable iff $\mu < \hat{\mu}$.

### A.8. *Proof of Proposition 5*

Denote respectively by $\hat{p}_k^{(i,j)}(w_1, w_2)$ and $p_k^{(i,j)}(w_1, w_2)$ manufacturer $M_k$'s prices under separation and integration when $M_1$ affiliates with platform $i$ and $M_2$ with $j$, with $(i,j) \in \{I, E\}^2$.

*(i)* We proceed in three steps.

STEP 1: ACCOMMODATION EFFECT. By the same argument than in the proof of Lemma 2, we have, for all $w \geq 0$,

$$p_1^{(I1,I1)}(0,w) \leq \hat{p}_1^{(I,I)}(0,w) \text{ and } p_2^{(I1,I1)}(0,w) \geq \hat{p}_2^{(I,I)}(0,w).$$

STEP 2: INCENTIVES TO JOIN AND INCENTIVES TO SERVE. Let us prove that, for all $w > 0$, $\pi_2^{(I1,E)}(0,w) < \pi_2^{(I1,I1)}(0,w)$. We have:

$$\begin{aligned}\pi_2^{(I1,E)}(0,w) &= \hat{\pi}_2^{(I,E)}(0,w) \\ &< \hat{\pi}_2^{(I,I)}(0,w) && \text{(by Assumption 5)} \\ &< \pi_2^{(I1,I1)}(0,w) && \text{(by Step 1 and revealed preferences)}\end{aligned}$$

The proof for establishing that platform $I1$ is better off serving manufacturer $M_2$ is similar to the proof of Lemma 3 and is therefore omitted.

STEP 3: OPTIMAL ROYALTY. The optimal royalty solves

$$\max_w \pi_{I1}^{(I1,I1)}(0,w) \text{ s.t. } \pi_2^{(I1,I1)}(0,w) \geq \pi_2^{(I1,E)}(0,0). \tag{A.8}$$

Since (i) $w \mapsto \pi_2^{(I1,I1)}(0,w)$ is decreasing in $w$, (ii) $\pi_2^{(I1,I1)}(0,0) > \hat{\pi}_2^{(I,E)}(0,0) > 0$ (by Step 2), (iii) $\exists \lim_{w \to +\infty} \pi_2^{(I1,I1)}(0,w) = 0$, there exists a unique $\overline{\overline{w}} > 0$ such that $\pi_2^{(I1,I1)}(0,w) > \hat{\pi}_2^{(I,E)}(0,0)$ iff $w < \overline{\overline{w}}$. Equation (A.8) then rewrites:

$$\max_w \pi_{I1}^{(I1,I1)}(0,w) \text{ s.t. } w \in [0, \overline{\overline{w}}].$$

Following the same argument as in the proof of Lemma 4 in Appendix A.4, one can show that $w \mapsto \pi_{I1}^{(I1,I1)}(0,w)$ is increasing in the neighborhood of 0. Therefore, $w \mapsto \pi_{I1}^{(I1,I1)}(0,w)$

is increasing in $w$ in the neighborhood of 0. It follows that the optimal royalty $w^{**}$ is positive and is defined by $w^{**} = \min\{\overline{\overline{w}}, w_m\}$, where $w_m = \arg\max_w \pi_{I1}^{(I1,I1)}(0,w)$ is given by the f.o.c. $\partial\pi_{I1}^{(I1,I1)}(0,w)/\partial w = 0$.

*(ii)* Since (i) $w \mapsto \pi_2^{(I1,I1)}(0,w)$ is decreasing in $w$, (ii) $w^{**} > 0$ and (iii) $\pi_2^{(I1,I1)}(0,0) = \hat{\pi}_2^{(I,I)}(0,0)$, it follows that $M_2$ is made worse-off by the merger.

*(iii)* Finally, by setting $w = 0$, the integrated platform can ensure a profit equal to the joint profit of platform $I$ and manufacturer $M_1$ under separation. By revealed preferences, the equilibrium profits of the integrated platform is higher and the vertical merger is always profitable.

### *A.9. Proof of Lemma 5*

We prove the case where $G(.)$ is concave. The case where $G(.)$ is convex follows similar steps.

We start by proving two intermediate lemmas.

LEMMA A.3. *For all $d \geq 0$, the iso-participation curve $\{(p_1, p_2) \mid D_S(p_1, p_2) = d\}$ is convex.*

*Proof.* By definition, for all $(p_1, p_2)$, $D_S(p_1, p_2)$ solves

$$D_S = Q_S(Q_B(p_1, D_S) + Q_B(p_2, D_S)).$$

Differentiating this equation, we obtain

(A.9)
$$dD_S = Q_S'(D_B)\left(\frac{\partial Q_B}{\partial p}(p_1, D_S)dp_1 + \frac{\partial Q_B}{\partial n_S}(p_1, D_S)dD_S + \frac{\partial Q_B}{\partial p}(p_2, D_S)dp_2 + \frac{\partial Q_B}{\partial n_S}(p_2, D_S)dD_S\right).$$

Let $\{(p_1, \psi(p_1)) \mid D_S(p_1, \psi(p_1)) = d\}$ be an iso-participation curve. By Equation (A.9), the slope of this curve is given by

$$\psi'(p_1) = \left.\frac{dp_2}{dp_1}\right|_{dD_S=0} = -\frac{\frac{\partial Q_B}{\partial p}(p_1, D_S(p_1, \psi(p_1)))}{\frac{\partial Q_B}{\partial p}(\psi(p_1), D_S(p_1, \psi(p_1)))}. \tag{A.10}$$

In particular $\psi'(p_1) < 0$. Taking the derivative wrt $p_1$ in Equation (A.10), we obtain

$$\begin{aligned}\psi''(p_1) \quad = \quad & -\frac{1}{\frac{\partial Q_B}{\partial p}(\psi(p_1), D_S)^2}\left\{\left(\frac{\partial^2 Q_B}{\partial p^2}(p_1, D_S) + \frac{\partial^2 Q_B}{\partial p \partial n_S}(p_1, D_S)\underbrace{\frac{dD_S}{dp_1}(p_1, \psi(p_1))}_{=0}\right)\frac{\partial Q_B}{\partial p}(\psi(p_1), D_S)\right. \\ & \left. - \left(\frac{\partial^2 Q_B}{\partial p^2}(\psi(p_1), D_S)\psi'(p_1) + \frac{\partial^2 Q_B}{\partial p \partial n_S}(\psi(p_1), D_S)\underbrace{\frac{dD_S}{dp_1}(p_1, \psi(p_1))}_{=0}\right)\frac{\partial Q_B}{\partial p}(p_1, D_S)\right\}.\end{aligned}$$

Since $G(.)$ is concave, $p \mapsto Q_B(p, n_S) = 1 - G(v + u_B n_S - p)$ is convex: $\forall (p, n_S)$, $\partial^2 Q_B/\partial p^2(p, n_S) > 0$. Therefore, the term in curly brackets in the right-hand side is negative since $\partial Q_B/\partial p < 0$ and $\psi' < 0$ and, therefore, $\psi''(\cdot) > 0$. This concludes the proof. □

LEMMA A.4. *Let $\Delta p_1 > \Delta p_2 > 0$. For all $p$, $D_S(p - \Delta p_1, p + \Delta p_2) > D_S(p, p)$.*

*Proof.* Notice that the iso-participation curve $\{(p_1, p_2) \mid D_S(p_1, p_2) = D_S(p, p)\}$ has slope -1 at some point $(p, p)$ (by Equation (A.10)). Since this iso-participation curve is convex (by Lemma

A.3), it is above its tangent at point $(p,p)$: for all $p_1$, $D_S(p,p) \leq D_S(p_1, 2p - p_1)$. To conclude, notice that

$$D_S(p - \Delta p_1, p + \Delta p_2) > D_S(p - \Delta p_1, p + \Delta p_1) = D_S(p - \Delta p_1, 2p - (p - \Delta p_1)) \geq D_S(p,p)$$

since $D_S(p_1,p_2)$ is decreasing in $p_2$ and $\Delta p_1 > \Delta p_2 > 0$. □

Under separation, $M_1$ and $M_2$ set prices $p_1 = p_2 = \hat{p}$. Suppose then that, following integration between $I$ and $M_1$, downstream prices are given by $\tilde{p}_1 = \hat{p} - \Delta p_1$ and $\tilde{p}_2 = \hat{p} + \Delta p_2$, with $\Delta p_1 > \Delta p_2 > 0$. Denote by $\hat{D}_S = D_S(\hat{p},\hat{p})$ and $\tilde{D}_S = D_S(\tilde{p}_1,\tilde{p}_2)$ the number of developers under separation and integration respectively.

It is immediate from Lemma A.4 that the merger increases developers' participation: $\tilde{D}_S > \hat{D}_S$.

Consider now the impact of the vertical merger on consumer surplus.

- Consider the variation in consumer surplus in market 1, that is

$$\Delta S_1 = \int_{u_B\hat{D}_S - \hat{p}}^{u_B\tilde{D}_S - \tilde{p}_1} G(\varepsilon)d\varepsilon.$$

  Since $u_B\tilde{D}_S - \tilde{p}_1 = u_B\tilde{D}_S - \hat{p} + \Delta p_1 > u_B\hat{D}_S - \hat{p}$, $\Delta S_1$ is positive.

- Consider the variation in total surplus $\Delta S = \Delta S_1 + \Delta S_2$, that is

$$\begin{aligned}
\Delta S &= \int_{v+u_B\hat{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}+\Delta p_1} G(\varepsilon)d\varepsilon + \int_{v+u_B\hat{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}-\Delta p_2} G(\varepsilon)d\varepsilon \\
&= 2\int_{v+u_B\hat{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}} G(\varepsilon)d\varepsilon + \int_{v+u_B\tilde{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}+\Delta p_1} G(\varepsilon)d\varepsilon + \int_{v+u_B\tilde{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}-\Delta p_2} G(\varepsilon)d\varepsilon, \\
&\geq 2\int_{v+u_B\hat{D}_S-\hat{p}}^{v+u_B\tilde{D}_S-\hat{p}} G(\varepsilon)d\varepsilon + \Delta p_1 G\left(v + u_B\tilde{D}_S - \hat{p}\right) - \Delta p_2 G\left(v + u_B\tilde{D}_S - \hat{p}\right),
\end{aligned}$$

  where the inequality stems from the fact that $G(\cdot)$ is non-negative and increasing. Since $v + u_B\tilde{D}_S - \hat{p} > v + u_B\hat{D}_S - \hat{p}$ and $\Delta p_1 > \Delta p_2$, the previous inequality implies that $\Delta S \geq 0$.

### A.10. Proof of Proposition 9

Simple manipulations show that $M_1$'s demand is given by

$$D_1(p_1,p_2) = \frac{1}{1-2\mu}\left(v - p_1(1-(1+\gamma)\mu) + p_2(\gamma - (1+\gamma)\mu)\right) \tag{A.11}$$

The nature of strategic interaction is given by the sign of $\partial^2\pi_1/\partial p_1\partial p_2$, which has the sign of $\partial D_1/\partial p_2$ when demands are linear. From equation (A.11), it is then immediate that prices are strategic complements if $\gamma - (1+\gamma)\mu > 0$ and strategic substitutes otherwise.

The proof of the second part of Proposition 9 involves tedious calculations and is thus relegated to the Online Appendix.

### A.11. *Complementary Result: Discussion of Assumption 4*

For the sake of completeness, we study here, first, the consequences of a vertical merger that leads to a decrease of the royalty paid by $M_2$ below its pre-merger level (that is, $w^* \leq \delta$), and, second, the case where $\delta$ is sufficiently small and show that, then, the royalty increases following the merger.

Since the formal arguments follow closely those developed in the proofs of Propositions 2, 3 and 5, we content ourselves with a graphical representation of the intuition.

Points $a$, $b$ and $c$ in Figure 5 represent respectively the equilibrium prices under integration when $M_2$ affiliates with $E$ and pays a royalty $\delta$ (point $a$), when $M_2$ affiliates with $I1$ and pays a royalty $\delta$ (point $b$) and when $M_2$ affiliates with $I1$ and pays a royalty 0 (point $c$). By construction, for all $w^* \in [0, \delta]$, the equilibrium prices lie somewhere in the shaded area. In particular, manufacturer $M_1$'s price always decreases following the vertical merger ($p_1^{(I1)}(w^*) < \hat{p}_1$) and manufacturer $M_2$'s may as well increase or decrease. If both prices decrease, then the analysis is immediate. If $p_2$ increases, the variation in $p_1$ is always larger than the variation in $p_2$. Our results on consumer surplus and welfare obtain accordingly.

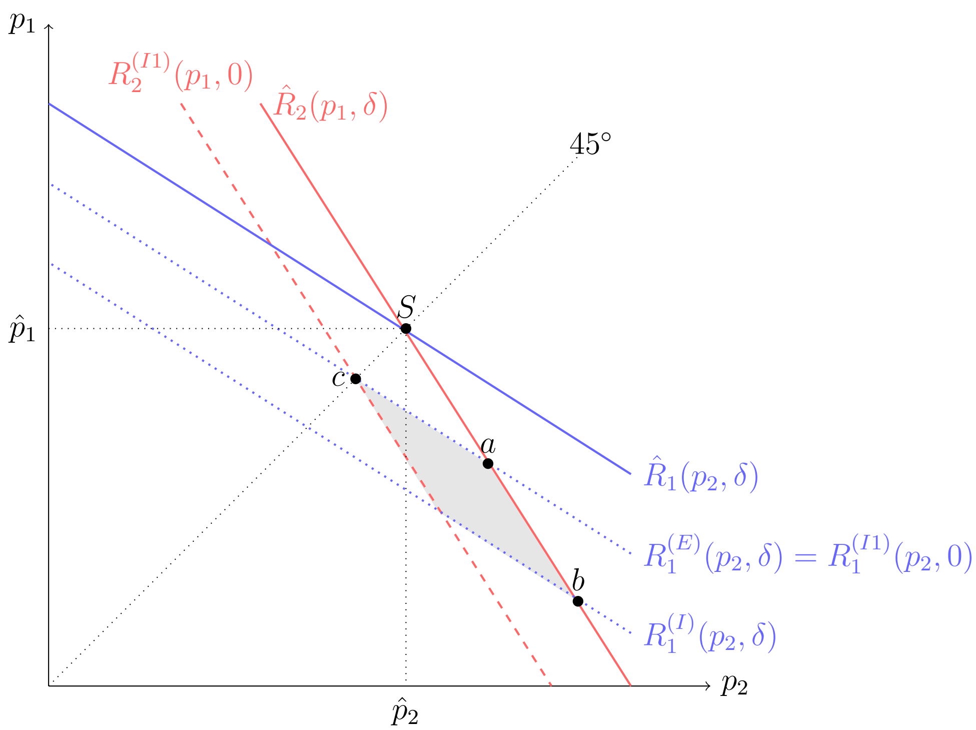


Figure 5 – The impact of vertical integration on equilibrium downstream prices when $M_2$'s royalty decreases following the merger ($w^* \in [0, \delta]$).

We now prove the following complementary result.

LEMMA A.5. *For $\delta$ positive but sufficiently close to 0, $w^* = \overline{w}$.*

*Proof.* We already know that $\pi_{I1}^{(I)}(w)$ is increasing in $w$ in the neighborhood of 0 (see the proof of Lemma 4 in Appendix A.4). It follows that $\pi_{I1}^{(I1)}(w)$ is increasing in $w$ on $(0, \overline{w})$ if $\overline{w}$ is small enough.

To conclude, let us prove that $\overline{w}$ goes to 0 when $\delta$ goes to 0. Recall that $\overline{w}$ is uniquely defined by

$$\pi_2^{(I1)}(\overline{w}) = \pi_2^{(E)}(\delta). \tag{A.12}$$

Notice first that, since $w \mapsto \pi_2^{(I1)}(w)$ and $w \mapsto \pi_2^{(E)}(w)$ are both strictly decreasing in $w$, $\overline{w}$ is strictly increasing in $\delta$. Then, if $\delta = 0$, the only solution for equation (A.12) is $\overline{w} = 0$. It is indeed apparent from the first order conditions on $p_1$ and $p_2$ that, for all $k = 1, 2$, $p_k^{(I1)}(0) = p_k^{(E)}(0)$, so that $\pi_2^{(I1)}(0) = \pi_2^{(E)}(0)$. These two observations together show that $\overline{w}$ is in a right neighborhood of 0 when $\delta$ is small enough. This concludes the proof.

□