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Abstract:

While the central bank observes the market activity to assess economic fundamentals, it shapes the market outcome through its policy interventions. The more the central bank influences the market, the more it spoils the informational content of economic aggregates. How should the central bank act and communicate when it derives its information from observing the market? This paper analyses the optimal central bank's action and disclosure under endogenous central bank's information for three operational frameworks: pure communication, action and communication, and signaling action. When the central bank takes an action, it would be optimal for the central bank to be fully opaque to prevent its disclosure from deteriorating the information quality of market outcomes. However, in the realistic case where central bank's action is observable, it may be optimal to refrain from implementing any action.

Keywords: heterogeneous information, public information, endogenous information, overreaction, transparency, coordination.

JEL codes: D82, E52, E58
Observing and shaping the market:  
the dilemma of central banks*

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Abstract

While the central bank observes market activity to assess economic fundamentals, it shapes the market outcome through the conduct of monetary policy. A dilemma arises from this dual role because the more the central bank shapes the market, the more it influences the informational content of market prices.

This paper analyses the optimal monetary policy action and disclosure when central bank information is endogenous for three operational frameworks: pure communication, action and communication, and signalling action. Although taking the endogenous nature of central bank information into account calls for less activism from the central bank, full transparency remains optimal when the weight assigned to price dispersion in social welfare takes on its micro-founded value.

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Keywords: endogenous information, overreaction, central bank communication.

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1 Introduction

To conduct monetary policy, the central bank observes market outcomes to glean information on the state of the economy. At the same time, the central bank shapes the economy through the conduct of monetary policy by taking actions and issuing statements. This paper analyses optimal monetary policy when the information accuracy of the central bank depends on its disclosure strategy.

A growing literature addresses the issue of central banks’ communication in coordination games with heterogeneous information. Morris and Shin (2002) (henceforth MS) present a Keynesian beauty contest game where the equilibrium behaviour of economic agents is driven by both a fundamental and a coordination motive. The focal role that public information exerts on higher order beliefs of agents gives rise to an overreaction, which may be detrimental to welfare. If public information is not accurate, it distorts the market outcome away from the economic fundamental. Although MS refer to the case where the provider of public information only considers the possibility of disclosing information, further works have applied the beauty contest mechanism to more realistic and complex environments. These extensions can be classified along two lines.

First, the beauty contest game has been extended by allowing the provider of public information to take an action, as central banks do in reality. James and Lawler (2011) analyse the optimal disclosure strategy when the central bank also takes an action and find that full opacity is optimal. Indeed, by taking a hidden action, the central bank succeeds in stabilising the economy without creating overreaction to any disclosure.\(^1\) However, Baeriswyl and Cornand (2010) argue that taking an action inevitably provides public information because it signals the central bank belief to agents, qualifying the possibility of taking an action under opacity.\(^2\)

Second, the beauty contest game has been applied to the case where the provider of public information collects its own information about economic fundamentals from observing the economic outcome, rather than from directly observing fundamentals. In the model of MS, the central bank information is exogenous in the sense that it is independent of the existence and behaviour of the market. However, the central bank usually assesses the state of the economy by observing market outcomes because there is no such thing as an observable fundamental in reality. Market prices are not only exchange ratios between goods but also aggregators of information, as underlined by Hayek (1945). Prices play an informational role by aggregating agents’ knowledge and beliefs about the state of the economy. However, market prices (and especially financial prices) are influenced by the central bank itself. The central bank is thus both an observer and a shaper of the market outcome.

\(^1\)Angeletos et al. (2016) also study the welfare consequences of public disclosures in micro-founded business cycle model.

\(^2\)There is a large literature that focuses on the interrelation between central bank transparency and stabilisation policy as e.g. Cukierman and Meltzer (1986) and Faust and Svensson (2001). However, the framework here is different as it is characterised by heterogeneous information and central bank information is not perfect.
The ambivalent role of the central bank poses a dilemma in the implementation of monetary policy, which has been documented by Amato and Shin (2006) and Morris and Shin (2005): the more successfully a central bank influences market expectations, the less market outcomes are reliable indicators of the fundamental of the economy. Baeriswyl (2011) extends the model of MS to the case where the central bank observes the economic outcome to draw its information about the fundamental (endogenous information). He shows that although a higher degree of transparency strengthens the influence of the central bank on agents’ decisions, it deteriorates the accuracy of its information, making transparency less desirable. Central bank endogenous information therefore represents an argument challenging the presumed benefit of central bank transparency. As in MS, Baeriswyl (2011) only considers the possibility of the central bank disclosing information but ignores central bank action.

Paper contribution The contribution of the present paper is twofold. First, it derives the welfare implications of endogenous information in a micro-founded model when the central bank takes an action (as in James and Lawler (2011)) and when the action of the central bank provides market participants with a public signal (as in Baeriswyl and Cornand (2010)). Second, it compares the social welfare in the different operational frameworks (pure communication, action and communication, and signalling action) both under exogenous and endogenous information.

Although accounting for the endogeneity of central bank information calls for a higher degree of opacity in principle, full transparency remains optimal when the weight of price dispersion in social welfare takes on its micro-founded value. However, when the weight of price dispersion is lower than in the micro-founded case as in MS, endogenous information enlarges the range of parameter values for which less central bank activism is beneficial to welfare.

Related literature Our paper contributes to a growing literature that accounts for the endogenous nature of information. In coordination games under heterogeneous information, endogenous information often refers to the case where market participants have an incentive to acquire exogenous, market-independent information. This is the case in Burguet and Vives (2000), Hellwig and Veldkamp (2009), Myatt and Wallace (2012) or Paciello and Wiederholt (2014), where agents put effort into collecting private information. By contrast, our definition of endogenous information means that agents glean information from observing the market outcome, which results from the behaviour of market participants, as in the works by Grossman and Stiglitz (1980), Amador and Weill (2010), Hellwig and Venkateswaran (2011), Amador and Weill (2012), Atolia and Chahrour (2013), or Vives (2013). Whereas these papers focus on the learning by market participants from information conveyed by prices, we focus on the learning by the policy maker. We are not the first to draw attention to the issue of the decision maker learning from market prices. For instance, Blinder (1998), Blinder et al. (2001), and Woodford (2005) argue that central bank transparency strengthens the effectiveness of the expectations channel of monetary policy.
Bond et al. (2010) show that if the decision maker, such as the firm management, relies on market prices when deciding on corrective actions, prices may become less revealing about the true fundamental of the firm because the prices may already incorporate the corrective actions expected by market participants. They conclude that it may be desirable for the decision maker to rely less extensively on market-based information. Goldstein et al. (2011) analyse the theoretical implications of the informational feedback from market activities to policy decisions in a model of currency attacks. The central bank learns from the speculative trading in currency markets about the viability of its currency regime and uses the inferred information to guide its policy decisions. Again, the authors conclude that the central bank can improve the effectiveness of its policy by putting a lower weight on the information gleaned from the market. The same conclusion is drawn by Bond and Goldstein (2015), in the context of financial stability measures taken by governments such as bailouts or lending facilities.

Our paper departs from this recent literature because the central bank has no direct source of private information about the economic fundamental at all and, thus, must observe the market activity to glean any information. Although the central bank may have market-independent sources of information in the context of speculative attacks (i.e., the level of its foreign exchange reserves) or financial stability (i.e., the balance sheet of financial institutions), the conduct of monetary policy does not typically benefit from such direct information about macroeconomic fundamentals. Economic fundamentals are abstract constructs of the mind that do not exist and cannot be observed outside the interactions of economic agents in the market.³

An empirical literature has also highlighted that central bank transparency tends to deteriorate the informative value of market outcomes. In an analysis of U.S. data, Ehrmann and Fratzscher (2005) show that with increasing transparency “markets attach more importance to the statements and the balance-of-risk assessments at FOMC meetings and less importance to news about macroeconomic fundamentals.” They conclude that “the reaction of financial markets to the release of macroeconomic fundamentals can be an important source of information for the central bank about the markets’ diverse and possibly deviating views,” and that “under its new disclosure strategy, the Federal Reserve has less such information available.” Although their analysis does not question the benefit of central bank disclosure per se, it suggests that too much transparency may have drawbacks. Relying on an international analysis, Lustenberger and Rossi (2017) also find that, beyond a certain level, more communication from central banks tends to increase errors and dispersion of professional forecasters.

The remainder of this paper is structured as follows. Section 2 presents the micro-founded model. Section 3 derives and compares welfare for three operational frameworks

³For example, it is impossible to directly observe a productivity shock. Instead, economists estimate productivity with models that link output to capital used and hours worked. These inputs are themselves the result of market interactions, which are influenced by, among other things, the conduct of monetary policy. This rationalises why central banks have no direct, i.e., exogenous, information about fundamentals in our setup.
under exogenous information. Section 4 forms the core of the paper and reconsiders these operational frameworks in light of endogenous information. Finally, section 5 concludes.

2 The economy

This section derives the theoretical Keynesian beauty contest from a micro-founded economy with flexible prices but heterogeneous information. The model is based on Adam (2007). The economy is populated by a representative household, a continuum of monopolistic competitive firms indexed on the unit interval [0, 1], and a central bank. The economy is hit by stochastic labour supply shocks that shift the efficient level of output. The central bank seeks to stabilise the economy by taking a policy action that determines the nominal aggregate demand and/or by disclosing information about its economic assessment.

2.1 Representative household

The representative household chooses its aggregate composite good \( C \) and labour supply \( L \) to maximise its utility subject to its budget constraint,

\[
U(C) - \nu V(L),
\]

s.t. \( WL + \Pi = PC + T. \)

\( W \) denotes the competitive wage, \( \Pi \) the profits the household gets from firms, and \( T \) the nominal transfer from the central bank. The parameter \( \nu \) is a stochastic labour supply shock that induces variations in the efficient level of output. The composite good \( C \) is defined by the Dixit-Stiglitz aggregator

\[
C = \left[ \int_0^1 (C_i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}},
\]

where \( \theta > 1 \) is the parameter of price elasticity of demand and where \( C_i \) is the good produced by firm \( i \). Because \( \theta \) is constant, there is no mark-up shock. \( P \) is the appropriate price index which solves \( PY = \int_0^1 P_i C_i di. \)

2.2 Firms

Each firm \( i \) produces a single differentiated good \( C_i \) with one unit of labour \( L_i \) according to the production function

\[
L_i = C_i.
\]

The profit maximisation problem of firm \( i \) is given by

\[
\max_{P_i} E[(1 + \tau)P_i C_i(P_i) - WC_i(P_i)|\Gamma_i],
\]
where $\tau$ is an output subsidy that offsets the efficiency detrimental effect of the mark-up, and $\Gamma_i$ is the information set of firm $i$. Linearising the first order condition of firm $i$’s problem around its steady state delivers

$$p_i = \mathbb{E}_i[p + \xi(c - c^*)],$$  \hspace{0.5cm} (1)$$

where $\mathbb{E}_i$ is the expectation operator conditional on firm $i$’s information set $\Gamma_i$ and where small letters indicate percentage deviation from the steady state. The pricing rule (1) states that firms set their price as a function of their expectation of the overall price level $p$ and the real output gap $c - c^*$. The deviation of the efficient level of output $c^*$ from its steady state is determined by the stochastic labour supply shifter $\nu$. The parameter $\xi = -U''(\bar{Y})\bar{Y}U'(\bar{Y}) + V''(\bar{Y})\bar{Y}V'(\bar{Y})$ determines the sensitivity of the optimal price to the output gap and is increasing in the risk aversion of the household.

Using the fact that the nominal aggregate demand $g$ can be expressed as $g = c + p$, we rewrite the pricing rule (1) as

$$p_i = (1 - \xi)\mathbb{E}_i(p) + \xi\mathbb{E}_i(g - c^*).$$  \hspace{0.5cm} (2)$$

Parameter $\xi$ determines whether prices are strategic complements or substitutes. We assume that prices are strategic complements, i.e., $0 < \xi < 1$. This means that each firm tends to raise its own price when it expects other firms to do so.

The efficient level of output, i.e., the fundamental of the economy, is $c^* \in \mathbb{R}$. It reflects the aggregate level of firm’s specific efficient output $c^* = \int x_i \, di$. The firm’s specific efficient output is centered on the aggregated efficient level $c^*$ and has a normally distributed error term:

$$x_i = c^* + \epsilon_i, \text{ with } \epsilon_i \sim \mathcal{N}(0, \sigma^2_{\epsilon}),$$  \hspace{0.5cm} (3)$$

where $\epsilon_i$ are identically and independently distributed across firms. The firm’s specific efficient output $x_i$ is interpreted as a private signal about the aggregate efficient level of output $c^*$, which drives marginal costs and optimal pricing.

### 2.3 Welfare

The second-order approximation of the welfare of the representative household is given by

$$W_{micro} = -\frac{\theta}{\xi} \int_0^1 (p_i - p)^2 \, di - (c - c^*)^2 = -\frac{\theta}{\xi} \int_0^1 (p_i - p)^2 \, di - (g - p - c^*)^2.$$  \hspace{0.5cm} (4)$$

The welfare decreases in both the dispersion of prices across firms $\int_0^1 (p_i - p)^2 \, di$ and the distortion of the effective output from the efficient level of output $(c - c^*)^2$. The weight assigned to price dispersion is determined by the price elasticity of demand $\theta$ in the Dixit-Stiglitz composite good and the output gap elasticity of optimal prices $\xi$.

Angeletos and Pavan (2007) note that the debate about the social value of public information is driven by the relationship between the weight assigned to dispersion in
the welfare function, i.e., the efficient degree of coordination, and the weight assigned to dispersion in the optimal individual action, i.e., the equilibrium degree of coordination. For the sake of generality, welfare (4) is expressed as

\[ W = -\lambda \int_0^1 (p_i - p)^2 di - (g - p - c^*)^2, \]

(5)
to encompass several cases discussed in the literature. The micro-founded welfare in our model corresponds to (5) with \( \lambda_{\text{micro}} = \theta/\xi \). The welfare in Morris and Shin (2002), \(-\int (p_i - c^*)^2 di\) corresponds to our framework (5) with \( \lambda_{MS} = 1 \) and \( g = 0 \) because the central bank takes no action. As shown below, the micro-founded welfare function weights dispersion more than individual firms do in their equilibrium decision, whereas the MS welfare function weights dispersion less.

2.4 The central bank

The central bank seeks to maximise the unconditional expected welfare (5). To achieve this task, the central bank can disclose information about the fundamental to firms and/or take a policy action to determine the nominal aggregate demand, depending on the operational framework.

The central bank information about the fundamental is written \( y \), and the variance of the central bank expectation error is

\[ \text{Var}[\mathbb{E}(c^*|y) - c^*] = \sigma^2. \]

(6)

This definition allows us to solve generally for the equilibrium behaviour of firms before specifying whether the central bank information is exogenous (section 3) or endogenous (section 4).

On the one hand, the central bank provides firms with its viewpoint about the fundamental. Following the classification of central bank transparency by Geraats (2002), our model focuses on the desirability of economic transparency.\(^5\) The central bank communicates its information \( y \) with more or less ambiguity. We capture this ambiguity with the degree of opacity of its disclosure.\(^6\) The signal disclosed by the central bank and received

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\(^5\)Political transparency is granted because firms know the objective (welfare) function of the central bank. Procedural transparency is granted because firms know the reaction function of the central bank. Policy transparency is not necessarily granted because the central bank may not disclose any information about the action it takes. However, it goes back to economic transparency because the economy is only affected by one shock. Operational transparency is also granted because there is no possible error in the transmission of central bank action to nominal aggregate demand in our stylised model.

\(^6\)This specification is more general than the public signal considered in Morris and Shin (2002) and James and Lawler (2011), where the central bank chooses between disclosing a fully transparent signal or withholding information completely. Our formulation of the public signal, with both common and idiosyncratic noise, is more realistic. For more details on the characteristics of semi-public information in these games, see Cornand and Heinemann (2008), Baeriswyl and Cornand (2014), Myatt and Wallace (2012) and Myatt and Wallace (2014).
The dispersion of individual noises $\sigma^2_{\phi}$ determines the degree of opacity of the central bank. It implies by no means that the central bank discloses a specific signal to each firm. Instead, the idiosyncratic noise captures the notion that each firm may interpret differently the same equivocal statement made by the central bank. The signal $y_i$ can be considered as a “semi-public” signal. Under transparency, all of the firms get the same unequivocal signal ($\sigma^2_{\phi} = 0$). The central bank disclosure $y$ is a public signal that is common knowledge among firms. Under opacity, the individual signal received by each firm has an infinite idiosyncratic noise ($\sigma^2_{\phi} \rightarrow \infty$). The central bank disclosure thus does not contain any valuable information.

On the other hand, the central bank can take an action (when provided by the operational framework) to maximise welfare (5). Through its monetary policy action, the central bank determines the nominal aggregate demand as a linear function of its expected efficient level of output $y$:

$$g(y) = \rho \cdot y,$$

where $\rho$ is the monetary policy coefficient. The policy rule means that the central bank aims to accommodate the nominal aggregate demand to the efficient level of output $c^*$. The determination of price dispersion (and level) arises from the agents’ price setting rule derived below and is anticipated by the central bank.

### 2.5 Equilibrium

This section derives the perfect Bayesian equilibrium behaviour of firms. To calculate the optimal rule (2), we calculate the first-order and higher-order expectations of firm $i$ about the fundamental $c^*$ conditional on its information. Given the firms’ information (3), (6) and (7), the expectation of degree one about the fundamental $E_i(c^*)$ yields

$$E_i(c^*) = \frac{\sigma^2_{\mu} + \sigma^2_{\phi}}{\sigma^2_{\epsilon} + \sigma^2_{\mu} + \sigma^2_{\phi}} x_i + \frac{\sigma^2_{\epsilon}}{\sigma^2_{\epsilon} + \sigma^2_{\mu} + \sigma^2_{\phi}} y_i.$$ (8)

The best estimate of the fundamental by firm $i$ is an average of both its signals, whose weights depend on their relative precision. To compute the higher-order expectations of firm $i$, one needs also to know the expectation of degree one of the central bank average disclosure $E_i(y)$. This delivers

$$E_i(y) = \frac{\sigma^2_{\phi}}{\sigma^2_{\epsilon} + \sigma^2_{\mu} + \sigma^2_{\phi}} x_i + \frac{\sigma^2_{\epsilon}}{\sigma^2_{\epsilon} + \sigma^2_{\mu} + \sigma^2_{\phi}} y_i.$$ (9)

Because the equilibrium decision is a linear combination of expectations ($E_i(c^*)$) and
and that $E_i(c^*)$ is itself a linear combination of signals, the equilibrium decision of any firm $i$ is a linear combination of $x_i$ and $y_i$

$$p_i = \gamma_1 x_i + \gamma_2 y_i,$$

and the average equilibrium decision can be written

$$p = \gamma_1 c^* + \gamma_2 y$$

as $c^* = \int x_i di$ and $y = \int y_i di$. Inserting this in (2) and using the expressions of expectations (8) and (9), we get an expression of $p_i$ as a function of $x_i$ and $y_i$. This expression is also equal to (10), which allows us to identify coefficients $\gamma_1$ and $\gamma_2$:

$$\gamma_1 = -\frac{\xi \sigma^2 + (1 - \rho) \sigma^2}{\sigma^2 + \xi \sigma^2 + \sigma^2}, \quad \gamma_2 = -\frac{(1 - \rho) \sigma^2 - \rho \xi \sigma^2}{\sigma^2 + \xi \sigma^2 + \sigma^2}$$

with $\gamma_1 + \gamma_2 = \rho - 1$. (11)

Given the equilibrium behavior of firms (11) and central bank information, the unconditional expected social welfare (5) can be written as

$$E(W) = -\lambda(\gamma^2_1 \sigma^2 + \gamma^2_2 \sigma^2) - (\rho - \gamma_2)^2 \sigma^2$$

The central bank chooses its disclosure strategy $\sigma^2$ and action $\rho$ (depending on the operational framework) to maximise the unconditional expected welfare.

### 2.6 Operational frameworks

Optimal central bank policy is derived within three operational frameworks. An operational framework refers to the set of communication and action instruments at the disposal of the central bank.

- Within the pure communication framework (PC), the central bank can solely disclose information. This framework corresponds to that of Morris and Shin (2002), which features how the central bank should optimally communicate when it takes no action. Our framework departs nevertheless from MS in that it allows the central bank to choose an intermediate degree of opacity, whereas the original formulation of MS limits the choice between disclosing fully transparent information or withholding information completely.

- Within the action and communication framework (AC), the central bank can take an action and disclose information. This framework corresponds to that of James and Lawler (2011), which features how the central bank should optimally combine its action and disclosure.

- Within the signalling action framework (SA), the action taken by the central bank is common knowledge. This framework corresponds to that of Baeriswyl and Cornand
(2010), which features how the central bank should optimally take its action when its action is perfectly observable by firms.\footnote{Note that contrary to the present paper, in Baeriswyl and Cornand (2010), the signalling action does not unambiguously reveal central bank information because the economy is affected by two shocks.}

These three frameworks are nested. AC yields the unrestricted optimum because the central bank unrestrictedly chooses its action and disclosure. PC is nested in AC by excluding the possibility of taking an action (i.e., by setting \( \rho = 0 \)). SA is also nested in AC and PC by imposing full transparency of the central bank (i.e., by setting \( \sigma^2_t = 0 \)).\footnote{As the three operational frameworks are nested, they are (weakly) welfare ranked, with AC dominating PC and PC dominating SA.}

These three operational frameworks are solved for two processes of information gathering by the central bank. Under exogenous information, the central bank directly observes the fundamental with some noise. The precision of central bank information is independent of the behaviour of market participants (section 3). Under endogenous information, the central bank does not directly observe the fundamental but instead watches the economic outcome to evaluate the state of the fundamental. In this case, the precision of central bank information is determined by how the central bank influences the economy (section 4).

3 Exogenous information

This section analyses the welfare effects of disclosure and action when central bank information is exogenous. The central bank directly observes the fundamental with some noise. According to the error term of central bank information (6), the central bank receives a signal \( y \) on the fundamental that is centered on its true value \( c^* \) and contains an error term \( \mu \):

\[
y = c^* + \mu, \quad \text{with} \quad \mu \sim N(0, \sigma^2_\mu).
\]

The precision of central bank information \( \sigma^2_\mu \) is independent of its behaviour. The present section serves as a benchmark for the analysis under endogenous information presented in section 4.

3.1 Optimal central bank behaviour

We derive the optimal central bank disclosure and/or action for the three operational frameworks under scrutiny before comparing these operational frameworks with each other.

As emphasised by Angeletos and Pavan (2007), the discrepancy between the efficient and equilibrium degrees of coordination determines the welfare effect of public information. Deriving welfare (12) with respect to \( \xi \) yields the efficient degree of coordination: \( \xi^{exo}_{eff} = 1/\lambda \). Alternatively, for a given \( \xi \), the weight on dispersion in the welfare function for which the equilibrium degree of coordination is efficient is \( \lambda^{exo}_{equi} = 1/\xi \). For this relationship between \( \xi \) and \( \lambda \), firms following (11) make an efficient use of their private and public
information. The weight on dispersion in the welfare function of MS is thus smaller than the weight for which the equilibrium and efficient degrees of coordination coincide, which is smaller than the weight on dispersion in the micro-founded welfare function:

$$\lambda_{MS} = 1 < \lambda_{equi}^{exo} = \frac{1}{\xi} < \lambda_{micro} = \frac{\theta}{\xi}.$$  

### 3.1.1 Pure communication

The first operational framework corresponds to Morris and Shin (2002), where the central bank discloses information but takes no action. This case is captured in our general setup by imposing $\rho = 0$. Inserting the equilibrium response of agents (11) into (12) yields the unconditional expected welfare

$$E(W|\rho=0) = -\frac{\left(\lambda(\xi\sigma^2 + \sigma^2_\phi)^2 + \sigma^2_\phi(\lambda\sigma^2_\phi + \sigma^2_\mu)\right)\sigma^2_\epsilon}{\left(\sigma^2_\epsilon + \xi\sigma^2_\mu + \sigma^2_\phi\right)^2}. $$

Differentiating with respect to $\sigma^2_\phi$ and setting the resulting expression to zero gives the optimal degree of opacity

$$\sigma^2_\phi^* = \max\left[0, \left(\frac{2}{\lambda} - 3\xi\right)\sigma^2_\mu - \sigma^2_\epsilon\right].$$  

The intuition behind the optimal degree of opacity is driven by the dual effect of public information on welfare. Less opaque public information improves welfare by reducing price dispersion across firms. However, noisy public information is detrimental to welfare because it increases the distortion of output from its efficient level.

The optimal degree of opacity decreases with the weight assigned to price dispersion $\lambda$ in welfare. When the relevance of price dispersion increases compared to that of output distortion, a more transparent central bank disclosure helps firms to coordinate their price setting.

The optimal degree of opacity decreases with the inaccuracy of private information $\lambda^{exo}$ and coincides with the equilibrium degree of coordination, i.e., $\lambda^{exo} = 1/\xi$, full transparency is always optimal because $\theta > 1$ and $0 < \xi < 1$. This conforms to the result of Hellwig (2005) and Angeletos and Pavan (2007), according to which full transparency is optimal when the equilibrium degree of coordination does not exceed the efficient degree of coordination.

By contrast, when the equilibrium degree of coordination exceeds the efficient degree of coordination, as in the case with $\lambda_{MS} = 1$, the optimal degree of opacity can be positive and increases with the degree of strategic complementarities $1 - \xi$. Stronger strategic complementarities induce firms to assign a higher relative weight to their public information. Although this reduces the price dispersion across firms, this exacerbates the distortion from efficient output, in reaction to which the central bank finds it optimal to increase the degree of opacity of its disclosure.

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By contrast, when the equilibrium degree of coordination exceeds the efficient degree of coordination, as in the case with $\lambda_{MS} = 1$, the optimal degree of opacity can be positive and increases with the degree of strategic complementarities $1 - \xi$. Stronger strategic complementarities induce firms to assign a higher relative weight to their public information. Although this reduces the price dispersion across firms, this exacerbates the distortion from efficient output, in reaction to which the central bank finds it optimal to increase the degree of opacity of its disclosure.
When private information becomes less accurate ($\sigma_\epsilon^2$ increases), it is optimal for the central bank to reduce its opacity to enhance the coordination of firms through a relatively more transparent disclosure.

Finally, the effect of the inaccuracy of central bank information $\sigma_\mu^2$ is ambiguous. The optimal degree of opacity increases with the inaccuracy of central bank information when $2/3 > \lambda_\xi$. Because inaccurate central bank information yields a distortion of output, an increase in inaccuracy calls for more opacity when the relative weight of distortion in welfare is large (i.e., $\lambda$ small) and when firms strongly react to the central bank disclosure because of a large degree of strategic complementarities (i.e., $\xi$ small).

### 3.1.2 Action and communication

In the second operational framework, the central bank can both take an action and disclose information, as in James and Lawler (2011). First, the optimal policy action of the central bank is obtained by differentiating (12) with respect to $\rho$, holding $\sigma_\epsilon^2, \sigma_\mu^2, \sigma_\phi^2, \xi$, and $\lambda$ fixed. This yields the optimal policy action

$$
\rho^* = \frac{(\lambda \sigma_\epsilon^2 + (2\lambda_\xi - 1)\sigma_\mu^2 + \lambda \sigma_\phi^2) \sigma_\epsilon^2}{(\lambda \sigma_\epsilon^2 + \sigma_\mu^2) \sigma_\phi^2 + \lambda (\sigma_\epsilon^2 + \xi \sigma_\mu^2)^2}.
$$

(14)

We then derive the optimal disclosure when the central bank implements the optimal policy action $\rho^*$. Inserting $\rho^*$ in (12) yields the welfare function

$$
E(W|\rho^*) = -\frac{(\sigma_\epsilon^2 + \lambda_\xi \sigma_\mu^2 + \sigma_\phi^2) \lambda \sigma_\epsilon^2 \sigma_\phi^2}{(\lambda \sigma_\epsilon^2 + \sigma_\mu^2) \sigma_\phi^2 + \lambda (\sigma_\epsilon^2 + \xi \sigma_\mu^2)^2}.
$$

Differentiating this expression with respect to $\sigma_\phi^2$ yields

$$
\frac{\partial E(W|\rho^*)}{\partial \sigma_\phi^2} = \frac{(\lambda_\xi - 1)^2 \lambda \sigma_\epsilon^4 \sigma_\mu^4}{(\lambda \sigma_\epsilon^2 + \sigma_\mu^2) \sigma_\phi^2 + \lambda (\sigma_\epsilon^2 + \xi \sigma_\mu^2)^2}.
$$

(15)

which is always positive. For any parameter value, it is welfare improving for the central bank to increase the noise in the semi-public signal, and thus full opacity is always optimal. Even when the central bank has some information of very high quality, it is optimal not to communicate this information. Full opacity is optimal both when the efficient degree of coordination is smaller than the equilibrium degree of coordination ($\lambda_{MS} = 1$) and when it is larger ($\lambda_{\text{micro}} = \theta/\xi$). When the efficient and equilibrium degrees of coordination coincide ($\lambda_{\text{exo}}^{\epsilon} = \lambda_{\text{equi}}^{\epsilon} = 1/\xi$), the disclosure strategy is irrelevant because firms make an efficient use of information.

Under full opacity, i.e., $\sigma_\phi^2 \to \infty$, the optimal central bank action is given by

$$
\rho^*|_{\sigma_\phi^2 \to \infty} = \frac{\lambda \sigma_\epsilon^2}{\lambda \sigma_\epsilon^2 + \sigma_\mu^2}.
$$

(16)
and is independent of the degree of strategic complementarities $1 - \xi$ because there is no public information disclosed.

Taking an action is more efficient for maximising welfare than disclosing information. Although firms overreact to public disclosures because of strategic complementarities in price setting, they do not overreact to unobserved actions. Firms know that the central bank takes an action but ignore which action it exactly takes. Deprived of central bank disclosure, each firm builds its own expectation about central bank action based on its private information only. Accounting for privately expected central bank action, firms react less to their private information, which reduces price dispersion without inducing an overreaction to any noisy central bank disclosure. Reducing dispersion does not require public information per se but a weaker response to private information, which is achieved through a more accommodating action. Thus, even when the efficient degree of coordination is larger than the equilibrium degree of coordination ($\lambda_{\text{micro}} = \theta/\xi$), it is optimal for the central bank to promote coordination between firms without disclosing any public information, but only by adjusting its action.

3.1.3 Signalling action

Whereas the central bank finds it optimal to take an action without disclosing any information in the previous section, one may wonder whether it is realistic to keep an action secret from the public. We now consider a third more realistic operational framework where taking an action signals what the central bank believes about the state of the economy. Although the central bank may not explicitly communicate its beliefs, its action is always observable and, therefore, reveals its beliefs about the economic fundamental, as highlighted in Baeriswyl and Cornand (2010). For example, prior to 1994, the US Federal Reserve Bank did not publicly announce its targeted interest rate. Even so, participants to the money market could easily infer from Fed operations what the aimed target actually was. The action taken by the central bank on the money market revealed to market participants its assessment despite its lack of disclosure.

Consider the case where central bank action is perfectly observable by agents, i.e., $\sigma_\phi^2 = 0$. Differentiating (12) with respect to $\rho$ shows that the optimal action is indeterminate because any common knowledge policy coefficient yields the same optimal welfare. Optimal action is indeterminate when it is common knowledge because welfare (4) is independent of the price level in itself but decreases with price dispersion and the output gap. A common knowledge action therefore has no effect on welfare because it influences neither price dispersion nor the output gap. For the sake of simplicity, we stipulate that the central bank fully accommodates the expected fundamental with $\rho^* = 1$. The expected action

---

9One way to solve the indeterminacy of optimal action is to introduce frictions such that welfare depends on the price level. This can be done by introducing sticky prices or sticky information in the model and by taking the optimal action at the limit when price or information stickiness converges towards zero, as in Baeriswyl and Cornand (2010). This would however not affect welfare.
Table 1: Expected welfare in various operational frameworks

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\rho}^2$</th>
<th>$\rho$</th>
<th>$E(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No CB</td>
<td>$\infty$</td>
<td>0</td>
<td>$-\lambda \sigma_{\epsilon}^2$</td>
</tr>
<tr>
<td>PC if $(2/\lambda - 3\xi)\sigma_\mu^2 \leq \sigma_\epsilon^2$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{(\sigma_\epsilon^2 + \lambda \xi \sigma_\mu^2)\sigma_\epsilon^2 \sigma_\mu^2}{\left(\sigma_\epsilon^2 + \xi \sigma_\mu^2\right)^2}$</td>
</tr>
<tr>
<td>otherwise</td>
<td>$(2/\lambda - 3\xi)\sigma_\mu^2 - \sigma_\epsilon^2$</td>
<td>0</td>
<td>$-\lambda \sigma_{\epsilon}^2 + \frac{\lambda \sigma_{\epsilon}^2 \sigma_\mu^2}{\lambda - \lambda \xi \sigma_\mu^2}$</td>
</tr>
<tr>
<td>AC</td>
<td>$\infty$</td>
<td>$\frac{\lambda \sigma_{\epsilon}^2 \sigma_\mu^2}{\lambda \sigma_{\epsilon}^2 + \sigma_\mu^2}$</td>
<td>$\frac{-\lambda \sigma_{\epsilon}^2 \sigma_\mu^2}{\lambda \sigma_{\epsilon}^2 + \sigma_\mu^2}$</td>
</tr>
<tr>
<td>SA</td>
<td>0</td>
<td>1</td>
<td>$-\frac{(\sigma_\epsilon^2 + \lambda \xi \sigma_\mu^2)\sigma_\epsilon^2 \sigma_\mu^2}{\left(\sigma_\epsilon^2 + \xi \sigma_\mu^2\right)^2}$</td>
</tr>
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</table>

After having derived the optimal behaviour of the central bank in the three operational frameworks, the next section compares these frameworks at their respective optimum.

### 3.2 Optimal operational framework

Table 1 summarises the optimal central bank behaviour within the pure communication (PC), the action and communication (AC), and the signalling action (SA) frameworks. For the sake of comparison, the first line also shows the welfare when the central bank neither talks nor takes any action (no CB). In this section, we compare these operational frameworks to each other.

Figure 1 illustrates the weight assigned by firms to their private information $\gamma_1$, the weight assigned by firms to the central bank disclosure $\gamma_2$, the central bank action $\rho$, and the unconditional expected welfare $E(W)$ in the different operational frameworks. The plots on the left show equilibrium values as a function of $\lambda$. For $\xi = 0.25$, the efficient degree of coordination coincides with the equilibrium degree of coordination when $\lambda = 4$. The efficient degree of coordination is smaller (larger) than the equilibrium degree of coordination when $\lambda < 4$ ($\lambda > 4$). The plots on the right show equilibrium values as a function of $\sigma_\mu^2$ for $\lambda_{MS} = 1$.

First, when the efficient degree of coordination coincides with the equilibrium degree of coordination, i.e., when $\lambda = 4$ in Figure 1, the three frameworks examined are equivalent in terms of welfare. Because firms use their information efficiently, it is irrelevant whether the central bank discloses its information with full transparency (PC), takes an action while withholding its information (AC), or takes an action that fully reveals its information (SA).

Second, when the efficient degree of coordination deviates from the equilibrium degree, the action and communication framework always yields a higher welfare than the pure communication and signalling action frameworks. Though arguably unrealistic, choosing an
Figure 1: Optimal firms’ and central bank’s behaviour and unconditional expected welfare under exogenous information as a function of $\lambda$ and $\sigma^2$ for $\sigma^2_\epsilon = 1$, $\xi = 0.25$, $\sigma^2_\mu = 1$ (left plots), and $\lambda = 1$ (right plots)
optimal action under full opacity is the welfare superior strategy. When firms make an inefficient use of public information, it is optimal not to provide them with public information, regardless of whether the equilibrium degree of coordination is larger or smaller than the efficient degree. The central bank, however, uses its information efficiently when it takes its action. Firms anticipate the action taken by the central bank into their price setting decision without overreacting to its error term. This reduces the price dispersion across firms without creating a distortion due to the central bank error term. Taking an optimal action under full opacity is a theoretical means of avoiding the problem of misalignment between efficient and equilibrium degree of coordination. In reality, this may not be so easily achieved because policy actions are typically difficult to hide completely from market participants. Nevertheless, it provides intuition of the mechanisms at play.

Third, abstracting from the unrealistic action and communication framework, full transparency is optimal when the weight on dispersion in welfare is not too small, i.e., when \( \lambda > \frac{2}{(\sigma^2_\epsilon/\sigma^2_\mu + 3\xi)} = 1.143 \) in left plots, or when central bank information is not too noisy, i.e., when \( \sigma^2_\mu < 0.8 \) in right plots. The pure communication and the signalling action frameworks are then equivalent. However, when partial transparency is optimal within the pure communication framework, i.e., when \( \lambda < \frac{2}{(\sigma^2_\epsilon/\sigma^2_\mu + 3\xi)} \), it yields greater welfare than taking a signalling action. The pure communication framework allows a subtle control of the overreaction of firms to the central bank disclosure by varying the degree of opacity. By contrast, the signalling action framework specifies that the action taken by the central bank is perfectly observable, which makes controlling overreaction impossible. Therefore, whenever increasing the degree of opacity is optimal within the pure communication framework, it consequently yields a higher welfare than the signalling action framework.

Fourth, doing nothing (no CB) yields a higher welfare than taking a signalling action when \( (1 - 2\lambda\xi)\sigma^2_\mu - \lambda\sigma^2_\epsilon > 0 \) or \( \lambda < 0.666 \) on left plots or \( \sigma^2_\mu > 2 \) on right plots. When the weight on dispersion in welfare \( \lambda \) is low or when the inaccuracy of central bank information \( \sigma^2_\mu \) is high, unconditional expected welfare is higher when the central bank refrains from influencing the economy than when it takes an action that is perfectly observable. This result coincides with the original formulation of MS, whereby the central bank chooses between disclosing fully transparent information and withholding information completely. Nevertheless, Svensson (2006) observed that MS’s anti-transparency result only holds in the unrealistic case where central bank information is less accurate than that of private agents. Svensson’s critique applies in our exogenous model. When welfare equally weights price dispersion and distortion, i.e., when \( \lambda_{MS} = 1 \), taking a signalling action is inferior to the absence of central bank only in the unrealistic case where the accuracy of central bank information \( (\sigma^2_\mu > 2) \) is significantly worse than that of private firms’ information \( (\sigma^2_\epsilon = 1) \).

In a nutshell, the analysis suggests that full transparency improves welfare under realistic configurations. Withholding information completely would yield a superior outcome only in the implausible cases where the central bank would be able to entirely hide its
action from market participants, when the efficient degree of coordination would be significantly lower than the equilibrium degree, and when central bank information would be significantly less accurate than that of private agents.

4 Endogenous information

The endogenous information setup departs from the assumption that the central bank directly observes economic fundamentals. Fundamentals typically are nothing more than the result of human actions. While fundamentals such as the crop production or rainfall are independent of human behaviour, most economic fundamentals such as aggregated demand, supply, investment, inflation, or preferences reflect the behaviour of economic agents. Observing fundamentals therefore comes down to observing economic agents. In reality, the central bank has no direct source of information on economic fundamentals. Instead, it observes the aggregate market activity to assess the economic situation.

To capture the endogenous nature of information about fundamentals, we postulate that the central bank receives a signal $\Omega$ on the average decision $p$ with some noise $\eta$

$$\Omega = p + \eta = \gamma_1 c^* + \gamma_2 y + \eta, \eta \sim \mathcal{N}(0, \sigma^2_\eta).$$

Using (11), the central bank’s best estimate of the fundamental conditional on its observation is

$$y = \mathbb{E}(c^* | \Omega) = \frac{\Omega}{\rho - 1}.$$\hspace{1cm} (14)

The information of the central bank can thus be expressed as

$$\Omega = \frac{\gamma_1 c^* + \gamma_2}{\rho - 1} + \eta = (\rho - 1)c^* + \frac{\rho - 1}{\gamma_1} \eta.$$\hspace{1cm} (15)

Even if the central bank knows the signal that it discloses to firms, it cannot infer the true fundamental $c^*$ from its observation of the aggregate decision $\Omega$. This is because its observation contains an unknown error $\eta$. Because its observation is noisy, knowledge of its disclosure to firms does not allow the central bank to infer the true fundamental. According to the definition of (6), the variance of central bank expectation error under endogenous information becomes

$$\text{Var} [\mathbb{E}(c^* | \Omega) - c^*] = \frac{\sigma^2_\eta}{\gamma_1^2} \equiv \sigma^2_{\mu}.$$\hspace{1cm} (17)

The precision of central bank information is a function of the equilibrium response of firms $\gamma_1$, which depends on the action and communication strategy of the central bank.

The analysis focuses on the simultaneous equilibrium (fixed point) between the central bank and the market outcome. To implement its policy, the central bank observes the market outcome $\Omega$, which is, at the same time, influenced by its action and disclosure. Focusing on simultaneous equilibrium is a shortcut for analysing the steady-state
or continuous interaction between the central bank and firms. In reality, monetary policy continuously influences the economy, which feeds back into policy decisions. Thus, the central bank cannot first observe the economy uninfluenced by its policy and then take the appropriate decision to shape the economy. Instead, the feedback process between the economy and policy is continuous.

In the following subsections, we analyse the consequences of endogenous central bank information on its accuracy and on welfare within the three operational frameworks under scrutiny. We derive welfare for each operational framework before comparing these operational frameworks at their respective optimum.

4.1 Optimal central bank behaviour

Solving analytically the optimisation problem of the central bank under endogenous information is not straightforward because the relationship between the variance of central bank expectation error and firms’ response is non-linear. Inserting (17) into (11) yields

\[ \gamma_1 = -\frac{\xi \sigma^2_\eta / \gamma_1^2 + (1 - \rho) \sigma^2_{\phi}}{\sigma^2_\xi + \xi \sigma^2_\eta / \gamma_1^2 + \sigma^2_{\phi}}, \]

which exhibits three equilibria. The cubic equation has only one real root (and two complex conjugate roots) when its discriminant is negative, which is satisfied when \( \rho \in [0, 1] \) (and \( 0 < \xi < 1 \)). Although assuming a monetary policy coefficient \( \rho \in [0, 1] \) gets rid of the indeterminacy of multiple equilibria, it also means that the central bank, realistically, seeks to accommodate shocks to the fundamental rather than to amplify them (i.e., \( \rho < 0 \)) or over-accommodate them (i.e., \( \rho > 1 \)). Because algebraic solutions remain cumbersome, we apply numerical procedures in the following.

Endogenous information affects the relationship between the efficient and the equilibrium degrees of coordination. Figure 2 plots the efficient degree of coordination as
a function of $\lambda$. Under exogenous information, as derived above, the relationship between the efficient and equilibrium degrees of coordination is proportional and is given by $\lambda_{\text{equi}}^{\text{exo}} = 1/\xi$ or $\lambda_{\text{eff f i}}^{\text{exo}} = 1/\lambda$. Under endogenous information, for a given weight on dispersion in welfare $\lambda$, the efficient degree of coordination $1/\xi_{\text{eff f i}}^{\text{endo}}$ is always lower than under exogenous information. Because firms’ reaction to public disclosure deteriorates the accuracy of central bank information, firms should respond less strongly to public disclosure than in the exogenous setup for their use of information to be efficient. Increasing the weight on dispersion $\lambda$ yields a less than proportional increase in the efficient degree of coordination $1/\xi_{\text{eff f i}}^{\text{endo}}$. The relationship is also dependent on the degree of opacity $\sigma^2$. An increase in the degree opacity reduces the reaction of firms to the central bank disclosure and raises the efficient degree of coordination $1/\xi_{\text{eff f i}}^{\text{endo}}$.

The weight on dispersion in the welfare function of MS is thus smaller than the equilibrium degree of coordination, which is smaller than both the weight on dispersion in the micro-founded welfare function and the weight for which the equilibrium and efficient degrees of coordination coincide:

$$\lambda_{\text{MS}} = 1 < 1/\xi < \lambda_{\text{micro}} = \theta/\xi = \lambda_{\text{endo}}_{\text{equi}}$$

When $\theta$ is small, the micro-founded weight $\lambda_{\text{micro}}$ can be smaller than the weight for which the equilibrium and efficient degrees of coordination coincide $\lambda_{\text{equi}}^{\text{endo}}$. Both are nevertheless larger than the equilibrium degree of coordination $1/\xi$.

### 4.1.1 Pure communication

Let us consider the first operational framework, in which the central bank discloses information but implements no action ($\rho = 0$). Under exogenous information, central bank disclosure affects the accuracy of the average price set by firms as an indicator of the economic fundamental. The informative value of the average price $p$ is evaluated as the variance of the error of fundamental expectations conditional on the average price:

$$\text{Var} [E(c^*|p) - c^*] = \text{Var} \left[ E \left( c^*|c^* + \frac{\gamma_2}{\gamma_1} \eta \right) - c^* \right] = \frac{\gamma_2^2}{\gamma_1^2} \sigma^2_{\eta}.$$ 

The effect of central bank communication on the informative value of the average price is illustrated in Figure 3 in the case of pure communication. The information about the fundamental contained in the average price increases with central bank opacity $\sigma^2_{\phi}$. Increasing opacity reduces the influence of the central bank on the decision of firms, which improves the informative value of the average price about the fundamental. When the central bank is completely opaque, i.e., when $\sigma^2_{\phi} \to \infty$, firms do not react to the central bank disclosure at all, and the average price becomes a perfect indicator for the fundamental $c^*$. The degree of strategic complementarities $\xi$ drives the overreaction of firms to the central bank disclosure and, therefore, affects the information contained in the average price. A higher degree of strategic complementarities reduces the informative
value of the average price because firms react more to the central bank disclosure.

Central bank opacity exerts a dual effect on the accuracy of firms’ information. On the one hand, an increase in opacity raises the idiosyncratic noise of the central bank disclosure $\sigma_\phi^2$, which deteriorates the accuracy of firms’ information. On the other hand, an increase in opacity improves the informative value of the average price and the accuracy of central bank information and disclosure. Overall, opacity exerts an ambiguous effect on the accuracy of firms’ information. This is evaluated by the variance of the error of fundamental expectations conditional on both the private signal $x_i$ and the central bank disclosure $y_i$:

$$\text{Var} \left[ E(c^*|x_i, y_i) - c^* \right] = \text{Var} \left[ E(c^*|c^* + \epsilon_i, c^* + \frac{\eta_i}{\gamma_i} + \phi_i) - c^* \right] = \frac{(\sigma_\eta^2/\gamma_1^2 + \sigma_\phi^2)\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2/\gamma_1^2 + \sigma_\phi^2}.$$

The effect of central bank communication on the accuracy of firms’ information is illustrated in Figure 4. When the degree of strategic complementarities is low ($\xi = 0.5$), increasing opacity deteriorates the accuracy of firms’ information. By contrast, for a higher degree of strategic complementarities ($\xi = 0.25$ and $\xi = 0.15$), increasing opacity does not unambiguously deteriorate the accuracy of firms’ information. Below a certain threshold, increasing opacity improves the accuracy of firms’ information: the rise in the informative value of the average price overcomes the rise in the idiosyncratic noise.

We now turn to the effect of endogenous central bank information on welfare. Figure 5 plots the expected welfare as a function of the degree of opacity for $\sigma_\epsilon^2 = 1$, $\sigma_\eta^2 = 1$, $\lambda = 2$, and $\rho = 0$. It shows that, depending on the degree of strategic complementarities, full transparency may (or not) be preferable to full opacity and that there is an interior optimal degree of opacity. The optimal degree of opacity increases with the degree of strategic complementarities, and its algebraic solution is given by

$$\sigma_\phi^2 = \max \left[ 0, \left( \frac{2}{\lambda} - 3\xi \right) \sigma_\eta^2/\gamma_1^2 \right].$$

(19)

Endogenous information calls for a higher degree of opacity than exogenous information. Figure 6 compares the optimal degree of opacity under endogenous and exogenous informational setups as a function of the degree of strategic complementarities $\xi$. The dotted line plots the optimal degree of opacity when the variance of central bank expectation error is exogenous. This corresponds to (13) with $\sigma_\mu^2 = 1$. Accounting for the endogeneity of the central bank information exerts a twofold effect on the optimal degree of opacity. First, endogenous information raises the optimal degree of opacity because it deteriorates the accuracy of central bank information. Even if the central bank ignores the effect of its disclosure on the accuracy of its observation, it should increase opacity because of less accurate information. This effect is plotted by the dashed line: the central bank behaves as in the exogenous setup but with an information accuracy equivalent to that in the endogenous setup. Second, endogenous information raises the optimal degree of opacity when the central bank accounts for the effect of its disclosure on the accuracy.
Figure 3: Error variance of the fundamental expectation conditional on the average price for $\sigma^2_e = 1, \sigma^2_\eta = 1$, and $\rho = 0$

Figure 4: Error variance of the fundamental expectation conditional on firms’ information for $\sigma^2_e = 1, \sigma^2_\eta = 1$, and $\rho = 0$

Figure 5: Expected welfare with pure communication as a function $\sigma^2_\phi$ for $\sigma^2_e = 1, \sigma^2_\eta = 1$, $\lambda = 2$ and $\rho = 0$
of its observation. The solid line plots the optimal degree of opacity in the endogenous setup. Endogenous information creates an externality, which calls for more opacity than in the case of exogenous information. The difference between the optimal degree of opacity under endogenous information (19) and that under exogenous information (13) for a given level of information accuracy, that is for $\sigma^2_{\mu,exo} = \sigma^2_{\mu,endo} = \sigma^2_\eta / \gamma_1$, is $\sigma^2_\xi$.

Although accounting for endogenous information generally implies a higher optimal degree of opacity, full transparency is nevertheless optimal when the weight on dispersion in welfare $\lambda$ is not too low, i.e., when $\lambda > 2/3\xi$. In the case where the efficient and equilibrium degrees of coordination are equivalent, i.e., $\lambda^\text{endo}_\text{equi} > 1/\xi$, or where the efficient degree of coordination takes on its micro-founded value, i.e., $\lambda^\text{micro} = \theta/\xi$; full transparency is always optimal because its benefit on reducing price dispersion dominates the cost of rising output distortion.

### 4.1.2 Action and communication

We now discuss the effect of endogenous information on the second operational framework, in which the central bank implements a policy action in addition to disclosing information. Accounting for the endogeneity of central bank information does not alter the conclusion reached under exogenous information with respect to the optimal degree of opacity. Figure 7 plots the unconditional expected welfare when the central bank implements an optimal action as a function of the degree of opacity $\sigma^2_\phi$. Welfare is strictly increasing with the degree of opacity such that full opacity is optimal when the central bank takes an optimal action. This result corroborates that of James and Lawler (2011) derived under exogenous information.

Under full opacity, the optimal action is independent of the degree of strategic complementarities because there is no public information. Figure 8 traces the optimal action under full opacity for exogenous and endogenous central bank information. Endogenous
information weakens the accommodating action taken by the central bank. Because central bank information is less accurate in the endogenous setup, the central bank accommodates less strongly shocks to the fundamental.

4.1.3 Signalling action

We now turn to the third operational framework, in which the action taken by the central bank is common knowledge among firms and signals its information on the fundamental state of the economy. As in the exogenous setup, the optimal action is also indeterminate under endogenous central bank information. For the sake of simplicity, we stipulate that the central bank fully accommodates the expected fundamental with $\rho^* = 1$. Endogenous information deteriorates information accuracy of the central bank and of the firms as in
the two frameworks examined above.

The next section assesses the three operational frameworks under endogenous information with respect to welfare.

4.2 Optimal operational framework

Figure 9 summarises the equilibrium behaviour of firms $\gamma_1$ and $\gamma_2$, the optimal action of the central bank $\rho$, and the unconditional expected welfare $E(W)$ in the different operational frameworks under endogenous information. As in Figure 1, the plots on the left show equilibrium values as a function of $\lambda$. For $\xi = 0.25$, the efficient and equilibrium degrees of coordination are equivalent when $\lambda^{\text{endo}}_{\text{equi}} = 8$. The plots on the right show equilibrium values as a function of central bank error term $\sigma^2_{\eta}$ for $\lambda_{MS} = 1$.

Accounting for the endogeneity of central bank information does not alter the relative assessment of frameworks between each other: the same ranking for the different operational frameworks applies as under exogenous information.

First, when the efficient and equilibrium degrees of coordination coincide, i.e., $\lambda^{\text{endo}}_{\text{equi}} = 8$, the three frameworks are equivalent in terms of welfare. The efficient use of information by firms makes the disclosure strategy of the central bank irrelevant.

Second, when the equilibrium degree of coordination deviates from the efficient degree, taking an optimal action under full opacity in AC yields the highest welfare.

Third, abstracting from this unrealistic framework, full transparency is optimal when the weight on dispersion in welfare is not too small, i.e., when $\lambda > 2/3\xi = 2.66$. Full transparency is thus optimal for a smaller range of parameter values under endogenous than under exogenous information (compared with $\lambda > 1.143$ in the exogenous setup). It remains nevertheless always optimal with the micro-founded weight on dispersion $\lambda = \theta/\xi > 4$.

Fourth, when $\lambda$ is small, endogenous information qualifies the critique of Svensson (2006) because the absence of central bank policy yields a higher welfare than taking a signalling action even when the central bank observation error $\sigma^2_{\eta}$ is smaller than that of private firms $\sigma^2_{\epsilon}$. When price dispersion and output distortion are equally weighted in social welfare, i.e., when $\lambda_{MS} = 1$, right plots in Figure 9 show that taking a signalling action may be inferior to the absence of central bank even when $\sigma^2_{\eta}$ is smaller than $\sigma^2_{\epsilon} = 1$.

To summarise, accounting for the endogenous nature of central bank information does not challenge the pro-transparency result when the weight assigned to price dispersion in social welfare accepts its micro-founded value. However, when price dispersion and output distortion are equally weighted, as in MS, endogenous information challenges the critique of Svensson (2006) because the absence of central bank intervention may yield a higher welfare than a signalling action even if the central bank observation error is smaller than that of market participants.
Figure 9: Optimal firms’ and central bank’s behaviour and unconditional expected welfare under endogenous information as a function of $\lambda$ and $\sigma^2_\eta$ for $\sigma^2_\xi = 1$, $\xi = 0.25$, $\sigma^2_\eta = 1$ (left plots), and $\lambda = 1$ (right plots)
5 Conclusion

The central bank plays a dual role in the market: while it observes the market to assess economic fundamentals, it also shapes the market outcome through the conduct of monetary policy. A dilemma arises from this dual role because the more effectively the central bank shapes market outcomes, the less reliably market outcomes serve as an indicator of economic fundamentals. The accuracy of central bank information is thus endogenous to its policy.

This paper has analysed the impact of endogenous central bank information on the optimal monetary policy in a standard macro-economic model where private firms’ price setting is characterised by strategic complementarities. It focuses on three operational frameworks proposed in the literature: pure communication, action and communication, and signalling action.

In principle, accounting for the endogeneity of information calls for less activism from the central bank. Compared to the exogenous information setup, endogenous information calls for a higher degree of opacity in the pure communication framework and for a weaker accommodation to shocks in the action and communication framework. Endogenous information also enlarges the range of parameter values for which taking a signalling action delivers a lower welfare than the absence of central bank activism. Nevertheless, when the weight assigned to price dispersion in social welfare takes on its micro-founded value, full transparency remains optimal under endogenous information.

References


