When and Why Are the Values of Physical Quantities Expressed with Uncertainties?

Aude Caussarieu, Andrée Tiberghien

When and Why Are the Values of Physical Quantities Expressed with Uncertainties? A Case Study of a Physics Undergraduate Laboratory Course

Aude Caussarieu • Andrée Tiberghien

International journal of science and mathematics education, vol
Published online 10 march 2016

Abstract

The understanding of measurement is related to the understanding of the nature of science—one of the main goals of current international science teaching at all levels of education. This case study explores how a first-year university physics course deals with measurement uncertainties in the light of an epistemological analysis of measurement. The data consist of the course documents, interviews with senior instructors, and laboratory instructors' responses to an online questionnaire. During laboratory work, uncertainties are expressed in the large majority of the measurements made by the students but only in less than half of their calculation results. The instructors' expectations are that students systematically estimate uncertainties so that they become aware that measurements and calculations are never exact. However, since uncertainties are not specified for the values given in the laboratory guides, uncertainties are often missing from the results of students' calculations. The potential side effects of students' measurement understanding are discussed and suggestions for improvements are proposed.

Keywords: *first-year university physics teaching; laboratory course; measurement uncertainties; physics; teaching design*

Introduction

Students' understanding of the nature of science (NOS) is a major goal of contemporary science education (Bartos & Lederman, 2014; Urhahne, Kremer, & Mayer, 2011). However, NOS content to be taught has been debated amongst science education researchers because there is lack of consensus amongst philosophers (Abd-El-Khalick, 2013; Duschl & Grandy, 2013; Irzik & Nola, 2011). Salter and Atkins (2014) found six recurring NOS themes to be taught at school in published literature: empiricism, process, tentativeness, subjectivity, context, and creativity. Measurement uncertainties are central in the validation process of knowledge production and the tentativeness of the knowledge claims in science; therefore, students need to understand these uncertainties to construct adequate views of NOS.

This case study documents how uncertainty of measurements is incorporated into a first-year introductory physics course at a large French university and what related outcomes are expected by the course and laboratory instructors. Course documents, interviews of senior instructors, and questionnaire surveys of laboratory instructors were used to construct, illustrate, and justify assertions about the central research focus. A background about NOS, measurements, and uncertainty serves as a foundation for the interpretative framework for this case study.

Nature of Science and Scientific Measurements

Numerous science education studies have explored students' conceptions about NOS (Deng, Chen, Tsai, & Chai, 2011; Lederman, 2007) and data processing (Buffler, Allie, & Lubben, 2001; Day & Bon, 2011; Deardorff, 2001; Séré, Journeaux, & Larcher, 1993). These studies have shown that most students at all teaching levels hold inadequate views about NOS. Students often think that scientific knowledge is absolute and that a scientist's primary goal is to uncover natural laws and discover truths. The naive, positivist science epistemology often embedded in physics teaching may even prevent students from developing modern NOS views (Sin, 2014). The students' understanding of data processing aspects of NOS has been investigated mainly in the context of replicated measurements (Buffler et al., 2001; Day & Bon, 2011; Séré et al., 1993). One result from these studies is that students tend to choose the mode of a series of measurements rather than the mean of the set of measurements. Another finding is that students believe a true measurement exists and that proper use of high-quality measurement devices with no human error will provide exact measurements of this true value (Deardorff, 2001). The choice of the repeated value is often explained as a consequence of the belief in the true value: because one can measure the true value then, within a series of measurements, one should find more often the true value; other values are obtained because of some avoidable error. Buffler, Lubben, and Ibrahim (2009) found a correlation between a naïve view on NOS and a naïve view on measurement.

Recent studies on measurement teaching have dealt with the presentation and evaluation of innovative teaching sequences (Allie et al., 2003; Lippmann-Kung, 2005). These courses were designed to teach measurement explicitly while taking into account the students' difficulties involving measurement; such instructional approaches have been shown to improve students' measurement understanding. However, to our knowledge, there has been no study about the effective teaching practices of university physics instructors regarding measurement based on the underlined epistemic choices situated within the academic physics discipline.

Epistemological Analysis of Physics Measurement

Measurement is at the very heart of all empirical sciences since knowledge production requires comparing experiment results to theoretical claims: "The principle of science, the definition, almost, is the following: *The test of all knowledge is experiment.* Experiment is the *sole judge* of scientific 'truth'." (Feynmann, 1965, p. 1-1). However, physical measurements are always limited by several sources of uncertainty. Two such sources are: (a) material objects and phenomena vary and, thus, replicated measurements of the same physical quantity may differ; and (b) measurement instruments are limited by a finite resolution since they all rely on a physical comparison with a reference quantity. A third source of uncertainty is related to the difficulty in discerning the limits of an object or a phenomenon. For example, in an optical experiment in which a lens projects a real image of an object onto a screen, it is very difficult to tell the precise position of the image. The range of positions in which the image looks equally in focus—because the light is polychromatic and the lens is not a perfect thin lens—illustrates an important source of uncertainty.

Measurements of quantities in physics are made to allow comparisons. Hence, it is recommended to estimate the uncertainty associated with a measurement and to indicate it by reporting the result of a measurement with two numbers, as in $\tau = (0.25 \pm 0.5)$ s. One number refers to the value of the measurand (i.e., the quantity being measured) whereas the other refers to the uncertainty in the value of the measurand. The value of the measurement should be written with a reasonable number of significant figures. However, this notation is not the only one that can be found in scientific literature or in science textbooks. The notations used in physics publications can be grouped into three categories:

- *Interval notations*: When the information is given with two numerical values that specify the measurement and its uncertainty, as in $\tau = (0.25 \pm 0.5)$ s.
- *Point notations*: When the information is given with only one numerical value of the measurement with no explicit reference to the uncertainty, as in $g = 9.81$ m.s^{-2}.
- *Approximate notations*: When the information is given with one numerical value accompanied by a symbol ($\sim$ or $\approx$) or an adjective (e.g., about or approximately), as in $\tau \sim 0.3$ s, which acknowledges the existence of uncertainty within the last significant figure.

The interval notation is obviously the more precise notation. When physicists use the point or approximate notation, they have in mind the order of magnitude of the uncertainty to choose the number of significant figures, which reflect the precision of the methods and techniques used.

Interpretation and estimation of uncertainties can be made within two different frameworks: the historical classical approach (sometimes called the error approach) and the contemporary uncertainty approach. Bevington and Robinson (2003) and Taylor (1997) provided extensive descriptions of the classical approach to measurement in which uncertainty is defined as the difference between the measurement and the true value of a given physical quantity. The International Organization for Standardization published the Guide to the Expression of Uncertainty in Measurement (GUM; Group 1 of the Joint Committee for Guides in Metrology, 2008) that describes the uncertainty approach in which uncertainty is defined as a “parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand” (GUM, 2008, p. 14). When an experiment is replicated and the result of the measurement varies, both approaches estimate uncertainties in the same way. When the uncertainty cannot be estimated through statistical methods, the two approaches have different ways of estimating uncertainties but their final numerical values are somewhat close. On the contrary, the rules for combining different sources of uncertainties and for estimating uncertainty in the result of a calculation involving values of other physical quantities are very different. The classical approach estimates the upper limit of the absolute value of the total error with a cumulative process whereas the uncertainty approach takes into account the possibility that one uncertainty source offsets another. Hence, the two approaches refer to different propagation laws. These two approaches can be seen as having different epistemological families. The classical approach may be associated with a positivist view of science in which laws are to be unveiled and true values exist; the uncertainty approach seems closer to a modeling approach where “scientists use [probabilistic] models to represent aspects of the world for specific purposes” and interpret their results (Giere, 2004, p. 742).

This epistemological analysis reveals that the existence of unavoidable uncertainty in measurement implies that uncertainties are estimated when numerical values are to be used in a comparison. When precise uncertainty estimations have been carried out, the numerical value of a physical quantity is written in interval notation.

Research Questions

The purpose of this study is to investigate the teaching about measurement offered in a first-year physics laboratory course that has been redesigned and implemented in a large university. The following research questions (RQs) involving specific elements and explicit/implicit rules guided the investigation:

RQ1: What are the elements of measurement teaching in this course?

RQ2: When does the laboratory guide ask the students to estimate measurement uncertainties?

RQ3a: What are the instructors' goals regarding measurement and uncertainty teaching?

RQ3b: How do instructors justify the actual handling of uncertainties during laboratory work?

Methods

A multi-method case study was used to address the research questions. Case studies require a well-defined context to establish boundaries for rich data collection, interpretation of the data, and limited generalization of results.

Context of the Study

The RQs were investigated in the framework of an introductory physics course for students majoring in biology at an open university in a large French city. The recently redesigned and implemented laboratory course included lectures, tutorial sessions, and laboratory sessions. This multi-section physics course is offered during the fall and spring semesters with a total enrolment of about 500 students. The university and course are not selective and do not require entrance examinations for program entry.

The teaching team consists of supervising professors and about 30 instructors of whom one-third is renewed each year. A senior professor supervises the whole course and two other professors are in charge of the electricity laboratory and the optics laboratory sessions. The lectures are given by three senior instructors. Every instructor has responsibility for a tutorial group (i.e., optic or electricity-radioactivity) and six laboratory sessions.

A major revision of the laboratory part of the course (i.e., experiments to be performed and laboratory guides) occurred in 2010 after the Physics Department received new equipment. An educational design team composed of a small group (n ~ 3–4) of the instructors conducted the revision with three major aims guiding the redesign of the course. First, they wanted to harmonize the content of the laboratory work with what was addressed during lectures and tutorials. Second, they wanted to address the needs of the biology instructors concerning the physics concepts. Third, they wanted to specifically address the teaching of measurement and measurement uncertainties. Some adjustments to the original version of the course materials were implemented over the next two years to reach the current version, which is studied here.

The course consists of 12 lectures (1h30 each), 12 tutorials (1h30 each), and 6 laboratory sessions (2h each). On average, students are expected to attend 6.5 hours of teaching sessions (1 lecture, 2 tutorials, and 1 laboratory session) each week. The lectures and tutorials deal with the basics of geometrical optics, electricity, and radioactivity; the biweekly laboratory sessions deal only with geometrical optics (3 sessions) and electricity (3 sessions). The subjects addressed during the laboratories and tutorials are aligned with the lectures and the needs expressed by the biology instructors.

Data Collection

The corpus of data for this study consists of the written documents for the course given to the students, responses to three individual interviews with instructors, and the answers of other instructors to a short online questionnaire.

Written documents. Analyses of the course documents provided to students in the course digital workspace were helpful in exploring the first two RQs. The elements of measurement knowledge taught in the lecture slides, the tutorial booklet, the pre-laboratory assignments, and the laboratory guides were analyzed to address RQ1 (What are the elements of measurement teaching in this course?). RQ2 (When does the laboratory guide ask the students to estimate measurement uncertainties?) was explored using the tasks students were expected to perform during laboratory work described in the laboratory guides.

Lecture slides. There are 12 lectures in this course; four deal with electricity, two with radioactivity, and six with optics. Lectures are supported by 12 slide sets that are the same across the different lecture halls, which are available online for student use. There is one supplementary set of 16 slides dealing with measurement uncertainties that is no longer addressed during the lectures because of a reduction in the teaching hours allocated to this course. Students are encouraged to download these slides to read and study at home before the first laboratory session.

Tutorials booklet. During the tutorial sessions, students are divided into groups of 20 to 35 students. Students work on exercises developed by the design team in booklets distributed to each student at the beginning of the course; these booklets are also available online. There is one booklet dealing with electricity and radioactivity and another dealing with geometrical optics. Each booklet starts with one exercise on error estimations; however, after this initial exercise, errors are not mentioned again. There is one instructor per tutorial group whose role is to help the students and to give the answers to the exercises.

Laboratory guides. A 34-page booklet distributed to each student at the beginning of the course contains an introduction to the laboratory course and the six guides corresponding to each session. Each guide provides instructions and commentaries on the specific investigation. During the session, students follow the instructions and fill in the blanks by writing down their measurements and answering short questions; Figure 1 provides a sample question.

Calculate, from the previously determined positions, the following values with their uncertainties:

$\overline{O_1A}$ = (±) cm $\overline{O_1A'}$ = (±) cm

a

2.3- Influence of the resistance R on the charging time

Taking C=10nF and giving R as the two indicated values indicated in the table, coming back to E=6V and f~100Hz

R (kΩ)	10	100
$\tau_1 \pm \Delta\tau$ (give units)		
τ_1/R (give units)		

Conclude

b

Figure 1. Extracts from the laboratory guides: (a) an optical laboratory guide; (b) an electricity laboratory guide.

Pre-laboratory assignments. The restructuring of the course required that the laboratory sessions were shortened from three to two hours. Therefore, pre-laboratory assignments were designed to contain most of the theoretical developments present in the previous laboratory guides. They often contain background concerning uncertainty estimations and about ten questions to help students understand the experimental tasks to be conducted. These pre-laboratory assignments are not printed; students can find them on the digital workspace. Students are encouraged to prepare for the laboratory sessions with these assignments but they are neither collected nor graded by the instructors; therefore, very few students actually read or use them.

Professor and instructor feedback. Interviews and a questionnaire were used to collect information from the instructors about RQ3a (What are the instructors' goals regarding measurement and uncertainty teaching?) and RQ3b (How do instructors justify the actual handling of uncertainties during laboratory work?).

***Interviews*.** Semi-structured interviews were developed and conducted after the analysis of the teaching documents was performed with the help of three senior professors. The instructors interviewed were the current leaders of this physics teaching. They have been part of this course for more than 5 years; two were current lecturers and part of the design team. The first part of the interview asked the professors about the general instructions given to the students in the laboratory guides:

1. Why do students have to perform uncertainty estimations during laboratory work?
2. What are the specific constraints that you faced during the rewriting of the laboratory guides?
3. Why does it say in the introduction of the laboratory guides that students should give every numerical value with its associated uncertainty?
4. Is this rule the actual one used during laboratory work?

The second part of the interview asked the professors to comment on specific instructions in the laboratory guides. The interviews (~1h) were audio-recorded and transcribed.

Online questionnaire. An online questionnaire was composed of two questions on the laboratory instructors' administrative status (e.g., PhD, post doc, or permanent instructor) and their seniority in the teaching unit as background on the respondents. The next two questions focused on the laboratory work's goals to supplement the interview responses dealing with RQ3a and RQ3b. The first question about the general goals for laboratory work is taken from Welzel et al. (1998) and asks the instructors to rank-order five possible goals for laboratory work. The second question on measurement learning outcomes was an open-ended question: "Can you explain what, in your opinion, are the goals of these laboratory sessions as regards teaching measurement and uncertainties?" Some instructors did not answer the open-ended question when completing the questionnaire.

The questionnaire was sent to all instructors associated with the course during the previous academic year; only 16 instructors completed and returned the questionnaire. This low response rate (~50%) is due to the facts that it was sent after the spring semester and that about 30% of the instructors would not teach this course the following year.

Data Analysis

This introductory course is attended by a very large population (500 students), and there are many instructors (senior professors, young professors, PhD students) with a high turnover (30 instructors, a third replaced each year). The final examination is the same for all students; therefore, there is a strong concern that each student has received the same teaching. The strategy adopted by the design team consisted of writing down as much information as possible to avoid possible variations amongst the instructors' presentations.

A thematic analysis (Braun & Clarke, 2006) of the lecture slides, tutorial booklets, pre-laboratory assignments, and laboratory guides was used to determine the measurement framework used by the design team: Classical or uncertainty approach and the eventual choices made by the design team (RQ1). The themes were based on our epistemological analysis of measurement and uncertainty; in particular, the concepts associated with the definition of uncertainty and the rules associated with its estimation.

We looked for the context elements that are associated with the estimation of measurement uncertainties during laboratory work (RQ2). The epistemological analysis led us to distinguish among the three categories of notation for numerical values described above (i.e., interval, point, and approximate). We used these notation categories to code all the numerical values presented in the tasks of the laboratory guides. The examples from the electricity and optical laboratory guides (Fig. 1) illustrate that students may have to report a direct measurement (e.g., a reading on a scale as in Fig. 1a or on a measuring instrument as in Fig. 1b) or to calculate the value of a physical quantity from other values (e.g., Fig. 1b). They may also use numerical values given in the laboratory guide, such as the values of C, E, R and f shown in Figure 1b. Hence, we defined three context categories corresponding to the origin of the numerical value: given in the laboratory guide, measure performed by the student, or calculation performed by the student. The association of the notation of a numerical value and its origin provided an overview of the notational values and categorical origin throughout the laboratory guide.

We investigated the teaching goals of the instructors through the analysis of the interviews and the online questionnaire responses (RQ3a). We also wanted to understand the instructors' justifications for the actual practice of uncertainty estimations during laboratory work (RQ3b), which relied on the analysis of the interview responses. The responses from the interviews and the open-ended question were analyzed using an inductive qualitative research method (Strauss & Corbin, 1998) to reveal constructs emerging from the professors' and instructors' responses. Their responses were coded into themes or categories based on patterns observed through repeated words or phrases. The research data were gathered, organized, and analyzed in a computer spreadsheet program (Excel).

Results

Samples of the teaching documents and responses from the interviews and questionnaires are presented as evidence (*italics*) and illustrations for the assertions (**boldface**) related to each research question. The authors elaborate the results in normal font.

RQ1: WHAT ARE THE ELEMENTS OF MEASUREMENT TEACHING IN THIS COURSE?

Analysis of the written documents (lecture slides, laboratory tasks, and pre-laboratory assignments) revealed that the classical approach of measurement was privileged over the modern uncertainty approach, and several elements related to variability were not addressed in these documents. The lecture slides and pre-laboratory guides provided the most information about the elements of measurement uncertainty, but no elements on variability were mentioned in any of the documents analyzed.

The word *error*—a central characteristic of the classical approach—is used in the slides on measurement for the interpretation of the interval notation given as the result: *the 'true' value of the measurement lies in the error interval given* [in the result]. The concomitant use of the term *uncertainty* is defined in the measurement lecture slides as: *the difference between the measured value* g_m *and the exact value* g *of a quantity* G. This definition relies on the concept of true value, which is characteristic of the classical approach of measurement. The propagation law and the error combinations made in the different documents are also based on the classical approach in which the final uncertainty is the sum of the absolute contributions of each uncertainty source, as in the example found in the lecture's slides on measurement: *if* $R = R_1 + R_2$ *then* $\mathrm{d}R = dR_1 + dR_2$.

Table 1 contains the main knowledge elements (concepts or rules) on measurement uncertainties in the classical approach and the results of the thematic analysis indicating which knowledge elements were present in the documents given to the students. Examination of Table 1 reveals that the knowledge elements on measurement and uncertainties were spread across the different documents and that 5 of the 22 elements were absent. All of the absent elements are related to the variability of physical objects or phenomena, which was not discussed in the content of the measurement teaching. Also, there is no mention of curve fitting in which multiple measurements are combined to estimate the value of a parameter. Only two sources of uncertainty are presented in this course: the measurement instrument and the difficulty involved in defining the limits of the measurand.

Most of the information on measurement is present in documents on the digital workspace that students are only encouraged to consult (e.g., lecture slides on measurement and pre-laboratory guides). The lecture slides on measurement, which might have been expected to act as a reference document for the students, are incomplete since they address only 11 of the 17 (= 22 - 5) knowledge elements on measurement present in this course. Furthermore, the rules for assessing specific uncertainties, such as the uncertainty associated with the reading of a measurement instrument, are not given in the lecture slides. The rule for concluding the compatibility of two measurements was only present in the pre-laboratory guides: *Measurements* n_1*and* n_2 *are compatible with each other if the calculated relative error* $\frac{|n_2-n_1|}{(n_2+n_1)/2}$ *is less than the sum of the relative uncertainties of each measurement* $\frac{\Delta n_1}{n_1} + \frac{\Delta n_2}{n_2}$.

The epistemological analysis pointed out that the justification of the uncertainty estimation—necessary for making comparisons—is central to understanding the role of uncertainties in physics. We did not find any mention of the fact that error estimations are necessary to compare two numerical values in the documents provided to students. On the contrary, the introduction of the laboratory guide presents uncertainties estimations as a systematic rule justified by a practice of experimentalists: *good experimentalists always give their results with an uncertainty*.

Table 1. *Results of the Analysis of the Content of the Measurement Teaching Provided in Comparison to the Reference Content of the Classical Approach to Measurement*

Elements of knowledge			Lecture slides	Tutorial booklet	Pre-lab. guide	Lab. guide commentary
Concept						
1	Uncertainties (or measurement error)	Definition (with reference to the true value)	X			
2		**Random error**				
3		**Systematic error**				
4		Relative uncertainty/fractional uncertainty/precision	X		X	
5	Source of uncertainties	Variability of physical objects and phenomena				
6		Limits of objects/phenomena	X		X	
7		Limited resolution of measurement instruments			X	
8		Non accuracy of measurement instruments	X		X	
9		Instrument reading	X		X	
10	Interpretation of a measurement result		X			
Rule						
11	Assessment of uncertainties associated with	**Variability in the replication of a measurement**				
12		Limits of objects/phenomena			X	
13		Manufacturer indications on the measurement instrument			X	X
14		Instrument reading			X	X
15	Propagation of measurement uncertainties	Combination of different errors for the same measurement	X		X	X
16		Simple combination of errors on different measured values	X	X	X	X
17		Law of propagation of uncertainties	X			
18	Assessment of uncertainty on a parameter of curve fitting	**Linear regression**				
19		**Least square fitting**				

20	Presentation of measurement result	Number of significant figures	X	X	X	X
21		Notation with sign ±	X	X	X	X
22	Comparison of results using uncertainties				X	

Note. *X* = presence of this knowledge element in the corresponding written document. Missing elements are highlighted in **boldface**.

RQ2: When does the laboratory guide ask the students to estimate the uncertainties?

Students are explicitly instructed that they have to state uncertainties in all their results (measurements and calculations) whereas in fact the laboratory guide asks them to do so on most of their measurements but in less than half of their calculations. The analysis of the written documents reveals that incompatibility between expectations and outcomes is mainly due to the systematic absence of uncertainty of the numerical values given in the laboratory guide.

The introduction of the laboratory guide (a similar statement is present in the slides on measurement) states:

> Important: Good experimentalists always give their results with an uncertainty. Do the same: in each laboratory session, get used to giving your measurements and calculations with an uncertainty. This last point is important for the final examination.

This statement allowed us to identify an explicit rule in this course about the expression of uncertainties by the students during laboratory work: Students should always estimate uncertainties for their results during laboratory work.

An inspection of the laboratory guide shows that in some cases students are not asked explicitly to estimate the uncertainty associated with their results. This is clearly visible in the extract of the laboratory guide provided in Figure 1b, presented earlier. We determined how widespread this oversight was by counting the different numerical values to be estimated by the students in the laboratory guides as a function of their origin (measurement or calculation) and their notation (interval, point, or approximate). The result of the count revealed 137 measurement and calculation requests distributed across interval and point notations (Fig. 2).

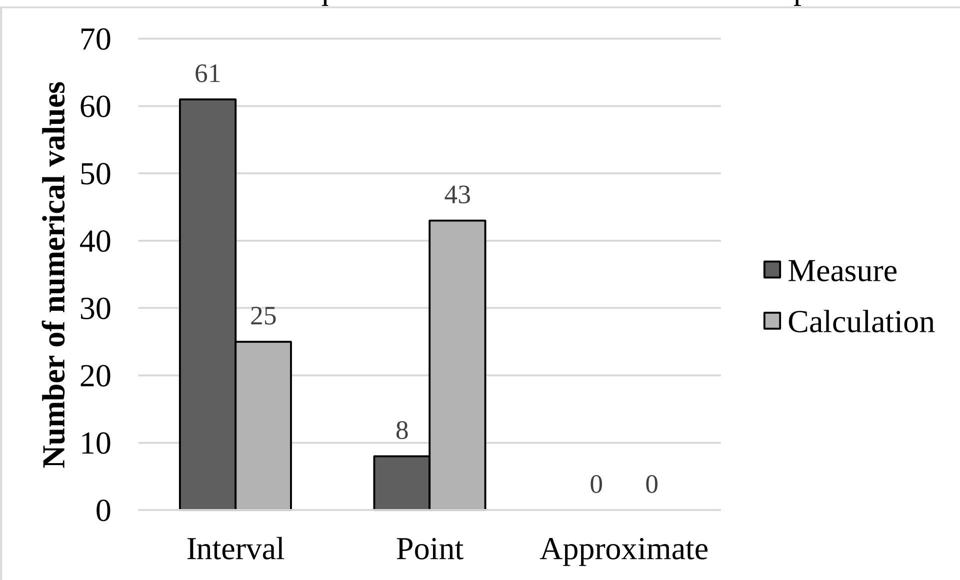

Figure 2. Number of results to be estimated by students depending on their notation (interval, point, approximate) and their origin (measure, calculation). Total possible number of requests = 137.

First, Figure 2 shows that students are never expected to use approximate notation, but they are expected to use both interval and point notation. They are asked to use interval notation for 63% of the occurrences (86 of 137 cases), which is not fully consistent with the explicit rule provided in the laboratory guide's introduction. Second, it shows that students should nearly always estimate the uncertainty in the direct measurements they perform (90%, 61 of 69 requests). Among the 8 anomalies (measures in the interval notation), four are introductory measurements (e.g., first measurement with an oscilloscope), two are measurements involved in the calculation of the power and magnification of a microscope for which students do not have the formula to propagate the uncertainty, and the last two are found in the final experiment of a very long optics practical.

Figure 2 also shows that students determine the uncertainty associated with the result of their calculations of physical quantity values less than half the time (~40%, 29 of 72 cases). Figure 1b, presented earlier, illustrates that students are not asked to estimate the uncertainty for the result of their calculation. The quantity to be evaluated through a calculation is the capacitance (C), which is a function of the charging time (τ_1) measured by the students and of the resistance (R) whose value is given by the laboratory guide: $C = \tau_1/R$. The uncertainty in the value of the resistance is not given in the laboratory guide; therefore, it is not possible to estimate the uncertainty in the result of the calculation (τ_1) without making an assumption about the uncertainty in the resistance.

Our second claim about RQ#2 is that the main cause of absence of uncertainty on the calculations is missing information. Thus, most of the data on results notations are explained by the following modified implicit rule: Students are expected to assess uncertainties on their results whenever it is possible. We identified all the numerical values involved in calculations to be performed by the students. Some values were the result of measurements or calculations already performed and others were numerical values of physical quantities provided (given) by the laboratory guides (Table 2). We identified 48 numerical values given in the laboratory guide: 41 are written down in point notation and 7 in approximate notation. Under the label *Calc1*, we grouped the results of students' calculations for which there is no missing information necessary for estimating the error for the calculated result. Under the label *Calc2*, we grouped the cases for which some information necessary for estimating the error of the result of the calculation is missing. This information might be the uncertainty on a value necessary for the calculation or the formula to propagate the uncertainty. Finally, we counted the number of results of calculation in each category and represented them as a function of their notation (interval or point).

Table 2. *The Associated Notations of Calculation Results and Numerical Values Involved*

Notation Origin	Interval	Point
Calc1: Result of calculation with no information missing	25	12
Calc2: Result of calculation with some information missing	0	31

Table 2 shows that our claim is supported by 56 (25 + 31) out of 68 calculations requested. There are 12 results of calculation in the point notation for which students had all the information necessary to estimate the uncertainty. Three of these results are preliminary results for a future complicated calculation for which students do not know the propagation formula. The other 9 results without uncertainties where students had all necessary information are in the last quantitative experiment of the session. Insights into these cases might be found later in the consideration of RQ3b.

Hence, we can formulate the following rule: Students are expected to estimate the uncertainty of the result of their measurement whenever it is possible, but it is not possible whenever a necessary uncertainty is missing. This reconstructed rule provides an explanation for 117 of the 137 students' results.

RQ3A: WHAT ARE THE INSTRUCTORS' GOALS REGARDING MEASUREMENT AND UNCERTAINTY TEACHING?

Teaching introductory physics and related laboratories can emphasize a variety of concepts, processes, practices, and NOS ideas. The instructors' responses to the online questionnaire and interviews indicate: **Teaching measurement is an important goal and instructional priority for them. These instructors want students to become aware that uncertainties are associated with every measurement and calculations using measurements. However, this priority for measurement uncertainty is embedded in the university science culture where knowledge and conceptual understanding is primary.**

The online questionnaire asked the instructors to rank five different goals one may assign to laboratory work (Welzel et al., 1998): A–for the student to link theory to practice; B–for the student to learn experimental skills, including measurement; C–for the student to get to know the methods of scientific thinking; D–for the student to foster motivation, personal development, social competency; and E–for the teacher to evaluate the knowledge of the students. Sixteen instructors responded to this question. Two respondents did not follow the instructions and assigned the same rank to several goals; their responses were omitted from further consideration. Figure 3 summarizes the instructors' responses to the online questionnaire.

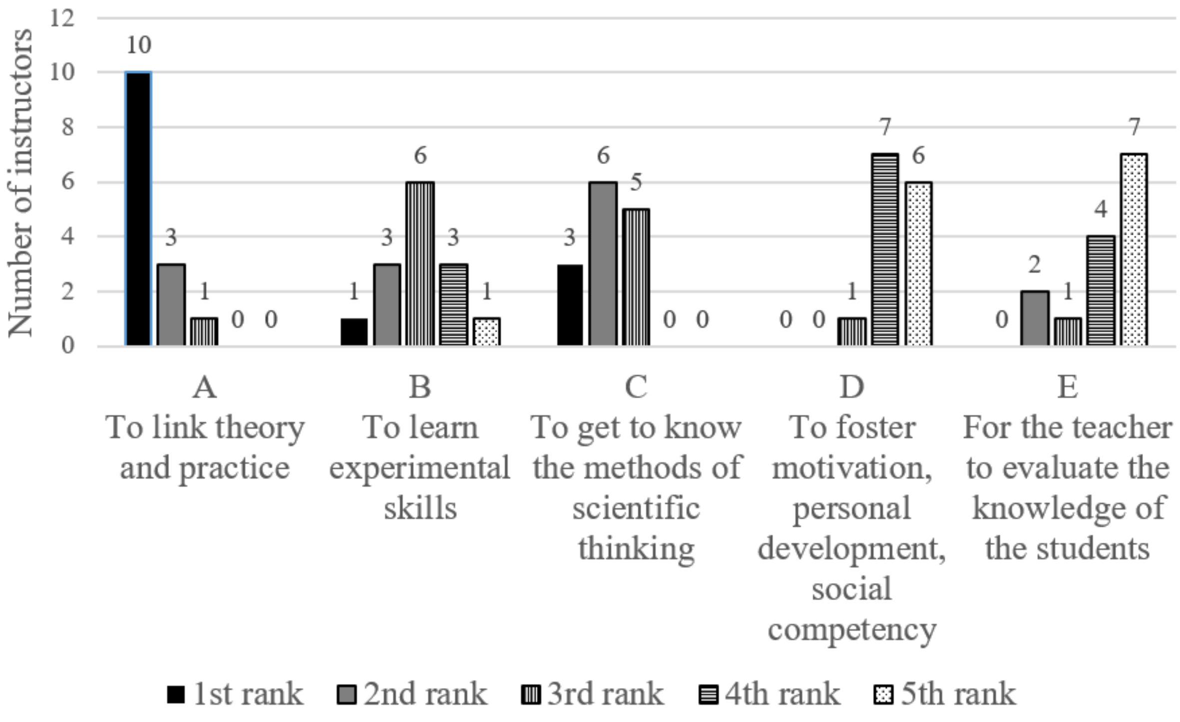

Figure 3. Goal-ranking responses to the online questionnaire by instructors (*n* = 14).

The data in Figure 3 illustrate clearly that these instructors' priority goal of laboratory work is Goal A–to link theory to practice. The second most important goals assigned nearly equal priority are B and C, which involve knowledge on measurement and uncertainties. Goals D and E are the lowest priorities for laboratory work for these instructors; Goal E was assigned a slightly higher priority than Goal D. The emphasis (i.e., ranked first by 10 of the 14) on connecting the knowledge taught during lectures to practice in other course components may explain why there is no time in the course devoted to explicit teaching of measurement.

We applied content analysis to the responses to the open-ended question (i.e., Can you explain what, in your opinion, are the goals of these laboratory sessions as regards teaching measurements and uncertainties?) to establish which aspects of measurement teaching were actually important to these instructors. The responses were first split into segments concerning a single idea (i.e., irreducibility of the segment to keep the idea). We identified a total of 48 segments in the 14 responses with an average of 3 ± 2 segments in each instructor's response. We focused further consideration only on the 36 segments related to goals concerning uncertainty teaching. First, we reformulated the segments into a formal sentence in which the goal is expressed from the students' point of view. Whenever the original segment was linked to a descriptive verb, then we kept it to build a formal sentence of the form: "One goal is that students [+ verb + subordinate clause]." (N.B. When there was no verb in the response, only the subordinate clause was retained.) Second, we grouped together the respondents' formal sentences that were similar. It appeared that most of the groups obtained thereby corresponded to the five elements of knowledge (i.e., definition of uncertainty, source of uncertainty, assessment of uncertainty, presentation of measurement result, and comparison of results using uncertainties) as themes that aligned with the earlier content analysis of the measurement teaching.

Two groups of segments (10 + 7) did not fit with these five themes. Hence, we constructed two new themes to accommodate these results: A–uncertainty is systematically associated to measurement and B–reliability of a result depends on its uncertainty. Theme A (10 segments) is straightforward and corresponds to the knowledge of a fact that is often part of the introduction of courses or textbooks on measurement (Taylor, 1997). What is interesting is that three instructors used verbs (*see/note/realize*) showing that these laboratory sessions were also aimed at convincing students of the systematic existence of uncertainties. In Theme B (7 segments), instructors' responses involved critical thinking about measurement results because of limited precision of measurements. In their responses, limits are related to the precision of a result, which is defined in the classical approach as relative uncertainty. One instructor said, *one goal is that students know that uncertainties are important in physics*; this statement could not be classified into the existing themes but appears to capture the overall, non-exact tenor of the nature of physics and science. The number of occurrences of each segment is presented in Table 3, indicating that the new themes (A and B) correspond to important teaching goals for the instructors. Surprisingly, only three segments correspond to the assessment of uncertainties theme whereas students are expected to assess 90 uncertainties during the laboratory course (Fig. 2).

Table 3. *Themes Identified in the Instructors' Responses*

Theme	Goals in instructors' responses	Segments (*n*)
A	Uncertainty is systematically associated to measurement	10
B	Reliability of a result depends on its uncertainty	7
C	*Source of uncertainty*	4
D	*Comparisons*	3
F	*Assessment of uncertainties*	3

G	*Definition of uncertainty*	3
H	*Propagation of uncertainties*	2
I	*Presentation of a measurement result*	3
J	Other	1
	Total	36

Note. Text in *italics* indicates elements defined in the epistemic analysis.

Further insights into the reasons why teaching elements about uncertainties is important to the instructors can be found in the interview responses. The three instructors interviewed were asked to justify from a pedagogical point of view *why* students are asked to estimate uncertainties during laboratory work (RQ1). Interpretation of their responses revealed the instructors' expectation that students should *be critical of their result* and that they should invoke the existence of uncertainties to explain why their actual measurement is different from the theoretical expected value.

Overall, these results show that in the instructors' responses the emphasis is on conceptual understanding about uncertainties rather than technical skill regarding uncertainties' estimations. This finding appears to contradict the outcomes from the document analysis that shows students are requested to estimate uncertainties regardless of their utility from the physics point of view.

RQ3B: HOW DO INSTRUCTORS JUSTIFY THE ACTUAL HANDLING OF UNCERTAINTIES DURING LABORATORY WORK?

The instructors justify that students should estimate the uncertainties of their results (i.e., measurements and calculations) as a norm of what must be done during physics laboratory work. The three instructors interviewed agreed on the explicit rule that students should always estimate uncertainty in laboratory sessions, which is given in the introduction of the laboratory guide. The statement *Good experimentalists always give their results with an uncertainty* appears to be the justification of the rule asking students to estimate uncertainties systematically. When asked about the justification of this systematic rule, two instructors suggested that in physics it is meaningless to give a result without its associated uncertainty. The normative aspect of the practice is also strengthened by the instructors, when one stated:

> Because you're doing physics, it must be meaningful, so, first you must give the result with a unit, it's not mathematics. [Then] because you are in a laboratory session, your measurement must have an associated uncertainty…. it is clearly indicated in the laboratory guide that each measurement without uncertainty will be penalized.

Another instructor stated that she justifies uncertainty estimations by her own research activity: *we do experiments in China and in France and we will try to compare our results and in order to compare our results it's necessary to know how precise they are*.

These instructors were probed during the interview to determine why in many cases students are not asked to estimate measurement uncertainties in which numerical values were to be written in point notation. We grouped their responses depending on the origin of the numerical value that was at stake.

Cases where the value is given. When asked about the absence of uncertainty in this case, each instructor was surprised: *I had not considered that problem* and *No no no no no, that's not a choice; in fact, we haven't done what we should do*. They immediately expressed the desire to change the laboratory guides so that the values are given with their uncertainty: *What must be done is that we give in the instructions the values with their uncertainties, we must remain coherent and indeed, here we are not.* Their surprise indicates clearly that there is no conscious rule that prevailed for the use of point notation for given values. Nevertheless, one instructor explained that he does not ask the students to take into account the error of the given quantity because his goal is that students understand errors associated with the reading of a digital or analogic instrument: *Well, there are already all the reading uncertainties with the number of divisions; it's okay if they manage to understand this notion. I'm happy*.

Cases where the value is the result of a calculation with some information missing. Most of the results of calculation expressed in point notation are explained by the instructors as the result of missing information: *Since we haven't given R with an uncertainty, they cannot* [estimate the uncertainty of the result] and *We should give them the formula, it would be easiest, that is something we can do*. The instructors confirmed that the implicit rule we posit in the results to RQ2 explained the absence of uncertainty on most of the calculation results.

Cases where the value is the result of a measurement or a calculation. When the instructors realized that in some cases students are asked to express the result of their measurements or calculations without uncertainty although it is possible to estimate them, the instructors saw an incoherence that should be fixed: *I didn't remember that we didn't ask for that*. They subsequently justify it with three different reasons. First, some cases are explained by the instructors as if they were omissions that should be fixed: *It's rather an omission*. Second, there are time constraints because the calculation is performed at the end of the session: *For me, I'd say that the problem is because we lack time.* and *We know that it is the end and that they won't finish, we don't ask them* [to estimate errors]. Third, the absence of measurement errors is put down to the nature and pedagogical purpose of the task involving the measurement or the calculation as an illustration or verification of the lecture's contents. In these cases, instructors just want the students to check if their result is of the same order of magnitude (power of ten) as the theoretical value rather than if their result is precisely consistent with the theoretical value. The laboratory activity is *truly applicative, it is just for illustrating the lecture but maybe there is an incoherence at this level* or because the instructors want the students to be aware of the order of magnitude: *It's the order of magnitude that matters, … if I find 10 microfarads, there is a problem; I mean the order of magnitude is bad*. Hence, the physics context of the numerical value is finally evoked to justify those problematic cases.

It appears at this introductory-level service course that conceptual understanding about uncertainty exists in primary measurements and awareness of this uncertainty is inherent in any calculations involving these data is the essential goal rather than the actual calculation of the uncertainty using complex formulae. Nevertheless, the instructors, for the sake of coherence, are willing that uncertainties are estimated in all cases even if it implies the use of a "turnkey" formula by the students.

Conclusion and Discussion

This case study of the redesign and implementation of an introductory physics course focused on the laboratory component and specifically the teaching of measurement and measurement uncertainty, which is essential to an informed view of the nature of science (physics). The research questions systematically considered the elements of measurement uncertainty, the intentions of the redesign team in developing course materials, and the actual use of these materials by course instructors.

Results of this study show that during laboratory sessions students actually express uncertainties on nearly all their measurements and about half of their calculations. This is mainly due to the absence of uncertainty on the data given in the laboratory guide, which creates an asymmetry between data obtained by the students and values given in the instructions. The risk is that it might lead students to think that the results of measurements made by scientists or manufacturers are of a distinct nature from those made by a student with the laboratory apparatus. Furthermore, there was a misalignment of the intentions and the resources regarding the determination of uncertainty in measurements and associated calculations since instructors wanted systematic estimation of uncertainties. These findings appeared to some members of the redesign team as an incoherence that should be fixed by providing students with all information necessary to estimate each uncertainty. We agree with the instructors that all the values given in the instructions of the laboratory guides should be provided with their associated uncertainties. Hence, their absence would not limit error estimation when they are necessary and would not reinforce the myth of the true value and physics as being an exact science. It is also coherent with the fact that if one communicates the numerical value of a physical quantity without knowing the future use—like a manufacturer does—one should still indicate the associated error.

In physics research, interval notation is nearly only used when the error is necessary for the comparison of two similar values; however, during daily routine in the laboratory, researchers do not systematically estimate the uncertainties. In the course studied, errors are always supposed to be estimated, even when they are unnecessary from the physics point of view. For example, in the first optics laboratory session, students are asked to measure the diameter of a spot of light on a screen after passing through a converging lens. This diameter (about one centimeter) is then compared to the diameter of the lens (about ten centimeters). The use of a ruler was useless in comparing these diameters and concluding that the lens action is to make the light rays converge (an observation with the naked eye was sufficient), and the uncertainties estimations are all the more useless. This may lead students to see uncertainty evaluation as a routine, meaningless, useless practice. A natural perspective of this study is the analysis of the teaching situations in which error estimations are necessary. A comprehensive study of the way physicists actually deal with uncertainties in their laboratories may help build a more realistic frame for uncertainty estimation in experimental physics courses and make students aware of uncertainty in physics.

The strength of systematic uncertainties estimation is that students will have estimated a high number of uncertainties during these sessions and, therefore, should have acquired solid technical skills. Nevertheless, we showed that instructors' main goals regarding uncertainties are that students *know what uncertainties are* and that they *get convinced that there are always uncertainties*. These goals are more related to conceptual understanding than technical skills and one may wonder if the actual teaching strategy is optimal to achieve conceptual understanding. Results of this case study also revealed that the design team and instructors have chosen the classical approach to measurement, rather than the contemporary uncertainty approach, leaving aside all concepts related to the variability of physical objects or phenomena. Variations across examples of objects and occurrences of events are not addressed in the course documents. These elements are crucial to develop a full understanding of measurement uncertainties and more generally of the NOS. Hence, the overall findings of this study indicate the need for new course designs compatible with the instructors' actual goals and constraints that enable an understanding of physics uncertainty.

A comprehensive overview of physics uncertainty includes the unavoidable nature of uncertainties and the relevant use of uncertainty for situations in which they are necessary to form a conclusion from the physics point of view. Designing such a course would imply a collaborative work between instructors and researchers to construct a conceptual framework of uncertainty adapted to introductory physics students with a hierarchy of the associated objectives. The design cycle should start based on these preliminary considerations, the implementation of the designed course would allow the collection of data in that authentic context that would then lead to refine the framework and revise the course. This design-based research will help instructors to become aware of the necessity of refining and aligning their goals and the intended goals, which will lead to better achievement within the new teaching activities. At a minimum, we recommend explicit teaching of the origin of errors (i.e., measurement and variability of physical phenomena) and explicit teaching of the role of uncertainties in physics research. Contrary to the actual rule in laboratory work, we suggest that instructors limit error computations performed by students to situations in which they are necessary to form a conclusion from the physics point of view.

References

Abd-El-Khalick, F. (2013). Teaching with and about nature of science, and science teacher knowledge domains. *Science & Education*, *22*(9), 2087–2107.

Allie, S., Buffler, A., Campbell, B., Lubben, F., Evangelinos, D., Psillos, D., & Valassiades, O. (2003). Teaching measurement in the introductory physics laboratory. *The Physics Teacher*, *41*(7), 394–401.

Bartos, S. A., & Lederman, N. G. (2014). Teachers' knowledge structures for nature of science and scientific inquiry: Conceptions and classroom practice. *Journal of Research in Science Teaching*, *51*(9), 1150–1184.

Bevington, P. R., & Robinson, D. K. (2003). *Data reduction and error analysis for the physical sciences* (3rd ed.). Boston, MA: McGraw-Hill.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101.

Buffler, A., Allie, S., & Lubben, F. (2001). The development of first year physics students' ideas about measurement in terms of point and set paradigms. *International Journal of Science Education*, *23*(11), 1137–1156.

Buffler, A., Lubben, F., & Ibrahim, B. (2009). The relationship between students' views of the nature of science and their views of the nature of scientific measurement. *International Journal of Science Education*, *31*(9), 1137–1156.

Day, J., & Bonn, D. (2011). Development of the concise data processing assessment. *Physical Review Special Topics - Physics Education Research*, *7*(1), 010114.

Deardorff, D. L. (2001). *Introductory physics students' treatment of measurement uncertainty* (Master's thesis, North Carolina State University). Retrieved from http://isobudgets.com/pdf/papers/10_DeardorffDissertation.pdf

Deng, F., Chen, D.-T., Tsai, C.-C., & Chai, C. S. (2011). Students' views of the nature of science: A critical review of research. *Science Education*, *95*(6), 961–999.

Duschl, R. A., & Grandy, R. (2013). Two views about explicitly teaching nature of science. *Science & Education*, *22*(9), 2109–2139.

Feynmann, R. (1965). The Feynmann's lecture on physics: Tome 1, mainly mechanics, radiation and heat. Boston, MA: Addison-Wesley.

Giere, R. N. (2004). How models are used to represent reality. *Philosophy of Science*, *71*(5), 742–752.

Group 1 of the Joint Committee for Guides in Metrology. (2008). *Evaluation of measurement data – Guide to the expression of uncertainty in measurement. (JCGM 100:2008).* Retrieved from http://www.bipm.org/en/publications/guides/gum.html

Irzik, G., & Nola, R. (2011). A family resemblance approach to the nature of science for science education. *Science & Education*, *20*(7-8), 591–607.

Lederman, N. G. (2007). Nature of science: Past, present, and future. In S. K. Abell & N. G. Lederman (Eds.), *Handbook of research on science education* (pp. 831-879). Mahwah, NJ: Lawrence Erlbaum Associates.

Lippmann-Kung, R. L. (2005). Teaching the concepts of measurement: An example of a concept-based laboratory course. *American Journal of Physics*, *73*(8), 771-777.

Salter, I. Y., & Atkins, L. J. (2014). What students say versus what they do regarding scientific inquiry. *Science Education*, *98*(1), 1-35.

Séré, M., Journeaux, R., & Larcher, C. (1993). Learning the statistical analysis of measurement errors. *International Journal of Science Education*, *15*(4), 427–438.

Sin, C. (2014) Epistemology, sociology, and learning and teaching in physics. *Science Education*, *98*(2), 342–365.

Strauss, A. L., & Corbin, J. M. (1998). Basics of qualitative research: Techniques and procedures for developing grounded theory (2nd ed.). Thousand Oaks, CA: SAGE.

Taylor, J. R. (1997). An introduction to error analysis: The study of uncertainties in physical measurements (2nd ed.). Sausalito, CA: University Science Books.

Urhahne, D., Kremer, K., & Mayer, J. (2011). Conceptions of the nature of science—Are they general or context specific? *International Journal of Science and Mathematics Education*, *9*(3), 707–730.

Welzel, M., Haller, K., Bandiera, M., Hammelev, D., Koumaras, P., Niedderer, H., … von Aufschnaiter, S. (1998). *Teachers' objectives for labwork: Research tool and cross country results* (Working Paper 6, European Commission Project PL 95-2005, Labwork in Science Education). Retrieved from University of Bremen, Institute of Science Education website: http://www.idn.uni-bremen.de/pubs/Niedderer/1998-LSE-WP6.pdf