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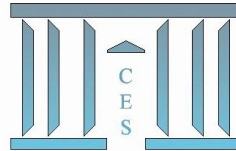
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**On the Measurement of Functional Income Distribution**

Marco RANALDI

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*Version révisée*



# On the Measurement of Functional Income Distribution\*

Marco Ranaldi<sup>†</sup>

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## Abstract

The present paper proposes a theoretical framework to examine the relationship between the functional and personal distribution of income. To this end, it introduces the concept of *inequality in income composition*. Inequality in income composition is high when two different sources of income are separately earned by the top and the bottom of the income distribution. On the contrary, it is low when each individual has the same population share of the two sources. This article designs an indicator to measure income composition inequality, named *income-factor concentration index*,  $\mathcal{I}_f$ . The sign of the indicator determines the condition of transmission for the rising share of income from any source to increase overall income inequality. This framework is then applied to several European countries on basis of EU-SILC data.

**JEL-Classification:** C430, E250

**Keywords:** Income Inequality, Income Composition Inequality, Income Distribution, Functional Income Distribution.

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# 1 Introduction

The study of the relationship between the functional and personal distribution of income has seen a revival of interest in the past years. Important scholars have given further impetus to the re-introduction of this relationship on today's economic research agenda (Atkinson, 1997, 2009; Glyn, 2011). According to Soci and Alacevich (2017), Atkinson and Freedman consider this to be 'the major gap in modern economic theory'. Several contributions have recently explored the empirical nature of this link (e.g. Piketty, 2014; Bengtsson and Waldenström, 2015; Francese and Mulas-Granados, 2015; Cirillo, Corsi and D'Ippoliti, 2017; Gabbuti, 2018), but little attention has been paid to its theoretical character. In his recent work, a chapter in the *After Piketty* book, Branko Milanovic (Milanovic, 2017) argues that in a context of rising share of capital income<sup>1</sup> the level of income inequality grows only under two conditions: (i) a high level of inequality in capital income and (ii) a high and positive association between capital-rich and overall income-rich people. These two conditions, operationalized by the Gini of capital income and the correlation coefficient between capital and total income respectively,<sup>2</sup> suggests an important theoretical connection between factor shares and income inequality. Particularly, the correlation coefficient between capital and total income, which is an elasticity of inter-personal income Gini to changes in capital income share, may act as a measure of such link.

Although other measures of association between capital and labor have also recently

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<sup>1</sup>Such context is determined by the fact that the rate of return on capital income tends to be greater than the growth rate of overall income (Piketty, 2014).

<sup>2</sup>These two variables emerge from the Yitzhaki-Lerman decomposition of the Gini coefficient (Yitzhaki and Lerman, 1984).

been proposed ([Atkinson and Lakner, 2017](#); [Aaberge, Atkinson and Knigs, 2018](#)), none of them directly discusses the relationship between functional and personal distribution of income. This article fills this gap by introducing the concept of *income composition inequality*. We say that inequality in income composition is high when two different sources of income are separately earned by the top and the bottom of the income distribution. On the contrary, it is low when each individual has the same population share of the two sources. This article shows that a high level of income composition inequality is associated with a strong relationship between functional and personal distribution of income.

To measure income composition inequality we design a novel indicator, called *income-factor concentration index*,  $\mathcal{I}_f$ . This indicator is constructed by means of specific concentration curves: the concentration curves for income source ([Ranaldi, 2017](#)). These curves cumulates the distribution of a given income source across the population, with individuals indexed by their income rank. Differently from usual concentration curves ([Kakwani, 1977a, 1977b](#)), these curves do not sum up to one, but to the total level of the factor share. In this way, the Lorenz curve for income can be decomposed into the sum of the concentration curves for each income source. This is a desirable property for the technical assessment of income composition inequality.

Following the way in which the Gini coefficient is computed,<sup>3</sup> we define the income-factor concentration index as the difference between the concentration curve for a given income source and its zero-concentration curve, suitably normalized. Both conditions of

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<sup>3</sup>The Gini is computed by taking the difference between the Lorenz curve for income and the egalitarian line, suitably normalized

minimum and maximum-concentration are defined in the article.

This indicator can be looked at in two ways. Firstly, from a technical perspective, it can be seen as an elasticity of personal income inequality to fluctuations of the functional income distribution. In other words, it mathematically links the functional and personal distribution of income. Secondly, from a political economy perspective, it measures the 'degree of capitalism' of a given social system. Specifically, following the classification proposed by Milanovic, under the maximum-concentration condition a society can be seen as a case of *classical capitalism*, while under the zero-concentration condition it can be regarded as a *new capitalism* (Milanovic, 2017). For instance, a fall in the indicator suggests that the corresponding society is moving towards a new form of capitalism, in which individuals have multiple sources of income at their disposal, and where there is a weaker relationship between functional and personal distribution of income.

The paper is structured as follows. Section 2 introduces the general framework and the income-factor concentration index. Section 3 derives the indicator in a two-person economy and describes its mathematical properties. Section 4 applied the proposed framework to several European countries. Section 5 concludes.

## 2 The Methodology

### 2.1 Concentration Curves for Income Source

Assume we have a  $n$ -sized population, where each individual is endowed with income  $Y_i$  with  $i = 1, \dots, n$ . We define each individual  $i$ 's income share as  $y_i = \frac{Y_i}{Y}$ , where

$Y = \sum_{i=1}^n Y_i$  is the total income of the population. Total income is divided into two sources, capital ( $\Pi$ ) and labor ( $W$ ), so that  $Y = \Pi + W$  and hence  $y = \pi + w$ , where  $\pi = \frac{\Pi}{Y}$  and  $w = \frac{W}{Y}$  are the capital and labor shares in income respectively.<sup>4</sup> As in Ranaldi (2017), we decompose individual  $i$ 's income as follows:

$$y_i = \alpha_i \pi + \beta_i w, \tag{1}$$

where  $\alpha_i = \frac{\Pi_i}{\Pi}$  and  $\beta_i = \frac{W_i}{W}$  are the relative shares of capital and labor of individual  $i$ , such that  $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i = 1$  while  $\Pi_i$  and  $W_i$  represent  $i$ 's total amount of capital and labor in the economy. Suppose that  $y_i \leq y_{i+1} \forall i = 1, \dots, n-1$  and  $y_0 = 0$ , so that individuals are indexed by their income rank. We define  $p = \frac{i}{n}$  as the proportion of the population with income less than or equal to  $y_p$ , so that  $p \in \mathbb{Q} := [0, 1]$ . Let  $\mathcal{L}(\mathbf{y}, p) = \sum_{j=1}^i y_j$ , with  $i = 1, \dots, n$ , be the Lorenz curve for income corresponding to the distribution  $\mathbf{y}$ .<sup>5</sup>

Following Ranaldi (2017), let  $\mathcal{L}(\boldsymbol{\pi}, p) = \pi \sum_{j=1}^i \alpha_j$ , with  $i = 1, \dots, n$  be the concentration curve for capital corresponding to the distribution  $\boldsymbol{\pi}$ , and  $\mathcal{L}(\mathbf{w}, p) = w \sum_{j=1}^i \beta_j$ , with  $i = 1, \dots, n$  be the concentration curve for labor corresponding to the distribution  $\mathbf{w}$ . The individuals are always indexed by their income rank (and not by their capital or labor rank). Figure 1 provides us with a graphical representation of the two curves just introduced.

The concentration curves allow us to get a general idea of whether a given income source is mainly concentrated at the bottom or at the top of the income distribution. Given the interdependence of the two concentration curves (i.e. when one source is con-

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<sup>4</sup>This method can also be used to compare different income factors from capital and labor.

<sup>5</sup>We define the Lorenz curve as in Shorrocks (1983).

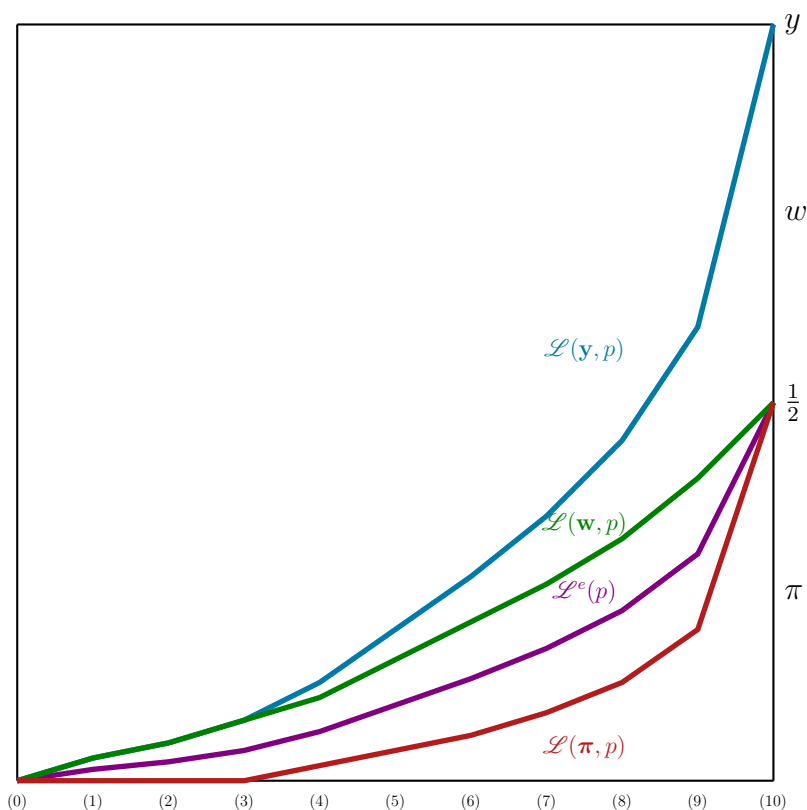
Income-Factor Concentration - A Graphical Representation with  $n = 10$ 

Figure 1: A graphical representation of the concentration curve for capital  $\mathcal{L}(\pi, p)$ , the Lorenz curve for income  $\mathcal{L}(y, p)$ , the concentration curve for labor  $\mathcal{L}(w, p)$  and the zero-concentration curve  $\mathcal{L}^e(p)$  with 10 individuals (or groups) and equal sources of income in the economy ( $\pi = w$ ).

centrated at the top the other is concentrated at the bottom), a single curve is sufficient to analyze the joint distribution of capital and labor. However, in order to precisely assess the extent to which capital and labor are polarized across the income distribution,<sup>6</sup> two benchmark conditions must be defined: the zero and maximum-concentration

<sup>6</sup>It is important to observe that there are some similarities between the concept of concentration that we use in this paper and that of polarization. Specifically, the articles by [Araar \(2008\)](#), [Deutsch and Silber \(2010\)](#) and [Deutsch, Fusco, and Silber \(2013\)](#), which have analyzed the extent to which income sources contribute to income polarization ([Esteban and Ray, 1991, 1994](#)) and income bi-polarization ([Foster and Wolfson, 1992, 2010](#)), may be regarded as similar to our analysis to a certain extent. However, these papers do not study the polarization of income sources in itself, independently from the issue of income polarization, which is exactly our objective. It can be noted that a positive level of income polarization always implies a positive level of income inequality. This is not necessarily the case once we compare income concentration with income inequality. In addition to that, the methodology



conditions. On the basis of these two conditions, the corresponding zero and maximum-concentration curves are hence introduced.

## 2.2 The Zero-Concentration Curve

Let us start by recalling the definition of zero-concentration of two income sources. For simplicity, in what follows we refer to capital and labor as the two income sources.

**Definition 2.1.** *We say that capital and labor are not concentrated across a population when each individual has the same population shares of the two sources. Formally, when*

$$\frac{W_i}{\Pi_i} = \frac{w}{\pi} \forall i, \text{ or, equivalently, when } \alpha_i = \beta_i \forall i.$$

Note that only two elements are needed in order to determine the zero-concentration condition, notably the functional and personal distribution of income. Two populations characterized by different Lorenz curves, or by different shares of capital income, have two different conditions of zero-concentration.

Interestingly, the distribution of capital and labor under the zero-concentration condition represents the long-run distribution of factors across individuals in a neoclassical framework, in which heterogeneity of both non-accumulated and accumulated factors are considered (Bertola et al., 2005). It also coincides with the underlying distribution of factors in the *New Capitalism 2* society as defined by Milanovic (2017).

At this point of the analysis, we define the zero-concentration curve,  $\mathcal{L}^e(\mathbf{z}, p)$ , corresponding to the distribution  $\mathbf{z}$ , as follows:  $\mathcal{L}^e(\mathbf{z}, p) = z \sum_{j=1}^i y_j$ , with  $i = 1, \dots, n$  and with  $z = \pi, w$ . Note that the zero-concentration curve is a scaled version of the 

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we construct is completely different from the one proposed by Esteban and Ray (1994) in their seminal article.

Lorenz curve for income. When the concentration curve for capital lies below the zero-concentration curve, then the capital is mainly concentrated at the top of the distribution; on the contrary, when the concentration curve for capital lies above the zero-concentration curve the reverse situation holds true, and the capital is mainly concentrated at the bottom of the distribution.

The zero-concentration curve describes the distribution of income sources such that an increase in any of the two factor shares of total income has no impact on inter-personal income inequality. This is equivalent to say that the Gini coefficient does not depend on the functional distribution of income (see [Ranaldi, 2017](#)). Additionally, we can observe that when the concentration curve for a given income source lies below the  $\mathcal{L}^e$  curve, then the concentration curve for the other income source lies above the  $\mathcal{L}^e$  curve.

We conclude this section with the following definition.

**Definition 2.2.** *We say that under zero-concentration of income sources, inequality in income composition is minimal.*

### 2.3 The Maximum-Concentration Curve

Let us focus our attention on the benchmark of maximum-concentration of two income sources, that we define as follows.

**Definition 2.3.** *We say that two income sources are maximum concentrated when the bottom  $p\%$  of the income distribution has an income consisting only of the source  $z$  and the top  $(1-p)\%$  of the income distribution has an income consisting only of the source  $z_-$ , where  $p$  s.t.  $y_p = \mathcal{L}(\mathbf{y}, p) = z$ ,  $1-p$  s.t.  $y_{1-p} = 1 - \mathcal{L}(\mathbf{y}, p) = z_-$ ,  $z_- = 1 - z$  and*

$z = \pi, w$ .

As for the condition of zero-concentration, also the condition of maximum-concentration is already present in the literature. In his recent article, Milanovic defines classical capitalism as a society in which "ownerships of capital and labor are totally separated, in the sense that workers draw their entire income from labor and have no income from the ownership of assets, while the situation for the capitalists is the reverse. Moreover, we shall assume that all workers are poorer than all capitalists. This gives us [...] two social groups, non-overlapping by income level" (Milanovic, 2017). We can therefore say that under the condition of maximum-concentration, specifically when the capital is owned by the top of the distribution and the labor by the bottom, a society is a classical capitalism á la Milanovic.<sup>7</sup>

From a formal point of view, we define the maximum-concentration curve,  $\mathcal{L}^{max}(\mathbf{z}, p)$ , corresponding to the distribution  $\mathbf{z}$ , as follows:

$$\mathcal{L}^{max}(\mathbf{z}, p) = \begin{cases} \mathcal{L}^M(\mathbf{z}, p) = \begin{cases} \mathcal{L}(\mathbf{y}, p) & \text{for } p \leq p' \\ z & \text{for } p > p' \end{cases} \\ \mathcal{L}^m(\mathbf{z}, p) = \begin{cases} 0 & \text{for } p \leq p'' \\ \mathcal{L}(\mathbf{y}, p) - z_- & \text{for } p > p'' \end{cases} \end{cases}, \quad (2)$$

with  $p'$  s.t.  $\mathcal{L}(\mathbf{y}, p') = z$ ,  $p''$  s.t.  $\mathcal{L}(\mathbf{y}, p'') = 1 - z$  and  $\mathbf{z} = \pi, w$ . In addition, we have:

- (i)  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$  if  $\mathcal{L}(\mathbf{z}, p) \geq \mathcal{L}^e(\mathbf{z}, p) \forall p$  and  $\exists p^*$  s.t.  $\mathcal{L}(\mathbf{z}, p^*) > \mathcal{L}^e(\mathbf{z}, p^*)$ ,

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<sup>7</sup>This type of society can also be found in the works by Kaldor (1955), Pasinetti (1962) or more recently by Stiglitz (2015), in which a class of capitalists is counterposed to a class of workers. However, these authors do not necessarily assume that the former class is poorer than the latter in terms of income.

- (ii)  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$  if  $\mathcal{L}^e(\mathbf{z}, p) \leq \mathcal{L}(\mathbf{z}, p) \forall p$  and  $\exists p^{**}$  s.t.  $\mathcal{L}^e(\mathbf{z}, p^{**}) < \mathcal{L}(\mathbf{z}, p^{**})$ .

To put it simply, we have that  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$  when the concentration curve lies above the zero-concentration curve, and that  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$  when the concentration curve lies below the zero-concentration curve.

However, the two conditions above-mentioned ((i) and (ii)) are rather strong, since they imply that the two curves cannot intersect along the distribution of income. A weaker condition is, instead, the one which considers the area covered by each curve, as follows:<sup>8</sup>

- (i)  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^M(\mathbf{z}, p)$  if  $\sum_{i=1}^n \sum_{j=1}^i \eta_j^k > \sum_{i=1}^n \sum_{j=1}^i y_j$ ,
- (ii)  $\mathcal{L}^{max}(\mathbf{z}, p) = \mathcal{L}^m(\mathbf{z}, p)$  if  $\sum_{i=1}^n \sum_{j=1}^i \eta_j^k < \sum_{i=1}^n \sum_{j=1}^i y_j$ ,

where  $\eta_j^k = \alpha_j$  if  $z = \pi$  and  $\eta_j^k = \beta_j$  when  $z = w$ .

As for the previous section, we conclude with the following definition.

**Definition 2.4.** *We say that under maximum-concentration of income sources, income composition inequality is maximum.*

## 2.4 The Income-Factor Concentration Index

In the previous sections, we defined the two benchmarks of zero and maximum inequality in income composition, together with their corresponding concentration curves.

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<sup>8</sup>Similarly, the first and second group of conditions can be regarded as of first and second-order stochastic dominance.

When the actual concentration curve is close to the zero-concentration curve, then income composition inequality is low. On the contrary, when the actual concentration curve is close to the maximum-concentration curve, then income composition inequality is high.

At this point, in order to precisely measure the level of income composition inequality, we introduce an indicator which serves this purpose. We call this metric as *income-factor concentration index*,  $\mathcal{I}_f$ , which we construct as follows.

Let us call  $\mathcal{A}(\mathbf{z})$  the area between the zero-concentration curve and the concentration curve for income source  $z$ , and  $\mathcal{B}(\mathbf{z})$  the area between the zero-concentration curve and the maximum-concentration curve.<sup>9</sup> We define the income-factor concentration index,  $\mathcal{I}_f(\mathbf{z})$ , corresponding to the distribution  $\mathbf{z}$ , as follows:

$$\mathcal{I}_f(\mathbf{z}) = \frac{\mathcal{A}(\mathbf{z})}{\mathcal{B}^{max}(\mathbf{z})}, \quad (3)$$

with  $\mathbf{z} = \boldsymbol{\pi}, \mathbf{w}$ .

This measure has considerable intuitive appeal: it is the area between the zero-concentration curve  $\mathcal{L}^e(\mathbf{z}, p)$  and the concentration curve for income source  $\mathcal{L}(\mathbf{z}, p)$ , divided by the area between the zero-concentration curve  $\mathcal{L}^e(\mathbf{z}, p)$  and the maximum-concentration curve  $\mathcal{L}^{max}(\mathbf{z}, p)$ .<sup>10</sup> This measure lies therefore between  $-1$  (when the people at the bottom own source  $z$  and the people at the top own source  $z_-$ ) and  $1$  (when the people at the bottom own source  $z_-$  and the people at the top own source  $z$ ).

It is equal to zero when the area of the concentration curve is the same as that of the

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<sup>9</sup>Formally,  $\mathcal{A}(\mathbf{z}) = \frac{1}{2n} \sum_{i=1}^n [(\mathcal{L}^e(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^e(\mathbf{z}, \frac{i-1}{n})) - (\mathcal{L}(\mathbf{z}, \frac{i}{n}) + \mathcal{L}(\mathbf{z}, \frac{i-1}{n}))]$  and  $\mathcal{B}^{max}(\mathbf{z}) = \frac{1}{2n} \sum_{i=1}^n [(\mathcal{L}^e(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^e(\mathbf{z}, \frac{i-1}{n})) - (\mathcal{L}^{max}(\mathbf{z}, \frac{i}{n}) + \mathcal{L}^{max}(\mathbf{z}, \frac{i-1}{n}))]$ , with  $max = m, M$ .

<sup>10</sup>Note that the areas between the curves  $\mathcal{L}^M(\mathbf{z}, p)$  and  $\mathcal{L}^e(\mathbf{z}, p)$ , and the curves  $\mathcal{L}^e(\mathbf{z}, p)$  and  $\mathcal{L}^m(\mathbf{z}, p)$  are the same for specific functional form of  $\mathcal{L}(\mathbf{y}, p)$  and for certain values of  $z$  (see the appendix for further details).

zero-concentration curve.<sup>11</sup> Furthermore, it can be noticed that  $\mathcal{I}_f(\mathbf{z}) = -\mathcal{I}_f(\mathbf{z}_-)$  (see the Appendix for further details).

In light of the relationship previously discussed between the concentration curves and the ideal-typical social systems proposed by Milanovic, we can also interpret such indicator as a measure of the degree of capitalism of a given social system. Furthermore, the new type of capitalism can also be seen as a multiple sources of income society.

The income-factor concentration index is not a rank-based measure of association between labor and capital (Atkinson and Lakner, 2017). Indeed, a monotone transformation in the marginal distributions would affect the index by changing the ranking in the distribution of total income.<sup>12</sup>

Although it may seem of little interest to consider negative values of the index, they have a powerful meaning in terms of income composition dynamics, as stated by the following definition.

**Definition 2.5.** *Let  $sign_{t,t+1}$  be the sign of  $\mathcal{I}_f^t(\mathbf{z}) \cdot \mathcal{I}_f^{t+1}(\mathbf{z})$ , where  $\mathcal{I}_f^t(\mathbf{z})$  is the income-factor concentration index at time  $t$ , while  $\mathcal{I}_f^{t+1}(\mathbf{z})$  the one at time  $t+1$ . We say that a change in the structure of income composition across the distribution of income occurs at time  $t$  if  $sign_{t,t+1} < 0$ .*

When a change in sign occurs at time  $t+1$  (i.e.  $sign_{t,t+1} < 0$ ), those who mainly own source  $z$  at time  $t$  earn source  $z_-$  at time  $t+1$  and vice versa.

The normalization coefficient  $\mathcal{B}^m(\mathbf{z})$  is a function of  $\mathcal{L}(\mathbf{y}, p)$ ,  $z$  and  $p''$ , while the coefficient  $\mathcal{B}^M(\mathbf{z})$  is a function of  $\mathcal{L}(\mathbf{y}, p)$ ,  $z$  and  $p'$ . To simplify the notation, let us

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<sup>11</sup>The latter may happen without that the two curves coincide.

<sup>12</sup>For a full discussion on rank-based measures of association, see Dardanoni and Lambert (2001), Atkinson and Lakner (2017), Aaberge, Atkinson and Knigs (2018).

generally call as  $\mathcal{B}(\mathbf{z})$  the denominator of the income-factor concentration index. A more compact expression for the index is, for  $z = \pi$ , the following:

$$\mathcal{I}_f(\boldsymbol{\pi}) = \frac{w\pi(\tilde{\mu}_w - \tilde{\mu}_\pi)}{\mathcal{B}(\boldsymbol{\pi})}, \quad (4)$$

where  $\tilde{\mu}_\pi = \frac{1}{2n} \sum_{i=0}^n \left( \sum_{j=0}^i \alpha_j + \sum_{j=0}^{i+1} \alpha_j \right)$  and  $\tilde{\mu}_w = \frac{1}{2n} \sum_{i=0}^n \left( \sum_{j=0}^i \beta_j + \sum_{j=0}^{i+1} \beta_j \right)$  are the areas of the concentration curves for labor and capital multiplied by  $\frac{1}{w}$  and  $\frac{1}{\pi}$  respectively.<sup>13</sup> Similarly, for  $z = w$ , we have:

$$\mathcal{I}_f(\mathbf{w}) = \frac{w\pi(\tilde{\mu}_\pi - \tilde{\mu}_w)}{\mathcal{B}(\mathbf{w})}. \quad (5)$$

Equations 4 and 5 simply mean to illustrate the functional forms of this indicator once we separately analyze the concentration of capital and labor at the top respectively. Specifically, when equation 4 is positive, then the capital is mainly concentrated at the top of the income distribution and the labor at the bottom. Conversely, when equation 5 is positive, then the labor is mainly concentrated at the top of the income distribution and the capital at the bottom. As we have previously discussed, the following relationship holds:  $\mathcal{I}_f(\boldsymbol{\pi}) = -\mathcal{I}_f(\mathbf{w})$ .

The two functions  $\tilde{\mu}_\pi$  and  $\tilde{\mu}_w$  have a precise dynamics: they increase (decrease) when the source in question moves towards the bottom (top) of the distribution. These areas can thus be considered as approximate metrics of income-factor concentration.<sup>14</sup> In a similar manner, the function  $\tilde{\mu}_y$  is a measure of income inequality: when it rises so does the surface of the Lorenz curve, by therefore reducing its distance from the egalitarian

<sup>13</sup>Note that one minus twice  $\tilde{\mu}_z$  gives the pseudo-Gini of income source  $z$  (see [Shorrocks, 1982](#)).

<sup>14</sup>We can also observe that the term  $\tilde{\mu}_\pi$  (and similarly  $\tilde{\mu}_w$  and  $\tilde{\mu}_y$ ) can be expressed as follows:  $\tilde{\mu}_\pi = \sum_{i=1}^n \alpha_i \left( \frac{2n-2i+1}{2n} \right)$ . It suffices to note that  $\tilde{\mu}_\pi = \frac{1}{2n} \sum_{i=0}^n \left( \sum_{j=0}^i \alpha_j + \sum_{j=0}^{i+1} \alpha_j \right) = \frac{1}{2n} \sum_{i=1}^n \left( 2 \sum_{j=1}^i \alpha_j + \alpha_i \right) = \frac{1}{n} \sum_{i=0}^n \sum_{j=0}^i \alpha_j + \frac{1}{2n} \sum_{i=0}^n \alpha_i$ , from which we obtain the result.

line.

## 2.5 From Functional to Personal Distribution of Income

In this section, we further investigate the relationship between functional income distribution and income inequality, in light of the novel metric previously illustrated. To this end, let us consider the well-known relationship between  $\tilde{\mu}_y$  (the area of the Lorenz curve) and the Gini coefficient:

$$\mathcal{G} = 1 - 2\tilde{\mu}_y. \quad (6)$$

The latter can be further developed, so to obtain:

$$\mathcal{G} = 1 - 2(z(\tilde{\mu}_z - \tilde{\mu}_{z_-}) + \tilde{\mu}_{z_-}). \quad (7)$$

The Gini coefficient can therefore be expressed as a function of the two approximate metrics of income-factor concentration  $\tilde{\mu}_z$  and  $\tilde{\mu}_{z_-}$  and of the factor share  $z$ . If we take the derivative of  $\mathcal{G}$  with respect to  $z$ , we obtain:

$$\frac{\partial \mathcal{G}}{\partial z} = 2(\tilde{\mu}_{z_-} - \tilde{\mu}_z). \quad (8)$$

The elasticity of personal income Gini to changes in the factor shares is (two times) the difference between the areas of the two concentration curves. Note that when  $\tilde{\mu}_{z_-} - \tilde{\mu}_z < 0$ , then an increase in the capital share reduces income inequality.

If we consider the standard decomposition of total income Gini into inequality contributed by each income source:<sup>15</sup>

$$\mathcal{G} = z\mathcal{R}_z\mathcal{G}_z + z_-\mathcal{R}_{z_-}\mathcal{G}_{z_-}, \quad (9)$$

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<sup>15</sup>See [Lerman and Yitzhaki \(1985\)](#) for further details.



where  $\mathcal{R}_z = \frac{\text{cov}(r(y),z)}{\text{cov}(r(z),z)}$  is the correlation ratio between the source  $z$  and total income,  $r(y)$  and  $r(z)$  are the individual's ranks according to total income and source  $z$  respectively, and  $\mathcal{G}_z$  is the Gini coefficient of income source  $z$ , we can write:

$$\frac{\partial \mathcal{G}}{\partial z} = \mathcal{R}_z \mathcal{G}_z - \mathcal{R}_{z-} \mathcal{G}_{z-}, \quad (10)$$

and by combining both equations 8 and 10 we get:

$$2 (\tilde{\mu}_{z-} - \tilde{\mu}_z) = \mathcal{R}_z \mathcal{G}_z - \mathcal{R}_{z-} \mathcal{G}_{z-}. \quad (11)$$

According to Milanovic, "for the rising share of capital income to increase overall income Gini, we need therefore to have two 'transmission' tools, Gini coefficient of capital income and  $\mathcal{R}_\pi$ , positive and high" (Milanovic, 2017), or, from a more formal point of view, the following condition must hold:  $\mathcal{R}_\pi \mathcal{G}_\pi > \mathcal{R}_w \mathcal{G}_w$ . It appears that such condition is well captured by the sign of the income-factor concentration index, which exclusively depends on  $\tilde{\mu}_{z-} - \tilde{\mu}_z$ . Therefore, equation 8 shows that, for the analysis of the relationship between the functional and personal distribution of income, the indicator we propose can be seen as a tool capable of linking these two concepts. For example, if the capital share of income was rising, then income inequality would grow only if the income-factor concentration index was greater than zero.

### 3 The Case of a Two-Person Economy

Let us now consider the scenario in which the population is divided into two groups (i.e.  $n = 2$ ) of equal size. This exercise is of interest for two main reasons. The first reason is that, due to the lack of data, it may be difficult in some cases to compute

the index previously illustrated, which requires information concerning the composition of individual incomes for the whole population.<sup>16</sup> The second reason is that the  $n = 2$  version of the index has some interesting mathematical properties which deserve to be exposed.

Let us call as  $y_p$  the income of the bottom  $p\%$  of the income distribution, and as  $y_{1-p}$  the income of the top  $(1 - p)\%$ , with  $y_p \in ]0, \frac{1}{2}[$ . Figure 2 provides us with a graphical representation of the concentration curves for  $n = 2$ .

The income-factor concentration index with  $n = 2$  takes the following mathematical form:

$$\mathcal{I}_{f,2}(\mathbf{z}, p) = b_{z,p} w \pi (\eta_p^{z-} - \eta_p^z) = t_{z,p} \rho I_f(\mathbf{z}, p), \quad (12)$$

where  $\rho = w\pi$ ,  $I_f(\mathbf{z}, p) = \eta_p^{z-} - \eta_p^z$ , and the normalization coefficient  $b_{z,p}$  is defined as follows:

$$b_{z,p} = \begin{cases} \frac{1}{y_p^z} & \text{if } y_p > \eta_p^z \\ \frac{1}{[\min(y_p, z) - y_p^z]} & \text{if } y_p < \eta_p^z \end{cases}.$$

This version can thus be regarded as the product of three elements, notably  $t_{z,p}$ ,  $\rho$  and  $I_f(\mathbf{z}, p)$ . An interesting way to grasp their meaning is to rewrite the index as follows:

$$\mathcal{I}_{f,2}(\mathbf{z}, p) = b_{z,p} \begin{vmatrix} w & 0 \\ 0 & \pi \end{vmatrix} \begin{vmatrix} \eta_p^{z-} & \eta_p^z \\ \eta_{1-p}^{z-} & \eta_{1-p}^z \end{vmatrix},$$

where the product of the two determinants  $\rho$  and  $I_f(\mathbf{z}, p)$  is simply the determinant of

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<sup>16</sup>This information is generally provided by the surveys, which however tend to underestimate the income of individuals at the top of the distribution.

**Income-Factor Concentration - A Graphical Representation with  $n = 2$**

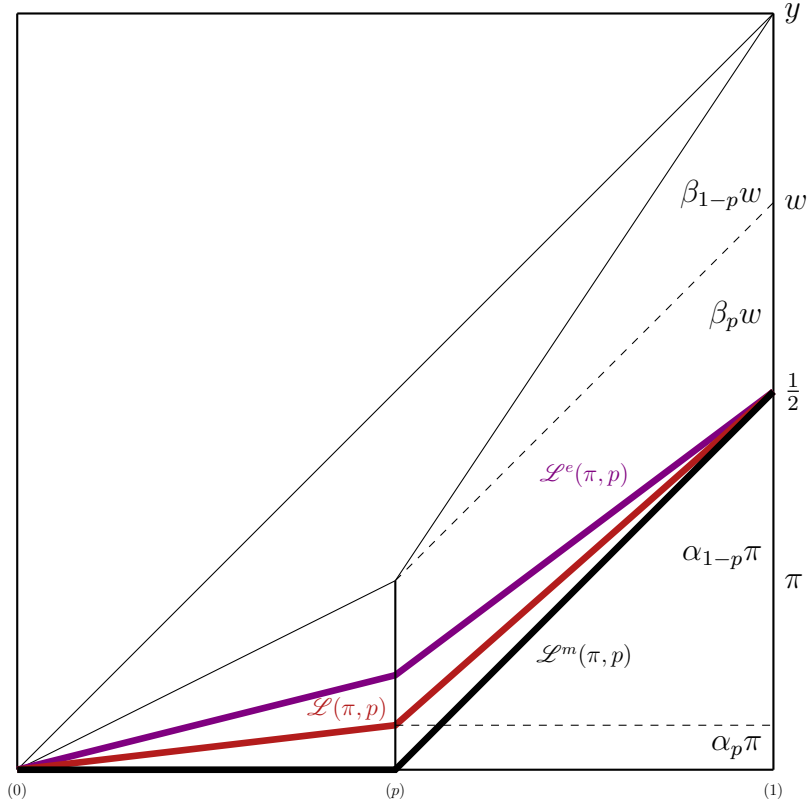


Figure 2: A graphical representation of the methodology in which two people (or groups) with different income ( $y_p < y_{1-p}$ , with  $p = \frac{1}{2}$ ), and two sources of the same amount ( $\pi = w$ ) are compared. The carnelian line  $\mathcal{L}(\pi, p)$  is the concentration curve for capital, while the violet line  $\mathcal{L}^e(p)$  is the zero-concentration curve. The following values have been here assigned:  $y_p = 0.25$ ,  $\pi = w = \frac{1}{2}$ ,  $\alpha_p = 0.12$  and  $\beta_p = 0.38$ .

the following matrix  $A$ :

$$A = \begin{pmatrix} \eta_p^{z-w} & \eta_p^z \pi \\ \eta_{1-p}^{z-w} & \eta_{1-p}^z \pi \end{pmatrix}.$$

The  $\mathcal{S}_{f,2}(\mathbf{z}, p)$  can hence be rewritten as the product between the determinants of two matrices and a normalizing coefficient. The first determinant,  $\rho$ , adjusts the degree of concentration for the level of income sources. The second determinant,  $I_f(\mathbf{z}, p)$ , is, instead, the channel through which the issue of income-source concentration is addressed.

Interestingly, we can notice that the following matrix  $A^*$ :

$$A^* = \begin{pmatrix} \beta_p & \alpha_p \\ \beta_{1-p} & \alpha_{1-p} \end{pmatrix},$$

whose determinant equals the component  $I_f(\mathbf{z}, p)$ , comes from the following relationship:

$$\bar{y} = A^* \bar{x},$$

where  $\bar{y} = \begin{pmatrix} y_p \\ y_{1-p} \end{pmatrix}$  and  $\bar{x} = \begin{pmatrix} w \\ \pi \end{pmatrix}$ , which in turns is equivalent to the following system of equations:

$$\begin{cases} y_p = \beta_p w + \alpha_p \pi \\ y_{1-p} = \beta_{1-p} w + \alpha_{1-p} \pi \end{cases}.$$

When the matrix  $A^*$  is nonsingular (i.e.  $\det A^* \neq 0$ , thus  $I_f(\mathbf{z}, p) \neq 0$ ), then we can write:  $\bar{x} = (A^*)^{-1} \bar{y}$ . It is of interest to observe that when  $\det A^* = 1$ , then ownerships of labor and capital are separated between individual 1 and 2. This explains why the coefficient  $I_f(\mathbf{z}, p)$  can be seen as a proxy of  $\mathcal{I}_f(\mathbf{z})$ .

Another way of writing the  $n = 2$  version of this indicator is the following. Assume that  $y_1 < w$  and  $\mathbf{z} = \boldsymbol{\pi}$ , then we have: <sup>17</sup>

$$\mathcal{I}_{f,2}(\boldsymbol{\pi}, p) = 1 - \frac{\alpha_1}{y_1}. \quad (13)$$

Equation 13 illustrates that the level of income composition inequality is in this very case determined by the ratio  $\frac{\alpha_1}{y_1}$ . This ratio combines individual 1's endowments of

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<sup>17</sup>This is a plausible assumption: in the contrary case, the labor share of income would have been lower than the capital share. In fact, if  $y_1$  was greater than  $w$ , given that  $y_1 < \frac{1}{2}$  by assumption, than we would have  $w < \pi$ . The latter is not supported by the empirical evidences concerning the developed countries (Stockhammer, 2013), with the exception of Mexico (Negrete, 2015).

capital and overall income. When the ratio is greater than one, then individual 1 is more capital poor than income poor. When it is equal to one, then she is as capital poor as income poor, and when it is lower than one, then she is more capital poor than income poor. Therefore, income composition inequality is positive when the poorest part of the society is more capital poor than income poor, and negative in the opposite case.

Let us now illustrate several properties of the  $I_f(\mathbf{z}, p)$ . First of all, the capital to labor ratio can be expressed as follows:

$$\frac{\pi}{w} = \frac{\frac{1}{1+\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}, \quad (14)$$

where  $\varphi = \frac{y_p}{y_{1-p}}$ , from which we simply derive the following result.

**Proposition 3.1.** *A variation of  $\varphi$  has no effect on  $\frac{\pi}{w}$  iff  $I_f(\mathbf{z}, p) = 0$ . Formally:*

$$\frac{\partial \frac{\pi}{w}}{\partial \varphi} = 0 \iff I_f(\mathbf{z}, p) = 0. \quad (15)$$

This result sheds light on the relationship between income inequality ( $\varphi$ ) and factor shares of income ( $\frac{\pi}{w}$ ).<sup>18</sup> A variation of  $\varphi$  does not affect the ratio  $\frac{\pi}{w}$  when the determinant of the matrix  $A^*$  is equal to zero.

Let us now consider the relationship between the determinant  $I_f(\mathbf{z}, p)$  and the between-group Gini coefficient  $G$ .<sup>19</sup> Precisely:

$$\frac{\partial G}{\partial z} = I_f(\mathbf{z}, p) p. \quad (16)$$

An increase in the factor share  $z$  reduces the between-group inequality  $G$  according to the degree of income-source concentration and the share of poor people,  $p$ . If we let

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<sup>18</sup>It is easy to notice that  $\frac{\partial \frac{\pi}{w}}{\partial \varphi} > 0$  when  $I_f(\mathbf{z}, p) > 0$ . In particular, when  $I_f(\mathbf{z}, p) = 1$ , an increase of  $\varphi$  raises the ratio  $\frac{\pi}{w}$  of the same amount. Indeed, when  $I_f(\mathbf{z}, p) = 1$  then  $y_p = \pi$  and  $y_{1-p} = w$ , thus  $\frac{\pi}{w} = \frac{y_p}{y_{1-p}}$ .

<sup>19</sup>See the appendix for further details.

$p$  be equal to  $\frac{1}{2}$  (thus we divide the population into two groups of equal size), and if we set  $z = \pi$ , then we get:

$$\frac{\partial G}{\partial \pi} = \frac{\alpha_{\frac{1}{2}} - \beta_{\frac{1}{2}}}{2}. \quad (17)$$

Equation 17 bears resemblance with equation 8. Specifically, in a two-person economy the condition for the rising share of capital income to increase income Gini is  $I_f(\mathbf{z}, p) > 0$ , or  $\det A^* > 0$ .

## 4 Empirical Application

In this section we apply the method previously illustrated to the case of six European economies, notably Finland, France, Germany, Italy, Norway and The Netherlands. The data we use come from the European Union Statistics of Income and Living Conditions (EU-SILC), which provide a representative sample of the European population. This data are firstly produced by the national statistical offices and later harmonized and released by Eurostat. In our analysis, we consider the period between 2007 and 2016, for which the information we need are available for all the six countries. The country samples vary between 7000 and 19000 units, and the unit of analysis is the household. The choice of these specific countries is determined by their puzzling trends. Indeed, while income composition inequality follows a U-shaped trend for Germany, Norway and The Netherlands over the period considered, the reverse pattern is traced by Finland, France and Italy, as it will be later explained.

Our analysis relies on a specific definition of capital and labor incomes.<sup>20</sup> Precisely,

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<sup>20</sup>The definitions of both capital and labor income can be, to a certain extent, arbitrary. For instance, Cirillo, Corsi and D'Ippoliti (2017), who conduct an empirical study of the functional and personal distribution of income at the European level before and after the crisis, and which also rely on EU-SILC data, provide a slightly different definition of the two sources from our own. Their definition of

we define capital income as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income, which is not directly furnished by EU-SILC, is imputed by means of the procedure proposed by Glyn (2011). We attribute the average payroll income of the entire sample to represent the labor income component of the self-employers, and "the margin of value added per head [...] is then regarded as accruing to the [self-employer] as property income" (Glyn, 2011, p. 8).<sup>21</sup> Labor income is defined as the difference between total household gross income minus capital income.<sup>22</sup>

To overcome the issue of negative values, we replace the bottom part of the concentration curves for which such problem occurs with the horizontal line (i.e. the  $x$ -axis).<sup>23</sup>

Figures 3 and 4 show the overall dynamics of the income-factor concentration index for the two groups of European countries respectively. The first group is composed by Germany, Norway and The Netherlands, while the second by Italy, France and Finland.

To begin with, we can notice that the indicator ranges between .3 and .5 in all countries.

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income, for example, does not include self-employment remuneration, which is an important component of households' budget.

<sup>21</sup>Precisely, we firstly compute the country average payroll income per household,  $\mu_{payroll}$ . Then, we decompose self-employment income into its capital and labor components in the following way. Let us call as  $y_{se}$  the income from self-employment provided by EU-SILC. We can write that  $y_{se} = y_{se}^{\pi} + y_{se}^w$ , where  $y_{se}^w = \begin{cases} y_{se} & \text{if } y_{se} \leq \mu_{payroll} \\ \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$ , while  $y_{se}^{\pi} = \begin{cases} 0 & \text{if } y_{se} \leq \mu_{payroll} \\ y_{se} - \mu_{payroll} & \text{if } y_{se} > \mu_{payroll} \end{cases}$ .

<sup>22</sup>The sources of labor income that we consider are: gross employee cash or near cash income, gross non-cash employee income, employers' social insurance contributions, value of goods produced for own consumption, unemployment benefits, old-age benefits, survivor' benefits, sickness benefits, disability benefits, education-related allowances, family/children related allowances, social exclusion not elsewhere classified, housing related allowances, regular inter-household cash transfers received and income received by people aged under 16.

<sup>23</sup>When a given variable at stake displays negative values, the bottom part of the corresponding concentration curve lies below the horizontal axe.

## Income Composition Inequality

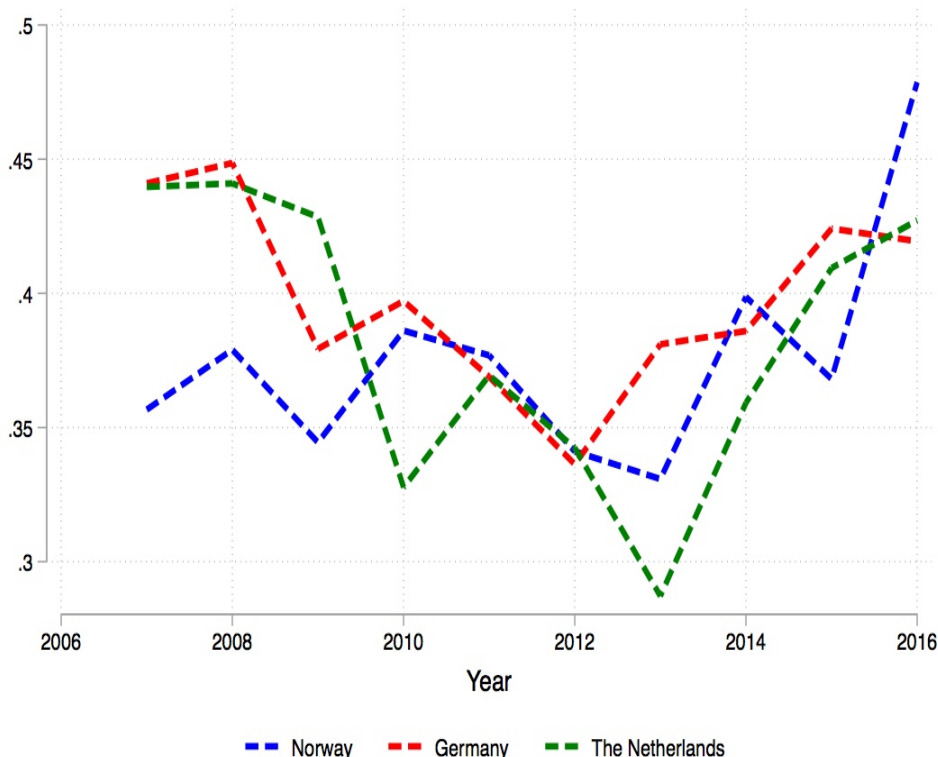


Figure 3: The series of income composition inequality for Norway, Germany and The Netherlands between 2007 and 2016. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). Labor income is defined as the difference between total household gross income minus capital income. **Source:** Author's computation on basis of EU-SILC.

However, as we stated before, the two groups follow different trends over the time considered. As regards to the first group, income composition inequality follows a U-shaped pattern, with its major peaks in 2007 and 2016 and its lowest levels in 2013-2014 (figure 3).

According to the second group, income composition inequality follows an inverted U-shaped motion, with its peaks in 2011 and 2012 and its lowest levels at the beginning



## Income Composition Inequality

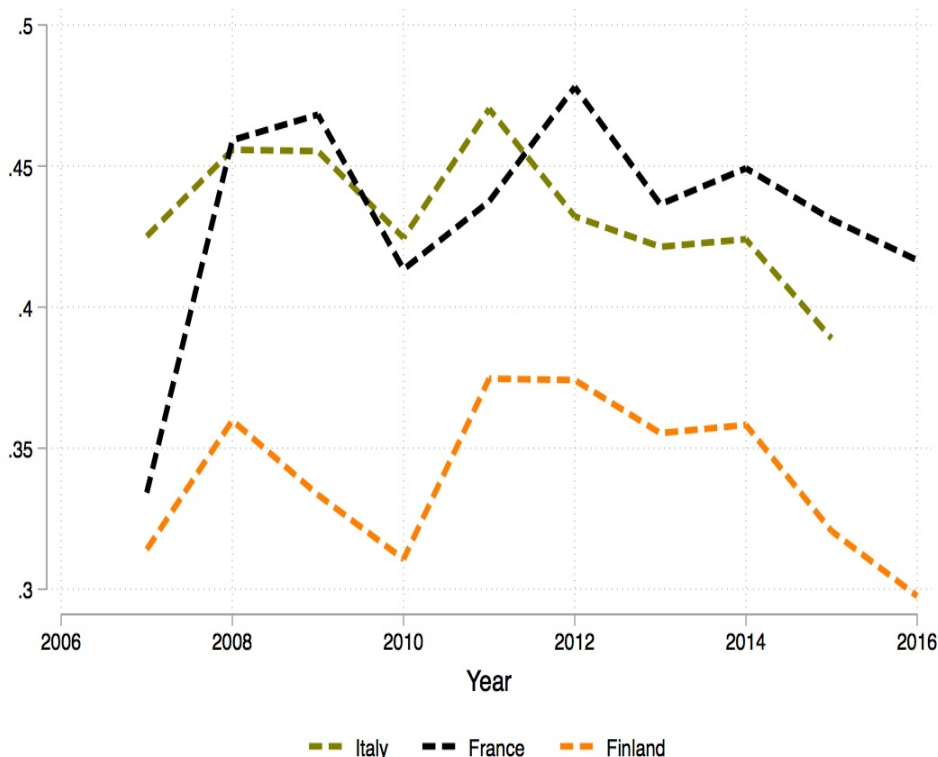


Figure 4: The series of income composition inequality for Italy, France and Finland between 2007 and 2016. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). Labor income is defined as the difference between total household gross income minus capital income. **Source:** Author's computation on basis of EU-SILC.

and at the end of the period (figure 4).

The patterns of Germany and The Netherlands almost coincide in both levels and trends, while that of Norway starts from a lower level in 2007, and ends with a higher level in 2016. On the contrary, all the countries of the second group share the same dynamics of income composition inequality. Although France and Italy exhibit higher levels of the  $\mathcal{I}_f(\boldsymbol{\pi})$  compared to Finland.

Following the political economy framework previously discussed, we can say that the first three countries considered are moving towards a classical capitalism, characterized by a group of rich people owning capital income and a group of poor people owning labor income. This type of society allows for a greater transmission of changes in the functional distribution of income into personal income inequality. Conversely, the second group of countries is moving towards a new capitalism, in which both sources of income are better distributed across the entire population. In the latter society, the relationship between functional and personal distribution of income is relatively weak, implying that fluctuations of both the capital and labor share of income have a less severe impact on the dynamics of income inequality.

At this point of the analysis, it is of utmost importance to analyze the role played by the two components of the income-factor concentration index, notably  $\tilde{\mu}_w$  and  $\tilde{\mu}_\pi$ , in shaping its overall dynamics. The evolution of the areas of the concentration curves for capital and labor are illustrated by figures 5 and 6 for the first group, and 7 and 8 for the second. As expected, the two metrics  $\tilde{\mu}_w$  and  $\tilde{\mu}_\pi$  follow completely independent patterns. Let us start with the first group. For all countries the area of the concentration curve for capital rises up to 2013, and falls afterwards (figure 5). We remind that an increase (decrease) of  $\tilde{\mu}_\pi$  implies that the capital income moves towards the bottom (top) of the income distribution. Therefore, we can state that Germany, Norway and The Netherlands saw their capital income flowing in the hands of the bottom part of the income distribution up to 2013, and afterwards coming back into possession of the rich part of the population. At the same time, the almost flat motion of the area of the concentration curve for labor  $\tilde{\mu}_w$  for all the countries (figure 6) clearly suggests that the

## Area of the Concentration Curve for Capital

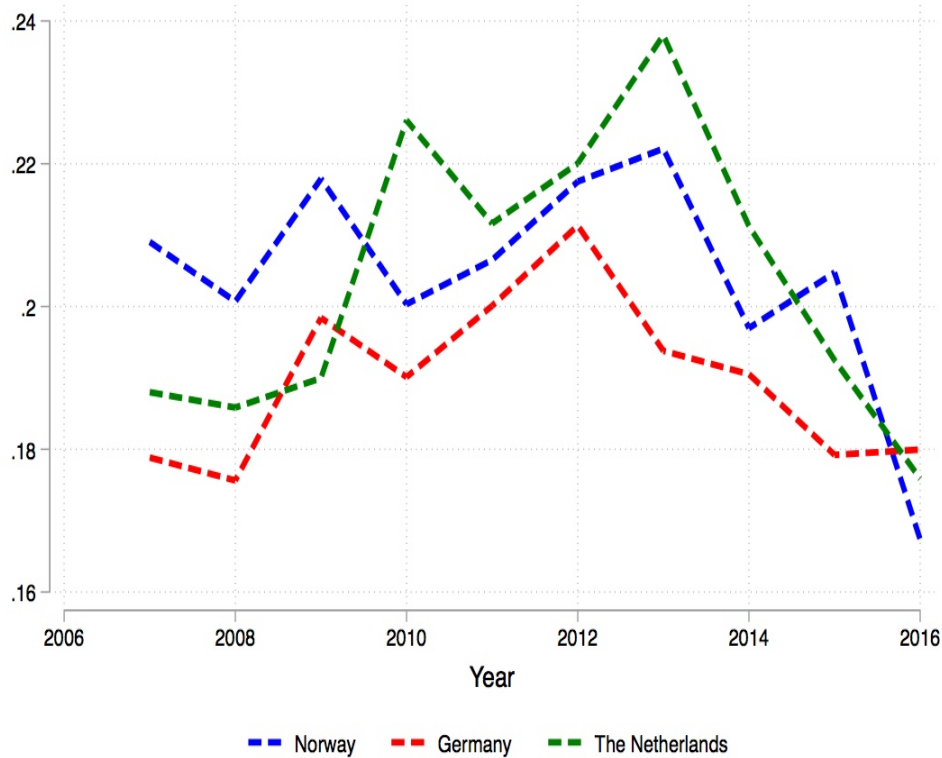


Figure 5: The series of the area of the concentration curves for capital income for Norway, Germany and The Netherlands between 2007 and 2016. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). **Source:** Author's computation on basis of EU-SILC.

principal driver of income composition inequality was the capital income. A different story can be told with regards to the second group of countries. The evolution of income composition inequality for Finland, France and Italy, also here dictated by the behavior of capital income (see figures 7 and 8), has been characterized by the latter moving firstly towards the top (up to 2013), and later towards the bottom of the income distribution (from 2013 onwards).

## Area of the Concentration Curve for Labor

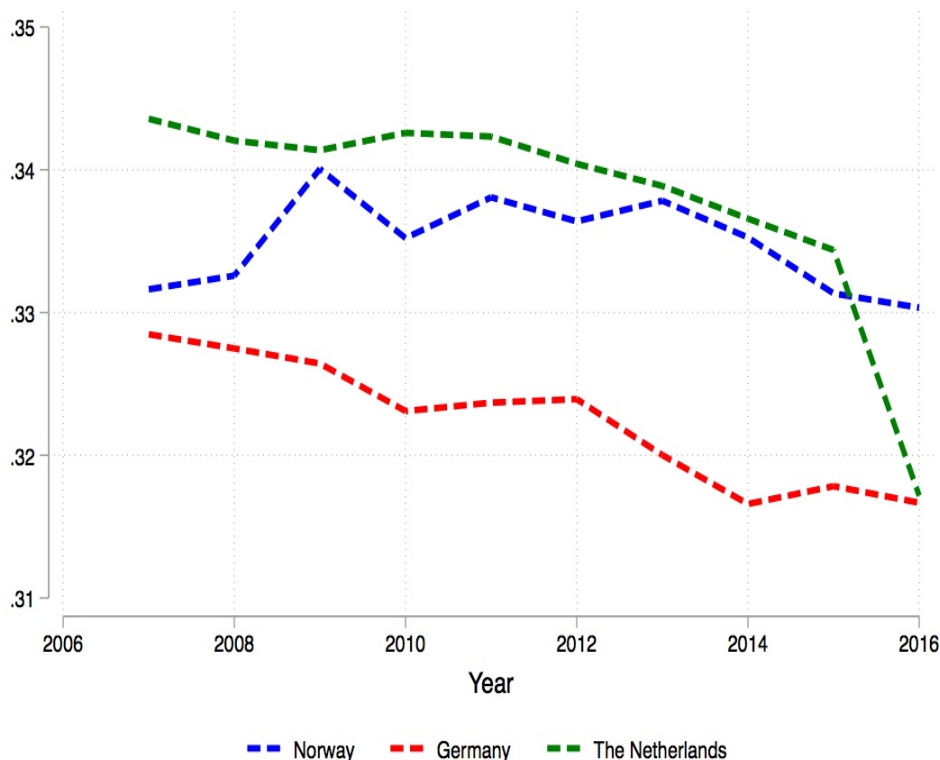


Figure 6: The series of the area of the concentration curves for labor income for Norway, Germany and The Netherlands between 2007 and 2016. Labor income is defined as the difference between total household gross income minus capital income. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). **Source:** Author's computation on basis of EU-SILC.

Having said that, in order to explain the major causes behind the dynamics of income composition inequality in the countries considered, we should further explore their underlying political contexts. In addition to that, given the nature of the data we use, which tend to underestimate the income of those at the top of the distribution,<sup>24</sup> our

<sup>24</sup>See [Jenkins \(2017\)](#) for an overview of the issues which raise the survey data.

## Area of the Concentration Curve for Capital

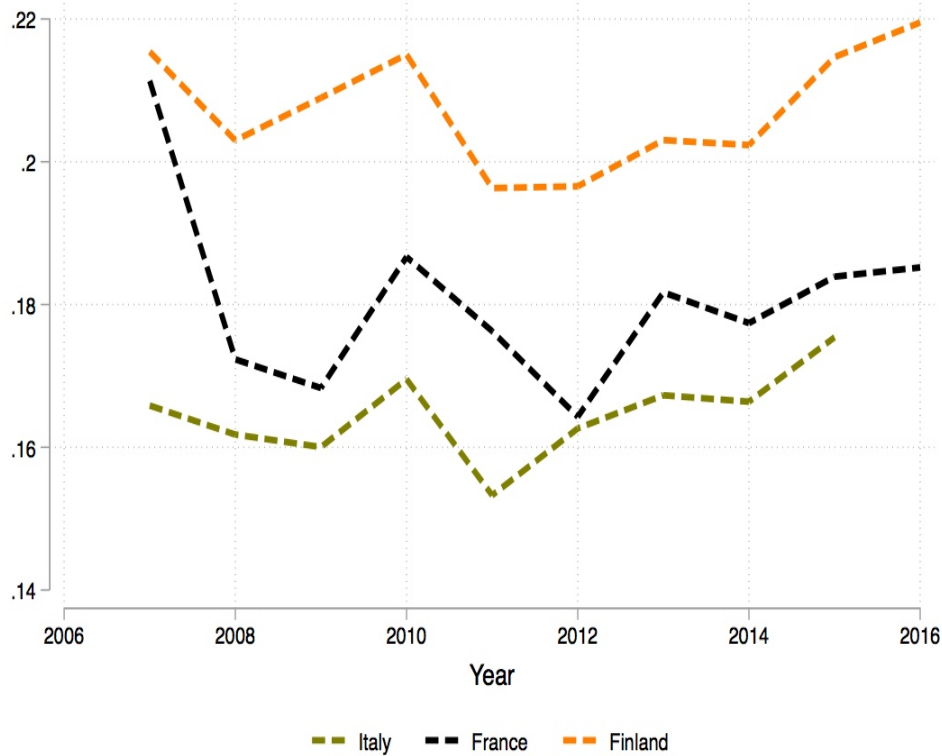


Figure 7: The series of the area of the concentration curves for capital income for Italy, France and Finland between 2007 and 2016. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). **Source:** Author's computation on basis of EU-SILC.

results should be taken with extreme caution. However, the theoretical nature of the present work entails that a deeper analysis would go well beyond the scopes of our research, paving therefore the way towards future investigations on the issue.

## Area of the Concentration Curve for Labor

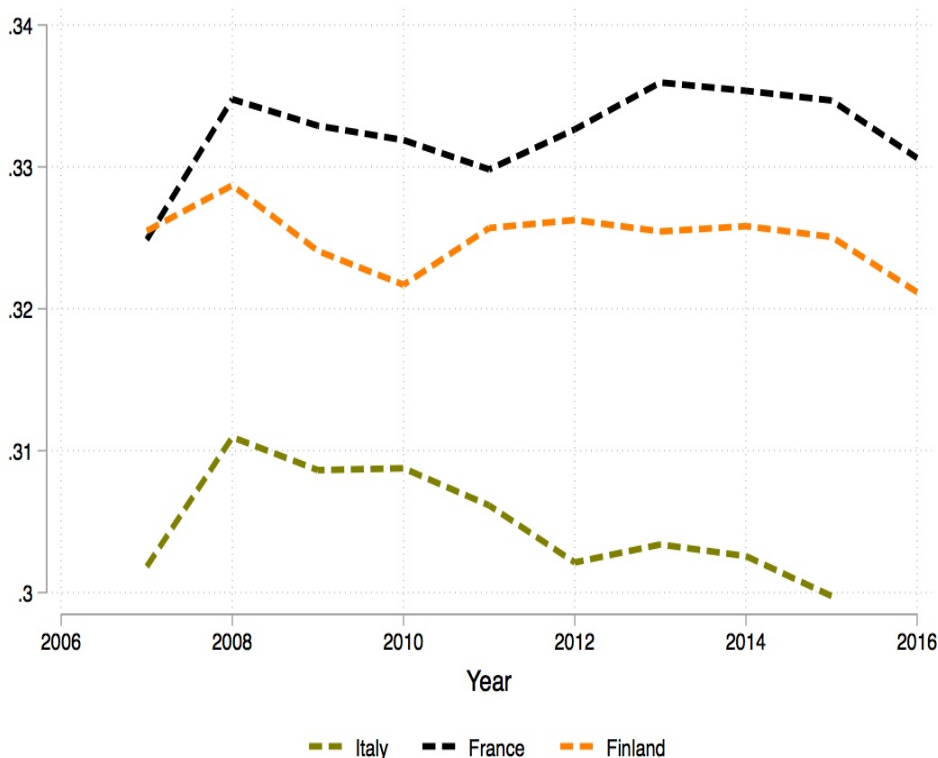


Figure 8: The series of the area of the concentration curves for labor income for Italy, France and Finland between 2007 and 2016. Labor income is defined as the difference between total household gross income minus capital income. Capital income is defined as the sum of household income from rental of a property or land, interests, dividends, capital from capital investments in unincorporated business, the capital component of gross cash benefits or losses from self-employment (including royalties) and pensions from individual private plans. The capital component of self-employment income is imputed following [Glyn \(2011\)](#). **Source:** Author's computation on basis of EU-SILC.

## 5 Conclusion

The present paper proposed a theoretical framework to examine the relationship between the functional and personal distribution of income. To this end, it introduced the concept of inequality in income composition, as well as an indicator to measure this issue: the income-factor concentration index. We showed that under a high level of

income composition inequality the link between the functional and personal distribution of income is stronger. We illustrated that this indicator can be looked at in two ways. It can be seen as an elasticity of personal income inequality to fluctuations of the functional income distribution, and as a measure of the degree of capitalism of a given social system. The dual nature of the income-factor concentration index makes it appealing from both a technical and a political economy perspective. We then applied this framework to several European countries on basis of EU-SILC data.

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## A Sign of the Indicator

In order to show that  $\mathcal{I}_f(\mathbf{z}) = -\mathcal{I}_f(\mathbf{z}_-)$ , we need to proof that  $\mathcal{B}^m(\mathbf{z}) = \mathcal{B}^M(\mathbf{z}_-)$ . The latter relationship states that the denominator of  $\mathcal{I}_f(\mathbf{z})$  equals that of  $\mathcal{I}_f(\mathbf{z}_-)$ . To this end, we should consider two different maximum-concentration curves.

From equations 4 and 5 we can simply notice that  $\mathcal{A}(\mathbf{z}) = -\mathcal{A}(\mathbf{z}_-)$ , thus that the numerator of the income-factor concentration index changes its sign (and not its absolute level) according to the source we analyze. Without loss of generality, if we assume that source  $z$  is mainly concentrated at the top, and that source  $z_-$  at the bottom, then the relationship  $\mathcal{B}^m(\mathbf{z}) = \mathcal{B}^M(\mathbf{z}_-)$  can be written as follows:

$$\int_0^1 z \mathcal{L}(\mathbf{y}, p) dp - \int_{p^*}^1 [\mathcal{L}(\mathbf{z}, p) - z_-] dp = \int_0^{p^{**}} \mathcal{L}(\mathbf{y}, p) dp + (1 - p^{**}) z_- - \int_0^1 z_- \mathcal{L}(\mathbf{y}, p) dp.$$

Considering that  $p^*$  and  $p^{**}$  are such that  $\mathcal{L}(\mathbf{y}, p^*) = \mathcal{L}(\mathbf{y}, p^{**}) = z_-$ , then  $p^* = p^{**}$ , and the relationship holds true.

## B Normalization Coefficient

As stated before in this article, we show that for specific functional forms of the Lorenz curve for income  $\mathcal{L}(\mathbf{y}, p)$ , and for specific values of  $z$  (and, thus,  $z_-$ ), the following relationship holds true:

$$\mathcal{L}^M(\mathbf{z}) - \mathcal{L}^e(\mathbf{z}) = \mathcal{L}^e(\mathbf{z}) - \mathcal{L}^m(\mathbf{z}). \quad (18)$$

For simplicity, let us move to the continuous space. Suppose, therefore, that we have three continuous distribution functions:  $\mathbf{y}$ ,  $\boldsymbol{\pi}$ ,  $\mathbf{w}$ . The relationship 18 is equivalent to

the following one:

$$z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp - \int_{p''}^1 (\mathcal{L}(\mathbf{y}, p) - z_-) dp = \int_0^{p'} \mathcal{L}(\mathbf{y}, p) dp + (1 - p') z - z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp. \quad (19)$$

We remember that  $p'$  s.t.  $\mathcal{L}(\mathbf{y}, p') = z$ ,  $p''$  s.t.  $\mathcal{L}(\mathbf{y}, p'') = 1 - z$  and  $\mathbf{z} = \pi, w$ . From equation 19 we can write:

$$2z \int_0^1 \mathcal{L}(\mathbf{y}, p) dp = \int_{p''}^1 (\mathcal{L}(\mathbf{y}, p) - z_-) dp + \int_0^{p'} \mathcal{L}(\mathbf{y}, p) dp + (1 - p') z.$$

If we call  $p' = f(z)$  and  $p'' = f(z_-)$ , where  $f(y) = \mathcal{L}^{-1}(\mathbf{y}, p)$  then, after further manipulations, we get:

$$\int_0^1 \mathcal{L}(\mathbf{y}, p) dp = 1 + \frac{1}{z - z_-} \int_{f(z)}^{f(z_-)} \mathcal{L}(\mathbf{y}, p) dp + \frac{zf(z) - z_-f(z_-)}{z - z_-},$$

which is true only if the following relationship is satisfied:

$$(z - z_-) \int_0^1 f(y) dy = \int_z^{z_-} f(y) dy. \quad (20)$$

Note that equation 20 is true for  $\pi = w$ ,  $\pi = 1$ ,  $w = 1$ , regardless of the functional form of  $\mathcal{L}$ , and for the family of functions  $f$  of the form  $f(x) = x^n$ , for  $n = 1, +\infty, -\infty$  only.

## C Result 3.1

Provided that  $y_p = \alpha_p \pi + \beta_p w$ , and  $y_{1-p} = \alpha_{1-p} \pi + \beta_{1-p} w$ , where  $y_p + y_{1-p} = y = \pi + w$ , we can write:

$$\begin{aligned} \varphi &= \frac{\beta_p w + \alpha_p \pi}{\beta_{1-p} w + \alpha_{1-p} \pi}, && \iff \\ y_p(\beta_{1-p} w + \alpha_{1-p} \pi) &= y_{1-p}(\beta_p w + \alpha_p \pi), && \iff \\ \frac{\pi}{w} &= \frac{\beta_p y_{1-p} - \beta_{1-p} y_p}{\alpha_{1-p} y_p - \alpha_p y_{1-p}}, && \iff \\ \frac{\pi}{w} &= \frac{\beta_p - \varphi \beta_{1-p}}{-\alpha_p + \varphi \alpha_{1-p}}, && \iff \\ \frac{\pi}{w} &= \frac{1 - (1 - \varphi) \beta_{1-p}}{\varphi - (1 - \varphi) \alpha_p}, && \iff \\ \frac{\pi}{w} &= \frac{\frac{1}{1-\varphi} - \beta_{1-p}}{\frac{\varphi}{1+\varphi} - \alpha_p}. \end{aligned}$$

If we now take the first derivative of  $\frac{\pi}{w}$  with respect to  $\varphi$  and we further manipulate, we obtain result 3.1.

## D Relationship between Gini and IFC for $n = 2$

Let us rewrite  $y_1$  (from equation 1) as follows:

$$y_1 = \beta_1 w \pm \alpha_1 w + \alpha_1 \pi.$$

After some algebraic manipulations, we get:

$$y_1 = I_f w + \alpha_1,$$

where  $I_f$  is the distribution's component of the index  $\mathcal{I}_f$  when  $n = 2$ . Let us now recall the expression of the Gini coefficient:

$$G = 1 - \sum_{k=1}^n (x_{k+1} - x_k)(y_{k+1} + y_k),$$

where the whole population is divided into  $n$  groups, and  $x_k, y_k$  represent the bottom  $x_k\%$  of the population, and their cumulative income respectively. When  $n = 2$  we can write:

$$G = 1 - py_p - (1 - p)y_p,$$

where  $p$  is the share of the poor group, and  $(1 - p)$  the share of the rich. The following can be derived:

$$G = 1 - p(I_f w + \alpha_p) - (1 - p)$$

whence:<sup>25</sup>

$$G = p(\alpha_{1-p} - I_f w)$$

from which, by taking the derivative with respect to  $w$ , we obtain equation 16.

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<sup>25</sup>It can be noticed that  $G = p(\alpha_{1-p} - I_f w) = p(1 - y_p) = py_{1-p}$ , which is a different way to express the two-groups Gini coefficient. Indeed, it clearly appears from the equation that inequality rises when either the share of poor people increases, or when the income share of the rich augments.