The missing corporate investment, are low interest rate to blame ?

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Abstract

The aim of this paper is to understand the small effect of a long period of low real interest on corporate investment. I challenge the idea that corporate investment is sensitive to the wedge between the marginal product of capital and the user cost of capital. I provide a theoretical justification for an investment which is unresponsive to the wedge between marginal product of capital and real interest rate. I build an infinite horizon adverse selection model. Investment is constrained by cash flows. Next, I try to determinate if investment is a decreasing or an increasing function of interest rate once user cost channel is removed. I build a macroeconomic model where investment is a linear function of cash flows. I identify two channels by which real interest rate still affects investment: the entrepreneurial net worth channel and the precautionnary channel. The first one is well known and induces a negative relation between real interest rate and investment. The second is often neglected by the literature and can induce a positive relation. Under some calibration, investment response to real interest fall is negative. Then , I endogenize the credit constraint in the model. The response becomes unambiguously negative. I conclude by arguing that such a counterintuitive response should be taken seriously.
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Introduction

Between 2008 and 2016, the federal fund rate was at the zero lower bound. As inflation was moderate but positive, short term real interest rate have been negative for nearly a decade. In standard investment theory, the marginal product of capital should equalize the user cost of capital. Low real interest should have triggered a significant increase in desired capital stock and thus in corporate investment.

This rise did not happen. I represent the evolution of the net corporate investment in the US between 1960 and 2015 in figure 1. Net corporate investment in the US was negative for more than a year when the financial crisis occurs and remains very low until 2012. After 2012, it was more in line with historical level but remains lower than during previous recoveries in which user cost of capital was much higher. The only comparable period is the post dotcom recovery, a period already characterized by a long period of low real interest rate.

This disappointing performance of corporate investment has not been unnoticed by influential economists. Low investment in the US was the starting point of the secular stagnation literature initiated by Summers(2013) and continued for example by Eggertson et al.(2014) or by Gordon(2016). The literature goes well beyond corporate investment to adress productivity growth decline, hysteresis and long run effect of financial shocks. The aim of this paper is more modest. My goal is to understand the small effect of a long period of low real interest on corporate investment.

There is an obvious explanation for that puzzle. Low investment can be driven by low marginal product of capital. The marginal product of capital is not directly observable. But, we can look at some proxys. For example with Cobb Douglass production function, it will be equal to the net operating surplus over the capital stock. Capital stock is hard to measure accurately but assuming a stable capital income ratio (a reasonable assumption in the short/medium run), the evolution of the marginal product of capital can be approximated by changes in the ratio of the net oprating surplus over the value added of corporate sector. I plot the result in figure 2. The result harshly endorses the investment opportunity explanation. Our proxy is at his highest level since the sixties!

Our proxy may represent a bad measure for marginal product of capital. Monopolistic position, uncertainty can create a significant wedge between average capital product and marginal one. Prominent economists have suggested that market power have increased in the US in recent years. Rise in uncertainty have been obvious during the financial crisis (see Stock and Watson 2008 for example) and have certainly caused the spike in risk premiums on corporate bonds during the crisis. It is still unclear if these two factors can quantitatively explain the investment dynamic during the recovery. Competitive structure evolves slowly. Perhaps uncertainty remained high compare to previous episode but some market measure of uncertainty like risk premiums were only slightly went back close to normal levels shortly after the financial crisis (see credit spread figure). risk premimum did not compensate for the loose monetary policy. Yield on corporate bonds were actually at a low level (see corporate yield figure). Whereas, uncertainty and market power are legitimate and promising line of research, datas also suggest to explore alternative explanation.

I look at an alternative in this paper. I raise a simple question. Do investment necessarily increase when real interest falls ? I have two motives.

First, the implications of this mechanism are important. Aggregate demand is supposed to increase when real interest rate falls. Consumption is not very responsive to lower real rates in datas and there could be theoretical reasons for that. If corporate investment is not responsive either, aggregate demand becomes an ambiguous function of real rates. Aggregate demand can be locally increasing. An expansionnary monetary policy could locally have a contractionnary effect on output.

Second, empirical evidences that corporate investment reacts to the changes in interest rate are not overwhelming. The consensus in the literature is that estimating short run elasticity of investment to interest rate with aggregate datas do not provide any evidence backing a significant effect of interest rate on business investment (Blanchard 1986, Caballero 1994, Bernanke and Gertler 1994, Chirinko 1993, Sharpe and Suarez 2015). When measuring the different channels of monetary policy, Bernanke and Gertler (1994) shows that the response of business investment to a recessionnary FED fund rate shock is negative but small and lagging behind the large response of residential investment and nondurable consumption. The lag and the size of the response actually suggest more a side effect of the residential investment response through accelerator phenomena than a user cost effect. Estimates of long run elasticity (Caballero 1994, Schaller 2006) and studies using microeconomic datas (Cummins, Hassett and Hubbard 1994, Chirinko Fazzari and Meyer 1998, Mojon, Smets and Vermeulen 2001) provide better evidences for a user cost of capital effect on investment. However, it is still unclear if their findings imply a high elasticity of investment to interest rate as noted by Sharpe and Suarez. For example, the estimate of user cost elasticity in Cummins, Hassett and Hubbard (1994) is unchanged when real interest rate is replaced by a fixed discount rate in the measure of the user cost (Sharpe and Suarez 2015). Direct measure of investment sensitivity to interest rate are not better. For example Kothari and Warner(2015) finds that interest rate is unable to predict corporate investment whereas for example interest rate predicts noncorporate investment. Facing these very mixed empirical results, Sharpe and Suarez(2015) have proposed a completely different approach. Instead of using econometric techniques to identify correlation, they directly ask to CFO (Chief Financial Officers) in what extent their investment decision is sensitive to interest rate. Results are very instructive. 68 % of CFO says that their investment plans will remain unchanged if interest rate falls.

The contribution of this paper is twofold. Second, I challenge the idea that corporate investment depends on the wedge between the marginal product of capital and the user cost of capital. I build an infinite horizon adverse selection model where investment is unresponsive to this wedge. Investment becomes a linear relation of cash flows reinvested in the firm. This friction have three advantages. First, few other models are able to kill the user cost channel. Financial frictions are a natural candidate. If investment is constrained by borrowing limit, a deeper wedge does not affect the limit and investment remains unchanged. But all financial frictions does not kill this channel. The costly state verification model is the more popular friction in applied macroeconomic models. Entrepreneurs will equalize the marginal product of capital with the user cost of capital plus a cost of external finance which depends on risk and entrepreneurial net worth. A lower real interest rate still have a significant impact on investment. Collateral constraint are more interesting. The level of debt raised by firms is limited by its asset. The problem is that the constraint only binds for highly indebted and they are not so common in reality. If investment is directly constrained by cash flow, firm with low debt level may also be constrained. Second, investment is strongly correlated to cash flow in many empirical studies (see for example Fazzari, Hubbard and Petersen 1988). Whereas the interpration of this correlation is controversial, our model offer a clear theoretical explanation for this empirical result. Third, adverse selection is not very popular in macroeconomics but is important in corporate finance and is one of the foundation of the Myers and Maljuf (1984) Pecking order theory.

The second contribution of this paper is to determine if investment is a decreasing or an increasing function of interest rate once user cost channel is killed. Investment is determined by firm savings reinvested but interest rate may also affect the amount of firm savings reinvested. I identify two channels: the entrepreneurial net worth channel, the precautionnary channel. The first one induces a negative relationship between interest rate and investment. When interest rates are low, entrepreneurs keep a bigger part of capital income. Additionnal profits can be reinvested in the firm. This channel is well known in the literature has been emphasized by Bernanke(1994)and Bernanke Gertler and Gilchrist (1999). It may help to generate hump shaped response to monetary shock. The second one is ambiguous. The story is the following. Risk averse entrepreneurs chooses between a risky asset generating high return and a safe asset whose return is equal to real interest rate. When the real interest rate is low, the return of the safe asset is low or equivalently the price of future consumption good in bad state of the world (when the risky asset holds by entrepreneurs generates a low return) is high. If consumption good in bad and good states are complement for the entrepreneur, the income effect of this higher price dominates the substitution effect and the demand for the risky asset by the entrepreneur will be lower. It is equivalent to say that entrepreneurs reduce the share of profits which is reinvested in the firm or to say they increase their dividend. Because total investment is a function of these reinvested profits, it falls. Unlike the first one, this channel was largely neglected by the literature. Papers usually assume risk neutral entrepreneurs. It means future consumption good are perfect substitute across states of the world. Entrepreneurs will always choose to reinvest all their profit in the risky asset. This assumption is considered as a purely simplifying one. I disagree with that viewpoint. It is true that risk neutrality allows to solve the agency problem easily but it has strong implications for the effect of interest rate on investment and thus for economic policy. In my model, I study the economic dynamic when both of these channels are operating whereas the wedge channel plays no role. Fall in real interest rate have ambigous effect on investment. Under some calibration, the decline of investment can be quantitatively significant and persistent.

The paper is divided in three sections. In the first one, I show that the user cost channel is no longer effective when capital markets features adverse selection. In the second one, I analyze a model of investment dynamic when investment is constrained by firm cash flows. In the third one, I allow the constraint to be endogenous and to react to real interest rate.

1 How to kill the user cost channel ?

1.1 Corporate investment and interest rate through the user cost channel

The standard neoclassical investment thory implies that marginal product of capital should be equal to the user cost of capital. Without adjustment costs, with a production $y_t = A_t K_t^{\alpha}$ and with no depreciation, capital taxes or other distortions it means

$$\alpha A_t K_t^{\alpha-1} = rr_t = r_t$$

where rr is the rental rate of capital equal in that simplified case to the real interest rate r (capital stock is measured in term of consumption good). Investment over capital is equal to the growth of capital stock

$$\frac{I_t}{K_t} = \frac{K_{t+1} - K_t}{K_t} = \left(\frac{r_t}{r_{t+1}} \frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\alpha}} - 1$$

Consider a 1/3 value for α, a drop in real interest rate from 5 percent to 4 percent for a period of ten years. Let's also assume a ten percent rise in productivity. The capital stock should grow 44 percent rise in capital stock. With a capital income ratio of 4. The investment should be equal to 176 percent of annual income over ten years. The annual effort lies between 15 and 18 percent of annual GDP. During the slow recovery, the actual effort was around *six* percent.

In reality, you are likely to face adjustment cost and irreversibility issue. However, large adjustment costs over ten years period seem implausible. The drop in real interest rate looks more like the drop in rate of long term loans for which irreversibility seems less relevant. Explaining the behavior of corporate investment during the slow recovery with the standard framework is not impossible. But this basic computation suggests that it would require strong assumption. It seems interesting to consider model in which the user cost channel is completely ineffective. A possibility is to introduce financial frictions.

1.2 Financial frictions and user cost

The equality between marginal product and user cost of capital holds if entrepreneurs can borrow at will. If they face a debt limit, their capital stock could be constrained by the borrowing limit and unable to maximize their profit. Thus, financial frictions seem an obvious solution. Unfortunately it is not so simple. Popular financial frictions in macroeconomics like the costly state verification model of Townsend (1979) used by Bernanke, Getler and Gilchrist(1999) and then Christiano, Motto, Rostagno(2015) keeps the user cost channel.

Costly state verification Indeed, in the costly verification model, lenders can only oberve the firm outcome if they support an auditing cost. If they do not audit, entrepreneurs would have an incentive to default in any cases and to keep all the capital income. A simple strategy to avoid the issue is for lenders to audit only if the firm declares itself bankrupt. Because lenders should be indifferent between corporate loans and safe bonds, this auditing cost is actually supported by the borrower. It creates a wedge between the user cost of capital and the cost of external finance equal to the auditing cost times the default probability. The entrepreneur will equalize the marginal of capital with the cost of external finance which includes the user cost of capital and the external finance premium. Thus, effects of a fall in user cost triggered by lower real interest rate are similar to the standard model.

Simple collateral constraint Collateral constraint are an alternative to the costly state verification model. Firm debt level are constrained by the value of their assets $Q_{t+1}K_{t+1} > R_{t+1}B_t$. This constraint was used in Kiyotaki and Moore (1997) for example. If the constraint is binding, capital stock is determined by future asset prices, interest rates. I denote S_t entrepreneurial savings reinvested in the firm. $B_t = B_{t-1} + I_t - S_t$ and $K_{t+1} = K_t + I_t$.

The investment becomes

$$\frac{I_t}{K_t} = \frac{Q_{t+1}}{R_{t+1} - Q_{t+1}} - \frac{R_{t+1}}{R_{t+1} - Q_{t+1}}(\frac{B_{t-1}}{K_t} - \frac{S_t}{K_t})$$

I do not choose this model because it has significant drawbacks. First, Interest rate still have significant effect on investment through the first term at the right side of the equation. Second, a theory of asset price is needed. But the whole purpose of financial friction is to make capital harder to sell. Kiyotaki and Moore solves the problem by introducing an unconstrained inefficient sector but asset prices becomes a decreasing function of real interest rate reinforcing the effect of real interest rate. Third, average corporate debt is not very high in datas. Corporate debt represents between 40 and 45 percent of total assets. The level was quite stable over in the postwar era (see figure). Obviously, this measure is not uniform across firms. Most firms have a lower and reasonable debt level whereas firm in trouble are much higher one. It seems not plausible that credit constraint of normal firms are binding. They can integrate the probability that their constraint will be binding if they become distressed (see Khan and Thomas for an example). The outcome is very closed to a costly state verification framework with similar effects from user cost.

1.3 Adverse selection in infinite horizon and user cost

Usual frictions in macroeconomics does not offer good prospects for killing the user cost channel. I look at adverse selection. Adverse selection are common in finance literature, less common in macroeconomics. The seminal paper of Stigltz and Weiss (1981) and recent papers from Pablo Kurlat are two notable exceptions. I use a simple adverse selection problem in infinite horizon. I consider separating equilibrium. The advantage of this choice is to find a linear relation between investment and entrepreneurial savings reinvested in the firm. The total corporate investment becomes equal to entrepreneurial investment times a leverage term. The drawback is that the leverage term itself is not independant from real interest rate. I come back to that point later.

The market environment There are two type of firms Bad and Good. Each type of firm have the same AK production function $y_t^i = \pi k$. A good firm has a probability κ to become a bad firm at the next period. A bad firm continue to produce $(\pi + \mu)k_t^i$ but has a probability $1 - \lambda$ to be bankrupt at the next period. When it is bankrupt, a firm produces nothing and its remaining capital stock have zero value. Neither lender nor borrower recovers anything.

Investment Firms finance their investment using debt and internal funds. The type of the firm is private information. So, lenders face an adverse selection problem. We assume a separating equilibria will hold. In such equilibrium, good firm investment is limited by their internal funds times a certain leverage and bad firm are not willing to invest.

Loans Our assumption regarding loan arrnagement between lenders and borrowers differ from the literature. Usually, loans and interests are assumed to be repaid at the next period. I assume that loans have infinite maturity. Interest rates on past loans are fixed. Only interest rate on new loans may vary. Thus for an amount B_0 borrowed at period 0 , the firm should pay the lender r_0 at each of the following period. These assumption allows me to derive a very simple credit constraint. Allowing for shorter maturity are interesting but introduces complex issues about optimal maturity design which is not the core of this paper. However, it is worth noting it is better for firms to accumulate short term assets and long term liabilities (see appendix).

The incentive compatibility constraint For the bad firm, the value of distributing all its profit as dividend and not investing whereas paying a high interest rate on its debt should be superior to the value of paying a lower interest rate whereas investing the same fraction of its earnings than good firms. Let's denote $V_t^{l,l}$ the value of the first strategy for the bad firm. The associated program is

$$\begin{aligned}
\text{Max}_{d_t,S_t,E_t,I_t,b_{t+1},K_{t+1}} \quad & V_t^{l,l} d_t + \frac{1}{1+r_{t+1}} E_t V_{t+1}^{l,l}(K_{t+1}, b_{t+1}) && \text{(1a)}\\
w.r.t \quad & K_{t+1} = K_t + I_t && \text{(1b)}\\
& b_{t+1} = b_t + r_t^l E_t && \text{(1c)}\\
& \pi K_t - b_t = d_t + S_t && \text{(1d)}\\
& I_t = E_t + S_t && \text{(1e)}\\
& I_t = 0 && \text{(1f)}\\
& S_t = 0 && \text{(1g)}\\
& d_t \geq (1-s)(\pi K_t - b_t) && \text{(1h)}
\end{aligned}$$

Where K_t is the capital stock of the firm, b_t are interest repayments at the period t, d_t are dividends, I_t is the total investment of the firm. S_t is the part of firm earnings which is reinvested, r_t is the short term real interest rate, and E_t the amount of borrowing at the period t.

If the bad entrepreneurs reveal their type, they pay the long term real interest rate associated to high risk of bankruptcy r_t^l. The return of investment π is lower than this interest rate meaning there is no incentive to invest and to borrow. Once the firm has revealed her type, it is supposed to be common knowledge among market participants.

I introduce now the value of emulating the good firm $V_t^{l,h}$

$$\begin{aligned}
\text{Max}_{d_t,S_t,E_t,I_t,B_{t+1},b_{t+1},r_t^f,K_{t+1}} \quad & V_t^{l,h} d_t + \frac{1}{1+r_{t+1}} \lambda V_{t+1}^{l,l}(K_{t+1}, b_{t+1}) && \text{(2a)}\\
w.r.t \quad & K_{t+1} = K_t + I_t && \text{(2b)}\\
& b_{t+1} = b_t + r_t^h E_t && \text{(2c)}\\
& \pi K_t - b_t = d_t + S_t && \text{(2d)}\\
& I_t = E_t + S_t && \text{(2e)}\\
& d_t \geq (1-s)(\pi K_t - b_t) && \text{(2f)}
\end{aligned}$$

Bad firms does not reveal their type and pay the real interest rate associated with low risk. To confuse lenders, they have to invest and borrow like a good firm. Bad entrepreneurs always assume the separating equilibria will hold at the next period and they will have to reveal their true type.

I assume that both bad firms maximizes the discounted sum of distributed dividends. In other words investors are risk neutral and their elasticity of intertemporal substitution is infinite. Combined with the AK production function, it allows to derive a linear value function for $V_t^{l,l}$ and $V_{t+1}^{l,l}$.The solution method is straightforward. I guess that the value is a linear function of capital stock and interest repayments. $V_t^{l,l} = Q_t^{l,k} K_t - Q_t^{l,b} b_t$. Using undetermined coefficients method, I solve for $Q_t^{l,k}$ and $Q_t^{l,b}$.

The value function can be rewritten in the following way.

$$V_t^{l,l} = Q_t^{l,k} K_t - Q_t^{l,b} b_t = \pi K_t - b_t + \lambda \frac{1}{1+r_t} \left[Q_{t+1}^{l,k} K_t - Q_{t+1}^{l,b} b_t \right] \tag{3}$$

I deduce

$$Q_t^{l,k} = \pi + \lambda \frac{1}{1+r_{t+1}} Q_{t+1}^{l,k} \tag{4a}$$

$$Q_t^{l,b} = 1 + \lambda \frac{1}{1+r_{t+1}} Q_{t+1}^{l,b} \tag{4b}$$

The incentive compatibility constraint implies

$$V_t^{l,l} \geq V_t^{l,h} \tag{5}$$

I can now rewrite it

$$\pi K_t - b_t + \lambda \frac{1}{1+r_t} \left[Q_{t+1}^{l,k} K_t - Q_{t+1}^{l,b} b_t \right] \geq \pi K_t - b_t - S_t^h + \lambda \frac{1}{1+r_t} \left[Q_{t+1}^{l,k} (K_t + I_t^h) - Q_{t+1}^{l,b} (b_t + r_t^h (I_t^h - S_t^h)) \right]$$

By simplifying, I get

$$\frac{1+r_t}{\lambda} S_t^h \geq + \left[Q_{t+1}^{l,k} I_t^h - Q_{t+1}^{l,b} r_t^h (I_t^h - S_t^h) \right] \tag{6}$$

Using (4A) and (4B), I have $Q_t^{l,k} = \pi Q_t^{l,b}$. Moreover, lenders should be indifferent between lending to bad firms and buying short term bonds. Thus, I have the no arbitrage equation

$$r_t^l \lambda Q_{t+1}^{l,b} = 1 + r_{t+1}$$

I get the ICC under a very compact form

$$I_t^h = \frac{\pi - r_t^h}{r_t^l - r_t^h} S_t^h \tag{7}$$

Interpreting the ICC The explanation for this formula is simple. Imitating good firm has a benefit. Bad firms may borrow and repay r^h at the following period if they survive. With their borrowing, they invest and will get a return π at each period. Because $\pi \geq r^h$, it is profitable for them. But imitating good firms also has a cost. Firms need to reinvest a part of their benefit $\pi K_t - b_t$. This reinvestment generates π at each period but is risky. The expected return of this investment is lower than the return of safe bonds meaning it has a negative net present value. The incentive compatibility constraint makes sure that this cost exceeds the benefit.

What is observable by the lender and the possibility of pooling equilibria We assume that past choices and current financial structure of the firm are observable by the lender. So, in particular, b_t, B_t, and the history of K_t, I_t and S_t. It means that once a bad firm have revealed his type at period T, his type is known by lenders forever. Thus at each period, the only bad firms able to cheat in separating equilibria are the new ones, those which were good until then but faces a bad shock. The consequence is that at each period the pool of potentially cheating firms is pretty low. As a consequence, the interest rate in a pooling equilibria would be quite similar to good firm interest rate in a separating one if previous periods were characterized by separating equilibria.

We argue however, that separating equilibrium remains relevant. Investment is the signal in this game. there are three possible signals for two types, zero investment, investment constrained by saving and a maximal investment $\overline{I}$. For the good firm, choosing the constrained investment requires at least that the interest rate charged for the maximal investment to be high enough to compensate the higher profit generated by the bigger investment. If we are in a separating equilibria, the probability of sending the maximal investment signal is zero, so we do not know the response of the lender to that signal. This response will also be affected by risk aversion of lenders. We have assumed risk neutrality for simplicity but it seems not be the case in real world. Moreover, sending this signal have an additionnal cost for a good firm. At the next period, lenders will be uncertain about his past behavior and will not know for sure if it was a good firm in the past.

1.4 Leverage and interest rate

The leverage term $\frac{\pi - r_t^h}{r_t^l - r_t^h}$ is not invariant with respect to real interest rate. In fact, lower real rates makes the constraint *tighter*. Indeed, The opportunity cost of reinvesting benefits is reduced. Moreover, r^h falls with r. The long term profit generated by each unit of capiatl borrowed and reinvested increases. In a nutshell, lower rates reduce the cost and increase the benefit of the imitating strategy.

This effect is interesting and in a first version of the paper, I focus on it. I built a macroeconomic model and verify that real interest rate have a strong negative impact on investment. Nevertheless, there are empirical reasons for being skeptical. The negative effect on investment is rather large (see section 4). Positive effects of interest rate are not easy to identify in datas but negative effects cannot be seen either. On this ground, our adverse selection friction seems giving a counterfactual prediction. I do not think it is a definitive argument against adverse selection. First, all financial frictions have serious empirical weaknesses.For example, the costly state verification model does not allow for the introduction of equity. Our friction have allows to have a direct relation between investment and retained earnings, a regular feature of empirical work. Simple extension gives a similar friction for equity.

I choose to ignore that effect in first approximation. In the next section, I focus on the determinants of the entrepreneurial savings S_t. I develop a macroeconomic model where the leverage term $\frac{\pi - r_t^h}{r_t^l - r_t^h}$ is hold constant equal to ψ. Obviously, the true value of the ψ coefficient is probably not invariant to interest rate and more generally to systematic economic policy. However, many macroeconomic models share similar issues. For example Kiyotaki and Moore (2012) develop a model with an exogenous resaleability constraint on capital.I reintroduce an endogenous section in the last section of the model.

2 Does interest rate decrease or increase retained earnings ?

Suppose that corporate investment is not sensitive to user cost of capital. It does not mean corporate investment is not sensitive to real interest rates. The latter affects the former through several other channels.

2.1 How does interest rate affect retained earnings ?

2.1.1 The income channel

The income channel is the simplest and more intuitive one. When real rates are lower, interests repayment will go down, increasing progressively shareholders earnings. This effect was emphasized by Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999). Because the fall in interest repayment is not immediate, the effect on investment is delayed. This is why financial frictions model often generates hump shaped response of investment to monetary policy shocks (see BGG 1999).

2.1.2 The precautionnary channel

The precautionnary channel is the second effect. Once they have got earnings, entrepreneurs face a choice between reinvesting them in the firm with a high return but with risks of capital losses or accumulating safe assets. Most macroeconomic models implies that entrepreneurs chooses the first option. Ineed, they assume that firms maximizes the expected value of their profit which is equivalent to assume that entrepreneurs are risk neutral. One rationale for such assumption is that shares of a given firm are a small part of entrepreneurs asset. In my opinion this rationale is contradictory with financial frictions. Financial frictions are often presented a an inability for entrepreneurs to borrow as much as they want. But, they also mean entrepreneurs are unable to sell income stream generated by capital stock through debt or equity contract. If they are not able to sell this capital stock, they are unable to diversify perfectly their assets. The firm they manage or they own as a large shareholder will represent a large part of their wealth. This wealth is sensitive to capital losses from this firm. Thus, investors will likely adopt a precautionnary behavior.

A second rationale for this risk neutrality assumption is that it makes the model more tractable. I agree with that assessment but in my opinion, this is not only a simplifying assumption. It deeply modifies the effect of real interest rate on retained earnings and corporate investment. Lower real interest rate decreases the return of safe bonds and increases the return of reinvested earnings. If safe and risky assets are substitutes, it should increase the fraction of earnings which are retained in the firm. But, if they are complement, entrepreneurs will prefer reinvest less earnings and accumulate more safe assets.

2.1.3 The intertemporal channel

A third channel is the consumption savings decision of investors. I neglect this channel because it is probably ambiguous in our case. Indeed, I consider investors whose wealth mostly comes from capital income, either from a specific firm of from safe assets. If an investor only holds safe assets whose return is equal to the real interest rate, a rate fall would reduce capital income and reduce discount rate. The final effect on consumption is undetermined. If she holds both type of assets, rate fall reduces the income from safe assets but increases the income of risky assets. Because of these ambigous effects, I choose to abstract from that channel in the model.

2.2 The model

Framework In this section, I outline a macroeconomic model which allows to study the response of investment to a lower real interest rate.

In a nutshell, there is a continuum (of mass one) of firms. Each firms is hold by an investor which can either invest in the firm or in a safe asset. Investing in the firm allow the investor to borrow proportionnaly to his own investment. At the beginning of each period, a fraction γ of $t-1$ investors exits the investor pool. When an exit occurs, there are two possibilities. With a probability $1-\kappa$ their firm is still productive and they can sell their firm. With a probability κ, their firm is unproductive. Their capital stock produces zero forever. In that case, they default on their debt.

The production function of a firm $i$is

$$Y_t^i = (\pi + \mu)K_t^i \tag{8}$$

π are constant. Only investors can own the capital stock. The choice of an AK production function is unusual for a business cycles model. This assumption has several purpose. First,I want is to focus on the investment dynamic when real interest rates are low. It seems logical at least in first approximation to abstract from labor supply consideration. the assumption also makes sure that the credit constraint binds at the steady state. It also allows me to aggregate more easily. As I want to study both the precautionnary behavior of investors and the income dynamic of entrepreneurs, I use an overlapping generation structure. The linear production function allow to aggregate more easily the entrepreneurial sector. Our model is not the first financial friction model to use such production function. For example the farmer sector of the Kiyotaki and Moore model have also a linear production function relative to land.

capital accumulation equation is standard.

$$K^i_{t+1} = K^i_t + I^i_t) \tag{9}$$

There is no depreciation of capital here. Capital income πK_t should be interpreted as gross capital income minus depreciation.

At each period, investors which are not exiting consume a fraction δ of the firm earnings and a fraction λ of their safe assets. The remaining part is divided between reinvestment in the firm and safe assets. the budget constraint is

$$(1-\delta)(\pi K_t - r^h_t B_t) + (1-\lambda)A_t = S_t + q_t A_{t+1} \tag{10}$$

Leaving Investors consume all their net worth after exiting. Their capital stock K^i_t generates $(1+\phi)K^i_t$ consumption good in the good state of the world. In that state, they have to repay their debt. In both cases, they consume their safe assets They maximize the following utility function. Thus, investors maximizes

$$Max \;\; \frac{1-\kappa}{1-\rho}\left[(1+\phi)K^i_{t+1} - (1+r^h_{t+1})B^i_{t+1})\right]^{1-\rho} + \frac{1}{1-\rho}A^{1-\rho}_{i,t+1}$$

This function is not an expected utility. Instead of smoothing between state of nature, investors directly smooth between lotteries. I choose this unusual objetcive function because a more conventionnal isoelastic expected utility generates unexpected problem. In particular, investors diversify too much in steady state. The consequence is that for some calibration, investment is very sensitive to small changes in interest rate or accumulated assets. My function allow a smoother behavior whereas retaining key intuition. It is important to note that these issues are not specific to my model but would appear for any other financial friction. I solve the model with a conventionnal expected utility in appendix 4 and display results in appendix 5. These results are discussed below.

Investors accumulate capital by using their own internal fund S_t and by borrowing. At each period, the total investment in the firm is limited by the saving which are reinvested in the firm

$$I^i_t = \psi S^i_t \tag{11}$$

ψ is the leverage and it is constant as discussed above. I assume that the maturity of debt is infinite. At each period, investors repays the sum of interests over past debt. These interest repayments evolves according to the law of motion

$$B^i_{t+1} = B^i_t + r^h_t(I^i_t - S^i_t) \tag{12}$$

At each period t, a new generation of investors emerges and is endowed with $\theta\mu K_t$ consumption good. At the period t, these new investors divide their endowment between investment in a risky asset and long term bonds.

The investor program The investor maximizes the program

$$
\begin{aligned}
&Max \;\; \frac{(1-\kappa)}{1-\rho}\left[(1+\phi)K^i_{t+1} - (1+r^h_{t+1})B^i_{t+1})\right]^{1-\rho} + \frac{1}{1-\rho}A^{1-\rho}_{i,t+1} \\
&w.r.t \;\; (1-\delta)(\pi K^i_t - r^h_t B^i_t) + (1-\lambda)A^i_t = S_t + q_t A^i_{t+1} \\
&w.r.t \;\; K^i_{t+1} = K^i_t + I^i_t \\
&w.r.t \;\; B^i_{t+1} = B^i_t + I^i_t - S^i_t \\
&w.r.t \;\; I_t = \psi S_t
\end{aligned}
$$

The first order condition is

$$A^i_{t+1} = \frac{R_{t+1}}{(1-\kappa)L_t}^{\frac{1}{\rho}}\left[(1+\pi)K^i_{t+1} - (1+r^h_{t+1})B^i_{t+1}\right] \tag{13}$$

By combining with the budget constraint, I get expression of S_t and A_{t+1}

$$[1+F_t q_t L_t]S_t = (1-\delta)(\pi K_t - r^h_t B_t) - F_t q_t\left[(1+\phi)K_t - (1+r^h_{t+1})B_t\right] + [1-\lambda]A_t \tag{14a}$$

$$q_t A_{t+1} = (1-\delta)(\pi K_t - r^h_t B_t) + (1-\lambda)A_t - S_t \tag{14b}$$

where

$$
\begin{aligned}
L_t &= \psi(1+\phi) - (1+r^h_{t+1})(\psi-1) \\
F_t &= \frac{R_{t+1}}{(1-\kappa)L_t}^{\frac{1}{\rho}}
\end{aligned}
$$

Choice variable are linear function of individual income. This feature is important and allows a straightforward aggregation of the corporate sector.

I specify now the environment in which firm operates.

Real interest rate follows an exogenous process.

$$R_t = Rz_t \tag{15}$$

This exogenous process may represent the effect of central bank policy on real interest rate.

The price of the safe asset q_t is related to the interest rate. Indeed, savers should be indifferent between selling such a security to investors and borrowing.

$$q_t = \frac{1}{1+r_{t+1}} \tag{16}$$

I close the model with a good market clearing condition.

$$Y_t = C_t + I_t + \delta(1-\gamma)(\pi K_t - r^h_t B_t) + (\lambda(1-\gamma)+\gamma)A_t + \gamma(1-\kappa)[(1+\phi)K_t - (1+r^h_t)B_t] \tag{17}$$

The solution strategy My goal is to understand the relation between interest rate and investment and especially to understand if investment is a decreasing or an increasing funtion of interest rate. To do that, I adopt the following strategy. First I solve the model under the perfect foresight hypothesis. Current investment is a function of the expected path of the real interest rate. I want to study the response of investment to a shock on interest rate. Formally, the variable z_t takes a value different from one and then follows the law of motion

$$z_{t+1} = z_t^{\varphi} \tag{18}$$

The initial shock comes as a surprise but the following sequence of real interest rate is perfectly forecasted by investors.

The complete model First, I summarize real interest rate and asset prices equation which can be solved independantly

$$q_t = 1 + \frac{1}{R_{t+1}} q_{t+1} \tag{19a}$$

$$L_t = \pi\psi - r_t^h(\psi - 1) \tag{19b}$$

$$F_t = \frac{R_{t+1}}{(1-\kappa)L_t}^{\frac{1}{\rho}} \tag{19c}$$

$$(1 + r_t^h)(1 - \gamma\kappa) = 1 + r_{t+1} \tag{19d}$$

$$R_{t+1} = Rz_t \tag{19e}$$

$$z_{t+1} = z_t^{\varphi} \tag{19f}$$

$$\tag{19g}$$

At each period, several generation of investors coexists. Because policy are linear with respect to quantity variables, aggregation is straightforward.

$$A_{t+1} = \frac{1-\gamma}{q_t}\left[(1-\delta)(\pi K_t - r_t^h B_t) + (1-\lambda)A_t - S_t\right] + \theta\mu K_t \tag{20a}$$

$$K_{t+1} = (1-\gamma)(K_t + I_t \tag{20b}$$

$$B_{t+1} = (1-\gamma)(B_t + I_t - S_t) \tag{20c}$$

$$I_t = \psi S_t \tag{20d}$$

$$[1 + F_t q_t L_t]S_t = (1-\delta)(\pi K_t - r_t^h B_t) - F_t q_t\left[(1+\phi)K_t - (1 + r_{t+1}^h)B_t\right] + [1-\lambda]A_t \tag{20e}$$

I close the model with the market clearing condition for consumption good and the aggregate production function

$$Y_t = C_t + I_t + \delta(1-\gamma)(\pi K_t - r_t^h B_t) + (\lambda(1-\gamma) + \gamma)A_t + \gamma(1-\kappa)[(1+\phi)K_t - (1+r_t^h)B_t] \tag{21}$$

$$Y_t = (\pi + \mu)K_t \tag{22}$$

stationnarization Because of the AK production function, the model features endogenous growth. Equations for quantity variables have to be stationnarized. I divide all quantity variables at period t by aggregate capital stock at period t. The outcome is displayed in appendix 3.

Results I simulated the model. I displayed the result of the simulation of the expected utility model in annex E. Corporate investment represents nine percent of the value added at the steady state. The shock of real interest rate generates an increase in investment in impact but a long period of low investment thereafter. Indeed, the rise in price of safe assets combined to the initial increase generates a significant fall in safe assets on impact. This fall generates a negative income effect the next period, lowering both safe assets and retained earnings. The consequence is a long period of depressed investment.

3 A macroeconomic model with the leverage channel

In the previous section , I analyzed investment dynamics under an exogenous cash flows constraint. In this section, I allow the retained earnings leverage denoted ψ in the previous model to be time varying. The leverage ψ is given by equation (7).

Macroeconomic environment The model is a growth model with an AK production function. More precisely, we set

$$Y_t = (\pi + \mu)\, K_t \tag{23}$$

Where π and μ are constant. The national income is divided in two parts. πK_t goes to entrepreneurs and μK_t goes to workers. Workers do not appear in the production function. We make some implicit assumption there is some form of perfect complement production function. These feature is not very realistic for understanding the evolution of output at business cycles frequencies but I want to focus on the investment dynamic and not on output dynamic.

Workers and capitalist does not belong to the same household. Capitalists hold firms. The profit of each firm is divided between dividends consumed by capitalists and new investment. Workers consues and lends to capitalists. They optimize according to an Euler Equation.

Productive sector The productive sector contains several generation of firms. At each period, a fraction θ of the national income is devoted to create new firms. Because of some moral hazard problem (implicit in our model), these newborn firms cannot borrow at all. They can start to borrow at the second period of their existence. Each firm i produces at each period t an amount $(\pi + \mu)k_t^i$. Firms differ by their riskiness. there are two type of firms. A good firm has a probability κ to become a bad firm at the next period. A bad firm continue to produce $(\pi + \mu)k_t^i$ but has a probability $1 - \lambda$ to be bankrupt at the next period. When she is bankrupt, a firm produces nothing and its remaining capital stock have zero value.

Investment Firms finance their investment using debt and internal funds. The type of the firm is private information. So, lenders face an adverse selection problem. We assume a separating equilibria will hold. In such equilibrium, good firm investment is limited by their internal funds times a certain leverage and bad firm are not willing to invest. At macroeconomic level, capital accumulation only comes from good firms.

Loans Our assumption regarding loan arrnagement between lenders and borrowers differ from the literature. Usually, loans and interests are assumed to be repaid at the next period. This is impossible in our model, because total repayment enter into our incentive compatibility constraint. I assume that loans haver infinite maturity but are repaid at a rate ϕ. Interest rates are fixed. Thus for an amount B_0 borrowed at period 0 , the firm should pay the lender $r_0 + \phi$ at each period.

Constant dividend At each period, good entrepreneurs consumes a fixed fraction of their capital income

$$d_t^H = (1-\gamma)(\pi K_t^H - b_t^H) \tag{24}$$

A constant saving rate from entrepreneurs is not very satisfcatory because the rate is unlikely to be invariant to interest rate in reality but linear policy rules are convenient in an endogenous growth framework.

3.1 Aggregation

Because of the complete linearity of the problem, aggregation is straightforward. We denote by G investing firm and B non investing firm. At each period a mass $\frac{1}{\kappa}$ of new firms are created. New entrepreneurs uses only their own funds to invest. These funds are a constant fraction of the household income. teh transfer is denoted T

Investing firm At each period, a fraction κ of investing becomes non investing firm. The investment of investing firms comes from incumbent firms and new firms. the investment of new firms is cosntrained by moral hazard. So, it is equal to the transfer to new entrepreneurs which is exogenous. The relevant saving value is the aggregate saving of investing firms at the ned of the period excluding newly created ones. We get 6 equations describing the behavior of investing firms.

$$K_{t+1}^H = (1-\kappa)(K_t^H + I_t^G) + \frac{1}{\kappa}T_t \tag{25a}$$

$$B_{t+1}^H = (1-\kappa)(B_t^H + I_t^H - S_t^H) \tag{25b}$$

$$b_{t+1}^H = (1-\kappa)\left((1-\phi)b_t^H + r_t^h(I_t^H - S_t^H - \phi B_t^H)\right) \tag{25c}$$

$$\pi K_t^H - b_t^H = S_t^h + d_t^h \tag{25d}$$

$$d_t^H = (1-\gamma)(\pi K_t^H - b_t^H) \tag{25e}$$

$$I_t^H = \psi_t\left(S_t^h - \phi B_t\right) \tag{25f}$$

Where $\eta_{1,t}$, $\eta_{2,t}$, are derived from the ICC

Non investing firm At each period, a fraction λ non investing firm disappear whereas non investing firms inherits of a fraction κ of the debt, capital stock and repayments values from investing firms. It is the only factor in debt and capital accumulation by non investing firm. All their income is distributed in dividends.

$$K_{t+1}^L = \kappa K_t^H + (1-\lambda)K_t^L \tag{26a}$$

$$B_{t+1}^L = \kappa B_t^H + (1-\lambda)B_t^L \tag{26b}$$

$$b_{t+1}^L = (1-\lambda)b_t^L + r_t^l\phi(1-\lambda)B_t^L + \kappa b_t^H \tag{26c}$$

$$d_t^L = (1-\lambda)(\pi K_t^L - b_t^L) \tag{26d}$$

Computing market interest rate are more challenging.

The aggregate capital stock is

$$K_t = K_t^G + K_t^B \tag{27}$$

The production function is

$$Y_t = \mu K_t + \pi K_t \tag{28}$$

I need the market clearing condition on good market to close the model

$$Y_t = T_t + I_t^g + d_t^H + d_t^L + C_t \tag{29}$$

where T_t is the amount dedicated to newly created firm with $T_t = \theta Y_t$

Household sector We consider two possibility to model household consumption. In the first one, Consumers maximizes their discounted utility over an infinite horizon. Their income is equal to the labor income, the debt repayment minus the transfer to the new entrepreneurs.

$$E_t C_{t+1} = \beta X_t (1 + r_t) C_t \tag{30}$$

Stationnarization The AK model have no proper steady state. All quantities variables have an endogenous state. We divide all by the capital stock K_t. The growth rate g_{t+1} appear explicitly in the model.

There are multiple steady state with no analytical solution. Numerical results and simulation suggest the existence of a stable steady state featuring a credible growth rate.

The complete stationnarized model is given in appendix.

Linearization method Our quantity variables in the stationnarized models are ratio. Using the percentage deviation to steady state would give spurious results. So, we simply take the deviation from steady state ratio and percentage deviation for asset prices not close to one.

The complete linearized model is given in appendix.

steady state Steady state should be computed numerically. Moreover, computations suggest strongly the existence of two steady state with positive values for variables. Computations of eigenvalues with Dynare also suggest that one with reasonable values for growth and interest rate is stable. A saddle path converge to him whereas no stable equilibrium converges to the high growth steady state. Make numerical exercise.

3.2 Simulation

Calibration We have to calibrate the transition rate from good firm to bad firm κ, the bad firm survival rate λ, the rate of dividend distribution γ, the rate of time preference β, the capital income per unit of capital π, the labor income per unit of capital μ, and the fraction of income dedicated to newborn firm θ.

π and μ can be calibrated by using net operating surplus and wage compensations in the corporate sector. γ is calibrated to match both dividend distribution and corporate income tax. The two first parameter governs the risk premium and the leverage jointly with β and π. We can target default rate, risk premium between investment and speculative grade investment and leverage for a given value of π.

We also control our parameter gives a credible value for the growth rate of the economy. Because this growth rate should be computed through numerical values, targets are hard to match in a completely satisfactory way. Moreover, the leverage value is highly sensitive to small changes in parameters. However, our results are roughly robust to alternative calibration. The more sensitive aspect is the persistence of the initial investment fall.

Our growth rate target is the average real growth of net value added of corporate sector over the post Volcker era 1982-2014. We find an average value of 2.4 per year.

We calibrate γ at 0.67. This the average value over the post volcker era of the sum of dividends and taxes paid over net operating surplus plus interests and dividends received minus interests paid. Using net dividends and net interest paid only slightly modify the value. Without taxes, distribution of dividends is about 37 percent of the net operating surplus.

In average over the period $1998 - 2007$, the effective yield of AA corporate grade was five percent whereas the effective yield of B corporate grade was in average 10 percent.

Measuring the leverage on flows is non trivial. We can measure directly the relation between internal funds and investment, corporate firms saving and investment, the implicit value of interests paid over net operating surplus or debt over total assets. An alternative is to target the interests paid by firms in percent of their net operating surplus.

The drawback of the first approach is that internal savings and new debt issuance are roughly equivalent and each very close to productive investment. The difference is generated by the strong accumulation of financial assets by US firms. These liquid assets does not appear in our model. The treatment of these financial assets is quite complicated.

Results Results are displayed in annex G. The figure 1 shows the response of several variables to a patience shock. This is the response of stationnarized variable. For example for investment, this is the response of the investment to capital ratio. Both the numerator and the denominator are affected by the shock. Figure 2 and 3 displays the true response of investment and leverage in "gross" variables.

The patience shock generates a rise in desired savings and in the supply of loanable funds. The interest rate drops as it could be seen on the response of r. This fall of interest rate reduces the leverage of firms. The fall is quite large. Leverage falls from its steady state value of 3.4 to a value equal 1.6 (see figure 3). This fall of leverage reduces sharply the demand of funds by firms creating a new fall of interest rate. The effect of the initial patience shock on interest rate is strongly amplified at such point that consumption **increases** in response to the shock despite a **rise** in consumer patience. Patience indeed increases by one percent but the final fall fall of real interest rate is four percent making real interest rate close to bind the zero lower bound.
The fall of interest rate has two separate effects on investment. The fall in leverage triggers a large drop in total investment which is contemporaneous to the shock. The fall is also quite large. Net investment jump from 3 percent of GDP to 1.8 percent equivalent to a fourty percent fall in investment.

A second effect intervenes later. The fall of interest rate reduces the transfer from firm to lenders. After seven periods, interest repayments are down by 1.5 percent of GDP, inducing a large increase in internal funds availabale to firms whereas the leverage recovers from the initial shock. As a result, investment becomes slightly higher than the steady one after 10 periods before slowly going back to its normal value.

This second effect is the channel through interest rate increases investment in traditionnal financial frictions model like BGG or CMR. Lower real rates are not efficient because they reduce the cost of capital but because they transfer wealth from lenders to borrowers allowing for an higher borrower net worth. It should be noted it is much smaller due to the infinite maturity assumption. Indeed, interests on loans contracted before the shock are the same. Only, interests on newly contracted loans are lower. A one period loan assumption significantly magnifies this effect.

4 Conclusion(Provisionnal)

In this paper, I have shown that adverse selection in capital markets may undo the user cost of capital effect of real interest rate. I have built two models with this type of financial friction. In the first one, the constraint is exogenous but investors may choose between reinvesting earnings into firms or accumulating safe assets. Real interest rate determines the return of the safe asset. Effects depends on the calibration but lower rates have a depressive effect on investment if precuationnary behavior is important. This depressive effect is moderate but persistent. In the second model, the constraint is endogenous. Lower rates makes the constraint tighter. The negative impact on investment is substantial.

Like all theoretical models, these results are obtained under some simplifying assumption. I think however that the possibility of a negative response of investment to lower real rates should be taken seriously. The intuition for large positive effects of lower rates on corporate investment mostly relies on the user cost of capital channel. These effects are not apparent at least in macroeconomic datas. I offer a plausible explanation for that. If adverse selection is important on capital markets (and given what we know about corporate finance, the pecking order theory of external finance, the sensitivity of investment to cash flows, it seems hard to deny it), investment of **all** firms, not only the more indebted one, will be constrained by cash flows. This constraint will not necessarily bind which allow for the possibility that small movements in real interest affect user cost of capital and thus investment. But large fall of real interest rate are likely to make the constraint binding. This phenomena would be compatible with a stronger response of investment to interest rate hike than to interest rate fall. Such assymetric response is clearly apparent in the Sharpe and Suarez study on CFO behavior.

Once the constraint is binding, lower real interest may lower investment through at least two channels. First, lower return of safe assets may also push investors to allocate more of their earnings to these assets to limit the risk of their portfolio. Second, Lower rates may make the constraint tighter, reducing firm borrowing possibilities for a given amount of cash flows. It induces a strong negative response of investment. I am skeptical about the latter but in my opinion the former si a real possibility. It seems a little bit abstract but imagine the following story. Take a fund manager which divide its portfolio between safe assets and shares. At the end of the year, the safe asset generates zero return and shares generates for example 20 percent return. Will he reinforce its success by buying more shares or will he try to rebalance its portfolio by selling shares and buying safe assets ? I suspect the second strategy is more common. If fund managers sells shares, share value will fall and firm manager can react by distributing more dividends or retaining more cash to deter hostile offer. In both cases, there will be less cash flows available for investment.

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A Datas on corporate investment

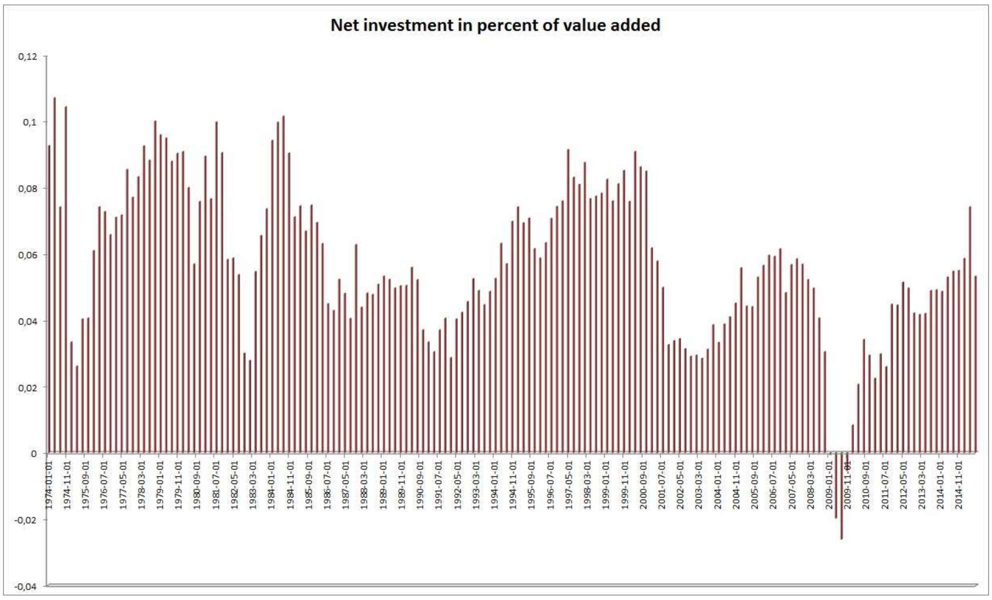


Figure 1: net corporate investment

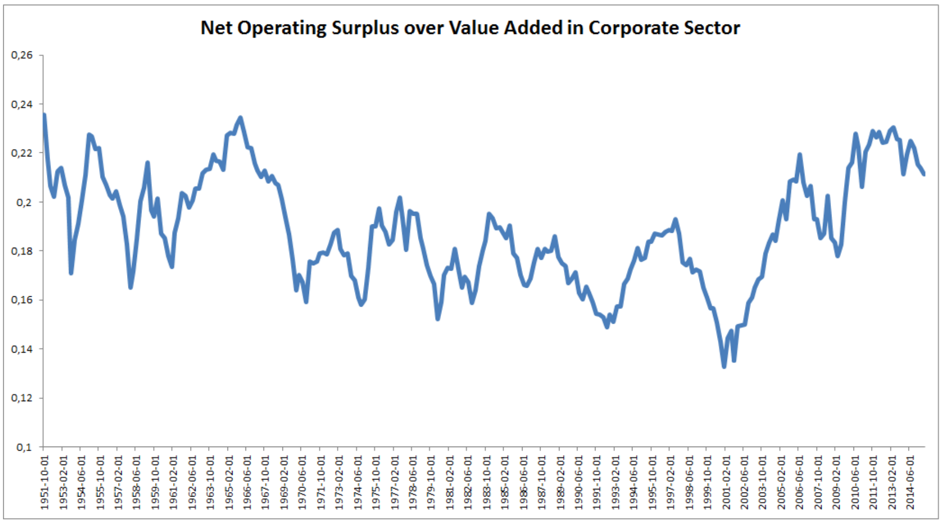


Figure 2: Net operating surplus over corporate value added

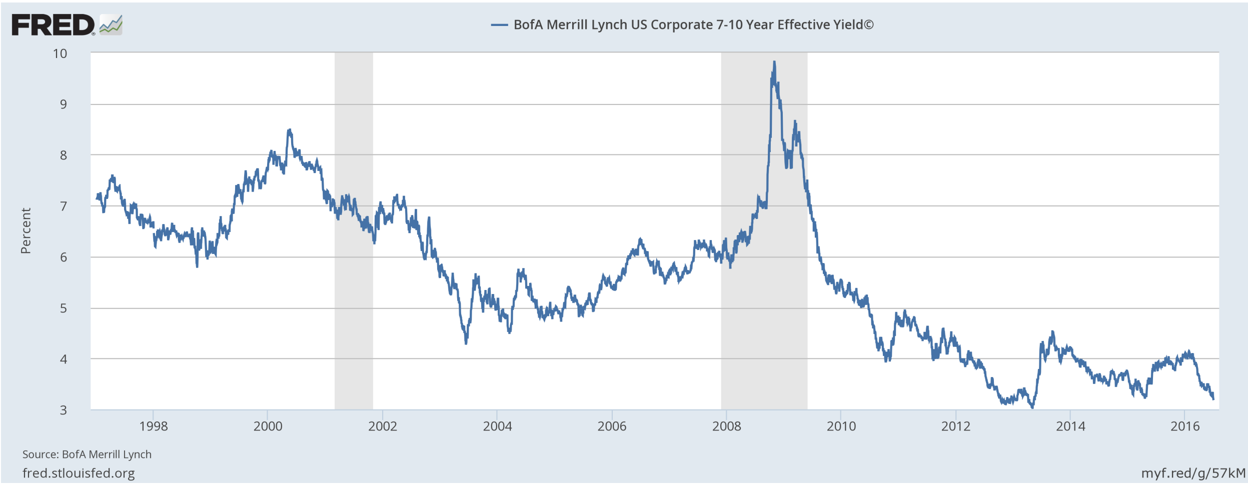


Figure 3: corporate yields

B Model 1

B.1 Optimization from entrepreneurs: OLG + indirect utility

Entrepreneurs maximizes the discounted sum of their utility. Utility depends from consumption.

$$
\begin{aligned}
&Max \;\; V_{t+1} = \frac{(1-\kappa)}{1-\rho}\left[(1+\phi)K_{t+1} - (1+r^h_{t+1})B_{t+1})\right]^{1-\rho} + \frac{1}{1-\rho}A^{1-\rho}_{t+1} && \text{(31a)}\\
&w.r.t \;\; (1-\delta)(\pi K_t - r^h_t B_t) + (1-\lambda)A_t = S_t + q_t A_{t+1} && \text{(31b)}\\
&w.r.t \;\; K_{t+1} = K_t + I_t && \text{(31c)}\\
&w.r.t \;\; B_{t+1} = B_t + I_t - S_t && \text{(31d)}\\
&w.r.t \;\; I_t = \psi S_t && \text{(31e)}
\end{aligned}
$$

Balanced growth requires $\rho = \alpha$

B.1.1 FOC

$$
\begin{aligned}
&w.r.t\ K \;\; (1-\kappa)(1+\phi)\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1}\right]^{-\rho} + \Lambda_{2,t} = 0 && \text{(32a)}\\
&w.r.t\ B \;\; -(1+r^h_{t+1})(1-\kappa)\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1})\right]^{-\rho} + \Lambda_{3,t} = 0 && \text{(32b)}\\
&w.r.t\ A \;\; A^{-\rho}_{t+1} - q_t\Lambda_{1,t} = 0 && \text{(32c)}\\
&w.r.t\ S \;\; -\Lambda_{1,t} - \psi\Lambda_{2,t} - (\psi-1)\Lambda_{3,t} = 0 && \text{(32d)}
\end{aligned}
$$

I denote $L_t = (1+\phi)\psi - (\psi-1)(1+r^h_{t+1})$

$$A_{t+1} = {\frac{R_{t+1}}{(1-\kappa)L_t}}^{\frac{1}{\rho}}\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1}\right] \tag{33a}$$

I denote

$$F_t = {\frac{R_{t+1}}{(1-\kappa)L_t}}^{\frac{1}{\rho}} \tag{34}$$

Reaaranging the equation, I find a relation between A_{t+1} and $(\pi K_{t+1} - B_{t+1})$

$$q_t A_{t+1} = F_t q_t \left[(1+\phi)K_{t+1} - (1+r^h_{t+1})B_{t+1}\right] \tag{35a}$$

It allows us to express A_{t+1} with respect to S_t and state variables

$$q_t A_{t+1} = F_t q_t((1+\phi)K_t - (1+r^h_{t+1})B_t + L_t S_t) \tag{36a}$$

I find S_t and A_{t+1}

$$[1 + F_t q_t L_t]S_t = (1-\delta)(\pi K_t - r^h_t B_t) - F_t q_t \left[(1+\phi)K_t - (1+r^h_{t+1})B_t\right] + [1-\lambda]A_t \tag{37a}$$

B.2 Aggregation

Aggregation is straightforward because the policy rule of the agent is linear with respect to individual state variables.

The trouble is to aggregate the corporate sector. Indeed, at each period a fraction γ of investors disappear whereas new investors bringing an amount of consumption good $\theta\mu K_t$ appears. Note that this consumption good is not completely invested. A fraction will be kept under the form of safe securities. The disparition of firms occur at the beggining of the period t. For capital stock, debt and safe assets, values at the end of the previous period and at the beginning of the current one are different. K_t is the value at the beginning of the period t and not at the end of the period $t-1$. Idem for B and A.

$$q_t = 1 + \frac{1}{R_{t+1}} q_{t+1} \tag{38a}$$
$$L_t = \pi\psi - r^h_t(\psi - 1) \tag{38b}$$
$$F_t = \frac{R_{t+1}}{(1-\kappa)L_t}^{\frac{1}{\rho}} \tag{38c}$$
$$(1 + r^h_t)(1 - \gamma\kappa) = 1 + r_{t+1} \tag{38d}$$
$$R_{t+1} = R z_t \tag{38e}$$
$$z_{t+1} = z_t^{\varphi} \tag{38f}$$
$$\tag{38g}$$

$$A_{t+1} = \frac{1-\gamma}{q_t}\left[(1-\delta)(\pi K_t - r^h_t B_t) + (1-\lambda)A_t - S_t\right] + \theta\mu K_t \tag{39a}$$
$$K_{t+1} = (1-\gamma)(K_t + I_t \tag{39b}$$
$$B_{t+1} = (1-\gamma)(B_t + I_t - S_t) \tag{39c}$$
$$I_t = \psi S_t \tag{39d}$$
$$[1 + F_t q_t L_t]S_t = (1-\delta)(\pi K_t - r^h_t B_t) - F_t q_t \left[(1+\phi)K_t - (1+r^h_{t+1})B_t\right] + [1-\lambda]A_t \tag{39e}$$

The market clearing condition for consumption good and the production function closes the model

$$Y_t = C_t + I_t + \delta(1-\gamma)(\pi K_t - r_t^h B_t) + (\lambda(1-\gamma)+\gamma)A_t + \gamma(1-\kappa)[(1+\phi)K_t - (1+r_t^h)B_t] \tag{40}$$

$$Y_t = (\pi+\mu)K_t \tag{41}$$

B.3 Stationnarization

There is endogenous growth in the model. So, I need to stationnarize quantity variables. I divide them by the value of the aggregate capital stock

$$(1+g_{t+1})a_{t+1} = \frac{1-\gamma}{q_t}\left[(1-\delta)(\pi - r_t^h b_t) + (1-\lambda)a_t - s_t\right] + \theta\mu \tag{42a}$$

$$1+g_{t+1} = (1-\gamma)(1+i_t \tag{42b}$$

$$(1+g_{t+1})b_{t+1} = (1-\gamma)(b_t + i_t - s_t) \tag{42c}$$

$$i_t = \psi s_t \tag{42d}$$

$$[1+F_t q_t L_t]s_t = (1-\delta)(\pi - r_t^h b_t) - F_t q_t\left[(1+\phi) - (1+r_{t+1}^h)b_t\right] + [1-\lambda]a_t \tag{42e}$$

Asset prices equation are unchanged

$$q_t = 1 + \frac{1}{R_{t+1}}q_{t+1} \tag{43a}$$

$$L_t = \pi\psi - r_t^h(\psi - 1) \tag{43b}$$

$$F_t = \frac{R_{t+1}}{(1-\kappa)L_t}^{\frac{1}{\rho}} \tag{43c}$$

$$(1+r_t^h)(1-\gamma\kappa) = 1 + r_{t+1} \tag{43d}$$

$$R_{t+1} = Rz_t \tag{43e}$$

$$z_{t+1} = z_t^{\varphi} \tag{43f}$$

$$\tag{43g}$$

$$y_t = c_t + i_t + \delta(1-\gamma)(\pi - r_t^h b_t) + (\lambda(1-\gamma)+\gamma)a_t + \gamma(1-\kappa)[(1+\phi) - (1+r_t^h)b_t] \tag{44}$$

$$y_t = \pi + \mu \tag{45}$$

C Simulation results : model 1

D Model 2:OLG + expected utility

D.1 Optimization from entrepreneurs

Entrepreneurs maximizes the discounted sum of their utility. Utility depends from consumption.

$$
\begin{aligned}
&Max \;\; V_{t+1} = \frac{(1-\kappa)}{1-\rho}\left[(1+\phi)K_{t+1} - (1+r^h_{t+1})B_{t+1}) + A_{t+1}\right]^{1-\rho} + \frac{\kappa}{1-\rho}A^{1-\rho}_{t+1} && (46a)\\
&w.r.t \;\; (1-\delta)(\pi K_t - r^h_t B_t) + (1-\lambda)A_t = S_t + q_t A_{t+1} && (46b)\\
&w.r.t \;\; K_{t+1} = K_t + I_t && (46c)\\
&w.r.t \;\; B_{t+1} = B_t + I_t - S_t && (46d)\\
&w.r.t \;\; I_t = \psi S_t && (46e)
\end{aligned}
$$

Balanced growth requires $\rho = \alpha$

D.1.1 FOC

$$
\begin{aligned}
&w.r.t\ K \;\; (1-\kappa)(1+\phi)\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1} + A_{t+1}\right]^{-\rho} + \Lambda_{2,t} = 0 && (47a)\\
&w.r.t\ B \;\; -(1+r^h_{t+1})(1-\kappa)\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1}) + A_{t+1}\right]^{-\rho} + \Lambda_{3,t} = 0 && (47b)\\
&w.r.t\ A \;\; (1-\kappa)\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1} + A_{t+1}\right]^{-\rho} + \kappa(R_{t+1}q_t A_{t+1})^{-\rho} - q_t\Lambda_{1,t} = 0 && (47c)\\
&w.r.t\ S \;\; -\Lambda_{1,t} - \psi\Lambda_{2,t} - (\psi-1)\Lambda_{3,t} = 0 && (47d)
\end{aligned}
$$

I denote $L_t = (1+\phi)\psi - (\psi-1)(1+r^h_{t+1})$

$$
A_{t+1} = \frac{\kappa R_{t+1}}{(1-\kappa)(L_t - R_{t+1})}^{\frac{1}{\rho}}\left[(1+\pi)K_{t+1} - (1+r^h_{t+1})B_{t+1} + A_{t+1}\right] \quad (48a)
$$

I denote

$$
F_t = \frac{\kappa R_{t+1}}{(1-\kappa)(L_t - R_{t+1})}^{\frac{1}{\rho}} \quad (49)
$$

Reaaranging the equation, I find a relation between A_{t+1} and $(\pi K_{t+1} - B_{t+1})$

$$
q_t A_{t+1} = \frac{F_t}{1-F_t} q_t\left[(1+\phi)K_{t+1} - (1+r^h_{t+1})B_{t+1}\right] \quad (50a)
$$

It allows us to express A_{t+1} with respect to S_t and state variables

$$
q_t A_{t+1} = \frac{F_t}{1-F_t} q_t((1+\phi)K_t - (1+r^h_{t+1})B_t + L_t S_t) \quad (51a)
$$

I find S_t and A_{t+1}

$$
[1 - F_t + F_t q_t L_t]S_t = [(1-\delta)(1-F_t)](\pi K_t - r^h_t D_t) - F_t q_t[(1+\phi)K_t - (1+r^h_{t+1})B_t] + [(1-\lambda)(1-F_t)]A_t \quad (52a)
$$

D.2 Aggregation

$$q_t = \frac{1}{R_{t+1}} \tag{53a}$$
$$L_t = \pi\psi - r_t^h(\psi - 1) \tag{53b}$$
$$F_t = \frac{\kappa R_{t+1}}{(1-\kappa)(L_t - R_{t+1})}^{\frac{1}{\rho}} \tag{53c}$$
$$(1 + r_{t+1}^h)(1 - \gamma\kappa) = 1 + r_{t+1} \tag{53d}$$
$$R_{t+1} = Rz_t \tag{53e}$$
$$z_{t+1} = z_t^{\varphi} \tag{53f}$$
$$\tag{53g}$$

$$A_{t+1} = \frac{1-\gamma}{q_t}\left[(1-\delta)(\pi K_t - r_t^h B_t) + (1-\lambda)A_t - S_t\right] + \theta\mu K_t \tag{54a}$$
$$K_{t+1} = (1-\gamma)(K_t + I_t \tag{54b}$$
$$B_{t+1} = (1-\gamma)(B_t + I_t - S_t) \tag{54c}$$
$$I_t = \psi S_t \tag{54d}$$
$$[1 - F_t + F_t q_t L_t]S_t = [(1-\delta)(1-F_t)](\pi K_t - r_t^h D_t) - F_t q_t[(1+\phi)K_t - (1 + r_{t+1}^h)B_t] + [(1-\lambda)(1-F_t)]A_t \tag{54e}$$

$$Y_t = C_t + I_t + \delta(1-\gamma)(\pi K_t - r_t^h B_t) + (\lambda(1-\gamma)+\gamma)A_t + \gamma(1-\kappa)[(1+\phi)K_t - (1 + r_t^h)B_t] \tag{55}$$
$$Y_t = (\pi + \mu)K_t \tag{56}$$

D.3 Stationnarization

$$(1 + g_{t+1})a_{t+1} = \frac{1-\gamma}{q_t}\left[(1-\delta)(\pi - r_t^h b_t) + (1-\lambda)a_t - s_t\right] + \theta\mu \tag{57a}$$
$$1 + g_{t+1} = (1-\gamma)(1 + i_t \tag{57b}$$
$$(1 + g_{t+1})b_{t+1} = (1-\gamma)(b_t + i_t - s_t) \tag{57c}$$
$$i_t = \psi s_t \tag{57d}$$
$$[1 - F_t + F_t q_t L_t]s_t = [(1-\delta)(1-F_t)](\pi - r_t^h b_t) - F_t q_t[(1+\phi) - (1 + r_{t+1}^h)b_t] + [(1-\lambda)(1-F_t)]a_t \tag{57e}$$

$$q_t = \frac{1}{R_{t+1}} \tag{58a}$$
$$L_t = \pi\psi - r_t^h(\psi - 1) \tag{58b}$$
$$F_t = \frac{\kappa R_{t+1}}{(1-\kappa)(L_t - R_{t+1})}^{\frac{1}{\rho}} \tag{58c}$$
$$(1 + r_{t+1}^h)(1 - \gamma\kappa) = 1 + r_{t+1} \tag{58d}$$
$$R_{t+1} = Rz_t \tag{58e}$$
$$z_{t+1} = z_t^{\varphi} \tag{58f}$$
$$\tag{58g}$$

$$y_t = c_t + i_t + \delta(1-\gamma)(\pi - r_t^h bB_t) + (\lambda(1-\gamma)+\gamma)a_t + \gamma(1-\kappa)[(1+\phi) - (1+r_t^h)b_t] \tag{59}$$

$$y_t = \pi + \mu) \tag{60}$$

E Simulation results : model 2

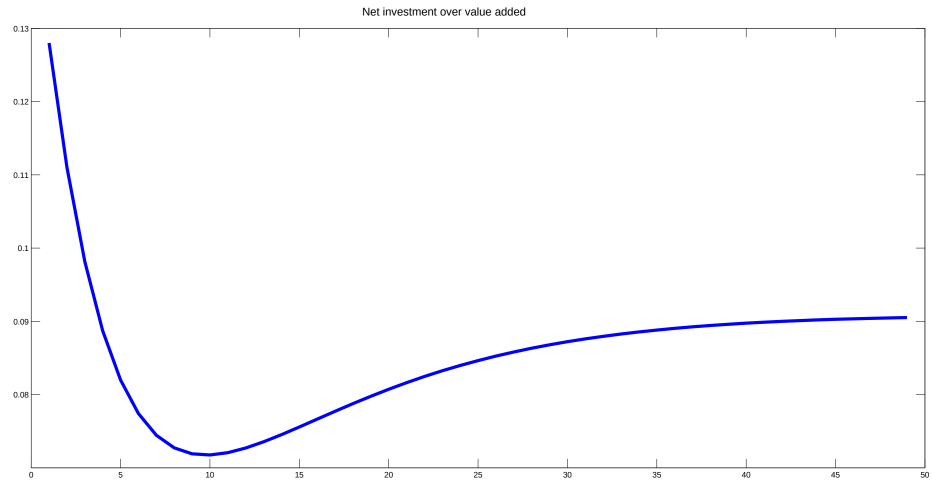


Figure 4: net corporate investment

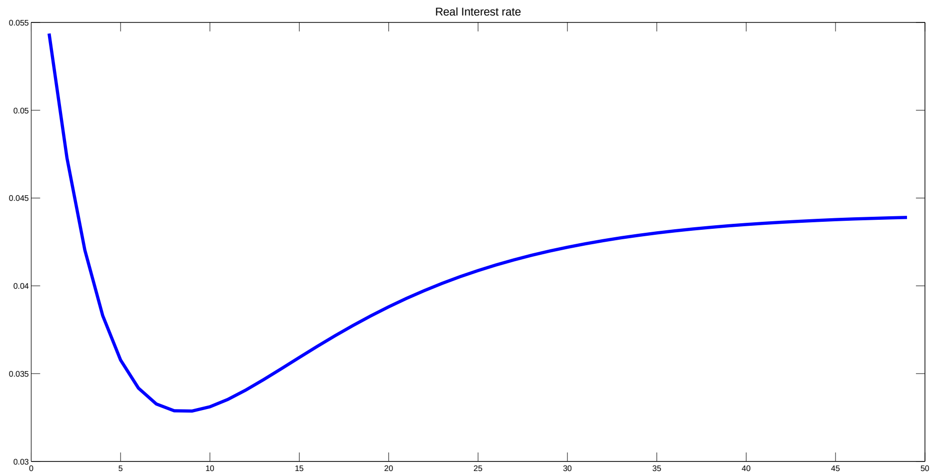


Figure 5: Real interest Rate

F Macroeconomic model with adverse selection

F.1 Asset Values

First, we compute implicit asset values for capital good of "good" type and "bad" type before computing the constraint. Under adverse selection, by definition the market price cannot be used. It creates diffculties. We need some assumptions to make the problem tractable for a simple macroeconomic model.

Because of the AK production function, discounted values of profits associated to a given firm will be linear function from the capital stock of the firm. The discounted value of interest and repayment flows will also be a linear function of repayment and interest at date t.

First, we compute the values of bad capital stock. If a separating equilibrium is reached, the bad firm does not invest. Capital stock and debt valued at historical cost remains stationnary. So, asset prices are quite easy to derive.

The challenge is to compute the value of bad firms under separating equilibria assumption So, the value function is defined

$$V_t^l = \pi K_t - b_t + \beta \lambda V_{t+1}^l \tag{61}$$

$$w.r.t \quad E_t = \phi B_t \tag{62}$$

$$b_{t+1} = (1-\phi) b_t + r^l E_t \tag{63}$$

The value of non investing firm in $t+1$ is a linear function of K_{t+1} , B_{t+1} and b_{t+1}

$$V_{t+1}^l(K_{t+1}, B_{t+1}, b_{t+1}) = q_{t+1}^{l,k} K_{t+1} - q_{t+1}^{l,B} B_{t+1} - q_{t+1}^{l,b} b_{t+1} \tag{64}$$

We use a simple method of undetermined coefficients to find asset prices with

$$Q_t^{l,k} = \pi + \frac{1}{1+r_{t+1}} \lambda Q_{t+1}^{l,k}$$

$$Q_t^{l,B} = \frac{1}{1+r_{t+1}} \lambda \left[r_t^l \phi Q_{t+1}^{l,b} + Q_{t+1}^{l,B} \right]$$

$$Q_t^{l,b} = 1 + \frac{1}{1+r_{t+1}} \lambda (1-\phi) Q_{t+1}^{l,b}$$

Computing the value of high quality capital stock. If we compute the value of a good firm, high capital/debt ratio means better income and thus a relaxed credit constraint later. This effect may make the investment a nonlinear function of capital stock. It raises significant issues for our model. Young firms will probably accumulate more capital in order to relax their future constraints. On another hand, we does not really integrate benefits of diversification for the entrepreneur. The initial lack of diversification means that an important part of its net worth belongs to a specific firm. Being risk averse, he has incentive to sell its capital stock. But, adverse selection implies that the only way to diversify is to pay more dividends. Facing this problems, we decide to abstract from these two problems completely.

Thus, we compute the value of a good asset as if it was sellable

We get

$$V_t^h = \pi K_t - b_t + \beta \left[(1-\kappa) V_{t+1}^h + \kappa V_{t+1}^l \right] \tag{65}$$

The value of investing firm in t is a linear function of K_t , B_t and b_t

$$V_t^h(K_t, B_t, b_t) = q_t^{h,k} K_t - q_t^{h,B} B_t - q_t^{h,b} b_t \tag{66}$$

We use a simple method of undetermined coefficients to find asset prices with

$$
\begin{aligned}
Q_t^{h,k} &= \pi + \frac{1}{1+r_{t+1}} \left[(1-\kappa) Q_{t+1}^{h,k} + \kappa Q_{t+1}^{l,k} \right] \\
Q_t^{h,B} &= \frac{1}{1+r_{t+1}} \left(r_t^l \phi \left[(1-\kappa) Q_{t+1}^{h,b} + \kappa Q_{t+1}^{l,b} \right] + \left[(1-\kappa) Q_{t+1}^{h,B} + \kappa Q_{t+1}^{l,B} \right] \right) \\
Q_t^{h,b} &= 1 + \frac{1}{1+r_{t+1}} (1-\phi) \left[(1-\kappa) Q_{t+1}^{h,b} + \kappa Q_{t+1}^{l,b} \right]
\end{aligned}
$$

We have to define two more things. First, we compute the equilibrium values for interest rate on good and bad firms

$$(r_t^l + \phi) \lambda Q_{t+1}^l = 1 + r_t \tag{67a}$$

$$(r_t^h + \phi)(\kappa Q_{t+1}^l + (1-\kappa) Q_{t+1}^h) = 1 + r_t \tag{67b}$$

Then, we define the expected asset values for good firms

$$V_{t+1}^{h,k} = (1-\kappa) Q_{t+1}^{h,k} + \kappa Q_{t+1}^{l,k} \tag{68a}$$

$$V_{t+1}^{h,B} = (1-\kappa) Q_{t+1}^{h,B} + \kappa Q_{t+1}^{l,B} \tag{68b}$$

$$V_{t+1}^{h,b} = (1-\kappa) Q_{t+1}^{h,b} + \kappa Q_{t+1}^{l,b} \tag{68c}$$

Stationnarized model

$$q_t^l = 1 + \frac{1}{1+r_{t+1}} \lambda q_{t+1}^l \tag{69a}$$

$$q_t^h = 1 + \frac{1}{1+r_{t+1}} \left[(1-\kappa) q_{t+1}^h + \kappa q_{t+1}^l \right] \tag{69b}$$

$$\lambda(r_t^l) q_{t+1}^l = 1 + r_{t+1} \tag{70a}$$

$$r_t^h((1-\kappa) q_{t+1}^h + \kappa q_{t+1}^l) = 1 + r_{t+1} \tag{70b}$$

$$G_{t+1} K_{t+1}^H = (1-\kappa)(K_t^H + I_t^H + T_t) \tag{71a}$$

$$G_{t+1} b_{t+1}^H = (1-\kappa) b_t^H + (1-\kappa) r_t^h (I_t^H - S_t^H) \tag{71b}$$

$$\pi K_t^H - b_t^H = S_t^h + d_t^h \tag{71c}$$

$$d_t^H = \gamma(\pi_t K_t^H - b_t^H) \tag{71d}$$

$$(r_t^l - r_t^h) S_t^h = I_t^h (\pi - r_t^h) \tag{71e}$$

$$G_{t+1} K_{t+1}^L = \kappa(K_t^H + I_t^H + T_t) + \lambda K_t^L \tag{72a}$$

$$G_{t+1} b_{t+1}^L = \lambda b_t^L (1-\phi) + \kappa b_t^H + \kappa r_t^h (I_t^H - S_t^H) \tag{72b}$$

$$d_t^L = \pi K_t^L - b_t^L \tag{72c}$$

$$\pi_t = \pi \tag{73}$$

$$y_t = \pi + \mu \tag{74}$$

where μ is the workers part.

$$1 = K_t^H + K_t^B \tag{75}$$

$$G_{t+1}C_{t+1} = \beta(1 + r_t)C_t \tag{76}$$

$$T_t = \theta y_t \tag{77}$$

$$y_t = T_t + I_t^g + d_t^H + d_t^L + C_t \tag{78}$$

Linearized model

As we have ratio, it will be hard to linearize using percentage deviation. So we use absolute deviation from non asset price equation. So, we denote $k_t^h = K_t^h - K^h$ where K^h is the steady state ratio. Asset prices are not ratio and can be far away from one so, we take the percentage deviation We linearize in the case $\phi = 0$ because of the analytical simplicity

$$\tilde{r}_{t+1} + (1 + r)\tilde{q}_t^l = \frac{1}{q^l}\tilde{r}_{t+1} + \lambda \tilde{q}_{t+1}^l \tag{79a}$$

$$\tilde{r}_{t+1} + (1 + r)\tilde{q}_t^h = \frac{1}{q^h}\tilde{r}_{t+1} + (1 - \kappa)\tilde{q}_{t+1}^h + \kappa\frac{q^l}{q^h}\tilde{q}_{t+1}^l \tag{79b}$$

$$\lambda \tilde{r}_t^l + \lambda r^l \tilde{q}_{t+1}^l = \frac{1}{q^l}\tilde{r}_{t+1} \tag{80a}$$

$$(1 - \kappa + \kappa\frac{q^l}{q^h})\tilde{r}_t^h + (1 - \kappa)r^h \tilde{q}_{t+1}^h + \kappa r^h \frac{q^l}{q^h}\tilde{q}_{t+1}^l = \frac{1}{q_h}\tilde{r}_{t+1} \tag{80b}$$

$$(1 + g)k_{t+1}^h + K^h g_{t+1} = (1 - \kappa)(k_t^h + i_t^h + \tilde{T}_t) \tag{81a}$$

$$(1 + g)\tilde{b}_{t+1}^h + b^h g_{t+1} = (1 - \kappa)b_t^h + (1 - \kappa)r^h(i_t^h - s_t^h) + (1 - \kappa)(I^h - S^h)r_t^h \tag{81b}$$

$$\pi k_t^h - b_t^h = s_t^h + d_t^h \tag{81c}$$

$$d_t^H = \gamma(\pi k_t^h - b_t^h) \tag{81d}$$

$$\psi_t S^h + s_t^h \psi = i_t^h \tag{81e}$$

$$(1 + g)k_{t+1}^l + K^h g_{t+1} = \kappa(k_t^h + i_t^h + \tilde{T}_t) + \lambda k_t^l \tag{82a}$$

$$(1 + g)\tilde{b}_{t+1}^l + b^l g_{t+1} = \lambda \tilde{b}_t^L + \kappa \tilde{b}_t^h + \kappa r^h(i_t^H - s_t^H + \kappa(I^h - S^h)r_t^h) \tag{82b}$$

$$d_t^L = \pi K_t^L - b_t^L \tag{82c}$$

$$\psi_t = \frac{1}{\pi - r^h}\left(r_t^l - r_t^h + \psi r_t^h\right) \tag{83}$$

$$\tilde{\pi}_t = 0 \tag{84}$$

$$\tilde{y}_t = 0 \tag{85}$$

where μ is the workers part.

$$0 = k_t^h + k_t^l \tag{86}$$

$$(1+g)c_{t+1} + Cg_{t+1} = \beta C \tilde{r_{t+1}} + \beta(1+r)c_t + \beta C(1+r)x_t \tag{87}$$

$$\tilde{T}_t = 0 \tag{88}$$

$$0 = i_t^g + d_t^h + d_t^l + c_t \tag{89}$$

G simulation results for the macroeconomic model with adverse selection

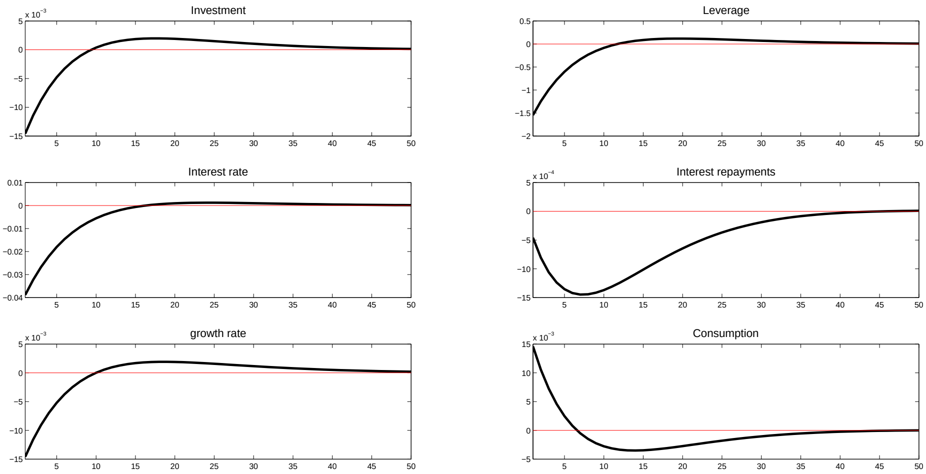
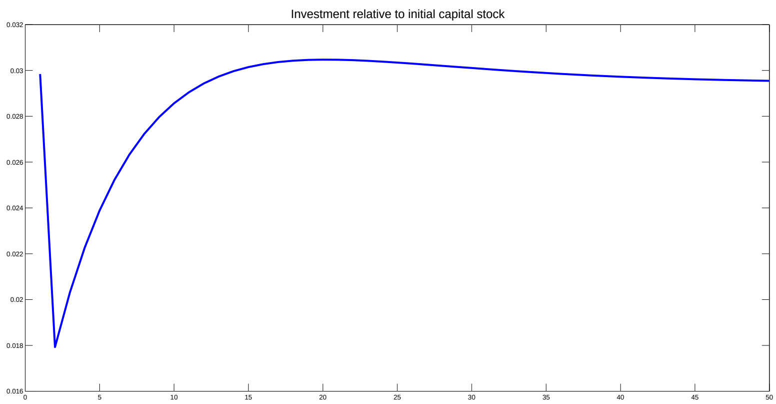
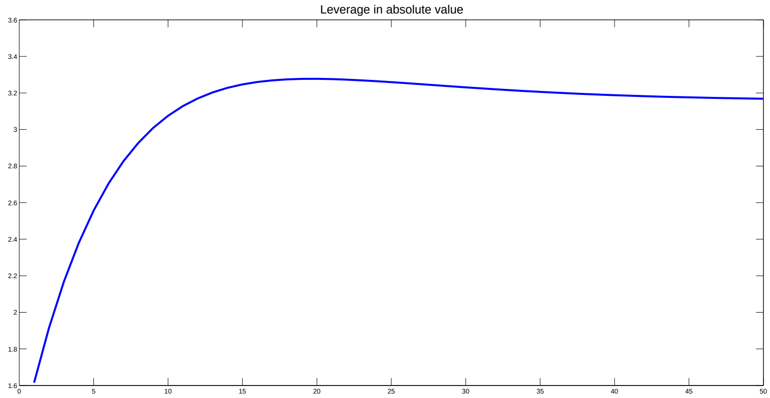


Figure 6: IRF





H Adverse selection problem: a three period analysis

H.1 Equity Contract

We consider the case of a firm facing an investment opportunity. This investment will provide a return $\overline{R}$ The firm already holds a certain amount of capital K which will provide a return R. There are two types of firm. Good firm holds productive capital with return R^h and have a valuable investment project providing a return $\overline{R}^h$. We assume that $R^h > \overline{R}^h$. Bad firms holds a depreciated capital stock which generates a low return R^l. they have also an opportunity of investment giving a return $\overline{R}^l$. We assume $\overline{R}^l > R^l$.

The equity of the firm is divided in two part. The first E is hold by a large stockholder which has the ability to control the management of the firm and which holds private information about the type of the firm. The second is hold by uninformed investor, either the public or some sort of mutual fund. These mutual funds may either invest in firm equities or in an alternative asset providing an interest rate R. We have $R^h > \overline{R}^h > R > \overline{R}^l > R^l$

At the beginning of the period, the large stockholder holds firm equity E and another liquid asset (cash for example) denoted M. She chooses the scale of firm investment I. He uses a part of his cash-holdings denoted S to finance the investment, the remaining being kept under the form of cash. The stockholder is supposed to understand the impact of her action on the price of its specific stock.

The problem we expose is similar to Myers and Maljuf (1984) but by contrast to their approach, we focus on separating equilibrium. The stockholder of good firm maximizes under the constraint that her chosen investment and leverage will not be adopted by the stockholder of bad firm.

Optimal allocation First, the type of the firm is supposed to be observable by the public.

The large stockholder of the good firm maximize its future wealth. Her cash flow is divided between the investment on her own firm and cash. She understands that the price of her stock is determined by the arbitrage opportunities of external investor which should be indifferent between buying stock and holding cash.

$$\text{Max}_{I^h,S^h,M'} \quad \left(R^h K + \overline{R}^h I_h\right)\left(\frac{Q^h E + S^h}{Q^h K + I_h}\right) + RM' \tag{90a}$$

$$M - S^h = M' \tag{90b}$$

$$\frac{R^h K + \overline{R}^h I_h}{Q^h K + I} = R \tag{90c}$$

$$S^h \geq 0 \tag{90d}$$

$$M' \geq 0 \tag{90e}$$

Equation (1d) is the arbitrage equation between investing in the shares of the firm and holding cash. Equation (1e) and (1f) implies that debt contract is not available to agents.

The program of the bad firm is the same. Using the price equation, the objective function can be rewritten in a more friendly way.

The price of equity is

$$Q^h = \frac{R^h K + I_h(\overline{R}^h - R)}{RK} \tag{91}$$

Reintroducing in (1a)

$$\left(R^h K + \overline{R}^h I_h\right)\left(\frac{Q^h E + S^h}{Q^h K + I_h}\right) + RM' = R^h E + I\frac{E}{K}(\overline{R}^h - R) + RS^h + RM' \tag{92}$$

The return of the saving invested in the firm and cas return is the same. The result is similar for large stockholders of bad firm. So, without private information the investor is indifferent between investing in the firm and holding cash.

The return of investment of the good firm is $I\frac{E}{K}(\overline{R}^h - R)$. Because $R^h > R$, the good firm will invest as much as possible. Conversely, investment in the bad firm generate a negative return. So, it is equal to zero.

Private Information Now, we suppose that mutual funds cannot observe the type of the firm, but they can observe the amount of firm equity hold by the large stockholder and the amount invested by the firm. The large stockholder of the good firm has the same maximizing goals but, now she should set the firm investment level and the reinvested saving in order to deter large stockholder of bad firms to imitate her choice and getting higher price for her newly issued equity.

The program is similar to (1)-(3) but with the additional incentive compatibility constraint

$$\left(R^l K + \overline{R}^l I_l\right)\left(\frac{Q^l E + S^l}{Q^l K + I_l}\right) + R(S^h - S^l) \geq \left(R^l K + \overline{R}^l I_h\right)\left(\frac{Q^h E + S^h}{Q^h K + I_h}\right) \tag{93}$$

The program of the bad firm at the separating equilibrium and with no private information is the same. So, we know that the bad firm will not invest at all. $I^l = 0$, $S^l = 0$. Moreover, the equity

The ICC becomes

$$R^l E + RS^h \geq \left(R^l K + \overline{R}^l I_h\right)\left(\frac{Q^h E + S^h}{Q^h K + I_h}\right) \tag{94}$$

$$\Rightarrow EI^h\left(R^l - \overline{R}^l Q^h\right) + KS^h\left(RQ^h - R^l\right) + I^h S^h\left(R - \overline{R}^l\right) \geq 0 \tag{95}$$

We define now the inverse leverage on investment denoted ψ. ψ is the amount of investment financed by the large stockholder saving, $\psi^h \equiv \frac{S^h}{I_h}$. We also define other variable relative to the capital stock, $i^h \equiv \frac{I^h}{K}$,$e \equiv \frac{E}{K}$,$m \equiv \frac{M}{K}$.

The program of the good firm can be rewritten

$$\text{Max}_{i^h,\psi^h,M'} \quad R^h e + i^h e(\overline{R}^h - R) + R\psi^h i^h + Rm' \tag{96a}$$

$$w.r.t \quad m - \psi^h i^h = m' \tag{96b}$$

$$Q^h = \frac{R^h}{R} + i^h \frac{\overline{R}^h - R}{R} \tag{96c}$$

$$e\left(R^l - \overline{R}^l Q^h\right) + \psi^h \left(RQ^h - R^l\right) + \psi^h i^h \left(R - \overline{R}^l\right) \geq 0 \tag{96d}$$

$$\psi^h \geq 0 \tag{96e}$$

$$i^h \geq 0 \tag{96f}$$

$$M' \geq 0 \tag{96g}$$

By combining (12C) and (12D), we can express the inverse of the investment leverage with respect to the investment level and the past leverage on total asset

$$\psi^h \geq \frac{e}{R} \left[\frac{\overline{R}^l R^h - R^l R + i^h(\overline{R}^h - R)\overline{R}^l}{R^h - R^l + i^h(\overline{R}^h - \overline{R}^l)} \right] \tag{97}$$

The inverse leverage for new investment is an increasing linear function of past inverse leverage (with a coefficient inferior to one) and an ambiguous function

Proposition 1 *Given the maximal value for the leverage of new investment ψ^h, The good investor invest its cash in the project $S^h = M$ and invest $I^h = \frac{S^h}{\psi^h}$*

H.2 Debt Contract

Advantage of debt The adverse selection problem of the previous section can be seriously relaxed if we introduce debt contract. The intuition is straightforward. If the firm is bad, either they will repay R which is superior to the return of the new investment, or they will not be able to repay the loan, so the entire return of capital stock and new investment will go to the lender. There is no incentive for the bad type to invest using debt contract in that environment. Only an equity contract allow them to transfer their bad capital stock to an outside investor. By contrast, the good firm has the same incentive to invest. Thus, issuing debt is an alternative way to signal the firm is good. In this framework, debt is preferred to equity.

Consider the program (1) without the two last constraints, thus making debt contracts available but with the ICC (4). S_h is no longer limited by the amount of cash initially hold by the large stockholder. She will adjust S_h to the desired level of investment by issuing safe debt. The ICC continue to exist but does not bind. The debt strategy is also available to the bad stockholder, but she will have to repay R, whereas receiving $\overline{R^l}$ at the second period. Thus, this strategy is not interesting for him.

Cost of debt in a stochastic framework In the framework of the first section, However, is only true if only the type of the firm is the only source of uncertainty. Let's assume that there is not only difference in the average return of capital stock but also in the risk of very low return.

The problem have been analyzed for a long time, especially by Stiglitz and Weiss (1981). There are however some differences with our approach. Stiglitz and Weiss study risk difference between new investment project with a similar NPV. We consider a firm with an initial level of capital and debt and difference in both average return and risk. Our goal is to show that leverage constraint is also about new investment and not total assets.

We consider two type of firms. Both type start with an amount of capital K and a level of debt B. A fraction ϕ of this debt should be repay at the investment period. The value of ϕ have a considerable importance for the result.

A good firm will get a rturn R^h with probability one. A bad firm will get a return R^h with probability $\lambda < 1$. Lenders can observe R^h but not λ. Both lenders and borrowers are assumed to be risk neutral. The assumption bias upward the leverage on new investment. For simplicity, only debt contract is available, so we do not really study the repartition between debt and equity. However, it should be kept in mind that an equity contract face the same problem that occurs in the previous section because the average return of bad firm is lower than those of good firm[1].

The program of the good firm is

$$\begin{aligned}
\text{Max}_{I^h,S^h,M'} \quad & R^h(K+I) + RM' - \phi B - (1+r^h)B' && \text{(98a)}\\
w.r.t \quad & M = M' + S^h && \text{(98b)}\\
& I^h = S^h - (1-\phi)B + B^h && \text{(98c)}\\
& \lambda[R^h(K+I^l) - (1+r^l)B^l - \phi B] + R(S^h - S^l) \geq \lambda[R^h(K+I^h) - (1+r^h)B^h - \phi B] && \text{(98d)}\\
& S^h \geq 0 && \text{(98e)}\\
& I^h \geq 0 && \text{(98f)}\\
& M' \geq 0 && \text{(98g)}
\end{aligned}$$

In the separating equilibria, the bad firm invest zero, borrow only to refinance the existing debt $(1-\phi)B$ and keep the same amount of cash $M = M'$ (More precisely, it is indifferent between keeping the same amount in cash and using cash for repaying the debt but I consider this particular case for convenience). Under separating equilibria, the lender should be indifferent between lending to the bad firm and holding cas, so

$$(1+r^l) = \frac{R}{\lambda} \tag{99}$$

and

$$(1+r^h) = R \tag{100}$$

The ICC can be rewritten

$$S^h \geq B(1-\phi) + \frac{\lambda(R^h - R)}{R - \lambda R} I^h \tag{101}$$

With change in variables similar to the previous section, we get

$$\psi^h \geq \frac{b}{i^h}(1-\phi) + \frac{\lambda(R^h - R)}{R - \lambda R} \tag{102}$$

Here the level of new investment inverse leverage is an increasing function of past debt level (unlike in the equity case where a larger share of external equity allow a looser constraint), a decreasing function of the investment

[1] We will also need a slightly different return between bad firm capital stock and new investment which is not present here. However, introduce such a difference would not alter substantially the result. It will actually lead to a tighter constraint

plus a constant. The assymetric impact of debt and equity on the leverage value is a very interesting feature. It would provide an explanation for the choice between debt and equity for firm finance.

The main difference with stiglitz and weiss is the role played by past debt. If the whole debt should be refinance in the short run, a large stockholder should invest the whole value of the debt into the firm to be credible.

We will mainly use this debt problem in further sections concerned by macroeconomic problems. The debt problem is much more simple. We verify in appendix the effects of the equity problem

The problem exposed in the previous section was a firm level problem. Before adressing macroeconomic considerations, we highlight some key points to understand the nature of the friction and under what condition it is relevant.