The mathematical chapter of the Āryabhaṭīya: A compilation of mnemonic rules?

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THE MATHEMATICAL CHAPTER OF THE *ĀRYABHAṬĪYA*: A COMPILATION OF ORAL MNEMONIC RULES?[1]

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ABSTRACT

Āryabhaṭa (a fifth century astronomer)'s mathematical rules (sūtras) composed in Sanskrit are often considered to be oral with a mnemonic aim and an educational purpose. In this paper, a close look at how these rules were crafted will be carried out in an attempt to specify the discussion on how such rules would have been composed, displayed, stored and transmitted in a context of mixed orality. The question of their authorship is raised along the way.

Les règles (sūtras) mathématiques Sanskrites d'Āryabhaṭa, un astronome du cinquième siècle, sont souvent considérées comme mnémoniques, issues d'un enseignement oral. Nous regarderons minutieusement comment ces textes sont rédigés, afin de tenter de mieux cerner comment de telles règles ont pu être composées, présentées, stockées et transmises dans un contexte d'oralité mixte. La question de qui est l'auteur de ces vers sera aussi un fil de la discussion.

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1. Introduction: Āryabhaṭa, astral science and oral texts

Āryabhaṭa's *Āryabhaṭīya* (Ab), is a 5th century cornerstone astronomy treatise. It was continuously commented upon in the Indian sub-continent until the end of the 19th century [2]. It gave rise to "schools" (*pakṣa*) who adopted its parameters[3]. However, like most texts from that period in the Indian subcontinent, we have little knowledge of its context of production, through what medium its author thought his text would be preserved, transmitted or displayed. The text is composed in versified *sūtra*s. These compact aphorisms are difficult to understand alone.

The earliest commentary preserved of this text was written by Bhāskara, at the beginning of the 7th century. Explaining why the author of the treatise would give his name, and why he would speak of himself in the third person, among a list of possible arguments he writes[4]:

> *athavā yas tejasvī puruṣaḥ samareṣu nikṛṣṭāsitejovitānacchuritabāhuś śatrusaṅghātam prakāśaṃ praviśya praharan evam āha – "ayam asāv udito 'ditikulaprasūtaḥ samareṣv anivāritavīryo yajñadattaḥ praharati | yadi kasyacic chaktiḥ pratipraharatv" iti | evam asav apy ācāryo gaṇitakālakriyāgolātiśayajñānodadhipārago vitsabhām avagāhya "āryabhaṭaḥ trīṇi gadati gaṇitam kālakriyām golam" iti uktavān |*
>
> Or as a heroic man on battle fields, whose arms have been copiously lacerated by the strength of vile swords, having entered publicly a battle with enemies, who proclaims the following, as he kills: 'This Yajñadatta here ascended, a descendant of the Aditis, having undaunted courage in battle fields, <now> strikes. If someone has power, let him strike back!'
> In the same way, this master also, who has reached the other side of the ocean of excessive knowledge about Mathematics, Time-reckoning and the Sphere, having entered an assembly of wise men, has declared:
> ‘Āryabhaṭa tells three: Mathematics, Time-reckoning, the Sphere'

With whom does Bhāskara imagine Āryabhaṭa battling? Is it with past authors? In the known context of public courtly debates? In the Indian equivalent of an university *disputatio*, were Āryabhaṭa would have been a known and celebrated master (*ācārya*)? Indeed, it seems that a century and a half years after the text was composed, the context of production of Āryabhaṭa's treatise was already a riddle. Bhāskara 's commentary however leaves no doubt to the fact that the display of the *Āryabhaṭīya* was oral.

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2 [Shukla & Sarma 1976, xxxv-lxii]

3 [Pingree 1981, 13-16]

4 [Shukla 1976; 5]

If we take the text alone, using commentaries as guides, what can be retrieved? Can anything at all be said, although we know practically nothing of the context of the text's production and posterity? Can we learn anything about the way the *Āryabhaṭīya* was composed? Do we have clues concerning the medium through which this text was to be preserved, displayed and used?

Secondary literature has often imagined for this text, and other astral *siddhānta*s an educational background, juxtaposing two tropes, which feed one another: first, an educational situation is described within which the treatise is composed, transmitted and commented upon, second the mnemonic character of the *sūtra*s is emphasized. It is assumed that education was mostly face to face. Orality appears first as the way the text is transmitted: spoken by Āryabhaṭa to his pupils. Was this under the form of a formal speech [5]? Indeed, another form of orality is usually evoked, in the informal speech that written commentaries provide an echo of. For example[6]:

> It is significant that the earliest prose commentaries in exact sciences in Sanskrit was that on the *Āryabhaṭīya*. As we have seen above, the verses in this text are so brief and condensed that they are very difficult to understand. The situation must have been the same when the text was orally communicated by Āryabhaṭa to his students. The students could have memorized the 120 verses correctly even without understanding them. Then the teacher might have given explanation (sic), sometimes putting down numeral symbols (*nyāsa*), giving examples (*udāharaṇa* or *uddeśaka*) and drawing figures (*parirekha*).

Here then, Āryabhaṭa's text is seen as having been transmitted orally, personally by the author himself. Explanation of the rules seems to require both informal oral explanations, but also the writing of numbers and the drawing of diagrams. In other words, explanations involve tracing activities which might not be text writing per se, but suggest that writing is not alien to such a culture. In this context, the writings themselves would be "informal", ephemeral, by contrast to a formal, written manuscript, made to be copied and transmitted. Michio Yano points out to us then that the *Āryabhaṭīya* belongs to what Ong would have termed a context of second orality[7]. A striking feature of this reconstruction is the idea that verses were learnt by heart before they were understood. Are such reconstructions validated by the sources?

In the following we will investigate first what can be said of traditions of oral texts in Sanskrit literature at large, we will then look closely at what the text itself states, before attempting a conclusion.

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5 [Waquet 2003] defines the class room speech of a teacher as a "formal" discourse, by opposition to conversations held informally in other contexts. This could further involve the recitation or chanting of versified text.

6 [Yano 2006; 153].

7 [Ong 1982, 11]: I style the orality of a culture totally untouched by any knowledge of writing or print, ‘primary orality’. It is ‘primary’ by contrast with the secondary orality’ of [the] present-day (...) in which a new orality is sustained by (...) devices that depend for their existence and functioning on writing and print.

2. SANSKRIT ORALITIES

Sanskrit scholarly orality remains an object of inquiry. There is no doubt that oral speech has been hauled in Sanskrit beliefs, scholarly culture and literature, as the prime way of displaying, teaching, and learning texts to be stored in one's memory, as epitomized by the corpus of sacred texts referred to as *śruti* ("what has been heard") and *smṛti* ("what has been remembered") which every Brahman should study[8]. Historically, after the antique Indus valley cultures for which engraved seals are still un-deciphered, traces of writing for administrative purposes only appear by 300 BCE with the Aśokan edicts. How much then are Sanskrit oral cultures inherited from a moment of primary orality? Was the culture of primary orality preserved in elite circles? Did this affect scholarly culture?

2.1. The debated oralities of the Vedas

It is usually claimed that the Vedas have been and still are transmitted orally in forms that enable it to be conserved without even the slightest change. They would be like an autonomous island of primary orality still preserved in today's world[9]. This idea has been challenged in many ways, and the amount of primary orality Vedic cultures testifies of is debated today. Jack Goody has repeatedly claimed that the Vedas and its associate scholarship necessarily require a literate culture of writing for its composition[10]. However, his arguments rest on non-attested claims of a continuity between the writing culture of the Indus valley culture and the Aśokan edicts. Indeed, it remains to be proved that the yet undeciphered signs on Indus seals constitute a writing. Further the claim for continuity is not consistent with simple chronology since it is likely that there is nearly a thousand year gap in between both[11]. The combinatorial and mnemonic feats of the Vedic caste cultures of recitation have also been understood as a sign of this primary orality[12]. However, mnemonic feats could just as well be signs on the contrary of a contested oral traditional in a context of mixed orality. Spectacular recitations could be a way of displaying an orality whose values and cultures could have been challenged by the spreading of a writing. Further, recordings of contemporary Vedic chantings show that even with the utmost care and complexity, oral transmissions are liable to changes : does this

8 [219, Olivelle 2008]. For the distrust in writing, especially of the vedas, [Brown 1986, 69; 71].

9 See for instance [91; Witzel 1995] quoted and discussed in [note 8, Bronkhorst 2002].

10 Consistently from [127; Goody 1968] to [166-169; Goody 2010].

11 [107-108; Falk 1990]

12 [p. 256 sqq, Staal 1986]

mean the Vedas were also transmitted through written sources? Anthropological field work shows that despite the emphasis on revelation by "seeing" and "hearing", texts written on palm-leafs can be used and presented even during prestigious performances[13]. They can further be used to ascertain verbatim memorization[14]. We can set aside for our discussion the contested context of composition and fixation of the Vedas (in or not a context of primary orality). The Vedas are transmitted with prescriptions to preserve them in a context which maintains as much primary orality as possible. Such prescriptions seem to indicate that such a primary orality was already in danger. Indeed, there is no doubt that the transmission of the Vedas has to be understood from the Aśokan inscriptions onwards as being progressively immersed in a culture of writing. In other words, from the beginnings of the common era onwards, there is no doubt that the scholarly culture was that of mixed orality.

The Vedas have served as a model for historians to think about the modes of display and transmission of other texts in the Indian subcontinent. For instance Frits Staal's observation and recording of Nambudiri Brahmin's recasting of Vedic rituals in the 1970's- in which the teaching and display of Vedic poems was notably filmed[15]- has often served as the implicit background from which secondary literature has represented the use of other ancient Sanskrit texts. However, Vedic texts are transmitted in a highly exclusive (a restricted cast of Brahmins) religious context. Especially, if we believe Staal, the emphasis is on the performativity of the chanting not on meaning[16]. Mathematical texts, and more generally astral sciences, although part of Vedic lore, were transmitted in less exclusive, less religiously orthodox contexts [17]. And their meaning is what could make such texts performative, not their chanting alone.

2.2.Scholary Sanskrit oralities

What do we know of oralities in other spheres of scholarly lore in the Indian subcontinent? Studies have examined the importance of mnemonics and recitation in buddhist millieus[18], the Puraṇas as

13 See for incance [Narayanan 1984; 144].

14 [Fuller 2001]

15 https://www.youtube.com/watch?v=RYvkYk7GvJ0 (as of october 2016)

16 [Staal 1986; 256 sqq]

17 C. Minkowski has thus chronicled how siddhāntic cosmology could conflict with puranic cosmologies, [Minkowski 2001].

18 [Guyatso 1992], [Anālayo 2007].

Hindu written scriptures[19] or the possibility that the Mahābhārata be a written text [20]. All of these studies highlight different ways in which various form of vocalized speech can be associated with written texts; and how memorization need not be disjoined from the existence of a written text used to learn, store or compose.

The meaning of texts is essential in the case of the worldly technical knowledge embodied in *śāstras*[21]. The mingling of oral speech, written compositions and transmissions in such context has been much less studied and debated. Arguments often mix several facts, assuming they are linked: the existence or not of written texts, the use of memorization, and testimonies to the oral display of the text are thus often used in arguments to argue in favor of "oral" or "non-oral" literature. Thus, it is usually admitted now that Pāṇini's oeuvre was not composed in a context of primary orality. This need not imply that grammatical treatises were not still meant to be memorized, even if they could also be transmitted through a written medium[22]. Further, philosophical school texts describe fierce court oral battles, of the kind evoked by Bhāskara above. Such texts testify to the importance of the oral display of arguments and reasoning, but testify little about the medium in which texts were composed, stored and transmitted[23].

Close studies show then that many different configurations can be imagined for the complex relations between oral and written compositions, mnemonic and written storage, vocal or written display and performance of texts, whether they are sacred lore, literary oeuvres or technical texts in the Indian subcontinent, in a context of mixed orality. They provide a backdrop against which we can try to imagine how Āryabhaṭa's text could have been thought of and used.

3. ĀRYABHAṬA'S TEXT

The treatise provides some information on its context of composition. The use of versification in the *Āryabhaṭīya* could be the foremost sign that the text was composed to be if not transmitted at least displayed orally. Of course, the use of versification, could also be thought of as just a convention of the genre of the treatise, not necessarily implying vocalization.

3.1. What the Āryabhaṭīya states explicitly

19 [Brown 1986]

20 [12; Hiltebeitel 2002] According to the still debated position of Alf Hiltebeitel, writing was essential to the coining of the *Mahābhārata*, for which as in Leonardo da Vinci's atelier, he imagines a head bard, instructing helpers to write different portions of a very structured text. Not denying the incorporation of motifs and stories of oral literature, he does not however believe that the text itself was an oral composition derived from a smaller poem a simple *bhārata.*

21 The word should be understood as a scholarly discipline, a text and a *regime de savoir*, that of systematic knowledge [Pollock 2007; 203-205] .

22 [11-14, Bronkhorst 2002]

23 [17-20, Bronkhorst 2002]

The first verse of the treatise runs as follows:

> Ab.1.1. *praṇipatya ekam anekam kaṃ satyāṃ devatām paraṃ brahma | āryabhaṭaḥ trīṇi gadati ganitaṃ kālakriyāṃ golam ||*
>
> Having bowed to Ka (Brahma) who is one and many, who is the true god, the supreme Brahman| Āryabhaṭa proclaims three: Mathematics, Time-reckoning , and the Sphere||

This verse, after the customary propitiatory evocation announces the structure of the treatise and gives the name of its author. The verb indicating how the treatise is spelled out, *gad-* refers to oral speech, and more specifically to the speech of one who will relate an action that has taken place. The *Āryabhaṭīya* here is thus a standard Sanskrit treatise self-proclaiming its vocation to be vocalized; here it specifies that the text was vocalized by its author. Therefore, the use of the verbal root *gad-* would indicate a kind of formal speech when displaying the text. Characteristically, this verse uses (and maybe abuses) the *holoioteleuton* (that is the repetion of a final sound) of *am* and *kam*, further giving the impression that such a verse was indeed made to be vocalized[24]. This statement is followed by a three termed list that has been understood as a kind of table of contents of the treatise.[25]

The second verse that evokes the context of redaction is the opening of the mathematical chapter of the *Āryabhaṭīya*[26].

> Ab.2.1. *brahmakuśaśibudhabhṛguravikujagurukoṇabhagaṇān namaskṛtya|*
> *Āryabhaṭas tv iha nigadati kusumapure 'bhyarcitaṃ jñānam||*
>
> Having paid homage to Brahma, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the group of stars| Here Āryabhaṭa proclaims the knowledge honored in Kusumapura||

As in verse Ab.1.1 the verb used here for "proclaiming", *nigad-*, *gad-* with a prefix *ni-*, means "announcement", "declaration" etc. So that the vocabulary still refers to what could have been a formal speech. Note that the knowledge (*jñāna*) is not attributed to the author himself but implicitly

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24 For the use of phonic repetitions as a criteria for memorization and oral display, see [Anālayo 2007; 5-6].

25 The religious invocation of Brahma will be developed in Āryabhaṭa's posterity, underlining that he belongs to a school who's teachings were derived from the teaching of this god (*brahma-pakṣa*).

26 The first chapter and the three others were also transmitted as two different treatises [Shukla & Sarma 1976, xxv-xvi].

to Brahma and especially to a place, Kusumapura, thought to have been the place were Āryabhaṭa learnt his lore[27]. Note that Āryabhaṭa does not however evoke teaching but reverence to knowledge. We thus know that, according to these verses, the text was composed to be displayed orally, that is to be chanted. It is a text of second orality: it does not belong to a purely oral context, but one where the oral and the written exist and complete each other. We can also note that the question of the authorship of the text has to do with proclamation not with contents.

3.2. What commentaries state

We have seen that Bhāskara imagines for the text something like a public debate, and gives it the form of a battle cry. Bhāskara's commentary indeed testifies of a complex context in which written and memorized texts coexist with working surfaces on which numbers could be noted and computed with, diagrams drawn, explanations given orally maybe informally, texts quoted from memory, and computations and drawings to be represented mentally[28]. Michio Yano's vision has thus a grounding in what he knows of how Āryabhaṭa's commentators used his text. The formal speech of the *Āryabhaṭīya* when we read Bhāskara's commentary is implicitly contrasted with the apparent informality of the staged dialogs of his *written* commentary.

But what can be retrieved from Āryabhaṭa's text alone? What does the text tell us of how Āryabhaṭa imagined his text was to be stored and transmitted? What does it let escape on how it was composed?

4. APPROACHING THE PROBLEM CRABWISE

Āryabhaṭa's rules are a paradox: they transmit knowledge but cryptically. The way that such a text was composed seems then to provide indirectly information on the context for which the text was composed: were such texts made intentionally cryptic ? And if so, why?

An analysis of the composition of the text is thus set forth, with the hope to find elements that could indicate whether the verses were composed so as to be stored in one's memory or by writing, wether

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27 Kusumapura is identified by 7th century Bhāskara with a place of learning in Magadha, Paṭaliputra, aka the modern city of Patna. Much later commentators, such as Parameśvara (late 14th beginning of the 15th century), and Raghunātha (16th century), further identify this place as the place where the text was composed. [Shukla & Sarma 1976, xvii-xviii].

In a third verse Āryabhaṭa gives us his age, but this does not tell us anything about the composition of the text :

Ab.3.10. *śaṣṭyabdānāṃ śaṣṭir yadā vyatītās trayaś ca yugapādāḥ|*

tryadhikā viṃśatirabdās tadeha mama janmano 'tītaḥ||

When sixty times sixty years and three quarter *yugas* (of the current *yuga*) had elapsed, twenty three years had then passed since my birth.

[Shukla & Sarma 1976, 95] We thus know that Āryabhaṭa was born on March 21 476 AD, and that he was twenty-three in 499. But this verse indicates nothing on the time of composition of the text. It was interpreted in this way first by Sūryadeva (b. 1191) and then in the Kerala school by Parameśvara and Nīlakaṇṭha (late 15th, beginning of the 16th century) [Shukla & Sarma 1976, 98].

28 Evoked in [Keller 2006, xl-li].

they were made to be transmitted orally or not, and maybe to indicate wether the text was composed orally or not. Āryabhaṭa's verses for the mathematical chapter are mainly about procedures. All of these verses are quoted, translated and characterized in the Appendix, at the end of this article. In the following an analysis of the different kinds of statements about algorithms made by Āryabhaṭa in the mathematical chapter of his treatise will be made. We will not discuss here the mathematical contents of these algorithms, which can be found elsewhere. Our analysis then will be indirect, progressing sideways like a crab; trying to retrieve from modes of algorithm statement, some indications about the *Āryabhaṭīya'*s oralities. The aim here will not be to comment on the different kinds of algorithm statements found in the text, but on what this indirectly tells us of how the text was composed, and its aim in terms of transmission and storage[29]. This study is not about speech acts in the sense that it does not aim at recovering indirect and implicit understandings of the rules that Āryabhaṭa would have assumed part of his text[30]. However it is inspired by speech acts since it will characterize statements using the kind of verbal forms used. It will further contain an analysis of whether the statement is a direct invitation to modify the world, or a description of the world, with an implicit invitation to make the statement and the world coincide.

Āryabhaṭa's *sūtra*s which are very cryptic will be understood through the lenses provided by Bhāskara's reading of the verse. Therefore the following analysis has to be taken as concerning Bhāskara's understanding of Āryabhaṭa's text.

Āryabhaṭa's *sūtra*s as read through the lenses provided by Bhāskara, can be classified into four groups:

1.Prescriptive statements dealing with algorithms

2.Formulaic description of procedures

3.Prescriptive descriptions of the world

4. Mixing

Two kinds of explicit statements about algorithms can be found in the mathematical chapter of the Ab. The first belongs to a category of voiced prescriptions, with just a choice of the most important elements of the algorithm. The second category is constituted by straight forward lists of computations listed one after the other: these statements contain no explicit prescription just a description of a state of the world, and an implicit invitation to make the world correspond to the statement. As we will see, these two types of statements are best contrasted when opposing the rules

29 [Keller 2015] deals with some aspects of algorithmic statement in the mathematical chapter of the Ab.

30 [Austin 1962], [Searle 1969], also discussed in [166-167, Ong 1982].

for root extractions (Ab.2.4, Ab.2.5) and the ones for series (Ab.2.19-Ab.2.23). A third category of rules are descriptions or theoretical statements, that authoritatively state a truth about the mathematical world, and may implicitly be a way of prescribing an algorithm. This is notably the case of a set of verses dealing with elementary geometrical figures (Ab.2.6-9). Finally some of the rules do not fit in one of these categories but appear as hybrids. They form a fourth category.
Let us look at examples of each kind of statement, characterize them, and investigate what they may yield as information on their oralities.

4.1.Prescriptive statements

Five verses out of thirty-two are prescriptive statements of algorithms. Their main syntactical feature is the use of the optative for a mathematical operation. They all concern recursive algorithms. They use puns, repetitions, and all sorts of striking rhetorical devices.

As studied elsewhere[31], the prescriptive statement about the algorithm to square roots (Ab. 2.4) states the heart of an iterative process, overlooking how the algorithm is initiated or ended. This gives the surprising impression that the algorithm is described by its end[32]. It rests on a pun linking the square powers of ten to the positional places were squares are subtracted: this pun points precisely to the mathematical groundings of this process. The rule for extracting cube roots (Ab.2.5) is constructed in the same way[33]:

> Ab.2.5. *aghanād bhajed dvitīyāt triguṇena ghanasya mūlavargeṇa*|
> *vargas tripūrvaguṇitaḥ śodhyaḥ prathamād ghanaś ca ghanāt*||
> One should divide the second non-cube <place> by three times the square of the root of the cube|
> The square <of the quotient> multiplied by three and the former <quantity> should be subtracted from the first <non-cube place> and the cube from the cube <place>||

Only the heart of the process is prescribed (as seen in Figure 1), the iteration beginning (A) with what can appear as the end of the process. The division (*bhajed*) is the conjugated verb of the rule, in the optative. It is thus given a central role: both as being at the heart of the iterative process, and also as being what will undo the multiplication that a root extraction undoes. The confusion between cube (*ghana*) and non-cube (*aghana*) numbers and cube and non-cube places is played out as a riddling statement which when unravelled points to how the algorithm works.

Figure 1: The stated steps of a cube-root extraction in the Ab

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31 [Keller 2015].

32 Unless you consider that the division is what begins the recursive process; as C. Morice-Singh in her phd thesis [Morice-Singh 2015].

33 For a mathematical analysis of this rule and a tentative construction, see [Keller 2006, II, 18-22].

The two other rules use differently the same devices. When prescribing the cutting (*chindyād*, in the optative) of a circle to derive half-chords, once again the rule points but to crucial elements of a process, here carried out in a diagram[34]. The reference to trilaterals and quadrilaterals in a circle is first an indication of the process- apt to bring to mind a standard diagram according to Bhāskara's interpretation of the text- but also points to how it is grounded mathematically (using the pythagorean procedure stated in Ab.2.17). The repetition of *sama*, could be a way of insisting that the other key to the process is uniformity, specifically the uniform subdivisions of a circle, seen as a regular/uniform curve. Such a repetition certainly is as an element of rhythm in the verse, participating then in its mnemonic features as well as adding style to what could have been its oral performance[35].

The two verses prescribing a pulverizer would probably need a separate thorough examination. Here also the heart of a process is given. The main operation of the rule, we understand with the repetition, is a series of divisions (*chind-, bhaj-*) and their remainders (*agra,* notably *agra-cheda* the 'divisor of the remainder' and what is sought *chedāgra* 'the remainder of (two) divisors')[36].

These prescriptions, voiced by the optative of an active verb, then are not so much a description of algorithms as theoretical statements about them. As such, to be used, they suppose that those that read, heard or recalled them already know the algorithm. The composition itself is weaved with what we may recognize as mnemonic elements: repetitions, puns, surprising statements that can act like a knot to a kerchief and bring back together the algorithm[37]. But if properly understood, the verse also contains clues to the reason why such an algorithm works and

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34 Ab.2.11. One should divide the quarter of the circumference of an evenly-circular <figure>. And, from trilaterals and quadrilaterals|
As many half-chords of an even <number of> unit arcs as one desires <are produced>, on the semi-diameter.||
samavṛttaparidhipādaṃ chindyāt tribhujāc caturbhujāc caiva|
samacāpajyārdhāni tu viṣkambhārdhe yatheṣṭāni||
For a discussion of the mathematical contents see [Keller 2006, II, 54-69].

35 [Anālalyo 2007; 8]. Note that such repetitions never become the pericopes, often associated with litterature belonging to a primary oral context.

36 One should divide the divisor of the greater remainder by the divisor of the smaller remainder.|
The mutual division <of the previous divisor> by the remainder <is made continuously.
The last remainder> having a clever <thought> for multiplier is added to the difference of the <initial> remainders <and divided by the last divisor>.||
33. The one above is multiplied by the one below, and increased by the last. When <the result of this procedure> is divided by the divisor of the smaller remainder|
The remainder, having the divisor of the greater remainder for multiplier, and increased by
the greater remainder is the <quantity that has such> remainders for two divisors||
Ab.2.32 adhikāgrabhāgahāraṃ chindyād ūnāgrabhāgahāreṇa|
*śeṣaparaspara**bhaktaṃ** matiguṇam agrāntare kṣiptam||*
*Ab.2.33 adhaupari**guṇita**m antyayugūnāgracchedabhājite śeṣam|*
adhikāgracchedaguṇaṃ dvicchedāgram adhikāgrayutam||
For a discussion of the mathematical content, see [Keller 2006, II, 142-185].

37[Severi 2007, 26-27]

even how it is inserted for instance in a system of operations (multiplication and division being opposite pairs for instance). Such algorithms then can probably be seen like small portable libraries for a practitioner: knowing them and understanding them provides many different elements about them.

As a contrast to the preceding rules, formulaic descriptions seem on the contrary to closely follow the different steps of an algorithm's execution.

4.2.Formulaic descriptions

Formulaic descriptions state in due order the computational steps of a curried algorithm: that is, each step is built from the previous and their execution in a linear sequence leads to a result. Some steps are with one operation (Ab.2.19, Ab.2.22), others with several operations (Ab.2.14, Ab.2.20, Ab.2.21). The main syntactical feature of such rules is the absence of a conjugated verb, and the repeated use of verbal adjectives. I count six out of thirty-two verses of this kind. Some of these lists are formed of sub-steps that can be quoted and read independently from the sequence they belong to (Ab.2.24). Ab.2.19 epitomizes this kind of rule, and at the same time is a mirage: read literally it provides false rules, selecting elements of the sequence enables one to read five mathematical algorithms in one rule[38]. This compact rule has been studied in [Keller 2006][39], but let us just examine how it states one of its algorithms to characterize these kinds of mathematical rules. Indeed the algorithm for the computation of the mean value of an arithmetical sequence is read selecting (in bold) the following steps in the verse:

> Ab.2.19 ***iṣṭaṃ vyekaṃ dalitaṃ*** *sa-pūrvam* ***uttara-guṇaṃ sa-mukhaṃ madhyam***/
> *iṣṭa-guṇitam iṣṭa-dhanaṃ tv athādy-antaṃ padārdha-hatam//*
> **The desired <number of terms>, decreased by one, halved**, increased by the previous <number of terms>, **having the common difference for multiplier, increased by the first term, is the mean <value>**|
> <The result>, multiplied by the desired, is the value of the desired <number of terms>. Or else, the first and last <added together> multiplied by half the number of terms <is the value>.||

38 *Ab.2.19. iṣṭaṃ vyekaṃ dalitaṃ sapūrvam uttaraguṇaṃ samukhaṃ madhyam*|
iṣṭaguṇitam iṣṭadhanaṃ tv athādyantaṃ padārdhahatam||
The desired <number of terms>, decreased by one, halved, increased by the previous <number of terms>, having the common difference for multiplier, increased by the first term, is the mean <value>|
<The result>, multiplied by the desired, is the value of the desired <number of terms>. Or else, the first and last <added together> multiplied by half the number of terms <is the value>.||

39 [Keller 2006, II, 106-110]

In other words, if (Ui) is an arithmetical sequence, of first term U1, and of increase a; M the mean value of the sequence M=[∑i=1 Ui]/n= [a(n-1)/2] + U1[40]. But the process describes a series of computation: first taking the desired numbers of terms, n, then decreasing it by one, n-1, then halving the result (n-1)/2, then multiplying it by the increase, [(n-1)/2]xa and increasing by the first term: [(n-1)/2]xa+U1 provides the result. Here then verbal adjectives (past participles) serve as building blocks for a sequence that will construct an algorithm. The rule (that one should read appropriately) provides the blocks, and implicitly one is invited to carry out the process to obtain the desired result. No indication from the commentaries or within the rule itself indicate how such processes are grounded mathematically. Here the rule and its very simple "detachable" syntax seems to have been crafted to state computational algorithms, and maybe compact them. Memorizing such rules then, can provide a library of several "ready to use" sequences. They do not however contain theoretical information on the rules.

Another set of *sūtra*s seems to contains prescriptive descriptions about mathematical objects.

4.3.Prescriptive descriptions of the world

Such prescriptive statements are characterized by the used of the verb "to be" (*bhū, as*) conjugated in the optative or simply omitted in nominal sentences. I count seven of these out of thirty-two. They essentially concern rules in geometry *(kṣetra-gaṇita*), in which figures are both defined and their areas and volumes computed (Ab.2.3; Ab.2.6-9). They form a homogenized set, characterized by what appears to be a theory of uniformity enabling the extension of computations of areas to volumes[41]. Puns often highlight how a figure and its computed area and volume are to be understood. Thus a square *varga* and a cube *ghana* have in Sanskrit the same double meaning as in English: a square is both a geometrical figure and the multiplication of a number by itself (and thus characterizes the area of such a figure); similarly a cube is both a geometrical solid, a number multiplied three times by itself, and the volume of such a geometrical figure. Ab.2.6, provides a rule to compute the area of trilaterals and then the volume of "six edged" solids:

> *tribhujasya phalaśarīraṃ samadalakoṭībhujārdhasaṃvargaḥ|*
> *ūrdhvabhujātatsaṃvargārdhaṃ sa ghanaḥ ṣaḍaśririti||*
> The bulk of the area of a trilateral is the product of half the base and the perpendicular|
> Half the product of that and the upward side, that is <the volume of> a solid called `six-edged'||

40 Because [∑i=1 Ui]/n=[U1n+an(n-1)/2]/n.

41 [Keller 2006; xxxii-xxxiii].

Areas of trilaterals are the product (*saṃvarga*) of the perpendicular with half the base (*bhujārdha*), and this situation is falsely extended to the solid[42]. This extension is marked by repetition and variation, the upraised base (*ūrdhvabhuja*) and half the product (*saṃvargārdha*) of the area, is the volume/solid (*ghana*). Here again, repetitions and double entendre are some of the rhetoric devices used to express a mathematical idea. These rules directly make a theoretical claim: the world is like the rule's statement; implicitly this might be an injunction to adjust the world to the statement. Therefore, such rules additionally are of course read as providing algorithms to compute areas and volumes. As in the prescriptive statements about algorithms, prescriptive descriptions of the world contain both theoretical statements and ready to use formulas to compute values of segments, areas or volumes. The last kind of *sūtra* is made of verses that are hybrids of the above.

4.4.Mixing

Indeed, the fourteen remaining *sūtra*s contain verses that mix several features of the above. Thus verse 8 of the mathematical chapter in the first half is a description of a trapeze, and in the second half the prescription of an algorithm to compute areas and inner segments[43]. It further uses the *homoioteleuton* of *e*:

> Ab.2.8. *āyāmaguṇe pārśve tadyogahṛte svapātalekhe te*|
> *vistarayogārdhaguṇe jñeyaṃ kṣetraphalamāyāme*||
> The two sides, multiplied by the height <and> divided by their sum are the ``two lines on their own fallings".| When the height is multiplied by half the sum of both widths, one will know the area.||

Verse 10 is both a description and a formulaic statement of an algorithm, providing the approximate ratio of the circumference of a circle to its diameter:

> 10. *caturadhikaṃ śatam aṣṭaguṇaṃ dvāṣaṣṭis tathā sahasrāṇām*|
> *ayutadvayaviṣkambhasyāsanno vṛttapariṇāhaḥ*||
> A hundred increased by four, multiplied by eight, and also sixty-two thousand|
> Is an approximate circumference of a circle whose diameter is two *ayuta*s||

In most cases the hybridity of such verses is not used as a mathematical device, but sometimes it is, as in the case of the definition of the place value notation:

> Ab.2.2. *ekaṃ ca daśa ca śataṃ ca sahasraṃ tv ayutaniyute tathā prayutam*| *koṭyarbudaṃ ca vṛndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt*||

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42 For further discussion see [Keller 2006, II, 22-30]

43 [Keller 2006, II, 34-40]

> One and ten and a hundred and one thousand, now ten thousand and a hundred thousand, in the same way a million|Ten million, a hundred million, and a thousand million. A place should be ten times the <previous> place||

The first part of this verse is a list of (names of) increasing power of tens. The second part, seemingly unrelated is a prescriptive descriptive statement, indicating that each place is ten times the previous. The relation between the two parts of the verse embodies concepts that are all mingled into the word *sthāna* (place): the idea of place that has both a rank and a value. The link in between both parts of the verse establishes the decimal place value notation as a notational system. The hybridity is used here to make a mathematical statement.

4.5.Little libraries

The mathematical rules of the *Āryabhaṭīya* have features that we can recognize as mnemonic and apt for an oral display in a formal setting: versified aphorism using puns, repetitions and *homoioteleutons*. A certain number of them can be understood as little "libraries" containing several rules, or englobing within them an algorithm and a theoretical statement. Further they are characterized by a great stylistic diversity: prescriptions of algorithms disguising theoretical statements, theoretical statements providing implicit algorithms, algorithms spelled out with assembled sequences of nouns and verbal adjectives, hybrids of all these. Was Āryabhaṭa the author of such a diverse range of mathematical statements? Was this a part of a literary effort? Can anything be further said then on the context of mixed orality such statements belonged to?

5.CONCLUSION

We have seen that the text itself yielded little information on its context of production or display: was the text intended for use in teaching? In scholarly courts? These questions remain open. But a close study of the text has first confirmed, and maybe highlighted stylistic features of what could have been an attempt to compile rules in a homogenized versified form.

5.1.Recasting meanings in a homogenized versified form

[Olson 1994] notes that in an oral culture the exact word need not be retained but only the intention. Texts in Sanskrit are often self-described as synthetic recasting of orally heard truths. In doing so, it is usually not mentioned whether such reworking required a written medium. We have seen that indeed, the *Āryabhaṭīya* presents itself as a recasting of knowledge. Should these different

kinds of statements on algorithms be read as a compilation of different rules? As a re-rendering of them? Does this say anything about the orality it belonged to?

The fourth and last verse is the one which ends the treatise (and the last chapter of the treatise) is as follows:

> Ab.4.50 *āryabhaṭīyaṃ nāmnā pūrvaṃ svāyambhuvaṃ sadā nityam*|
> *sukṛtāyuṣoḥ praṇāśaṃ kurute pratikañcukaṃ yo 'sya*||
> He who makes a criticism (*pratikañcuka)*[44] of the *Āryabhaṭīya* which is by name the ancient perpetual continual [astral science] of Svayambhū (Brahma) destroys his good deeds and longevity.

The last verse repeats thus what has been stated before that its contents is that of an immemorial divine tradition. Such transmissions in the form of synthesis and recasting is evoked in commentaries as well, while evoking the genealogy of the text and the discipline (*śāstra*) of astral science (*jyotiṣa*). Thus Sūryadeva a XIIth century south Indian commentator of Āryabhaṭa, provides a genealogy of this text[45]:

> *... saṃsmṛtyādau bhagavatā brahmaṇā bahuvistaraṃ jyotiḥśāstraṃ kṛtam| brahmaṇaḥ sakāśād adhītatacchāstro vṛddhagargas tat saṃkṣipya anyac cakāra | tasmād api labdhatadvidyāḥ parāśarādayo munayo' apyanyāni jyotiḥ śāstrāṇi cakruḥ / tathā ca vṛddagargaḥ (...)*
> *mattaś cānyān ṛṣīn prāptaṃ pāraparyeṇa puṣkalam*|
> *tais tathā ṛṣibhir bhūyo granthaiḥ svaiḥ svair udāhṛtam*||
> ... having at first recollected, lord Brahmaṇ made a treatise on Astral Science (*jyotiṣa*) whose subject is vast[46]. Garga the old (*Vṛddagarga*) learned that science from Brahmaṇ in person, having synthetised it (*saṃkṣip*), he made another [treatise]. Parāśara and others sages (*muni*) who also obtained that knowledge (*vidyā*) from him also made other treatises on *jyotiṣa*. Thus [according to] the older Garga:*(...)*
> And other seers, one after the other, have obtained from me the best [elements of this science]| Then, [this science] once more has been retold (*udāhṛta*) by these seers in their own compositions (*grantha*)||

In Sūryadeva's view a composition, which could very well be the scholarly knowledge itself, is made of recasting, homogenizing and maybe reduction. An author is then someone who has absorbed older knowledge and finds a way to recast it. It makes sense to understand Āryabhaṭa's

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44 Someśvara , who flourished sometime in between the end of the 10th and the 13th century, a commentator of Āryabhaṭa who mostly paraphrases Bhāskara, glosses this word with the term *pratibimba*, which means "a reflected image" and from there "a replicant". The term can thus be understood as referring to plagiarism . Michio Yano or before him Clark understand the verse in this way. In other words, an alternative translation could be: "He who gives a distorted image of the *Āryabhaṭīya* which is by name the ancient perpetual continual [astral science] of Svayambhū (Brahma) destroys his good deeds and longevity".

45[Sarma 1976; 2-3].

46 Another interpretation of this expression could be: "made a science which is *jyotiṣa*", because of the double meaning of *śāstrā,* which is both systematic knowledge, and a text propounding such systematic knowledge.

authorship of the text in this way[47]. Indeed, on the one hand the text itself is understood as a recasting and a compilation. On the other an examination of the *sūtra*s of the mathematical chapter has shown that a larger half of them are composed in very distinct ways, the other part being hybrids of the different kind of possible rules. Therefore, it is possible to imagine that the rules provided in the mathematical chapter were intially composed in several different contexts, by different authors, and then recompiled, and stylistically homogenized by Āryabhaṭa[48]. In other words, Āryabhaṭa would be the author of a compilation and recasting of rules. Does this tell us anything of the text's oralities?

5.2."Looking oral"

We have seen that the verses of the mathematical chapter were probably composed to be learnt by heart and vocalised in the context of a “formal orality”, and in a society of secondary orality. Paul Zumthor distinguishes, within Ong's subdivision of primary and secondary orality, a finer strata within secondary orality: mixed orality would be a culture of oral text existing in a world using writing but in which writing does not affect directly the contents of the oral text, while secondary orality (*une oralité seconde d'une culture lettrée*), although oral would be affected by the presence of writing[49]. The mathematical chapter of the *Āryabhaṭīya* is a paradox, it has all the features of secondary orality, however no direct reference to writing can be found in the text. Nonetheless some of his mathematical rules have to be understood in a world where a graphical device whether effectively traced or represented mentally is used. This is notably the case of the definition of the decimal place value notation and of the operations to extract square and cube roots which rest on such a notation; this is also the case of other processes such as the "cut" figure of verse 11.

Indeed, the *Āryabhaṭīya* presents none of the striking characteristics of a text of primary orality, but some of its features seem to have been devised to stage something like a primary orality, somehow

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47 K. Plofker says this in more general terms, in [Plofker 2009, 213]. One may further notice that most of the verses that explicitly state Āryabhaṭa's authorship of the text, open and close sections of the text: are these verses part of the original composition? Could they have been added by a person editing the text, giving it coherence, in between the composition of the text and Bhāskara's commentary?

48 Further exploration then, would involve surveying the whole of the treatise. M. Husson has suggested that Bible studies could help characterize better the eventual different strata of the text.

49 [Zumthor 1983, p. 36] je propose de réduire à quatre espèces idéales l'extrême diversité des situations possibles
-une oralité *primaire* et immédiate ou *pure*, sans contact avec l'"écriture": j'entends par ce mot tout système visuel de symbolisation exactement codée et traductible en langue;
-une oralité coexistant avec l'écriture et qui, selon le mode de cette coexistence, peut fonctionner de deux manières: soit comme oralité *mixte*, quand l'influence de l'écrit y demeure externe, partielle et retardée (ainsi, de nos jours, dans les masses analphabètes du tiers monde (! sic)); soit comme oralité *seconde* qui se (re)compose, qui se (re)compose à partir de l'écriture et au sein d'un milieu où celle-ci prédomine sur les valeurs de la voix dans l'usage et dans l'imaginaire; en inversant le point de vue on poserait que l'oralité mixte procède de l'existence d'une culture *écrite* (au sens de "possédant une écriture"); l'oralité seconde, d'une culture *lettrée* (où toute expression est marquée par la présence de l'écrit);
-une oralité mécaniquement *médiatisée*, enfin, donc différée dans le temps et/ou l'espace.

"hiding" some of its written features[50]. For instance, P. Zumthor (p. 138-140) explores the phonic pleasures of oral poetry, which he notes sometime give rise to absurd texts made of phonemes without meanings. Āryabhaṭa's code for noting numbers with syllables makes for strange verses, with un-understandable onomopatic sequences in the middle of plain text. At times, some of the conjunct consonants seem very difficult to pronounce[51]. This system for coding numbers is not used in the mathematical chapter. Nonetheless, this artefact feeds the impression that Āryabhaṭa's text was made to look oral, look like a text of primary or mixed orality, as a composer's stylistic *coqueterie*[52].

5.3. Back to the "educational context"

The fact that the text itself comes as eponymous to its authors, gives further the feeling of what could have been if not a posthumous compilation at least a posthumous title[53].

However, Āryabhaṭa's text is not a vestige of a class room context, it is a well polished gem. As we have seen, maybe his authorship is all about the polishing. So that the first "educational" context which arises has to do not with those who used the *Āryabhaṭīya*, but on Āryabhaṭa's side: as a student retelling with brilliance what he has previously known. We saw that Michio Yano considered that the word of the *sūtra* was taught before its meaning. This is quite common and we find this again in P. -S. Filliozat's account of the relations of mathematical texts with orality[54]:

> With high probability we can assume the teaching master to have been the most common type among ancient pandits. The typical composition produced for teaching is the *sūtra*, or a composition of the same kind of style, which the master explains orally in his own way. The general rule is that the disciple memorizes the letter of the *sūtra* and remembers the contents, if not the very wording, of the oral explanation. (...) Even if oral transmission is always appreciated, even if a composition in *sūtra* style and in verse is an aid to memorization, the pandits never refused writing, never neglected the help they could derive from it. (...) Metrical

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50 Notably the formal repetition of fixed phrases as spelled out in [Anālayo 2007] insprired by Milman Pary. How universal however are such criteria,and how much could they also in the end be stylistic features made to imitate primary orality in a context of mixed orality remains open.

51 For instance, Ab.1.12 contains Āryabhaṭa's famous "Sine table", mainly a list of onomatopic syllables providing a value, if you know how to decode it:
makhi, bhakhi, dhari, ṇakhi ñakhi, ṅakhi, hasbha, skaki, kiṣga, śghaki, kighva|
ghlaki, kigra, hakya, ghaki, kica, sga, śbha, ṅva, kla, ghta, cha are the half-chords in minutes||
Onomatopic sequences like śbha or ghta seem difficult to pronounce, but not impossible...

52 Notably the absence of strings of synonyms and pericopes confirm that at the time of composition of the *Āryabhaṭīya*, memory was not the only medium to store texts.

53 This remark was nicely given to me by D. Morgan.

54 [Filliozat 2004, 148] Here P. S. Filliozat evokes what he surely has experienced himself of the twentieth century pandits' way of transmitting knowledge. An informal orality is described here: that of the explanation of the rule. Concerning the rules themselves, nothing is clearly stated. They are considered here as "transmitted through memory, used mentally": this seems almost to imply that they are learnt threw thought transmission! This fuzziness of P. -S Filliozat's statements concerning how the rules would be stated (formally orally or through a written medium) and learnt (using a written text or by repetition of a chanted rule) especially underlines the fact that both constitute a blind angle of our historical knowledge.

> form and brevity rend it all the more easy to memorize. Under this form the mathematical text remains an "oral text", which can be transmitted through memory, used mentally, being well-adapted to such functions. (...) In a general manner we can say that the verses have preserved the stye of an oral exposition, and the commentary is an expansion of the memorized knowledge using all the facilities provided by writing.

ī¯As underlined in this quotation as well, this idea of learning first the word and then the meaning, if demonstrated, could testify to the presence of orality as a medium of learning. Implicitly, the memorized verses are considered to be displayed orally[55]. Students would have been made to learn the verses to hear them and retain them through an oral repetition, rather then using a written text. Such a representation is certainly inspired by what is known of the transmission of the Vedic corpus, although as we have underlined meaning in our case is central to the text. We have found no elements in the *Āryabhaṭīya* alone that could infirm or confirm this point. In commentaries, rules are considered to be known although they have not yet been commented upon. However, it is the *meanings* of the rules that are considered known. So that here again it is often impossible to decide whether this is actually a feature of the written medium or not. In other words, nothing can be inferred from our study about the educational context in which the *Āryabhaṭīya* would have been composed to be used in. This of course does not exclude that there was no such aim when the treatise was composed, nor that it was not used in a teaching context.

5.4. Collective composition and understanding

To conclude, we can assume that the *Āryabhaṭīya* was composed to be both memorized and displayed by chanting, with commentarial clues to the fact that indeed it belonged to a context in which formal and informal orality could be found together with informal writings and drawings. Some of Āryabhaṭa's text seems to imitate a primary oral text but this could be an effect of style. Our indirect attempt to uncover elements of the context in which Āryabhaṭa composed his text, has told us more about his authorship than about the text's oralities. Indeed, it makes sense to understand the authorship of the mathematical rules of the *Āryabhaṭīya* as a collective endeavor, of recasting, reorganizing and reformulating. If the mathematical rules of the *Āryabhaṭīya* are the result of a collective composition, their interpretation also rests on collective readings and understandings[56]. And they were probably crafted with this in mind.

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55 [Zumthor 1983, p. 27] notes that by definition an oral speech is first a sound and only, secondly, afterwards and then a meaning ('Le phônè ne tient pas de façon immédiate au sens, elle lui prépare le milieu où il se dira'). However, one could similarly state that a written text is first a set of signs, before being a meaning.

56 This analysis of the *Āryabhaṭīya* was grounded on a first lense: Bhāskara's understanding of the rules. Bhāskara's interpretation opened meanings for me into the terse verses of the mathematical chapter of the *Āryabhaṭīya.* Without it no analysis of how the verse were composed would have been possible. Although my analysis need not agree with Bhāskara's understandings of the rules, a first key into how the rules can be read were required to open the text. This is why I understand that the text was understood to be included in a collective reading and understanding of the text.

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APPENDIX

Verses of the mathematical chapter of the Ab	*Type as understood by me with Bhāskara's lense*
Ab.2.1. Having paid homage to Brahma, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and the group of stars| **Here Āryabhaṭa proclaims the knowledge honored in Kusumapura||** ***brahmakuśaśibudhabhṛguravikujagurukoṇabhagaṇān namaskṛtya|*** ***āryabhaṭas tv iha nigadati kusumapure 'bhyarcitam jñānam||***	Does not concern algorithms. A description of the display of the text.
Ab.2.2. One and ten and a hundred and one thousand, now ten thousand and a hundred thousand, in the same way a million| **Ten million, a hundred million, and a thousand million.A place should be ten times the <previous>** **place||** ***ekaṃ ca daśa ca śataṃ ca sahasraṃ tv ayutaniyute tathā prayutam|*** ***koṭyarbudaṃ ca vṛndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt||***	The first part is a list. As such it is can be thought of as a description of a state of the world. The second part, seemingly unrelated is a prescriptive description: the world should be adjusted to the statement that is made.
Ab.2.3 A square is an equi-quadrilateral and the area/result (*phala*) is the product of two identicals| A cube is the product of a triple of identicals as well as a twelve edged <solid>|| ***vargaḥ samacaturaśraḥ phalaṃ ca sadṛśadvayasya saṃvargaḥ|*** ***sadṛśatrayasaṃvargo ghanas tathā dvādāśāśriḥ syāt||***	Prescriptive description
Ab.2.4. One should divide, constantly, the non-square <place> by twice the square-root| **When the square has been subtracted from the square <place>, the quotient is the root in a different** **place||** ***bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena|*** ***vargād varge śuddhe labdhaṃ sthānāntare mūlam||***	Prescriptive statement of an algorithm

Verses of the mathematical chapter of the Ab	*Type as understood by me with Bhāskara's lense*
Ab.2.5. One should divide the second non-cube <place> by three times the square of the root of the cube| **The square <of the quotient> multiplied by three and the former <quantity> should be subtracted from the first <non-cube place> and the cube from the cube <place>||** ***aghanād bhajed dvitīyāt triguṇena ghanasya mūlavargeṇa|*** ***vargas tripūrvaguṇitaḥ śodhyaḥ prathamād ghanaś ca ghanāt||***	Prescriptive statement of an algorithm
Ab.2.6 The bulk of the area of a trilateral is the product of half the base and the perpendicular| **Half the product of that and the upward side, that is <the volume of> a solid called `six-edged'||** ***tribhujasya phalaśarīraṃ samadalakoṭībhujārdhasaṃvargaḥ|*** ***ūrdhvabhujātatsaṃvargārdhaṃ sa ghanaḥ ṣaḍaśririti||***	Description of a state of the world.
Ab.2.7 Half of the even circumference multiplied by the semi-diameter, only, is the area of a circle| That multiplied by its own root is the volume of the circular solid without remainder.|| ***samapariṇāhasyārdhaṃ viṣkambhārdhahatam eva vṛttaphalam|*** ***tan nijamūlena hataṃ ghanagolaphalaṃ niravaśeṣam||***	Description of a state of the world.
Ab.2.8. The two sides, multiplied by the height <and> divided by their sum are the ``two lines on their own fallings''.| When the height is multiplied by half the sum of both widths, one will know the area.|| ***āyāmaguṇe pārśve tadyogahṛte svapātalekhe te|*** ***vistarayogārdhaguṇe jñeyaṃ kṣetraphalam āyāme||***	The first half is a description of the world. The second is a promise.
Ab.2.9. For all fields, when one has acquired the two sides, the area is their product | The chord of a sixth part of the circumference, that is equal to the semi-diameter|| ***sarveṣāṃ kṣetrāṇāṃ prasādhya pārśve phalaṃ tadabhyāsaḥ|*** ***paridheḥ ṣaḍbhāgajyā viṣkambhārdhena sā tulyā||***	Explicit general statement in the first half. Description of the world.
Ab.2.10. A hundred increased by four, multiplied by eight, and also sixty-two thousand| **Is an approximate circumference of a circle whose diameter is two *ayutas*||** ***caturadhikaṃ śatam aṣṭaguṇaṃ dvāṣaṣṭis tathā sahasrāṇām|*** ***ayutadvayaviṣkambhasyāsanno vṛttapariṇāhaḥ||***	A description of the world and a formulaic statement of an algorithm.
Ab.2.11. One should divide the quarter of the circumference of an evenly-circular <figure>. And, from trilaterals and quadrilaterals| **As many half-chords of an even <number of> unit arcs as one desires <are produced>, on the semi-diameter.||** ***samavṛttaparidhipādaṃ chindyāt tribhujāc caturbhujāc caiva|*** ***samacāpajyārdhāni tu viṣkambhārdhe yatheṣṭāni||***	Prescriptive statement of a geometrical construction and promise.
Ab.2.12. The segmented second half-<chord> is smaller than the first half-chord of a <unit> arc by certain <amounts>| **The remaining <segmented half-chords> are smaller <than the first half-chord, successively> by those <amounts> and by fractions of the first half-chord accumulated.||** ***prathamāc cāpajyārdhād yair ūnaṃ khaṇḍitaṃ dvitīyārdham|*** ***tatprathamajyārdhāṃśais tais tair ūnāni śeṣāni||***	Description of the world.
Ab.2.13. A circle should be brought about with a pair of compasses, and a trilateral and a quadrilateraleach <are brought about> with two diagonals| **Flat ground should be brought about with water, verticality (litterally: top and bottom) with just a plumb-line||** ***vṛttaṃ bhrameṇa sādhyaṃ tribhujaṃ ca caturbhujaṃ ca karṇābhyām|*** ***sādhyā jalena samabhūr adho ūrdhvaṃ lambakenaiva||***	Prescription.
Ab.2.14. Having summed the square of the size of a gnomon and the square of the shadow | **The square root of that <sum> is the semi-diameter of one's own circle ||** ***śaṅkoḥ pramāṇavargaṃ chāyāvargeṇa saṃyutaṃ kṛtvā|*** ***yat tasya vargamūlaṃ viṣkambhārdhaṃ svavṛttasya||***	Description of the world and formulaic statement of an algorithm.

Verses of the mathematical chapter of the Ab	*Type as understood by me with Bhāskara's lense*
Ab.2.15. The distance between the gnomon and the base, multiplied by the <height of> the gnomon, is divided by the difference of the <heights of the> gnomon and the base.| **What has been obtained should be known as that shadow of the gnomon <measured> indeed from its foot.||** ***śaṅkuguṇaṃ śaṅkubhujāvivaraṃ śaṅkubhujayor viśeṣahṛtam|*** ***yal labdaṃ sā chāyā jñeyā śaṅkoḥ svamūlād hi||***	Formulaic description of an algorithm and prescriptive description of the world.
Ab.2.16. The upright side is the distance between the tips of the <two> shadows multiplied by a shadow divided by the decrease.| **That upright side multiplied by the gnomon, divided by <its> shadow, produces the base||** ***chāyāguṇitaṃ chāyāgravivaram ūnena bhājitaṃ koṭī|*** ***śaṅkuguṇā koṭī sā chāyābhaktā bhujā bhavati||***	Formulaic description of an algorithm and prescriptive description of the world.
Ab.2.17. That which is the square of the base and the square of the upright side is the square of the hypotenuse.| **In a circle, the product of both arrows, that is the square of the half-chord, certainly, for two bow <fields>||** ***yaś caiva bhujāvargaḥ koṭīvargaś ca karṇavargaḥ saḥ|*** ***vṛtte śarasaṃvargo 'rdhajyāvargaḥ sa khalu dhanuṣoḥ||***	Description of the world.
Ab.2.18. One should divide separately the <diameter of> the two circles decreased by the *grāsa* and having the *grāsa* for multiplier,| **The two quotient <of the division> by the sum of <the diameter> decreased by the *grāsa* are the two arrows at the meeting, which are <in relation to> one another||** ***grāsone dve vṛtte grāsaguṇe bhājayet pṛthaktvena|*** ***grāsonayogalabdhau saṃpātaśarau parasparataḥ||***	An incomplete division stated in the first half verse is prescribed, the second half verse completes the missing part and describes what is obtained.
Ab.2.19. The desired <number of terms>, decreased by one, halved, increased by the previous <number of terms>, having the common difference for multiplier, increased by the first term, is the mean <value>| **<The result>, multiplied by the desired, is the value of the desired <number of terms>. Or else, the first and last <added together> multiplied by half the number of terms <is the value>.||** ***iṣṭaṃ vyekaṃ dalitaṃ sapūrvam uttaraguṇaṃ samukhaṃ madhyam|*** ***iṣṭaguṇitam iṣṭadhanaṃ tv athādyantaṃ padārdhahatam||***	Five mathematical rules in one. Appears as a formulaic description of an algorithm.
Ab.2.20 The value of the terms multiplied by eight and the common difference, increased by the square of the difference of twice the first term and the common difference,| **<Its> square root, decreased by twice the first term, divided by its common difference, increased by one and halved.||** ***gaccho 'ṣṭottaraguṇitād dviguṇādyuttaraviśeṣavargayutāt* |** ***mūlaṃ dviguṇādyūnaṃ svottarabhajitaṃ sarūpārdham* ||	List of operations in due order: formulaic description of an algorithm.
Ab.2.21 The product of three <quantities> starting with the number of terms of the sub-pile whose common difference **and first term is one, and increasing by one,|** **Divided by six, that is the solid <made> of a pile, or the cube of the number of terms increased by one, decreased by <its cube>root, <divided by six produces the same result>||** ***ekottarādyupaciter gacchādyekottaratrisaṃvargaḥ||*** ***ṣaḍbhaktaḥ sa citighanaḥ saikapadaghano vimūlo vā||***	Formulaic description of an algorithm.
Ab.2.22. One sixth of the product of three <quantities which are>, in due order, the number of terms, <that> increased by one, and <that increased> by the <number of> terms| **That will be the solid <made> of a pile of squares, and the square of a pile should produce the solid <made> of a pile of cubes||** ***saikasagacchapadānāṃ kramāt trisaṃvargitasya ṣaṣṭho 'ṃśaḥ|*** ***vargacitighanaḥ sa bhavec citivargo ghanacitighanaś ca||***	Prescriptive formulaic description of an algorithm.

Verses of the mathematical chapter of the Ab	*Type as understood by me with Bhāskara's lense*
Ab.2.23 Indeed, one should merely subtract from the square of the sum, the sum of two squares| **That which is its half should be known as the product of two multipliers||** ***samparkasya hi vargād viśodhayed eva vargasamparkam|*** ***yat tasya bhavaty ardhaṃ vidyād guṇakārasaṃvargam||***	Prescriptive formulaic description of an algorithm.
Ab.2.24. The square root of the product <of two quantities> with the square of two for multiplier, increased by the square of the difference of the two,| **Is increased or decreased by the difference, and halved, <this will produce> the two multipliers of that <product>.||** ***dvikṛtiguṇāt saṃvargād dvyantaravargeṇa saṃyutān mūlam|*** ***antarayuktaṃ hīnaṃ tadguṇakāradvayaṃ dalitam||***	Formulaic description of an algorithm.
Ab.2.25 The interest on the capital, together with the interest <on the interest>, with the time and capital for multiplier, increased by the square of half the capital| **The square root of that, decreased by half the capital and divided by the time, is the interest on one's own capital||** ***mūlaphalaṃ saphalaṃ kālamūlaguṇam ardhamūlakṛtiyuktam|*** ***tanmūlaṃ mūlārdhonaṃ kālahṛtaṃ svamūlaphalam||***	Formulaic description of an algorithm.
Ab.2.26. Now, when one has multiplied that fruit quantity in the rule by the desire quantity| **What has been obtained from that divided by the requisite should be this fruit of the desire||** ***trairāśikaphalarāśiṃ tam athecchārāśirāśinā hataṃ kṛtvā|*** ***labdhaṃ pramāṇabhajitaṃ tasmād icchāphalam idaṃ syāt//***	Prescriptive description of the world and algorithm.
Ab.2.27 The denominators are respectively multiplied to the multipliers and the divisor.| **One and the other <quantity> with a denominator has the denominator for multiplier that is the state of** **having the same category||** ***chedāḥ parasparahatā bhavanti guṇakārabhāgahārāṇām|*** ***chedaguṇaṃ sacchedaṃ parasparaṃ tat savarṇatvam||***	Definition. Prescriptive description of the world.
Ab.2.28 In a reversed <operation>, multipliers become divisors and divisors, multipliers | **And an additive <quantity> becomes a subtractive <quantity>, a subtractive <quantity> an additive <quantity>.||** ***guṇakārā bhāgaharā bhāgaharās te bhavanti guṇakārāḥ |*** ***yaḥ kṣepaḥ so'pacayo'pacayaḥ kṣepaś ca viparīte ||***	Description of the world, definition.
Ab.2.29. The value of the terms decreased by <each> quantity, separately added| **Is divided by the terms decreased by one, in this way, that becomes the whole value||** ***rāśyūnaṃ rāśyūnaṃ gacchadhanaṃ piṇḍitaṃ pṛthaktvena|*** ***vyekena padena hṛtaṃ sarvadhanaṃ tad bhavaty evam||***	Formulaic description of an algorithm and prescriptive description of the world.
Ab.2. 30. One should divide the difference of coin <belonging> to two men by the difference of beads.| **The result is the price of a bead, if what is made into money <for each man> is equal.||** ***gulikāntareṇa vibhajed dvayoḥ puruṣayos tu rūpa|*** ***labdhaṃ gulikāmūlyaṃ yady arthakṛtaṃ bhavati tulyam||***	Prescription of an algorithm, and promise as description of the world.
Ab.2.31 When the distance of <two bodies moving in> opposite directions is divided by the sum of two motions; <or> when the distance of two <bodies moving in> the same direction <is divided>| **By the difference of two motions, the two <quotient> obtained are the past or future meeting time of the two.||** ***bhakte vilomavivare gatiyogenānulomavivare dvau|*** ***gatyantareṇa labdhau dviyogakālāv atītaiṣyau||***	Description of the world.

Verses of the mathematical chapter of the Ab	*Type as understood by me with Bhāskara's lense*
Ab.2.32. One should divide the divisor of the greater remainder by the divisor of the smaller remainder.| **The mutual division <of the previous divisor> by the remainder <is made continuously. The last remainder> having a clever <thought> for multiplier is added to the difference of the <initial> remainders <and divided by the last divisor>.**|| **Ab.2.33. The one above is multiplied by the one below, and increased by the last. When <the result of this procedure> is divided by the divisor of the smaller remainder**| **The remainder, having the divisor of the greater remainder for multiplier, and increased by the greater remainder is the <quantity that has such> remainders for two divisors**|| ***adhikāgrabhāgahāraṃ chindyād ūnāgrabhāgahāreṇa***| ***śeṣaparasparabhaktaṃ matiguṇam agrāntare kṣiptam***|| ***adhauparigunitam antyayugūnāgracchedabhājite śeṣam***| ***adhikāgracchedaguṇaṃ dvicchedāgram adhikāgrayutam***||	Prescriptive algorithm.