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Managing relational contracts

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Managing relational contracts

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Abstract

Relational contracts are typically modeled as being between a principal and an agent, such as a firm owner and a supplier. Yet in a variety of organizations relationships are overseen by an intermediary such as a manager. Such arrangements open the door for collusion between the manager and the agent. This paper develops a theory of such managed relational contracts. We show that managed relational contracts differ from principal-agent ones in important ways. First, kickbacks from the agent can help solve the manager’s commitment problem. When commitment is difficult, this can result in higher agent effort than the principal could incentivize directly. Second, making relationships more valuable enables more collusion and hence can reduce effort. We also analyze the principal’s delegation problem and show that she may or may not benefit from entrusting the relationship to a manager.

JEL classifications: D73, D86, L14.

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In 2006, the American retailer Aéropostale accused its chief merchandising manager Christopher Finazzo of receiving more than $25 million in kickbacks from a supplier, South Bay. Aéropostale argued that Finazzo had paid inflated prices to South Bay in exchange. Finazzo responded that he had favoured South Bay since they provided higher quality and a willingness to adapt to Aéropostale’s procurement needs. He argued that Aéropostale often remained “loyal” and “committed” to long-time “vendors even when those vendors charged higher prices” (Droney, 2017, p.30). In 2013, a jury found Finazzo and South Bay guilty of fraud. They appealed the restitution amount and in 2017 the Court of Appeals for the Second Circuit demanded a recalculation. Judge Droney argued that it was possible that Aéropostale did not lose money as a result of the kickback scheme. He argued that instead Finazzo’s “conduct may have reduced transactions costs for South Bay” and the relationship may have made it profitable for South Bay to pay kickbacks even at non-inflated prices (Droney, 2017, p.77).

Relational contracts between organizations are ubiquitous and are crucial for enforcing promises. Indeed, “lack of trust and commitment” is behind most supplier collaboration failures (Webb, 2017). The task of maintaining these relationships is often delegated to a manager like Finazzo. As illustrated by Aéropostale’s case, the firm can never guarantee that the manager will exclusively act in the firm’s best interest. Managers can exploit the (otherwise very valuable) trust relationship with their suppliers to collude with them. Indeed, a recent report shows that 20% of surveyed firms had been victims of procurement fraud in the past year, with most fraud incidents involving firm insiders (Kroll, 2018). This paper aims to understand how relationships behave when they are managed on the principal’s behalf. In particular, we focus on the following main questions: First, does collusion between the manager and agent crowd out quality? Second, is collusion always detrimental for the principal? Third, how should the principal constrain a manager who might collude?

To answer these questions, we develop a theory of managed self-enforcing relational contracts. A manager and an agent can exchange side payments, which represent kickbacks, bribes or other favors. This collusion can disincentivize quality if the manager pays a discretionary price premium regardless of quality. In particular, she may do so when she trusts that the agent
will respond by making a side payment. More surprisingly, side payments can enhance a manager’s ability to commit, and hence allow higher quality. This is because the supplier will renego on paying side payments if the manager reneges on the promised price. This is consistent with evidence that side payments can help contract enforcement. Cole and Tran (2011) analyze informal payments in Asiatic country and find that when contract payments are dependent on non-contractible quality, “the kickback is paid only after all contract payments have been made”. Side payments are thus not necessarily detrimental for the firm when commitment is scarce. This theory thus provides an instance of the “reduced transaction costs” mentioned by Judge Drony.

In the standard principal-agent model of relational contracts, more trustworthy relationships (i.e. those with higher discounted value) produce higher quality. In managed relational contacts, we show that the opposite may happen. More valuable relationships produce higher effort, and hence higher quality, only up to a point. Once the relationship is sufficiently valuable, extra value facilitates collusion, which reduces effort. In particular, it allows the manager to pay the agent a high price in exchange for a side payment even when quality is low. This non-monotonicity result is consistent with evidence on firms’ use of guanxi, a system of trust-based “informal social relationship” in China which is often used to ensure “that a contract is honored” (Chow, 1997). Vanhonacker (2004) observes that “it would be naive to think—as many Western executives do—that the more guanxi you have on the front lines in China, the better”. Instead, he argues too much guanxi can “divide the loyalties of the sales and procurement people”.

Our model features a manager and an agent who have a bilateral relational contract over an infinite number of periods. To model that the relationship is managed on behalf of a third party, we assume that profits generated by the relationship are shared between the manager and a principal who is inactive during the relational contract. Every period, the

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1In a similar case, Paine (2004) describes how “a purchasing official called about an overdue payment for items already received, [explaining] ‘we can get you a check by next week if you can give us a discount – in cash so we can distribute it to employees’”. Likewise Freeman, Hernadi and Tolbert (2000) describe how a supplier insisted the purchasing manager accept kickbacks as otherwise “the supplier’s invoices would not be paid on time”, arguing that was the way that business was done in the country.
agent privately exerts costly effort to produce a quality which cannot be formally contracted on. To motivate effort, the manager promises to reward high quality with a price premium. This price is bounded above and is paid in part by the principal. The manager and agent can also make side payments after the quality has been realized and the price has been paid. The payment of both the price and side payments needs to be self-enforced.

In Section 2, we begin by considering the principal as completely passive and characterize the surplus maximizing stationary contract between the manager and agent. The manager can incentivize effort through variation in prices or side payments. Variation in prices is more credible than variation in side payments. This is because the manager pays only a fraction of the price, and can receive side payments from the agent in return. When the discounted value of the relationship is low, the manager cannot trust the agent to return much of the price paid as a side payment. She thus prefers to use variation in prices to incentivize the agent. When the discounted value of the relationship is high, the manager can trust that the agent will return a large side payment if quality is low. She can thus pay a price premium regardless of quality and motivate effort through variation in side payments. This increases the manager-agent joint surplus, since part of the price is paid for by the principal. Paying a premium price regardless of quality, however, means effort no longer has an impact on the price level. This decreases the value of quality to the manager and agent, and they prefer a contract with less effort. Hence effort can decrease in the value of the relationship.

The Aéropostale case provides an example of how collusion can result in prices becoming insensitive to quality. In 2005 and 2006, South Bay had delays “that cost Aéropostale approximately $1.8 million in lost sales”. A product manager suggested that they should ask South Bay for discounts “to compensate for the delays” (Droney, 2017). Yet Finazzo, knowing that this would reduce future kickbacks, refused.

Section 3 analyzes an environment where the principal can choose the profit share and the maximum price at the beginning of the relationship. We show that she chooses these parameters to prevent the manager colluding to pay the agent a premium price for low quality. An interesting feature of the optimal contract is that, when the discounted value in the
manager-agent relationship is high, the principal limits the manager’s discretion and the manager varies side payments to motivate effort. This is consistent with evidence documented by Ledeneva (2013) that, in Russian government procurement, kickbacks were “linked to performance and facilitated the quality of service”. In this case, potential collusion is still costly for the principal because it forces her to decrease the discretion available to the manager, lowering quality.

This is the first paper that studies the impact of collusion on relational contracts. It bridges two large theoretical literatures - that on relational contracts and that on collusion in hierarchies. Models in the relational contracting literature have typically not considered hierarchies with intermediary layers (see Malcomson, 2013, for a survey). A recent exception is the paper by Fong and Li (2017a) whose model includes a non-strategic supervisor carrying out subjective performance evaluations of the agent. They show that by garbling the evaluations intertemporally, the supervisor can relax the self-enforceability problem, thereby benefiting the principal. The possibility of collusion, however, is not the focus of their analysis, and hence they conclude that “formal study of how collusion affects relational contracts is an interesting line of future research”.

Collusion between supervisors and agents has been the focus of a large literature including seminal works by Tirole (1986), Milgrom (1988) and Kofman and Lawarree (1993).² A few papers have studied how the need for self-enforcement impacts such collusion - see, for instance, Martimort (1999), Martimort and Verdier (2004) and Dufwenberg and Spagnolo (2015) - but these do not consider any interaction with other commitment problems.

Our model also relates to a literature investigating how delegation can solve commitment problems. Early papers in this literature include Vickers (1985) and Katz (1991).³ An important difference between this literature and our paper is that it is not a contract between the principal and the supervisor that is driving the commitment effect, but rather a relational

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²Within this literature, Olsen and Torsvik (1998) find that potential supervisor-agent collusion can mitigate a commitment problem in a two-period setting with adverse selection.

³Spagnolo (2005) uses a repeated game framework to show how delegation can enhance the enforcement of tacit collusion between firms.
contract between the supervisor and agent. In this sense, our paper shares the spirit of Hermalin (2015), who builds a model whereby “wining and dining” helps sustain a productive relationship between two firms’ managers. He also considers managers colluding against their principals, but this collusion is not sustained through relational contracting and only occurs when side payments are costless. As a result, in his model, allowing cross-firm managerial rewards always benefits the principals.

1 The model

Our model focuses on a manager and an agent who interact repeatedly over an infinite horizon of discrete periods. The manager works on behalf of a principal who is inactive during the game. We refer to the manager and principal as female and the agent as male.

The timeline for each period is shown in Figure 1. The manager first proposes a pricing scheme and a set of side payments to the agent (to be defined below). The agent either rejects or accepts - let \( d_t \in \{0, 1\} \) denote the agent’s decision. If the agent accepts, then he chooses an effort \( e_t \in [0, 1] \) incurring a cost \( c(e_t) \), where \( c(0) = 0 \), \( c'(0) = 0 \) and \( c''(\cdot) > 0 \). The agent’s effort generates a binary stochastic quality, which is high \((h)\) with probability \( e_t \) and low \((l)\) otherwise. High quality produces a revenue \( Y_t \) of \( y > 0 \), while low quality produces a revenue of 0. If the agent rejects, then for the rest of the game the firm receives low quality goods and the agent receives a per-period payoff of \( u \leq \max_e \mathbb{E}[Y_t - c(e_t)] \).

The agent is compensated with a discretionary payment \( p_t \) that can depend on quality. The price the manager can pay to the agent is limited by a maximum price \( \overline{p} \) that does not change over time, such that \( p_t \leq \overline{p} \). This represents the limit of the manager’s discretion. One way to consider this maximum price is as a function of the market price set by other suppliers, above which a manager might attract suspicion. Indeed, one of the triggers of the Aéropostale case was Finazzo’s reluctance to provide “a price breakdown of South Bay’s T-shirts by component to compare to another vendor” (Droney, 2017). In other settings, \( \overline{p} \) can be thought as a reservation price above which the manager needs to seek the principal’s approval for the transaction to go through.
In addition to the pricing scheme, the manager also suggests a package of side payments that will be made between the agent and the manager. The side payment is paid in two parts: the first part, $s_t^F$, is paid before quality is realized, while the second part, $s_t^A$, is paid after quality is realized.\footnote{We split the side payment into two parts to allow for purely stationary contracts to be optimal - with only one part, optimal contracts would have to either differ in the first period or depend on the prior period’s quality.}

Let $S_t = s_t^F + s_t^A$ denote the total side payment made. The payment of both $p_t$ and $s_t$ need to be self-enforced.

The total profit $Y_t - p_t$ is split between the principal and the manager, who receive shares $1 - \alpha$ and $\alpha$ respectively, where $\alpha \in [0, 1]$.\footnote{We hence assume that the principal and manager receive an equivalent to zero effort as their outside option, as in Levin (2002). This ensures our results are not driven by differences between the principal and manager in the relative valuations of the outside option.} All players have the same discount factor $\delta \in (0, 1)$. The expected payoff functions of the principal, the manager and the agent can thus be written as follows:

$$
\pi_t = (1 - \delta) E \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau} (1 - \alpha) (Y_{\tau} - b_{\tau}) \right]
$$

$$
v_t = (1 - \delta) E \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau} [\alpha(Y_{\tau} - b_{\tau}) + S_{\tau}] \right]
$$

$$
u_t = E \left[ (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ d_{\tau} [b_{\tau} - S_{\tau} - c(e_{\tau})] + (1 - d_{\tau}) u_{\tau} \right\} \right]
$$

Figure 1: Timeline of period $t$ in manager-agent game
The information structure is one of moral hazard. Effort is the agent’s private information, while the quality and agent’s compensation are observed by both the manager and the agent, but not the principal.

The assumption that the manager, but not the agent, receives a fraction $\alpha$ of the total profit is important. One way of rationalizing such an assumption is to think of the agent as representing a group of individuals, such as a firm’s workforce or suppliers. In this way, the profit provides information on the effort exerted by the group, but cannot be used to motivate a single agent. In the case of Aéropostale, for instance, the manager Finazzo received bonuses and stock options linked to the firm’s profit, but no such incentives were provided to the firm’s suppliers (Department of Justice, 2013). In the Online Appendix, we microfound this argument in a multi-agent model where we distinguish between non-contractible individual outputs and a contractible aggregate output. We show that if the number of agents is above a threshold it is not optimal for the manager to share her profit with the agents because of the free-riding problem. Another potential reason for the disparity between the manager’s and the agent’s incentives is that the manager may be intrinsically motivated or entitled to more information than the agent. A private company, for instance, may not want profit to be revealed to low-level employees or actors outside of the company.

In Section 2, we solve for the optimal manager-agent relational contract, and assume that the principal takes no actions. This analysis provides useful insights for contexts where principals do not have any control over managers’ incentives or discretion, or where these variables cannot be changed frequently enough to adapt to changes in the relational context. For instance, the manager could be a controlling shareholder who deals with suppliers and employees. In this case, the principal represents the remaining dispersed shareholders, and $\alpha$ is determined by the pre-existing equity structure and $\overline{p}$ is constrained by the corporate governance in place.

In Section 3, we then consider the case where the principal can set two key parameters at the beginning of the game, $\overline{p}$ and $\alpha$, and then takes no further action. This allows us to draw inferences as to how principals should incentivize and constrain managers as a function of the potential value of the manager-agent relationship. We focus on a context where the
principal cannot demand a transfer from the manager or agent to extract their surplus.\(^6\)

Managed relational contracts occur in many contexts beyond procurement, such as employment relationships. In some cases the principal might have a richer set of instruments with which to contract with the manager and agent, while in other cases she may have a more restricted set. We consider some alternative contracts in Section 4. In general, more instruments allow the principal to constrain collusion and approach first best, while fewer will make managed relational contracting less attractive. We use a profit-sharing rule for our main analysis since it is a simple and widely used rule (Kruse, Freeman and Blasi, 2010) which is sufficient to demonstrate the key insights of the paper.

2 Managed relational contracts

We begin this section by considering the case when \(\alpha = 1\), i.e. when the manager receives the total benefit of the agent’s effort. This serves as a benchmark as it is equivalent to a standard principal-agent model. We then consider the model with \(\alpha < 1\) and solve for the optimal manager-agent contracts.

2.1 Managed contracts with \(\alpha = 1\)

We can consider the outcome of direct principal-agent relational contracting by considering the case where \(\alpha = 1\). In this case, the manager receives the total benefit of the agent’s effort and pays the full cost of any payments made to the agent (i.e. price and side payments are substitutable), making the model equivalent to that of Levin (2003). We can thus treat the results of this case as the ‘no delegation’ benchmark. Then, if the manager and agent could write a third-party enforced contract on \(Y_t\), it would be optimal to induce the value of effort \(e_t\) that maximizes the joint surplus, \(ye_t - c(e_t)\). Defining this first-best effort as \(e^{FB}\), we then have \(c'(e^{FB}) = y\).

\(^6\)This is somewhat equivalent to a limited liability constraint where the principal cannot force the manager to have a negative payoff in any period, assuming that price premiums are only paid when high quality has been realized.
When the manager and agent cannot contract on $Y_t$, a self-enforcing contract is needed. We follow the definition of a self-enforcing contract given by Levin (2003) and similarly define a self-enforcing contract as optimal if no other self-enforcing contract generates higher expected surplus for the manager and agent. Levin (2003) shows that, if we are concerned with optimal contracts, then there is no loss of generality in focusing on stationary optimal contracts. Let $p_h$ be the price and $s_h$ be the side payment when quality is high. Similarly, let $p_l$ be the price and $s_h$ be the side payment when quality is low. Then effort will be determined by the following binding incentive compatibility constraint:

$$c'(e) = p_h - p_l - s_h + s_l$$  \hspace{1cm} (IC)

where we have dropped the $t$ subscript. This payment variation is constrained by the following inequality:

$$c'(e) \leq \frac{\delta}{1 - \delta}(e_y - c(e) - u)$$  \hspace{1cm} (1)

This inequality states that effort incentives (the left hand side) are limited by the future gains from the relationship. The tightness of this constraint depends on the value of the future relationship. When it is not valuable enough, the manager cannot credibly pay enough to implement the first-best effort. Instead, the effective reward for high quality will be the largest that can be credibly promised. Effort will therefore be increasing in the discounted joint surplus.

### 2.2 Managed contracts with $\alpha < 1$

In this section, we first show that we can still restrict our attention to stationary contracts that maximize joint manager-agent surplus. We then outline the key constraints that will potentially bind in any optimal manager-agent contract. This allows us to derive the main proposition in this section, which characterizes the optimal contract as a function of the discounted joint surplus. In particular, it details the varying ways in which effort is motivated and how the relationship between effort and surplus is non-monotonic.
A first notable point is that the manager-agent surplus depends directly on the price paid. This is because the manager only pays for part of the price \( p_t \) that the agent receives in full. If contracting on \( Y_t \) were possible, the manager and the agent would maximize their joint surplus by setting the price at the maximum \( \bar{p} \) regardless of quality and then use side payments to induce an effort level \( e_{MA}^{FB} \), where \( c'(e_{MA}^{FB}) = \alpha y \).

Side payments can be used to divide surplus between the manager and each agent. We can therefore focus on relational contracts that generate the largest possible manager-agent surplus. We follow Levin (2003) in defining a self-enforcing contract as strongly optimal if the continuation contract is optimal for all potential histories, even those off-equilibrium. We then obtain the following lemma:

**Lemma 1.** If an optimal contract exists, there are stationary contracts that are strongly optimal.

The intuition behind this stationarity result is that any variation in promised continuation values can be transferred into side payments in the same way that, in the principal-agent case studied by Levin (2003), any variation can be transferred to bonus payments.

We therefore focus on stationary contracts and drop the \( t \) subscripts on our variables. Effort is incentivized via variation in prices and side transfers as established by \( (IC) \).

We define \( g(e, p_h, p_l) \) as the expected manager-agent surplus in any stationary contract that has prices \( p_h \) and \( p_l \) and induces effort \( e \). This is given by the following equation:

\[
g(e, p_h, p_l) = \alpha ey + (1 - \alpha)(ep_h + (1 - e)p_l) - c(e) - u
\]

Note that, within stationary contracts, there are two ways that the manager can motivate effort: through variation in the price or through variation in side payments. The following lemma shows that the price will never be negative and, if the price or side payments vary as a function of quality, then they will do so in a way that encourages effort.

**Lemma 2.** In any optimal contract, the price is always non-negative, i.e. \( p_h \geq 0 \) and \( p_l \geq 0 \). Moreover, the price is weakly higher when quality is
high \((p_h \geq p_l)\) and side payments are weakly lower \((s_h \leq s_l)\).

If the manager wants to take surplus from the agent, then she prefers to do so using side payments rather than the price. This is because the two are equivalent for the agent, but the manager captures the whole value of any side payments given.

The need for the contract to be self-enforcing can be expressed in terms of dynamic enforcement constraints which demand that the future benefits of continuing the relationship are larger than the static gains from reneging on promises. Lemma 2 pins down the binding dynamic enforcement constraints that potentially bind. Since the price is never negative, only the manager has a reason to deviate when it comes to the paying \(p\). This temptation will be greatest when quality is high, as this is when the price is greatest. On the other hand, only the agent may wish to deviate from paying the agreed side payments, because if the manager does not wish to pay the side payment, she already would have deviated by not paying \(p\). The agent will be most tempted to renege when quality is low, as this is when the side payment is greatest. We therefore need to concentrate on the two following dynamic enforcement constraints:

\[
(1 - \delta) (-\alpha p_h + s_h) + \delta v \geq 0 \quad (DE_S)
\]

\[
-(1 - \delta)s_l + \delta u \geq \delta u \quad (DE_A)
\]

From these constraints, we can see that variation in prices is easier to sustain than variation in side payments. Increasing \(p_h\) by 1 only tightens \((DE_S)\) by an amount \(\alpha\), but increasing \(s_l\) or decreasing \(s_h\) by 1 tightens the constraints by 1. In other words, motivating effort through prices is easier than motivating effort through side payments.

It is useful to compare the manager’s dynamic enforcement constraint here with that in Section 2.1. When \(\alpha = 1\), the manager’s dynamic enforcement constraint is \((1 - \delta)(-p_h + s_h) + \delta v \geq 0\). Contrasting this with \((DE_S)\), we see that an \(\alpha < 1\) will make the manager’s dynamic enforcement constraint easier to satisfy. Intuitively, by paying the promised price, the manager can capture some surplus from the principal via the side payments. In other words, the manager will be more willing to pay a high price if she knows she will get part of this back from the agent as a ‘kickback’.

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A lower value of $\alpha$ increases the relative amount of surplus that can be captured from the principal in this way, making a high price more credible.

Summing $(DE_S)$ and $(DE_A)$ together and substituting in $(IC)$ then gives us the following constraint:

$$c'(e) + \alpha p_h - [p_h - p_l] \leq \frac{\delta g(e, p_h, p_l)}{1 - \delta}$$  \hspace{1cm} (IC - DE)

Comparing this to (1), the equivalent in the principal-agent case, we see that the requirement for contracts to be self-enforcing has a more complex impact in the manager-agent game. In particular, as the surplus in the relationship decreases, a reduction in effort is now only one possible effect. The manager and agent may instead choose to keep effort constant by replacing variation in side payments with variation in prices. This is possible because it is more credible for the manager to induce effort using prices rather than side payments. A further option is for the manager to decrease the prices, and hence make them more credible, but keep their variation constant.

We define $\delta^{FB}$ as the critical level of $\delta$ at which the manager and the agent can implement their first-best contract, i.e. $\frac{\delta^{FB}}{1 - \delta^{FB}} = \frac{\alpha y + \alpha p}{g(e_{FB}, p, p_{LB})}$. Then we obtain the following lemma:

**Lemma 3.** If $\delta \leq \delta_{FB}$, then $(IC - DE)$ is binding.

The ability to transfer utility through side payments ensures that there cannot be a second-best optimal contract where one of the dynamic enforcement constraints has slack. Hence, in any optimal contract that does not achieve first best, both $(DE_S)$ and $(DE_A)$ will be binding, and therefore so will $(IC - DE)$.

The following proposition characterizes the optimal contract as a function of $\delta$ and shows that the relationship between effort and $\delta$ is non-monotonic. We can think of $\delta$ as a determinant of the potential discounted joint surplus, and indeed the proposition could be written similarly in terms of the agent’s outside option $u$.

**Proposition 1.** Agent’s effort may be a non-monotonic function of the surplus. In particular, there exist values $\delta^0$, $\delta^L$ and $\delta^H$, where $\delta^0 \leq \delta^L \leq$
\( \delta^H \), such that \( e > 0 \) if and only if \( \delta > \delta^0 \), and the optimal manager-agent relational contract can be characterized as follows:

- **High surplus:** If \( \delta \geq \delta^H \), then prices are not used to induce effort, i.e. \( p_h = p_l = \overline{p} \), and effort is weakly increasing in \( \delta \).

- **Intermediate surplus:** If \( \delta^H > \delta > \delta^L \), then both side payments and prices are used to induce effort, i.e. \( p_l < p_h \) and \( s_h < s_l \), and prices are at the maximum when quality is high, i.e. \( p_h = \overline{p} \). When \( p_l > 0 \), then effort is decreasing in \( \delta \), and otherwise it is increasing in \( \delta \).

- **Low surplus:** If \( \delta \leq \delta^L \) then side payments are not used to induce effort, i.e. \( s_h = s_l \), and effort is weakly increasing in \( \delta \).

The basic intuition behind the non-monotonicity result can be understood as follows. When surplus is high, the relationship can sustain both large unvarying prices and a large variation in side payments to induce effort. When surplus falls below a certain level, the manager replaces some of the variation in side payments with variation in prices, since these are easier to sustain. Doing so means that effort benefits the manager and the agent more, since high quality now not only triggers \( y \) but also a higher price. The manager therefore increases variation in prices further to induce more effort. Lower surplus makes inducing effort more difficult, but this is more than compensated for by the increase in the value of effort for the two parties.

Note that the moral hazard in the model is important in generating this non-monotonicity. If the manager could observe the agent’s effort directly, she would make the price and side payment depend directly on effort, and hence neither would vary in equilibrium. An increase in \( \delta \) would then simply ease the dynamic enforcement constraint, allowing for larger prices or side payments, and hence weakly increase effort.

In order to better understand the nature of the optimal manager-agent contract, we now briefly detail the three cases outlined in Proposition 1. We also depict in Figure 2 the optimal contract for particular parameter values when \( c(e) = \frac{1}{2}ce^2 \). Figures 2a and 2b plot the prices and side payments as a function of \( \delta \). Figure 2c then plots the induced effort levels, and for
comparison we also include the effort level that would be exerted with principal-agent contracting. Finally, Figure 2d plots the principal’s payoff and the manager-agent surplus $g$.

**Figure 2:** Optimal manager-agent contract as a function of the discount factor

\[ c(e) = 0.54 \times e^2, \quad \bar{p} = 0.39, \quad y = 0.85, \quad \alpha = 0.67 \text{ and } \underline{u} = 0.0015 \]

### 2.2.1 High surplus

When surplus is slightly below the level that allows the manager and agent’s first-best contract, effort will be below first best but prices will remain
at the maximum regardless of quality. In particular, since \((IC - DE)\) is binding, effort will be determined by the equation \(c'(e) = \frac{\delta g(e,p,h)}{1-\delta} - \alpha \bar{p}\). Effort is reduced before prices because, when \(e = e_{MA}^{FB}\), a marginal reduction in effort leads to a second-order reduction in manager-agent surplus, while the cost of reducing the prices is first-order. There will thus always be a range of surplus for which the optimal contract involves \(p_l = p_h = \bar{p}\) and \(e < e_{MA}^{FB}\). Hence \(\delta^H < \delta^{FB}\).

When surplus falls further, what happens depends on the relative value that the manager places on quality, \(\alpha\). If \(\alpha\) is low, then she will continue to cut effort rather than prices until no effort is sustainable. In this case \(\delta^H = \delta^L\) and there is no ‘intermediate’ range of surplus. If \(\alpha\) is high, then \(\delta^H > \delta^L\), and there will be an intermediate surplus range where prices are used to induce effort.

### 2.2.2 Intermediate surplus

The manager and agent face a trade-off in deciding upon the price paid when quality is low, \(p_l\). A higher \(p_l\) generates greater surplus directly, but it also decreases effort. Maximizing joint surplus gives us the following expression for \(p_l\) when \(\bar{p} > p_l > 0\):

\[
    p_l = \frac{1 - \alpha}{\alpha} (1 - e) c''(e) - y + \frac{1}{\alpha} \frac{\delta g(e,p,h)}{1-\delta}
\]

The first term of this expression stems from the direct gain in manager-agent surplus that an increase in \(p_l\) produces; the more likely low quality is to occur, the higher this gain. The second term is the result of the reduction in expected quality that an increase in \(p_l\) produces through the change in effort induced. The third term comes from the relational contracting constraint; higher surplus means that more effort can be induced through side payments, and hence \(p_l\) can be higher.

Since \((IC - DE)\) is binding, the effort level \(e\) is given by \(c'(e) = \bar{p} - p_l + \frac{\delta g(e,p,h)}{1-\delta} - \alpha \bar{p}\). If we substitute in (2), we can see that effort is weakly decreasing in the discounted surplus if and only if \(p_l > 0\). To see this, note that \(c'(e) + \frac{1-\alpha}{\alpha} (1 - e) c''(e)\) is increasing in \(e\) by equation (6) in the proof of Proposition 1.
manager have a greater incentive to increase effort. Instead, when \( p_l = 0 \), a lower surplus forces the manager to reduce the variation in side payments.

### 2.2.3 Low surplus

When the discounted surplus becomes low, i.e. \( \delta = \delta^L \), the manager can only just promise to pay \( p_h = \bar{p} \) and will not be able to combine this with any variation in side payments. When \( \delta \leq \delta^L \), the price \( p_h \) will be the maximum that the manager can credibly promise, i.e. \( p_h = \frac{\delta g(p_h, p_l)}{1-\delta} \), and \( p_l \) will again be either equal to \( p_h \), zero or a solution to (2).

### 2.3 Discussion

An important implication of Proposition 1 is that there is sometimes, but not always, a trade-off between increasing quality and reducing the surplus captured by intermediaries. The previous literature on relational contracts suggests that non-contractible production can be improved by increasing the discounted joint-surplus within relationships, for instance by increasing tenure or decreasing competitive pressure (Calzolari and Spagnolo, 2009; Board, 2011; Gibbons and Henderson, 2013). Yet those concerned with collusion argue that such policies will facilitate side payments (Martimort, 1999; Lambsdorff and Teksoz, 2005). Our analysis implies that in some cases both effects may indeed occur simultaneously. Examples of such a trade-off can be found in public procurement, where in some instances policies designed to reduce corruption appear to have a negative impact on performance or quality (Coviello, Guglielmo and Spagnolo, 2018; Lichand, Lopes and Medeiros, 2017). We also find, however, that in other cases there is no such trade-off, and decreasing discounted surplus will reduce collusion without any negative impacts. An example of this can perhaps be found in the public procurement reforms studied by Lewis-Faupel et al. (2016) who find reducing discretion and decreasing interactions appears to reduce corruption with a non-negative effect on quality. In our model, these correspond to situations where the price is positive every period and the manager is sometimes, but not always, paying the maximum possible price.

Furthermore, the example plotted in Figure 2 demonstrates that the
ability of the manager and agent to make side payments may facilitate the productive relational contract. In particular, we can see from Figure 2(c) that, for a certain range of $\delta$, the agent’s effort is higher when he is incentivized by the manager. This is because, when relational contracting is relatively difficult, the ability of the manager to receive kickbacks from the agent helps to make a higher price credible. It is therefore possible that the principal may benefit from the existence of side payments, even if they lead to the manager capturing surplus. Indeed, in a recent appeal of the Aéropostale case, the court ruled that the profit that Finazzo made through receiving side-payments did not necessarily come at Aéropostale’s expense, since the kickback scheme may have created value greater than the cost of the kickbacks (Droney, 2017). For instance, South Bay was willing to go beyond the contract terms by holding and storing inventory. This allowed Aéropostale to “quickly start printing new styles”, which was very valuable to adapt to its “fickle” teenage customer base (Droney, 2017, p.30). Indeed, we find that when the discounted surplus in the manager-agent relationship is low, a manager who is motivated by side payments can facilitate higher quality.

3 Governing managed relational contracts

In the previous section, we ignored the role of the principal and treated the parameters $\alpha$ and $\overline{p}$ as exogenous. In this section, we consider that the principal sets these parameters at the beginning of the game.

The principal faces the following trade-off when choosing the optimal $\alpha$. A lower $\alpha$ has two benefits for the principal - it implies giving up less surplus to the manager, and it also makes it easier for the manager to commit to pay the prices. If $\alpha$ is too low, however, then the manager will wish to pay prices even when quality is low. This both costs the principal surplus directly and weakens the agent’s effort incentives.

The following proposition describes how the principal sets the parameters $\alpha$ and $\overline{p}$ to maximize her payoff. In particular, the principal sets $\alpha$ at the lowest possible value that ensures the manager does not misuse prices, and then uses $\overline{p}$ to limit how much surplus is appropriated by the manager and the agent.
Proposition 2. The principal will set $\alpha$ and $\bar{p}$ such that the optimal manager-agent contract has $p_l = 0$. Moreover, there exist $\delta_0$, $\delta_l$ and $\delta_h$, where $\delta_0 < \delta_l < \delta_h < 1$, such that $e > 0$ if and only if $\delta > \delta_0$, and the agent is incentivized in the following way:

- If $\delta \geq \delta_h$, only side payments are used to induce effort and $\bar{p} = 0$.
- If $\delta_h > \delta > \delta_l$, prices and side payments are used to induce effort.
- If $\delta_l \geq \delta > \delta_0$, only prices are used to induce effort.

Note that when $\delta$ is low, the principal may not need to use a maximum price - in this case, the price that the manager can pay is sufficiently limited by the relational contracting constraint. For higher $\delta$, however, the principal will use a maximum price to ensure that she does not have to give too large a share of her profit to the manager to achieve $p_l = 0$. By setting a more stringent maximum price, the principal forces the manager to use more variation in side payments as an incentive device. Indeed, if the surplus is very high, the principal will optimally give no discretion to the manager (i.e. $\bar{p} = 0$) and all the effort incentives will be provided through side payments.

In general characterizing the principal’s optimal solution is complex, but in the case where $c(e) = \frac{ce^2}{2}$ we can derive relatively simple expressions for the principal’s optimal behavior. In particular, in this case the principal sets $\alpha$ such that there is zero marginal benefit to the manager of increasing $p_l$ above zero. This is done by setting $\alpha$ such that $p_l = 0$ in equation (2). The maximum price $\bar{p}$ is then set to maximize the principal’s profit function.

We display in Figure 3 the optimal contract when the principal sets $\alpha$ and $\bar{p}$ and $c(e) = ce^2$. For $\delta < \delta_l$, we see that the price paid when quality is high is increasing in $\delta$, and in this range the agent is not constrained by the maximum price. As $\delta$ increases, the principal increases $\alpha$ to ensure that the manager will be willing to maintain a larger variation in prices. Between $\delta_l$ and $\delta_h$, the principal combines an increase in $\alpha$ with a decrease in $\bar{p}$ in order to ensure $p_l = 0$. Once $\bar{p} = 0$, at $\delta_h$, the manager can no longer pay a positive $p_l$ and it is optimal for the principal to reduce $\alpha$. Finally, when $\delta$ is sufficiently high, the relational contracting constraint no longer
binds, the manager sets effort to her preferred level, where $ce = \alpha y$, and then effort decreases with $\delta$.

Figure 3: Optimal contract as a function of the discount factor

(a) High quality price, $p_h$

(b) Variation in side payments, $s_l - s_h$

(c) Effort, $e$

(d) Profit sharing, $\alpha$

$$c(e) = 0.54 \times e^2, \; y = 0.85 \text{ and } w = 0.0015$$

Having characterized the principal’s optimal behavior, we can now ask when the principal benefits from delegating to a manager. In some situations, employing a manager may be obliged; the leader of a government or large firm may simply be unable to manage all relevant relational contracts herself. In other situations, the principal may have the choice between delegating the relational contract to a manager or managing it herself. In
these cases, it is interesting to consider when such delegation may be in the principal’s best interest.

The following proposition describes when the principal should delegate to a manager, assuming she sets $\alpha$ and $\beta$ optimally. If she does not delegate, we assume that she undertakes direct relational contracting with the agent, achieving the results outlined in Section 2.1. The proposition states that, in the case of the quadratic cost function, there exists a range of discount factors for which delegating is strictly preferable and a higher range when direct relational contracting is strictly preferable.

**Proposition 3.** If $c(e) = \frac{ce^2}{2}$, then there exist values $\delta, \hat{\delta}$ and $\check{\delta}$ with $0 \leq \check{\delta} < \delta < \hat{\delta} < 1$ such that:

- If $\delta > \check{\delta}$, then the principal’s payoff from the optimal managed relational contract is strictly below that from direct relational contracting.

- If $\hat{\delta} > \delta > \check{\delta}$, then the principal’s payoff from the optimal managed relational contract is strictly above that from direct relational contracting.

If the discount factor is high, then relational contracting poses no problem, and there is no reason to delegate. The principal and agent can implement a large level of effort on their own, and the principal has no reason to share surplus with a manager. On the other hand, if the discount factor is low, then direct relational contracting is difficult and cannot sustain much effort. The principal would therefore prefer to delegate and thus generate more effort, since the extra surplus generated more than compensates for the part given to the manager.

Figure 4 demonstrates this result graphically by plotting the best payoffs that the principal can achieve with and without delegation when $c(e) = \frac{ce^2}{2}$. A similar logic applies for other variables affecting the potential discounted surplus, including the agent’s outside option $u$. Note that the range of $\delta$ for which delegation increases the principal’s profit is much smaller than that for which effort increases. Since the principal cannot extract surplus from the manager, she will refrain from delegating on some occasions when doing so would lead to higher quality.
3.1 Discussion

Proposition 2 tells us that the principal would like a manager whose payoffs are partly, but not completely, aligned with her own. The principal needs the manager to care somewhat about profit because otherwise no effort will be induced. This makes manager-agent relational contracting costly, which means that to get more effort the principal has to give up more surplus. As a result, the principal will not wish to induce first-best effort when delegating, and hence there is no need to have a manager whose incentives align completely with her own. Instead, the principal would rather have a manager who cares less than her about profit in order to facilitate relational contracting.

An example of such behavior can perhaps be seen in the way in which businesses in China deal with the practice of Guanxi, a system of informal relationships often formulated through gift exchange. Many firms are well aware of the risks stemming from procurement and sales managers’ personal relationships, since these can facilitate side payments and other malpractice (Millington, Eberhardt and Wilkinson, 2005). Yet, when it comes to hiring such personnel, Wiegel and Bamford (2015) find evidence that firms specifically hire people with personal Guanxi, and they cite the ability of
Guanxi to smooth inter-firm relationships as an important factor. Indeed, Schramm and Taube (2003) argue that while Guanxi networks facilitate corruption, this corruption itself helps to strengthen the legitimate transactions coordinated through Guanxi, in a way similar to that described in our model above.

4 Alternative specifications and extensions

In this section we consider the sensitivity of our results to a number of extensions or alternative assumptions. We begin by considering other contexts beyond the case of inter-firm supplier relationships where relational contracts are managed on a third party’s behalf. We then consider the principal’s optimal actions if we were to extend her set of instruments in a number of ways, including making relationship dependent transfers and sharing costs and revenues at different rates. We then allow for side payments to be costly actions such as favors, rather than simple transfers. Finally, we consider how our model may translate into a setting where the agent is multitasking and one task benefits only the manager.

4.1 Adapting the model to other contexts

We have focused our model on supply relationships between firms, but there are many other contexts where relational contracts are important and are managed on a third party’s behalf. Adapting our model appropriately may generate useful insights for these situations.

In the main model considered in this paper we have assumed the manager’s incentives are aligned with the principal’s through some form of profit sharing, and hence incentivizing the manager comes at a direct cost to the principal. In some contexts, such as large government bureaucracies, this is unlikely to be the case, and instead a manager’s motivation may come from intrinsic motivations or reputation. In this case, \( \alpha \) represents how much the manager cares about the organization’s goal and hence the principal’s payoff function would not depend on \( \alpha \) directly. The results of Section 2 would remain unchanged, while if the principal could choose \( \alpha \) (through, for instance, hiring decisions) than the optimal \( \alpha \) would be weakly greater
than that found in Section 3. Note, however, that it would still not be optimal for the principal to set $\alpha = 1$ unless $\delta$ was sufficiently large that first-best effort could be achieved. This is because, when $e < e^{FB}$, it is always optimal for the principal to have $\alpha < 1$ to facilitate relational contracting.

Another setting where self-enforcing contracts are important is in relationships between employers and employees (Levin, 2003). As with procurement, these relational contracts are frequently managed by mid-level managers within an organization, and hence there is the possibility of collusion through ‘influence activities’ (Milgrom, 1988). For instance, Nkamleu and Kamgnia (2014) document that in African governments per-diems are “mainly given to provide financial incentives to employees in order to increase their motivation” but managers may “expect the staff member to share or kickback a portion of the per-diem”.

The case of Credit Suisse First Boston described by Stewart (1993) suggests that some insights of our model are likely to carry over to the labor relationship setting. The investment banking firm First Boston (FB), which we can consider as the manager, had been very successful at maintaining long-term relationships with its bankers (the agent). At the same time, it financed very precarious transactions that brought bonuses for managers and bankers at FB at the expense of decreasing the firm’s long-term value, thereby hurting shareholders like Credit Suisse (CS), the principal. CS introduced measures to change the “freewheeling atmosphere (...) notable for an absence of the layers of controls... [and] for huge salaries and bonuses.” In particular, it imposed a tighter bonus cap (akin to our max-

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8Examples of bribing for promotions can be found in a recent study by Weaver (2017), while Bebchuk and Fried (2004, p.93) note that directors give CEOs large retirement gifts partly “to express gratitude for what the CEO has done for them”.

9We thank Jin Li for suggesting this case study.

10Following the 1987 Wall Street crash, CS had to rescue FB “by sinking more than $300 million” and removing “more than $400 million in troubled loans.” Moreover, FB’s reputation with its clients was damaged: “some corporations are asking why they should seek advice from a firm that managed its own finances so disastrously, and helped arrange such ill-fated deals” (Greenhouse, 1991).

11Hierarchical collusion to oversell was a common feature in the Wall Street’s 1990s-era: “So much of communication wasn’t captured in e-mails or directly mentioned in meetings. It was implicit—understood without words. If your chairman asked you to take a look at a stock, (...) you didn’t need to be told explicitly what to say or write. It was understood, (...) that you were to comply by lavishing the stock or the deal with
imum price), and as a result, FB top management could no longer pay bonuses that they felt were sufficient to reward their employees. Stewart (1993) then notes that at least one manager “dipped into his own pocket to pay them more”, which mirrors what happens in our model when the manager’s discretion is reduced.\footnote{\textit{positive comments”} (Prins, 2006).}

4.2 Additional instruments for the principal

We previously assumed that the manager simply receives a share \( \alpha \) of the profit. In some contexts, however, the principal may have additional tools at their disposal to motivate the manager. If the principal can receive an upfront or contractible transfer from the manager, then delegation becomes always weakly preferable for the principal, since at worst she can always give the manager identical incentives to her own. If such a transfer is not possible, we may ask how sensitive our results above are to the particular tools given to the principal. In this section, we therefore consider how our results change if the principal can incite the manager to weigh revenues and prices differently, as well as pay the agent and manager a wage that depended on their relationship continuing. This would give the parties the following payoff functions:

\[
\pi_t = (1 - \delta) \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau} \left( (1 - \alpha_y) Y_{\tau} - (1 - \alpha_p) p_{\tau} - w_S - w_A \right) \right]
\]

\[
v_t = (1 - \delta) \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau} \left[ \alpha_y Y_{\tau} - \alpha_p p_{\tau} + S_{\tau} + w_S \right] \right]
\]

\[
u_t = \mathbb{E} \left[ (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ d_{\tau} \left[ p_{\tau} - S_{\tau} - c(e_{\tau}) + w_A \right] + (1 - d_{\tau}) u \right\} \right]
\]

where \( \alpha_Y \) and \( \alpha_p \) are the weights placed on revenue and prices, \( w_S \) is the wage paid by the principal to the manager and \( w_A \) is the wage paid by the principal to the manager. These wages are conditional on the manager-agent relationship continuing, so they require the principal to observe

\footnote{Cases of managers paying bonuses from their own money are rarely documented due to their informal nature, but anecdotes suggest that the phenomenon may be fairly common - see, for instance, Green (2013, 2017).}
whether cooperation has taken place. We assume that $\alpha_p \geq 0$ and $\alpha_Y \leq 1$.

A first point to note is that, when these new parameters are taken as given, these changes would in general not change the nature of managed relational contracts studied in Section 2. For the manager and agent, an increase in either wage has a similar effect to a decrease in the agent’s outside option, while an increase in $\alpha_y$ is equivalent to an increase in $y$. Manager-agent optimal contracts therefore would not change substantially, and Proposition 1 would remain unchanged.

The principal, however, can use these additional tools to her advantage when governing the relational contract. In particular, the following lemma notes that by calibrating wages appropriately, she can eliminate side payments in equilibrium.

**Lemma 4.** *If the principal can pay the agent and manager relationship-dependent wages, she can eliminate side payments in equilibrium at no cost.*

Side payments benefited the principal in the main model by increasing the value that the manager placed on maintaining the relationship with the agent. If the principal can pay the manager directly for this, then they lose their benefit. Moreover, the wages enable the principal to control the manager-agent surplus without changing effort incentives. Thus she does not need to reduce the manager’s discretion as $\delta$ increases. Instead, she can decrease the wages. Since using surplus to sustain variation in side payments is a waste for the principal, she will reduce the wages so that this type of variation is not used to induce effort in equilibrium (i.e. only low surplus contracts will arise in equilibrium). This may be one reason why side payments in the public sector receive more attention than side payments in the private sector - a greater number of instruments may allow private firms to replace the positive aspect of side payments and hence crack down on collusion.

One reason why, in reality, managed relational contracts may be sustained by side payments rather than payments from the principal is that the principal has limited information. Hermalin (2015) argues that, within firms, principals may imperfectly observe when their intermediaries are cooperating with other agents. If the manager might continue to receive payments from the principal after termination, then payments would also
increase the manager’s outside option. Moreover, to induce an optimal contract without side payments, the principal needs to know the relative bargaining powers of the manager and agent. For these reasons, side payments may be a more effective tool for sustaining managed relational contracts.

The principal may also rule out direct transfers to the manager to reduce the potential for future renegotiations. Katz (1991) shows that if unobservable renegotiation is possible, then delegation loses much of its ability to solve commitment problems. Hence managed relational contracts sustained by side payments may be more credible than those with relatively flexible principal-manager transfers.

Although these additional instruments mean that the principal can induce better contracts, they do not necessarily allow her to achieve first-best or to make delegation always weakly preferable. It is optimal for the principal to set $\alpha_y = 1$ because a higher $\alpha_y$ encourages the manager to induce more effort. There is no cost to the principal in increasing $\alpha_y$ since she can extract surplus through the wage $w^S$. It is not optimal, however, for the principal to set $\alpha_p = 1$, just as it was not optimal for her to set $\alpha = 1$ in the simpler model.

The principal will set $\alpha_p$ at the lowest level at which it is possible to induce a contract with $p_l = 0$ and effort at the desired level. If this implies $\alpha_p > 0$, then the effort level will be below first-best since greater effort requires giving more surplus to the manager. Delegation will therefore be inferior to direct principal-agent contracting when $\delta$ is sufficiently large that first-best could be achieved directly. It may alternatively be optimal for the principal to set $\alpha_p = 0$. This makes relational contracting between the manager and agent unnecessary because the manager has no temptation not to pay promised prices. In this case, the principal only needs to give the manager and agent their outside options, and delegation will be optimal so long as effort is as high as in the direct principal-agent contracting case. The threat of the manager paying prices when quality is low still exists, however, and hence the amount of effort that can be induced this way is limited, and may be below $e^{FB}$.

\[^{13}\text{To see that obtaining } e^{FB} \text{ may or may not be possible when } \alpha_p = 0, \text{ consider the case where } c(e) = ce^2. \text{ Since } \alpha_p < 1/2, \text{ the manager and agent’s payoff is convex in } p_l, \text{ and hence the principal is constrained by having to set } p \text{ such that the manager and} \]
Of course, with further additional instruments the principal would be able to achieve first best. This would be possible, for instance, if she could set $\alpha_Y > 1$, though this would mean she would receive a negative share of profits each period, which is likely to be difficult to implement in practice (Rayo, 2007). Similarly, allowing an initial fee to be paid to the principal by the manager or agent would mean she could set $\alpha_p = 1$, pay a very large wage $w_S$ and extract all the surplus ex-ante. But, if direct principal-agent relational contracting cannot also produce the first best, then achieving the first-best with managed relational contracting requires a wage larger than the total surplus generated each period. Overall, the fact that we observe binding relational contracts in many instances suggest there must be important restrictions on the tools principals can use when entrusting relational contracts to managers.

4.3 Costly side payments

We have assumed for simplicity that side payments between the manager and agent are costless except to the extent that they need to be self-enforced. In reality, however, side payments may be intrinsically costly. For instance, there may be a risk of punishment, and payments may be made in an inefficient way to avoid detection. Alternatively, side payments may not represent cash transactions, but favors that the agent can do for the manager, where the manager’s benefit is not necessarily equal to the agent’s cost.

In this subsection we consider how our model would change if we make side payments costly. In particular, we assume that a side payment which costs an agent $S$ only gives a benefit of $\kappa S$ to the manager, where $0 < \kappa \leq 1$.\footnote{Such a model is most appropriate for a context where, at least in net terms, side payments are paid from the manager to the agent. This will happen, for instance, if the manager has all the bargaining power and the maximum price is sufficiently high. For other contexts, we could alternatively assume that the side payment $S$ was always positive, or that a cost was born by the agent for receiving side payments. Either assumption would lead to optimal contracts being potentially non-stationary, since the agent prefer to set $p_l = 0$ than $p_l = \bar{p}$. If $p_l = 0$, then $e = \frac{\bar{p}}{\kappa}$, and hence the constraint on $\bar{p}$ is $\frac{\bar{p}}{\kappa} (y + \bar{p}) - \frac{\bar{p}}{2} \left( \frac{\bar{p}}{\kappa} \right)^2 \geq \bar{p}$, which implies $\bar{p} \leq 2(c - y)$. Since $e^{FB} = \frac{c - y}{2}$, this implies first best effort can only be achieved if $e^{FB} \leq \frac{2}{3}$.}

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The optimal manager-agent contracts will not change significantly with these new assumptions. In particular, a monotonic transformation of the manager’s payoff function tells us that introducing a cost of side payments $\kappa$ is equivalent to the costless case where she receives a share of profit $\alpha/\kappa$.

**Proposition 4.** The impact on managed relational contracts of an increase in the cost of side payments, i.e. a decrease in $\kappa$, is equivalent to an increase in $\alpha$. When side payments are more costly, the principal will benefit from delegation more often.

In other words, Proposition 1 will not change, and in this important sense we can interpret the parameter $\alpha/\kappa$ as both a measure of how closely aligned incentives are and a measure of the manager’s corruptibility.

The principal’s optimal behavior will be impacted by $\kappa$. In particular, when $\kappa$ is smaller, the risk of collusion is reduced, and hence the principal can set a lower value of $\alpha$ and still avoid $p_l > 0$. Delegation is therefore more beneficial for the principal since she can achieve the same level of effort by sharing a smaller share of the profit. This result is consistent with Bloom, Sadun and Van Reenen (2012) and Bloom et al. (2013) who find that firms delegate more when there is either stronger rule of law or management practices which allow better monitoring of managers. The impact of potential collusion on delegation can have important consequences - Ákcigit, Alp and Peters (2016), for instance, show that the relative difficulty of delegation in India compared to the US can account for 15% of the difference in income between the two countries.

If side payments were impossible ($\kappa = 0$), then the principal could set $\alpha = 0$ and induce first-best effort. Thus in general the principal prefers for collusion to be more costly.\textsuperscript{15}

Our results thus contrast with Strausz (1997), who in an alternative model of intermediation finds outcomes are the same whether or not supervisor-agent collusion is possible (i.e. $\kappa = 1$ or $\kappa = 0$). One reason for this difference is that Strausz (1997) considers a model where the supervisor’s monitoring creates verifiable information, and hence the principal can write a contract with the supervisor that depends both on her effort and the agent’s performance.
4.4 An application to multitasking

We have so far modeled manager-agent collusion in our model through the transfer of side-payments, as is typical in models of collusion in hierarchies. An alternative approach is in the style of a multitasking model where the agent can undertake two types of activity - one which benefits the principal, and one which benefits only the manager. In this section, we demonstrate with a simple model of multitasking that this form of collusion can also aid relational contracting.

Let us suppose that the agent cannot pay side payments to the manager, but instead can now exert effort on two alternative tasks. He exerts effort $e_1$, which probabilistically generates a revenue of $Y$ accruing to the principal as before, and $e_2$ which benefits the manager by an amount $f(e_2)$. The two tasks are substitutes, and hence the cost to agent of exerting effort is $c(e_1 + e_2)$. For simplicity, we assume now that the manager observes $e_1$ and $e_2$ directly. The principal’s only action is to set $\alpha$ at the beginning of the game, such that the manager’s payoff function is

$$v_t = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} d_t [\alpha Y_t - \alpha b_t + f(e_{2t})].$$

If the manager’s contract with the agent is enforceable by a third party, then effectively she can set $e_1$ and $e_2$ and pay the agent a compensation of $b = c(e_1 + e_2)$. She will therefore choose $e_1$ and $e_2$ such that $\alpha c'(e_1 + e_2) = \alpha y = f'(e_2)$. From the principal’s point of view, therefore, in this setting increasing $\alpha$ decreases the ‘collusive’ effort $e_2$ and increases the ‘productive’ effort $e_1$.

Now, alternatively, suppose that the manager’s contract with the agent is relational. In this case we will still have $b = c(e_1 + e_2)$, but this will now be subject to a dynamic enforcement constraint $b \leq \frac{\delta}{1 - \delta} (\alpha y + f(e_2) - \alpha c(e_1 + e_2))$. Solving the manager’s constrained maximization problem gives us that, when this dynamic enforcement constraint is binding, she will set $\alpha y = f'(e_2)$ and $\alpha c(e_1 + e_2) = \delta (\alpha y e_1 + f(e_2))$. This implies that, if $\delta < \min \left\{ \frac{c'(e_1 + e_2)}{g}, \frac{\alpha y c'(e_1 + e_2) + f''(e_2) c(e_1 + e_2)}{g(e_1 f'(e_2) + f'(e_2))} \right\}$, then $\frac{de_1}{d\alpha} < 0$. In other words, if relational contracting is sufficiently difficult, reducing the weight which the manager places on the principal’s payoff can facilitate effort that is beneficial for the principal. The mechanism is the same one as in our model with side payments - by increasing the value the manager places
on collusion as compared to the principal’s payoff, the manager has more credibility in promising to reward the agents effort.

5 Conclusion

This is the first paper that studies the impact of collusion on relational contracts. The main take away messages are the following: First, collusion can crowd out productive effort when the relationship between manager and agent is too strong. Second, when trust is a scarce resource, managed relational contracts are more credible and can incentivize more quality than direct relational contracts. Third, a principal delegating optimally will constrain the manager’s discretion so as to prevent overpayment, and in this case delegation may or may not be beneficial for the principal.

Before the most recent Aéropostale judgement, it was common to use “the value of the kickbacks” as “a reasonable measure of the pecuniary loss suffered” by the third party (Droney, 2017, p.70). Judge Droney, however, argued that this “negative correlation” between kickbacks and loss should not be taken for granted. Indeed, our model has shown when this negative correlation may not exist. Hence, our conclusions may help explain why politicians and firm owners frequently turn a blind eye to employees accepting side payments (Banfield, 1975). On the other hand, our model also identifies when side payments undermine effort, and hence emphasize the complex relationship between kickbacks and productive relational contracts.

The model produces a number of testable implications. We could, for instance, test directly for a non-monotonic relationship between output and trust in situations where the principal is constrained in her ability to govern the manager-agent relationship, such as the public sector. In some circumstances it may also be possible to observe the extent to which managers use their discretion and test for the type of misuse predicted by the model. For instance, Rasul and Rogger (2015) find Nigerian public projects to be better implemented when the overseeing bureaucrats are ethnically diverse - might this be because collusion is harder in such contexts? In other contexts, we may use the model to analyze the principal’s behavior by observing variation in the incentives and discretion given to
managers. Variation in the value of manager-agent relationships may be obtained by considering connections between individuals outside work or the importance of contracts in light of the business cycle or changes in competition. A potentially under-explored area may be investigating firm owners’ concerns with employee fraud in procurement, particularly in developing countries where courts are weak.

There are also multiple theoretical extensions to the model that would be valuable to pursue. For instance, we have assumed that the manager’s preferences are known, but in reality there is uncertainty as to ‘how corrupt’ any individual is. Removing this assumption, in the spirit of Chassang and Padró i Miquel (2014) or through uncertainty over the manager’s outside option as in Halac (2012), may reveal insights into how corruption and effort evolve over time. We may also ask whether collusive relational contracts make managers more likely to stick with the same firm over time. In this regard, the papers by Board (2011) and Calzolari and Spagnolo (2009) that consider relational contracts with potential competitors may provide useful approaches.
Appendix

Proof of Lemma 1. We provide a sketch of the proof since it is analogous to that of Theorem 2 in Levin (2003).

Consider a manager-agent contract that in its first period calls for payments \( p(y) \), \( s^F \), \( s(y) \) and effort \( e \). If the offer is made and accepted and the discretionary payments made, the continuation contract gives payoffs \( u(y) \), \( v(y) \) as a function of the observed outcome \( y \). Let \( u \), \( v \) be the expected payoffs under this contract:

\[
\begin{align*}
    u &\equiv (1 - \delta) E \left[ p(y) - s^F - s(y) - c(e) \mid e \right] + \delta E \left[ u(y) \mid e \right] \\
    v &\equiv (1 - \delta) E \left[ \alpha(y - p(y)) + s^F + s(y) \mid e \right] + \delta E \left[ v(y) \mid e \right]
\end{align*}
\]

We follow Levin (2003) in defining this contract as self-enforcing if and only if the following conditions hold:

i. Parties willing to initiate the contract: \( u \geq u \) and \( v \geq 0 \)

ii. The agent is willing to choose \( e \): \( e \in \arg\max_e E \left[ p(y) - s(y) + \frac{\delta}{1 - \delta} u(y) \mid e \right] - c(e) \)

iii. For all \( y \), both parties willing to pay \( p \):

\[
\begin{align*}
    (1 - \delta) (-\alpha p(y) + s(y)) + \delta v(y) &\geq 0 \\
    (1 - \delta) (p(y) - s(y)) + \delta u(y) &\geq \delta u
\end{align*}
\]

iv. For all \( y \), both parties willing to pay \( s \):

\[
\begin{align*}
    (1 - \delta) s(y) + \delta v(y) &\geq 0 \\
    -(1 - \delta) s(y) + \delta u(y) &\geq \delta u
\end{align*}
\]

v. Each continuation contract is self-enforcing: \( u(y) \), \( v(y) \) correspond to a self-enforcing contract that will be initiated in the next period.

Let \( g^* \) be the maximum surplus generated by any self-enforcing contract. Consider an optimal non-stationary contract with continuation payoffs \( u(y) \) and \( v(y) \) (such that \( u(y) + v(y) = g^* \)), a side payment \( s(y) \) and a price \( p(y) \).
We must define new side payments to produce the stationary contract that gives $u^*$ to the agent and $v^*$ to the manager, where $v^* = g^* - u^*$

$$s^*(y) = s(y) - \frac{\delta}{1 - \delta} u(y) + \frac{\delta}{1 - \delta} u^*$$

$$u^* = \mathbb{E}_Y \left[ p(y) - s^F - s^*(y) - c(e) | e \right].$$

Proof of Lemma 2. For the first part, consider an optimal contract with $p_h < 0$. Then consider an alternative contract with price $p'_h = 0$ and side payment $s'_h = s_h - p_h$. It is simple to check that all the self-enforcing constraints are still satisfied. Moreover, this contract has a higher surplus, and therefore the original contract cannot be optimal. The same logic holds if $p_l < 0$.

For the second part, first suppose that $s_h > s_l$. If positive effort is being made, we must have $p_h > p_l$. Then, consider an alternative contract with $s'_l = s_h, p'_l = p_l + s_h - s_l$. This alternative contract must also be self-enforcing, yet surplus is greater. Hence the original contract is not optimal. In the case of prices, if $p_h < p_l$, then we can similarly consider an alternative contract with $p'_h = p_l$ and $s'_l = s_h + p_l - p_h$.

Proof of Lemma 3. First, consider an optimal contract with $(DE_A)$ not binding. If $e < e_{MA}^{FB}$, then consider an alternative contract with $s'_l = s_l + \epsilon$. This contract induces higher effort and, for some $\epsilon > 0$, is self-enforcing. Thus we must have $e \geq e_{MA}^{FB}$. If $p_l < \overline{p}$, then consider a contract with $b'_l = p_l + \epsilon$ and $s'_l = s_l + \epsilon$. This contract generates higher surplus and, for some $\epsilon > 0$, is self-enforcing. Thus we must have $p_l = \overline{p}$. Lemma 2 then implies $p_h = \overline{p}$.

Second, consider an optimal contract with $(DE_S)$ not binding. If $e < e_{MA}^{FB}$, then consider an alternative contract with $s'_h = s_h - \epsilon$. This contract generates higher effort and, for some $\epsilon > 0$, will be self-enforcing. Hence we must have $e \geq e_{MA}^{FB}$. If $p_h < \overline{p}$, then we must have $s_l = s_h$, since otherwise we can construct an alternative self-enforcing contract that generates higher surplus with $p'_h = p_h + \epsilon$ and $s'_h = s_h + \epsilon$, for some $\epsilon > 0$. Since $e > 0$, it therefore follows that $p_l < p_h$, but now we can construct a self-enforcing
contract with \( p'_h = p_h + \epsilon \) and \( p'_l = p_l + \epsilon \), for some \( \epsilon > 0 \). Hence we must have \( p_h = \bar{p} \). Finally, if \( p_l < \bar{p} \), then we can consider a contract with \( p'_l = p_l + \epsilon \) and \( s'_h = s_h - \epsilon \) (since \( s_l \geq s_h \) from Lemma 2). But this contract is self-enforcing for some \( \epsilon > 0 \) and has higher surplus. Hence we must have \( p_l = \bar{p} \).

Therefore, if either \((DE_A)\) or \((DE_S)\) is not binding, we must have \( p_h = p_l = \bar{p} \) and \( e \geq e_{FB}^{MA} \). Summing \((DE_A)\) and \((DE_S)\) and substituting into \((IC)\) gives \( \alpha \bar{p} + c'(e) < \delta \frac{1}{1-\delta} \). But, since \( e \geq e_{FB}^{MA} \), we must have \( c'(e) \geq \alpha y \), which implies \( \delta \geq \delta_{FB}^{FB} \).

**Proof of Proposition 1.** If \( \delta \geq \delta_{FB}^{FB} \), then the first-best contract is self-enforcing. This contract is ‘high surplus’ in the sense of the proposition. For the rest of the proof we consider the case when \( \delta < \delta_{FB}^{FB} \) and hence \((IC - DE)\) binds. We first consider how the variation in prices and side payments in the optimal contact change as a function of \( \delta \), and then how effort \( e \) changes as a function of \( \delta \) for each contract type.

First, note that both the surplus and effort level are increasing in \( p_h \), and hence \( p_h = \min \{ \bar{p}, \frac{1}{\alpha} \frac{\delta g^*}{1-\delta} \} \), where the second term is the bound imposed by \((IC - DE)\) when \( s_h = s_l \).

If \( \bar{p} > \frac{1}{\alpha} \frac{\delta g^*}{1-\delta} \), then \( p_h < \bar{p} \). It must therefore be that \( s_h = s_l \), and otherwise we could consider an alternative contract with \( p'_h = p_h + \epsilon \) and \( s'_h = s_h + \epsilon \) and other values as before. In this alternative contract, surplus is greater since effort is unchanged and prices are higher. Hence we can define \( \delta_L \) such that \( \bar{p} = \frac{1}{\alpha} \frac{\delta g^*}{1-\delta} \). There is a unique value of \( \delta \) that solves this equation since \( g^* \) is weakly increasing in \( \delta \). We now consider in turn the cases when \( \delta \geq \delta_L \) and \( \delta \leq \delta_L \).

If \( \delta \geq \delta_L \), then \((IC - DE)\) determines \( p_l \): \( p_l = \frac{\delta g(e, \bar{p}, p_l)}{1-\delta} + (1-\alpha)\bar{p} - c'(e) \). This gives that, if \( \delta > \delta_L \), then \( \frac{\delta g(e, \bar{p}, p_l)}{1-\delta} > \alpha \bar{p} \), and hence \( c'(e) > \bar{p} - p_l \), implying \( s_l > s_h \). The manager and agent’s problem is to maximize the surplus function \( g_1(e) = g(e, \bar{p}, p_l) \) with respect to \( e \) subject to the boundary conditions on \( p_l \), i.e. \( 0 \leq p_l \leq \bar{p} \). Let \( \bar{g}_1 \) and \( g_1 \) be the surpluses at the upper and lower boundaries. Effort levels \( \bar{e}_1 \) and \( e_1 \) are determined by \((IC - DE)\) at these two potential solutions. Let \( \bar{g}_1 \) be the surplus at the interior solution that maximizes surplus. This will involve effort level
\( \hat{e}_1 \) where \( g'_1(\hat{e}_1) = 0 \). Differentiating \( g_1(e) \) gives

\[
g_1'(e) = \frac{\alpha y + (1 - \alpha) (\alpha \bar{p} - (1 - e)c''(e)) - \alpha c'(e) - (1 - \alpha) \frac{\delta g_1(e)}{1 - \delta}}{1 - \delta - \delta (1 - e)(1 - \alpha)}
\] (3)

Setting this to zero together with \( (IC - DE) \) gives us the value of \( p_i \) in this case, which is written in equation (2).

Since we wish to characterize the optimal contract as a function of \( \delta \), we differentiate each of these surpluses with respect to \( \delta \), giving the following equations:

\[
\begin{align*}
\frac{d\bar{g}_1}{d\delta} &= \frac{\bar{g}_1}{1 - \delta} \left( \frac{(1 - \alpha) (\alpha \bar{p} - (1 - e)c''(e))}{1 - \delta - \delta (1 - e)(1 - \alpha)} - \delta (\alpha y - c'(\bar{e}_1)) \right) \\
\frac{d\tilde{g}_1}{d\delta} &= \frac{\tilde{g}_1}{1 - \delta} \left( \frac{(1 - \alpha) (1 - \tilde{e}_1)}{1 - \delta - (1 - \alpha)(1 - \tilde{e}_1)\delta} \right) \\
\frac{d\bar{g}_1}{d\delta} &= \frac{\bar{g}_1}{1 - \delta} \left( \frac{\alpha y + (1 - \alpha) \bar{p} - c'(\bar{e}_1)}{1 - \delta - \delta (1 - e)c''(e)} - \delta (\alpha y + (1 - \alpha) \bar{p} - c'(\bar{e}_1)) \right)
\end{align*}
\]

From (3), at any interior solution we have \( c'(\hat{e}_1) = y - \frac{1 - \alpha}{\alpha} (1 - \hat{e}_1)c''(\hat{e}_1) - \frac{1 - \alpha}{\alpha} (\frac{\delta g_2}{\delta} - \alpha \bar{p}) \). Thus, at any \( \delta \) where \( \bar{g}_1 = \tilde{g}_1 \), we have \( c'(\bar{e}_1) < c'(\hat{e}_1) \) and hence \( \frac{d\bar{g}_1}{d\delta} > \frac{d\tilde{g}_1}{d\delta} \). There therefore exists a single value of \( \delta \) such that for all higher values the upper boundary is preferable to an interior solution, and for all lower values the interior solution is preferable. We can show similarly that, at any \( \delta \) where \( \bar{g}_1 = g_1 \), we have \( \frac{d\bar{g}_1}{d\delta} > \frac{dg_1}{d\delta} \). Hence there exists a value \( \delta^H \) such that the optimal solution has \( p_i = \bar{p} \) if and only if \( \delta \geq \delta^H \).

If \( \delta \leq \delta^L \), then we can write the surplus as \( g_2(e, \delta) \) where

\[
g_2(e, \delta) = g(e, \frac{1}{\alpha} \bar{p}, \frac{1}{\alpha} \tilde{e}_1, \frac{1}{\alpha} \tilde{e}_1, \delta) = g_2(e) - c'(e)
\]

Expanding and rearranging gives:

\[
g_2(e, \delta) = \frac{\alpha (1 - \delta)}{\alpha - \delta} (\alpha ye + (1 - \alpha) (w - (1 - e)c'(e)) - c(e) - y)
\] (4)

Now define \( \delta^0 \) to be the maximum \( \delta \) such that all optimal contracts have \( e = 0 \). Note that \( \delta^0 \leq \delta^L \) since if \( \delta > \delta^L \) we have \( p_i = \bar{p} \) and hence \( e > 0 \) from the binding \( (IC - DE) \). Now suppose that there exists a value of \( \delta < \delta^0 \) such that the optimal contract has \( e > 0 \). Hence \( g_2(0, \delta) = g_2(0, \delta^0) > g_2(0, \delta^0) \), and it follows that \( g_2(e, \delta^0) = \frac{(1 - \delta^0)(\alpha - \delta)}{(\alpha - \delta^0)(1 - \delta)} g_2(e, \delta) > \frac{(1 - \delta^0)(\alpha - \delta)}{(\alpha - \delta^0)(1 - \delta)} g_2(0, \delta) = g_2(0, \delta^0) \), which contradicts the definition of \( \delta^0 \). Hence
we must have $e = 0$ in all optimal contracts when $\delta \leq \delta^0$.

Finally, let us consider the relationship between $e$ and $\delta$ in the optimal contracts. For high surplus contracts, a binding $(IC - DE)$ implies that the LHS of the equation $c'(e) + \alpha p = \frac{\delta}{1 - \delta}$ is increasing in $e$. For low surplus contracts with $p_l = 0$ we can similarly transform the binding $(IC - DE)$ to be $\frac{c'(e)}{g(e,p_l,0)} = \frac{\delta}{1 - \delta}$. In each case, the LHS does not depend directly on $\delta$ and hence it is straightforward to see that $\frac{dc}{d\delta} > 0$. For low surplus contracts with $p_l > 0$, combining equation (2) and the binding $(IC - DE)$ gives

$$c'(e) = y - \frac{1 - \alpha}{\alpha} (1 - e)c''(e)$$

and hence $\frac{dc}{d\delta} = 0$.

For intermediate surplus optimal contracts with $p_l > 0$, we have $e = \tilde{e}_1$ where $g'_1(\tilde{e}_1) = 0$ and $g''_1(\tilde{e}_1) < 0$. Differentiating (3) by $e$ and using $g'_1(\tilde{e}_1) = 0$ gives:

$$g''_1(\tilde{e}_1) = (1 - \delta) \frac{(1 - 2\alpha)c''(\tilde{e}_1) - (1 - \alpha)(1 - \tilde{e}_1)c'''(\tilde{e}_1)}{1 - (1 - \alpha)\delta(1 - \tilde{e}_1)}$$

Note that for this contract to be optimal we must have $1 - \delta - (1 - \alpha)\delta(1 - \tilde{e}_1) > 0$, since otherwise increasing $p_l$ and $s_l$ simultaneously relaxes $(IC - DE)$. $g''_1(\tilde{e}_1) < 0$ therefore implies

$$(1 - 2\alpha)c''(\tilde{e}_1) - (1 - \alpha)(1 - \tilde{e}_1)c'''(\tilde{e}_1) < 0$$

We then differentiate $g'_1(\tilde{e}_1) = 0$ implicitly by $\delta$ to obtain:

$$\frac{d\tilde{e}_1}{d\delta} = \frac{g_1(\tilde{e}_1)}{(1 - \delta)^2 ((1 - 2\alpha)c''(\tilde{e}_1) - (1 - \alpha)(1 - \tilde{e}_1)c'''(\tilde{e}_1))}$$

This expression is negative in any optimal intermediate contract with $p_l > 0$, and this thus completes the proof.

**Proof of Proposition 2.** Suppose first that $p_l = p_h$. Then $(IC - DE)$ is $c'(e) = \frac{\delta \bar{g}}{1 - \delta} - \alpha p_h$ and the principal can do better by decreasing $\bar{p}$ to zero, increasing $\alpha$ to keep $g$ constant.
Now, suppose that $0 < p_l < p_h$. It must be that $g'_1(e) = 0$, since otherwise the manager would marginally increase or decrease $p_l$. We hence have that the principal is maximizing $\pi = ey - c(e) - g$ subject to $(IC-DE)$ and (2). To simplify the algebra, we define $\gamma = \frac{d}{1-\delta}$. We now consider first the case where $\alpha \leq \delta$, and then the case where $\alpha \geq \delta$.

If $\alpha \leq \delta$, then substituting out for $p_l$ and $p_h$ gives

\[
g = \alpha ey + e \left( c'(e) + \frac{1-\alpha}{\alpha} (1-e) c''(e) - y + \gamma \frac{1-\alpha}{\alpha} g \right)
+ (1-\alpha)(1-e) \left( \frac{1-\alpha}{\alpha} (1-e) c''(e) - y + \frac{\gamma}{\alpha} g \right) - c(e) - u \tag{7}
\]

Rearranging gives

\[
(\alpha - \gamma(1-\alpha))g = \alpha^2 ey - \alpha c(e) - \alpha u - \alpha(1-\alpha)(1-e)c'(e)
+ (1-\alpha + \alpha e)(\alpha c'(e) + (1-\alpha)(1-e)c''(e) - \alpha y)
\]

Note that, since $c''(e) \geq 0$, the right hand side is strictly increasing in $e$, and hence, since $(\alpha - \gamma(1-\alpha)) < 0$, $g$ is strictly decreasing in $e$. This implies that $\pi$ is strictly increasing in $e$, since $\pi = ey - c(e) - g - u$, and hence the principal can do better by decreasing $g$, which she does through lowering the maximum price $\bar{p}$.

If $\alpha > \delta$, then we can go one step further and substitute out for $g$ in the principal’s profit function, giving

\[
\pi = ey - c(e) - \frac{\alpha^2 ey + \alpha ec'(e) + (e + (1-\alpha)(1-e))((1-\alpha)(1-e)c''(e) - \alpha y) - \alpha c(e) - \alpha u}{\alpha - \gamma(1-\alpha)}
\]

If $\pi$ is being maximized, we must have $\frac{d\pi}{d\alpha} = 0$ (since it is clearly not maximized at either corner solution). Solving out for $\frac{d\pi}{d\alpha}$ gives us that it has the same sign as the following expression:

\[
[1 + \alpha - \gamma + \gamma \alpha] [(1 - \alpha)(1-e)c''(e) + c'(e) - \alpha y]
+ e[1 - (\alpha - 2\gamma + \gamma \alpha)(1-\alpha)] (1-e)c''(e)
+ ec''(e) - c'(e) + \gamma[ec'(e) - c(e)] + [\alpha + \alpha \gamma - \gamma](y - c'(e))
\]

Note that, since $c''(e) \geq 0$ and $(1-\alpha)(1-e)c''(e) + c'(e) - \alpha y = (1-$
\[ \alpha)(p_h - p_l) > 0, \] together these terms are strictly positive, and we have a contradiction. It there must be that \( p_l = 0 \).

To see that \( \delta < 1 \), consider \( \delta \) sufficiently high that \((IC - DE)\) is not binding for a given \( p \), e.g. \( c'(e^{FB}) < \frac{\delta g(e^{FB},0,0)}{1-\delta} - \alpha \overline{p} \). In this case, \( p_h = p_l = \overline{p} \) and hence the principal can set \( \overline{p} = 0 \) without changing effort.

To see that \( \delta_l > \delta_0 \), let \( \alpha_0 \) and \( \overline{p}_0 \) be the optimal values of \( \alpha \) and \( \overline{p} \) at \( \delta = \delta_0 + \epsilon \), where \( \epsilon \) is some small positive number. Suppose that \( \overline{p}_0 < \frac{\alpha_0}{1 - \delta_0} \frac{\delta g(\alpha_0, \overline{p}_0,0)}{1 - \delta_0} \), where \( c'(e_0) = (1 - \alpha_0)\overline{p}_0 + \frac{\delta_0 g(\alpha_0, \overline{p}_0,0)}{1 - \delta_0} \). Now, consider a contract where \( \alpha = \alpha_0 \) and the maximum price is increased to \( c'(e_0) \). If \( \delta_l = \delta_0 \), then there exists sufficiently small \( \epsilon \) such that there exists some \( \delta < \delta_0 \) such that \( c'(e_0) = \frac{\alpha_0}{1 - \delta} \frac{\delta g(\alpha_0, c'(e_0),0)}{1 - \delta} \), and hence there is a managed relational contract which generates positive effort at this level of \( \delta \).

Finally, to see that \( \delta_l < \delta_h \), note that the principal’s problems and constraints are continuous, and hence for some range of \( \delta \), \( \overline{p} \) will decrease continuously in \( \delta \).

\[ \square \]

**Proof of Proposition 3.** The existence of \( \tilde{\delta} \in (0,1) \) is straightforward. Direct relational contracting is only a constraint when \((1)\) does not hold at \( e^{FB} \). When \( e^{FB} \) is achievable without delegation, it is better for the principal to contract directly than to involve a manager, since this way she does not have to cede surplus to the manager.

To show the existence of \( \hat{\delta} \in (0,\tilde{\delta}) \), we first consider the case where \( u > 0 \), and show that there exists a range of \( \delta \) such that the principal can achieve positive profit with delegation but no effort without. With direct relational contracting, there will be effort if and only if there exists a positive solution to the equation \( ce = \frac{\delta}{1 - \delta}(ey - ce^2/2 - u) \), which is equivalent to the condition that \( \delta \geq \frac{cy+c-\sqrt{2cy^2}}{y+c-\sqrt{2cy}} \). With optimal managed relational contracting, the principal always receives a strictly positive payoff when \( e > 0 \), since \( \pi = (1-\alpha)e(y-\bar{b}) \). The optimal solution for the principal involves setting \( \alpha \) such that \( b_l = 0 \) and, for low \( \delta \), not using the maximum price, implying \( 2ce = y + c - \frac{1}{\alpha}(1 - e)c \). Combining this with the relational contracting constraint \( c = \frac{\delta}{1 - \delta}\left((y - ce) + \frac{ce-2u}{2a}\right) \) gives us an expression for \( e \) which is positive when \( \delta > \frac{c}{(c+y)(c-y)} \). Since \( u < \frac{y^2}{2c} \) and \( y < c \), this expression is strictly smaller than \( \frac{c}{y+c-\sqrt{2cy}} \).

If \( u = 0 \), then in both cases profit is zero at \( \delta = \frac{c}{y+c} \) and positive for
larger $\delta$. We therefore compare $\frac{d\pi}{d\delta}$ at $\delta = \frac{c}{y+c}$ in both cases. With direct relational contracting, $\frac{d\pi}{d\delta}\bigg|_{\delta = \frac{c}{y+c}} = 2 \frac{y}{\delta}$. With managed relational contracting, $\frac{d\pi}{d\delta}\bigg|_{\delta = \frac{c}{y+c}} = \frac{2yc}{\delta(c-y)}$. Hence for $\delta$ just larger than $\delta = \frac{c}{y+c}$, managed relational contracting is more profitable for the principal than direct relational contracting.

**Proof of Lemma 4.** First, it is straightforward to see that in any equilibrium where $s_l = s_h$, the principal can ensure $s_l = s_h = s^F = 0$ by adjusting the wages appropriately. It therefore remains to show that the principal will set parameters such that $s_l = s_h$. We begin by considering how the principal should set $p$ and $p_l$ if she wishes to induce a contract with $s_l = s_h$. We then show that inducing such a contract is indeed optimal.

From Proposition 1, the principal can induce a contract with $s_l = s_h$ by setting $p = \frac{1}{\alpha p_0} \frac{\delta y}{1-\delta}$. Then, from the binding ($IC - DE$), we have $c'(e) = \frac{1}{\alpha p_0} \frac{\delta y}{1-\delta} - p_l$, where $p_l$ either takes the value 0, $\bar{p}$ or a solution to (2). The optimal contract has $p_l = 0$. To see this, suppose $p_h > p_l > 0$. Then effort is given by (5) which is not directly affected by $g$. The principal can then increase her payoff by decreasing $w_A$. This change will decrease $g$ along with $p_l$ given by (2) but will not affect effort. Now suppose that $p_h = p_l > 0$. Since $s_h = s_l$, no effort is induced. The principal can improve her payoff by decreasing $w_A$ until $g = 0$ so no price can be credibly paid.

Now, suppose that parameters induce a manager-agent contract with $s_l > s_h$. Then, from the proof of Proposition 1, we have $p_h = \bar{p}$. We will consider the three possibilities for $p_l$, and show that in each case the principal can adjust parameters to improve her payoff. First, suppose that $p_l = 0$. Then from the binding ($IC - DE$), we have $c'(e) = \frac{\delta g}{1-\delta} + (1 - \alpha p)\bar{p}$ and hence by increasing $\bar{p}$ and reducing $w_A$ to keep $g$ constant the principal can increase effort while holding surplus constant through reducing $p$ and increasing $w_A$. If $p > 0$, then from the binding ($IC - DE$), we have $c'(e) = \frac{\delta g}{1-\delta} - \alpha p \bar{p}$ and the principal can increase effort while holding surplus constant through reducing $\bar{p}$ and increasing $w_A$. If $\bar{p} = 0$, then the ($IC - DE$) may or may not be binding, and hence $c'(e) = \min \{ \alpha g y, \frac{\delta g}{1-\delta} \}$. If $g > \frac{(1-\delta)\alpha g y}{\delta}$, then the principal can
reduce $g$ without reducing effort. If $g \leq \frac{(1-\delta)\alpha y}{\delta}$, then $c'(e) = \frac{\delta g}{1-\delta}$, and hence for any given $g$ effort is equal to or less than the effort achieved in the optimal contract with $s_l = s_h$. Hence the principal can always weakly improve on any contract with $s_l > s_h$.

**Proof of Proposition 4.** By dividing the manager’s payoff by $\kappa$, we can see that the manager-agent contract with a cost of corruption $\kappa$ and profit sharing $\alpha$ will be equivalent to one with costless corruption and profit sharing $\alpha/\kappa$. It is therefore straightforward to see that the principal can react to any decrease in $\kappa$ by a similar decrease in $\alpha$, and the manager-agent contract will remain unchanged. Since this new contract involves the principal keeping a larger share of the surplus, the principal’s payoff is improved, and $\frac{d\pi}{d\kappa} < 0$. \hfill \Box
Online appendix

In this extension, we consider a model with a principal, a manager, and $N$ identical agents where aggregate revenue is contractible. As in Section 4.2, we allow the principal to pay relationship dependent transfers to the manager and agent, and to share revenue and prices with the manager in different proportions. Contracts between the manager and agents are bilateral, so we treat the relationship between the manager and each agent as a separate game (Levin, 2002). The timing of the game is unchanged.

Denote $p_{it} \leq \bar{p}$ the price paid to agent $i$ and $s^F_{it}$ and $s_{it}$ the set of side payments. If agent $i$ accepts, $d_{it} = 1$, he chooses an effort $e_{it} \in [0,1]$ which generates stochastic revenue $Y_{it} \in \{0,y\}$. If an agent $i$ rejects, $d_{it} = 0$, then no revenue is produced and he receives a per-period payoff of $u_i$. Collective revenue $Y_t$ is then the sum of these individual revenues and some noise, i.e. $Y_t = \sum_i Y_{it} + \epsilon_t$, where $\epsilon_t$ i.i.d. with $\mathbb{E}[\epsilon] = 0$. A share $\alpha_S$ of this revenue is shared with the manager, while a share $\alpha_i$ is given to agent $i$. The manager also pays for a fraction $\alpha_p$ of the prices and wages received by the agent.

The expected payoff functions are therefore:

$$\Pi_t = (1 - \delta) \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left( 1 - \alpha_S - \sum_{i=1}^{N} \alpha_i \right) Y_{\tau} - \sum_{i=1}^{N} \left( (1 - \alpha_p) p_{i\tau} + w_i \right) \right]$$

$$V_t = (1 - \delta) \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[ \alpha_S Y_{\tau} + w_S - \sum_{i=1}^{N} \left( \alpha_p p_{i\tau} + S_{i\tau} \right) \right] \right]$$

$$u_{it} = \mathbb{E} \left[ (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ d_{i\tau} [\alpha_i Y_{\tau} + w_i + p_{i\tau} - S_{i\tau} - c(e_{i\tau})] + (1 - d_{i\tau}) u \right\} \right]$$

Effort is the agent’s private information, while the individual quality and agent’s compensation are observed by both the manager and the particular agent. Agents cannot observe the individual quality of the other agents.

If, as we have assumed in the main article, the agents do not receive a share of revenue (i.e. $\alpha_i = 0 \ \forall i$), then we achieve identical results to before. Since the relationships between the manager and each agent are technologically independent of one another and contracts are bilateral, we
can treat the manager’s relationship with each agent as a separate game. As a result, Propositions 1, 2 and 3 go through unaffected.

We now consider the case where it is possible to share revenue with the agents. We first consider how a manager would optimally share revenue with agents were she to have this tool at her disposal in addition to the relational contract. In particular, consider the case where the principal sets the share of revenue received by the manager, but the manager can then give some part of this share to each agent. If there is only one agent, then clearly the manager can obtain her preferred effort level by passing all her revenue share to the agent. If there are multiple agents, however, Rayo (2007) shows that relational contracting may be optimal when it is impossible for each agent to receive the entire revenue. To see this in our model, consider the behavior of the manager and agents when the parameters set by the principal are given. Suppose that the manager can reward the agent with a share \( \alpha_i \) to agent \( i \), given the constraint that \( \sum_i \alpha_i \leq \alpha_Y \), where \( \alpha_Y \) is the share of revenue that the manager receives. Then let \( g_R(\delta) \) be the manager-agent surplus generated at a given level of \( \delta \) by the optimal manager-agent relational contract when there is no sharing of revenue, i.e. \( \alpha_i = 0 \), and let \( e_R(\delta) \) be defined by the equation \( c'(e_R(\delta)) = \frac{\delta g_R(\delta)}{1-\delta} \). \( e_R(\delta) \) is therefore a minimum level of effort that any optimal manager-agent contract will generate. The following lemma then shows that, when \( N \) is large, it is optimal to share no revenue with the agents unless \( \delta \) is small, and in particular smaller than the range of \( \delta \) which Proposition 1 focuses on.\(^{16}\)

**Lemma 5.** For a given \( N \), if \( \delta \geq \tilde{\delta}(N) \), where \( \frac{\tilde{\delta}(N)}{1-\tilde{\delta}(N)} = \frac{1}{(N-1)e_R(\delta(N))} \), then it is optimal for the manager to share no revenue with the agents. Moreover, if \( N > 1 + \frac{g_R(\delta_L)}{e_R(\delta_L)} \), then \( \tilde{\delta}(N) < \delta_L \).

**Proof of Lemma 5.** Since agents are identical, there is no reason to give them different shares of profit, and we therefore consider that each agent receives \( \frac{\alpha_A}{N} Y \), where \( \alpha_A \leq \alpha_S \). Suppose that \( \delta \geq \tilde{\delta}(N) \) and that the manager has an optimal contract with each agent with \( \alpha_A > 0 \) generating

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\(^{16}\)For lower \( \delta \) a combination of revenue sharing and relational contracts will be used and, as in Baker, Gibbons and Murphy (1994), these instruments may be complements or substitutes. A complete characterization of such contracts is beyond the scope of this paper.
effort $e$. Now, consider an alternative contract with $\alpha_A$ reduced by $\epsilon$ and $s_t - s_h$ increased by $\frac{1}{N}e\epsilon y$. Such a contract induces the same amount of effort and the expected surplus within each each manager-agent relationship is increased by $\frac{N-1}{N}e\epsilon y$. The new contract is therefore self-enforcing if this increase in surplus is sufficient to allow the increased difference in side payments, i.e. if $\frac{\delta}{1-\delta} \frac{N-1}{N}e\epsilon y \geq \frac{1}{N}e\epsilon y$. If $\delta \geq \hat{\delta}(N)$, this holds for all $e \geq e_R(\delta)$, and hence any optimal manager-agent contract. There is therefore always an optimal contract with $\alpha_A = 0$. Finally, for the second part of the lemma, note from the proof of Proposition 1 that $\delta^L$ is defined by $\frac{\delta^L}{1-\delta^L} = \frac{p}{\theta_R(\delta^L)}$.

The intuition behind this lemma is that sharing revenue with agents has two opposing effects on the dynamic enforcement constraints. First, the enforcement constraints are effectively relaxed, since agents are motivated to exert effort with non-discretionary incentives. Second, the enforcement constraints are tightened, since the joint surplus in the relationship between a given agent and manager diminishes because part of the manager’s revenue share has been given to the remaining agents. The size of the first effect is diminishing in $N$, while the size of the second effect is increasing in $N$. Hence, for a given discount factor $\delta$, there is a critical number of agents $N$ above which giving any share of revenue to agents makes inducing effort more difficult.

Turning to the decisions of the principal, it is clear that, if there is only one agent, the principal can achieve the first best. In particular, by setting $\alpha_Y = \alpha_p = 1$, the manager will have the same incentives as the principal. Then the manager will use only contractible incentives by sharing all the revenue with the agent. Since relational incentives are not used, the principal can extract all the surplus via a negative $w^S$ or $w$. For large $N$, however, there will be relational contracting and, as before, it will be optimal to set $\alpha_p < 1$ to reduce the cost of this relational contracting.

References


