



# Daniel P Morgan

**Reflections upon the Presentation of Parallel Algorithms Across the Astral and Mathematical Sciences in First-Millennium China**

Int. Conf. History of Ancient Mathematics and Astronomy,  
Northwest University, August 23 – 29, 2015

The research leading to these results has received funding from the European research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC Grant agreement n. 269804.



# The Rule of Three: Astronomy vs. Mathematics

## Triple Concordance *li* 三統曆, c. 5 CE (cit. *HS* 21B.1001)

推天正：以章月乘入統歲數，盈章歲得一，名曰積月，不盈者名曰閏餘。

### Calculate astronomical [month] l:

Mount ( $\times$ ) the years entered into current concordance ( $y$ ) by [235], overflowing [19], get one.  
Name [this] 'accumulated months'; name that which does not overflow 'intercalary remainder'.

## Nine Chapters 九章算術, 2.1

術曰：以粟求糲米，三之，五而一。

### Procedure:

To find milled grain from unhulled grain, three it, five then one.

# The Rule of Three: Astronomy vs. Astronomy

Daniel  
Patrick  
Morgan

## Triple Concordance *li* 三統曆, c. 5 CE (cit. *HS* 21B.1001)

推天正：以章月乘入統歲數，盈章歲得一，名曰積月，不盈者名曰閏餘。

### Calculate astronomical [month] I:

Mount ( $\times$ ) the years entered into current concordance ( $y$ ) by [235], overflowing [19], get one. Name [this] 'accumulated months'; name that which does not overflow 'intercalary remainder'.

## Quarter-remainder *li* 四分曆, 85 CE (cit. *HHS zhi* 3, 3062)

推天正術：置入薨年減一，以章月乘之，滿章法得一，名為積月，不滿為閏餘。

### Procedure for calculating astronomical [month] I:

Set out the years entered into the current obscuration ( $y$ ), diminished ( $-$ ) one, mount ( $\times$ ) it by [235], filling [19], get one. The name [for this] is 'accumulated months'; [that which does] not fill is the 'intercalary remainder'.

## Supernal Image *li* 乾象曆, 206 CE (cit. *JS* 17.505)

推朔：置入紀年，外所求，以章月乘之，章歲而一，所得為定積月，不盡為閏餘。

### Calculate new moon:

Set out the years..., excluding that sought, mount ( $\times$ ) it by [235], [19] then one. That which is obtained is accumulated months; [that which is] not exhausted is the intercalary remainder.

## Luminous Inception *li* 景初曆, 237 CE (cit. *JS* 18.540)

推朔積月術曰：置壬辰元以來，盡所求年，外所求，... 以章月乘之，如章歲而一，為積月，不盡為閏餘。

### Procedure for calculating new moon accumulated months:

Set out the [years]..., excluding that sought, mount ( $\times$ ) it by [235], then one per [19], making accumulated months; [that which does] not fill is the intercalary remainder.

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# The Rule of Three: Astronomy vs. Astronomy

## Triple Concordance *li* 三統曆, c. 5 CE (cit. *HS* 21B.1001)

推天正：以章月乘入統歲數，盈章歲得一，名曰積月，不盈者名曰閏餘。

### Calculate astronomical [month] I:

Mount ( $\times$ ) the years entered into current concordance ( $y$ ) by [235], overflowing [19], get one. Name [this] 'accumulated months'; name that which does not overflow 'intercalary remainder'.

...

## Epochal Excellence *li* 元嘉曆, 443 CE (cit. *Song shu*, 13.273)

推積月術：置入紀年數，算外，以章月乘之，如章歲為積月，不盡為閏餘。

### Procedure for calculating accumulated months:

Set out the years..., outside the count, mount ( $\times$ ) it by [235], and make accumulated months per [19]; [that which is] not exhausted is the intercalary remainder.

## Ascendant Harmony *li* 興和曆, 539 CE (cit. *WS* 107B.2702)

推積月術曰：置入紀以來盡所求年減一，以章月乘之，章歲如一，所得為積月，不盡為閏餘。

### Procedure for calculating accumulated months:

Set out the [years]..., diminished ( $-$ ) one, mount ( $\times$ ) it by [235], and [19] is like one. That which is obtained is accumulated months; [that which is] not exhausted is the intercalary remainder.

# The Rule of Three: Mathematics vs. Mathematics

## *Nine Chapters* 九章算術, 2.1

術曰：以粟求糲米，三之，五而一。

### **Procedure:**

To find milled grain from unhulled grain, **three it, five then one.**

## *Suan shu shu* 算數書 MS, s116 (TAQ 186 BCE)

以米求粟、因而五之，三成一 (DIV!)。

Seeking hulled from unhulled grain: take and **five it, three becomes one.**

## *Nine Chapters* 九章算術, 6.3

術曰：以一里僦價，乘至輪所里，以一車二十五斛除之。

### **Procedure:**

**Mount** ( $\times$ ) the [number of] *li* to the toll office **by** the rental price per *li*, then **eliminate** it **by** 25 *hu* per vehicle...

# The Rule of Three: Division

## 'Then/get one per'

### Abbreviations (?)

$x$  如  $y$  得一

For  $x$ , get one per  $y$ .

$x$  如  $y$  而 (得) 一

For  $x$ , then [get] one per  $y$ .

$x$  (如)  $y$  得一

For  $x$ , get one [per]  $y$ .

$x$  (如)  $y$  而 (得) 一

For  $x$ , then [get] one [per]  $y$ .

$x$ ,  $y$  如一

For  $x$ , 'one-per'  $y$ .

### Variations

$x$  如  $y$  爲  $z$

$x$  per  $y$  makes  $z$ .

$x$  盈  $y$  得一

For  $x$ , overflowing  $y$  get one.

$x$  滿  $y$  得一

For  $x$ , filling  $y$  get one.

## 'Eliminate'

以  $y$  除  $x$

Eliminate  $x$  by  $y$ .

# 'Elimination' (*chú* 除) in Ast. Procedure Texts

## Triple Concordance *li* 三統曆 (c. 5 CE)

Procedure 1: 'Calculate the luni-solar origin & concordance' 推日月元統

置太極上元以來，外所求年。

Set out the [number of years elapsed] since grand culmen high origin (143,231 BCE XI-1 00:00, WS), excluding the year sought.

盈元法除之。

[If] overflowing the origin divisor (4617), **eliminate** it.

餘不盈統者，則天統甲子以來年數也；

Any remainder that does not overflow a concordance (1539 years) is 'the number of years since Heaven concordance' at *jiǎ-zǐ*.<sub>01/60</sub>

盈統，除之，餘則地統甲辰以來年數也。

[If] overflowing a concordance, **eliminate** it, and the remainder is 'the number of years since Earth concordance' at *jiǎ-chén*.<sub>41/60</sub>

又盈統，除之，餘則人統甲申以來年數也。

[If] overflowing yet another concordance, **eliminate** it, and the remainder is 'the number of years since Man concordance' at *jiǎ-shēn*.<sub>21/60</sub>

各以其統首日為紀。

Each [concordance] takes the (sexagenary) head day of the concordance as the mark (from which to count subsequent sexagenary dates in the procedures to follow).

# 'Elimination' (*chú* 除) in Ast. Procedure Texts

## Triple Concordance *li* 三統曆 (c. 5 CE)

Procedure 1: 'Calculate the luni-solar origin & concordance' 推日月元統

置太極上元以來，外所求年。

Set out the [number of years elapsed] since grand culmen high origin (143,231 BCE XI-1 00:00, WS), excluding the year sought.

盈元法除之。

[If] overflowing the origin divisor (4617), eliminate it.

餘不盈統者，則天統甲子以來年數也；

Any remainder that does not overflow a concordance (1539 years) is 'the number of years since Heav

year sought = 2015

盈統，除之

the 'since high origin' = 145,245 (= 2015 - -143,230)

[If] overflowing a concordance, eliminate it, and the remainder is 'the number of years since Earth concordance' at *jiǎ-chén*.<sup>41/60</sup>

又盈統，除之，餘則人統甲申以來年數也。

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各以其統首日為紀。

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# 'Elimination' (*chú* 除) in Ast. Procedure Texts

## Triple Concordance *li* 三統曆 (c. 5 CE)

Procedure 1: 'Calculate the luni-solar origin & concordance' 推日月元統

置太極上元以來，外所求年。

Set out the [number of years elapsed] since grand culmen high origin (143,231 BCE XI-1 00:00, WS), excluding the year sought.

盈元法除之。

[If] overflowing the origin divisor (4617), **eliminate** it.

餘不盈統者，則天統甲子以來年數也；

Any remainder that does not overflow a concordance (1539 years) is 'the number of years since Heav

**eliminate 4,617 from 145,245 = 2,118 (= 145,245 - 31 × 4,617)**

盈統，除之

[If] overflowing a concordance, eliminate it, and the remainder is 'the number of years since Earth concordance' at *jiǎ-chén*.<sup>41/60</sup>

又盈統，除之，餘則人統甲申以來年數也。

[If] overflowing yet another concordance, eliminate it, and the remainder is 'the number of years since Man concordance' at *jiǎ-shēn*.<sup>21/60</sup>

各以其統首日為紀。

Each [concordance] takes the (sexagenary) head day of the concordance as the mark (from which to count subsequent sexagenary dates in the procedures to follow).

# 'Elimination' (*chú* 除) in Ast. Procedure Texts

## Triple Concordance *li* 三統曆 (c. 5 CE)

Procedure 1: 'Calculate the luni-solar origin & concordance' 推日月元統

置太極上元  
Set out the  
00:00, WS)

2,118 overflows 1,539? Yes.

Eliminate 2,118 by 1,539 = 579 (= 2,118 - 1,539)

BCE XI-1

579 overflows 1,539? No

盈元法除之  
[If] overflowing

Result: Earth concordance, *jiǎchén*.<sup>41/60</sup>

餘不盈統者，則天統甲子以來年數也；

Any remainder that does not overflow a concordance (1539 years) is 'the number of years since Heaven concordance' at *jiǎ-zǐ*.<sup>01/60</sup>

盈統，除之，餘則地統甲辰以來年數也。

[If] overflowing a concordance, eliminate it, and the remainder is 'the number of years since Earth concordance' at *jiǎ-chén*.<sup>41/60</sup>

又盈統，除之，餘則人統甲申以來年數也。

[If] overflowing yet another concordance, eliminate it, and the remainder is 'the number of years since Man concordance' at *jiǎ-shēn*.<sup>21/60</sup>

各以其統首日為紀。

Each [concordance] takes the (sexagenary) head day of the concordance as the mark (from which to count subsequent sexagenary dates in the procedures to follow).

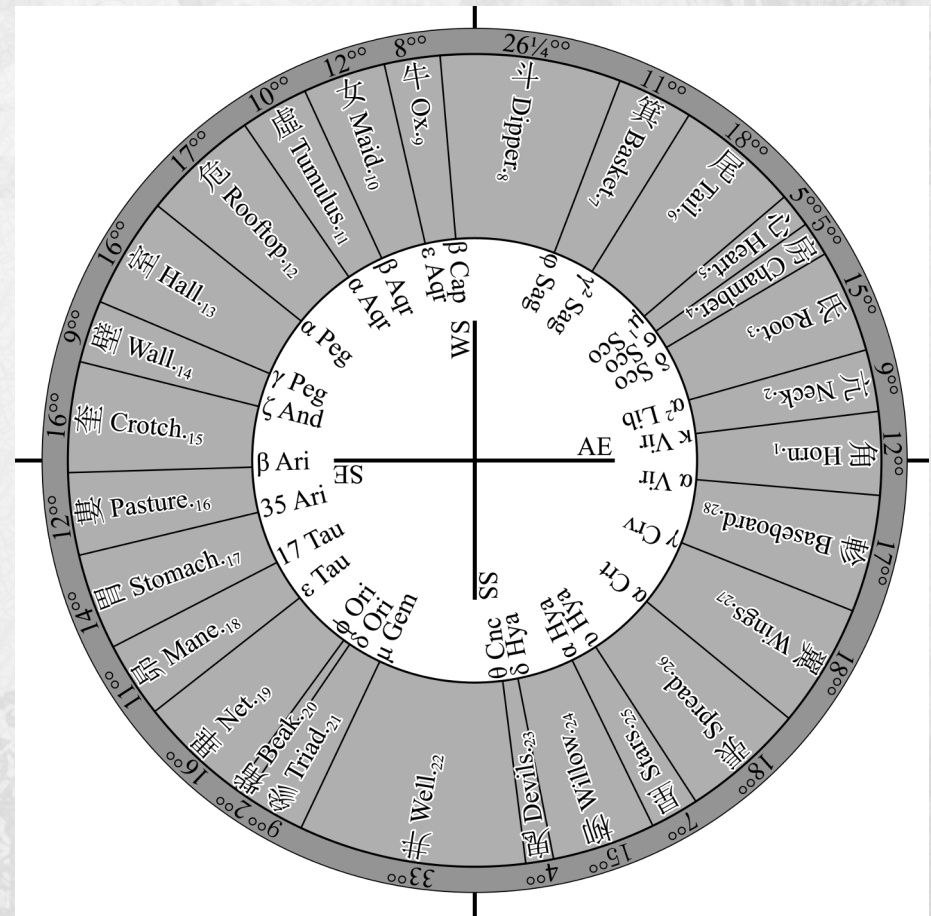
# 'Elimination' (*chú* 除) in Ast. Procedure Texts

## Quarter-remainder *li* 四分曆 (85/86 CE)

Procedure: 'Calculate the *du*-position of syzygy' 推合朔所在度

... 積度加斗二十一度，加二百三十五分，以宿次除之，不滿宿，則日月合朔所在星度也。

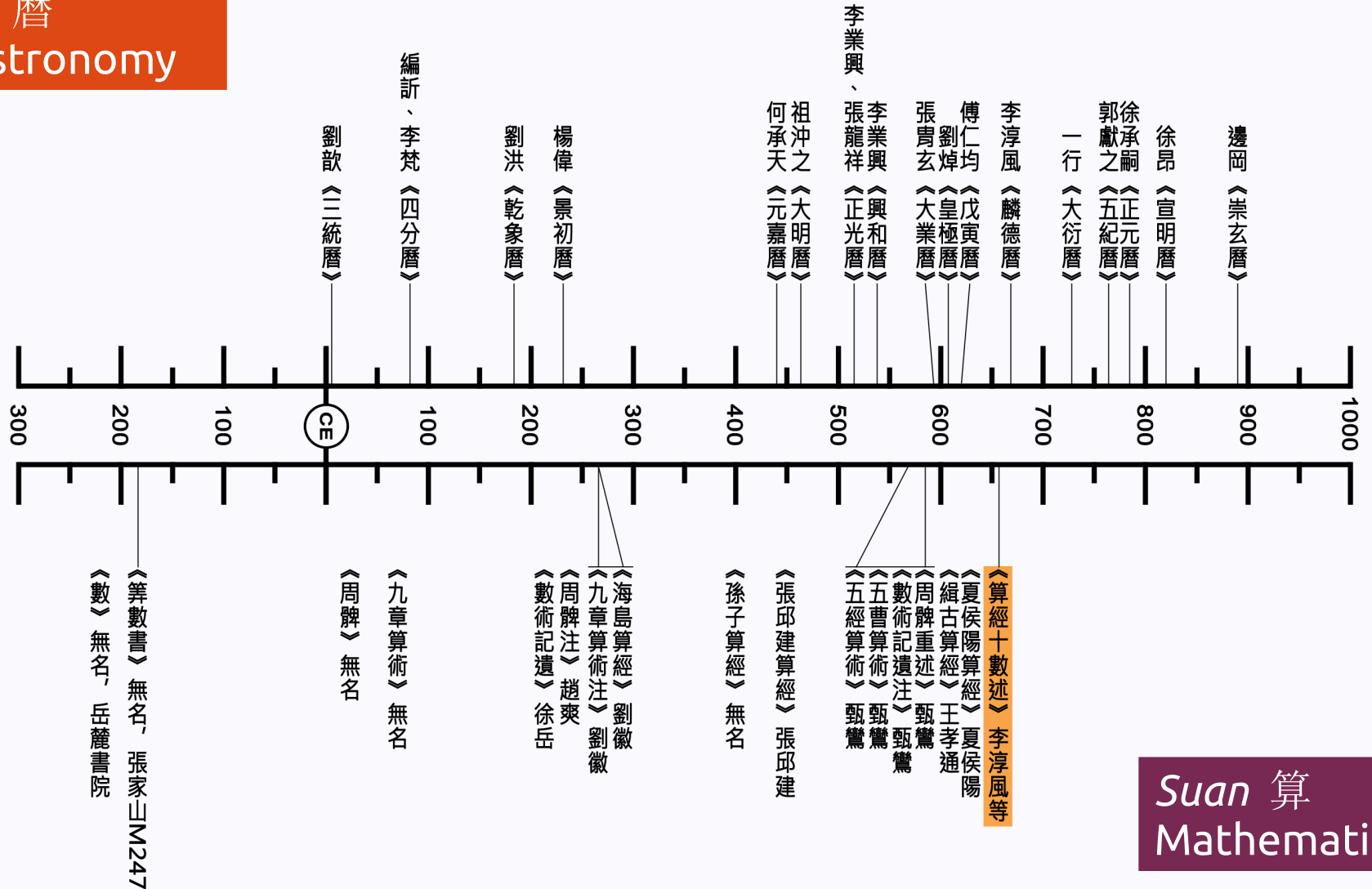
... add the accumulated *du* to the 21 *du* of Dipper and add two-hundred and thirty-five parts. **Sequentially eliminate (*ci chu*)** it by lodges, and that which does not fill [the last] lodge is the star & *du* position of the sun & moon at syzygy.



# Sources

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## Li 曆 Astronomy



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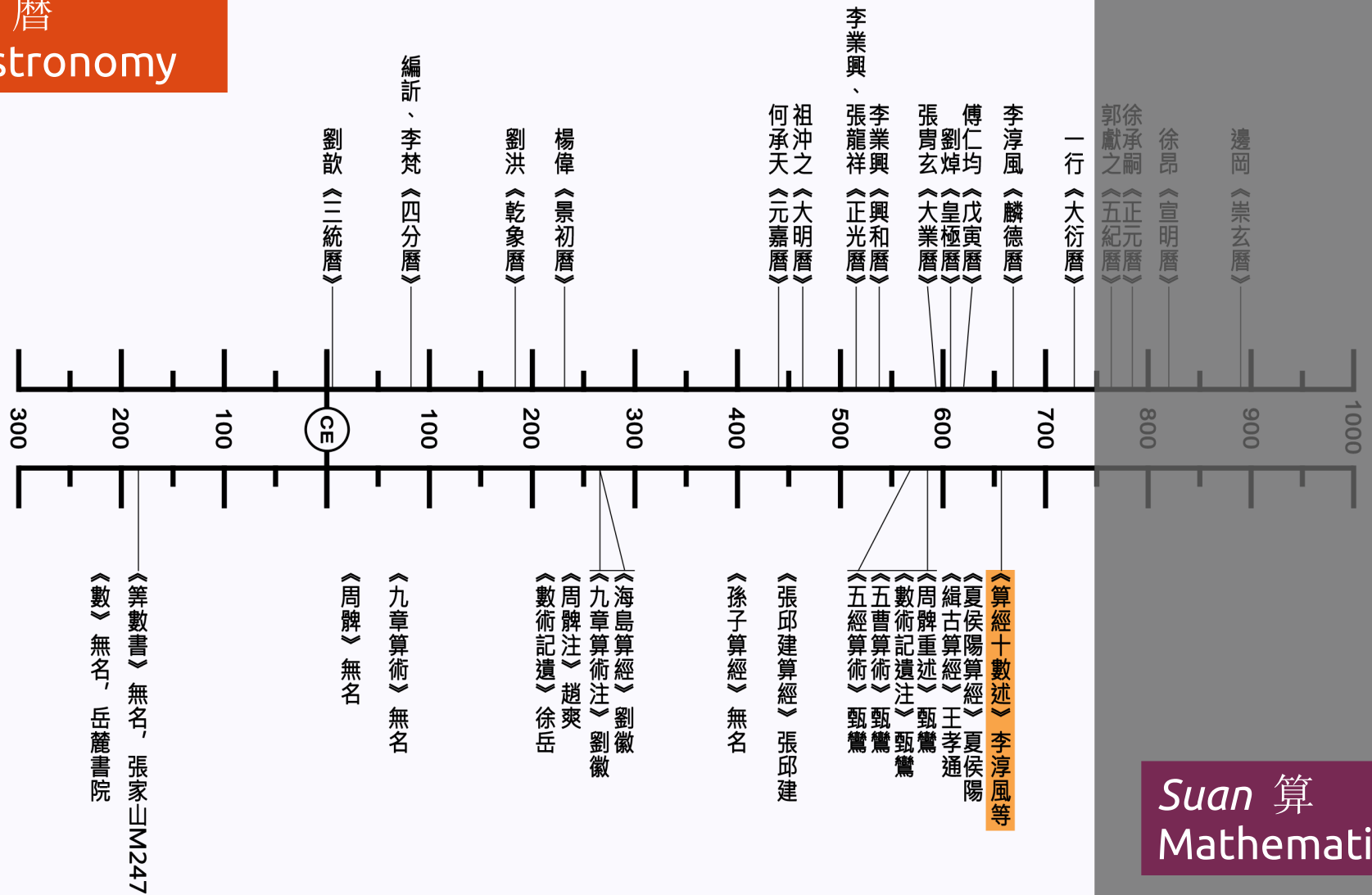
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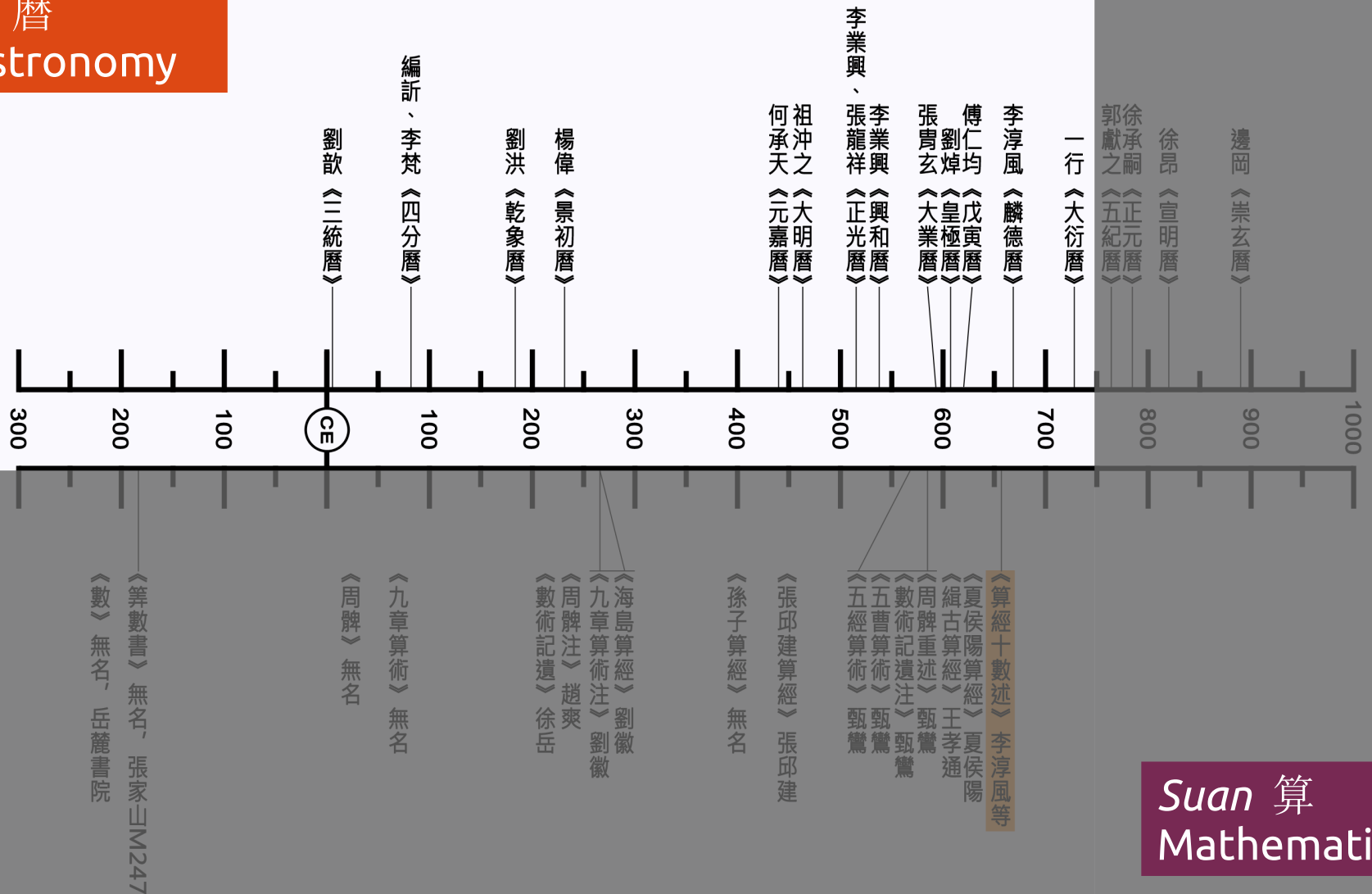


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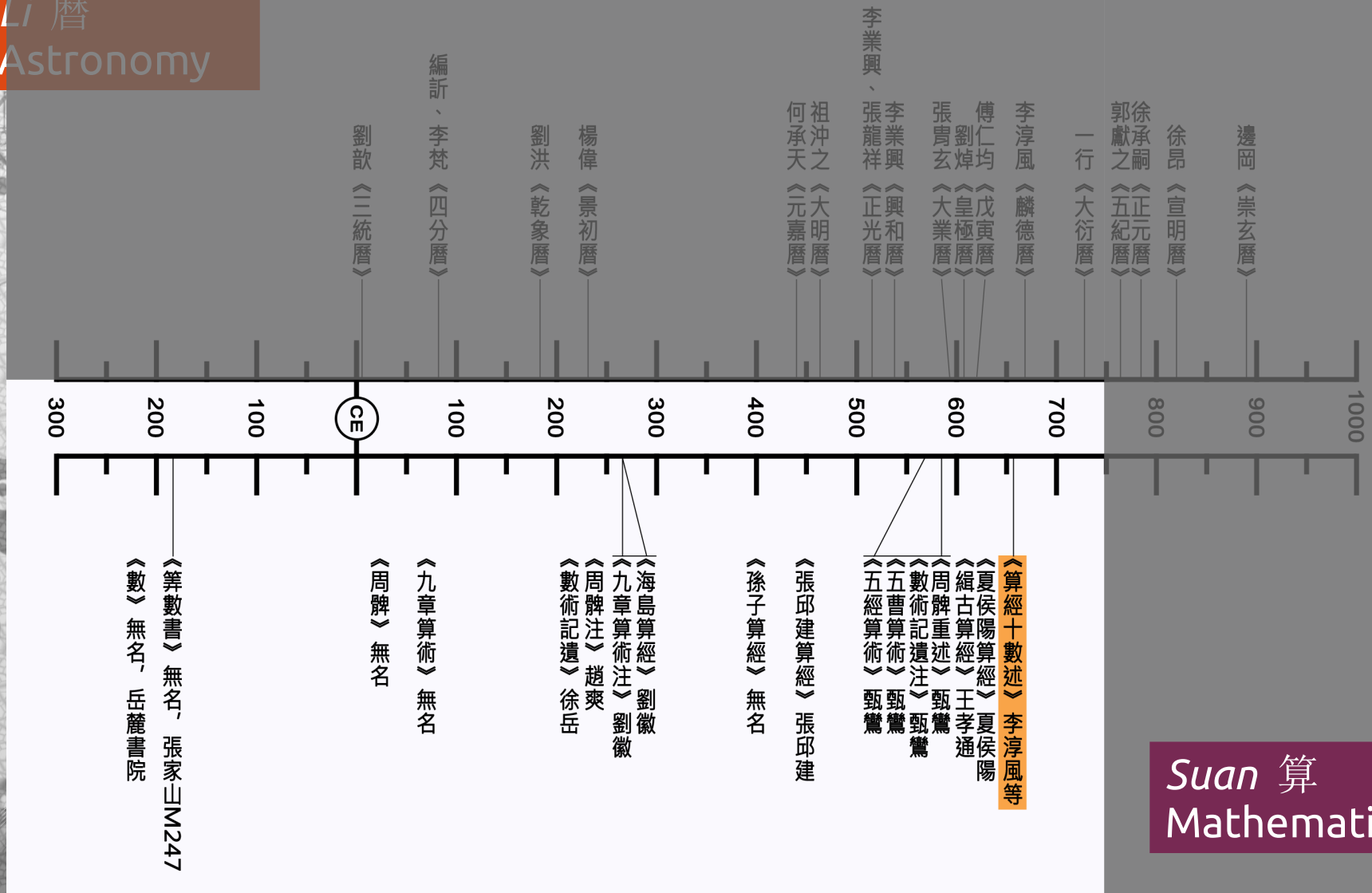


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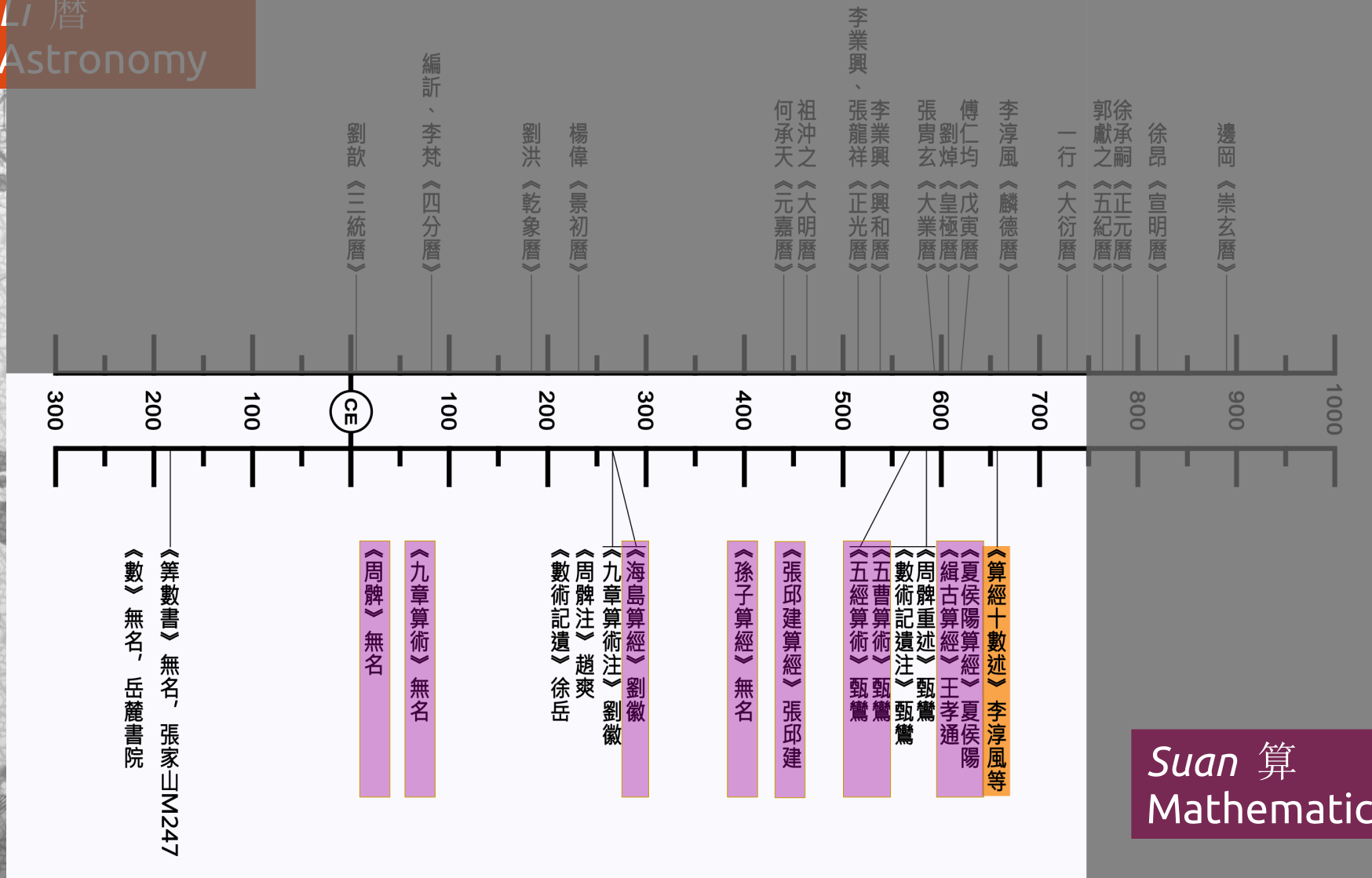


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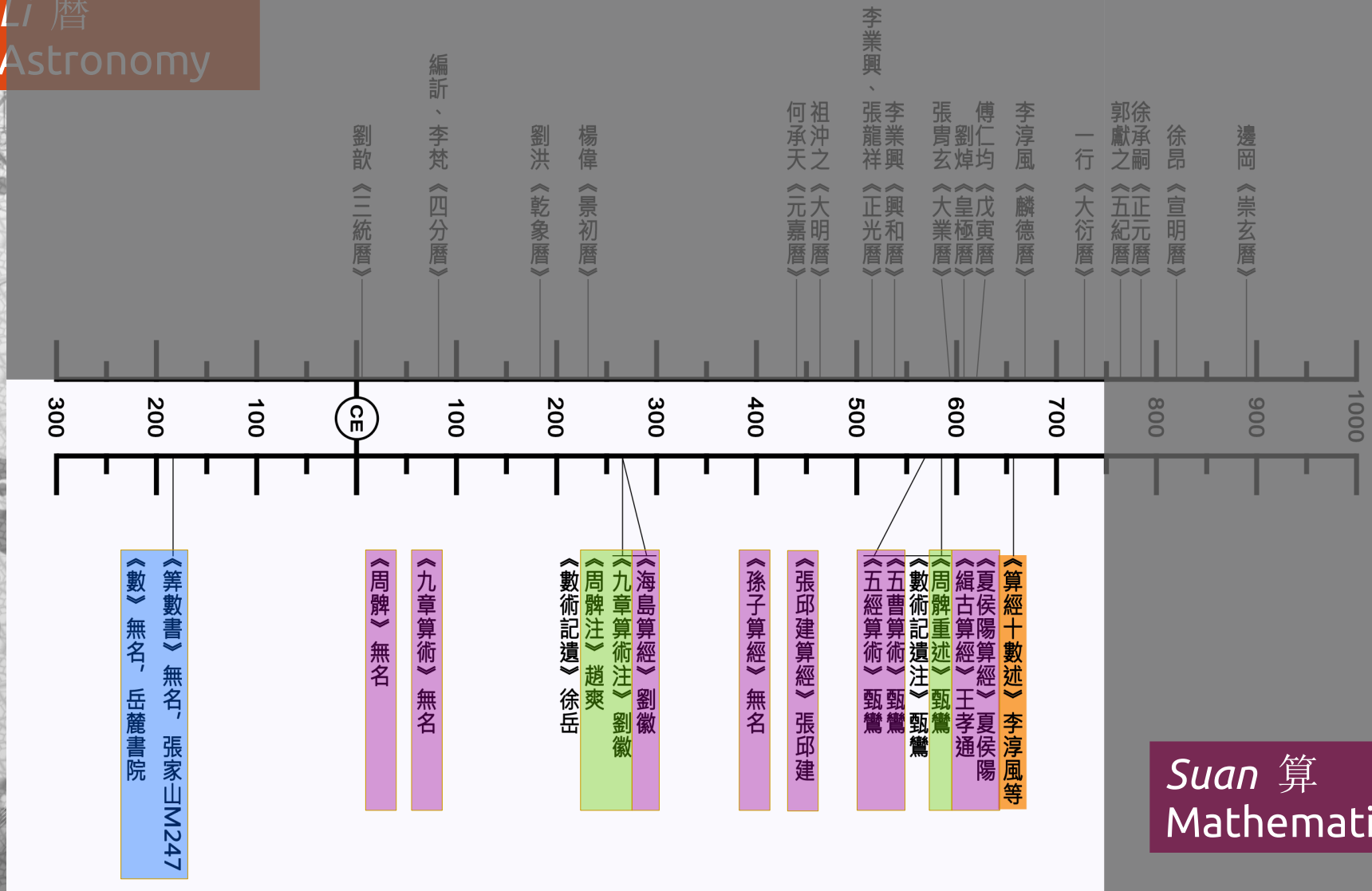
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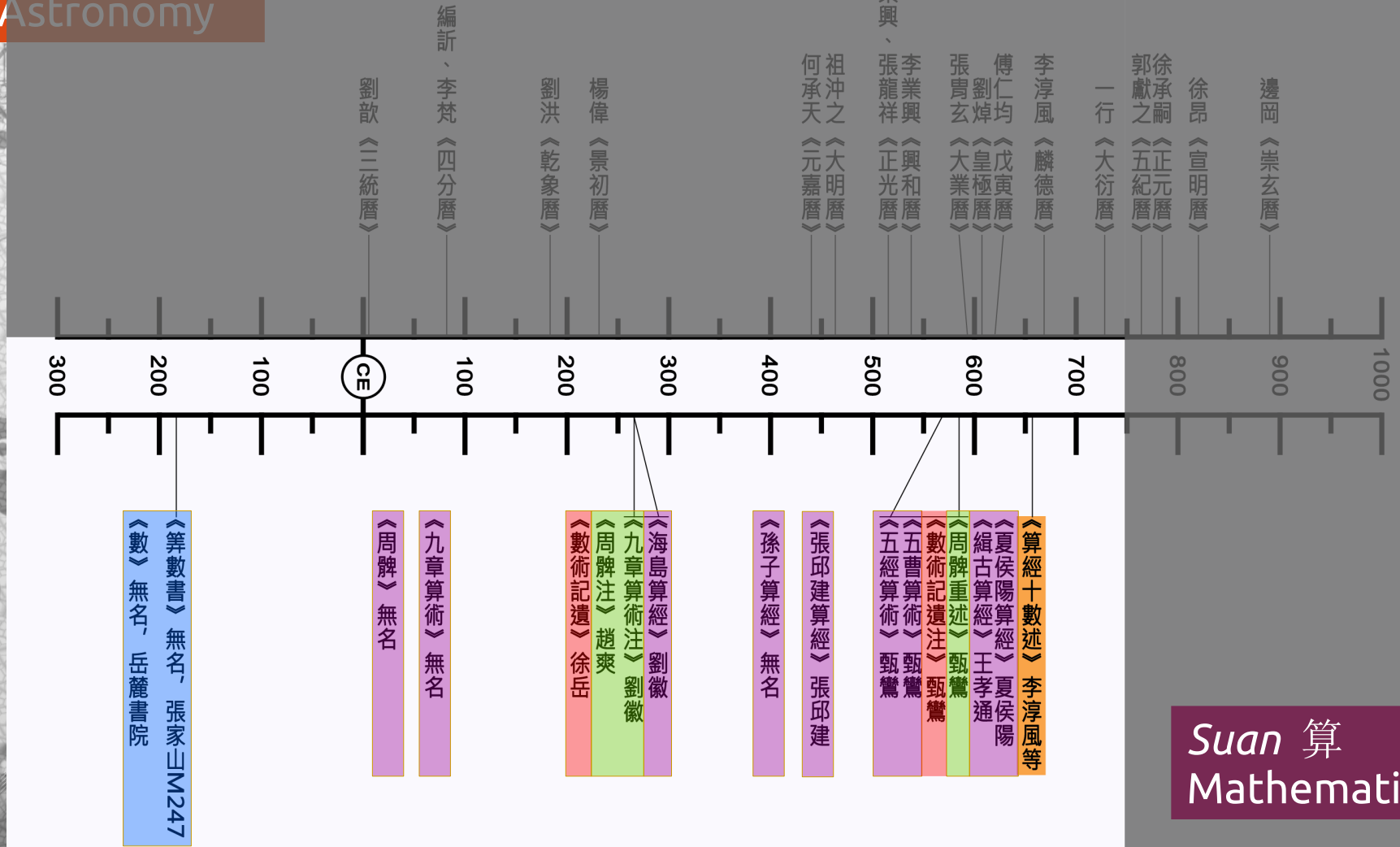


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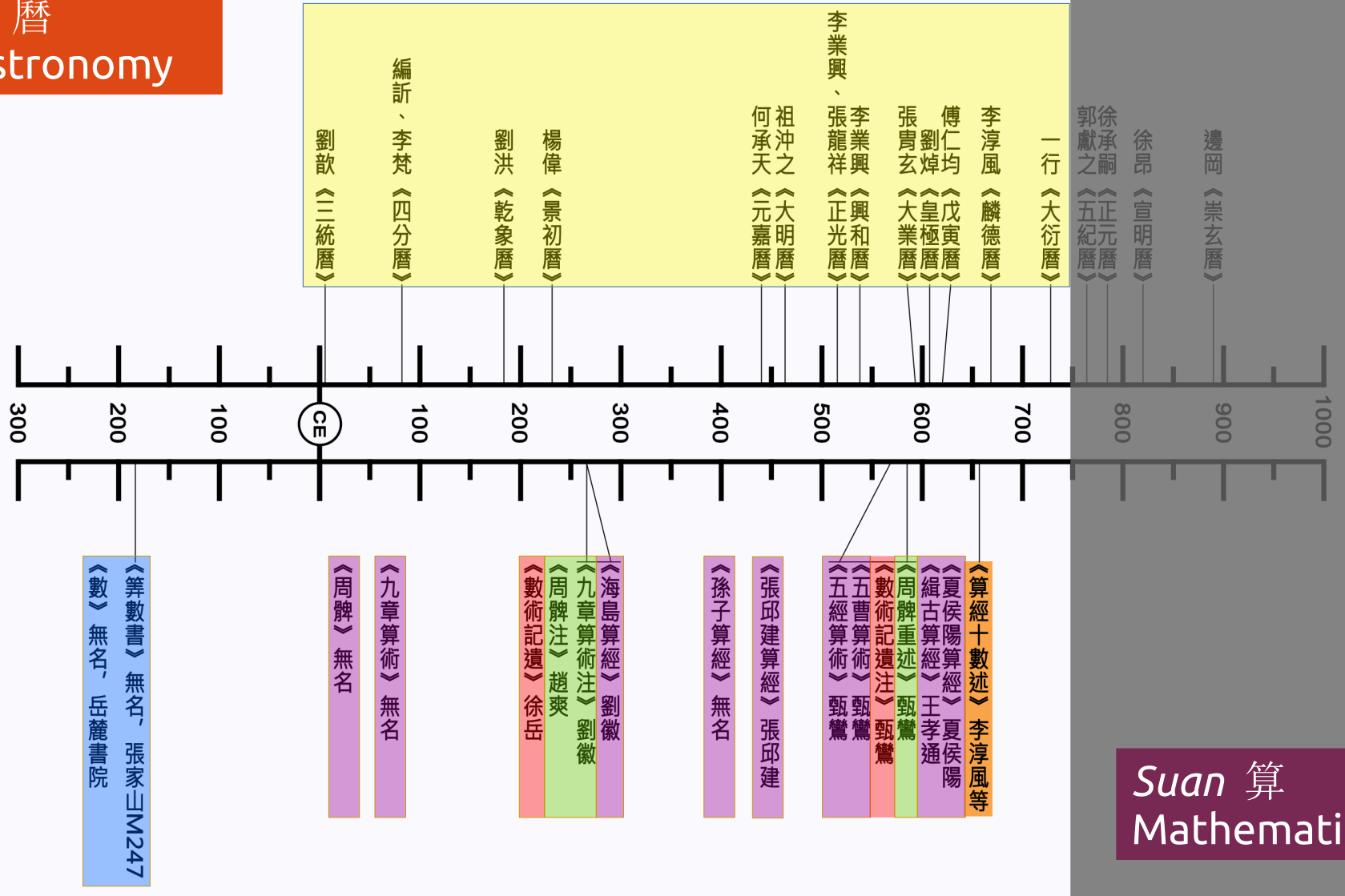


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## Li 曆 Astronomy



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# Methodology

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Morgan

The image shows two overlapping windows. The top window is a PDF viewer displaying a page from a document titled 'Chemla, Guo Shuchun - 2004 - Les neuf chapitres le classique mathématique de la Chine ancienne et ses commentaires.pdf'. The page number is 225. The text on the page is in French: 'Petit mil et grains décortiqués' and 'ON MULTIPLIE, PAR LA QUANTITÉ (SHU) DE CE QUE L'ON A, LE LÜ DE CE QU'ON CHERCHE, CE QUI FAIT LE DIVIDENDE. ON PREND LE LÜ DE CE QU'ON A COMME DIVISEUR <sup>21</sup>. Le peu est le commencement du beaucoup <sup>22</sup>; 1 est la source des quantités (shu) <sup>23</sup>. La raison en est que ce qui est pris comme lü, il faut l'égaliser à 1. En s'appuyant sur le fait que le lü du petit mil <sup>24</sup> est 5 et que le lü du grain grossièrement décortiqué est 3, alors de 5 de petit mil, ...'. The bottom window is LibreOffice Writer showing a document titled '九章算術 distribution.odt'. The text is in Chinese: '又有環田，中周六十二步、四分步之三，外周一百一十三步、二分步之一，徑十二步、三分步之二。問為田幾何？答曰：四畝一百五十六步、四分步之一。術曰：并中外周而半之，以徑乘之為積步。密率術曰：置中外周步數，分母、子各居其下。母互乘子，通全步，內分子。以中周減(SI)外周，餘半之，以益中周。徑亦通分內子，以乘周為實。分母相乘為法，除(DIVI)之為積步，餘積步之分。以畝法除(DIVI)之，即畝數也。《粟米》粟米之法：粟率五十；糲米三十。糲米二十七；糲米二十四。御米二十一；小糲十三半。' The status bar at the bottom of the LibreOffice window shows 'Page 3 / 23', '24689 words, 26723 characters', and 'Chinese (simplified)'. A note at the bottom of the screenshot reads: 'fait constamment le dénominateur » : c'est une erreur par omission.'

**Step 1:** Establish intended operation for each instance of actors' categories 除、去、滿、而一、減, etc. based on modern studies.

# Methodology

Daniel Patrick Morgan

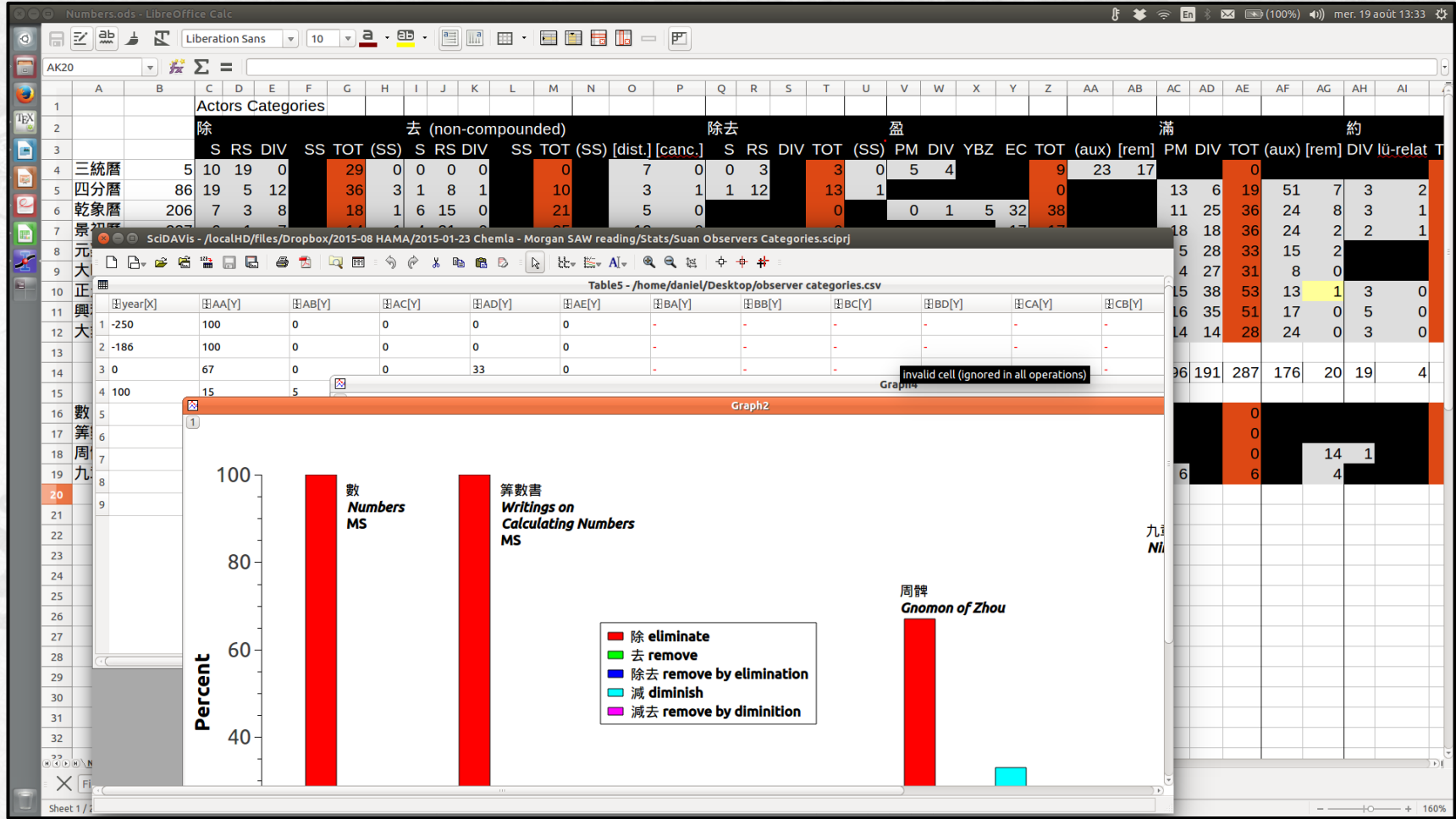
The screenshot displays a spreadsheet titled 'Actors Categories' with columns for various operations and their counts. The text document below it contains Chinese text explaining the methodology, including terms like '粟米' and '減(SI)'.

粟米之法：粟率五十；糲米三十  
 糲米二十七；糲米二十四  
 御米二十一；小糲十三半  
 大糲五十四；糲飯七十五  
 糲飯五十三。糲飯四十八

**Step 2:** Tabulate distribution of (1) each word's intended usage and (2) each operation's word choice.

# Methodology

Daniel Patrick Morgan



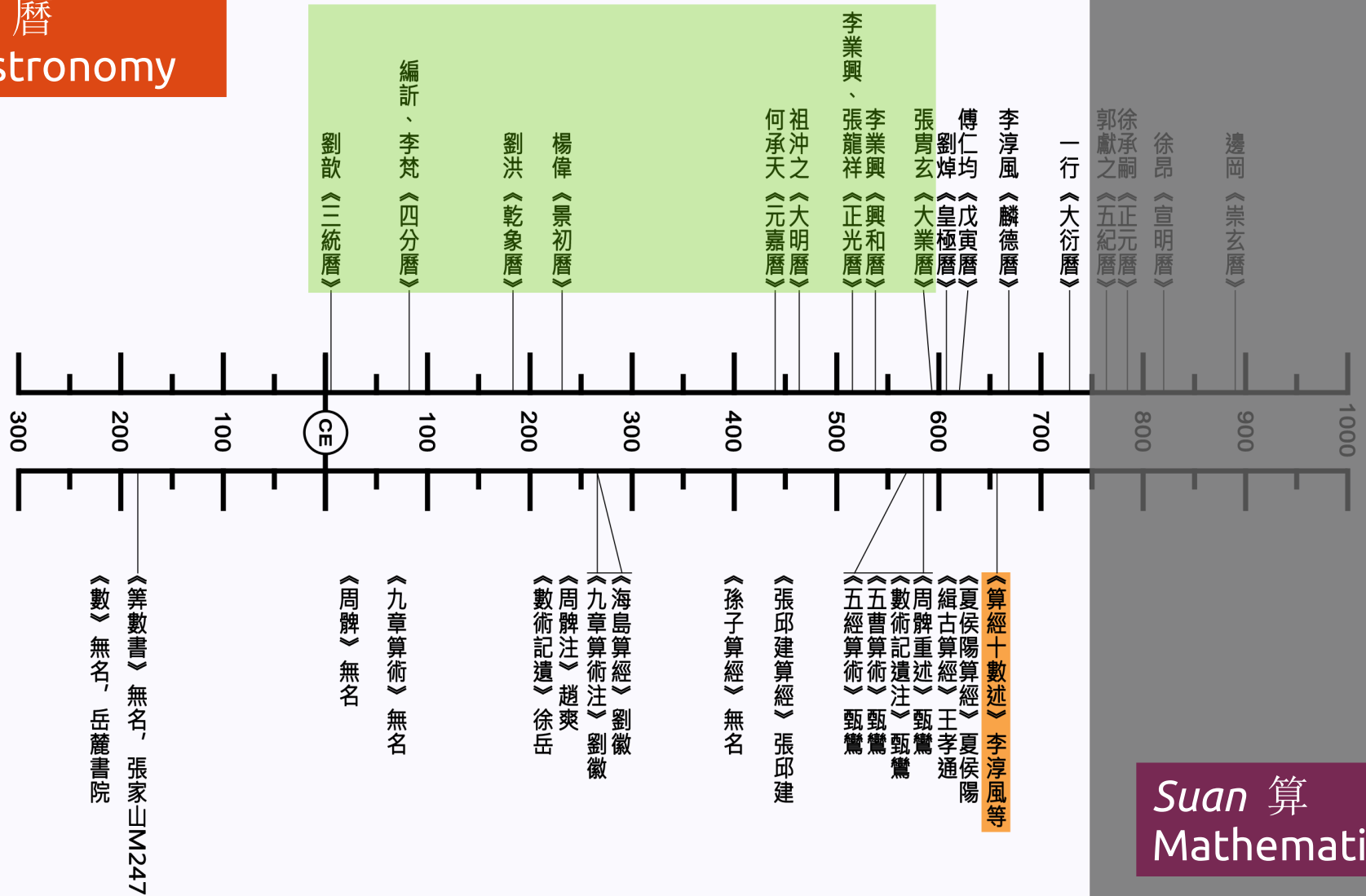
**Step 3:** Plot graphs to visualise synchronic differences and diachronic trends



# Sources

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## Li 曆 Astronomy



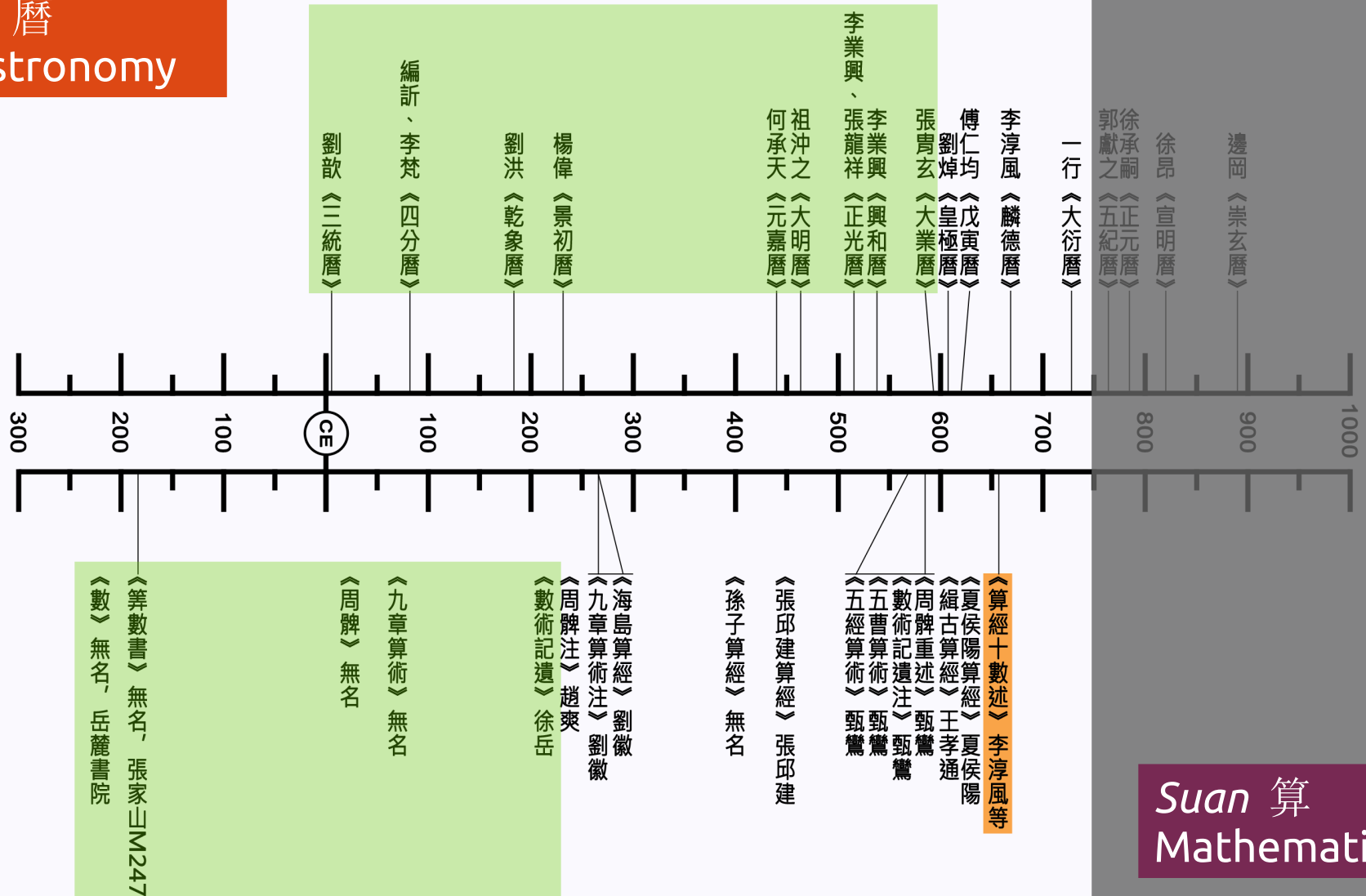
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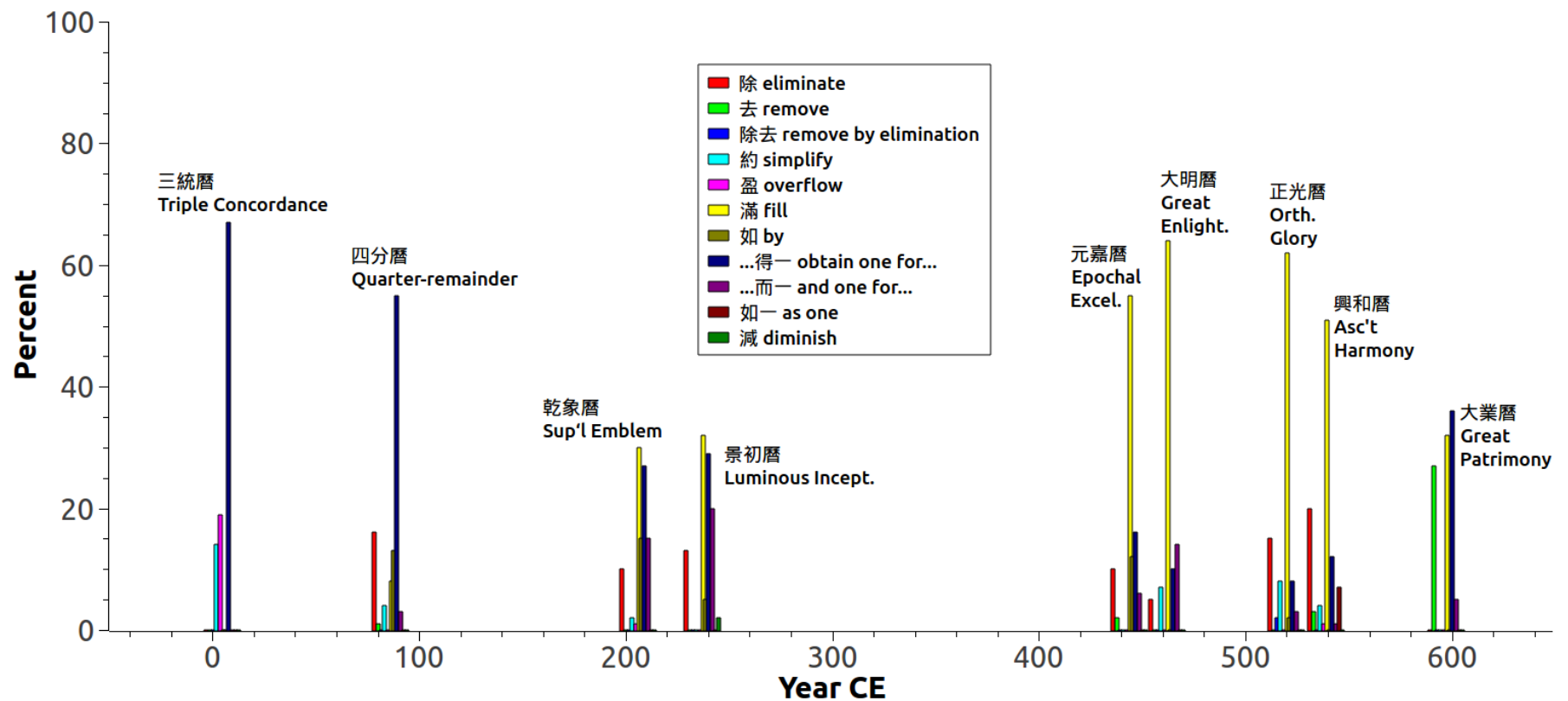


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# Distribution

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## Division, astronomical texts



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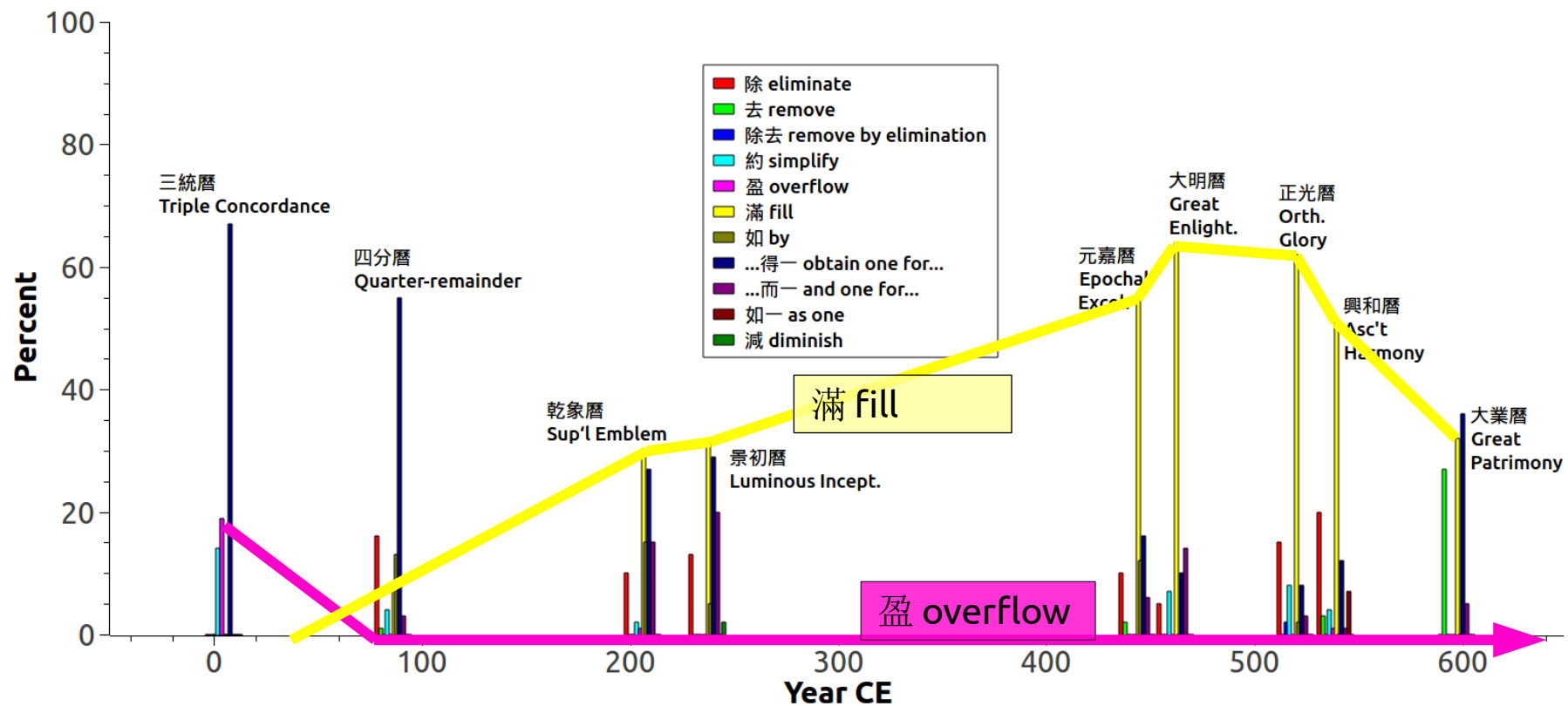
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## Division, astronomical texts



**Observation:** 'fill' replaces 'overflow' in 83 CE.

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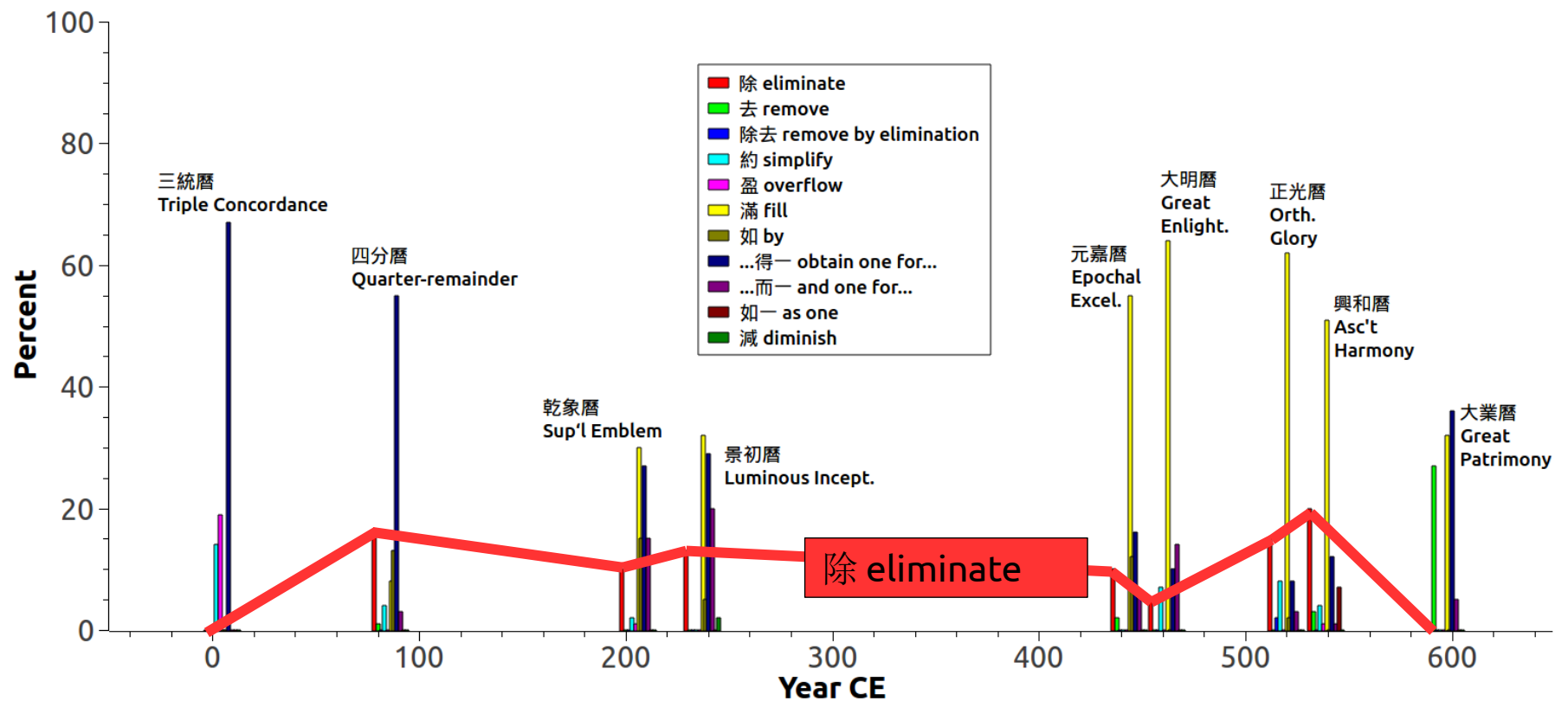
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## Division, astronomical texts

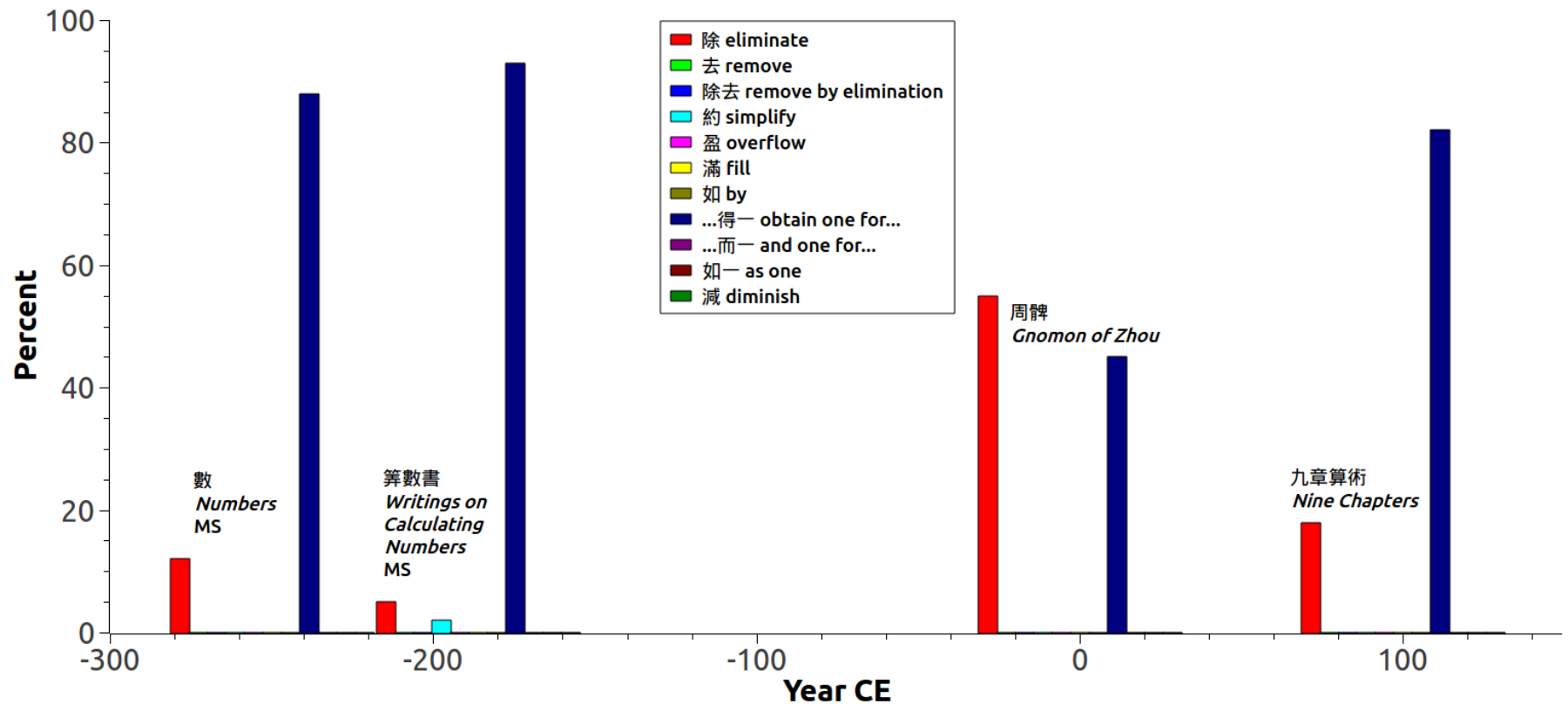


**Observation:** 'eliminate' appears in 85 CE; c.12% of cases thereafter; disappears in 600

# Distribution

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## Division, mathematical texts



**Observation:** 'eliminate' appears as division three centuries earlier than in ast.

**Observation:** 'eliminate' appears as division more frequently than in ast.

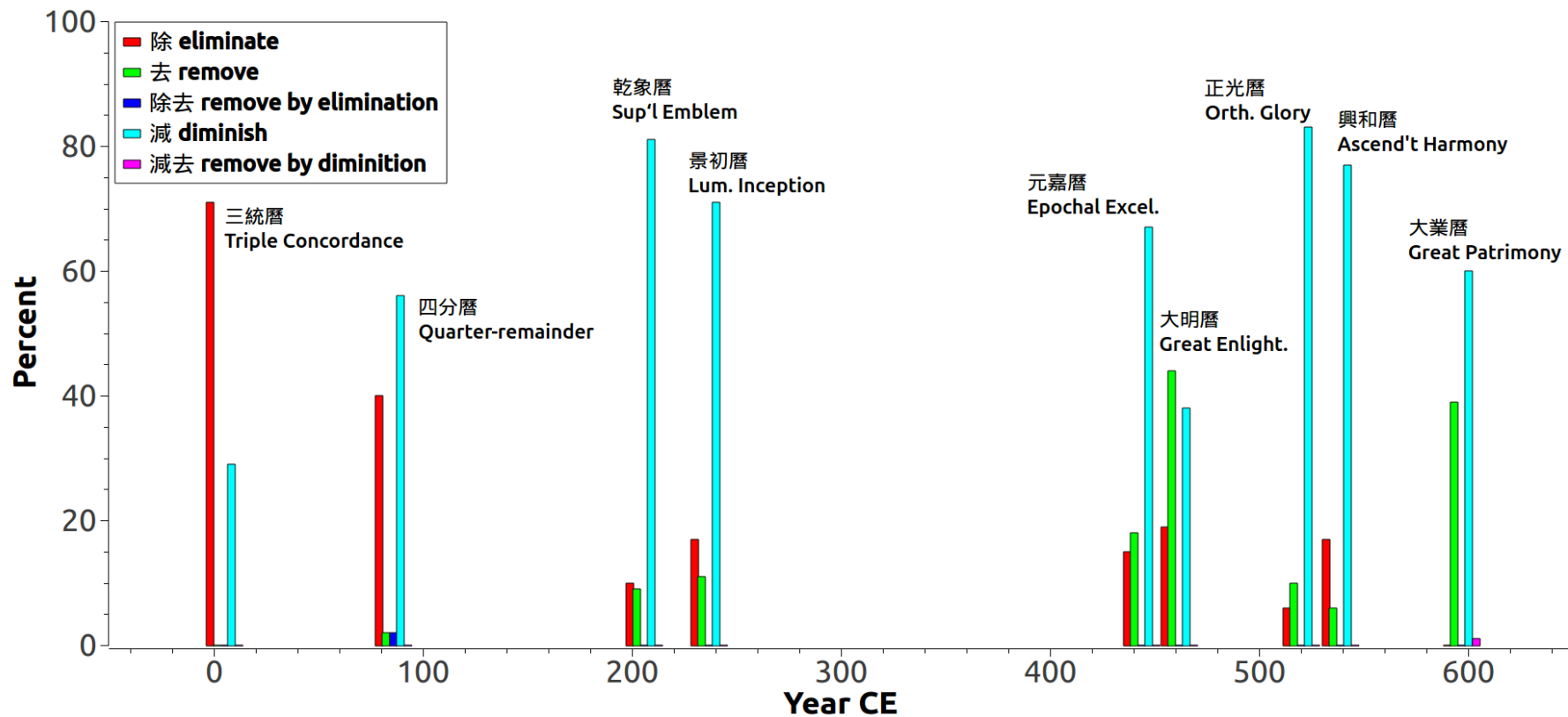
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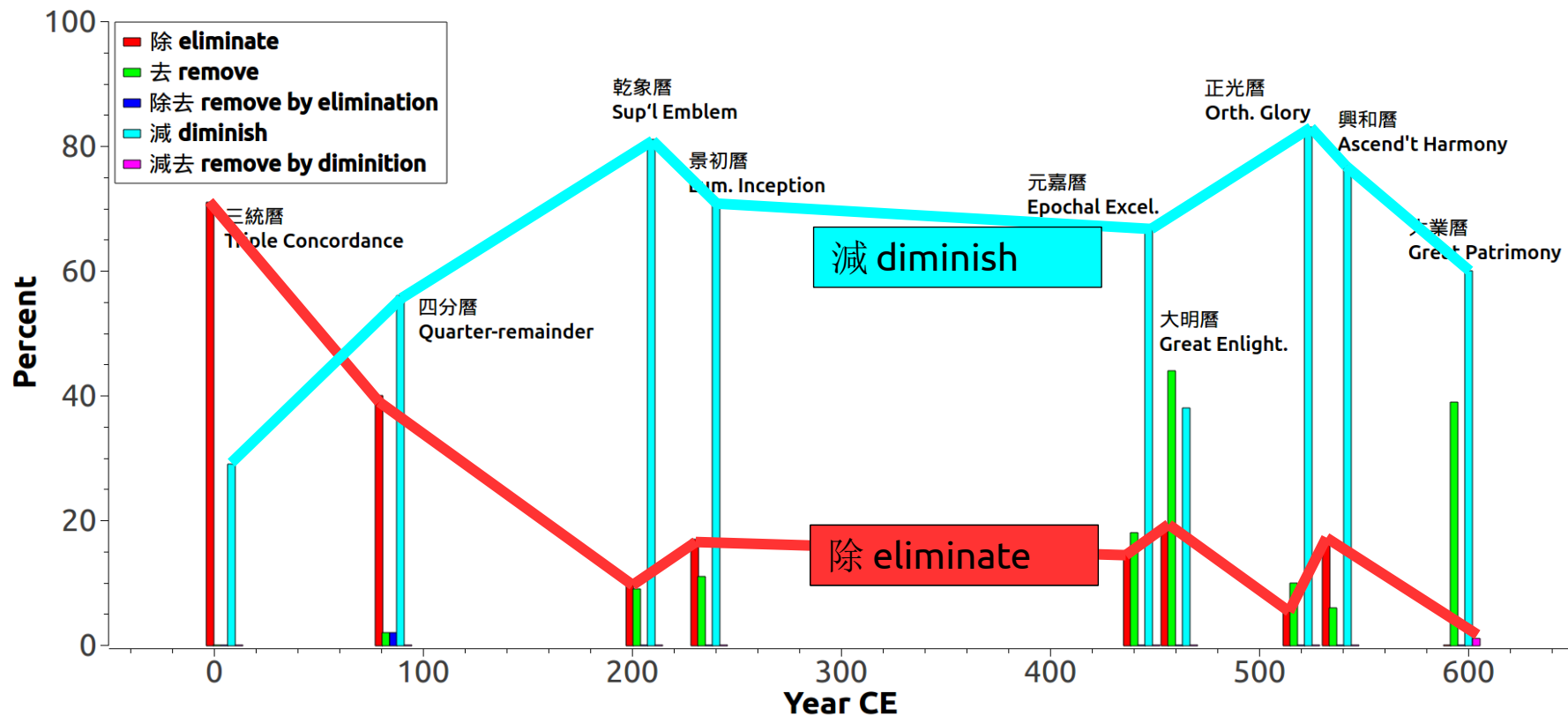
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## Subtraction, astronomical texts



# Distribution

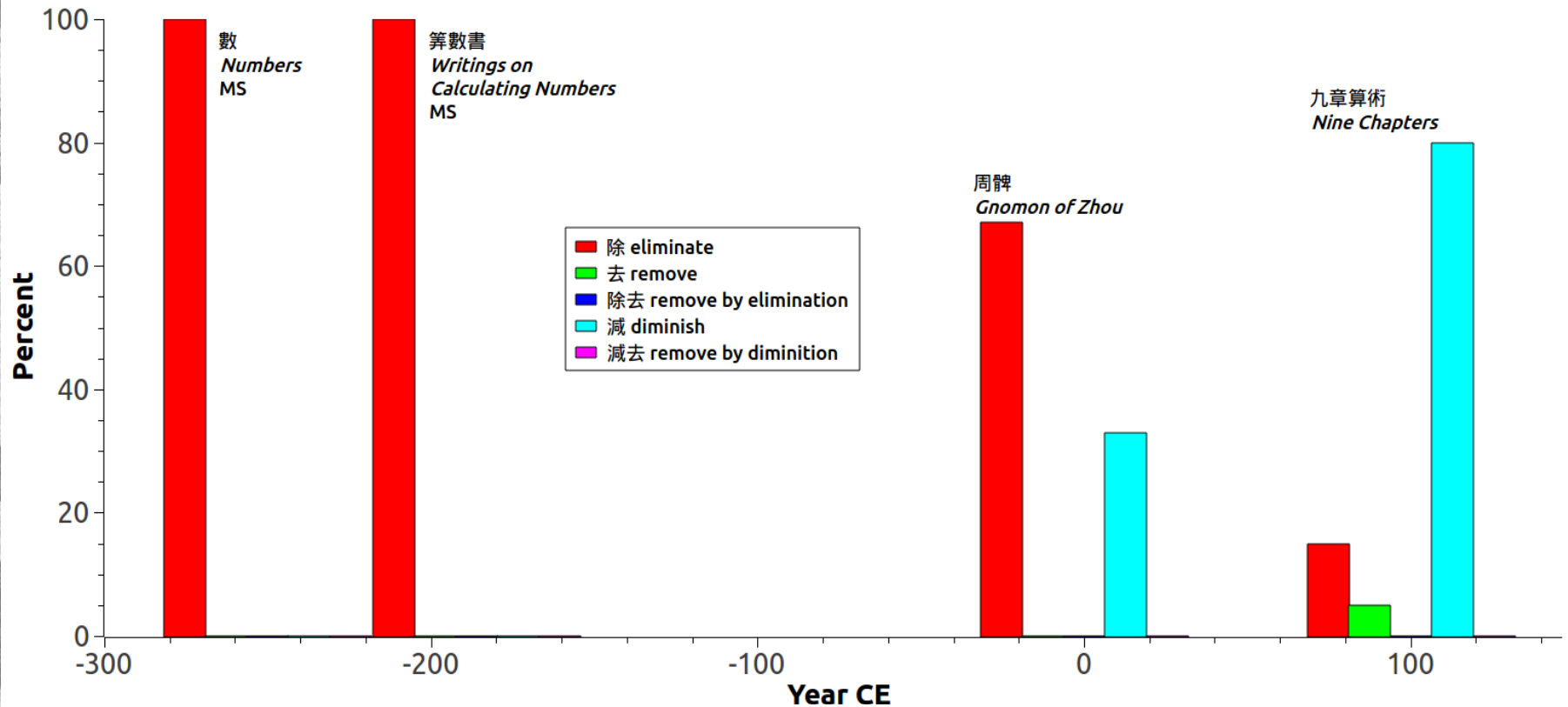
## Subtraction, astronomical texts



**Observation:** 'diminish' gains popularity over 'eliminate'

# Distribution

## Subtraction, mathematical texts



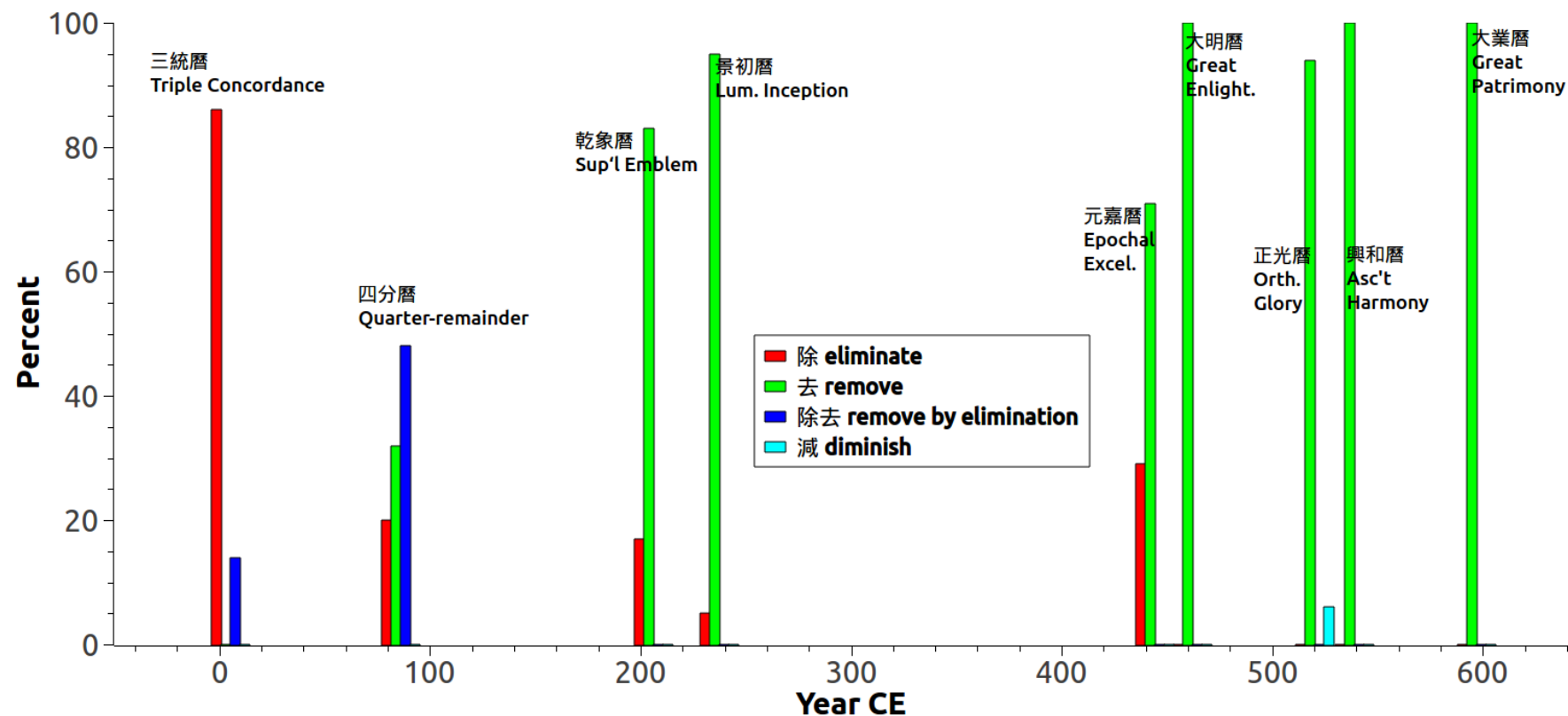
**Observation:** 'diminish' appears post 186 BCE, becomes quickly popular (in parallel to ast. texts)



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## Modulo, astronomical texts



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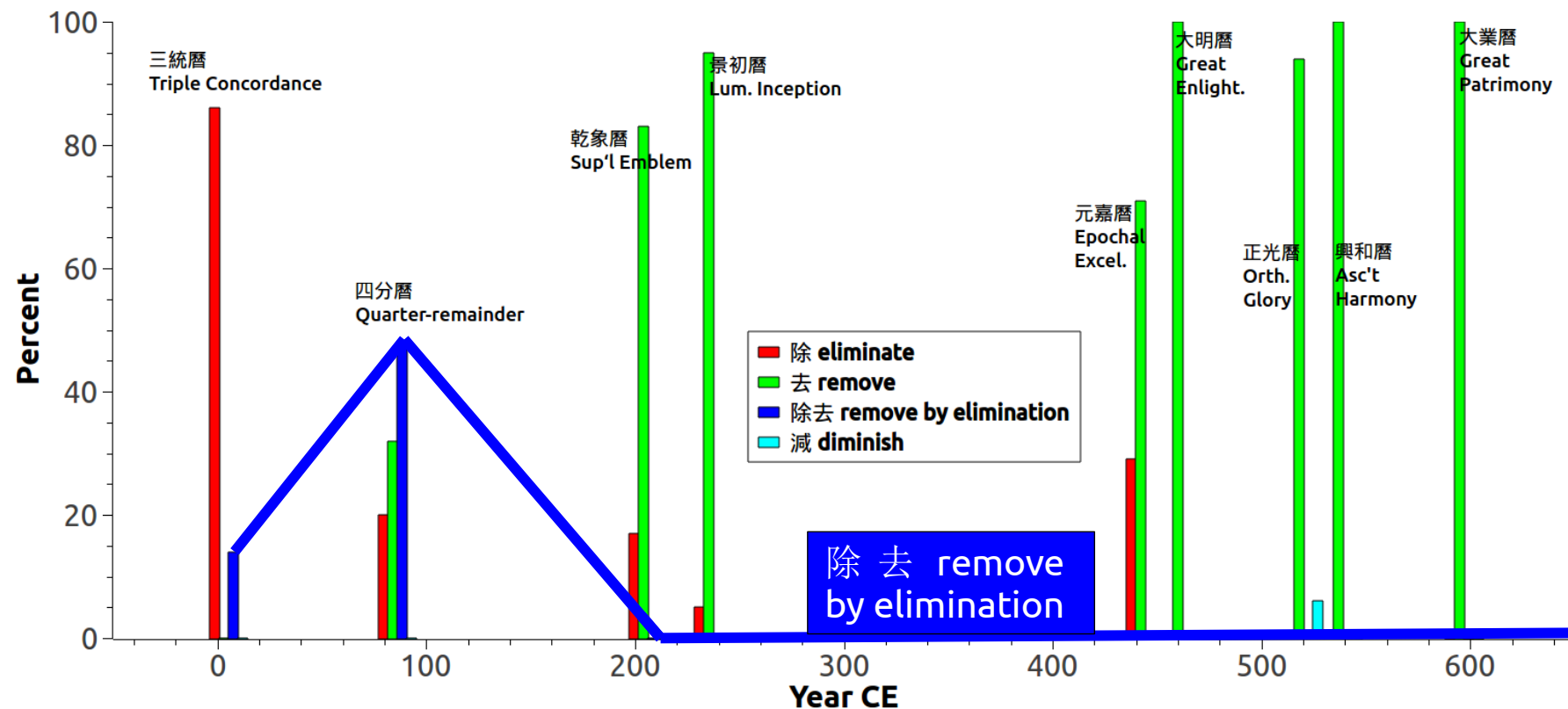
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## Modulo, astronomical texts



**Observation:** hybrid 'remove by elimination' fails to take off.

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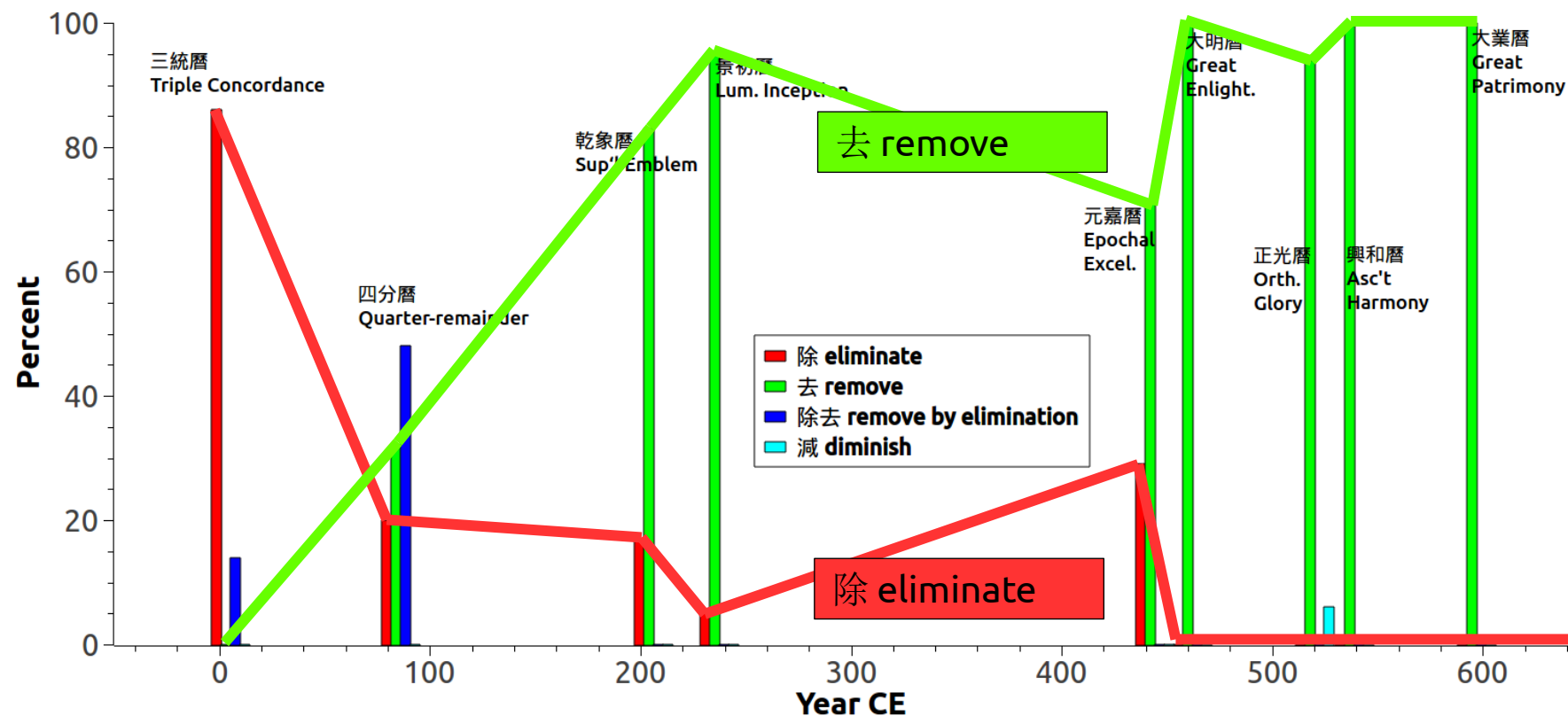
SOHERE  
SCIENCE, PHILOSOPHY, HISTORY  
1800-1900

erc  
UNIVERSITÉ  
PARIS  
DIDEROT  
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# Distribution

Daniel Patrick Morgan

## Modulo, astronomical texts

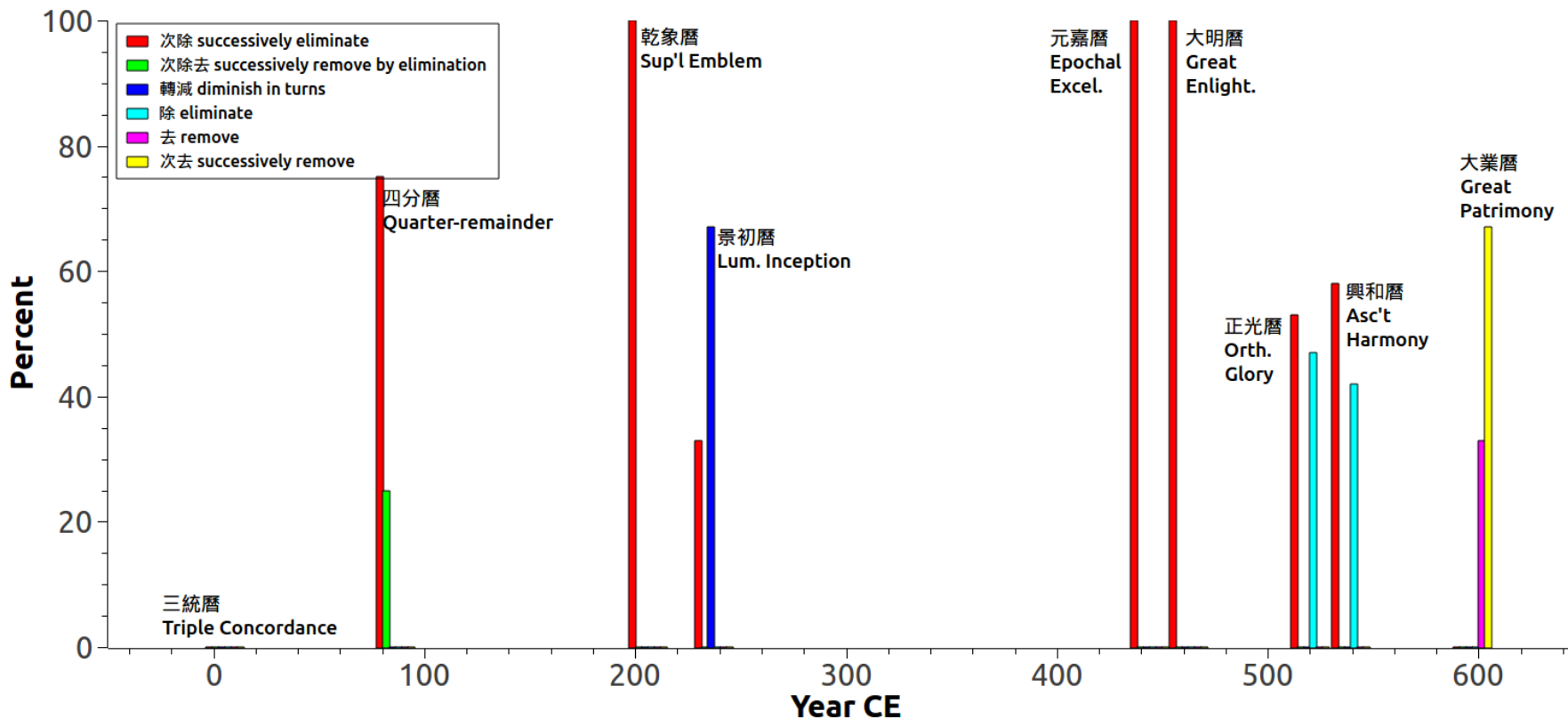


**Observation:** 'remove' quickly and completely replaces 'eliminate'



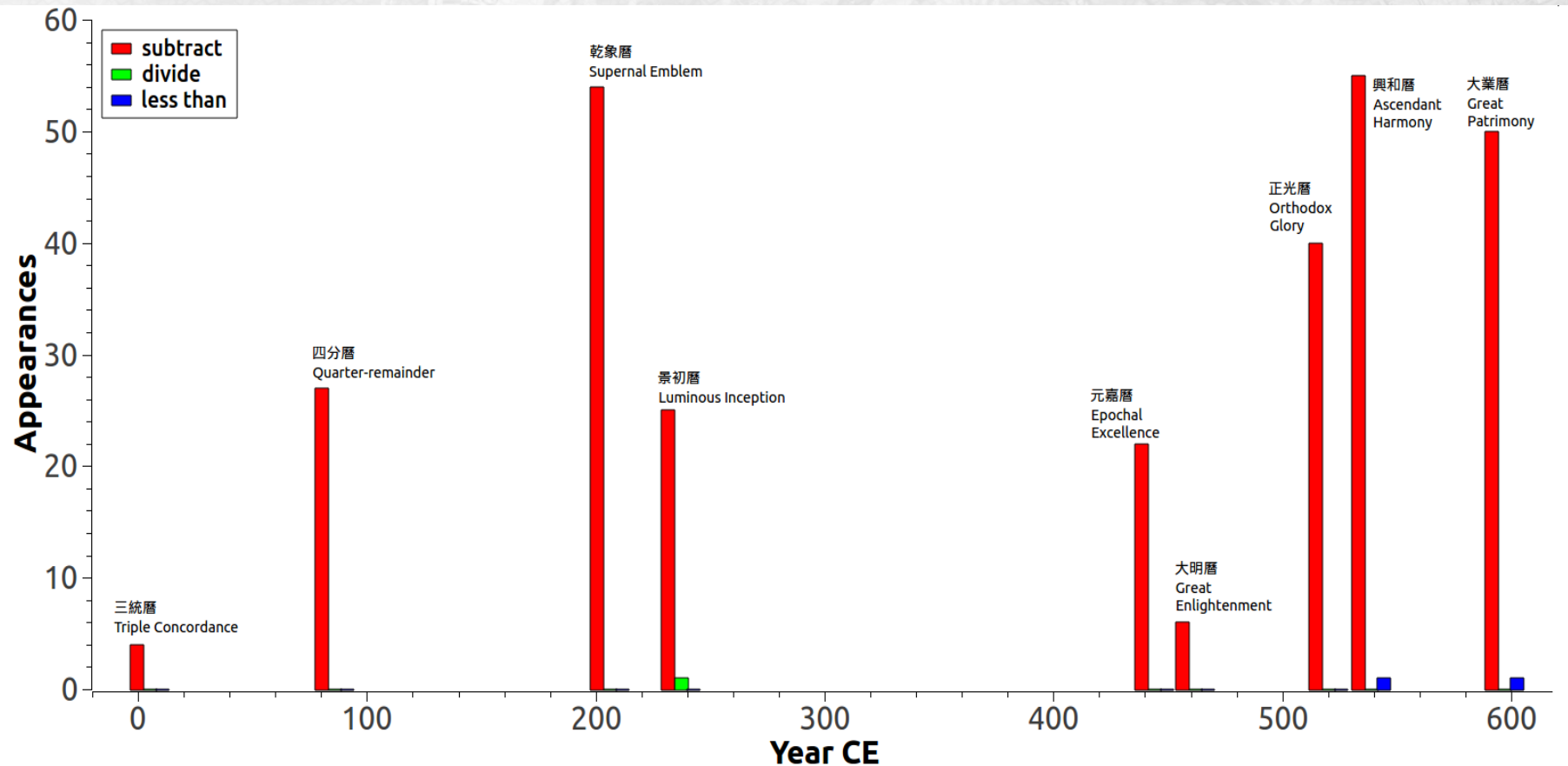
# Distribution

## Sequence subtraction, astronomical texts



# Distribution

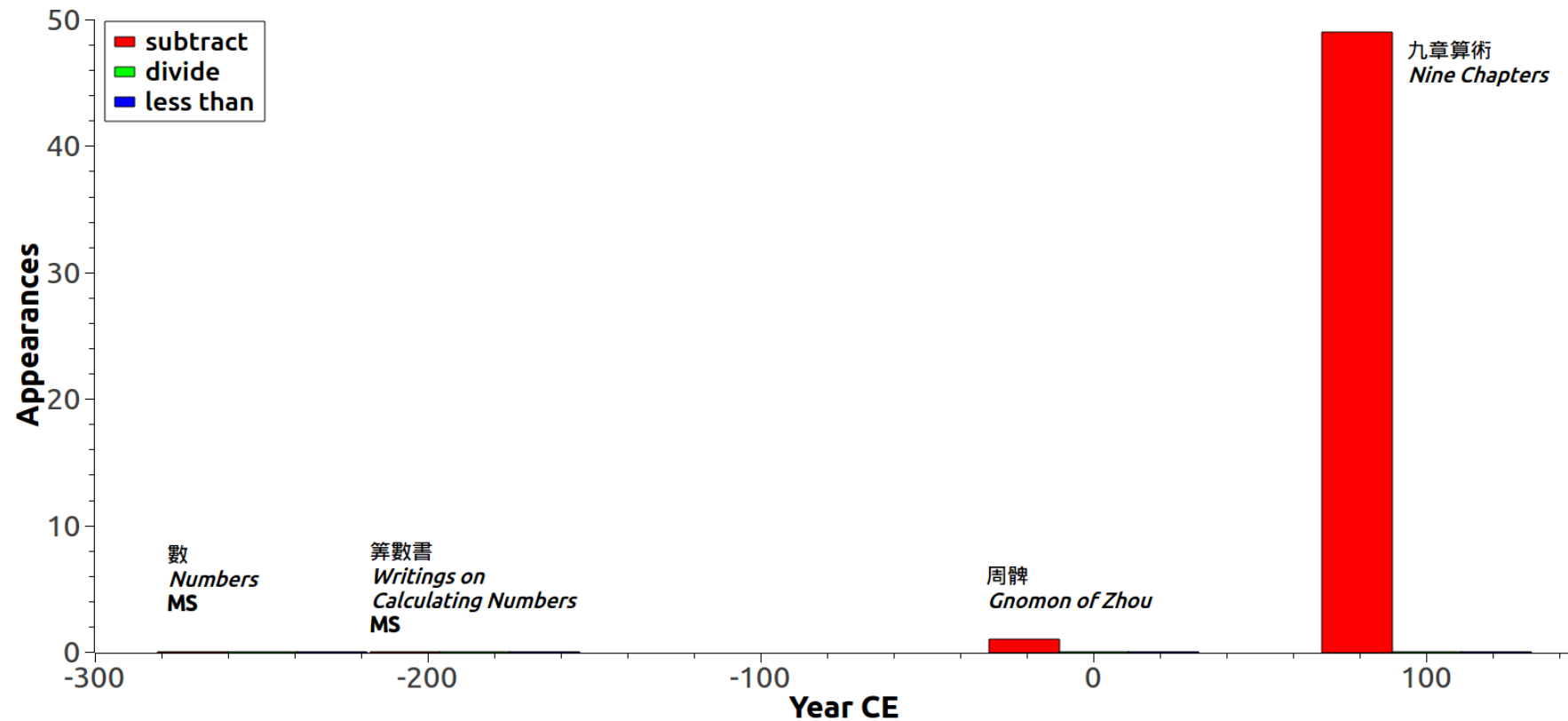
## 減 Diminish, astronomical texts



# Distribution

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## 減 Diminish, mathematical texts



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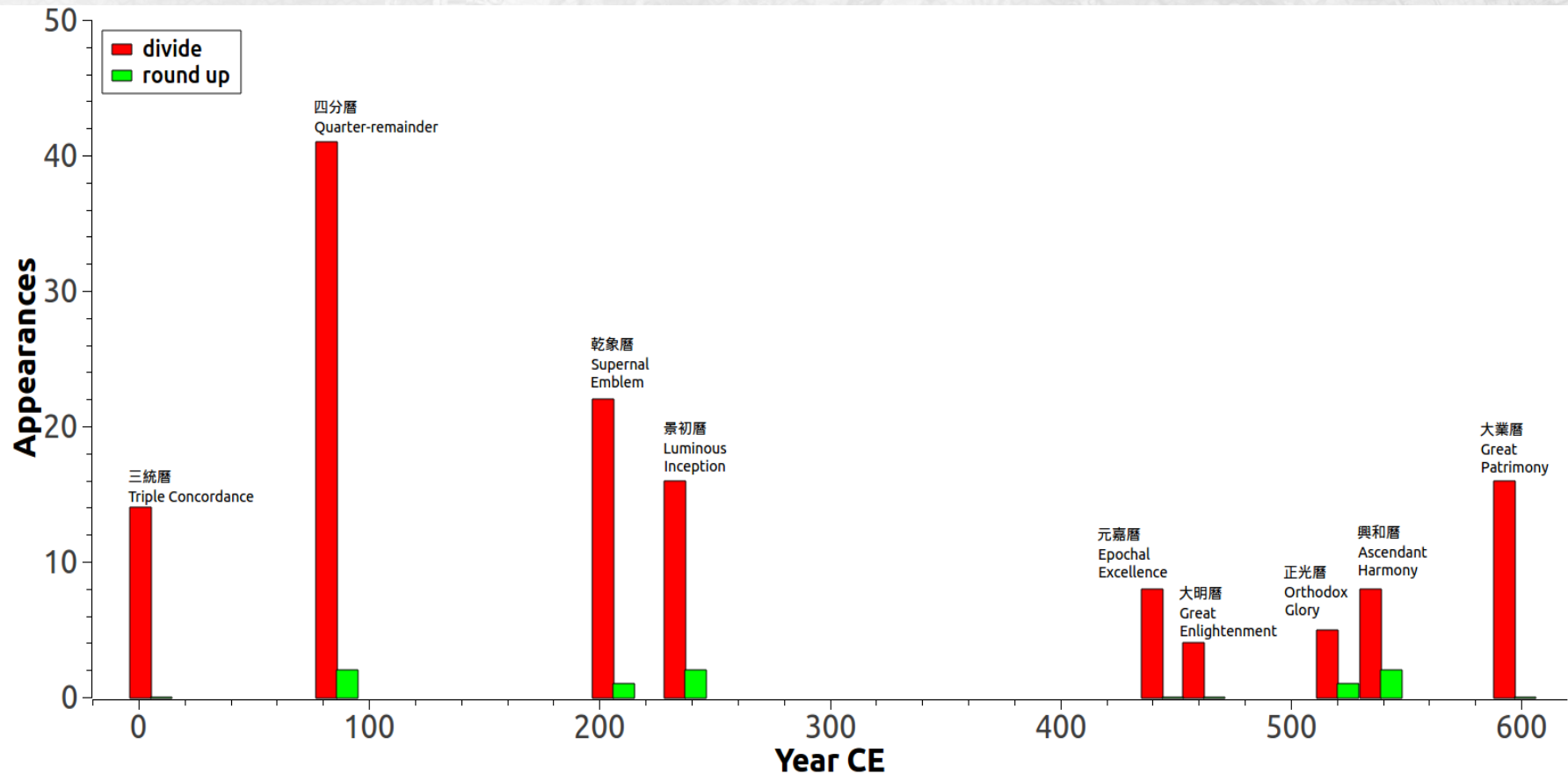
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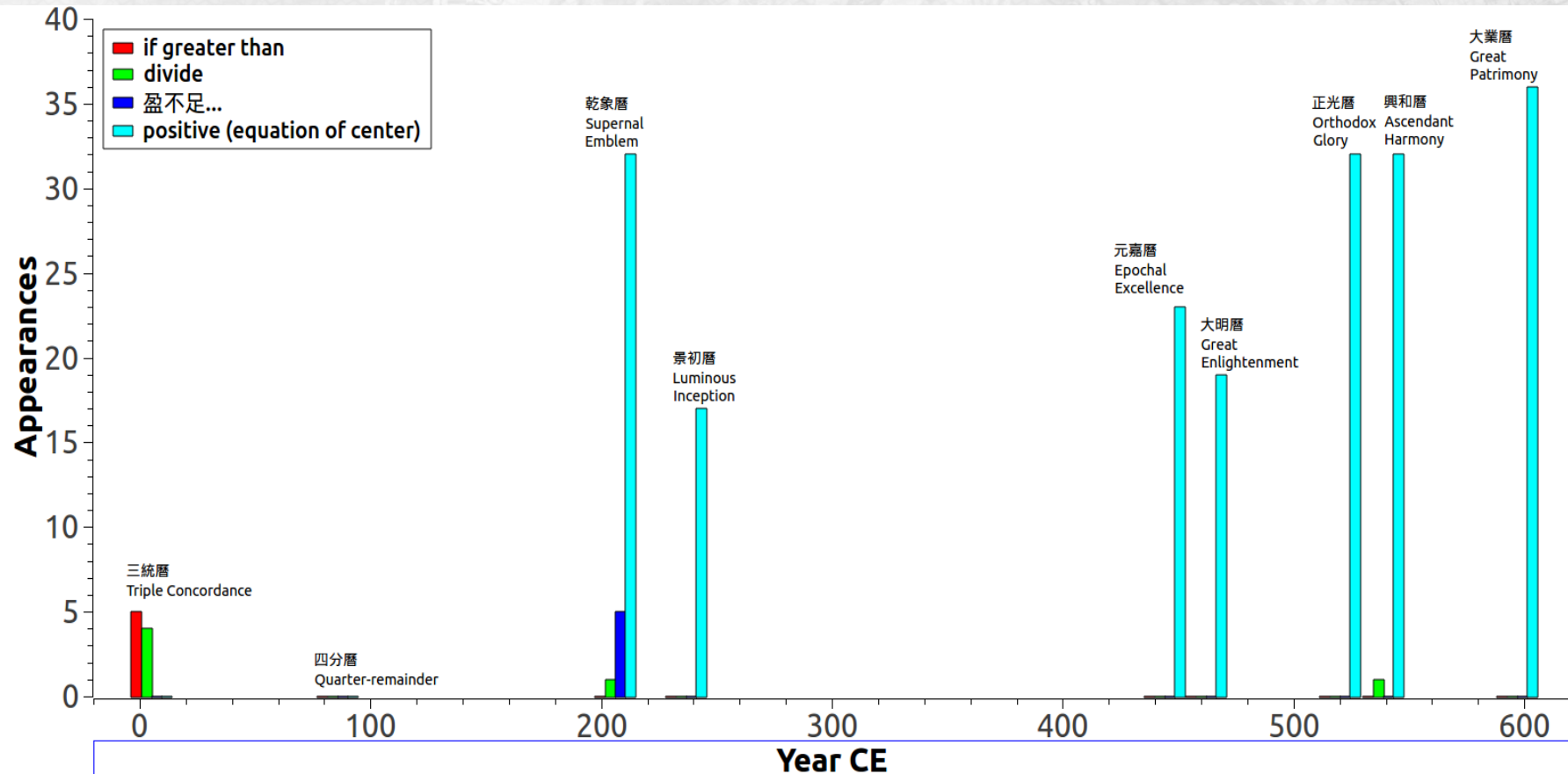
# Distribution

## 得一 Get one, astronomical texts



# Distribution

## 盈 Overflow, astronomical texts

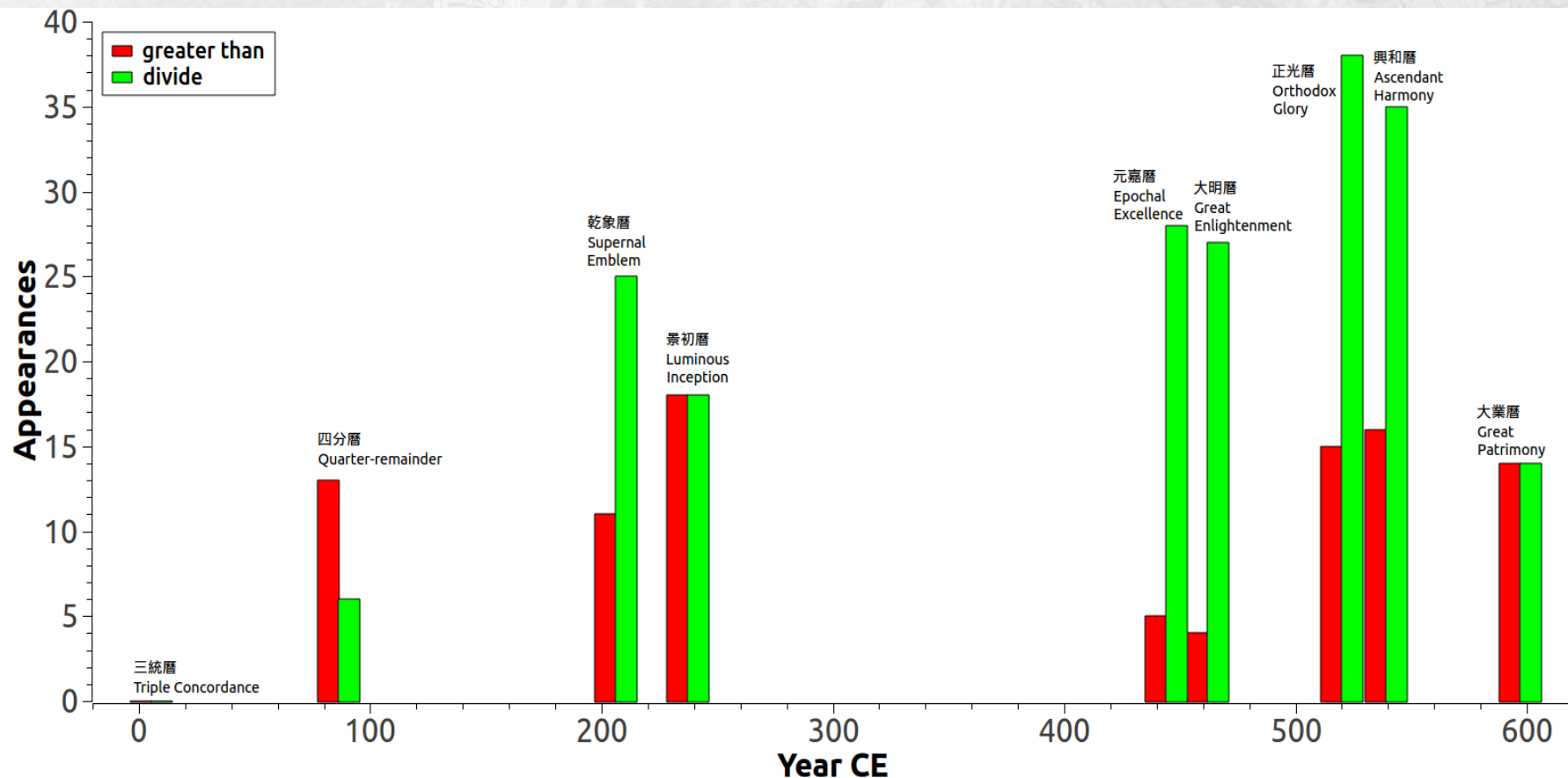


**Observation:** clear historical break between usages.



# Distribution

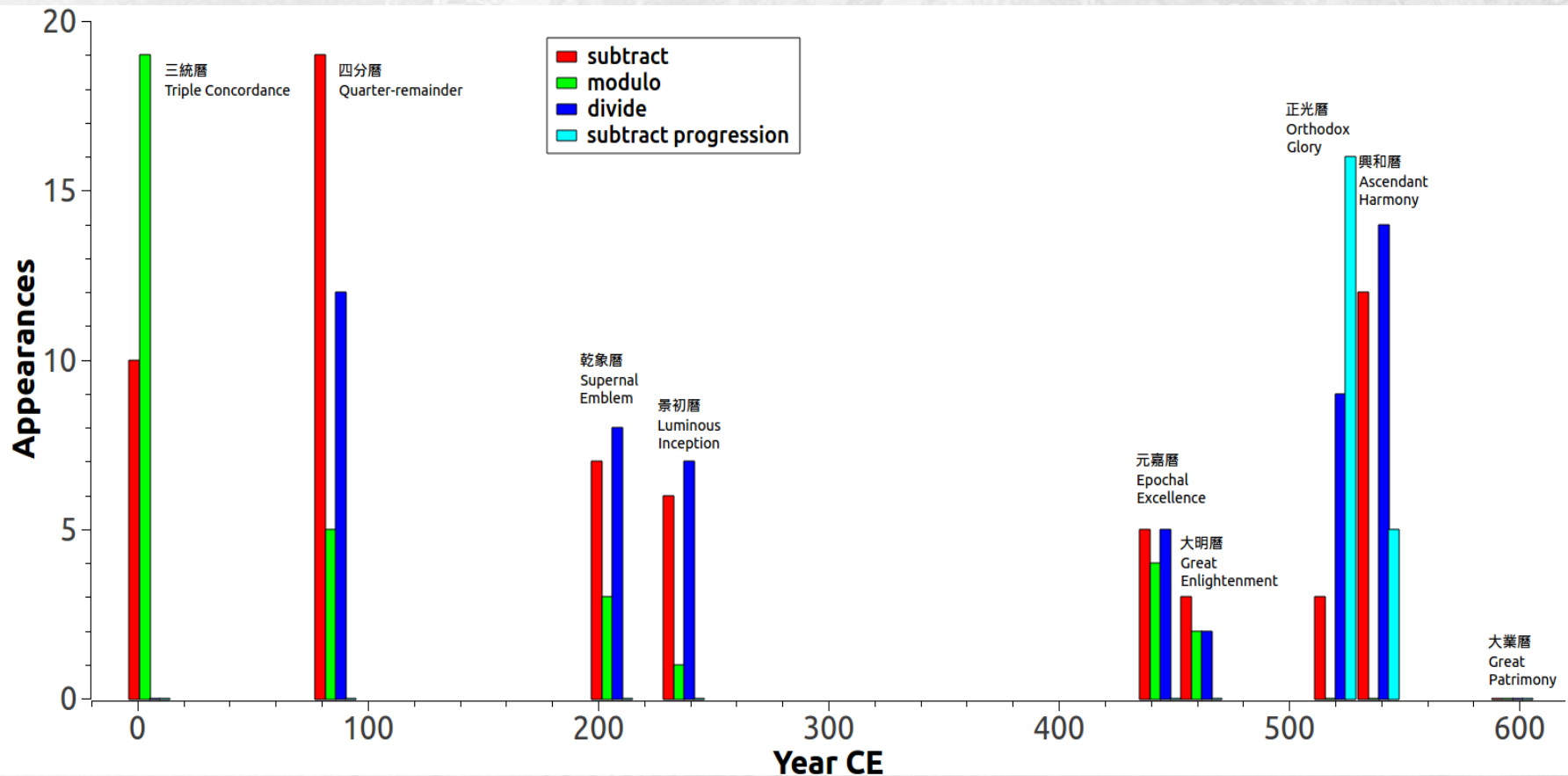
## 滿 Fill, astronomical texts



**Observation:** at the same time, 'fill' takes over 'overflow' (because of imperial taboo)

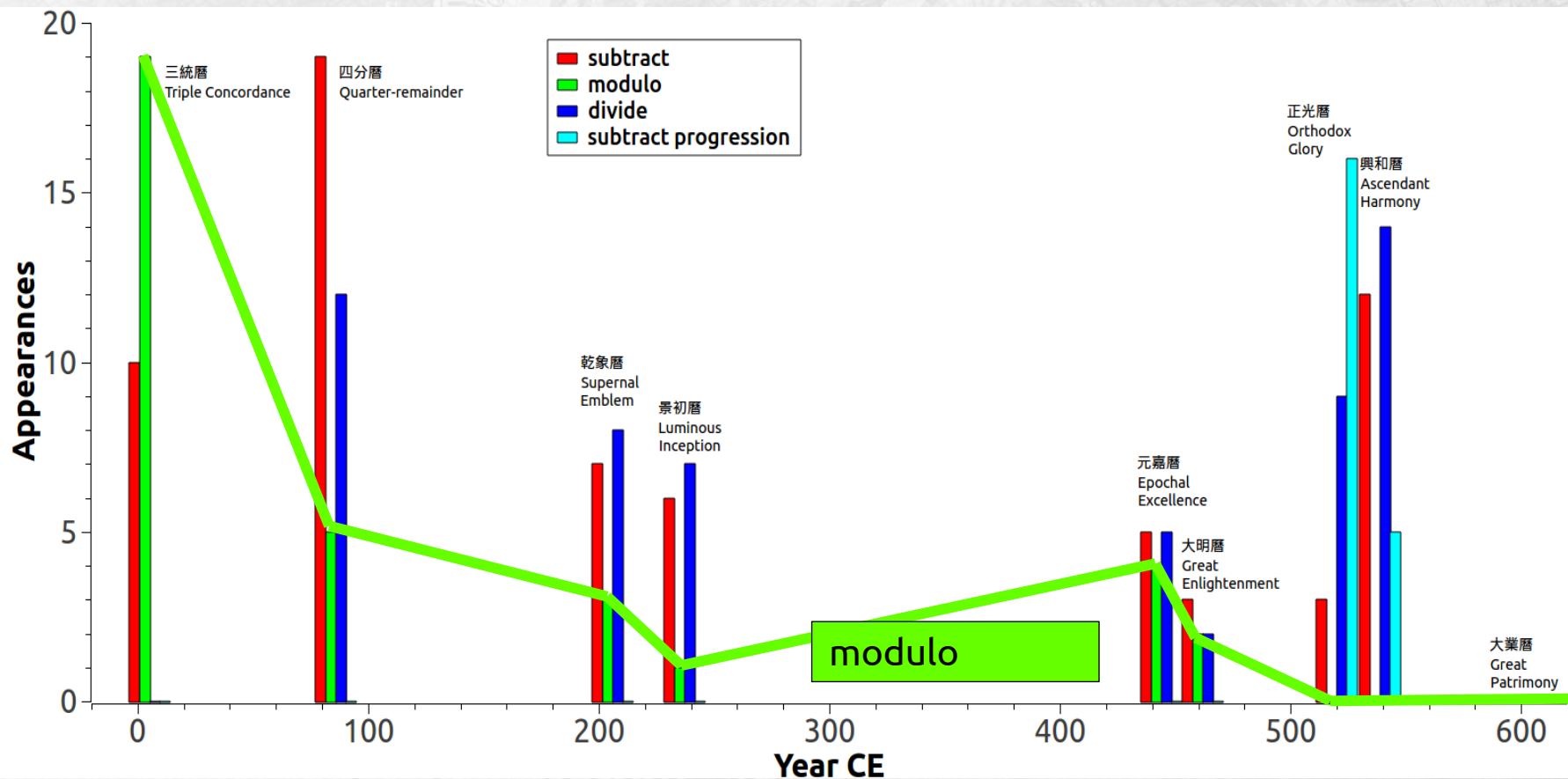
# Distribution

## 除 Eliminate, astronomical texts



# Distribution

## 除 Eliminate, astronomical texts

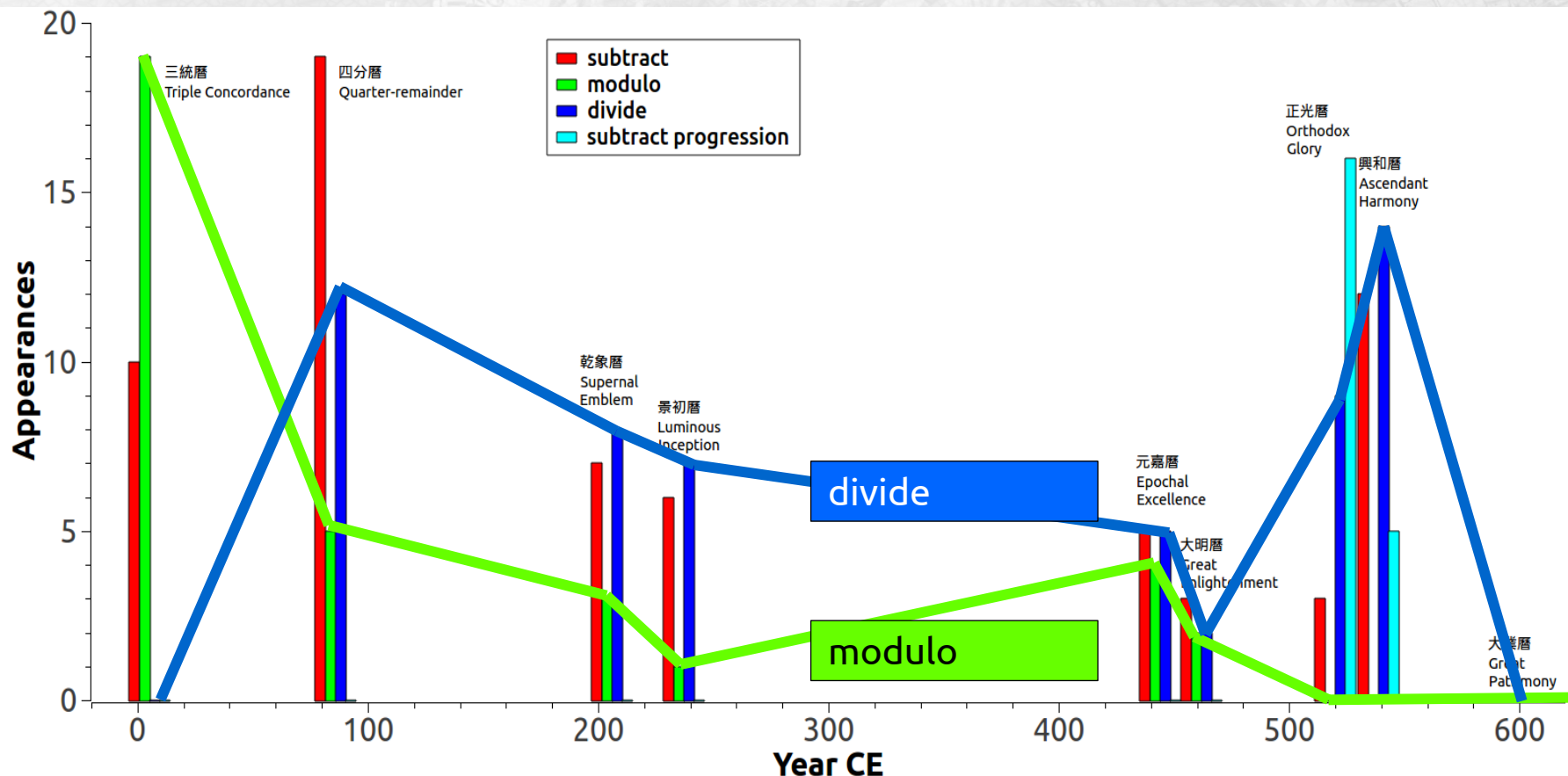


**Observation:** 'eliminate' used less for *modulo* (replaced by 'remove')

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## 除 Eliminate, astronomical texts



**Observation:** 'eliminate' used for division in 85 CE; this becomes main meaning thereafter.

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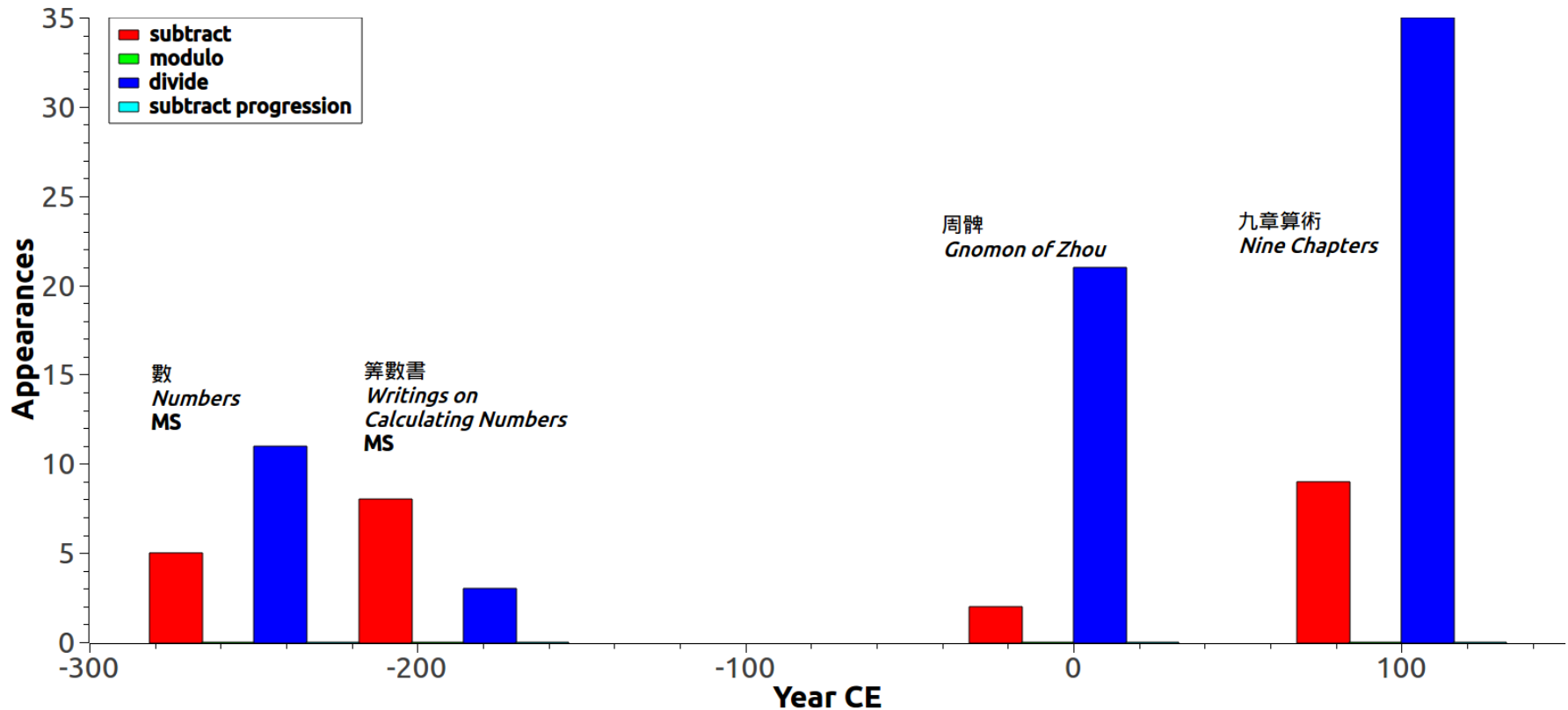
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# Distribution

## 除 Eliminate, mathematical texts



**Observation:** 'eliminate' used for division in **third century BCE**; this becomes main meaning after the first century CE (also).

# March 2015 Conclusions & New Problems

## March 2015 Conclusions

- The operational vocabulary of *li* 曆 proc. text is a mess:
  - one word describes multiple operations,
  - one operation described by multiple words
- Patterns of usage experience historical trends:
  - emergence,
  - disappearance,
  - redistribution.
- Therefore, we can say that we have a *diachronic* plurality of mathematical cultures on our hands.

## Now, with maths!

Math texts exhibit the same ambiguity, but they experience *less* of it.

Math texts exhibit trends as well, and some of those trends correspond: the emergence of 'diminish' for subtraction, the shift of 'eliminate' to division, the replacement of 'overflow' with 'fill'. Some parallel trends happen *first* in mathematics, suggesting cross-pollination; further research might reveal trends moving in the opposite direction.

Comparatively, we can say that there was *synchronic* plurality as well. Further research should be done on synchronic pluralities in sub-genres of mathematics.

# Confusion!

## Supernal Image *li* 乾象曆 (206 CE)

凡言如、盈、約、滿，皆求實之除也；去及除之，取盡之除也。

Any talk of 'as per', 'overflow', 'simplify' or 'fill' all [refers to] **elimination to seek the solid** (integer quotient); 'remove' as well as 'eliminate it' [refer to] **elimination to take the exhausted** (remainder).

## *Suan shu shu* 算數書 MS, s116 (TAQ 186 BCE)

少廣 =：（廣）一步、半步，以一為二 $\perp$ ，半為一，同之，三以為法，即直（置）二百冊（四十）步，亦以一為二，**除，如法得從（縱）一步** $\perp$ ，為從（縱）百六十步。

Reduced width: for a width of 1 *bu* &  $\frac{1}{2}$  *bu*, make the 1 into 2, the  $\frac{1}{2}$  into 1 and combine them, making 3 the divisor. Immediately set out 240 *bu*, also making 1 into 2, **eliminate and get one *bu* of length per the divisor**, making a length of 160 *bu*.

## *Gnomon of Zhou* 周髀 (TAQ 3rd cent. CE)

術曰：置十一萬九千里為實，以半歲一百八十二日、八分日之五為法，而通之，得九十五萬二千，為實。所得一千四百六十一為法，**除之。實如法得一里**。不滿法者，三之，如法得百步。不滿法者，十之，如法得十步。不滿法者，十之，如法得一步。不滿法者，以法命之。

Procedure: Set out 119,000 *li* to make the dividend, make the 182 days &  $\frac{5}{8}$  day of a half-year the divisor, then make them communicate, getting 952,000 as the dividend. [Taking] the 1461 obtained as the divisor, **eliminate it and get one *bu* per the divisor**. That which does not fill the divisor, triple it and get 100 *bu* per the divisor. That which does not fill the divisor, ten it and get 10 *bu* per divisor. That which does not fill the divisor, ten it and get 1 *bu* per divisor. That which does not fill the divisor, name it by the divisor.

# Algorithm as Vector

## Quarter-remainder *li* 四分曆 (85 CE)

推合朔所在度：... 經斗除二百三十五分。

**Calculate *du* position of syzygy:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** 235 parts.

推日所在度：... 經斗除十(九)分。

**Calculate *du* position of sun:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** 19 parts.

推月所在度：... 經斗除十九分。

**Calculate *du* position of moon:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** 19 parts.

求合度：... 經斗除如周率矣。

**Calculate *du* position of [planetary] conjunction:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** per circuit lü.

## Supernal Image *li* 乾象曆 (206 CE)

推日度：... 經斗除分。

**Calculate solar *du*:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** [its] parts.

推合朔度：... 經斗除大分。

**Calculate syzygy *du*:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** [its] big parts.

五星曆步術：... 經斗除分。

**Procedure for *li*-pacing the five [planets]:**

... If passing through Dipper<sub>L08</sub>, **eliminate (-)** [its] parts.



# Algorithm as Vector

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## Luminous Inception *li* 景初曆 (237 CE)

推日度術： ... 經斗除分。

**Procedure to calculate solar *du*:**

... If passing through Dipper.<sub>L08</sub>, **eliminate (-)** [its] parts.

推合朔度術： ... 經斗除其分。

**Procedure to calculate syzygy *du*:**

... If passing through Dipper.<sub>L08</sub>, **eliminate (-)** its parts.

五星曆步術： ... 斗除分。

**Procedure for *li*-pacing the five [planets]:**

... **Eliminate (-)** Dipper.<sub>L08</sub> parts.

## Epochal Excellence *li* 元嘉曆 (443 CE)

推日所在度法： ... 經室去度分。

**Method for calculating the *du* position of the sun:**

... If passing through Hall.<sub>L13</sub>, **remove (-)** [its] *du* parts.

推月所在度法： ... 經室去度分。

**Method for calculating the *du* position of the moon:**

... If passing through Hall.<sub>L13</sub>, **remove (-)** [its] *du* parts.

推合朔度： ... 經室除度分。

**Calculate syzygy *du*:**

... If passing through Hall.<sub>L13</sub>, **eliminate (-)** [its] *du* parts.

求星見度法： ... 經室去分，不足減者，破全度。

**Method for finding [planetary] appearance *du*:**

... If passing through Hall.<sub>L13</sub>, **remove (-)** [its] parts; if insufficient to **diminish (-)**, break a whole *du*.

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# Algorithm as Vector

## Great Enlightenment *li* 大明曆 (462 CE)

入虛去度分。

If entering Tumulus.<sub>L11</sub>, **remove (-)** [its] *du* parts.

入虛去行分六，小分百四十七。

If entering Tumulus.<sub>L11</sub>, **remove (-)** motion parts 6 & small parts 147.

入虛去度分也。

If entering Tumulus.<sub>L11</sub>, **remove (-)** [its] *du* parts.

虛去分如上法。

Tumulus.<sub>L11</sub>, **remove (-)** [its] *du* parts as per above.

去虛去度分，命如前

If entering Tumulus.<sub>L11</sub>, **remove (-)** [its] *du* parts and name as per above.

從行入虛，去行分六，小分百四十七。

If entering Tumulus.<sub>L11</sub> in prograde, **remove (-)** motion parts 6 & small parts 147.

## Orthodox Glory *li* 正光曆 (520 CE)

逕斗去其分一千四百七十七。

If passing through Dipper.<sub>L08</sub>, **remove (-)** its parts by 1477.

逕斗除其分。

If passing through Dipper.<sub>L08</sub>, **eliminate (-)** its parts.

逕斗去其分。

If passing through Dipper.<sub>L08</sub>, **remove (-)** its parts.

逕斗去其分一千四百七十七。

If passing through Dipper.<sub>L08</sub>, **remove (-)** its parts by 1477.

# Algorithm as Vector

## Orthodox Glory *li* 正光曆 (520 CE)

逕斗除其分。

If passing through Dipper.<sub>L08</sub>, **eliminate (-)** its parts.

逕斗去其分

If passing through Dipper.<sub>L08</sub>, **remove (-)** its parts.

逕斗除其分。

If passing through Dipper.<sub>L08</sub>, **eliminate (-)** its parts.

除斗分。

**Eliminate (-)** Dipper.<sub>L08</sub> parts.

## Great Patrimony *li* 大業曆 (597 CE)

經斗去其分。

If passing through Dipper.<sub>L08</sub>, **remove (-)** its parts.

經斗去轉分十，小分四百六十六。

If passing through Dipper.<sub>L08</sub>, **remove (-)** [its] turn parts by 10 & small parts by 466.

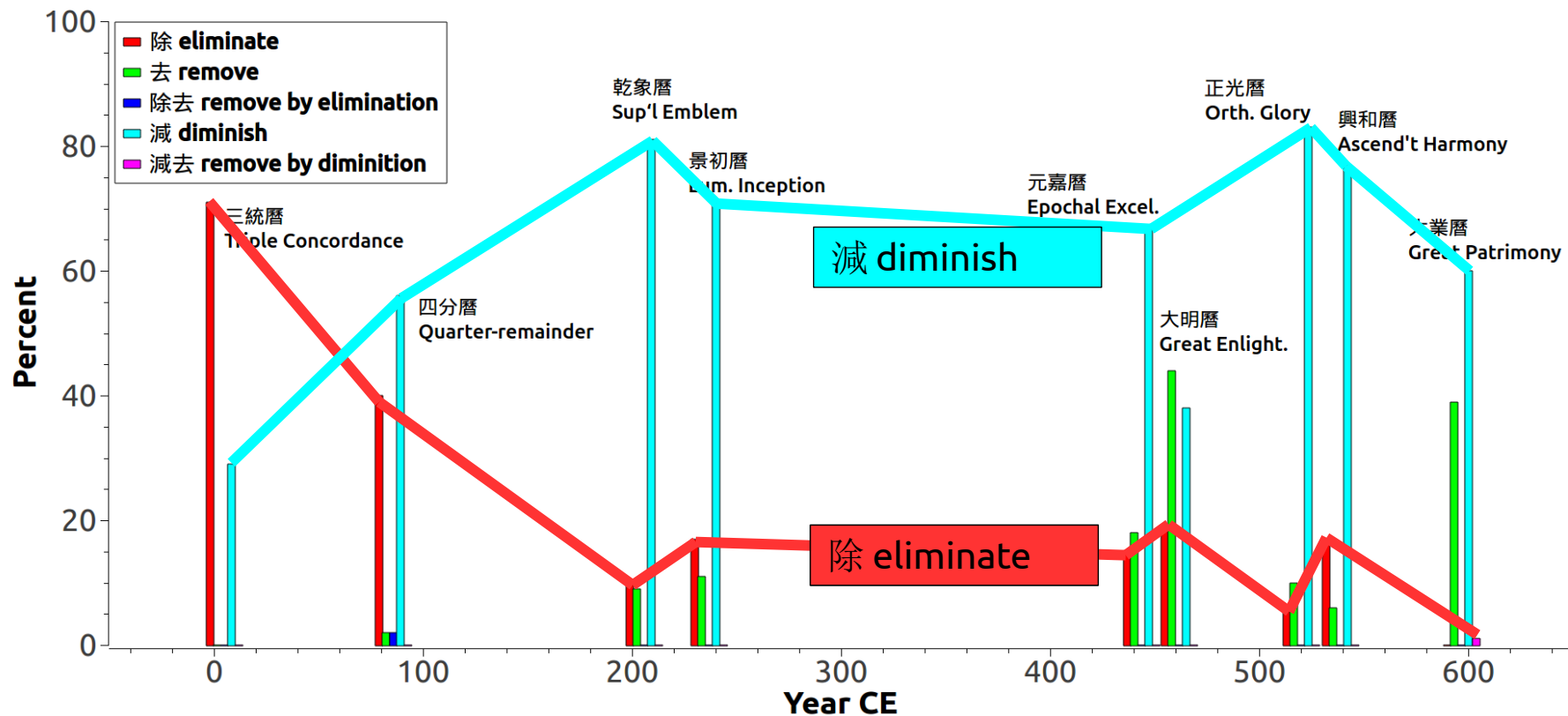
出斗去其分

If exiting Dipper.<sub>L08</sub>, **remove (-)** its parts.

# Algorithm as Vector

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## Subtraction, astronomical texts



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# Aesthetics of the Procedure

## **Nine Chapters 九章算術 (TAQ 3rd cent CE)**

術曰：置生絲兩數，除耗數，餘，以為法。三十斤乘乾絲兩數為實。實如法得生絲數。

少廣術曰：... 各以其母除其子 ... 實如法而一，得從步。

開方術曰：... 以除。除已 ... 以除 ... 復除 ... 其母報除 ... 訖，令如母而一。

術曰：... 除 ... 約 ... 實如法得一。

術曰：... 實如法得一日 ... 除之 ... 約 ... 實如法得一斛。

術曰：除 ... 實如法得主人馬一日行。

# Conclusion

## Analysis at word-level:

- Operation words are ambiguous across *li* astronomy and *suan* mathematics corpora.
  - We see certain synchronic patterns of distribution distinct to each corpus
  - Each corpus redistributes operations and words over time.
- The corpora thus reflect cultures/idioms that are *distinct* but *in communication*.

## Analysis at procedure-level:

- One culprit of ambiguity is old procedure and sub-procedure language copied from one text to the next.
- Procedures more helpful to identify change than looking in the aggregate.

## Stuff to do:

- Analyse remaining text.
- Identify specific ruptures in each corpus.
- Attempt tracing ruptures from one corpus to the other.