Luminous Inception System Draft Translation with Apparatus

Daniel P. Morgan
ERC project SAW (UMR 7219), CNRS - Université Paris Diderot

March 15, 2014

Luminous Inception system (*Jingchu li* 景初曆)
by
Yang Wei 楊偉, 237 CE[1]

Contents

[1]Sources: *Song shu* 宋書 (Zhonghua shuju ed.), 12.232–264; *Jin shu* 晉書 (Zhonghua shuju ed.), 18.535–562.

1 Preface

楊偉表曰：

Yang Wei 楊偉 petitioned:

「臣攬載籍，斷考曆數，時以紀農，月以紀事，其所由來，遐而尚矣。乃自少昊，則玄鳥司分，顓頊帝嚳，則重、黎司天，唐帝、虞舜則羲、和掌日。三代因之，則世有日官。日官司曆，則頒之諸侯，諸侯受之，則頒于境內。夏后之代，羲、和湎淫，廢時亂日，則書載胤征。由此觀之，審農時而重人事者，歷代然也。

"Your humble servant has scrolled through records & works, and decisively examined calendro-astronomy (*li*) numbers; it is by the seasons that we schedule agriculture, and it is by the months that we schedule [sacrificial] affairs—the origin of these [truths] is far off and high up [out of reach]. From the time of Shaohao 少昊, the Dark Bird served as director(s) of equinoxes; with Zhuanxu 顓頊 and Diku 帝嚳, Chong 重 & Li 黎 served as directors of heaven; and with Thearch Tang 唐 and Shun of Yu 虞舜, Xi 羲 & He 和 managed the sun. The Three Dynasties (Xia 夏, Shang 商, & Zhou 周) followed suit, and so the age saw the office of the sun; the office of the sun directed the calendar (*li*), and so disseminated it to the marquises; and the marquises received it, and so disseminated it throughout their realms. In the age of the rulers of the Xia, Xi & He debauched themselves in drink, forsaking the seasons & confounding the days, and so do the *Documents* record "Yin's Campaign."[2] Contemplating it from this, care in agricultural timing & attention to human affairs (sacrifices) has been [esteemed] as such throughout the ages.

逮至周室既衰，戰國橫騖，告朔之羊，廢而不紹，登臺之禮，滅而不遵。閏分乖次而不識，孟陬失紀而莫悟，大火猶西流，而怪蟄蟲之不藏也。是時也，天子不協時，司曆不書日，諸侯不受職，日御不分朔，人事不恤，廢棄農時。仲尼之撥亂於春秋，託褒貶糾正，司曆失閏，則譏而書之，登臺頒朔，則謂之有禮。自此以降，暨于秦、漢，乃復以孟冬為歲首，閏為後九月，中節乖錯，時月紕繆，加時後天，蝕不在朔，累載相襲，久而不革也。

After the fall of the house of Zhou (771 B.C.), warring states vied one against the other, [and so] the [sacrificial] lamb of the announcement of the new moon was abandoned and discontinued, and the [monthly] rite of ascending the terrace was abolished and unobserved.[3] The INTERCALARY PARTS strayed from their station yet went unrecognised, the *meng-zou* [correspondence] lost its order without anyone realizing, and while Great Fire (Antares) still "flowed" westward, they puzzled at how the hibernating insects did not hide.[4] At this

[2]"Yin zheng" 胤征 (Yin's Campaign) is the chapter of the *Book of Documents* that records the Marquis of Yin's punitive campaign against the debauched Xi & He.

[3]On "lost" calendar-related rituals ascribed to the Zhou in the classics, see Daniel P. Morgan, "Knowing Heaven: Astronomy, the Calendar, and the Sagecraft of Science in Early Imperial China" (Ph.D. diss., University of Chicago, 2013), 242–268.

[4]In other words, the fall of the Western Zhou 西周 (1045–771 B.C.) meant a disruption of the state calendar's intercalation schedule, the result of which was the misalignment of lunar and solar/sidereal time. On the INTERCALARY PARTS or REMAINDER, see procedure (1.1) below.

time, neither did the son of heaven mediate the seasons, nor did the director of the calendar write the days; neither did the marquises accept their duties, nor did their day secretaries [announce] the new moon, [and so] were human affairs (sacrifices) given no concern and did they disregard & discard agricultural timing. As to Confucius' eradication of chaos in the *Spring and Autumn Annals*, he relied upon praise & blame to rectify & correct, and where the director of the calendar missed an intercalation, he wrote about and censured it; and where they ascended the terrace to announce the new moon, he said of it that it was ritually appropriate. From this point onward, on down into the Qin 秦 (221–207 B.C.) and Han 漢 (206 B.C. – A.D. 220), the beginning of the year was returned to the first month of winter, and the intercalary [month] was made month IX^2; the medial & nodal [*qi*] were thrown into disarray, and the seasons & months into confusion; the [predicted] hour was behind heaven, and the [observed] eclipse was not at syzygy; [and so] year-upon-year did it continue on, perpetually without reform.

至武帝元封七年，始乃寤其繆焉。於是改正朔，更曆數，使大才通人，造太初曆，校中朔所差，以正閏分，課中星得度，以考疏密，以建寅之月為正朔，以黃鍾之月為曆初。其曆斗分太多，後遂疏闊。至元和二年，復用四分曆，施而行之。至于今日，考察日蝕，率常在晦，是則斗分太多，故先密後疏而不可用也。

It was only in year 7 of Wudi's 武帝 Yuanjia-reign (104 B.C.) that [they] finally awoke to the absurdity of [all] this. And at that they changed the first month of the year and switched calendro-astronomical numbers; they ordered experts of great talent to construct the Grand Inception system (*Taichu li* 太初曆), and they examined the discrepancy between medial and syzygy to correct the INTERCALARY PARTS, and they tested the meridian stars and got their *du* 度 to verify looseness & tightness (accuracy); they took the month established at $yin._{\mathrm{B03}}$ as the first month, and they took the month of the HUANGZHONG as the beginning of the calendar (*li*). Their calendro-astronomy's (*li*) DIPPER PARTS was too big, and later it gradually loosened & widened (became inaccurate). In Yuanhe, year 2 (A.D. 85) they restored the Quarter-remainder system (*Sifen li* 四分曆) and instituted and carried it out. To this day, [however], if you investigate solar eclipses, they are almost always on the *last* day of the month, which is because the DIPPER PARTS is too big. Therefore [are these systems] first tight & later loose and, thus, inadvisable for use.

是以臣前以制典餘日，推考天路，稽之前典，驗之食朔，詳而精之，更建密歷，則不先不後，古今中天。以昔在唐帝，協日正時，允釐百工，咸熙庶績也。欲使當今國之典禮，凡百制度，皆韜合往古，郁然備足，乃改正朔，更曆數，以大呂之月為歲首，以建子之月為曆初。臣以為昔在帝代，則法曰顓頊，曩自軒轅，則曆曰黃帝。暨至漢之

Meng 孟 refers to the first month of each season, and, according to the *Erya* 爾雅 and the Chu Silk Manuscript from Zidanku 子彈庫, Changsha, *zou* 陬 refers to the first month of the civil year; see Wang Zhiping 王志平, "Chu boshu yueming xintan" 楚帛書月名新探, *Huaxue* 華學 3 (1998): 181–188.

孝武，革正朔，更曆數，改元曰太初，因名太初曆。今改元為景初，宜曰景初曆。臣之所建景初曆，法數則約要，施用則近密，治之則省功，學之則易知。雖復使研桑心算，隸首運籌，重、黎司晷，羲、和察景，以考天路，步驗日月，究極精微，盡術數之極者，皆未如臣如此之妙也。是以累代曆數，皆疏而不密，自黃帝以來，改革不已。

It is for this [reason] that your humble servant has previously, in my days off from attending to the canons [of state], calculated & tested the heavenly road examining it in prior literature, and verifying it in eclipse syzygies; detailing and honing it, and establishing anew a tight calendro-astronomy (*li*); and so is it neither ahead nor behind, on the mark in heaven, past & present. Now, Thearch Tang 唐 of old mediated the days & corrected the seasons, and "verily did he regulate the hundred officers that they fully performed their various works."[5] If you desire to make the ritual canon and the hundred institutions of the current state all match like a scabbard the ancient past, and be complete & sufficient & flourishingly so, then [you must] change the first month and renew the calendro-astronomy (*li*) numbers; take the month of DALÜ as the head of the year, and take the month established at $zi._{\mathrm{B01}}$ as the beginning of the calendar (*li*). It is your humble servant's understanding that in the age of the thearchs of old, their method was called Zhuanxu's 顓頊, and all the way back to Xuanyuan 軒轅, their calendro-astronomy (*li*) was called the Yellow Thearch's; and in the time of Xiaowu[di] 孝武帝 (r. 140–87 B.C.) of the Han, they reformed the first month, and they renewed the calendro-astronomy (*li*) numbers; they changed the [reign period] to "Grand Inception" and, thus, named it the Grand Inception system. This [year] we have changed the [reign period] to Luminous Inception, so it would be proper to call this the Luminous Inception system. This Luminous Inception system established by your humble servant, its methods & numbers are concise & essential, and its implementation & use is close & tight (accurate); work on it, and you will find it saves time, study it, and you will find it easy to understand. Now, even if you made [Ji] Yan 計研 & Sang [Hongyang] 桑弘羊 perform mental calculations, Li Shou 隸首 operate the counting rods,[6] Chong & Li direct the gnomon, and Xi & He examine the shadow to investigate the road(s) of heaven, and calculate & verify the sun & moon, to plumb the extremes of fineness & subtlety, and to exhaust the limits of techniques & numbers—none of this would nearly so marvellous as [the marvel that is] I, your humble servant. It is for this [reason] that the calendro-astronomy (*li*) numbers of the accreted ages are all loose and not tight (inaccurate), and why, since the time of the Yellow Thearch, they have been reformed without end."

[5] *Shangshu zhushu* 尚書注疏 (Siku quanshu 四庫全書 ed.), 1.11b.

[6] Ji Yan, Sang Hongyang, and Li Shou are all renowned experts from the history of mathematics.

2 Parameters

壬辰元以來，至景初元年丁巳，歲積四千四十六，算上。此元以天正建子黃鍾之月為曆初，元首之歲夜半甲子朔旦冬至。

From the *renchen*.$_{29}$-[year] origin to Luminous Inception year 1, *dingsi*.$_{54}$ (A.D. 237), the year accumulation is 4046, counted up (inclusively).[7] This origin sets the month of the HUANGZHONG and established at *zi*.$_{S01}$ (the month of the winter solstice) as the astronomical first month, and the year of origin head [sees the coincidence of] midnight, [day] *jiazi*.$_{01}$, new moon, and winter solstice.

Notes:

1. On sexagenary binomes, see p. 47, Table 17
2. Origin falls on the (hypothetical) coincidence of midnight, day *jiazi*.$_{01}$, new moon, and winter solstice in the year 3809 B.C. (= 237 − 4046), BCE 3809 January 05 00:00:00 LT Monday

Table 1: [Astronomical constants]

no.	name		trans.	val.
1	元法	*yuan fa*	ORIGIN FACTOR	11 058
2	紀法	*ji fa*	ERA FACTOR	1 843
3	紀月	*ji yue*	ERA MONTHS	22 795
4	章歲	*zhang sui*	RULE YEARS	19
5	章月	*zhang yue*	RULE MONTHS	235
6	章閏	*zhang run*	RULE INTERCALATIONS	7
7	通數	*tong shu*	COMMUNICATION NUMBER	134 630
8	日法	*ri fa*	DAY FACTOR	4 559
9	餘數	*yu shu*	REMAINDER NUMBER	9 670
10	周天	*zhou tian*	CIRCUITS OF HEAVEN	673 150
11	歲中	*sui zhong*	YEAR MEDIALS	12
12	氣法	*qi fa*	QI FACTOR	12
13	沒分	*mo fen*	DISAPPEARANCE PARTS	67 315
14	沒法	*mo fa*	DISAPPEARANCE FACTOR	967
15	月周	*yue zhou*	LUNAR CIRCUITS	24 638
16	通法	*tong fa*	COMMUNICATION FACTOR	47
17	會通	*hui tong*	COINCIDENCE COMMUNICATION	790 110
18	朔望合數	*shuowang-he shu*	NEW-FULL CONJUNCTION NUMBER	67 315
19	入交限數	*rujiao xian shu*	CROSSING-ENTRY LIMIT NUMBER	722 795
20	通周	*tong zhou*	COMMUNICATION CIRCUITS	125 621
21	周日日餘	*zhouri ri yu*	CIRCUIT OF DAYS DAY REMAINDER	2 528
22	周虛	*zhou xu*	CIRCUIT VOID	2 031
23	斗分	*dou fen*	DIPPER PARTS	455

Notes:

[7] Note: Cullen suggests “above the count,” but Chemla points out that 算 as a noun is not common.

1. RULE: 19 years : 235 months : 7 intercalations, the length of time required for the year (winter solstice) and month (new moon) to coincide. The RULE is an *a priori* ratio. Note: $235 = 19 \times 12 + 7$.

2. Solar year (*sui*): CIRCUITS OF HEAVEN (673 150) : ERA FACTOR (1843) = $365\frac{455}{1843}$ days. The solar year is, in this case, an observation-derived or *a priori* ratio.

   (a) The solar year is divided into 24 *qi*, which alternate between 12 *jie* 節 "nodal" 12 and 12 *zhong* 中 "medial" *qi*. The "medial *qi*" is also a unit of solar time equal to 1/12 solar year or approximately 30 days.

   (b) Circumference of heaven (in *du*): $365\frac{455}{1843}$ *du*, because the (mean) sun travels 1 *du* per day. (*a priori*)

   (c) DIPPER PARTS (455): the fractional *du* (or day) part at the end of the circumference of heaven (or solar year), named after the lodge to which it is affixed. (*a priori*)

   (d) REMAINDER NUMBER (9670): the discrepancy between the solar year and six sexagenary cycles (360 days)—i.e. the number of days through the sexagenary cycle that two consecutive winter solstices move—as expressed in ERA FACTOR: $365\frac{455}{1843} - 360 = 5\frac{455}{1843} = \frac{9670}{1843}$.

   (e) DISAPPEARANCE (non-astronomical): DISAPPEARANCE PARTS (67315) : DISAPPEARANCE FACTOR (967), which is the quotient of the length of a year divided by its REMAINDER: $365\frac{455}{1843} \div 5\frac{455}{1843} = \frac{67\,315}{967}$.[8]

3. Synodic month (a "new-full" month): COMMUNICATION NUMBER (134630) : DAY FACTOR (4559) = $29\frac{2419}{4559}$ days. The DAY FACTOR is an (arbitrarily chosen?) *a priori*, the COMMUNICATION NUMBER is derived from this and the previous two constants.[9]

4. Lunisolar resonance periods

   (a) ERA: 1843 years : 22 795 months : 673 150 days, smallest integer ratio between the year, month, and day, and thus the time required for the winter solstice, new moon (syzygy), and midnight to coincide. (extrapolated)

   (b) ORIGIN: 11 058 years (6 ERAS), the smallest integer ratio between the year, month, day, *and* sexagenary cycle, and thus the time required for the winter solstice, new moon, midnight, *and* day *jiazi*$_{01}$ to coincide. (extrapolated)

[8] On the DISAPPEARANCE, see Jean-Claude Martzloff, *Le calendrier chinois: structure et calculs, 104 av. JC-1644: indétermination céleste et réforme permanente: la construction chinoise officielle du temps quotidien discret à partir d'un temps mathématique caché, linéaire et continu*, Sciences, techniques et civilisations du Moyen Âge à l'aube des Lumières 11 (Paris: Champion, 2009), 221–240. Note: Chemla points out that the DISAPPEARANCE PARTS is exactly 1/10 of the CIRCUITS OF HEAVEN, and, at one point offered an explanation.

[9] Specifically, the DAY FACTOR (4559) is the product of the COMMUNICATION FACTOR (47) and the "magic number" 97. The COMMUNICATION NUMBER is the product of the DAY FACTOR (4559) multiplied by the quotient of the number of days per year (previous item) divided by the number of months per year (RULE MONTHS [235] : RULE YEARS [19]).

5. Sidereal month (a "circuit around heaven" month): according to the RULE, in 19 years, the sun & moon experience 235 syzygies; in 19 years, the sun makes 19 circuits of heaven. Therefore, in 19 years, the moon makes 254 circuits of heaven (= 235+19), and in one ERA (1843 years) it makes 24638 circuits—the LUNAR CIRCUITS (24 638). 673 150 days : 24 638 circuits = $27\frac{7924}{24\,638}$ days per circuit.

6. Eclipse month (a "crossing-coincidence" month): when the synodic and eclipse month coincide—when syzygy occurs at the "crossing" of the lunar and solar paths—an eclipse occurs. The Luminous Inception system posits one eclipse every $5\frac{116\,960}{134\,630} = \frac{790\,110}{134\,630}$ months in the ratio COINCIDENCE COMMUNICATION (790 110) : NEW-FULL CONJUNCTION NUMBER (67 315) ×2. The difference between these two constants (790110−67315 = 722795) is the CROSSING-ENTRY LIMIT NUMBER (722 795).[10]

7. Anomalystic month: the Luminous Inception system posits an anomalystic month of $27\frac{2528}{4559} = \frac{125\,621}{4559}$ days in the ratio COMMUNICATION CIRCUITS (125 621) : DAY FACTOR (4559). In addition, the fractional remainder of the anomalystic month is the CIRCUIT OF DAYS DAY REMAINDER (2528), which equals out to an integer when added to the CIRCUIT VOID (2031) (= DAY FACTOR [4559] − CIRCUIT OF DAYS DAY REMAINDER [2528]). (I do not recall how this datum is derived).[11]

***

***

交會紀差，十萬三千六百一十。求其數之所生者，置一紀積月以通數乘之，會通去之，所去之餘，紀差之數也。以之轉加前紀，則得後紀。加之未滿會通者，則紀首之歲天正合朔，月在日道裏。滿去之，則月在日道表。加表滿在裏，加裏滿在表。

CROSSING-COINCIDENCE ERA DIFFERENCE: 103 610. To find the origin of this number, set out the number of months accumulated over one ERA (i.e. the ERA MONTHS [22 795]), multiply this by the COMMUNICATION NUMBER (134 630), and remove (modulo) the COINCIDENCE COMMUNICATION (790 110) from it. The remainder of that which has been removed [from] is the number of the ERA DIFFERENCE (103 610). Take this, and revolvingly add it to the prior era, and you will obtain [the CROSSING-COINCIDENCE difference LÜ for] the later era.

加之未滿會通者，則紀首之歲天正合朔，月在日道裏。滿去之，則月在日道表。加表滿在裏，加裏滿在表。

[10] As of 2014-01-22 I don't remember exactly how this works. I am excerpting here from Liu Hongtao 劉洪濤, *Gudai lifa jisuanfa* 古代曆法計算法 (Tianjin: Nankai daxue chubanshe, 2003), 189. For an early attempt to explain early Chinese eclipse methods through eclipse cycles, see Nathan Sivin, "Cosmos and Computation in Early Chinese Mathematical Astronomy," *T'oung pao* 2d ser., 55, no. 1/3 (1969): 1–73.

[11] Christopher Cullen explains the second-century A.D. origins of the methods for calculating the sidereal and draconic months seen here in "The First Complete Chinese Theory of the Moon: The Innovations of Liu Hong c. A.D. 200," *Journal for the history of astronomy* 33 (2002): 21–39.

Table 2: [Era heads & lunar epacts]

era (head date) A	no. B	ins./outs. C	C.C. dif. LÜ D	speed dif. LÜ E
jiazi.$_{01}$	1st	inside	412 919	103 947
jiaxu.$_{11}$	2nd	inside	516 529	73 767
jiashen.$_{21}$	3rd	inside	620 139	43 587
jiawu.$_{31}$	4th	inside	723 749	13 407
jiachen.$_{41}$	5th	[outside]	37 249	108 848
jiayin.$_{51}$	6th	[outside]	140 859	78 668

NOTE: Column A is the sexagenary date of era head. Column C tells us, "at syzygy at era head, the moon is inside/outside the solar *dao* (ecliptic)" 紀首合朔，月在日道表 • 裏; the editors of the Zhonghua shuju edition of the *Jin shu* version of the Luminous Inception system have emended the last two values to "outside." Column D gives the "CROSSING-COINCIDENCE difference LÜ" 交會差率, i.e. the eclipse month epact at era head. Column E gives the "slow-fast difference LÜ" 遲疾差率, i.e. the anomalistic month epact at era head.

> In the case that, upon adding, it does does not fill a COINCIDENCE COMMUNICATION (790 110), then at the syzygy of the astronomical first month of the year of era head, the moon is inside the solar *dao*. If it fills and has [the COINCIDENCE COMMUNICATION (790 110)] removed from it, then the moon is outside the solar *dao*. If you fill [the COINCIDENCE COMMUNICATION (790110)] in adding to an "outside," then it's on the inside; if you fill [the COINCIDENCE COMMUNICATION (790 110)] in adding to an "inside," then it's on the outside.

Notes:

1. This section explains the tabulation of Table 2, column C & D.
2. The difference between subsequent values in column D is 103 610, the "era difference."[12]

$$\text{ERA MONTHS} \times \text{COMM. NUMBER} \mod \text{COINCIDENCE COMM.} = \text{era difference} \quad (1)$$

or

$$22\,795 \times 134\,630 \mod 790\,110 = 103\,610$$

3. When the value of the "CROSSING-COINCIDENCE difference LÜ" (column D) exceeds 790 110—the COINCIDENCE COMMUNICATION—you "cast" (modulo) out said value and switch between "inside" and "outside" in column C. So, moving from the 4th to 5th era, because

$$723\,749 + 103\,610 > 790\,110$$

[12] As to *why*, Liu Hongtao explains that The COMMUNICATION NUMBER is the parts of a synodic month. Multiplied by ERA MONTHS, we get the number of month parts in one era. COINCIDENCE COMMUNICATION is the number of possible lunar eclipses in COMMUNICATION NUMBER months (COINCIDENCE COMM. : COMM. NUMBER = 5.6...). So, this LÜ gives the difference, in month parts, between the length of an era in synodic and eclipse months. See *Gudai lifa jisuanfa*, 190.

we thus switch from “inside” to “outside.” Then,

$$723\,749 + 103\,610 \mod 790\,110 = 37\,249$$

***

遲疾紀差，三萬一百八十。求其數之所生者，置一紀積月，以通數乘之，通周去之，餘以減通周，所減之餘，紀差之數也。以之轉減前紀，則得後紀。不足減者，加通周。

FAST-SLOW ERA DIFFERENCE: 30 180. To find the origin of this number, set out the number of months accumulated over one ERA (i.e. the ERA MONTHS [22 795]), multiply this by the COMMUNICATION NUMBER (134 630), remove (modulo) the COMMUNICATION CIRCUITS (125621) from it, and reduce the remainder from the COMMUNICATION CIRCUITS (125 621). The remainder of that which has been reduced is the number of the era difference (30 180). Take this, and revolvingly add it to the prior era, and you will obtain [the CROSSING-COINCIDENCE difference LÜ for] the later era. To anything insufficient for reduction, add the COMMUNICATION CIRCUITS (125 621).[13]

求次元紀差率，轉減前元甲寅紀差率，餘則次元甲子紀差率也。求次紀，如上法也。

To find the era difference LÜ for the next origin, revolvingly reduce from the *jiayin*.$_{51}$ era difference LÜ of the previous origin, and the remainder shall be the *jiazi*.$_{01}$ era difference LÜ of the next origin. To find the next era, follow the previous method.

Notes:

1. This section explains the tabulation of Table 2, column E.

2. The difference between subsequent values in column E is 30 180, the “era difference.” (explanation temporarily omitted).[14]

$$\begin{aligned}\text{COMM. CIRCUITS} - (&\text{ERA MONTHS} \times \text{COMM. NUMBER} \\ &\mod \text{COMM. CIRCUITS}) \\ &= \text{era difference}\end{aligned} \tag{2}$$

or

$$125\,621 - (22\,795 \times 134\,630 \mod 125\,621) = 30\,180$$

***

[13] As to *why*, see Liu Hongtao, *Gudai lifa jisuanfa*, 191.
[14] Liu Hongtao, *Gudai lifa jisuanfa*, 191–193.

3 Procedure

3.1 The calendar

推朔積月術曰：置壬辰元以來，盡所求年，外所求，以紀法除之，所得算外，所入紀第也，餘則入紀年數。年以章月乘之，如章歲而一為積月，不盡為閏餘。閏餘十二以上，其年有閏。閏月以無中氣為正。

(1.1) Technique for calculating accumulated months at new moon: Set out [the accumulated years] from the *renchen*.29 origin all the way through to the year sought but excluding the [year] sought (x). Cast out (divide by) the ERA FACTOR (1843),[15] and that obtained, counted exclusively, is the entered era number (e); the remainder is the era-entry year number (e_r). Multiply the [era-entry] year [number] by the RULE MONTHS (235) and count one for each RULE YEARS (19), which gives you the accumulated months [since era head] (m); that which is not exhausted is the intercalary remainder (m_r). If the intercalary remainder is 12 or above there is an intercalation that year, the correct [assignment] of the intercalary month being by the absence of a medial *qi*.

Notes:

1. Unknown values:

x	accumulated years from high origin
e	entered era number (ordinal)
e_r	era-entry year number
m	accumulated months from era head (integer)
m_r	accumulated months from era head (remainder)

2. Let us solve for N. Wei Taiping zhenjun 太平真君 year 12 (A.D. 451). Winter solstice, A.D. 451 lies 214 years after winter solstice A.D. 237 ($451 - 237 = 214$), which lies 4046 years after the *renchen*.29-year origin (see p. 5). Counting exclusively, a total number of 4259 years have elapsed since high origin ($4046 + 214 - 1 = 4259$).

$$x = 4259$$

3. Divide by the number of years in an ERA (i.e. the ERA FACTOR [1843]). The integer quotient (e), counted exclusively, is the ordinal number of the current ERA; the remainder (e_r) is the number of years entered said ERA:

$$x \div \text{ ERA FACTOR} = (e - 1) + \frac{e_r}{\text{ERA FACTOR}} \quad (3)$$

In this case,

$$e = 3$$
$$e_r = 573$$

[15] In the context of *li* 曆 mathematical astronomy, *chu* 除 typically refers to the operation of modulo—"casting out"—but here we find it used unambiguously to refer to division.

which is to say that we are 573 years into the third ERA. According to Table 2 (p. 8), the 3rd ERA begins on day *jiashen*.$_{21}$.

4. Next, multiply the era-entry year number (e_r) by the ratio of months to years in a RULE (235 months : 19 years). The integer product (m) and remainder (m_r) are the number of months elapsed from era head to winter solstice, A.D. 451:

$$e_r \times \text{ RULE MONTHS } \div \text{ RULE YEARS } = m + \frac{m_r}{\text{RULE YEARS}} \qquad (4)$$

   in this case

$$\begin{aligned} m + \frac{m_r}{19} &= 573 \times \frac{235}{19} \\ &= 7087\frac{2}{19} \end{aligned}$$

5. If the intercalary remainder (m_r) is less than 12, there is no intercalary month this year. Why? Every year the intercallary remainder increases by 7, which is to say that every year the 12 months (lunar) fall $\frac{7}{19}$ month earlier *vis-à-vis* the 24 *qi* (solar). As such, any year that begins with an intercalary remainder of 12 or higher—any year where syzygy, astronomical first month, falls $\frac{12}{19}$ month or more prior to winter solstice—will trip over to 0 by the end of the year, which means 13 months.

6. In the case of A.D. 451, $m_r < 12$, thus there is no intercalary month. Were there an intercalary month, *placement* is decided in technique 1.4 (p. 15).

***

推朔術曰：以通數乘積月，為朔積分，如日法而一為積日，不盡為小餘。以六十去積日，餘為大餘。大餘命以紀，算外，所求年天正十一月朔日也。

(1.2) Technique for calculating new moon(s): Multiply the accumulated months (m) by the COMMUNICATION NUMBER (134630) to get the new moon accumulated parts, which you count one for each DAY FACTOR (4559) to get the accumulated days (d); that which is not exhausted is the lesser remainder (d_r). Remove (modulo) complete sets of 60 from the accumulated days (d), the remainder of which is the greater remainder (g). Count the greater remainder (g) from era [head], counting exclusively, [and you get] the new moon day of the astronomical first month, month XI, for the year sought.

求次月，加大餘二十九，小餘二千四百一十九，小餘滿日法從大餘，命如前，次月朔日也。小餘二千一百四十以上，其月大也。

To find subsequent months, add 29 to the greater remainder (g) & 2419 to the lesser remainder (d_r), and if the lesser remainder (d_r) fills the DAY FACTOR (4559), it goes together with (is added to) the greater remainder (g), and we count like before for the new moon day of the next month. If the lesser remainder (d_r) is 2140 or above, that month is large.

Notes:

1. Unknown values:

d	accumulated days from era head (integer)
d_r	accumulated days from era head (remainder)
g	accumulated days from era head *in the sexagenary cycle* (integer)

2. Convert the integer number of months elapsed from era head to month XI (m) into days via the ratio COMMUNICATION NUMBER : DAY FACTOR (134 630 days : 4559 months), again separating out integer days (d) and the remainder (d_r):

$$m \times \text{ COMM. NUMBER } \div \text{ DAY FACTOR} = d + \frac{d_r}{\text{DAY FACTOR}} \quad (5)$$

In this case,

$$\begin{aligned} d + \frac{d_r}{4559} &= 7087 \times \frac{134\,630}{4559} \\ &= 209\,283\frac{1613}{4559} \end{aligned}$$

3. Cast out full sexagenary cycles to find the greater remainder (g):

$$d \mod 60 = g \quad (6)$$

In this case

$$g = 3$$

Thus, we have cycled $3\frac{1613}{4559}$ days through the sexagenary cycle since era head. We are currently in the 3rd era, which begins on day *jiashen*.$_{21}$ (see p. 11). $21 + 3 = 24$, which places new moon day on day *dinghai*.$_{24}$.

4. Now, sequentially add $29\frac{2419}{4459}$ days to produce a *shuo-run* 朔閏 (new & intercalary moon) table as in Table 3.

***

推弦望，加朔大餘七，小餘千七百四十四，小分一，小分滿二從小餘，小餘滿日法從大餘，大餘滿六十去之，餘命以紀，算外，上弦日也。又加得望、下弦、後月朔。其月蝕望者，定小餘，如所近中節間限，限數以下者，算上為日。望在中節前後各四日以還者，視限數；望在中節前後各五日以上者，視間限。

To calculate quarter & full [moons], add 7 to the greater remainder (g) & 1744 & 1 lesser part to the lesser remainder (d_r), and when the lesser parts fills 2, it goes together with (is added to) the lesser remainder (d_r), and when the lesser remainder (d_r) fills the DAY FACTOR (4559), it goes together with (is added to) the greater remainder (g), and when the greater remainder (g) fills 60, remove

Table 3: *Shuo-run* table computed for A.D. 451

month		g	d_r	size
ast.	civ.			
1	XI	*dinghai*.$_{24}$	1613	short
2	XII	*bingchen*.$_{53}$	4032	long
3	I	*bingxu*.$_{23}$	1892	short
4	II	*yimao*.$_{52}$	4311	long
5	III	*yiyou*.$_{22}$	2171	long
6	IV	*yimao*.$_{52}$	31	short
7	V	*jiashen*.$_{21}$	2450	long
8	VI	*jiayin*.$_{51}$	310	short
9	VII	*guiwei*.$_{20}$	2729	long
10	VIII	*guichou*.$_{50}$	589	short
11	IX	*renwu*.$_{19}$	3008	long
12	X	*renzi*.$_{49}$	868	short

(modulo) this; count what remains from the era [head date], counting exclusively—that is the date of first quarter. Add again to get full moon, third quarter, and the subsequent new moon. If there is an eclipse at full moon that month, fix (apply equation of center to) the lesser remainder (d_r). if it is below the LIMIT NUMBER of the INTERVAL LIMIT of the medial or nodal [*qi*] to which it is nearest, count it as the previous day. If new moon is within four days of either medial or nodal [*qi*], see the LIMIT NUMBER; if new moon is more than five days from either medial or nodal [*qi*], see the INTERVAL LIMIT (Table 6).

Notes:

1. Calculate the date of first quarter, full moon, and third quarter by successively adding $7\frac{1744\frac{1}{2}}{4559}$ days ($\frac{1}{4}$ synodic month) as in Table 4.

Table 4: New, quarter, & full moons computed for A.D. 451

month		phase	g	d_r	size
ast.	civ.				
1	XI	●	*dinghai*.$_{24}$	1613	short
”	”	☽	*jiawu*.$_{31}$	$3357\frac{1}{2}$	”
”	”	○	*renyin*.$_{39}$	543	”
”	”	☾	*jiyou*.$_{46}$	$2287\frac{1}{2}$	”
2	XII	●	*bingchen*.$_{53}$	4032	long
”	”	☽	*jiazi*.$_{01}$	$1217\frac{1}{2}$	”
”	”	○	*xinwei*.$_{08}$	2962	”
”	”	☾	*jimao*.$_{16}$	$147\frac{1}{2}$	”
			...		

2. The LIMIT NUMBERS and INTERVAL LIMITS are found on Table 6. In brief, both “limits” are the fraction of a day past midnight, expressed in DAY

FACTORS (4559), at which "daytime" begins. The LIMIT NUMBER is used ±4 days from a *qi*; the INTERVAL LIMIT is used > 4 days from a *qi*. If the lesser remainder (d_r) is smaller than the appropriate "limit," it means that the phenomena occurred prior to daytime, *and that that phenomena is counted as having occurred on the previous "day."* Note, however, that the text instructs us to determine this *only* in the case of a lunar eclipse and *only* after having "fixed" the lesser remainder (see the section on Lunar Astronomy below).

***

推二十四氣術曰：置所入紀年，外所求，以餘數乘之，滿紀法為大餘，不盡為小餘。大餘滿六十去之，餘命以紀，算外，天正十一月冬至日也。

(1.3) Technique for calculating the twenty-four *qi*: Set out the era-entry year number (e_r), excluding the [year] sought. Multiply this by the REMAINDER NUMBER (9670) and [the amount of times] it fills the ERA FACTOR (1843) is the greater remainder (g); that which is not exhausted is the lesser remainder (d_r). If the greater remainder (g) fills 60, remove (modulo) it; count what remains from the era [head date], counting exclusively—that is the date of winter solstice, astronomical first month, month XI.

求次氣，加大餘十五，小餘四百二，小分十一，小分滿氣法從小餘，小餘滿紀法從大餘，命如前，次氣日也。

To find subsequent *qi*, add 15 to the greater remainder (g) & 402 & 11 lesser parts to the lesser remainder (d_r), and when the lesser parts fills the QI FACTOR (12), it goes together with (is added to) the lesser remainder (d_r), and when the lesser remainder (d_r) fills the ERA FACTOR (1843), it goes together with (is added to) the greater remainder (g). Count like before, and that is the date of the next *qi*.

Notes:

1. Unknown values:

g	accumulated days from era head (integer)
d_r	accumulated days from era head (remainder)

   *Note that these differ from the terms introduced in Technique 1.2.

2. Multiply the number of years elapsed by the (annual) difference between the length of the solar year and six sexagenary cycles and modulo out full cycles to find the number of days elapsed between winter solstices *in the sexagenary cycle*, distinguishing integer days (g) and the remainder (d_r):

$$x \times \text{ REMAINDER NUMBER} \mod \text{ERA FACTOR} \mod 60 = g + \frac{d_r}{\text{ERA FACTOR}} \tag{7}$$

In this case,

$$g + \frac{d_r}{1843} = (4259 \times 9670) \mod 1843 \mod 60$$
$$= 6\frac{852}{1843}$$

Thus, we have cycled $6\frac{852}{1843}$ days through the sexagenary cycle since era head. We are currently in the 3rd era, which begins on day *jiashen*.$_{21}$ (see p. 11). $21 + 6 = 27$, which places winter solstice day on day *gengyin*.$_{27}$.

3. Calculate the date of subsequent *qi* by successively adding $\frac{1}{24}$ of a solar year, or $15\frac{402\frac{11}{12}}{1843}$ days as in Table 5.

Table 5: *Qi* table computed for A.D. 451

no.	name	g	d_r
1	冬至 winter solstice	*gengyin*.$_{27}$	$852\frac{0}{12}$
2	小寒 lesser cold	*yisi*.$_{42}$	$1254\frac{11}{12}$
3	大寒 greater cold	*gengshen*.$_{57}$	$1657\frac{10}{12}$
4	立春 enthronement of spring	*bingzi*.$_{13}$	$217\frac{9}{12}$
5	雨水 rainwater	*xinmao*.$_{28}$	$620\frac{8}{12}$
6	驚蟄 awake from hibernation	*bingwu*.$_{43}$	$1023\frac{7}{12}$
7	春分 spring equinox	*xinyou*.$_{58}$	$1426\frac{6}{12}$
8	清明 clear & bright	*bingzi*.$_{13}$	$1829\frac{5}{12}$
9	穀雨 grain rain	*renchen*.$_{29}$	$389\frac{4}{12}$
10	立夏 enthronement of summer	*dingwei*.$_{44}$	$792\frac{3}{12}$
11	小滿 small but full	*renxu*.$_{59}$	$1195\frac{2}{12}$
12	芒種 bearded grain	*dingchou*.$_{14}$	$1598\frac{1}{12}$
13	夏至 summer solstice	*guisi*.$_{30}$	$158\frac{0}{12}$
14	小暑 lesser heat	*wushen*.$_{45}$	$560\frac{11}{12}$
15	大暑 greater heat	*guihai.*.$_{60}$	$963\frac{10}{12}$
16	立秋 enthronement of spring	*wuyin*.$_{15}$	$1366\frac{9}{12}$
17	處暑 abiding heat	*guisi*.$_{30}$	$1769\frac{8}{12}$
18	白露 hoarfrost	*yiyou*.$_{46}$	$329\frac{7}{12}$
19	秋分 autumn equinox	*jiazi*.$_{01}$	$732\frac{6}{12}$
20	寒露 cold dew	*jimao*.$_{16}$	$1135\frac{5}{12}$
21	霜降 frost settles	*jiawu*.$_{31}$	$1538\frac{4}{12}$
22	立冬 enthronement of winter	*gengxu*.$_{47}$	$98\frac{3}{12}$
23	小雪 lesser snow	*yichou*.$_{02}$	$501\frac{2}{12}$
24	大雪 greater snow	*gengchen*.$_{17}$	$904\frac{1}{12}$

***

推閏月術曰：以閏餘減章歲，餘以歲中乘之，滿章閏得一月，餘滿半法以上亦得一月。數從天正十一月起，算外，閏月也。閏有進退，以無中氣御之。

(1.4) Technique for calculating the intercalary month: Subtract the the intercalary remainder (m_r; see Technique 1.1) from the RULE YEARS (19), multiply what remains by the YEAR MEDIALS (12), and get one month for each time it fills the RULE INTERCALATIONS; what remains [from that], if it fills half or more of the *fa* "divisor" you also obtain one month (i.e. in the case of a remainder, round to the nearest month). Count off from astronomical first month, month XI, counting exclusively, and there's your intercalary month. [Note:] Intercalations experience advance & retreat, manage this via the absence of medial *qi*.[16]

Notes:

1. Unknown values:

m_r	accumulated months from era head (remainder) (Technique 1.1)
i	intercalary month (ordinal)

2. Explanation: In technique 1.1, m_r : RULE YEARS (19) is the fractional portion of a month elapsed from the syzygy at astronomical first month and the winter solstice of the year sought; m_r increases each year by 7, and when m_r trips over to 0—when it exceeds the RULE YEARS (19)—said year has an extra months. Technique 1.4 helps us identify *what month* it is that m_r trips over to 0, because *that* is the intercalary month.

3. If m_r : RULE YEARS (19) is the fractional portion of a month elapsed from the syzygy at astronomical first month and the winter solstice of the year sought, then (RULE YEARS − m_r) : RULE YEARS is the fractional portion of a month *from winter solstice to the next syzygy.* Because lunar time is sliding backwards *vis-à-vis* solar time, the shorter this latter interval is, the sooner we will experience a month without a medial *qi*, which should be counted as intercalary.

   Next, multiplying this by the inverse of the ratio of years : intercalations (i.e. RULE YEARS [19] : RULE INTERCALATIONS [7]) produces the fraction of the civil year that will pass prior to tripping to 0—the intercalary month. Note, however, that since the intercalary remainder (m_r) implies a divisor of RULE YEARS (19), this cancels out the RULE YEARS (19) by which we multiply:

$$\frac{m_r}{\text{RULE YEARS}} \times \frac{\text{RULE YEARS}}{\text{RULE INTERCAL.}} = \frac{m_r}{\text{RULE INTERCAL.}}$$

   Finally, we convert years to months by multiplying this fractional part of a year by the number of medial *qi*/months in one year. Thus we have the rule:

$$\lfloor(\text{ RULE YEARS } - m_r) \times \text{ YEAR MEDIALS } \div \text{ RULE INTERCAL.}\rfloor = i \quad (8)$$

[16] Interesting research question: does it ever actually happen that the results of these two rules for intercalation vary? Over the centuries that the Luminous Inception system saw official use, is there any evidence from historical sources of the preference of one method over the other in the placing of intercalary months?

In the case of A.D. 451:

$$\begin{aligned} i &= \lfloor (19-2) \times \frac{12}{7} \rfloor \\ &= \lfloor 29\frac{1}{7} \rfloor \\ &= 29 \end{aligned}$$

In other words, the intercalary month with fall on the 29th month counted from astronomical first month. As there are only 12 or 13 months in a year, however, the 29th month falls not in A.D. 451 but two years later.

***

Table 6: *Qi* limit table

no.	name	mo.	type	LIMIT NO.	INT. LIMIT
24	大雪 greater snow	XI	nod.	1242	1248
1	冬至 winter solstice	”	med.	1254	1245
2	小寒 lesser cold	XII	nod.	1235	1224
3	大寒 greater cold	”	med.	1213	1192
4	立春 enthronement of spring	I	nod.	1172	1147
5	雨水 rainwater	”	med.	1122	1093
6	驚蟄 awake from hibernation	II	nod.	1065	1036
7	春分 spring equinox	”	med.	1008	979
8	清明 clear & bright	III	nod.	951	925
9	穀雨 grain rain	”	med.	900	879
10	立夏 enthronement of summer	IV	nod.	857	840
11	小滿 small but full	”	med.	823	813
12	芒種 bearded grain	V	nod.	800	799
13	夏至 summer solstice	”	med	798	801
14	小暑 lesser heat	VI	nod.	805	815
15	大暑 greater heat	”	med.	825	843
16	立秋 enthronement of spring	VII	nod.	859	883
17	處暑 abiding heat	”	med.	907	935
18	白露 hoarfrost	VII	nod.	962	992
19	秋分 autumn equinox	”	med.	1021	1051
20	寒露 cold dew	IX	nod.	1080	1107
21	霜降 frost settles	”	med.	1133	1157
22	立冬 enthronement of winter	X	nod.	1181	1198
23	小雪 lesser snow	”	med.	1215	1229

NOTE: (1) The “type” column indicates “nodal” (*jie* 節) vs. “medial” (*zhong* 中) *qi*. (2) The LIMIT NUMBER (*xian shu* 限數) is one half the length of night at said *qi*, as expressed in parts of a DAY FACTOR (4559). For example, at the winter solstice, “daytime” begins $\frac{1254}{4559}$ day (i.e., $100 \times \frac{1254}{4459} = 27.5$ *ke*, or $24 \times \frac{1254}{4459}$ =6h36m) after midnight. According to Technique 1.2, the LIMIT NUMBER is used only ±4 days from said *qi*. (3) The INTERVAL LIMIT (*jian xian* 間限) functions the same way as the LIMIT NUMBER, but it is used in the “interval” between LIMIT NUMBERS, i.e. > 4 days from a given *qi*. The INTERVAL LIMIT is the average of the LIMIT NUMBERS of two consecutive *qi*.

***

推沒滅術曰：因冬至積日有小餘者，加積一，以沒分乘之，以沒法除之，所得為大餘，不盡為小餘。大餘滿六十去之，餘命以紀，算外，即去年冬至後沒日也。

(1.5) Technique for calculating disappearances and annihilations: Take (*yin*) the accumulated days from winter solstice that has a lesser remainder (Technique 1.3), add one, multiply by the DISAPPEARANCE PARTS (67 315), and cast out (divide by) the DISAPPEARANCE FACTOR (967). What you obtain is the greater remainder (g); that which is not exhausted is the lesser remainder (d_r). If the greater remainder (g) fills 60, remove (modulo) it; count what remains from the era [head date], counting exclusively—that's the "disappearance" day after the winter solstice of the prior [civil] year.

求次沒，加大餘六十九，小餘五百九十二，小餘滿沒法得一，從大餘，命如前。小餘盡，為滅也。

To find subsequent disappearances, add 69 to the greater remainder (g) & 592 to the lesser remainder (d_r), and when the lesser remainder fills the DISAPPEARANCE FACTOR (967), it goes together with (is added to) the greater remainder (g). Count like before. When the lesser remainder is exhausted, it is an "annihilation."

Notes:

1. Unknown values:

d	accumulated days from era head (integer)
g	accumulated days from era head (integer), mod 60
d_r	accumulated days from era head (remainder)

   *Note that these differ from the terms introduced in Technique 1.2, &c.

2. Note that the text instructs the user to use the "accumulated days from winter solstice with lesser remainder" (d), which we have presumably calculated in Technique 1.3. That said, Technique 1.3 actually solves for a greater remainder (g), which is $d \mod 60$ (see p. 14, Equation 7).

   In this case,

   $$d = 3006$$
   $$d_r = 852$$

3. The "disappearance" and "annihilation," as explained on p. 6, are non-astronomical phenomena computed for the purposes of hemerology (the science of selecting lucky days). In short, the practitioner takes the difference between the solar year and 6 sexagenary cycles ($365\frac{455}{1843} - 360 = 5\frac{455}{1843} = \frac{9670}{1843}$ days), and divides the solar year by that number ($365\frac{455}{1843} \div 5\frac{455}{1843} = \frac{67\,315}{967} = 69\frac{592}{967}$), from which one posits one "disappearance" every $69\frac{592}{967}$ days. When a disappearance falls an integer number of

days past era head—i.e., at midnight—it is an "annihilation." The rule is as follows:

$$(d + \frac{d_r}{\text{ERA FACTOR}} + 1) \times \text{ DIS. PARTS } \div \text{ DIS. FACTOR} \\ = g + \frac{g_r}{\text{DIS. FACTOR}} \tag{9}$$

In this case,

$$g = (3006\frac{852}{1843} + 1) \times 67\,315 \div 967 \\ = 209\,356\frac{72}{967}$$

Cast out full sexagenary cycles, and count $16\frac{72}{967}$ ($= 209\,356\frac{72}{967} \mod 60$) days from era head to arrive at *gengzi*.$_{37}$.

4. From this, calculate Table 7.

Table 7: Disappearance & annihilation table calculated for A.D. 451

no.	date	g_r	type
1	*gengzi*.$_{37}$	72	disappearance
2	*jiyou*.$_{46}$	664	disappearance
3	*jiwei*.$_{56}$	289	disappearance
4	*wuchen*.$_{05}$	881	disappearance
5	*wuyin*.$_{15}$	506	disappearance

5. Note that the text seems to instruct the user to calculate using both d and d_r, but this is very atypical.

***

推五行用事日：立春、立夏、立秋、立冬者，即木、火、金、水始用事日也。各減其大餘十八，小餘四百八十三，小分六，餘命以紀，算外，各四立之前土用事日也。大餘不足減者，加六十；小餘不足減者，減大餘一，加紀法；小分不足減者，減小餘一，加氣法。

(1.6) Calculate the dates of the ascendancies of the five agents: the enthronement of spring.$_{Q01}$, summer.$_{Q07}$, autumn.$_{Q13}$, & winter.$_{Q19}$ are the first days of the ascendancy of wood, fire, metal, & water, [respectively]. For each one, subtract 18 from its greater remainder (g) & 483 & 6 lesser parts from its lesser remainder (d_r); count what remains from the era [head date], counting exclusively—that is the date prior to each "enthronement" at which earth is in the ascendancy. For a greater remainder (g) that is insufficient to subtract from, add 600; for a lesser remainder (d_r) that is insufficient to subtract from, subtract 1 from the greater remainder (g) and add an ERA FACTOR (1843); and for a lesser parts that is insufficient to subtract from, subtract 1 from the lesser remainder (d_r) and add a QI FACTOR (12).

Notes:

1. Unknown values: Take g and d_r from Table 5 on p. 15.
2. These dates are non-astronomical phenomena computed for purposes of hemerology. In short, the four agents wood, fire, metal, and water begin at the enthronements of the respective seasons, which we have already computed on Table 5, and earth begins $18\frac{483\frac{6}{12}}{1843}$ days prior to *all four*.
3. From this, calculate Table 8.

Table 8: Ascendancies of the five agents table calculated for A.D. 451

agent	date (g)	d_r
earth	*dingsi*.$_{54}$	$1577\frac{3}{12}$
wood	*bingzi*.$_{13}$	$217\frac{9}{12}$
earth	*jichou*.$_{26}$	$308\frac{9}{12}$
fire	*dingwei*.$_{44}$	$792\frac{3}{12}$
earth	*gengshen*.$_{57}$	$883\frac{3}{12}$
metal	*wuyin*.$_{15}$	$1366\frac{9}{12}$
earth	*xinmao*.$_{28}$	$1457\frac{9}{12}$
water	*gengxu*.$_{47}$	$98\frac{3}{12}$

***

推卦用事日：因冬至大餘，六其小餘，坎卦用事日也。加小餘萬九十一，滿元法從大餘，即中孚用事日也。

(1.7) Calculate the dates of the ascendancies of the hexagrams: Take (*yin*) the greater remainder (g) of winter solstice and six-fold its lesser remainder (d_r)—that is the date of the ascendancy of the *kan* 坎 (29) ䷜ hexagram. Add 91 to the lesser remainder (d_r), and if it fills the ORIGIN FACTOR (11 058), it goes together with (is added to) the greater remainder (g)—that is the date of the ascendancy of the *zhongfu* 中孚 (61) ䷼ hexagram.

求次卦，各加大餘六，小餘九百六十七。其四正各因其中日，六其小餘。

To find the next hexagram, each time add 6 to the greater remainder (g) & 967 to the lesser remainder (d_r). The four cardinal points are each according to the date of their medial [*qi*]; six-fold the lesser remainder (d_r).

Notes:

1. Unknown values: Again, take g and d_r from Table 5 on p. 15.
2. Again, these dates are non-astronomical phenomena computed for purposes of hemerology. In short, the four cardinal hexagrams—*kan* 坎 (29) ䷜ (water), *zhen* 震 (51) ䷲ (wood), *li* 離 (30) ䷝ (fire), and *dui* 兌 (58) ䷹

(metal)—fall on the solstices and equinoxes corresponding to their agent (see Technique 1.6). From each solstitial & equinoctial point, subsequent hexagrams are then counted off in $6\frac{967}{11\,058}$-day intervals.

3. From this, calculate Table 9.

Table 9: Ascendancies of the hexagrams table calculated for A.D. 451

hexagram	date (g)	d_r
䷜ (29) *kan* 坎 (water)	*gengyin*.$_{27}$	5112
䷼ (61) *zhongfu* 中孚	*xinmao*.$_{28}$	4145
䷗ (24) *fu* 復	*dingyou*.$_{34}$	5112
䷂ (03) *tun* 屯	*guimao*.$_{40}$	6079
	...	

***

推日度術曰：以紀法乘朔積日，滿周天去之，餘以紀法除之，所得為度，不盡為分。命度從牛前五起，宿次除之，不滿宿，則天正十一月朔夜半日所在度及分也。

(1.8) Technique for calculating the *du* of the sun: Multiply the new moon accumulated days (d) by the ERA FACTOR (1843), and if it fills the CIRCUITS OF HEAVEN (673 150), remove (modulo) it, then cast out (divide) the ERA FACTOR (1843) from what remains—what is obtained is the *du* (t), and what is not exhausted is the parts (t_r). Count out the *du* from 5 *du* before Ox.$_{L09}$, casting out (subtracting) successive lodge [width]s, and the lodge that it does not fill, that is the *du* (t) and parts (t_r) of the position of the sun at midnight of new moon day, astronomical first month, month XI.

求次日，日加一度，分不加，經斗除斗分，分少退一度。

To find the next day, add one *du* per day. Parts are not added, [however], when you pass through Dipper.$_{L08}$, cast out (subtract) the DIPPER PARTS (455), and if the parts (t_r) are too few, then retreat one *du* (t).

Notes:

1. Unknown values:

d	accumulated days from era head to month XI(integer) (from Technique 1.2)
t	accumulated *du* from era head (integer)
t_r	accumulated *du* from era head (remainder)

2. Explanation: The number of days elapsed since era head equals the number of *du* traveled by the sun in the same interval, but to arrive at a useful datum we need to modulo out complete circuits, then subtract out complete lodges.

3. Convert the *integer* number of days/*du* elapsed from era head to syzygy, astronomical first month into *parts* by multiplying by the ERA FACTOR (1843). Then, modulo out complete circuits of heaven (expressed in ERA FACTOR parts), and divide again by the ERA FACTOR to convert back to *du*:

$$(d\times \text{ ERA FACTOR}) \mod \text{ CIRCUITS OF HEAVEN} \\ \div \text{ ERA FACTOR } = t + \frac{t_r}{\text{ERA FACTOR}} \tag{10}$$

In this case,

$$\begin{aligned} t + \frac{t_r}{1843} &= (209\,283 \times 1843) \mod 673\,150 \div 1843 \\ &= \frac{666\,769}{1843} \\ &= 361\frac{1446}{1843} \end{aligned}$$

or,

$$\begin{aligned} t &= 361 \\ t_r &= 1446 \end{aligned}$$

4. Note: using only the *integer* days elapsed from era head allows us to calculate for midnight, since era head begins, by definition, at midnight. Here and elsewhere, *li* manuals tend to instruct the user to calculate for midnight.

5. Because we begin counting from 5 *du* before Ox.$_{\text{L09}}$, subtract 5 from this total ($361\frac{1446}{1843} - 5 = 356\frac{1446}{1843}$). Subtract subsequent lodge widths (Fig. 4) as in Table 10.

   The result, in this case, is Dipper.$_{\text{L08}}$ $17\frac{1446}{1843}$ *du*.

6. Note that by placing the fractional part of the circumference of heaven in the lodge of the winter solstice (Dipper.$_{\text{L08}}$) and counting *du* from the next, integer lodge (Ox.$_{\text{L09}}$), we are able to effectively avoid dealing with this fraction in the "casting out" operation.

7. Note that once we have calculated a single position in this manner, subsequent positions are easily computed: add one and cast out full lodges.

***

推月度術曰：以月周乘朔積日，滿周天去之，餘以紀法除之，所得為度，不盡為分，命如上法，則天正十一月朔夜半月所在度及分也。

(1.9) Technique for calculating the *du* of the moon: Multiply the new moon accumulated days (d) by the LUNAR CIRCUITS (24638), and if it fills the CIRCUITS OF HEAVEN (673150), remove (modulo) it, then cast out (divide) the ERA FACTOR (1843) from what remains—what is obtained is the *du* (t), and what is not exhausted is the parts (t_r). Count according to the previous method, and this is the *du* (t)

Table 10: Example of casting out subsequent lodges

Lodge	width	remainder
Ox.$_{\text{L09}}$	8	$348\frac{1446}{1843}$
Maid.$_{\text{L10}}$	12	$336\frac{1446}{1843}$
Tumulus.$_{\text{L11}}$	10	$326\frac{1446}{1843}$
Rooftop.$_{\text{L12}}$	17	$309\frac{1446}{1843}$
Hall.$_{\text{L13}}$	16	$293\frac{1446}{1843}$
Wall.$_{\text{L14}}$	9	$284\frac{1446}{1843}$
Crotch.$_{\text{L15}}$	16	$268\frac{1446}{1843}$
Pasture.$_{\text{L16}}$	12	$256\frac{1446}{1843}$
Stomach.$_{\text{L17}}$	14	$242\frac{1446}{1843}$
Mane.$_{\text{L18}}$	11	$231\frac{1446}{1843}$
Net.$_{\text{L19}}$	16	$215\frac{1446}{1843}$
Beak.$_{\text{L20}}$	2	$213\frac{1446}{1843}$
Triad.$_{\text{L21}}$	9	$204\frac{1446}{1843}$
Well.$_{\text{L22}}$	33	$171\frac{1446}{1843}$
Devils.$_{\text{L23}}$	4	$167\frac{1446}{1843}$
Willow.$_{\text{L24}}$	15	$152\frac{1446}{1843}$
Stars.$_{\text{L25}}$	7	$145\frac{1446}{1843}$
Spread.$_{\text{L26}}$	18	$127\frac{1446}{1843}$
Wings.$_{\text{L27}}$	18	$109\frac{1446}{1843}$
Baseboard.$_{\text{L28}}$	17	$92\frac{1446}{1843}$
Horn.$_{\text{L01}}$	12	$80\frac{1446}{1843}$
Neck.$_{\text{L02}}$	9	$71\frac{1446}{1843}$
Root.$_{\text{L03}}$	15	$56\frac{1446}{1843}$
Chamber.$_{\text{L04}}$	5	$51\frac{1446}{1843}$
Heart.$_{\text{L05}}$	5	$46\frac{1446}{1843}$
Tail.$_{\text{L06}}$	18	$28\frac{1446}{1843}$
Basket.$_{\text{L07}}$	11	$17\frac{1446}{1843}$
Dipper.$_{\text{L08}}$	–	–

and parts (t_r) of the position of the moon at midnight of new moon day, astronomical first month, month XI.

求次月，小月加度二十二，分八百六；大月又加一日，度十三，分六百七十九；分滿紀法得一度，則次月朔夜半月所在度及分也。其冬下旬，月在張心署之。

To find the next month, for a short month, add 22 to the *du* (t) & 806 to the parts (t_r); and for a long month, add another day, [another] 13 to the *du* (t), & 679 to the parts (t_r); and if the parts fill the ERA FACTOR (1843), then you get 1 *du*—this is the *du* (t) and parts (t_r) of the position of the moon at midnight of new moon day on the subsequent month. In the last decade (10-day period) of winter, note if the moon is in Spread.$_{\text{L26}}$ or Heart.$_{\text{L05}}$.

Notes:

1. Unknown values: see Technique 1.8

2. Explanation: According to the RULE ratio, the moon makes $235+19 = 254$ circuits of heaven in 19 years (see p. 7). As such, it travels at a speed of

$\frac{254}{19} = 13\frac{6}{19}$ *du* per day. Therefore, over the course of a 29-day civil month, the moon travels

$$29 \times 13\frac{7}{19} = 387\frac{13}{19}\text{days}$$

And casting out full circuits of $365\frac{455}{1843}$ *du*, we arrive at the following synodic arc a:

$$\begin{aligned} a &= \frac{7366}{19} \mod \frac{673\,150}{1843} \\ &= \frac{7366 \times 97}{19 \times 97} \mod \frac{673\,150}{1843} \\ &= \frac{714\,502 \mod 673\,150}{1843} \\ &= 22\frac{806}{1843} \end{aligned}$$

Therefore it is, also, that the difference between the synodic arcs described in a 29- and 30-day civil month is $13\frac{679}{1843} = 13\frac{7\times 97}{19\times 97}$ *du*.

3. The text instructs the user to perform the identical procedure as for the sun, the difference being that before casting out complete circuits and complete lodges, we first multiply the number of days elapsed since origin d by the moon's daily rate of travel:

$$\begin{aligned} &(d \times \frac{254}{19} \times \text{ ERA FACTOR}) \mod \text{ CIRCUITS OF HEAVEN} \\ &\quad \div \text{ ERA FACTOR } = t + \frac{t_r}{\text{ERA FACTOR}} \end{aligned}$$

   And since

$$\text{LUNAR CIRCUITS } = \frac{254}{19} \times \text{ ERA FACTOR}$$

   the procedure is represented thus:

$$\begin{aligned} &(d \times \text{ LUNAR CIRCUITS}) \mod \text{ CIRCUITS OF HEAVEN} \\ &\quad \div \text{ ERA FACTOR } = t + \frac{t_r}{\text{ERA FACTOR}} \end{aligned} \qquad (11)$$

   In this case,

$$\begin{aligned} t + \frac{t_r}{1843} &= (209\,283 \times 24\,638) \mod 673\,150 \div 1843 \\ &= 357\frac{753}{1843} \end{aligned}$$

   Casting out complete lodges, we thus arrive at a position of Dipper.$_{\text{L08}}$ $13\frac{753}{1843}$ *du* for *midnight*, new moon day, month XI.

4. One presumably adds this data to a table like Table 3 or 4 on p. 13.

***

推合朔度術曰：以章歲乘朔小餘，滿通法為大分，不盡為小分。以大分從朔夜半日度分，分滿紀法從度，命如前，則天正十一月合朔日月所共合度也。

(1.10) Technique for calculating the *du*[-position] of syzygy: Multiply the new moon lesser remainder (d_r) by the RULE YEARS (19), and that which fills (divide by) the COMMUNICATION FACTOR (47) makes the greater parts (p); that which is not exhausted gives you the lesser parts (p_r). Make the greater parts (p) go together with (add to) the *du* (t) and parts (t_r) of the sun at midnight, new moon day, and if the parts fills the ERA FACTOR (1843), then it goes together with (adds to) the *du* (t).

求次月，加度二十九，大分九百七十七，小分四十二，小分滿通法從大分，大分滿紀法從度。經斗除其分，則次月合朔日月所共合度也。

To find the subsequent month, add 29 to the *du* (t) & 977 to the greater parts & 42 to the lesser parts; and if the lesser parts (p_r) fills the COMMUNICATION FACTOR (47), it goes together with (is added to) the greater parts (p), and if the greater parts (p) fills the ERA FACTOR (1843), it goes together with (is added to) the *du* (t). When passing through Dipper.$_{\text{L08}}$, cast out (subtract) its parts (455), and that's the conjunction *du* common to sun & moon at syzygy of the subsequent month.

Notes:

1. Unknown values:

d_r	accumulated days from era head to month XI(remainder) (from Technique 1.2)
t	accumulated *du* from era head to I-1, 00h (integer) (from Technique 1.8)
t_r	accumulated *du* from era head to I-1, 00h (remainder) (from Technique 1.8)
p	the denominator d_r expressed in ERA FACTORS (1843)
p_r	the fractional remainder from converting d_r to p
t'	accumulated *du* at syzygy (integer)
t'_r	accumulated *du* at syzygy (remainder)

2. Explanation: In Technique 1.8, we found the *du*-position of the sun at *midnight*, new moon day, astronomical first month. We did this by converting the *integer* number of days elapsed from era head into *du*. To find its position *at syzygy*, we thus need to convert the *non-integer* part of a day elapsed since era head (d_r) into *du*, as well, and add this to the integer number of *du* elapsed. Unfortunately, the lesser remainder d_r produced in Technique 1.2 is given in terms of DAY FACTORS (4559), so we must convert it first to ERA FACTORS (1843) using the ratio

$$47 \times \text{ ERA FACTOR } = 19 \times \text{ DAY FACTOR}$$

3. This gives us the following procedure:

$$d_r \times \frac{\text{RULE YEARS}}{\text{COMM. FACTOR}} = p + \frac{p_r}{\text{COMM. FACTOR}} \tag{12}$$

$$t + \frac{t_r + p + \frac{p_r}{\text{COMM. FACTOR}}}{\text{ERA FACTOR}} = t' + \frac{t'_r}{\text{ERA FACTOR}} \tag{13}$$

In this case,

$$\begin{aligned} p + \frac{p_r}{47} &= 1613 \times \frac{19}{47} \\ &= 652\frac{3}{47} \end{aligned}$$

thus,

$$\begin{aligned} t' + \frac{t'_r}{1843} &= 361 + \frac{1446 + 652 + \frac{3}{47}}{1843} \\ &= 362\frac{255\frac{3}{47}}{1843} \end{aligned}$$

Or, once we cast out complete lodges, Dipper.$_{\text{L08}}$ $18\frac{255\frac{3}{47}}{1843}$ *du*.

4. In one synodic month—$29\frac{2419}{4559}$ days—the sun will have traveled $29\frac{2419}{4559}$ *du*, where it will be once again in conjunction with the moon. Therefore, to find the position of this subsequent moon, we must convert the synodic month from DAY FACTORS (4559) to ERA FACTORS (1843)

$$29\frac{2419}{4559} \times \frac{19}{47} = 29\frac{977\frac{42}{47}}{1843}$$

and then, as in Technique 1.3, successively add this arc to the moon's initial position.

***

推弦望日所在度：加合朔度七，大分七百五，小分十，微分一，微分滿二從小分，小分滿通法從大分，大分滿紀法從度，命如前，則上弦日所在度也。又加得望、下弦、後月合也。

Calculate the *du* at which the sun is located at quarter and full moon: Add 7 to the syzygy *du* (t') & 705 to the greater parts (p) & 10 to the lesser parts (p_r) & 1 to the minute parts; when the minute parts fills 2, it goes together with (is added to) the lesser parts (p_r), when the lesser parts fills the COMMUNICATION FACTOR (47), it goes together with (is added to) the greater parts (p), and when the greater parts (p) fills the ERA FACTOR (1843), it goes together with (is added to) the *du* [at syzygy] (t'). Count like before, and that is the *du* at which the sun is located at first quarter. Add again to obtain full moon, third quarter, & the next month's conjunction.

推弦望月所在度：加合朔度九十八，大分千二百七十九，小分三十四，數滿命如前，即上弦月所在度也。又加得望下弦後月合也。

Calculate the *du* at which the moon is located at quarter & full moon: Add 98 to the *du* of syzygy (t') & 1279 to the greater parts (p) & 34 to the lesser parts (p_r), and if the numbers fill [their respective units], then count like before, and this is the *du* at which the moon is located at first quarter. Add again to obtain full moon, third quarter, & the next month's conjunction.

Notes:

1. The first paragraph instructs the user to calculate the position of the sun at each new, quarter, and full moon based on the fact that it travels $7\frac{705\frac{10\frac{1}{2}}{47}}{1843}$ *du* (= $\frac{1}{4}$ synodic month, with 1 day = 1 *du*) between consecutive points.

2. The second paragraph then instructs the user to calculate the position of the moon at each new, quarter, and full moon based on *its* constant rate of travel over the same period:

$$7\frac{705\frac{10\frac{1}{2}}{47}}{1843} \times \frac{254}{19} = 98\frac{1279\frac{34}{47}}{1843}$$

3. Note that the later technique prefers that the user calculate the moon's rate of travel rather than resorting to a more spatial shortcut, like placing the moon $\frac{1}{4}$, $\frac{2}{4}$, & $\frac{3}{4}$ of the sky away from it. This makes one wonder why it is that we need to calculate the *sun's* position at quarter and full moon at all.

***

推日月昏明度術曰：日以紀法，月以月周，乘所近節氣夜漏，二百而一，為明分。日以減紀法，月以減月周，餘為昏分。各以加夜半，如法為度。

(1.11) Technique for calculating *du*[-position] of the sun & moon at dusk & dawn: Multiply the night clepsydra [marks] (c) of the nearest nodal [or medial] *qi* by the ERA FACTOR (1843), if for the sun, or the LUNAR CIRCUITS (24 638), if for the moon, and count 1 for each 200 for the dawn parts (j). Reduce (subtract) this from the ERA FACTOR (1843), if for the sun, or the LUNAR CIRCUITS (24 638), if for the moon, and the remainder makes the dusk parts (k). Add each to midnight, using [the appropriate] divisor (*fa*) for *du*.

Notes:

1. Unknown values:

c	night clepsydra marks (from a solar table, not included)
j	the number of marks (*ke*) from midnight to dawn
k	the number of marks from from midnight to dusk

2. Note, again, that the user must consult a solar table not included in the Luminous Inception system to find the value c for the right *qi*, e.g. that recorded in the Han Quarter-remainder system of A.D. 85/86. The "night clepsydra marks" are the number of marks (out of 100) allotted to the night at any given *qi*. Night is defined as sunset to sunset minus $2\frac{1}{2}$ marks each for dusk and dawn. Let's say that we want to calculate for winter solstice, A.D. 451. The Quarter-remainder system solar table[17] gives us a ratio of 45 : 55 marks between day & night at winter solstice. Thus, for our current purposes, let's take

$$c = 55$$

3. The procedure uses different factors depending on whether the user is operating in solar or lunar terms:

   for the sun,

$$j_{\odot} = c \times \text{ ERA FACTOR } \div 200 \tag{14}$$

   and

$$k_{\odot} = \text{ ERA FACTOR } - j_{\odot} \tag{15}$$

   while for the moon,

$$j_{☾} = c \times \text{ LUNAR CIRCUITS } \div 200 \tag{16}$$

   and

$$k_{☾} = \text{ LUNAR CIRCUITS } - j_{☾} \tag{17}$$

   Note that one divides by 200 rather than 100 (the number of marks in one nychthemeron) because $c/2$ is the time elapsed from *midnight* to the end of night, and thus half the length of night.

   In the case of winter solstice, A.D. 451:

$$j_{\odot} = 506\frac{165}{200}$$
$$k_{\odot} = 1336\frac{35}{200}$$

   and

$$j_{☾} = 6775\frac{90}{200}$$
$$k_{☾} = 17862\frac{110}{200}$$

4. What do you do with these values? System manuals do not tell us. However, the function seems to be to give the user a pre-computed index (like the LIMIT NUMBERS in Table 6) for quickly determining when a phenomena with a lesser remainder (d_r) fell *vis-à-vis* seasonal daylight hours. In

[17] On which, see Christopher Cullen, "Huo Rong's Observation Programme of AD 102 and the *Han Li* Solar Table," *Journal for the history of astronomy* 38, no. 1 (2007): 75–98.

the case of eclipses, for example, this index is helpful since neither lunar eclipses that occur during [the Chinese observer's] daylight hours nor solar eclipses that occur during [his] nighttime hours will be visible [to him].

***

3.2 Lunar astronomy

推合朔交會月蝕術曰：置所入紀朔積分，以所入紀交會差率之數加之，以會通去之，餘則所求年天正十一月合朔去交度分也。以通數加之，滿會通去之，餘則次月合朔去交度分也。以朔望合數各加其月合朔去交度分，滿會通去之，餘則各其月望去交度分也。朔望去交分如朔望合數以下，入交限數以上者，朔則交會，望則月蝕。

(2.1) Technique for calculating conjunction crossing-coincidence & lunar eclipse: set out the new moon accumulated parts into the era entered (p), add to this the number of the CROSSING-COINCIDENCE difference LÜ for the era entered (D_e), and remove from it [full] COINCIDENCE COMMUNICATIONS (790 110), the remainder (quotient) of which is the the *du*-part distance from crossing at the syzygy of the astronomical first month, month XI, for the year sought (n_{XI}). Add to this the COMMUNICATION NUMBER (134 630), and if [the sum] fills the COINCIDENCE COMMUNICATIONS (790 110), remove it (i.e. modulo out the latter), and what remains is the next month's conjunction-crossing DU-part distance (n_m). Add each month's conjunction-crossing *du*-parts distance to the NEW-FULL CONJUNCTION NUMBER (67 315), and if [the sum] fills the COINCIDENCE COMMUNICATIONS (790 110), remove it (i.e. modulo out the latter), and what remains is each month's opposition-crossing DU-part distance (f_m). For those *du*-part distances from crossing at either new or full moon that are like (equal to) or less than the NEW-FULL CONJUNCTION NUMBER (67 315) or [equal to or] greater than the CROSSING-ENTRY LIMIT NUMBER (722 795), if it is new moon, there will be a crossing-coincidence, while if it is full moon, there will be a lunar eclipse.

Notes:

1. Unknown values:

p	new moon accumulated parts into the era entered (from Technique 1.2; expressed in DAY FACTOR [4559] parts)
D_e	CROSSING-COINCIDENCE difference LÜ for era entered (from Table 2)
n_m	eclipse period epact at syzygy for month m (expressed in COMMUNICATION NUMBER [134 630] parts)
f_m	opposition-crossing DU-part distance for month m (expressed in COMMUNICATION NUMBER [134 630] parts)

2. Take p, the number of days/*du* elapsed from era head to syzygy, month XI, as expressed in DAY FACTOR (4559) parts from Technique 1.2—in other

words, the product of the multiplication of the integer number of accumulated months from era head by the COMMUNICATION NUMBER (134 630). We use this unit because, if we remember, the eclipse period is expressed in the ratio COINCIDENCE COMMUNICATION (790 110) : COMMUNICATION NUMBER (134 630), which is to say that the nodical and synodic months coincide every $5\frac{116\,960}{134\,630}$ (synodic) months (see p. 7). We add this to the eclipse cycle epact at era head D_e—the CROSSING-COINCIDENCE difference LÜ, from Table 2, as expressed also in COMMUNICATION NUMBER (134630) parts—from which we cast out complete COINCIDENCE COMMUNICATION (790 110) eclipse cycles to find the age of the current eclipse cycle.

$$n_{\text{XI}} = (p + D) \quad \text{mod COINCIDENCE COMM.} \tag{18}$$

In this case,

$$n_{\text{XI}} = (954\,122\,810 + 602\,139) \quad \text{mod } 790\,110$$
$$n_{\text{XI}} = 290\,069$$

3. For each subsequent month, simply add another COMMUNICATION NUMBER (134630) and cast out full COINCIDENCE COMMUNICATIONS (790110) to find the distance between the point of conjunction and the nearest lunar node.

$$n_{\text{XI}+x} = (n_{\text{XI}} + \text{COMM. NUMBER}) \quad \text{mod COINCIDENCE COMM.} \tag{19}$$

For results computed for A.D. 451, see Table 11.

4. To calculate the distance between the nearest lunar node and the point of *opposition*, i.e. full moon, then add *half* of a COMMUNICATION NUMBER (134 630)—i.e. one NEW-FULL CONJUNCTION NUMBER (67 315)—to each value n at conjunction, i.e. new moon, and, again, cast out full COINCIDENCE COMMUNICATIONS (790 110).

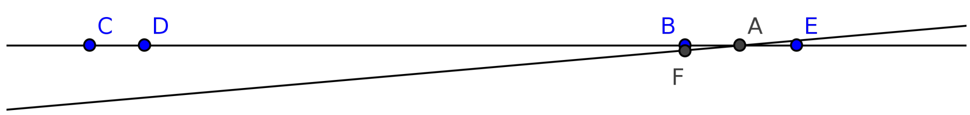
$$f_m = (n_m + \text{NEW-FULL CONJ. NUMBER}) \quad \text{mod COINCIDENCE COMM.} \tag{20}$$

For results computed for A.D. 451, see Table 11.

5. Eclipses occur when the moon is sufficiently close to one of the lunar nodes—the points where the orbit of the sun & moon intersect—represented in Figure 1 by point A. Let point C represent the other node, the distance AC being 180°—the Luminous Inception system represents this distance as COINCIDENCE COMMUNICATION (790 110) (expressed in COMMUNICATION NUMBER [134 630] parts). The Luminous Inception system further posits a point B NEW-FULL CONJUNCTION NUMBER (67 315) distant from "crossing" at and within which eclipse occurs. The Luminous Inception system also gives us a point D CROSSING-ENTRY LIMIT NUMBER (722 795)

distant from “crossing,” which is the difference between the two previous values ($722\,795 = 790\,110 - 67\,315$), and thus a point equidistant from, but on the opposite side of, the other node. Point D thus corresponds to point E *vis-à-vis* the node at point A, and we can imagine a zone of invisibility stretching from point B to point E around node A.

Figure 1: White (lunar) & Yellow (solar) roads and crossing



Therefore, there is a “crossing-coincidence” (possible solar eclipse) if either

$$n_m \leq 67\,315$$
$$n_m \geq 722\,795$$

And there is a lunar eclipse if either

$$f_m \leq 67\,315$$
$$f_m \geq 722\,795$$

6. Liu Hongtao explains the choice of NEW-FULL CONJUNCTION NUMBER (67 315) or, on Figure 1, point B, as follows. The lunar and solar paths intersect at an angle of 6 *du*, which means that travelling at a rate of $\frac{254}{19}$ *du* per day, the mean moon will be at its maximum latitude about seven days from the node or $\frac{254}{19} \times 7 \approx 93.6^{\circ\circ}$, at which point the ratio of latitude to longitude is $6^{\circ\circ} : 93.9^{\circ\circ}$. The distance AB, the NEW-FULL CONJUNCTION NUMBER, is $\frac{67\,315}{4559} \approx 14.7^{\circ\circ}$. Now, for the moon to be eclipsed, we should expect the distance BF to be in the environment of 1 degree/*du* since the earth casts a shadow of about 2° diameter, and this is indeed what we find:[18]

$$\frac{BF}{14.7} = \frac{6}{93.6}$$
$$BF \approx 0.9^{\circ\circ}$$

[18]Liu, *Gudai lifa jisuanfa*, 205.

***

推合朔交會月蝕月在日道表裏術曰：置所入紀朔積分，以所入紀下交會差率之數加之，倍會通去之，餘不滿會通者，紀首表，天正合朔月在表，紀首裏，天正合朔月在裏。滿會通去之，表在裏，裏在表。求次月，以通數加之，滿會通去之，加裏滿在表，加表滿在裏。先交會後月蝕者，朔在表則望在表，朔在裏則望在裏。先月蝕後交會者，看食月朔在裏則望在表，朔在表則望在裏。交會月蝕如朔望合數以下，則前交後會；如入交限數以上，則前會後交。其前交後會近於限數者，則豫伺之〔前月〕；前會後交近於限數者，則後伺之〔後月〕。

2.2 Technique for calculating whether conjunction crossing-coincidence & lunar eclipse are outside or inside the solar road: set out the new moon accumulated parts into the era entered (p), add to this the number of the CROSSING-COINCIDENCE difference LÜ for the bottom of the era entered (D_e),[19] and remove from this double the COINCIDENCE COMMUNICATION (790 110): if what remains does not fill the COINCIDENCE COMMUNICATION (790 110) and the era head is "outside," then the moon is on the outside at syzygy, astronomical first month, while if the era head is "inside," the moon is on the inside at syzygy, astronomical first month; if it (what remains) fills the COINCIDENCE COMMUNICATION (790 110), remove it (the latter), "outside" is on the inside, and "inside" on the outside. To find the next month, add to it (p) the COMMUNICATION NUMBER (134 630), and if it (the sum) fills the COINCIDENCE COMMUNICATION (790 110), remove it (the latter)—if filled adding to an "inside," it is on the outside, and if filled adding to an "outside," it is on the inside.

In the case of a crossing-coincidence followed by a lunar eclipse, if new moon is on the outside, then full moon is on the outside, and if new moon is on the inside, then full moon is on the inside. In the case of a lunar eclipse followed by a crossing-coincidence, look at the month of eclipse, and if new moon is on the inside, then full moon is on the outside, and if new moon is on the outside, then full moon is on the inside.

If crossing-coincidence or lunar eclipse are like (equal to) or less than the NEW-FULL CONJUNCTION NUMBER (67 315), then crossing proceeds coincidence; and if like (equal to) or greater than the CROSSING-ENTRY LIMIT NUMBER (722 795), then coincidence proceeds crossing.

When there is a crossing proceeding coincidence that is close to the limit number, then watch for it in advance; and when there is a coincidence proceeding a crossing that is close to the limit number, then watch for it later.[20]

---

[19]Note: here, the expression "the number of the CROSSING-COINCIDENCE difference LÜ for the bottom of the era entered" 所入紀下交會差率之數 differs from "the number of the CROSSING-COINCIDENCE difference LÜ for the era entered" 所入紀交會差率之數 seen in Technique 2.1 by the single graph *xia* 下 "below, next," which would seem to imply that the user should take the value for the *next* era (D_{e+1}) from Table 2; however, this is not the case.

[20]Note that I have emended the text here from the *Jin shu* version. The *Song shu* reads:

Notes:

1. Unknown values:

p	new moon accumulated parts into the era entered (from Technique 1.2; expressed in DAY FACTOR [4559] parts)
D_e	CROSSING-COINCIDENCE difference LÜ for era entered (from Table 2)
s	the side index

2. The COINCIDENCE COMMUNICATION (790 110) is the distance from one "crossing" (lunar node) to the next; after "crossing" the moon goes from "outside" (south) to "inside" (north) of the ecliptic and *vice versa.* As explained on p. 8 in the key to Table 2, each time we fill an odd number of COINCIDENCE COMMUNICATIONS (790 110), we alternate between "outside" (*biao* 表) & "inside" (*li* 裏); the elapse of an *even* number of COINCIDENCE COMMUNICATIONS (790 110) thus leaves the side unchanged. Therefore, to find what side of the ecliptic the moon is at syzygy, astronomical first month, we add the number of integer months elapsed since era head, expressed in COMMUNICATION NUMBER (134 630) parts (see Technique 2.1), to the eclipse cycle epact at era head and cast out *pairs* of COINCIDENCE COMMUNICATIONS (790 110); and if the result is greater than one COINCIDENCE COMMUNICATION (790 110), we know that an odd number of them have elapsed, thus signalling an alternation in sides (from era head).

$$s_{\mathrm{XI}} = (p + D_e) \mod 2 \times \text{COINCIDENCE COMM.} \tag{21}$$

And, if

$$s_{\mathrm{XI}} > \text{COINCIDENCE COMM.}$$

then switch "inside" ↔ "outside."

In this case,

$$\begin{aligned} s_{\mathrm{XI}} &= 954\,122\,810 + 602\,139 \mod 2 \times 790\,110 \\ &= 290\,069 \\ s_{\mathrm{XI}} &< 790\,110 \end{aligned}$$

3. As in Technique 2.1, the next month's side index value (s) is found by sequentially adding the COMMUNICATION NUMBER (134 630) and casting out full COINCIDENCE COMMUNICATIONS (790 110). For results computed for A.D. 451, see Table 11.

其前交後會近於限數者，則豫伺之**前月**；前會後交近於限數者，則後伺之**後月**。

When there is a crossing proceeding coincidence that is close to the limit number, then watch for it in advance **in the previous month**; and when there is a coincidence proceeding a crossing that is close to the limit number, then watch for it later **in the next month**.

Table 11: Eclipse data computed for A.D. 451

month		sex. date	crossing-coincidence				lunar eclipse			
ast.	civ.		n_m	side	event	ord.	f_m	side	event	ord.
1	XI	*dinghai*.$_{24}$	290069	in	-	-	357384	in	-	-
2	XII	*bingchen*.$_{53}$	424699	in	-	-	492014	in	-	-
3	I	*bingxu*.$_{23}$	559329	in	-	-	626644	in	-	-
4	II	*yimao*.$_{52}$	693959	in	-	-	761274	in	yes	◎ → ✠
5	III	*yiyou*.$_{22}$	38479	out	yes	✠ → ◎	105794	out	-	-
6	IV	*yimao*.$_{52}$	173109	out	-	-	240424	out	-	-
7	V	*jiashen*.$_{21}$	307739	out	-	-	375054	out	-	-
8	VI	*jiayin*.$_{51}$	442369	out	-	-	509684	out	-	-
9	VII	*guiwei*.$_{20}$	576999	out	-	-	644314	out	-	-
10	VIII	*guichou*.$_{50}$	711629	out	-	-	778944	out	yes	◎ → ✠
11	IX	*renwu*.$_{19}$	56149	in	yes	✠ → ◎	123464	in	-	-
12	X	*renzi*.$_{49}$	190779	in	-	-	258094	in	-	-

NOTE: ✠ denotes crossing, ◎ denotes coincidence/syzygy.

4. Next, the text provides us with the following rules of thumb:

   (a) *If crossing-coincidence proceeds lunar eclipse, the new & full moon will be on the same side of the ecliptic,* and, *if vice versa, an alternation.* I can't remember/figure out why this is true, but Table 11 verifies this rule. For an explanation, see Liu Hongtao (2003: 208–208).

   (b) *If the value* n *or* f *is less than or equal to the* NEW-FULL CONJUNCTION NUMBER *(67 315), then crossing (nodal transit) → coincidence (eclipse-conjunction); and if they are greater than the* CROSSING-ENTRY LIMIT NUMBER *(722 795), then coincidence → crossing.* Explanation: referring back to Figure 1, with the moon moving right-to-left, syzygies occurring *right after* nodal crossings fall between points A & B, a distance from A less than or equal to the NEW-FULL CONJUNCTION NUMBER (67 315), and those occurring *right before* fall between points D & C, a distance from A greater than or equal to the CROSSING-ENTRY LIMIT NUMBER (722 795).

   (c) *As concerns crossing-coincidence—potential solar eclipse—if crossing proceeds coincidence, then watch for it in advance, and vice versa.* Liu Hongtao (2003: 210) explains this rule as follows: crossing proceeding coincidence means that lunar eclipse proceeds crossing-coincidence because $f_m \geq 722\,795$, as we see in Table 11 and, conversely, coincidence proceeding crossing means crossing-coincidence proceeds lunar eclipse because $n_m \geq 722\,795$; therefore, if one is uncertain whether crossing-coincidence falls within the eclipse limit, one can quickly verify from the preceding/proceeding value for lunar eclipse. This explanation feels wrong to me for two reasons. First, this double-checking seems is unnecessarily redundant since it is just as easy to determine whether n_m is within the eclipse limit as it is for f_m. Second, the use of the terms *yu* 豫 "in advance" and *si* 伺 "observe" or "wait" imply a *temporal* rather than *spatial* sequence of checking, the object of reference probably being the sky rather than a table. Note, however, that this reading is only possible if we follow the *Jin shu* version of the text (see p. 32 n. 20).

***

求去交度術曰：其前交後會者，今去交度分如日法而一，所得則却去交度分也。其前會後交者，以去交度分減會通，餘如日法而一，所得則前去交度，餘皆度分也。去交度十五以上，雖交不蝕也。十以下是蝕，十以上虧蝕微少，光晷相及而已。虧之多少，以十五為法。

2.3 Technique to find the *du* distance from crossing: in the case of crossing proceeding coincidence, count one for each DAY FACTOR (4559) in the crossing distance *du*-parts (n_m & f_m), and what is obtained is is the backward distance in *du*-parts (sic.) to crossing ($-\delta$); in the case of coincidence proceeding crossing, subtract the crossing distance *du*-parts (n_m & f_m) from the COINCIDENCE COMMUNICATION (790 110), count one for each DAY FACTOR (4559) in the remainder (difference), and what is obtained is the forward distance in *du* from crossing ($+\delta$)—the remainders are all *du* parts. If the crossing distance *du* (δ) is greater than or equal to 15, then there is crossing but no eclipse; if it is less than or equal to 10, there is definitely an eclipse; and if it is greater than or equal to 10 (i.e. if $15 > \delta > 10$), the eclipse depletion will be minute, a meeting of light & shadow and that's it. For the extent of depletion, we use 15 as the denominator (*fa*).

Notes:

1. Unknown values:

n_m	crossing distance *du*-parts (new moon) (from Technique 2.2, Table 11)
f_m	crossing distance *du*-parts (full moon) (from Technique 2.2, Table 11)
δ	*du* distance from crossing

2. Convert the the crossing distance *du*-parts (n_m & f_m) from parts into *du* by dividing by the DAY FACTOR (4559):

   If n_m or f_m are less than or equal to the NEW-FULL CONJUNCTION NUMBER (67 315),

$$-\delta_n = n_m \div \text{ DAY FACTOR} \quad (22)$$
$$-\delta_f = f_m \div \text{ DAY FACTOR} \quad (23)$$

   If n_m or f_m are greater than or equal to CROSSING-ENTRY LIMIT NUMBER (722 795),

$$+\delta_n = (\text{COINCIDENCE COMM.} - n_m) \div \text{ DAY FACTOR} \tag{24}$$

$$+\delta_f = (\text{COINCIDENCE COMM.} - f_m) \div \text{ DAY FACTOR} \tag{25}$$

Whether the distance is forward or backward—the sign of δ—depends on whether crossing proceeds coincidence or vice versa, and for this we shall represent forward distance a positive value and backward distance by a negative value. Performing this operation on the four crossing-coincidence (solar) and lunar eclipse events calculated above in Table 11, we get the following:

Table 12: Eclipse data computed for A.D. 451 (continued)

month ast.	month civ.	sex. date	event	order	δ	mag.
4	II	*yimao*.52	lun.	◎ → ✠	$+6\frac{4480}{4559}^{\circ\circ}$	(full)
5	III	*yiyou*.22	sol.	✠ → ◎	$-8\frac{2007}{4559}^{\circ\circ}$	(full)
10	VIII	*guichou*.50	lun.	◎ → ✠	$+2\frac{3914}{4559}^{\circ\circ}$	(full)
11	IX	*renwu*.19	sol.	✠ → ◎	$-12\frac{1441}{4559}^{\circ\circ}$	minute

NOTE: the fractions are probably to be left off.

3. Now, because the absolute value of the *du* distance from crossing (δ) for two lunar eclipses (months II & VIII) and the crossing-coincidence at month III are all less than 10, we should expect complete eclipses; being between 10 and 15, we should expect the crossing-coincidence in month IX to be "minute."

***

求日蝕虧起角術曰：其月在外道，先交後會者，虧蝕西南角起；先會後交者，虧蝕東南角起。其月在內道，先交後會者，虧食西北角起；先會後交者，虧食東北角起。虧食分多少，如上以十五為法。會交中者，蝕盡。月蝕在日之衝，虧角與上反也。

2.4 Technique for finding the corner from which eclipse/depletion begins: If the moon is on the outer road, then in the case of crossing proceeding coincidence, eclipse/depletion begins from the south west corner, and in the case of coincidence proceeding crossing, eclipse/depletion begins from the south east corner. If the moon is on the inner road, then in the case of crossing proceeding coincidence, eclipse/depletion begins from the north west corner, and in the case of coincidence proceeding crossing, eclipse/depletion begins from the north east corner. The eclipse/depletion parts are quantified as [stated] above using 15 as the divisor (*fa*). If coincidence occurs right at crossing, then the eclipse is total. Lunar eclipses are opposite the sun, [so] their depletion corner is the reverse of that above.

Notes:

Table 13: Rules for determining eclipse corner

road	order	corner
crossing-coincidence		
out	✠ → ◎	SW
out	◎ → ✠	SE
in	✠ → ◎	NW
in	◎ → ✠	NE
lunar eclipse		
out	✠ → ◎	NE
out	◎ → ✠	NW
in	✠ → ◎	SE
in	◎ → ✠	SW

1. From the aforementioned rules, summarized it Table 13, we are able to calculate the direction of the four previous eclipse phenomena as in Table 14:

Table 14: Eclipse data computed for A.D. 451 (continued 2)

month ast.	month civ.	sex. date	event	road	order	corner	index
4	II	*yimao.*$_{52}$	lun.	in	◎ → ✠	NW	6/15
5	III	*yiyou.*$_{22}$	sol.	out	✠ → ◎	SW	8/15
10	VIII	*guichou.*$_{50}$	lun.	out	◎ → ✠	SW	2/15
11	IX	*renwu.*$_{19}$	sol.	in	✠ → ◎	NW	12/15

2. Note that it's odd that the eclipse direction & index is computed *before* applying the equation of center, which follows.

***

推合朔交會月蝕入遲疾曆術曰：置所入紀朔積分，以所入紀下遲疾差率之數加之，以通周去之，餘滿日法得一日，不盡為日餘，命日算外，則所求年天正十一月合朔入曆日也。求次月，加一日，日餘四千四百五十。求望，加十四日，日餘三千四百八十九。日餘滿日法成日，日滿二十七去之。又除餘如周日餘，日餘不足除者，減一日，加周虛。求次月，加一日，日餘四千四百五十。

2.5 Technique for calculating the speed sequence entry for conjunctions, crossing-coincidences, & lunar eclipses: set out the new moon accumulated parts into the era entered (p), add to this the number of the slow-fast difference LÜ for the era entered (E_e), remove from it [full] COMMUNICATION CIRCUITS (125621), and obtain one for each time the remainder fills the *day factor* (4559), while that which is not exhausted is the day remainder (s_r); count the day exclusively and that is the sequence entry day (s) for the

Table 15: Lunar speed sequence table

1 day	2 lun. motion *du* speed	3 increase /decrease LÜ	4 accumulated excess/deficit	5 lunar motion parts
01	$14\frac{14}{19}$	+26	-	280
02	$14\frac{11}{19}$	+23	+118 534	277
03	$14\frac{8}{19}$	+20	+223 391	274
04	$14\frac{5}{19}$	+17	+314 571	271
05	$14\frac{1}{19}$	+13	+392 074	267
06	$13\frac{14}{19}$	+7	+451 341	261
07	$13\frac{7}{19}$	-	+483 254	254
08	$13\frac{1}{19}$	−6	+483 254	248
09	$12\frac{16}{19}$	−10	+455 900	244
10	$12\frac{13}{19}$	−13	+410 310	241
11	$12\frac{11}{19}$	−15	+351 043	239
12	$12\frac{8}{19}$	−18	+282 658	236
13	$12\frac{5}{19}$	−21	+200 596	233
14	$12\frac{3}{19}$	−23	+104 857	231
15	$12\frac{5}{19}$	−21	-	233
16	$12\frac{7}{19}$	−19	−95 739	235
17	$12\frac{9}{19}$	−17	−182 360	237
18	$12\frac{12}{19}$	−14	−259 863	240
19	$12\frac{15}{19}$	−11	−323 689	243
20	$12\frac{18}{19}$	−8	−373 838	246
21	$13\frac{3}{19}$	−4	−410 310	250
22	$13\frac{7}{19}$	-	−428 546	254
23	$13\frac{12}{19}$	+5	−428 546	259
24	$13\frac{18}{19}$	+11	−405 751	265
25	$14\frac{5}{19}$	+17	−355 602	271
26	$14\frac{11}{19}$	+23	−278 099	277
27	$14\frac{7}{19}$	+19	−173 242	278
28	$14\frac{13}{19}+\frac{626}{86621}$	$+25+\frac{626}{86621}$	−63 826	$278+\frac{626}{86621}$

NOTE: **Column 1** gives the *ordinal* day of entry into the speed sequence. **Column 2**, "lunar motion *du* speed" 月行遲疾度, gives the moon's daily motion in *du* & parts. **Column 3**, "increase/decrease LÜ" 損益率, gives the difference between the true & mean daily motion in $\frac{1}{19}$ parts of a *du*, positive values denoting an *yi* 益 "increase," negative values denoting a *sun* 損 "decrease." **Column 4**, "accumulated excess/deficit parts" 盈縮積分, gives the accumulated difference between the true & mean speed—i.e. the equation of center—as expressed in *xiao fen* 小分 "lesser parts" of $\frac{1}{4559\times 19}=\frac{1}{86621}$ *du*; positive values denote an *ying* 盈 "excess," negative values denoting a *suo* 縮 "deficit." **Column 5**, "lunar motion parts" 月行分, gives the moon's daily motion in parts ($\frac{1}{19}$ *du*) only.

new moon day of the astronomical first month, month XI, for the year sought.

To find subsequent months, add 1 day and a day remainder of 4450. To find full moon, add 14 days and a day remainder of 3489. When the day remainder fills the *day factor* (4559) it becomes a day; and when the days fill 27 remove it (the latter from the former) and further cast out from the remainder like the (sic.) CIRCUIT OF DAYS DAY REMAINDER (2528), though if the day remainder is insufficient for casting out, reduce by one day and add the CIRCUIT VOID (2031).

Notes:

1. Unknown values:

p	new moon accumulated parts into the era entered (from Technique 1.2; expressed in DAY FACTOR [4559] parts)
E_e	slow-fast difference LÜ for era entered (from Table 2)
s	speed sequence entry day
s_r	speed sequence entry day remainder

2. In order to adjust for the inequality of lunar motion, the Luminous Inception system posits a "speed sequence" or anomalystic month of $27\frac{2528}{4559}$ days and provides us on Table 15 with a day-by-day progression of speed and equation of center—displacement from the position of the mean moon.[21] We begin in this step by locating the moment of syzygy at new moon, astronomical first month, month XI, of the year sought within the sequence. Take p, the number of days/*du* elapsed from era head to syzygy, month XI, as expressed in DAY FACTOR (4559) parts from Technique 1.2—in other words, the product of the multiplication of the integer number of accumulated months from era head by the COMMUNICATION NUMBER (134 630). We add this to the anomalystic epact at era head E_e—the slow-fast difference LÜ, from Table 2, as expressed also COMMUNICATION CIRCUITS (125 621)—from which we cast out complete COMMUNICATION CIRCUITS (125 621) to find the age of the current anomalystic month.

$$s + \frac{s_r}{\text{DAY FACTOR}} = \frac{(p + E_e) \mod \text{COMM. CIRCUITS}}{\text{DAY FACTOR}} + 1 \tag{26}$$

In this case,

$$\begin{aligned} s + \frac{s_r}{4559} &= \frac{(954\,122\,810 + 43\,587) \mod 125\,621}{4559} + 1 \\ &= 17\frac{1958}{4559} \end{aligned}$$

[21]This method was introduced in Liu Hong's 劉洪 (c. A.D. 135–210) late second-century Supernal Emblem system (*Qianxiang li* 乾象曆), on which see Cullen, "The First Complete Chinese Theory of the Moon."

3. For subsequent months, the text instructs the user to add $1\frac{4450}{4559}$ days, the difference between the synodic and anomalystic months: $29\frac{2419}{4559} - 27\frac{2528}{4559} = 1\frac{4450}{4559}$. For full moon, the text instructs the user to add $14\frac{3489}{4559}$ days, one half of a synodic month: $29\frac{2419}{4559} \div 2 = 14\frac{3489}{4559}$. In each case, the user is instructed to cast out complete anomalystic months using a somewhat convoluted method that deals with the integer and fractional components of the values separately.

***

推合朔交會月蝕定大小餘：以入曆日餘，乘所入曆損益率，以損益盈縮積分為定積分。以章歲減所入曆月行分，餘以除之，所得以盈減縮加本小餘。加之滿日法者，交會加時在後日；減之，不足者，交會加時在前日。月蝕者，隨定大小餘為日加時。

2.6 Calculate the fixed greater & lesser remainders for conjunctions, crossing-coincidences, & lunar eclipses: multiply the sequence-entry day remainder (s_r) by the increase/decrease LÜ appropriate to sequence-entry (x_s), and increase/decrease by this the accumulated excess/deficit parts (y_s), which gives you the fixed accumulated parts (a). Reduce the lunar motion parts appropriate to sequence-entry (z_s) by the RULE YEARS (19) and cast out from this (a, the previous sum) the remainder (of this operation), and then add (*jia*) to the original lesser remainder (d_r) using [the rule] "excess : reduce :: deficit : add." If you add it and it fills the DAY FACTOR (4559), then the appended hour of crossing-coincidence is on the next day; if you reduce it and it is insufficient, then crossing-coincidence is on the previous day. In the case of lunar eclipses, the date and appended hour follows from the fixed greater (g') & lesser remainder (d'_r).

入曆在周日者，以周日日餘乘縮積分，為定積分。以損率乘入曆日餘，又以周日日餘乘之，以周日日度小分并之，以損定積分，餘為後定積分。以章歲減周日月行分，餘以周日日餘乘之，以周日度小分并之，以除後定積分，所得以加本小餘，如上法。

If sequence entry is on the circuit day (the fractional day at the end of the sequence), multiply the accumulated deficit parts (y_s) by the CIRCUIT OF DAYS DAY REMAINDER (2528), which gives you the fixed accumulated parts (a), and multiply the sequence-entry day remainder (s_r) by the decrease LÜ (x_s). Further, multiply this by the CIRCUIT OF DAYS DAY REMAINDER (2528), add this to the circuit of days day remainder lesser parts (626), and fix the accumulated parts (a) through decrease. The remainder gives you the subsequently fixed accumulated parts (a'). Reduce the lunar motion parts (z_s) by the RULE YEARS (19), multiply the remainder by the CIRCUIT OF DAYS DAY REMAINDER (2528), add this to the circuit of days *du* lesser parts (626), and cast this out from the the subsequently fixed accumulated parts (a'). Add what is obtained to the original lesser remainder (d_r) as per the above method.

Notes:

1. Unknown values:

s	speed sequence entry day (from Technique 2.5)
s_r	speed sequence entry day remainder (from Technique 2.5)
g	(new moon date) greater remainder (from Technique 1.2)
d_r	(new moon date) lesser remainder (from Technique 1.2)
x_s	increase/decrease LÜ (from Table 15)
y_s	accumulated excess/deficit parts (from Table 15)
z_s	lunar motion parts (from Table 15)
a	fixed accumulated parts
d'_r	fixed lesser remainder

2. Table 15 gives us equations of center—accumulated excess/deficit parts—at one-day intervals. To find the equation of center for a mixed number of days, we must therefore interpolate. We begin by multiplying the fraction of a day elapsed by the difference between true & mean lunar motion, which either add to or subtract from the accumulated difference of true & mean speed for the integer number of days elapsed. This we then divide by the difference between lunar and solar velocities—254/19 and 19/19 *du* per day, respectively—and use the result to *ding* 定 "fix" the moment of mean conjunction into the moment of true conjunction.

$$a = y_s \pm s_r \times x_s \tag{27}$$

$$d'_r = d_r \pm a \div (z_s - \text{RULE YEARS}) \tag{28}$$

In this case,

$$\begin{aligned} a &= -182\,360 + 1958 \times -17 \\ &= -215\,646 \end{aligned}$$

$$\begin{aligned} d'_r &= 1613 + (-215\,646) \div (237 - 19) \\ &= 2602 \end{aligned}$$

Since the fixed lesser remainder is neither greater than the DAY FACTOR (4559) nor less than 0, the date, or the greater remainder (g), thus remains the same: in this case, $g = 3$. To convert to civil time, we then count off from the date of era head: being in the third era, our fixed greater and lesser remainders place us $3\frac{2602}{4559}$ days past midnight, day *jiashen*.$_{21}$, i.e. midway through day *dinghai*.$_{24}$.

3. For subsequent months, see Table 16.

4. The second half of the technique is devoted to the special case of an instant falling on the fractional $\frac{2528}{4559}$ of a day at the end of the speed sequence. The formula is as follows:

$$d'_r = d_r + \frac{\text{CDDR} \times y_s - (x_s \times s_r \times \text{CDDR} + 626)}{(z_s - \text{RULE YEARS}) \times \text{CDDR} + 626} \tag{29}$$

***

推加時：以十二乘定小餘，滿日法得一辰，數從子起，算外，則朔望加時所在辰也。有餘不盡者四之，如日法而一為少，二為半，三為太。又有餘者三之，如日法而一為強，半法以上排成之，不滿半法廢棄之。以強并少為少強，并半為半強，并太為太強。得二強者為少弱，以之并少為半弱，以之并半為太弱，以之并太為一辰弱。以所在辰命之，則各得其少、太、半及強、弱也。其月蝕望在中節前後四日以還者，視限數；五日以上者，視間限。定小餘如間限、限數以下者，以算上為日。

2.7 Calculate appended hour: multiply the fixed lesser remainder (d'_r) by 12 and get one chronogram (*chen*) for each time it fills the DAY FACTOR (4559); count off from *zi*.$_{\text{B01}}$, counting exclusively, and that is the chronogram of the appended hour of new & full moon (h). If there is a remainder that is not exhausted, quadruple it, and if the DAY FACTOR (4559) goes into it once, that gives you "a few" ($\frac{1}{4}$), if twice that gives you "half," and if thrice, that gives you "grand" ($\frac{3}{4}$). Further, if there is a remainder [from this], treble it, and if the DAY FACTOR (4559) goes into it once, that gives you "strong" ($+\frac{1}{12}$), and if [what remains] is over half the divisor (*fa*), then complete it by moving it up a row*, [otherwise] discard it. Combine "strong" with "a few" and you get "a few strong" ($\frac{4}{12}$), with "half" and you get "a half strong" ($\frac{7}{12}$), and with "great" and you get "great strong" ($\frac{10}{12}$). If you get two "strongs" you get "a few weak" ($\frac{2}{12}$); combine them with "a few," you get "a half weak" ($\frac{5}{12}$), combine them with "a half," you get "great weak" ($\frac{8}{12}$), and combine them with a "great," and you get "one chronogram weak" ($\frac{11}{12}$). Count these from the current chronogram to its "few," "great," & half as well as "strong" & "weak." If the full moon of lunar eclipse is within four days of either medial or nodal [*qi*], see the LIMIT NUMBER; if more than five days, see the INTERVAL LIMIT (Table 6). If the fixed lesser remainder (d'_r) is below the INTERVAL LIMIT or LIMIT NUMBER, it is counted as belonging to the previous day.

Notes:

1. Unknown values:

d'_r	fixed lesser remainder (from Technique 2.6)
h	chronogram (double-hour) of appended hour
h_r	chronogram remainder

2. Convert the fixed lesser remainder—the fractional part of a day beyond midnight of the instant of true new/full moon—from parts of a DAY FACTOR (4559) into double-hours:

$$h + \frac{h_r}{12} = \frac{d'_r}{\text{DAY FACTOR}} \times 12 \tag{30}$$

In this case,

$$\begin{aligned} h + \frac{h_r}{12} &= \frac{2602}{4559} \times 12 \\ &\approx 6 + \frac{10}{12} \end{aligned}$$

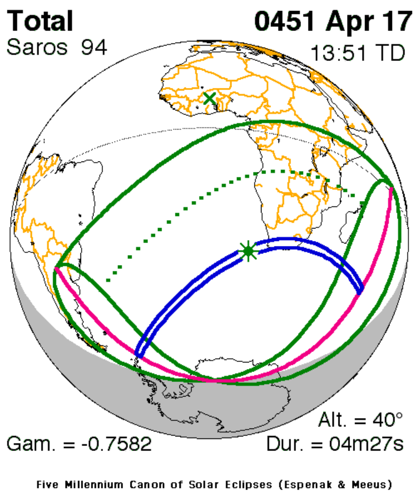
3. On "limits," see p. 13 item 2.

Table 16: Fixed new & full moon table computed for A.D. 451

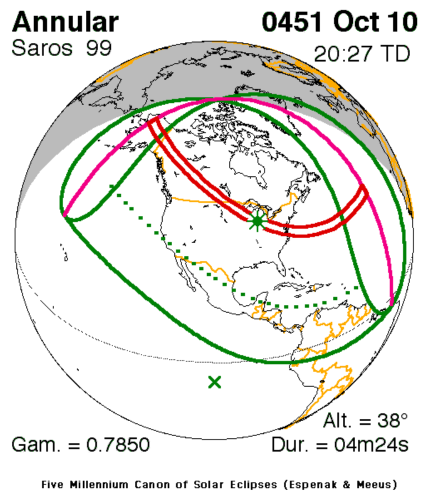
month		phen.	g	d_r	s	s_r	a	d'_r	h	h_r
1	XI	●	*dinghai*.24	1613	17	1958	−215646	2602	*wu*.B07	$\frac{10}{12}$
		○	*renyin*.39	543	4	2919	+364194	−902	*you*.B10	$\frac{8}{12}$
2	XII	●	*bingchen*.53	4032	19	1849	−344028	5568	*yin*.B03	$\frac{8}{12}$
		○	*xinwei*.08	2962	6	2810	+471011	1016	*yin*.B03	$\frac{8}{12}$
3	I	●	*bingxu*.23	1892	21	1740	−417270	3698	*you*.B10	$\frac{9}{12}$
		○	*xinchou*.38	822	8	2701	+467048	−1218	*shen*.B09	$\frac{10}{12}$
4	II	●	*yimao*.52	4311	23	1631	−420391	6063	*mao*.B04	$\frac{12}{12}$
		○	*gengwu*.07	3241	10	2592	+376614	1545	*chen*.B05	$\frac{1}{12}$
5	III	●	*yiyou*.22	2171	25	1522	−329728	3479	*you*.B10	$\frac{2}{12}$
		○	*gengzi*.37	1101	12	2483	+237964	4	*zi*.B01	$\frac{0}{12}$
				...						
10	VIII	●	*guichou*.50	589	7	3008	+483254	−1467	*shen*.B09	$\frac{2}{12}$
		○	*dingmao*.04	4078	22	1938	−428546	5902	*mao*.B04	$\frac{6}{12}$
11	IX	●	*renwu*.19	3008	9	2899	+426910	1111	*yin*.B03	$\frac{11}{12}$
		○	*dingyou*.34	1938	24	1829	−385632	3506	*you*.B10	$\frac{3}{12}$
				...						

... (more to come) ...

Figure 2: Solar eclipses in A.D. 451

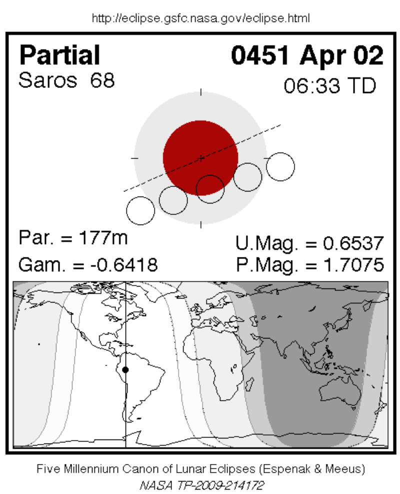


Apr 17 20:17 LAT (Luoyang) (inv.)
Prediction: 18:20/18:30

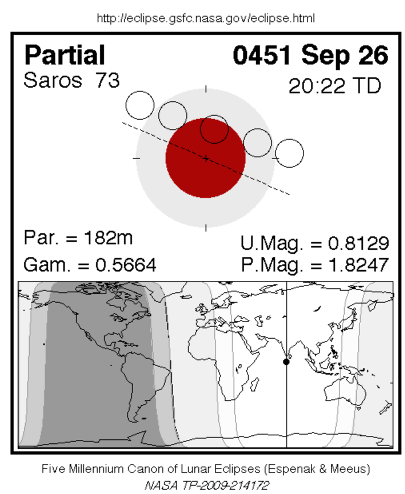


Oct 11 02:14 LAT (Luoyang) (inv.)
Prediction: 05:50/06:00

Figure 3: Lunar eclipses in A.D. 451



Apr 02 12:19 LAT (Luoyang) (inv.)
Prediction: 08:10/08:20 NW



Sep 09 02:20 LAT (Luoyang)
Prediction: 07:00/07:10 SW

Figure 4: The Han-era twenty-eight lodges polar-equatorial coordinate system

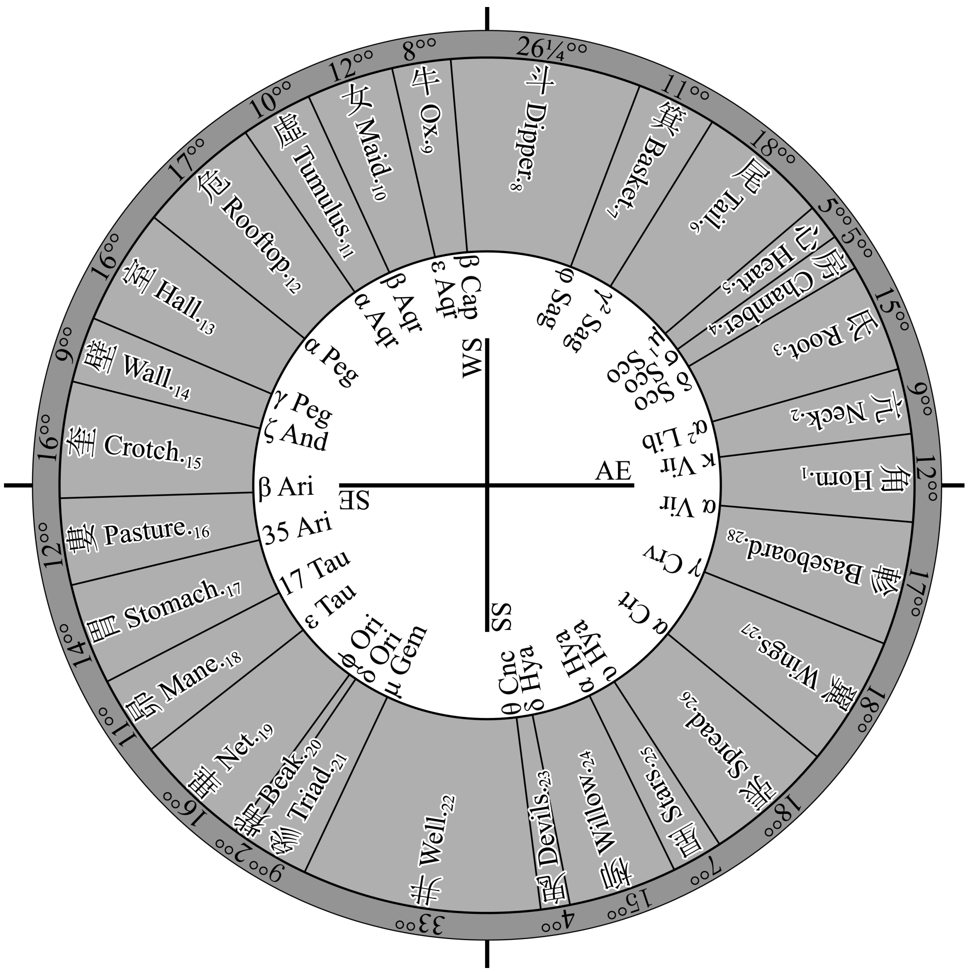


Figure 5: The twenty-four *qi* (epoch A.D. 5)

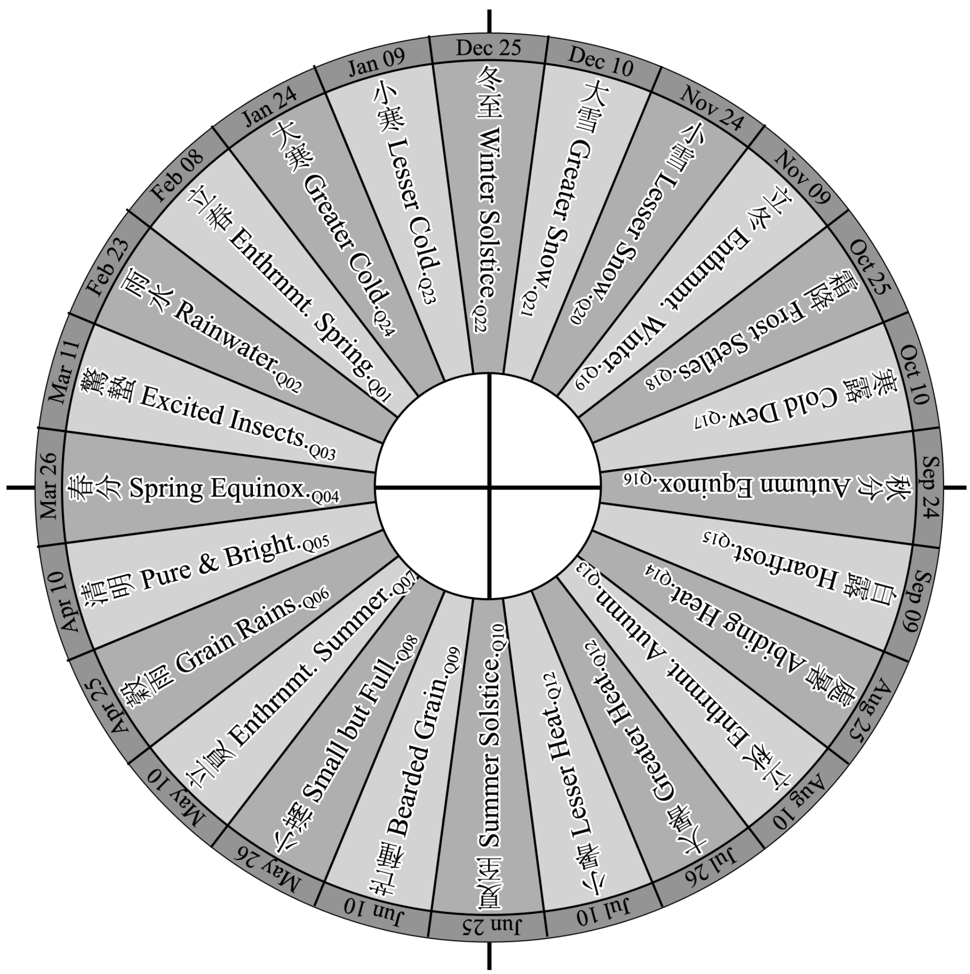


Table 17: Heavenly branches (*tiangan* 天干), earthly stems (*dizhi* 地支), and sexagenary cycle

Stems	Branches	Binomes					
甲	子	甲子	甲戌	甲申	甲午	甲辰	甲寅
jia.$_{\text{S01}}$	*zi*.$_{\text{B01}}$	*jiazi*.$_{01}$	*jiaxu*.$_{11}$	*jiashen*.$_{21}$	*jiawu*.$_{31}$	*jiachen*.$_{41}$	*jiayin*.$_{51}$
乙	丑	乙丑	乙亥	乙酉	乙未	乙巳	乙卯
yi.$_{\text{S02}}$	*chou*.$_{\text{B02}}$	*yichou*.$_{02}$	*yihai*.$_{12}$	*yiyou*.$_{22}$	*yiwei*.$_{32}$	*yisi*.$_{42}$	*yimao*.$_{52}$
丙	寅	丙寅	丙子	丙戌	丙申	丙午	丙辰
bing.$_{\text{S03}}$	*yin*.$_{\text{B03}}$	*bingyin*.$_{03}$	*bingzi*.$_{13}$	*bingxu*.$_{23}$	*bingshen*.$_{33}$	*bingwu*.$_{43}$	*bingchen*.$_{53}$
丁	卯	丁卯	丁丑	丁亥	丁酉	丁未	丁巳
ding.$_{\text{S04}}$	*mao*.$_{\text{B04}}$	*dingmao*.$_{04}$	*dingchou*.$_{14}$	*dinghai*.$_{24}$	*dingyou*.$_{34}$	*dingwei*.$_{44}$	*dingsi*.$_{54}$
戊	辰	戊辰	戊寅	戊子	戊戌	戊申	戊午
wu.$_{\text{S05}}$	*chen*.$_{\text{B05}}$	*wuchen*.$_{05}$	*wuyin*.$_{15}$	*wuzi*.$_{25}$	*wuxu*.$_{35}$	*wushen*.$_{45}$	*wuwu*.$_{55}$
己	巳	己巳	己卯	己丑	己亥	己酉	己未
ji.$_{\text{S06}}$	*si*.$_{\text{B06}}$	*jisi*.$_{06}$	*jimao*.$_{16}$	*jichou*.$_{26}$	*jihai*.$_{36}$	*jiyou*.$_{46}$	*jiwei*.$_{56}$
庚	午	庚午	庚辰	庚寅	庚子	庚戌	庚申
geng.$_{\text{S07}}$	*wu*.$_{\text{B07}}$	*gengwu*.$_{07}$	*gengchen*.$_{17}$	*gengyin*.$_{27}$	*gengzi*.$_{37}$	*gengxu*.$_{47}$	*gengshen*.$_{57}$
辛	未	辛未	辛巳	辛卯	辛丑	辛亥	辛酉
xin.$_{\text{S08}}$	*wei*.$_{\text{B08}}$	*xinwei*.$_{08}$	*xinsi*.$_{18}$	*xinmao*.$_{28}$	*xinchou*.$_{38}$	*xinhai*.$_{48}$	*xinyou*.$_{58}$
壬	申	壬申	壬午	壬辰	壬寅	壬子	壬戌
ren.$_{\text{S09}}$	*shen*.$_{\text{B09}}$	*renshen*.$_{09}$	*renwu*.$_{19}$	*renchen*.$_{29}$	*renyin*.$_{39}$	*renzi*.$_{49}$	*renxu*.$_{59}$
癸	酉	癸酉	癸未	癸巳	癸卯	癸丑	癸亥
gui.$_{\text{S10}}$	*you*.$_{\text{B10}}$	*guiyou*.$_{10}$	*guiwei*.$_{20}$	*guisi*.$_{30}$	*guimao*.$_{40}$	*guichou*.$_{50}$	*guihai*.$_{60}$
	戌						
	xu.$_{\text{B11}}$						
	亥						
	hai.$_{\text{B12}}$						

4 Bibliography

[1] Martzloff, Jean-Claude. *Le calendrier chinois: structure et calculs, 104 av. JC-1644: indétermination céleste et réforme permanente: la construction chinoise officielle du temps quotidien discret à partir d'un temps mathématique caché, linéaire et continu*. Sciences, techniques et civilisations du Moyen Âge à l'aube des Lumières 11. Paris: Champion, 2009.
Explains the means of computing mean and true lunisolar *dates* for a variety of famous *li* systems throughout Chinese history. While comprehensive, this work suffers from two limitations: (1) its focus is purely *calendrical*, and thus neglects, for example, positional, eclipse and planetary *li*; (2) Martzloff attempts to represent actors' procedures in symbolic algebra, which, in my opinion, makes an already difficult topic far more difficult.

[2] Sivin, Nathan. "Cosmos and Computation in Early Chinese Mathematical Astronomy." *T'oung Pao* 2d ser., 55, no. 1/3 (1969): 1–73.
This article has provided the field with the metaphor of interlocking gears with which we still talk about the subject, offering a more-or-less complete account of the Triple Concordance system (*Santong li* 三統曆) of circa A.D. 5 *in terms, once again, of calendrics*. It is also one of the only articles to speak seriously about the very real possibility that the Chinese utilized Mesopotamian-like eclipse cycles. This article is old and has several limitations: (1) it, again, neglects positional, eclipse and planetary *li*; (2) it, again, makes a difficult topic more difficult; (3) the discussion of eclipse cycles is completely speculative; (4) its socio-political conclusions are now very out of date.

[3] ———. *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of Its Many Dimensions and a Translation of Its Records*. New York: Springer, 2009.
A monograph-length complete study of every facet of the greatest achievement in Chinese *li*, which provides significant analysis, and offers nuanced and up-to-date conclusions about socio-political matters. Limitations: (1) the Season Granting system (*Shoushi li* 授時曆) of A.D. 1280 is the most complicated manual of Chinese *li*, and thus takes considerably more time to work through for a beginner; (2) Sivin offers no practice problems, and, in my experience, the most difficult steps of procedure tend to lack sufficient explanation to reconstruct on one's own.

[4] Yabuuti Kiyosi 藪內清. "Astronomical Tables in China, from the Han to the T'ang Dynasties." In *Chūgoku chūsei kagaku gijutsushi no kenkyū* 中國中世科學技術史の研究, edited by Yabuuti Kiyosi 藪內清, 445–492. Tōkyō: Kadokawa shoten, 1963.

[5] ———. "Astronomical Tables in China—from the Wutai to the Ch'ing Dynasties." *Japanese Studies in the History of Science* no. 2 (1963): 94–100.
These two English-language articles by Yabuuti Kiyosi provide the *parameters* for every single extant *li* system, allowing the savvy reader to judge their values and potentially reconstruct working elements of any system with which to perform his/her own calculations.

[6] Liu Hongtao 劉洪濤. *Gudai lifa jisuanfa* 古代曆法計算法. Tianjin: Nankai daxue chubanshe, 2003.
This wonderful little Chinese gem explains every parameter and every step of every system from the Han 漢 (206 B.C. – A.D. 220) to the Sui 隋 (A.D. 581–618). Limitations: relatively few practice examples.