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Sphere Confusion: a Textual Reconstruction of Instruments and Observational Practice in First-millennium CE China

Daniel Patrick Morgan

Abstract: This article examines the case of an observational and demonstrational armillary sphere confused, one for the other, by fifth-century historians of astronomy He Chengtian and Shen Yue. Seventh-century historian Li Chunfeng dismisses them as ignorant, supplying the reader with additional evidence. Using their respective histories and what few written sources for the history of early imperial armillary instruments survive independent thereof, this article tries to explain the mix-up by exploring the ambiguities of 'observation' (*guan*) as it was mediated through terminology, text, materiality and mathematics. Reconstructing the material features of the 'sight' (*yi*) and 'effigy' (*xiang*), the article will reflect upon the mathematics necessary for their operation. The 'effigy', as Li Chunfeng defines it, is a substitute for observation; the 'sight', however, is so mediated by the material and mathematical sphere as to confound Li's suggested distinction of looking *through* vs. looking *at*. In the end, the difference hardly matters, as the observational armillary spheres documented by our sources appear to have played very little role in the history of astronomy in first-millennium China, leaving us to wonder what instrument(s) *were* used for observation.

Introduction

The most important thing to know about the Chinese armillary sphere is that it was made of money. You could use iron, or even wood, but to do it right you needed bronze, and bronze was the basis of the currency. It is for this most mundane of reasons—liquid capital—that the history of the armillary sphere in China is a history of making do without. So too must the historian make do, for the earliest extant instrument is a fifteenth-century reproduction of Guo Shoujing's 郭守敬 (1231-1316) 'simplified instrument' at Purple Mountain Observatory, Nanjing, prior to which all we have is texts. Sinologists tend to speak of the armillary sphere as the apogee of scientific achievement, Needham (1959, p. 339), for example, calling it 'the indispensable instrument of all astronomers for the determination of celestial positions before the invention of the telescope'. If this at all sounds odd to the historian of Mediterranean or Arab astronomy—where the sphere was mostly relegated to demonstration—it is because she has not heard how our sources wax ecstatic about the thing. All we are really told about its operation, however, is that its users were 'observing' (*guan* 觀) or 'watching' (*hou* 候), and if the implications of this seem self-evident, studies of more technically forthcoming traditions like Włodarczyk (1987) remind us that it is not.

This paper is a preliminary attempt to account for the practice(s) of armillary-sphere 'observation' in first-millennium CE China. In the absence of the instrument, the question is one that we must

approach through text, for which we shall focus on written traces of Zhang Heng 張衡 (78-139) and Kong Ting's 孔挺 (fl. 323) 'sphere sights' (*hun yi* 渾儀). Our primary source in this regard will be the histories of astronomical instrumentation written by Li Chunfeng 李淳風 (602-670) and Shen Yue 沈約 (441-513) in their respective 'heavenly patterns' (*tianwen* 天文) monographs for the dynastic histories. Writing on the Five Dynasties (502-618) and Jin (265-420), in the case of Li Chunfeng, and the Liu-Song (420-479), for Shen Yue, their histories overlap as concerns the lead-up to the fifth century. Weaving lengthy descriptions and citations into a chronicle of first 'creations' (*zao* 造) and 'awakenings' (*wu* 悟), these histories preserve most of what we know about astronomical instrumentation prior to the seventh century.[i] Of the handful of sources to have survived independently of these histories, to which we will turn below, is Zhang Heng's *The Sphere Heaven Sight* (*Hun-tian yi* 渾天儀). Preserved in Li Xian's 李賢 (654-684) scholastic commentary to the *Book of Later Han*, the treatise documents Zhang's work constructing, measuring and extrapolating algorithms from the physical sphere.

Our reason for focusing on the Zhang Heng and Kong Ting spheres is to reveal some of the confusion surrounding this topic in the early imperial period—a confusion of two physical instruments bespeaking a greater confusion about what it means to 'observe'—it is faced with confusion, after all, that actors tend to define their terms. In brief, the one disappeared from Luoyang in the fog of war, and when the other was captured a century later from occupied Chang'an, fifth-century experts believed themselves to have recovered the wrong sphere. Their confusion is difficult for the seventh-century expert to understand, for the one sphere was built to look *through*, and the other, *at*. As different as that sounds, I will attempt to explain this confusion via the terminology, text, materiality and mathematics through which 'observation' is in this case mediated.

Before we begin, I must say a word about units. For the sake of concision, I reduce compound decimal length measures into the equivalent number of *chi* 尺 (e.g. '14.61 *chi*' for '1 *zhang* 4 *chi* 6 *cun* 1 *fen*'), providing metric equivalents as per the inflationary historical rates in Qiu (1992). For historical dates, I translate reign-years into the equivalent Julian year (e.g. '164' for 'Huandi, Prolongation of Brightness 7'). As to astronomical units, our subjects work in *du* 度 ('measure/crossing'): a linear measure of the circumference of a great circle where one *du* equals the distance travelled by the mean sun in one day, and, thus, the number of *du* in one 'circuit of heaven' (*zhou tian* 周天) equals the length in days of the tropical year (Huang, 1992; Cullen, 1996, pp. 35-66). For most intents and purposes, 360° ≈ 365¼ *du*.

Lost & Found

The term that sinologists translate as 'sphere' is *hun* 渾/混, invoking the 'confused' and 'undifferentiated' state of matter at the beginning of time to describe the *tian* 天 'heaven(s)'. The

rubric ‘sphere/confusion’ could not be more appropriate to the subject at hand. The earliest reference to a *hun-tian* comes from Yang Xiong 揚雄 (53-18 BCE), the vagueness of which makes it difficult to distinguish the cosmology from the instrument (Cullen, 1996, pp. 53-59):

> 或問渾天，曰：「落下閎營之，鮮于妄人度之，耿中丞象之，幾乎幾乎！莫之能違也。」
> Someone inquired about sphere heaven, [to which Yang Xiong responded]: ‘Luoxia Hong (fl. 104 BCE) worked it out, Xianyu Wangren (fl. 78-74 BCE) *du*-measured it, and Geng [Shouchang] the palace assistant (fl. 52 BCE) made an effigy of it. How exact it is! No one can contradict it (*Yangzi Fayan*, 7.2a-b).

It is only with the ‘Grand Clerk yellow-path bronze sight’ 太史黃道銅儀 of 103 that we see unequivocal evidence of something resembling an armillary sphere in China. Commissioned for the state observatory by imperial edict at the (late) behest of General Jia Kui 賈逵 (30-101), Cai Yong 蔡邕 (133-92) and Liu Hong’s 劉洪 (fl. 167-206) monograph in the *Book of Later Han* offers the following description of the device:

> 以角為十三度，亢十...凡三百六十五度四分度之一。冬至日在斗十九度四分度之一。史官以郭日月行，參弦望，雖密近而不為注日。儀，黃道與度轉運，難以候，是以少循其事。
> With Horn.$_{L01}$ as 13 *du*, Neck.$_{L02}$ [as] 10, (see fig. 1)... it totalled to 365 *du* & ¼ *du*. The winter solstice was at Dipper.$_{L08}$ 19 *du* & ¼ *du*. The Clerk’s Office perimetered (?) solar & lunar motion and checked quarter & full moons, and though it was tight & close (accurate), it was not used for noting the sun/days. As to the sight, the yellow path and *du* (equator ring) rotated; it was difficult to watch with, which is why [the order to use it] was rarely heeded (*Hou Han shu*, *zhi* 2, 3029-30).

The ‘rotating’ equator and ecliptic identify this as an armillary sphere, and ‘watching’, an observational model, but this is all we really know about the sphere prior to Zhang Heng.

As concerns instrumentation, Zhang Heng’s *Book of Later Han* biography attributes him with having ‘created [the] sphere heaven sight/s’ 作渾天儀 (*Hou Han shu*, 59.1898), likely referring to the treatise. Later sources like Li Chunfeng, however, highlight an installation:

> 至桓帝延熹七年，太史令張衡，更以銅製，以四分為一度，周天一丈四尺六寸一分。亦於密室中，以漏水轉之。令司之者，閉戶而唱之，以告靈臺之觀天者，璇璣所加，某星始見，某星已中，某星今沒，皆如合符。
> In 164, Prefect Grand Clerk Zhang Heng redesigned [a sphere] in bronze with 4 *fen* (9.4 mm) to the *du*, for a circuit of heaven of 14.61 *chi* (343.34 cm). It was placed in a sealed chamber and rotated by means of waterclock water. The person charged with watching called it out from behind closed doors to announce to the observers of heaven of the Numinous Terrace (observatory) the add[ed hour] (?) of the ‘rotating mechanism’, that such-and-such star was first visible, that such-and-such star was already

> [culminated], and that such-and-such star was currently setting—all of which were like matching [the two halves of] a tally (*Sui shu*, 19.516-17).[ii]

Unmentioned in his biography, the device is attributed to a date twenty-five years after its creator's death. Whatever that tells us, this sphere-clock turned *indoors*, separate from the activity of 'watching', which distinguishes it from the observational model of 103. Arai (1989, p. 325) labels the non-observational model a 'computer'.

Both the 103 and 164 spheres were installed at the Numinous Terrace observatory at Eastern Han (25-220) Luoyang. Excavated in 1974-1975, this 44,000 m^2 walled site revealed nothing but ruined foundations, floor tiles and the earthen terrace where the sphere once stood (*Kaogu* 1978.1, pp. 54-57). A lot had happened in the meantime. In 189, Military Governor Dong Zhuo 董卓 (d. 192) sacked the city in a succession struggle between the palace and civil service. With Luoyang in flames, a child emperor was installed in Xuchang while real power devolved upon a coalition of warlords fighting military rebellions, millenarian movements and one another in his name. In 220, the Han emperor abdicated to his generals, the Cao 曹 clan of Wei 魏, who abdicated to *their* generals, the Sima 司馬, in 265. The Cao and Sima re-established Luoyang as their capital over the Three Kingdoms (220-280), but only after massive reconstruction. Then, upon reunification, Jin Wudi 晉武帝 (r. 265-290) split the empire amongst his sons, who, upon his death, flooded the central plains with mercenary steppe tribes in a new war for the imperial seat. The fought and they fought, and soon they had to fight rebellions within the tribes who were doing their fighting, but they kept on fighting until in 311 an alliance of mercenary tribes sacked the capital and drove the Jin city-by-city into the south.

The heartland was lost, and so too in the fog of war and exodus had the spheres of the Luoyang observatory gone missing. A century later, in the 417 siege of the Qiang 羌 proto-Tibetan capital at Chang'an, General Liu Yu's 劉裕 (363-422) armies made an unexpected discovery amongst the city's ruins: a two-metre bronze sphere, intricate and imposing, inscribed with astral symbols along its rings. The general transported his find to the new capital at Jiankang in 418 (where, with his armies, he would usurp the Jin throne in 420). In 439, within the framework of Xu Yuan's 徐爰 (394-475) history project to legitimize the new Liu-Song dynasty, He Chengtian identified this instrument with Zhang Heng's 'sphere sight' of 164. In his new 493 history of the Liu-Song, Shen Yue reiterates He's identification, noting that 'though the sight was visibly intact, it was [no longer] ornamented with the canon stars or seven luminaries' 儀狀雖舉，不綴經星七曜 (*Song shu*, 23.678).

This, according to Li Chunfeng's *Book of Sui* monograph, is what they were looking at:

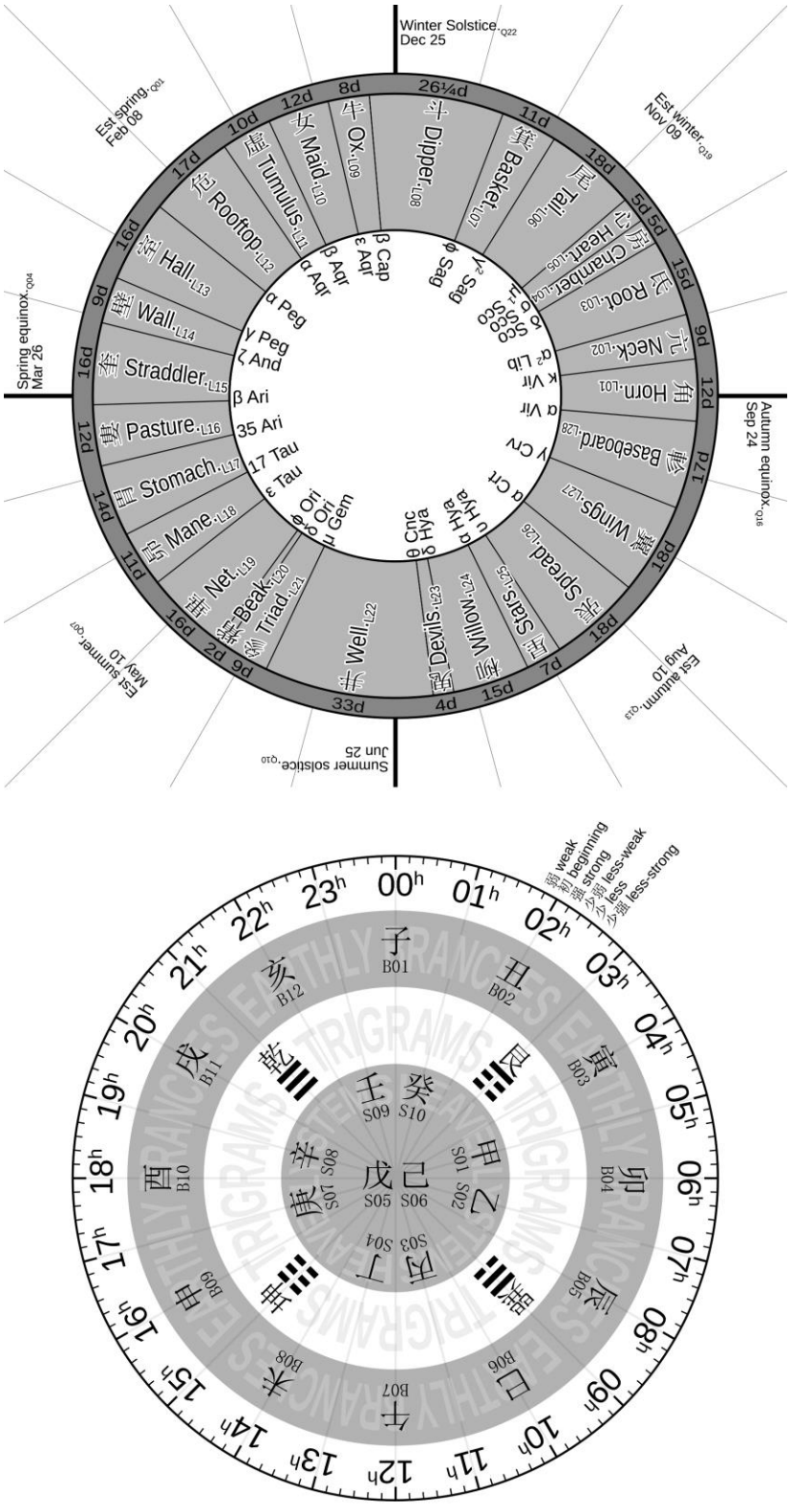

Fig. 1 Twenty-eight lodges and equatorial *du*-widths (above) vs. 'corners & chronograms' hour angles (below)

其制則有雙環規相並，間相去三寸許。正豎當子午。其子午之間，應南北極之衡，各合而為孔，以象南北樞。植楗於前後，以屬焉。又有單橫規，高下正當渾之半，皆周币分為度數，署以維辰之位，以象地。又有單規，斜帶南北之中，與春秋二分之日道相應。亦周币分為度數，而署以維辰，並相連著。屬楗植而不動。

[Six-joint sight:] Its construction featured double ring-circles joined [parallel] to one another with a space of roughly 3 *cun* (9.09 cm) between them. It stood upright to serve as the meridian. Between the meridian [plates], where coincides the traverse [between] south & north poles (i.e. where the diameter drawn through the N-S poles intersects the meridian), each joined to form a hole in effigy of the southern & northern pivots [of the celestial sphere]. Lock pins in the front & back (pivots) allowed joining [the second apparatus group] to it. **In addition**, it had a single horizontal circle at a height corresponding exactly with half the [vertical diameter of the] sphere, divided around its circumference into du numbers and inscribed with the positions of the corners & chronograms in effigy of earth (the horizon). **In addition**, it had a single circle that belted at an incline midway between south & north (i.e. at an incline to the horizon circle and perpendicular to the N-S axis), corresponding to the path of the sun at spring & autumn equinox. It too was divided around its circumference into *du* numbers and inscribed with the corners & chronograms, the two of which were written together in a single [band]. It was held in place by a connecting bolt and did not move.

其裏又有雙規相並，如外雙規。內徑八尺，周二丈四尺，而屬雙軸。軸兩頭出規外各二寸許，合兩為一。內有孔，圓徑二寸許。南頭入地下，注於外雙規南樞孔中，以象南極。北頭出地上，入於外雙規規北樞孔中，以象北極。其運動得東西轉，以象天行。

[Four-direction displacement sight:] Its interior further had double circles joined [parallel] to one another, like the outer double-circle [meridian ring]. Its inner diameter was 8 *chi* (242.4 cm), its circumference was 24 *chi* (727.20 cm), and it was connected to the axle pair. The two axle heads each protruded roughly 2 *cun* (6.06 cm) from the [four-displacement] circle, joining the two [parallel plates] as one. Inside of these were holes with a circular diameter of roughly 2 *cun*. The southern head went beneath 'the earth' (horizon circle), where it was inserted into the southern pivot hole of the outer double circle in effigy of the south [celestial] pole. The northern head protruded from 'the earth', going into the northern pivot hole of the outer double circle in effigy of the north [celestial] pole. Its movement allows for east-west rotation in effigy of Heaven's motion.

其雙軸之間，則置衡，長八尺，通中有孔，圓徑一寸。當衡之半，兩邊有關，各注著雙軸。衡既隨天象東西轉運，又自於雙軸間得南北低仰。所以準驗辰曆，分考次度，其於揆測，唯所欲為之者也。

[Sighting tube:] Between its two axles was installed a traverse 8 *chi* (242.4 cm) in length, through the centre of which was a [sighting] hole 1 *cun* (3.03 cm) in diameter. Halfway down the traverse was, on either side, a [pivot] bolt, each of which were inserted & connected to [another] axle pair (at the midpoint of an unmentioned crossbar). The traverse could both follow rotate

> east-west to follow heavenly phenomena and achieve of itself north-south lowering & raising between the axle pair. This is how one levelled & verified the chronograms & *li* (time) and distinguished & examined the stations & *du* (space). In regards to observation & measurement, it did truly everything that one could desire (*Sui shu*, 17.517-18; cf. Maspero, 1939, pp. 322-23).

A Sphere for Calculation

Of everything that is wrong with He Chengtian and Shen Yue's identification, Li Chunfeng points to the most obvious: 'Inspection of the engraving [reveals that] this was constructed by Clerk's Office Assistant Kong Ting of Nanyang in 323, under the rule of the [Xiongnu] imposter Liu Yao (r. 318-329)' 檢其鐫題，是偽劉曜光初六年，史官丞南陽孔挺所造 (*Sui shu*, 19.518). There is also the fact that Kong Ting sphere was fitted with a *sighting tube* for use *outdoors*. On this point, Li insists on a rectification of names:

> 渾天儀者，其制有機有衡。既動靜兼狀，以效二儀之情，又周旋衡管，用考三光之分。所以揆正宿度，準步盈虛，來古之遺法也。
> **The sphere heaven sight** is constructed with both engine (cage) and traverse. Not only in its at once moving & static state does it replicate the true situation of [yin & yang], the complete rotation of the transverse (sighting) tube allows examination of the fractions of the three lights (the sun, moon and stars). It is that by which one estimates & corrects the lodge *du* (widths) and levels & paces excess & void—a method handed down from antiquity (*Sui shu*, 19.517).

> 渾天象者，其制有機而無衡... 不如渾儀，別有衡管，測揆日月，分步星度者也。
> **The sphere heaven effigy** is constructed with engine and no traverse... It is inferior to the sphere sight, which has in addition a traverse tube—the thing that [allows] the measure & estimation of sun & moon and the division & pacing of stars & *du* (*Sui shu*, 19.519).

By Li Chunfeng's definition, Zhang Heng's indoor sphere was an 'effigy', Kong Ting's outdoor sphere was a 'sight', and 'the sight & effigy are two [distinct] devices with nothing whatsoever to do with one another' 儀象二器，遠不相涉 (*Sui shu*, 19.519). What room is there for confusion?

There is room for confusion in the terminology. The term *yi* 儀 derives from the graduated sight/range-finder pegs of early missile weapons, which, extended to the sphere, came to stand for sighting pegs, graduated rings and the instrument itself; *xiang* 象, on the other hand, refers to an 'effigy' or 'simulacrum' linking something in the world of man to a truth beyond his ken (Li, 2014, pp. 171-77; Schafer, 1977, pp. 54-56). In second-millennium parlance, 'sight' refers to an armillary sphere, and 'effigy', a celestial globe, but we mustn't read this distinction back on the first millennium.

For us, the difference is that between a hollow and solid sphere, for Li Chunfeng, looking *through* and looking *at*, but the fact that the demonstrational sphere had *rings* (and that the observational sphere was an effigy) afforded a certain ambiguity as to which applies to a given sphere (Wang, 2015).

Zhang Heng's reputation for having 'created [the] sphere heaven sight/s' (above) would seem to suggest the label for his unnamed (and posthumous?) computer of 164, and *The Sphere Heaven Sight*, for its part, deals with computation. At its core, the treatise is about measuring the ecliptic, which, without spherical trigonometry, means using a ruler:

> 是以作小渾，盡赤道黃道，乃各調賦三百六十五度四分之一，從冬至所在始起，令之相當值也。取北極及衡，各（誠）〔鍼〕掇之為軸，取薄竹篾，穿其兩端，令兩穿中閒與渾半等，以貫之，令察之與渾相切摩也。乃從減半起，以為〔百〕八十二度八分之五，盡衡減之半焉。又中分其篾，拗去其半，令其半之際正直，與兩端減半相直，令篾半之際從冬至起，一度一移之，視篾之半際（夕）多〔少〕黃赤道幾也。其所多少，則進退之數也。從（此）〔北〕極數之，則（無）〔去〕極之度也。
>
> For this, make a small sphere complete with red & yellow path, then allocate each with 365 *du* & 1/4 *du* and make sure to align their relative values starting from the position of winter solstice. Take the north pole and the transverse (here, the support base connected at the southern pole) and stick both with needles to form an axis. Take a thin bamboo strip and punch a hole at either end so that the distance between the two holes is exactly one half [of the circumference] of the sphere and that [the pins] may be run through them (affixing each end to opposite poles). Make sure to check that [the bamboo strip] rubs closely against [the inner surface of] the sphere. Then, starting from the diminished half-[way point] (the northern axis), make 182 *du* & 5/8 [*du*] [running] all the way down to the half-[way point] diminished at the transverse (the southern axis). Furthermore split the strip [along the] middle and remove its [one] half, making sure that the edge of its half (centreline) is true & straight and that it is aligned with the diminished half-[way points] (the poles) at both ends. Make sure to begin with the [centreline] edge of the bamboo strip at winter solstice and shift it one *du* at a time, looking at how much [is the north-polar distance of the ecliptic on] the half-edge of the bamboo strip and how many [*du* of longitude and RA have elapsed on] the yellow & red path. [The amount] by which [the latter] differ is the number of advance/retreat, while counting from the north pole [down the graduated bamboo strip] is (sic.) the *du* of polar distance (*Hou Han shu, zhi* 3, 3076 comm.).

Having thus derived the limits of 'advance/retreat' 進退, *The Sphere Heaven Sight* concludes with a step function for interpolating therefrom the correction needed to convert between any given ecliptic and equatorial 'lodge-entry *du*' (*ruxiu du* 入宿度). Deferring the reader to Western-language studies of this algorithm in Maspero (1939, pp. 337-52), Cullen (2000) and Lien (2012), the point that I want to make here is that Zhang Heng's 'small sphere', like his water-driven sphere of 164, is a material

means to a computational ends. These are spheres for looking *at*. Where the later was in a 'sealed chamber', Zhang explains the former thus:

> 本當以銅儀日月度之，則可知也。以儀一歲乃竟，而中閒又有陰雨，難卒成也。
> What one should do is *du*-measure these [numbers] over days and months via the bronze sight—*then* could [they] be known—[but as] this would take a year at the sight to complete, and [as] there would furthermore be overcast & rainy [days] interspersed therein, it would be difficult to bring to successful completion (*Hou Han shu*, *zhi* 3, 3076 comm.).

The sphere was a substitute for observation, and the algorithm, a substitute for the sphere. Tellingly, in 721, the answer to Monk Yixing's 一行 (683-727) petition that '[we] must know the yellow-path advance/retreat [numbers]' 須知黃道進退 was that '[the clerk's] office does not have a/the yellow-path displacement sight [and thus] has no means of measure-watching [it]' 官無黃道游儀，無由測候 (*Jiu Tang shu*, 35.1293-94).

A Sphere for Observation

Let us return to Kong Ting's sphere of 323 by point of comparison. As described above, the Kong Ting sphere was comprised of two of three component groups typical to later models (fig. 2). The first was the 'six-joint sight' (*liu he yi* 六合儀), a fixed outer cage 'joining' a horizon, meridian and equator ring at six points (and to a platform). Aligned at the horizon and celestial pole, the outer cage provided a fixed coordinate grid within which to turn interior rings. The second component group was the 'four-[directional] displacement sight' (*si you yi* 四遊儀), a meridian ring turning east-west around the polar axis and fitted with a sighting tube that pivoted north-south through its centre (Maspero, 1939, 306-27).

The key to any precision instrument is graduation, without which a cage of rings is no more an armillary sphere than a metal slat a ruler. Shen Gua 沈括 (1031-1095) offers the following meditation on the subject:

> 度不可見，其可見者星也。日、月、五星之所由，有星焉。當度之晝者凡二十有八，而謂之舍。舍所以絜度，度所以生數也。度在天者也，為之璣衡，則度在器。度在器，則日月五星可摶乎器中，而天無所豫也。天無所豫，則在天者不為難知也。
> *Du* cannot be seen; what can be seen are stars, and the course of the sun, moon, & five [planets] is replete with stars. Those [stars] that act as demarcations of *du*, they are twenty & eight in total, which we call lodges. Lodges are that by which *du* are measured out, and *du* are that by which numbers are born. *Du* are in heaven; but make a 'device-traverse' (sphere sight), and you have *du* on an apparatus. If you have *du* on an apparatus, then the sun, moon, & five [planets] can be moulded/modelled within the apparatus, and heaven will have no play. And if heaven has no play, then the

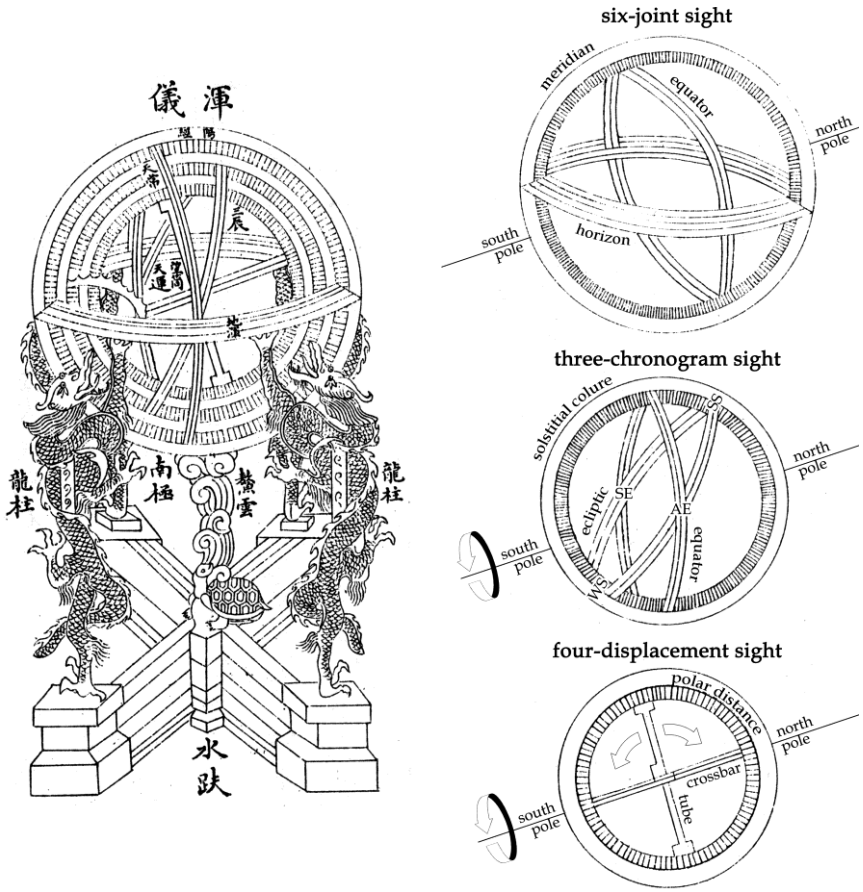

Fig. 2 Su Song's 蘇頌 (1020-1101) sphere heaven sight and component groups, from *Xin yixiang fayao*, A.9a, 11a, 13a, 14a.

> things in heaven will not be difficult to know (*Song shi*, 48.954-55).

If the sphere sight were to be a microcosm of the sphere heaven, one would expect that it be graduated accordingly—into the *du* of the mean sun's daily progress over one tropical year as counted from twenty-eight unevenly spaced reference stars (fig. 1). The fact that lodge-entry *du* are indeed the *only* coordinates of RA and longitude attested in *li* 曆 mathematical astronomy makes it difficult to imagine the alternative.

Luckily, we need not rely on imagination. Li Chunfeng reports that Kong Ting's (fixed) horizon and equator rings featured '*du* numbers and... the corners & chronograms', the latter a twenty-four point reference grid—twenty (stem and branch) 'chronograms' and four (trigram) 'corners'—counted 'leftward'—clockwise or E-W—from due north. Familiar from compass and divination boards, the 'corners & chronograms' scheme typically features in observational data and *li* procedure texts as an expansion of the standard duodenary (double-hour) civil day counted from midnight. On a fixed equator ring, this would give the user a Mediterranean-looking 'hour angle' counted from the opposite (midnight) meridian line (fig. 1). Corroboration for the use of these 'added hours' (*jia shi* 加時) as spatial coordinates furthermore appears in a set of eclipse data presented as evidence in a debate of 226 (*Jin shu*, 17.500; cf. Qu, 1994).

For observational data to be of any use to *li* calculation, one needs lodge-entry *du*. For equatorial *du*, one had two basic options: a sphere with a 'three-chronogram sight' (*san chen yi* 三辰儀)—a moving equator and ecliptic ring, mounted on the polar axis, that allowed one to align the stars of the instrument with those of heaven as per the description of Shen Gua (fig. 2)—*or* an algorithm for converting from 'added hours' and transit times. For ecliptic *du*, actors likewise speak of needing a 'three-chronogram sight' or an algorithm for converting from equatorial *du*. In both cases, one of these options was not actually an option.

Material availability would have left most state observatories with no choice but to rely on the algorithms. The last mention of the (unused) Luoyang observatory sphere of 103 comes in 178 (*Song shu*, 23.673), the instrument having likely been melted down between the sack of 189 and the loss of the city in 311. Judging from Shen Yue and Li Chunfeng's histories, the next observational armillary to grace a Chinese capital was the Xiongnu sphere of 323, brought to Jiankang in 418. After that was an iron version of the same design made in 412 for the Xianbei Tuoba-Wei 拓跋魏 (386-535) court at Luoyang. This was captured and moved to Chang'an by Sui 隋 (581-605) forces in 583, where it would see official use until replaced by Yixing's in 723 (Wu & Quan, 2008, pp. 433-40). Prior to 723, what observational spheres Chinese courts possessed were mainly barbarian hour-angle models *sans* lodges and *sans* ecliptic.

Availability, of course, depends as much on *allocation* as location. When we hear that '[the Clerk's] Office does not have a/the yellow-path displacement sight' in 721, for example, our

source is referring to *the* sphere by that name finished by Li Chunfeng in 633. Financed in 627 to replace the Xianbei observatory sphere, whose 'design & construction were loose & rough' 法制疏略 (*Jiu Tang shu*, 35.1293), Li's was the first observational model in 520 years to incorporate an ecliptic ring. Unfortunately,

> 其所造渾儀，太宗令置於凝暉閣以用測候，既在宮中，尋而失其所在。
> [Tang] Taizong (r. 627-649) ordered the sphere sight that [Li Chunfeng] had constructed installed in the Pavilion of Congealed Light so as to [personally] use it for measuring & watching, and though it was right there in the palace, when [later] looked for, [they] had lost track of where it went (*Jiu Tang shu*, 35.1293).

Li's was not the only priceless observational instrument to become a lawn ornament. What we know about the chain of custody for Kong Ting's Xiongnu sphere, for example, is that General Liu Yu 'donated it to the capital' 獻于京師 (*Song shu*, 2.42), bringing it 'to [a] royal palace' 及王府 in Jiankang (*Yiwen leiju*, 1.6a-b), where, by the sixth century, it would be installed within the closed imperial park at Hualin 華林園 (*Sui shu*, 17.517). It is not surprising that He Chengtian, Shen Yue and other fifth-century writers managed to miss the 'made in Chang'an' label—they probably never saw the thing.

Where and when an observational armillary sphere *was* accessible, experts would have had to make do with a fixed equatorial ring. To work with the ecliptic, one would have had no other option but 'advance/retreat numbers'. Cited both north and south, *The Sphere Heaven Sight* clearly saw interstate circulation, as did the 'advance/retreat numbers' in the tables of 174 (*Hou Han shu, zhi* 3, 3074). The period likewise saw an explosion of 'effigy' production, with which one could reproduce Zhang Heng's measurements (Wu & Quan, 2008, pp. 466-73). As to the equator, we do not know how actors converted from 'added hours' to equatorial lodge-*du*, as the Xiongnu and Xianbei spheres would have necessitated, but it would have certainly been the simpler of the two tasks. All one would need to do, for example, is note the 'meridian star' (*zhong xing* 中星) at the time of measurement and 'add the hour' to the lodge-entry *du* opposite it at the midnight meridian line. The fact that Kong Ting's equatorial ring featured '*du* numbers' suggests a provision for counting this 'hour' therein.

This raises the question of *du*-graduation and its precision. Shen Gua, above, juxtaposes the celestial and material *du*, but he fails to mention the third leg of the cauldron: the *mathematical du*, for which *li* experts used values like 365 385/1539 and 365 455/1843 *du* to the 'circuit'. In practice, there clearly must have been some compromise—some 'play' (*yu* 豫)—between the material and mathematical *du*, the question being how much. Pan (1989, pp. 271-72) argues that, up until the thirteenth-century, the Chinese armillary sphere was only ever graduated to the integer *du*, the trailing fractions of *shao* 少 ('lesser' = 1/4), *ban* 半 ('half') and *tai* 太 ('greater' = 3/4) seen in observational data being the

product of estimation. Pan's argument rests on three points. The first is that, in 1280, Guo Shoujing claims to have been the first to *really* empirically measure the twenty-eight lodges down to fractional widths. The second is the degree of precision witnessed in the observational record, where trailing fractions are rough and rare. The third is unequivocal documentation of 365-*du* observational spheres in late sources. The first two points are arguments from authority and absence, respectively, but the third gives us something to chew on.

The 365-*du* sphere sight appears in four sources relating to three devices. First, Shen Gua complains in 1074 that the observatory's observational sphere 'could only be allocated 365 *du* with no way to possess the remainder part' 但可賦三百六十五度而不能具餘分 (*Song shi*, 48.959). His description matches that of a 365-*du* sphere sight constructed in 995 'on the basis of the method inherited from [Li] Chunfeng and Monk Yixing' 本淳風及僧一行之遺法 (*Song shi*, 48.952). The *Old Book of Tang* indeed confirms that the Li Chunfeng sphere was graduated with '365 *du* in warp (RA) & weft (declination)' 經緯三百六十五度 (*Jiu Tang shu*, 79.2718). However, things only get weirder when we turn to Yixing's presentation of his design:

> 黃道單環：外一丈五尺四寸一分，橫八分，厚四分，直徑四尺八寸四分。日之所行，故名黃道。古人知有其事，竟無其器...臣今創置此環，置於赤道環內，仍開合使隨轉運，出入四十八度，而極畫兩方，東西列周天度數，南北列百刻，使見日知時，不有差謬。上列三百六十策，與用卦相準，度穿一穴，與赤道相交。
>
> Yellow-path single ring: exterior (perimeter) 15.41 *chi* (466.92 cm), traverse (width) 8 *fen* (2.42 cm), thickness 4 *fen* (1.21 cm), diameter 4.84 *chi* (146.65 cm). [This is] where the sun travels, thus is it named the yellow path. The ancients knew that there was such a thing and yet never possessed the apparatus... Your servant now creates & installs this ring, in-stalling it within the red path ring and then opening & closing [it to] make [it] rotate accordingly (i.e. locking it to the rotating equator ring), emerging & entering 48 *du* (the difference in declination from winter to summer solstice). The extremes (solstices) are drawn in two places, east-west are arrayed the *du*-numbers of the circuit of heaven, north-south are arrayed the 100 notches—making it so that one sees the sun and know the time without error or blunder—and atop are arrayed the 360 rods—levelled with the reigning hexagrams. At each *du* is drilled a hole (?) [where the ecliptic ring] crosses with the red path (*Jiu Tang shu*, 35.1297-98).

Here, the ecliptic ring is not only graduated with *du* but the 100 waterclock 'notches' (*ke* 刻) of the civil day and the 360 'rods' of *Book of Changes* numerology.

Whatever Yixing and Li Chunfeng's choice of 'circuit', we do see documentation of the 365¼-*du* sphere prior to Shen Gua's call to action. As to observational spheres, we have the 'Grand Clerk yellow-path bronze sight' of 103 (above), but it is the 'effigy', oddly enough, where one finds consistent evidence of 365¼-*du* rings. *The Sphere Heaven Sight*, as we saw, has one 'allocate each with 365 *du* & 1/4 *du*', making for '182 *du* & 5/8 du' per

hemisphere. The Shen Yue and Li Chunfeng histories cite Wang Fan 王蕃 (228-266) describing historical 'effigies' of 2, 3, and 4 *fen* to the *du*, the circumference of which works out in each case to 365¼, e.g. '[I] have redesigned the sphere effigy taking 3 *fen* to the *du*, for a total circuit of heaven of 1095 *fen* & 3/4 *fen* (= 365¼ × 3 *fen*)' 更制渾象，以三分為一度，凡周天一丈九寸五分四分分之三也 (*Song shu*, 23.677; *Jin shu*, 11.288). They likewise attribute Qian Lezhi 錢樂之 (fl. 436-443) with demonstrational spheres at 1 and 5 *fen* to the *du* that work out to the same total (*Song shu*, 23.678-79; *Sui shu*, 19.519-20). However we are to understand measures like '5/8 *du*' or '3/4 *fen*', the fact that contemporary *chi*-rules were graduated down to the *fen* does testify to the capacity for fractional graduation at a scale of at least 4 *fen*/*du* (Qiu, 1992).

If the potential for a 365¼-*du* sphere sight was there in the second century, why then would later models opt out? I think the answer lies in the way that the real-world practice of 'observation' was mediated by the material and arithmetic sphere. On the material end, there is always going to be 'play'. Whether or not you round the quarter *du*, the material 'circuit' will never meet the precision of its mathematical counterpart. Nor for that matter does *precision* translate into *accuracy*. Of the iron sphere of 412, for example, Yixing complains that 'the ring construction is crude & rough manner, and its *du* notches are uneven' 規制朴略，度刻不均, rendering an error of some ±2½ *du* when measuring lunar anomaly (*Jiu Tang shu*, 35.1295). Whatever the quality of construction, the fact that this and the Kong Ting sphere were war booty transported to new latitudes would have introduced further alignment errors (and damage). On the mathematical end, 'observation' was less spontaneous than our sources let on. For centuries, actors had developed 'effigies' and algorithms as a computational substitute for an ecliptic ring, and the rings they *did* build were graduated to unlikely integers, reminding us that the difference between a 365- and 365¼-*du* 'sight' is simply one of quotidian unit conversion. Either way, the absence of spherical trigonometry precludes corrections like refraction and parallax, otherwise necessary in, say, a Ptolemaic tradition (Włodarczyk, 1987), without which a precision of 1/4 *du* is frankly superfluous.

Conclusion

From what we read about the material 'sphere heaven' we can infer something of how the 'observation' of the celestial sphere was mediated by its material and mathematical counterpart. First-millennium sources tend to efface these processes of mediation, the inherency of which we recall when we turn to Ptolemaic writings, wherein 'observation' is mostly calculation. The difference, however, is less to do with 'East vs. West' than the way that early Chinese observational practices are, in turn, mediated by our sources. Treatises like *The Sphere Heaven Sight* go into the details of practice—be it the extraction of a

mathematical substitute for the material substitute for heaven—but the majority of what survives of such literature survives as excerpted in *histories*, the point of which is to provide names, dates and a narrative to what you (once) could read about somewhere else. Still, histories like Shen Yue and Li Chunfeng's leave us just enough to reconstruct some of what 'observation' entailed. 'Looking through', for one, was mediated by algorithms such as those for converting 'added hours' into 'lodge-entry *du*' on the equator, and moving equatorial *du* onto the ecliptic, and so too was it mediated by material factors such as the precision and accuracy of graduation.

The most important material factor as concerns the history of the 'sphere sight', however, is its *absence*. However our sources philosophise about the object, the history of the observational armillary sphere in first-millennium China was one of want, waste, confusion and foreign production. Prior to 723, the only state observatories in possession of such 'sights' were those of Han-Wei-Jin Luoyang (103-189/311), Xiongnu-Qiang Chang'an (323-417), Xianbei Luoyang (412-583) and Sui-Tang Chang'an (583 on), and those that did see use in Chinese hands were misaligned, 'loose & rough' and 'difficult to watch with'. It would have been simpler and cheaper to refine observational practice at the *computational* end, which might explain the relative outpouring of demonstrational 'computers' by the likes of Zhang Heng, Wang Fan, Qian Lezhi and others in the intervening centuries. There was no shortage of armillary spheres, but the majority, as in the West, were made for looking *at*. This qualifies them as 'effigies' by Li Chunfeng's definition, but others used these terms rather fluidly, leading one to wonder whether looking *at* is not incompatible with their idea of 'observation'. Either way, He Chengtian and Shen Yue had 'looked at' neither of spheres that they confused, for Zhang Heng's had long since turned into cash, and Kong Ting's, into an imperial lawn ornament.

Rather than leave things there, I would like to end on a question: What *did* actors rely upon for observational data all these centuries in the absence and dereliction of the 'sphere sight'? And what was this perfect armillary sphere that the Shen Guas of the world are describing?

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Bibliography

Arai Shinji 新井晉司 (1989) Chō Kō *Kontengi*, *Kontengi-chū* saikō 張衡『渾天儀』『渾天儀注』再考, in: Yamada Keiji 山田慶兒 (ed.) *Chūgoku kodai kagakushi ron* 中國古代科學

史論 (Kyōto: Kyōto daigaku jinbun kagaku kenkyūjo), pp. 317-36.

Cullen, C. (1996) *Astronomy and Mathematics in Ancient China: The* Zhou Bi Suan Jing (Cambridge: Cambridge UP).

Cullen, C. (2000) Seeing the Appearances: Ecliptic and Equator in the Eastern Han, *Ziran kexueshi yanjiu* 自然科學史研究 19(4), 352-82.

Ho, P. Y. (1966) *The Astronomical Chapters of the* Chin Shu (Paris: Mouton).

Hou Han shu 後漢書 (Zhonghua shuju ed., 1965).

Huang Yi-long 黃一農 (1992) Jixing yu gudu kao 極星與古度考, *Tsing Hua Journal of Chinese Studies* 22(2), 93-117.

Jin shu 晉書 (Zhonghua shuju ed., 1974).

Jiu Tang shu 舊唐書 (Zhonghua shuju ed., 1975).

Li Zhichao 李志超 (2014) *Tianren gu yi: Zhongguo kexueshi lungang* 天人古義：中國科學史論綱 (3d ed. Zhengzhou: Daxiang chubanshe).

Lien Y. E. (2012) Zhang Heng's Huntian yi zhu Revisited, *T'oung Pao* 98(1-3), 31-64.

Maspero, H. (1939) Les instruments astronomique des Chinois au temps des Han, *Mélanges chinois et bouddhiques* 6, 183-370.

Needham, J. (1959) *Science and Civilisation in China, vol.3: Mathematics and the Sciences of the Heavens and the Earth* (Cambridge: Cambridge University Press).

Pan Nai 潘鼐 (1989) *Zhongguo hengxing guance shi* 中國恆星觀測史 (Shanghai: Xuelin chubanshe).

Qiu Guangming 丘光明 (1992) *Zhongguo lidai du liang heng kao* 中國歷代度量衡考 (Beijing: Kexue chubanshe).

Qu Anjing 曲安京 (1994) Zhongguo gudai lifa zhong de jishi zhidu 中國古代曆法中的計時制度, *Hanxue yanjiu* 漢學研究 12(2), 157-72.

Schafer, E. H. (1977) *Pacing the Void: T'ang Approaches to the Stars* (Berkeley: University of California Press).

Song shi 宋史 (Zhonghua shuju ed., 1977).

Song shu 宋書 (Zhonghua shuju ed., 1974).

Sui shu 隋書 (Zhonghua shuju ed., 1973).

Wang Yumin 王玉民 (2015) 'Hun-tian yi' kao 「渾天儀」考, *Zhongguo keji shuyu* 中國科技術語 2015.3, 39-42.

Włodarczyk, J. (1987) Observing with the Armillary Astrolabe, *Journal for the History of Astronomy* xviii, 173-95.

Wu Shouxian 吳守賢 and Quan Hejun 全和鈞 (2008) *Zhongguo gudai tianti celiangxue ji tianwen yiqi* 中國古代天體測量學及天文儀器 (Beijing: Zhongguo kexue jishu chubanshe).

Xin yixiang fayao 新儀象法要 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983-1986).

Yangzi Fayan 揚子法言 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983-1986).

Yiwen leiju 藝文類聚 (Siku quanshu ed., rpt. Taiwan shangwu yinshuguan, 1983-1986).

[i] For a translation of Li Chunfeng's *Book of Jin* monograph, see Ho (1966).

[ii] Ge Hong 葛洪 (283-343) offers the same basic description of Zhang Heng's water-driven sphere as cited in *Jin shu*, 11.281-84; Ho (1966), pp. 55-56.