

# Supplementary material for the paper Cost-Aware Early Classification of Time Series

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## 1 Performing fair comparison with EarlyOpt.SVM

Method EarlyOpt.SVM [2] intends to minimize a cost function defined as:

$$C'(\mathbf{x}, y) = \alpha \times C_m(h_{\tau(\mathbf{x})}(\mathbf{x}), y) + (1 - \alpha) \times C'_d(\tau(\mathbf{x})), \quad (1)$$

while other three methods considered in our experimental evaluation optimize on:

$$C(\mathbf{x}, y) = C_m(h_{\tau(\mathbf{x})}(\mathbf{x}), y) + C_d(\tau(\mathbf{x})). \quad (2)$$

In practice, in the framework introduced in [1] and used in our paper, the experimental evaluation is conducted using the following form for  $C_d$ :

$$C_d(t) = \beta \times t, \quad (3)$$

while in [2], experiments are conducted using

$$C'_d(t) = \frac{t}{T}. \quad (4)$$

For a fixed  $\alpha$ , optimizing on Eq. (1) is then equivalent to optimizing on Eq. (2) with:

$$\beta = \frac{1 - \alpha}{\alpha \times T}. \quad (5)$$

In order to compare results published in [2] to those obtained in our experiments, we then selected  $\alpha$  so as to approximate  $\beta$  as well as possible. The approximation is denoted  $\hat{\beta}$  in the following. If no reasonable approximation was found (that is the difference between  $\beta$  and  $\hat{\beta}$  was greater than 20% of  $\beta$ ), no comparison was made since the objectives were considered to differ too much from each other.<sup>1</sup>

Then, for a given pair  $(\beta, \hat{\beta})$ , both following costs can be considered:

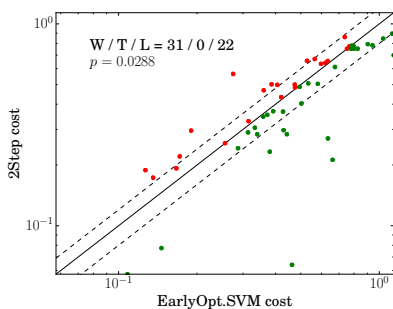
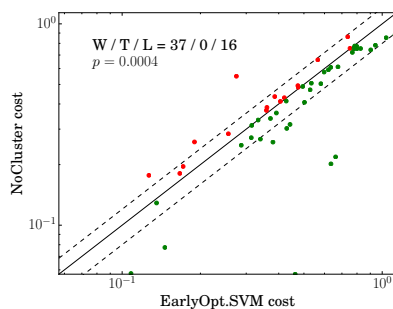
$$C_\beta(\mathbf{x}, y) = C_m(h_{\tau(\mathbf{x})}(\mathbf{x}), y) + \beta \times \tau(\mathbf{x}) \quad (6)$$

$$C_{\hat{\beta}}(\mathbf{x}, y) = C_m(h_{\tau(\mathbf{x})}(\mathbf{x}), y) + \hat{\beta} \times \tau(\mathbf{x}). \quad (7)$$

$C_\beta(\mathbf{x}, y)$  is the cost on which all methods derived from [1] optimize, while  $C_{\hat{\beta}}(\mathbf{x}, y)$  is proportional to the cost EarlyOpt.SVM attempts to minimize. Hence,

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<sup>1</sup> Matching between  $\beta$  and  $\alpha$  values, together with resulting  $\hat{\beta}$  values can be found on our project's GitHub repository: [https://github.com/rtavenard/CostAware\\_ECTS](https://github.com/rtavenard/CostAware_ECTS).

(a) *2Step* vs. EarlyOpt.SVM(b) *NoCluster* vs. EarlyOpt.SVMFig. 1: Comparisons with method EarlyOpt.SVM for  $C_\beta$  cost function.

in the following, we present comparisons between our proposed methods and EarlyOpt.SVM in terms of these two costs.

Fig. 1 reports comparison between our proposed methods and EarlyOpt.SVM in terms of  $C_\beta$  as introduced above. In the paper, results using  $C_\beta$  (that tend to favor EarlyOpt.SVM) are presented. It is striking to see that both costs give very similar results and, whatever the cost used, our proposed methods always outperform EarlyOpt.SVM. One-sided Wilcoxon signed rank test  $p$ -values indicate that differences observed for both our methods can be considered significant at the 5% level.

## References

- [1] Asma Dachraoui, Alexis Bondu, and Antoine Cornuéjols. “Early Classification of Time Series as a Non Myopic Sequential Decision Making Problem”. In: *Machine Learning and Knowledge Discovery in Databases*. Springer, 2015, pp. 433–447.
- [2] Usue Mori et al. “Early classification of time series from a cost minimization point of view”. In: *Proc. of the NIPS Time Series Workshop*. 2015.

## 2 Exhaustive cost results

Dataset	$\beta$	Baseline	NoCluster	2Step
ArrowHead	0.0005	0.648	0.326	<b>0.306</b>
	0.0050	0.673	<b>0.670</b>	0.684
	0.0010	0.652	0.420	<b>0.392</b>
	0.0100	0.739	0.686	<b>0.651</b>
BeetleFly	0.0005	0.380	<b>0.347</b>	0.406

Dataset	$\beta$	Baseline	NoCluster	2Step
	0.0050	<b>0.378</b>	0.386	0.461
	0.0010	<b>0.359</b>	0.425	0.396
	0.0100	0.408	<b>0.391</b>	0.393
BirdChicken	0.0005	0.402	<b>0.281</b>	0.404
	0.0050	0.448	0.454	<b>0.387</b>
	0.0010	0.429	<b>0.357</b>	0.410
	0.0100	0.500	0.509	<b>0.441</b>
Car	0.0005	0.446	0.389	<b>0.381</b>
	0.0050	0.727	0.806	<b>0.703</b>
	0.0010	0.625	0.488	<b>0.462</b>
	0.0100	0.821	0.823	<b>0.740</b>
ChlorineConcentration	0.0005	0.362	0.358	<b>0.285</b>
	0.0050	0.623	0.489	<b>0.487</b>
	0.0010	0.432	0.473	<b>0.364</b>
	0.0100	0.786	<b>0.507</b>	<b>0.507</b>
Coffee	0.0005	0.573	0.102	<b>0.075</b>
	0.0050	<b>0.591</b>	<b>0.591</b>	0.641
	0.0010	0.575	<b>0.057</b>	0.064
	0.0100	<b>0.611</b>	<b>0.611</b>	<b>0.611</b>
Computers	0.0050	0.513	<b>0.504</b>	–
	0.0100	0.538	<b>0.524</b>	–
Cricket_X	0.0005	0.882	0.788	<b>0.731</b>
	0.0050	1.044	<b>0.882</b>	0.894
	0.0010	0.896	0.872	<b>0.862</b>
	0.0100	1.037	<b>0.912</b>	0.919
Cricket_Y	0.0005	0.713	0.738	<b>0.698</b>
	0.0050	0.979	0.862	<b>0.861</b>
	0.0010	<b>0.751</b>	0.824	0.823
	0.0100	1.122	<b>0.881</b>	<b>0.881</b>
Cricket_Z	0.0005	0.843	0.855	<b>0.758</b>
	0.0050	0.911	0.853	<b>0.846</b>
	0.0010	<b>0.845</b>	0.868	0.856
	0.0100	0.955	0.873	<b>0.866</b>
DistalPhalanxOutlineAgeGroup	0.0005	0.229	0.232	<b>0.228</b>
	0.0050	0.300	0.230	<b>0.228</b>
	0.0010	0.224	<b>0.222</b>	0.254
	0.0100	0.303	0.256	<b>0.247</b>
DistalPhalanxOutlineCorrect	0.0005	0.542	0.494	<b>0.284</b>
	0.0050	0.600	0.580	<b>0.578</b>
	0.0010	0.557	0.560	<b>0.358</b>
	0.0100	0.665	0.598	<b>0.593</b>

Dataset	$\beta$	Baseline	NoCluster	2Step
ECG200	0.0005	<b>0.169</b>	0.192	0.206
	0.0050	0.250	0.250	<b>0.242</b>
	0.0010	0.223	<b>0.196</b>	0.221
	0.0100	0.321	<b>0.272</b>	0.290
Earthquakes	0.0005	0.299	<b>0.184</b>	0.251
	0.0050	0.427	<b>0.206</b>	0.212
	0.0010	0.327	<b>0.187</b>	0.231
	0.0100	<b>0.220</b>	0.230	<b>0.220</b>
FISH	0.0005	0.367	0.302	<b>0.298</b>
	0.0050	0.778	0.754	<b>0.754</b>
	0.0010	0.430	0.408	<b>0.403</b>
	0.0100	0.841	<b>0.783</b>	<b>0.783</b>
FaceAll	0.0005	<b>0.262</b>	0.321	0.304
	0.0050	0.511	<b>0.470</b>	0.657
	0.0010	<b>0.301</b>	0.361	0.369
	0.0100	<b>0.621</b>	0.662	0.670
Gun.Point	0.0005	0.513	<b>0.199</b>	0.261
	0.0050	0.564	<b>0.412</b>	0.499
	0.0010	0.507	0.250	<b>0.232</b>
	0.0100	0.597	<b>0.549</b>	0.565
Ham	0.0005	0.350	0.376	<b>0.338</b>
	0.0050	0.594	<b>0.506</b>	0.511
	0.0010	<b>0.395</b>	0.419	0.464
	0.0100	0.750	<b>0.526</b>	<b>0.526</b>
HandOutlines	0.0005	0.452	<b>0.330</b>	–
	0.0050	<b>0.337</b>	0.381	–
	0.0010	0.322	<b>0.319</b>	–
	0.0100	0.381	<b>0.357</b>	–
Haptics	0.0005	0.674	<b>0.641</b>	0.699
	0.0050	0.675	<b>0.673</b>	0.674
	0.0010	0.671	<b>0.636</b>	0.664
	0.0100	0.743	<b>0.699</b>	<b>0.699</b>
Herring	0.0005	0.498	<b>0.413</b>	0.527
	0.0050	<b>0.426</b>	0.427	0.441
	0.0010	0.466	<b>0.412</b>	0.556
	0.0100	<b>0.446</b>	<b>0.446</b>	<b>0.446</b>
InsectWingbeatSound	0.0005	0.774	0.455	<b>0.437</b>
	0.0050	0.819	<b>0.776</b>	0.843
	0.0010	0.738	0.511	<b>0.485</b>
	0.0100	0.867	0.872	<b>0.858</b>
ItalyPowerDemand	0.0005	0.096	<b>0.039</b>	0.059

Dataset	$\beta$	Baseline	NoCluster	2Step
	0.0050	0.185	<b>0.129</b>	0.173
	0.0010	0.111	<b>0.053</b>	0.083
	0.0100	0.290	<b>0.259</b>	0.296
LargeKitchenAppliances	0.0005	<b>0.582</b>	0.582	–
	0.0050	0.704	<b>0.588</b>	–
	0.0010	<b>0.555</b>	0.573	–
	0.0100	<b>0.613</b>	0.619	0.636
Lighting2	0.0005	0.443	<b>0.335</b>	0.423
	0.0050	0.405	<b>0.353</b>	0.354
	0.0010	0.455	<b>0.339</b>	0.354
	0.0100	<b>0.368</b>	0.435	0.425
Meat	0.0005	0.369	0.221	<b>0.203</b>
	0.0050	<b>0.387</b>	0.390	<b>0.387</b>
	0.0010	0.371	0.396	<b>0.249</b>
	0.0100	<b>0.407</b>	0.412	<b>0.407</b>
MiddlePhalanxOutlineAgeGroup	0.0005	0.236	<b>0.233</b>	0.265
	0.0050	0.797	<b>0.750</b>	0.755
	0.0010	0.258	<b>0.245</b>	0.277
	0.0100	<b>0.770</b>	<b>0.770</b>	<b>0.770</b>
MiddlePhalanxOutlineCorrect	0.0005	0.500	<b>0.486</b>	0.491
	0.0050	<b>0.551</b>	0.575	0.557
	0.0010	<b>0.516</b>	0.524	0.525
	0.0100	0.543	<b>0.393</b>	<b>0.393</b>
MoteStrain	0.0005	0.273	0.164	<b>0.157</b>
	0.0050	<b>0.244</b>	0.284	0.256
	0.0010	0.232	<b>0.177</b>	0.179
	0.0100	<b>0.302</b>	0.333	0.306
OSULeaf	0.0005	0.656	<b>0.576</b>	0.634
	0.0050	0.836	<b>0.721</b>	0.774
	0.0010	0.675	<b>0.608</b>	0.655
	0.0100	0.908	<b>0.742</b>	0.794
PhalangesOutlinesCorrect	0.0005	0.377	<b>0.363</b>	0.366
	0.0050	0.488	<b>0.407</b>	<b>0.407</b>
	0.0010	0.394	0.384	<b>0.383</b>
	0.0100	0.592	<b>0.427</b>	<b>0.427</b>
ProximalPhalanxOutlineAgeGroup	0.0005	<b>0.141</b>	0.181	0.185
	0.0050	<b>0.157</b>	0.219	<b>0.157</b>
	0.0010	<b>0.141</b>	0.195	0.188
	0.0100	<b>0.177</b>	0.189	0.179
ProximalPhalanxOutlineCorrect	0.0005	0.211	0.190	<b>0.189</b>
	0.0050	0.423	<b>0.337</b>	0.341

Dataset	$\beta$	Baseline	NoCluster	2Step
	0.0010	0.234	0.257	<b>0.212</b>
	0.0100	0.448	<b>0.356</b>	<b>0.356</b>
RefrigerationDevices	0.0005	0.698	<b>0.620</b>	0.718
	0.0050	0.769	<b>0.628</b>	0.629
	0.0010	0.691	<b>0.616</b>	0.693
	0.0100	0.789	<b>0.648</b>	0.712
ScreenType	0.0005	0.666	0.659	<b>0.657</b>
	0.0050	<b>0.681</b>	0.689	0.695
	0.0010	<b>0.668</b>	0.674	0.684
	0.0100	<b>0.704</b>	0.715	0.715
ShapeletSim	0.0005	0.499	<b>0.489</b>	0.551
	0.0050	0.552	<b>0.531</b>	0.561
	0.0010	0.521	<b>0.514</b>	0.534
	0.0100	<b>0.560</b>	0.571	0.612
SmallKitchenAppliances	0.0005	<b>0.441</b>	0.447	–
	0.0050	0.447	<b>0.441</b>	–
	0.0100	<b>0.461</b>	<b>0.461</b>	<b>0.461</b>
SonyAIBORobotSurfaceII	0.0005	0.244	0.228	<b>0.227</b>
	0.0050	0.279	0.303	<b>0.278</b>
	0.0010	0.237	<b>0.233</b>	0.236
	0.0100	0.317	<b>0.313</b>	0.330
StarLightCurves	0.0005	0.222	<b>0.177</b>	0.189
	0.0050	<b>0.192</b>	0.195	–
	0.0010	0.287	<b>0.181</b>	0.193
	0.0100	<b>0.212</b>	0.218	<b>0.212</b>
Strawberry	0.0005	0.131	0.093	<b>0.092</b>
	0.0050	0.394	0.525	<b>0.380</b>
	0.0010	0.181	0.129	<b>0.125</b>
	0.0100	<b>0.397</b>	<b>0.397</b>	<b>0.397</b>
SwedishLeaf	0.0005	0.390	<b>0.225</b>	0.226
	0.0050	0.505	<b>0.430</b>	0.434
	0.0010	0.402	<b>0.268</b>	0.284
	0.0100	0.593	<b>0.493</b>	0.500
ToeSegmentation1	0.0005	<b>0.462</b>	0.519	0.521
	0.0050	0.562	<b>0.490</b>	0.609
	0.0010	<b>0.516</b>	0.525	0.555
	0.0100	0.740	<b>0.495</b>	0.534
ToeSegmentation2	0.0005	<b>0.513</b>	0.606	0.541
	0.0050	0.789	0.715	<b>0.704</b>
	0.0010	0.609	0.611	<b>0.547</b>
	0.0100	0.753	<b>0.698</b>	0.713

Dataset	$\beta$	Baseline	NoCluster	2Step
Trace	0.0005	0.296	0.259	<b>0.233</b>
	0.0050	0.526	0.372	<b>0.347</b>
	0.0010	0.396	0.317	<b>0.284</b>
	0.0100	0.544	0.414	<b>0.368</b>
TwoLeadECG	0.0005	<b>0.085</b>	0.125	0.110
	0.0050	0.239	<b>0.202</b>	0.271
	0.0010	0.127	<b>0.119</b>	0.133
	0.0100	0.574	0.553	<b>0.449</b>
Two_Patterns	0.0005	0.253	<b>0.247</b>	0.255
	0.0050	0.790	0.756	<b>0.753</b>
	0.0010	0.317	<b>0.309</b>	0.312
	0.0100	0.789	<b>0.776</b>	0.780
UWaveGestureLibraryAll	0.0005	0.619	0.494	<b>0.492</b>
	0.0050	0.841	<b>0.775</b>	<b>0.775</b>
	0.0010	<b>0.724</b>	–	0.760
	0.0100	0.965	–	<b>0.795</b>
Wine	0.0005	<b>0.502</b>	0.539	0.560
	0.0050	<b>0.520</b>	0.523	0.522
	0.0010	<b>0.504</b>	0.507	0.527
	0.0100	<b>0.540</b>	<b>0.540</b>	0.550
WormsTwoClass	0.0005	0.428	<b>0.426</b>	0.486
	0.0050	0.496	<b>0.440</b>	<b>0.440</b>
	0.0010	0.435	<b>0.428</b>	0.539
	0.0100	0.589	<b>0.460</b>	<b>0.460</b>
synthetic_control	0.0005	0.270	<b>0.117</b>	0.118
	0.0050	0.285	<b>0.214</b>	0.241
	0.0010	0.272	0.153	<b>0.137</b>
	0.0100	0.308	0.281	<b>0.268</b>
uWaveGestureLibrary_X	0.0005	0.633	<b>0.471</b>	0.497
	0.0050	<b>0.773</b>	0.773	0.773
	0.0010	0.678	<b>0.534</b>	0.757
	0.0100	0.793	0.793	<b>0.793</b>
uWaveGestureLibrary_Y	0.0005	0.563	<b>0.502</b>	0.534
	0.0050	0.904	<b>0.753</b>	<b>0.753</b>
	0.0010	0.616	<b>0.585</b>	–
	0.0100	0.773	0.773	<b>0.772</b>
uWaveGestureLibrary_Z	0.0005	0.726	–	<b>0.549</b>
	0.0050	0.794	<b>0.752</b>	<b>0.752</b>
	0.0010	0.734	–	<b>0.651</b>
	0.0100	0.863	0.772	<b>0.771</b>
wafer	0.0050	0.077	<b>0.058</b>	<b>0.058</b>

Dataset	$\beta$	Baseline	NoCluster	2Step
	0.0010	0.047	<b>0.042</b>	–
	0.0100	0.114	<b>0.078</b>	<b>0.078</b>
yoga	0.0005	<b>0.380</b>	0.385	0.469
	0.0050	0.535	<b>0.484</b>	<b>0.484</b>
	0.0010	<b>0.404</b>	0.435	0.502
	0.0100	0.586	<b>0.504</b>	<b>0.504</b>

Table 1: Per-dataset cost for both proposed methods and the baseline.