Procyclicality and Bank Portfolio Risk Level Under A Constant Leverage Ratio

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Documents de travail GREDEG
GREDEG Working Papers Series

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GREDEG WP No. 2013-35
http://www.gredeg.cnrs.fr/working-papers.html

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Olivier Bruno[*], Alexandra Girod[+]

Abstract: *We investigate the impact the risk sensitive regulatory ratio may have on banks' risk taking behaviours during the business cycle. We show that the risk sensitivity of capital requirements introduce by Basel II adds either an "equity surplus" or an "equity deficit" on a bank that owns a fixed capital endowment and a constant leverage ratio. Depending on the magnitude of cyclical variations into requirements, the "surplus" may be exploited by the bank to increase its value toward the selection of a riskier asset or the "deficit" may restrict the bank to opt for a less risky asset. Whether the optimal asset risk level swings among classes of risk through the cycle, the risk level of bank's portfolio may increase during economic upturns, or decrease in downturns, leading to a rise in financial fragility or a "fly to quality" phenomenon.*

Keywords: Bank capital, Basel capital accord, risk incentive

JEL: G11 - G28

1. Introduction

Banking supervisors face the challenging task to preserve the safety of the financial system without hampering the key role banks play in the financing of the economic activity. Among the available tools to comply with these objectives, risk sensitive capital requirements stated by Basel II prudential regulation deserve a lot of attention. Contrary to the old framework, now conveniently named Basel I, the capital charges of Basel II are based on asset quality rather than on asset type. The main idea of the Basel Committee was to make the bank capital to asset ratio more sensitive to economic risk in order to achieve better credit risk management and to reduce the potential for regulatory arbitrage.

However, such a sensitive ratio seems to amplify the procyclicality of the regulatory requirements and exacerbates the business cycle (Kashyap and Stein 2004, Catarineu-Rabell et al. 2004, Heitfeild 2004, Heid 2007, Pederzolli et al. 2009). A change in the economic situation indeed affects borrowers' risk profile: as business conditions deteriorate for instance, it becomes harder for borrowers to honour their debt and defaults may increase during recession periods. Reverse happening while economic upturns, where the improvement in economic conditions may lead to over optimist expectations on the willingness and the capability of borrowers to honour their debt. Consequently, as business conditions improve, the regulatory ratio decreases to reflect the amelioration of borrowers' risk profiles, allowing an even greater expansion of banks' loan supply; whereas, during recession phases, the regulatory requirements increase as borrowers' risk profiles worsen, restricting further banks

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loan supply.[1] This mechanism may lead to excessive leverage ratio during expansion and to high "de-leveraging" during recession. Such a rise in leverage is problematic as it may increase the whole risk of the banking system and exacerbate financial fragility.

The theoretical literature has assigned great importance to procyclicality in debt leverage and emergence of financial fragility. H. S. Shin (2009) models a lending boom fuelled by declines in measured risk. He shows that in benign financial market conditions when measured risks are low, financial intermediaries expand their balance sheets as they increase leverage. There is, of course, a symmetrical process that accentuates the magnitude of the crisis when the measured risks are high, leading to sharp deleveraging, then resulting in a credit crunch. In the same vein, T. Adrian and H. S. Shin (2010a, 2010b) examine the role of leverage effects in the financial crisis of 2007-2009. They emphasize the pro-cyclicality of leverage and the positive relationship between leverage and the size of financial intermediaries' balance sheets, especially before the crisis. In these papers, financial fragility appears because of a rise in leverage due to an increase in banks' balance sheets.

With the will to improve the existing regulation and limit excessive leverage, the Basel Committee adjuncts three elements to the existing regulation, resulting from studies about the 2007 financial crisis: the leverage ratio, the conservative and the countercyclical buffers (Basel III macroprudential regulation).
First, the Committee introduces two countercyclical instruments: the buffer policies.[2] These buffer should increase during economic expansion (and decrease during recession) in order to accentuate (reduce) the need of regulatory capital and mitigate the procyclicality of capital requirements. Second, the Committee institutes a minimum leverage ratio of 3% (the leverage ratio is a measure of a bank's Tier 1 capital as a percentage of its assets plus off-balance sheet exposures and derivatives). These measures are based on the idea that procyclicality and excessive leverage during economic expansion are the main causes of financial fragility.

The objective of this paper is to show that such a policy may be inefficient as procyclicality and financial fragility can emerge even with a constant leverage ratio. In this end, we propose an original mechanism of risk taking incentive that results in increase of financial fragility with constant leverage for a bank that meets risk sensitive capital requirements.

The mechanisms behind this result stand on the interaction of the regulatory risk sensitivity ratio with the bank value maximizing objective. We show that bank lending decisions depends on two mains elements: its own target of default probability and its capital buffer defined as the difference between its constant amount of capital and the level of capital required by the regulation. According to theses values, the bank opts for a particular asset risk level that maximise its expected value. As regulatory capital requirements is risk sensitive, a change in the risk level of the asset chosen by the bank during the cycle may alter regulatory requirements, leading to an increase or a decrease into regulatory charges. With a constant level of capital, this change in the level of capital requirement leads to a change in the level of bank's capital buffer. When economic environment improves capital buffer increases, leading

[1] When recession occurs, banks may face increased borrowers' defaults that deepen their capital. To maintain their compliance with the regulatory capital to asset ratio, banks cut their loan supply, reducing the denominator of the regulatory ratio, rather than raising fresh expensive capital that is too costly.

[2] The conservative and the countercyclical buffers are both defined as a 2.5% additional requirement of the risk weighted asset.

to an "equity surplus". On the contrary, during recession period where regulatory requirements increase, the level of capital buffer is decreasing leading to an "equity deficit". What can be the reaction of a value maximizing bank which faces an equity "surplus" or "deficit"? Literature presented above supports the idea the bank would use this surplus to expand its loan supply and thus increase leverage (conversely, an "equity deficit" would lead to a credit contraction). When the bank is constrained and cannot increase its leverage, we show that this surplus can either be used to increase bank value toward the financing of a riskier asset that would absorb the equity surplus of the bank. Conversely, an "equity deficit" restricts the opportunity of investing into a high risk level asset. Depending on the capital buffer level, such a deficit may force the bank to reduce its risk exposure to comply with its internal target of default probability, leading to the selection of a less risky asset. Consequently, at a constant leverage, a riskier asset may be selected by the bank as business conditions improve leading to financial fragility of the banking system.[3] On the contrary, less risky asset may be selected by the bank when economic conditions deteriorate leading to a "fly to quality" phenomenon.

We stress the fact that in each of these scenarii, the bank complies with the regulatory requirements. However, the risk level of bank's portfolio is altered along the business cycle into its loans composition and might increase during economic upturns, leading to a fragility of the bank and the banking system as a whole. This result allows us to stress that the new Basel III macro prudential reform, based on a maximum leverage ratio, may appear inefficient to prevent financial fragility during economic expansion.

The rest of the paper is organised as follow. Section 2 presents the main assumption of the model whereas section 3 states the equilibrium condition of the model in terms of asset choice. In section 4 we analyse the impact of the business cycle on bank equilibrium and we comment our results in Section 5. Section 6 concludes.

2. The model

Our contribution is built on Heid (2007). We study the impact of the risk sensitivity of regulatory requirements on the choice of asset's risk level made by a bank. We appraise the risk level of assets by the means of the asymptotic single risk factor (hereafter ASRF) model used into the regulatory requirements and consequently we make the usual assumptions, presented under a formulation closed to Repullo and Suarez (2004).

2.1. Investment projects

Consider a two periods economy with a single systematic risk factor $z \sim N(0,1)$ and a continuum of firms, indexed by i. Each firm can undertake a risky investment project, requiring one unit of wealth at period 0, and we link the firm i to its investment. Moreover, we consider that each investment project is related to a specific class of risk. We assume firms lack capital and need to borrow from a bank to undertake their project. At period 1, the investment generates a gross return equal to $(1+R)$ in case of project success, or $(1-\lambda)$ in case of project failure. Project success depends on the value of a random variable x_i^c defined by:

[3] Financial fragility may increase since a riskier asset is financed whereas the level of capital of the bank is constant.

$$x_i^c = \mu_i^c + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i \tag{1}$$

Where the idiosyncratic risk $\varepsilon_i \sim N(0,1)$ is independently distributed across firms and independent of the systematic risk factor z.[4] We assume μ_i^c is an increasing function of the specific risk of the project i, which is dependent of the business cycle c. We assimilate an improvement in economic condition (rise in c) to a decrease in in the value of μ_i^c. It means that the risk of the project decreases as economic conditions improve with $\frac{\partial \mu_i^c}{\partial c} < 0$. Moreover, the correlation coefficient ρ reflects the sensitivity of the project to the systematic risk factor. Project i is successful only if $x_i^c \le 0$, thus x_i^c expresses the whole level of credit risk appraised by the ASRF model.

From (1) we note that the unconditional distribution of x_i^c is $N(\mu_i^c, 1)$, so the unconditional probability of default for a firm in class i is equal to:

$$\overline{p}_i^c = \Pr\left(\mu_i^c + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i > 0\right) = \phi\left(\mu_i^c\right) \tag{2}$$

Where ϕ denotes the cumulative distribution function of a standard normal random variable. The probability of default is an increasing function of the specific risk μ_i^c for firm i and we assume projects are such that $\overline{p}_i^c \in \left]0; \overline{p}_{\max}^c\right]$ with $\overline{p}_{\max}^c < 1$.

From (2) we can also determine the distribution of variable x_i^c conditional on the realization of the systematic risk factor z. This conditional probability of default or the default rate of project i is given by

$$p_i^c(z) = \Pr\left(\mu_i^c + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i > 0 \ \middle|\ z\right) = \phi\left(\frac{\phi^{-1}\left(\overline{p}_i^c\right) + \sqrt{\rho}\ z}{\sqrt{(1-\rho)}}\right) \tag{3}$$

The default rate of a project is an increasing function of its own class of risk given by its probability of default $\overline{p}_i^c$, but also depends on the realization of the systematic risk factor z.

Finally, recalling that z is normally distributed, the cumulative distribution function of the default rate is given by

$$F\left(p_i^c(z)\right) = \phi\left(\frac{\sqrt{(1-\rho)}\phi^{-1}\left(p_i^c(z)\right) - \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{\rho}}\right) \tag{4}$$

The mean of the default rate distribution is the probability of default related to the class of risk of project i, while the variance is determined entirely by the exposure (sensitivity) to the systematic risk measured by ρ.

[4] This is the usual formulation retained by the ASRF model and the IRB approach of Basel II.

2.2. The banking system

There is one bank in the economy that is funded with deposits D and equity capital $\overline{K}$, and we assume that $\overline{K}+D=1$. For simplicity, we make the assumption of no banking intermediation costs. Bank deposits are insured through a government-funded scheme and deposits receive the riskless interest rate δ. Bank capital is provided by shareholders who require an expected rate of return $(1+W)$, with $W=\delta+\omega$ and $\omega>0$, to capture the scarcity of shareholders' wealth as the agency and/or the asymmetric information problems they face.

At period 0, the bank decides to allocate its loans portfolio across the various investment projects, picking the investment project that maximizes its value with respect to its own internal target of default probability. We assume that the rate of return on a loan financed by the bank, R_i^c, is an increasing function of its credit risk level, represented hereafter by the class of risk it belongs to, $\overline{p}_i^c$, and is measured by a spread over the riskless interest rate, denoted $S(\overline{p}_i^c)>0$, such that $R_i^c=\delta+S(\overline{p}_i^c)$.

At period 1, shareholders receive the net value of the bank if it is positive (no bankruptcy), and zero otherwise (assumption of limited liability). Finally, net expected returns for a bank financing a project i, conditional on the realization of the systematic risk factor z is given by:

$$\pi^c(z)=(1-p_i^c(z))(1+R_i^c)+p_i^c(z)(1-\lambda)-(1-\overline{K})(1+\delta) \tag{5}$$

The first term $(1-p_i^c(z))(1+R_i^c)$ measure the expected payment the bank receives in the event of the success of the investment project, whereas it receives $p_i^c(z)(1-\lambda)$ in the case of failure with $(1-\lambda)$ the residual value of the project. Finally, the third term represents the costs of deposits $(1-\overline{K})(1+\delta)$. As we link project to firm, and firm to bank, the net expected profits are also the net expected value of the bank at period 1. Using the above notations and simplifying (5) we obtain:

$$\pi^c(z)=\left(1-p_i^c(z)\right)S(\overline{p}_i^c)-p_i^c(z)(\delta+\lambda)+\overline{K}\,(1+\delta) \tag{6}$$

We assume that the bank is subject to the regulatory requirements defined by the Basel II framework. In this framework, capital requirements are risk sensitive and depend on the legal category of the loan but also on its risk level i.e. on the class of risk the loan is assigned to. The regulatory risk weight function and the parameters applied into the computation of regulatory requirements depend on the legal category of the loan (Corporate, Small and Medium Enterprises, Sovereign debtor, etc...). Nevertheless, for a given category, a risk weight function determines the regulatory requirements which are increasing with the risk level of the asset. If we define $k_r(\overline{p}_i^c)$ as the capital requirements for a loan with a level of risk $\overline{p}_i^c$, this risk sensitivity leads to $\dfrac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c}>0$.

Moreover, for corporate loans, the regulatory weight function, constructed on the ASRF model, gets the reduced form:

$$k_r(\overline{p}_i^c)=EAD\times LGD\times(p_i^c(z_{reg})-\overline{p}_i^c) \tag{7}$$

With the exposure at default (EAD) equals to 1 in our model, the loss given default (LGD), λ in our framework (a regulatory parameter set to 45% under the foundation IRB approach) and $p_i^c(z_{reg}) = \phi\left(\dfrac{\phi^{-1}(\overline{p}_i^c) + \sqrt{\rho}\ \phi^{-1}(0.999)}{\sqrt{(1-\rho)}}\right)$ the default rate adjusted to the target of non-default probability fixed by the regulator (here 99.9%).

Following Heid (2007), we assume "*that regulatory requirements shift the bank's default point from 0* [the solvency constraint] *to the regulatory constraint*" and models his idea through the assumption that "*the bank sets itself a target probability of its own default*".[5] In this line of thinking, we assume that the level of bank's capital $\overline{K}$, is higher than the regulatory requirements $k_r(\overline{p}_i^c)$. Actually, as investments returns are random and regulatory infringements generate important economic costs,[6] the bank makes sure to operate with a higher level of capital than regulatory requirements. The difference between the actual level of bank capital and regulatory requirements is known as the capital buffer. It is important to note that banks actually choose their investments portfolio which in turn determines the level of capital buffer.[7] We also retain the assumption that banks choose their own target of default probability $\left(1-\alpha_B^*\right)$.[8] However, it seems to us that the intuition behind this internal target of default probability is more easily understood when it is presented under the non-defaulted perspective. Thus, α_B^* is defined as the probability of non default for bank and we use this formulation in the rest of the paper.

The bank's internal target of default probability requires that bank expected value at period 1, i.e. expected value of equity endowment and expected net returns generated by the investment made by the bank, must not be smaller than the regulatory requirements in α_B^* case which lead to the following condition:

$$\Pr\left[\pi^c(z) \geq k_r(\overline{p}_i^c)\right] = \alpha_B^* \tag{8}$$

with $\pi^c(z)$ defined by equation (6) as the net expected value of the bank at period 1, including net expected return and the future value of equity capital. Note that we model neither the choice of the optimal level of equity capital nor the selection of the optimal internal target of default probability. On the contrary, we consider them as given and we rather focus on the impact the regulatory requirements have on the risk level of the asset financed by the bank at equilibrium, given its capital endowment and its own target of default probability.[9]

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[5] Heid (2007) pp. 3888-3889.

[6] We do not model these costs.

[7] We assume the bank does not raise equity capital along the economic cycle and it operates most of the time with a given level of capital inherited from the previous business year. This assumption seems realistic for a short to medium horizon of analysis.

[8] The choice of the default probability depends on the degree of conservatism of the bank. Note also that banks must display a certain rating to access to the interbank market, constraint that can also be materialized through this internal target of default probability.

[9] We refer the lecturer to, among others, Jeitschko and Dong Jeung (2005) for an explicit formulation of the optimal risk target chosen by the bank, and to Leland and Bjerre Toft (1996) and Diamond and Rajan (2000) for an analysis of the optimal capital structure.

Finally, the objective of the bank is to maximize its expected present value at period 0, net of shareholders' initial infusion of capital, with respect to its own default probability target, that is:

$$\max_{\overline{p}_i^c} \quad V = -\overline{K} + \frac{1}{1+W} \int_0^{\hat{p}_i^c(z)} \pi^c(z)\, dF(p_i^c(z))$$
$$s.t. \quad \Pr\left[\pi^c(z) \geq k_r(\overline{p}_i^c)\right] = \alpha_B^*$$

Where $\hat{p}_i^c(z) \equiv \min\left\{\dfrac{S(\overline{p}_i^c) + \overline{K}(1+\delta) - k_r(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda}, 1\right\}$ is the adjusted bankruptcy rate of default i.e. the maximum rate of default that the bank must not exceed in order to be solvent. This rate is compute as the one for which the net value of the bank at period 1 is equal to $k_r(\overline{p}_i^c)$, i.e. $p_i^c(z)$ such that $\pi^c(z) = k_r(\overline{p}_i^c)$.

Note that in a normal situation, $\overline{K}(1+\delta) - k_r(\overline{p}_i^c) < \delta + \lambda$ with $\left(\overline{K}(1+\delta) - k_r(\overline{p}_i^c)\right)$ the capital buffer of the bank, and $\hat{p}_i^c(z) = \dfrac{S(\overline{p}_i^c) + \overline{K}(1+\delta) - k_r(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda} < 1$. On the contrary, in the case where the capital buffer $\left(\overline{K}(1+\delta) - k_r(\overline{p}_i^c)\right)$ covers all the credit loss of the bank $(\delta + \lambda)$, the bank will never default and $\hat{p}_i^c(z) = 1$. In the following, we exclude this case and assume that $\overline{K}(1+\delta) - k_r(\overline{p}_i^c) < \delta + \lambda$.

The value of the adjusted bankruptcy rate of default depends on the class of risk of the asset chosen by the bank. Actually, two opposite effects play for driving the evolution of the adjusted bankruptcy rate of default. On one hand, a rise in the class of risk of the asset chosen by the bank increases the value of the spread and plays positively on the value of the adjusted bankruptcy rate of default. On the other hand, the value of regulatory capital increases also reducing the capital buffer of the bank which plays negatively on the value of the adjusted bankruptcy rate of default. In the rest of the paper, we assume that the adjusted bankruptcy rate of default $\hat{p}_i^c(z)$ is a decreasing function of the class of risk of the asset chosen by the bank (*Lemma* 1). It means that the higher the risk of the asset chosen by the bank the lower the rate of default that leads the bank to bankruptcy. This assumption implies that, choosing an asset with a higher risk the bank will increase its fragility.

Lemma 1.

Assume that $0 < \dfrac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c} \Big/ \dfrac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c} < \dfrac{S\left(\overline{p}_i^c\right) + \delta + \lambda}{\left(\delta + \lambda - \left[\overline{K}\left(1+\delta\right) - k_r\left(\overline{p}_i^c\right)\right]\right)}$, we have $\dfrac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} < 0$.

Proof of *Lemma* 1. See appendix 1.

3. Optimal asset's risk level choice for defined business conditions

Given the assumption of a constant capital endowment, the objective of the bank is to find the risky asset $\left(\overline{p}_i^{c*}\right)$ that maximizes its value subject to its own target of probability of default $\left(\alpha_B^*\right)$. As we focus on banks' risk taking incentives, we concentrate the analysis on the well

known "higher-risk higher-return" (HRHR) assets and we make the parametric assumption on assets (given by *lemma* 2) such that the bank net expected value is increasing with the class of risk of the investment.
Recall that the maximization program of the bank is given by

$$\max_{\overline{p}_i^c} \; V = -\overline{K} + \frac{1}{1+W} \int_0^{\hat{p}_i^c(z)} \pi^c(z) dF(p_i^c(z))$$
$$s.t. \quad \Pr\left[\pi^c(z) \geq k_r(\overline{p}_i^c)\right] = \alpha_B^*$$

Lemma 2.
Assume that $\frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c} \Big/ \frac{\partial p_i^c(z)}{\partial \overline{p}_i^c} > \frac{S(\overline{p}_i^c) + \delta + \lambda}{(1 - p_i^c(z))} > 0$, we have $\frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} > 0$.

Moreover, assume that $\forall \overline{p}_i^c \in \left]0; \overline{p}_{\max}^c\right]$, $\int_0^{\hat{p}_i^c(z)} \frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} dF(p_i^c(z)) > -k_r(\overline{p}_i^c) \underbrace{\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c}}_{-} > 0$ and we have $\frac{\partial V}{\partial \overline{p}_i^c} > 0$.

Proof of *Lemma 2*. See appendix 2.

Lemma 2 means that, as the net expected present value of the bank is continuously increasing with the level of risk of the chosen asset, the bank retains the last asset's class of risk which satisfies its own target of default given by:

$$\Pr\left[\pi^c(z) \geq k_r(\overline{p}_i^c)\right] = \alpha_B^*$$

Substituting $\pi^c(z)$ by its value and rearranging, we obtain (see appendix 3)

$$\Pr\left[p_i^c(z) \leq \hat{p}_i^c(z)\right] = \alpha_B^*, \; with \; \hat{p}_i^c(z) \equiv \frac{\overline{K}(1+\delta) - k_r(\overline{p}_i^c) + S(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda} \tag{9}$$

As the default rate $p_i^c(z)$ is distributed under the cumulative distribution function $F\left(p_i^c(z)\right)$, we have (see appendix 4):

$$\Pr\left[p_i^c(z) \leq \hat{p}_i^c(z)\right] = F(\hat{p}_i^c(z)) = \alpha_B^* \text{ and}$$
$$\hat{p}_i^c(z) = \phi\left(\frac{\phi^{-1}(\overline{p}_i^c) + \sqrt{\rho}\;\phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right) \tag{10}$$

Recall that the first term of equation (10), $\hat{p}_i^c(z) \equiv \frac{\overline{K}(1+\delta) - k_r(\overline{p}_i^c) + S(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda}$ represents the maximum rate of default that the bank must not exceed in order to be solvent. The second term of equation (10), $\phi\left(\frac{\phi^{-1}\left(\overline{p}_i^c\right)+\sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right)$ measures the effective rate of default of the bank according to its own target probability of default and to the asset it finances.

The equilibrium asset's class of risk $\overline{p}_i^{c*}$ retained by the bank will be the one that satisfied the following equation:

$$\phi\left(\frac{\phi^{-1}\left(\overline{p}_i^{c*}\right)+\sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right) = \frac{\overline{K}(1+\delta)-k_r\left(\overline{p}_i^{c*}\right)+S(\overline{p}_i^{c*})}{S(\overline{p}_i^{c*})+\delta+\lambda} \tag{11}$$

Equation (11) means that the bank will choose the asset's class of risk that makes equal its maximum rate of default with its effective rate of default. Said differently, for this class of risk, the capital buffer of the bank is just sufficient to absorb its losses in order to ensure that the bank will not fail with a probability of α_B^*.

The default rate of the asset compatible with the bank's own target of default probability is increasing with both the bank's target and the class of risk the asset belongs to.

For convenience, we call $a\left(\overline{p}_i^{c*}\right)=\left(\frac{\phi^{-1}\left(\overline{p}_i^{c*}\right)+\sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right)$ and we have $\frac{\partial\phi\left(a\left(\overline{p}_i^{c*}\right)\right)}{\partial\overline{p}_i^c}>0$.

Proposition 1 gives the condition of existence of an equilibrium asset's class of risk for the bank at given conditions on business cycle.

PROPOSITION 1.

a. If $\overline{K}<\frac{\phi(a\left(\overline{p}_{\max}^c\right))(\delta+\lambda)-\left(1-\phi(a\left(\overline{p}_{\max}^c\right))\right)S\left(\overline{p}_i^c\right)+k_r\left(\overline{p}_{\max}^c\right)}{(1+\delta)}$, there is an interior solution and the equilibrium asset's class of risk retained by the bank will be such that $\overline{p}_i^{c*}\in\left]0;\overline{p}_{\max}^c\right[$.

b. If $\overline{K}>\frac{\phi(a\left(\overline{p}_{\max}^c\right))(\delta+\lambda)-\left(1-\phi(a\left(\overline{p}_{\max}^c\right))\right)S\left(\overline{p}_i^c\right)+k_r\left(\overline{p}_{\max}^c\right)}{(1+\delta)}$, the equilibrium asset's class of risk retains by the bank would be the riskiest one available $\overline{p}_{\max}^c$.

c. The asset's class of risk retains by the bank at equilibrium is an increasing function of the bank's level of capital and a decreasing function of the bank's own target of default probability.

Proof of PROPOSITION 1. See appendix 5.

As long as the default rate of the asset compatible with the bank's own target of default probability is lower than the adjusted bankruptcy rate of default, the capital buffer of the bank allows it to absorb a loss higher than the one compatible with its target of default. In other words, for the retained class of risk of the asset, the capital buffer is too "high" compared to the target of default of the bank. Consequently, the bank can increase its profitability by increasing the level of risk of the asset it finances while maintaining its rate of default compatible with its target. The equilibrium class of risk of the asset retained by the bank is the one for which bank's capital buffer just permits to absorb losses compatible with its own target of default. Figure 1 gives a graphical representation of part a. of proposition 1.

Case a. $\bar{K} < \dfrac{\phi(a(\overline{p}^c_{\max}))(\delta+\lambda) - \left(1-\phi(a(\overline{p}^c_{\max}))\right) S(\overline{p}^c_i) + k_r(\overline{p}^c_{\max})}{(1+\delta)}$.

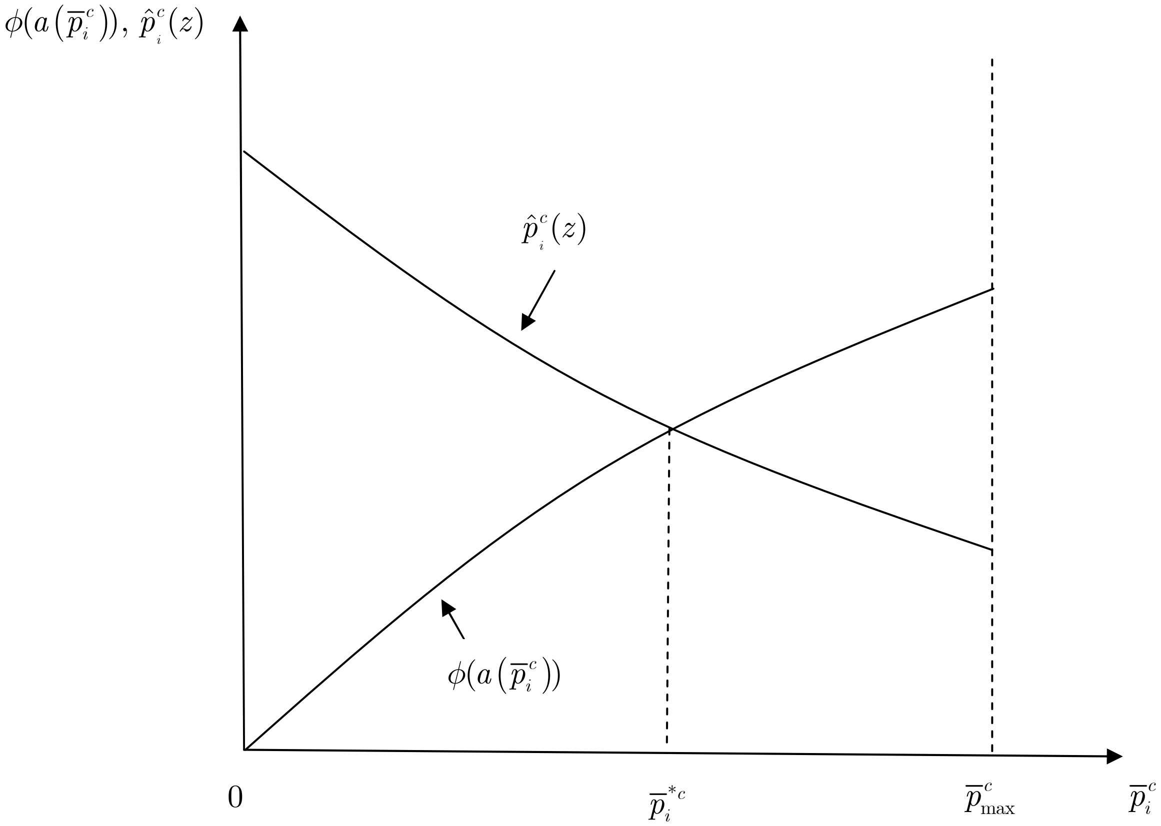

Figure 1

Figure 2 gives a graphical representation of part b. of proposition 1. It states that in some cases, the quantity of capital of the bank is so high that it will absorb the losses compatible with the bank's own target of default regardless the level of risk of the asset it finances. In that case, as the profitability is an increasing function of the class of risk of the asset, the bank will choose to finance the riskiest asset available $\overline{p}^c_{\max}$.

Case b. $\bar{K} > \dfrac{\phi(a(\overline{p}^c_{\max}))(\delta+\lambda) - \left(1-\phi(a(\overline{p}^c_{\max}))\right) S(\overline{p}^c_i) + k_r(\overline{p}^c_{\max})}{(1+\delta)}$.

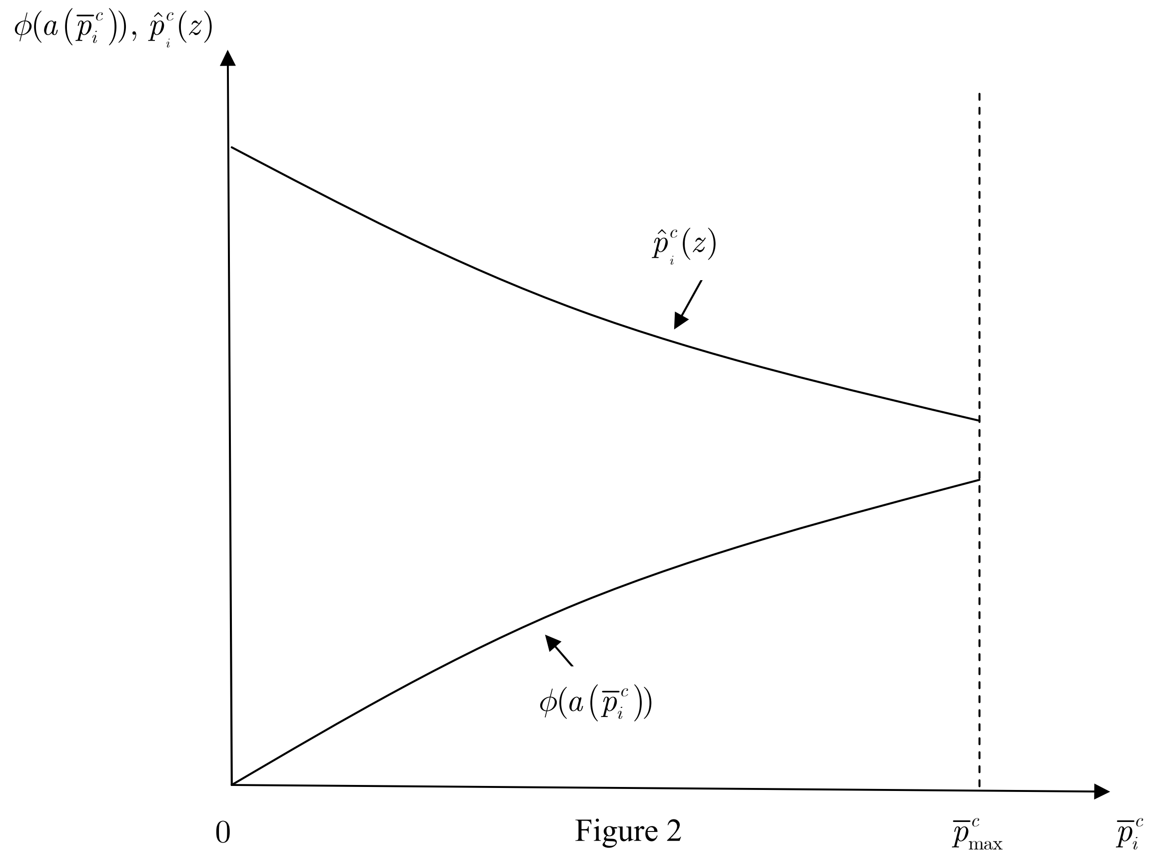

Figure 2

The equilibrium class of risk of the asset financed by the bank depends on two mains exogenous variables: the available quantity of bank's capital and the bank's own target of default probability. All things equal, a rise in the available quantity of capital increases the capital buffer of the bank and incites it to choose a riskier asset. Conversely, all things equal, a rise in the bank's own target of default probability increases the required level of capital buffer needed to cover the losses of the bank for a given class of risk of the asset and the bank will choose a less risky asset at equilibrium.

4. Optimal asset's risk level choice and business cycles

In the following, we study the impact of the cycle on the asset's risk level retained by banks at equilibrium. We show that the bank may be inclined to finance riskier assets during economic upturn. Such a mechanism, that appears while the level of bank capital is constant, may increase financial fragility since the rate of default that leads the bank to bankruptcy is decreasing.

We have assumed that firms' default probability assessed by the bank depends on the relative position of the economy within the business cycle: during economic upturns, estimations of firms' default probabilities are more favourable than during economic downturns. According to equation 2, the unconditional probability of default for a firm in class i is equal to:

$$\overline{p}_i^c = \Pr\left(\mu_i^c + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i > 0\right) = \phi\left(\mu_i^c\right) \qquad (2)$$

We have assumed that $\frac{\partial \mu_i^c}{\partial c} < 0$ and it follows that $\frac{\partial \overline{p}_i^c}{\partial c} < 0$. Thus, the probability of default for a firm increases during a recession and decreases when expansion periods occur.

Moreover, as Basel II capital requirements are risk sensitive and depend on the probability of default of the asset financed by the bank, the cyclicality of the economy affects the level of the regulatory requirements. We have $\frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c} > 0$ which means that $\frac{\partial k_r(\overline{p}_i^c)}{\partial c} = \frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c} \cdot \frac{\partial \overline{p}_i^c}{\partial c} < 0$. Thus, regulatory requirements decrease when expansion periods prevail, whereas they increase when recession periods occur. Note that because the amount of bank capital is constant and equal to $\overline{K}$, a decrease in regulatory requirements lead, all things equals, to a rise in bank's capital buffer $\left(\overline{K} - k_r(\overline{p}_i^c)\right)$. On the contrary, during a recession when regulatory requirements increase bank's capital buffer $\left(\overline{K} - k_r(\overline{p}_i^c)\right)$ is decreasing.

Finally, credit spreads also depend on the business cycle. As we make the HRHR asset assumption, credit spreads are negatively linked with the business cycle: while economic conditions improve, credit spreads decrease with the probability of default of the asset, reflecting an easier access to liquidity and higher expectations on investment returns; reverse happening while economic conditions deteriorate. This result is formally express by the following equation:

$$\frac{\partial S\left(\overline{p}_i^c\right)}{\partial c} = \frac{\partial S\left(\overline{p}_i^c\right)}{\partial \overline{p}_i^c} \cdot \frac{\partial \overline{p}_i^c}{\partial c} < 0 \text{ since } \frac{\partial S\left(\overline{p}_i^c\right)}{\partial \overline{p}_i^c} > 0, \forall c \tag{12}$$

These variations in probability of default, regulatory requirements and credit spread change the asset's class of risk retained by the bank at equilibrium. Proposition 2 summarizes the cyclical impact of risk sensitivity requirements on the equilibrium asset class of risk retained by the bank.

PROPOSITION 2.

1. If $\overline{K} < \frac{\phi(a(\overline{p}_{\max}^c))(\delta+\lambda) - \left(1-\phi(a(\overline{p}_{\max}^c))\right) S\left(\overline{p}_i^c\right) + k_r\left(\overline{p}_{\max}^c\right)}{(1+\delta)}$, the equilibrium asset's class of risk retained by the bank increases while economic conditions improve and decreases while economic conditions deteriorate.

2. If $\left|\frac{\partial \hat{p}_i^c(z)}{\partial c}\right| < \left|\frac{\partial \phi\left(a\left(\overline{p}_i^{c*}\right)\right)}{\partial c}\right|$ the rise in the level of risk chosen by the bank at equilibrium leads to a rise in financial fragility.

3. $\bar{K} > \dfrac{\phi(a(\overline{p}^{c}_{\max}))(\delta+\lambda)-(1-\phi(a(\overline{p}^{c}_{\max})))S(\overline{p}^{c}_{i})+k_{r}(\overline{p}^{c}_{\max})}{(1+\delta)}$, the equilibrium asset's class of risk retained by the bank remains unchanged with the cycle and will be the riskiest one available $\overline{p}^{c}_{\max}$.

Proof of PROPOSITION 2: see Appendix 6.

Following a change in business cycle conditions, the bank is incited to change its behaviour and choose a riskier or lower asset's risk level. Two situations are possible.

In the case of an economic expansion, the lightening in regulatory capital charges creates an "equity surplus" for the bank (its capital buffer increases). This softening of regulatory requirements impacts all classes of risk. Furthermore, the equity surplus available for the bank onto the particular class of risk initially retained, associated with the lightening of regulatory requirements for all classes of risk might incite the bank to opt for a new class of risk, exploiting its "equity surplus" to allocate across a riskier class of asset to increase its value while maintaining constant it target of default probability. Finally, when the choice of the new asset's class of risk leads to a decrease in the adjusted bankruptcy rate of default, financial fragility increases. This situation, corresponding to case 1 and 2 of proposition 2 is illustrated in figure 3.

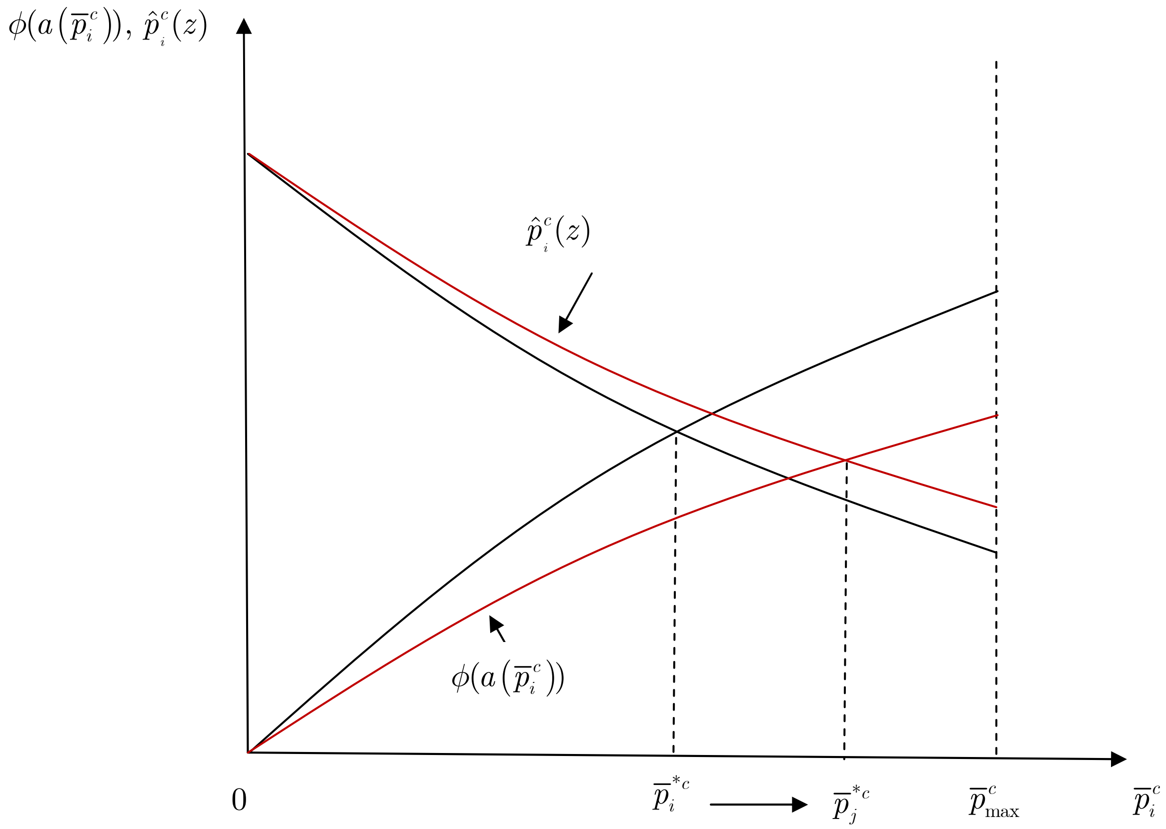

Figure 3

Within the same line of thinking, additional regulatory requirements due to worsen economic conditions impact all asset classes of risk. This "equity deficit" leads the bank to select for a new class of risk, reducing the risk level retained to comply with its own target of default probability.

This process of asset reallocation is always possible in our setting because we assume that the bank can choose its optimal level of risk in a continuous range of assets $\overline{p}_i^c \in \left]0; \overline{p}_{\max}^c\right]$. Actually, in a more realistic setting, the granularity of the risk bucket available to the bank is not infinite, and firms sharing different risk profile are to be clustered into the same risk bucket. The risk bucket Probability of Default[10] represents the regulatory input used to compute capital requirements, and leads to thresholds effects. In that case, for instance, the switch towards a riskier asset implies a discontinuous jump from a risk bucket to a riskier one. Depending on the granularity of the bank's risk bucket, this jump towards a riskier bucket may raise regulatory requirements of such an amount that bank cannot comply with the constraint for that risk level. In this case, where the next risk bucket leads to an infringement of the constraint, the bank keeps its current asset even if the risk level is sub-optimal (i.e. not binding with the constraint).

The traditional literature concludes to the pro-cyclical impact of capital requirements in terms of volume of lending, as on the exacerbation of this phenomenon by the risk sensitivity embodied into the revised regulatory ratio. Our analysis stresses the pro-cyclical impact in terms of risk level induced by the risk sensitivity of the capital asset ratio. We stress that the rise into the risk level of bank portfolio leads to a weakening of the bank which is similar to an increase in financial fragility. On the contrary, in case of deterioration in business conditions, the additional regulatory charges can constrain the bank to reduce its risk level. This lowering into the risk level of the bank portfolio represents a "flight to quality" phenomenon.

5. Discussion

Our result concerning the cyclical impacts of risk sensitive regulatory requirements derives from a comparative static analysis under a constant capital endowment. This constant capital assumption contrasts with other settings that assume a free adjustable leverage for the bank. Moreover our setting gives an interesting sight with regard to the Basel III novelties: the leverage ratio and the capital buffer conservative and countercyclical policies.[11]

The leverage ratio acts as a backstop ratio the bank capital shall not fall below (fixed at 3%). As it aims to give a complementary measure of the risk of the bank, the Basel Committee defines it as a risk insensitive measure: the capital (numerator) as exposure (denominator) are denominated in amount without being related to the risk weighted asset computed under the first pillar. Our setting examines how risk sensitive requirements impact along the business cycle the risk taking behaviour of a bank at constant capital endowment: the leverage ratio is constant in our setting and above the leverage ratio prescribed by the banking regulation.

Instituting such a leverage ratio aims to prevent from excessive leverage and tries to deal with systemic risk: it aims that capital would not fall below a minimum to maintain a shock absorbing capability from the bank. However, it does not interact with the cyclical impacts induced by the risk sensitive requirements as presented by our analysis. The cyclical

[10] Defined as the average or the mean of borrowers' PD.

[11] Basel Committee on Banking Supervision (2010)

variations into regulatory requirements affect bank's risk taking behaviour, fostering its risk appetite as regulatory requirements soften, restricting it as regulatory requirements strengthen, without modifying the leverage ratio in our setting (constant capital level and exposure set at one by assumption). The leverage ratio seems to be a complementary measure of the risk of the bank in a risk-wide managing perspective but does not prevent from a risk taking behaviour induced by the business cycle variations into the regulatory requirements.

The conservative and the countercyclical buffers are both defined as a 2.5% additional requirement of the risk weighted asset. Both buffers pursue the objective that banks operate with capital level well above the minimum required by the first pillar such that the bank can absorb financial shocks. More precisely both policies require the bank to build up capital buffer during economic upturns that can be draw down as losses occur during business downturns. The conservative buffer will applies systematically whereas the countercyclical buffer occurs only under the supervisory decision, when credit growth is perceived as excessive by the supervisory authorities. They define a corridor of additional capital requirements the bank must comply with, otherwise a penalty, consisting in restricting the earning to be distributed as dividend, applies. They complete the first pillar and the leverage ratio in their objective of avoiding a spill over of financial shocks to the real economy by maintaining a level of capital high enough to absorb shock.

Our analysis stresses the level of capital that excesses regulatory requirements plays a crucial role to allow the bank exploiting the regulatory "equity surplus" by means of a risk taking behaviour that increases its value as business conditions improve. Instituting a conservative buffer the bank must build up and maintain may mitigate the risk taking opportunity offered by the "equity surplus". Only highly capitalized banks could exploit the "equity surplus" under a conservative buffer policy. Less well capitalized banks that face this additional requirement may transfer the "surplus" obtained from an alleviate in the regulatory ratio to the conservative buffer requirement, otherwise they would have to raise capital to increase their optimal asset risk level and comply with these two requirements.

Although our setting cannot precisely examine what the countercyclical buffer implies on our result, we think that it reinforces the risk restrictive impact of the conservative buffer. Whether a highly capitalized bank could exploit the "equity surplus" and still complies with both regulatory ratio and conservative buffer requirements, the occurrence of this further countercyclical buffer may restrict the bank risk taking behaviour, inducing the bank to raise capital to comply with those three requirements that could equals 13% of the risk weighted asset.[12]

The "equity deficit" occurring as business conditions worsen leads the bank to lower its optimal risk level in our setting. The countercyclical buffer prevails only during expansion phase, the bank being no longer required to maintain these additional requirements during economic downturns. Moreover the conservative and the countercyclical buffers aim to be drawn down as bank incurs losses. These additional regulatory capital requirements, inherited from the expansion period, may help the bank to comply with the first pillar as its own target of default without modifying its asset choice. Those buffers may help avoiding or diminishing the "fly to quality" phenomenon that could occurs under the recession.

[12] Respectively 8% for the first pillar and 2.5% for each buffer.

The Committee defines the buffers to a risk-wide management perspective, aiming that the capital level of individual banks would be high enough to prevent from a widening of shocks to the financial as the real sectors. Therefore, the design of these policies, applying as economic conditions get better, helps counteracting the risk taking opportunities induced by the risk sensitive requirements that vary with the business cycle. However, highly capitalized bank could insulate from the restrictive impacts of the buffer policies and exploit the "equity surplus", if we reason under a constant capital level.

6. Conclusion

The risk sensitivity of the revised framework brings closer the respective objective of each owner of the bank liability and consequently seems prevent from regulatory trades-off and risk taking incentives embodied into a flat rate capital to asset ratio for the mean-variance ordering asset risk profiles. However the HRHR assets may still offer risk taking incentives under a risk sensitive regulatory framework.

Actually, under a risk sensitive regulation, regulatory requirements vary with the business cycle, strengthening while economic conditions worsen, softening as the economy gets better. The freeing of regulatory capital during upturns, the "equity surplus", can be exploited by the bank to increase its value toward the financing of a riskier asset that enhances the value of bank equities. With the same line of thinking, the "equity deficit" that grows as business conditions worsen can restrict the bank risk taking behaviour, pushing it to opt for a lower risk level of asset during recessions. Our analysis stresses the existence of procyclicality in terms of asset risk level induced by the risk sensitive regulation, and the resulting bank weakening: potential increase in financial fragility as the expansion prevails.

Moreover the novelties of Basel III do not seem to efficiently counteract the risk taking opportunity offered by the "equity surplus". As in Estrella (2004), our results highlight the crucial role played by the second pillar: the Supervisory Review Process, which encompasses risks that are not taken into the computation of the first pillar (regulatory capital to asset ratio) as some aspects of the bank's risk governance and management structure as a whole, to assess the relevant requirements that can be enforced to the bank. The Four Principles, where the second and the third are reminded below,[13] appear essential to insure a safety banking system:

<u>Principle 2</u>*: Supervisors should review and evaluate banks’ internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios. Supervisors should take appropriate supervisory action if they are not satisfied with the result of this process.*
<u>Principle 3</u>*: Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum.*

[13] Basel Committee on Banking Supervision (2005) page169 and page 170.

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APPENDIX

Appendix 1: proof of *Lemma* 1.

Recall that $\hat{p}_i^c(z) = \dfrac{S(\overline{p}_i^c) + \overline{K}(1+\delta) - k_r(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda} < 1$ for $\overline{K}(1+\delta) - k_r(\overline{p}_i^c) < (\lambda + \delta)$.

We have

$$\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} = \frac{\left[\frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c} - \frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c}\right]\left[S(\overline{p}_i^c) + \delta + \lambda\right] - \frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left[S(\overline{p}_i^c) + \overline{K}(1+\delta) - k_r(\overline{p}_i^c)\right]}{S(\overline{p}_i^c) + \delta + \lambda}$$

$$\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} = \frac{\frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left[\delta + \lambda - \left(\overline{K}(1+\delta) - k_r(\overline{p}_i^c)\right)\right] - \frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left[S(\overline{p}_i^c) + \delta + \lambda\right]}{S(\overline{p}_i^c) + \delta + \lambda}$$

As $\dfrac{\partial S\left(\overline{p}_i^c\right)}{\partial \overline{p}_i^c} > 0$, $\dfrac{\partial k_r\left(\overline{p}_i^c\right)}{\partial \overline{p}_i^c} > 0$, $\overline{K}(1+\delta) - k_r(\overline{p}_i^c) < (\lambda + \delta)$ we have $\dfrac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} < 0$ if $\frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left[\delta + \lambda - \left(\overline{K}(1+\delta) - k_r(\overline{p}_i^c)\right)\right] - \frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left[S(\overline{p}_i^c) + \delta + \lambda\right] < 0$ or put differently if

$$0 < \frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c} \Big/ \frac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c} < \frac{S\left(\overline{p}_i^c\right) + \delta + \lambda}{\left(\delta + \lambda - \left[\overline{K}\left(1+\delta\right) - k_r\left(\overline{p}_i^c\right)\right]\right)} \quad \blacksquare$$

Appendix 2: proof of *Lemma* 2.

Recall that $\pi^c(z) = \left(1 - p_i^c(z)\right)S(\overline{p}_i^c) - p_i^c(z)(\delta + \lambda) + \overline{K}\left(1+\delta\right)$

Consequently we have

$$\frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} = \frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left(1 - p_i^c(z)\right) - \frac{\partial p_i^c(z)}{\partial \overline{p}_i^c}\left(S(\overline{p}_i^c) + \delta + \lambda\right) \text{ with } \frac{\partial S\left(\overline{p}_i^c\right)}{\partial \overline{p}_i^c} > 0, \frac{\partial p_i^c(z)}{\partial \overline{p}_i^c} > 0.$$

And $\dfrac{\partial \pi^c(z)}{\partial \overline{p}_i^c} > 0$ if $\dfrac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c}\left(1 - p_i^c(z)\right) > \dfrac{\partial p_i^c(z)}{\partial \overline{p}_i^c}\left(S(\overline{p}_i^c) + \delta + \lambda\right)$

Or put differently if

$$\frac{\partial S(\overline{p}_i^c)}{\partial \overline{p}_i^c} \Big/ \frac{\partial p_i^c(z)}{\partial \overline{p}_i^c} > \frac{S\left(\overline{p}_i^c\right) + \delta + \lambda}{\left(1 - p_i^c(z)\right)} > 0$$

Moreover, we have $\dfrac{\partial V}{\partial \overline{p}_i^c} = \dfrac{1}{1+W} \cdot \left[\displaystyle\int_0^{\hat{p}_i^c(z)} \frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} dF(p_i^c(z)) + k_r\left(\overline{p}_i^c\right)\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c}\right]$

With $\displaystyle\int_0^{\hat{p}_i^c(z)} \frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} dF(p_i^c(z)) > 0$ as $\dfrac{\partial \pi^c(z)}{\partial \overline{p}_i^c} > 0$ and $k_r\left(\overline{p}_i^c\right)\dfrac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} < 0$ since $\dfrac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c} < 0$ according to *Lemma* 1.

Consequently, we have $\dfrac{\partial V}{\partial \overline{p}_i^c} > 0$ if

$$\forall \overline{p}_i^c \in \left]0; \overline{p}_{\max}^c\right], \int_0^{\hat{p}_i^c(z)} \frac{\partial \pi^c(z)}{\partial \overline{p}_i^c} dF(p_i^c(z)) > -k_r\left(\overline{p}_i^c\right) \underbrace{\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c}}_{-} > 0$$

This condition implies that the option value of the bank is always smaller than the expected gain due to a rise in the level of risk. It means that the rise in the expected gain from the financing of a riskier project is higher than the rise in the expected loss (due to a lower value of the adjusted bankruptcy rate of default) from the financing of this project ■

Appendix 3.

We have $\Pr\left[\pi^c(z) \geq k_r(\overline{p}_i^c)\right] = \alpha_B^*$
which is similar to
$\Pr\left[\pi^c(z) \leq k_r(\overline{p}_i^c)\right] = \left(1 - \alpha_B^*\right)$
Substituting $\pi^c(z) = S(\overline{p}_i^c) - p_i^c(z)\left(S(\overline{p}_i^c) + \delta + \lambda\right) + \overline{K}(1+\delta)$ in the previous equation we obtain $\Pr\left[\left(S(\overline{p}_i^c) - p_i^c(z)\left(S(\overline{p}_i^c) + \delta + \lambda\right) + \overline{K}(1+\delta)\right) \leq k_r\left(\overline{p}_i^c\right)\right] = \left(1 - \alpha_B^*\right)$.
Rearranging we have $\Pr\left[p_i^c(z) \geq \dfrac{\overline{K}(1+\delta) - k_r\left(\overline{p}_i^c\right) + S(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda}\right] = \left(1 - \alpha_B^*\right)$ or

$\Pr\left[p_i^c(z) \leq \dfrac{\overline{K}(1+\delta) - k_r\left(\overline{p}_i^c\right) + S(\overline{p}_i^c)}{S(\overline{p}_i^c) + \delta + \lambda}\right] = \alpha_B^*$ which correspond to equation (9) ■

Appendix 4.

We search the value of b such that $\Pr\left[p_i^c(z) \leq b\right] = \alpha_B^*$
or equivalently $\Pr\left[p_i^c(z) \leq b\right] = F(b) = \alpha_B^*$.
As the cumulative distribution function of the default rate is given by:
$F\left(p_i^c(z)\right) = \phi\left(\dfrac{\sqrt{(1-\rho)}\phi^{-1}\left(p_i^c(z)\right) - \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{\rho}}\right)$,

we have $F(b) = \phi\left(\dfrac{\sqrt{(1-\rho)}\phi^{-1}(b) - \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{\rho}}\right)$. And b must be solution to

$\phi\left(\dfrac{\sqrt{(1-\rho)}\phi^{-1}(b) - \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{\rho}}\right) = \alpha_B^*$ or $\dfrac{\sqrt{(1-\rho)}\phi^{-1}(b) - \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{\rho}} = \phi^{-1}(\alpha_B^*)$.

Rearranging we obtain $\phi^{-1}(b) = \dfrac{\sqrt{\rho}\ \phi^{-1}(\alpha_B^*) + \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{(1-\rho)}}$ and finally

$b = \phi\left(\dfrac{\sqrt{\rho}\ \phi^{-1}(\alpha_B^*) + \phi^{-1}\left(\overline{p}_i^c\right)}{\sqrt{(1-\rho)}}\right)$ which correspond to equation (10) ■

Appendix 5. Proof of PROPOSITION 1.

Proof of part a and b.

Recall that the optimal asset class of risk $\overline{p}_i^c \in \left]0;\overline{p}_{\max}^c\right]$ chosen at equilibrium by the bank must satisfy the following equation

$$\phi\left(\frac{\phi^{-1}\left(\overline{p}_i^{c*}\right)+\sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right) = \frac{\overline{K}(1+\delta)-k_r\left(\overline{p}_i^{c*}\right)+S(\overline{p}_i^{c*})}{S(\overline{p}_i^{c*})+\delta+\lambda} \qquad (11)$$

With $\phi(a\left(\overline{p}_i^c\right))=\phi\left(\frac{\phi^{-1}\left(\overline{p}_i^c\right)+\sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right)$, ϕ being a cumulative distribution function of a standard normal variable, strictly positively and increasing with risk by construction.

According to the HRHR asset assumption, which implies low spreads level for low risk asset, we assume that $\underbrace{\lim}_{\overline{p}_i\to 0} S(\overline{p}_i^c)\to 0$.

Note also that ρ is computed as in Basel II and it is decreasing with the asset probability of default with $\rho\in[0.12;0.24]$.

Let's begin by studying the limits of the two parts of equation (11) for $\overline{p}_i^c \in \left]0;\overline{p}_{\max}^c\right]$.

1. Limit of $\phi(a\left(\overline{p}_i^c\right))$ when $\overline{p}_i^c \to 0$.

$\lim\limits_{\overline{p}_i^c\to 0}\phi^{-1}\left(\overline{p}_i^c\right)\to-\infty$ which implies that $\lim\limits_{\overline{p}_i^c\to 0} a\left(\overline{p}_i^c\right)\to-\infty$ and $\lim\limits_{\overline{p}_i^c\to 0}\phi(a\left(\overline{p}_i^c\right))\to 0$

2. Limit of $\hat{p}_i^c(z)=\dfrac{S(\overline{p}_i^c)+\overline{K}(1+\delta)-k_r(\overline{p}_i^c)}{S(\overline{p}_i^c)+\delta+\lambda}$ when $\overline{p}_i^c\to 0$

The limit of $\overline{K}(1+\delta)-k_r\left(\overline{p}_i^c\right)$ depends on the regulatory capital requirements. In Basel II we have $\dfrac{\partial k_r(\overline{p}_i^c)}{\partial \overline{p}_i^c}>0$ and $\lim\limits_{\overline{p}_i^c\to 0} k_r(\overline{p}_i^c)\to 0$. Moreover, we assume that $\underbrace{\lim}_{\overline{p}_i\to 0} S(\overline{p}_i^c)\to 0$.

Consequently, we have $\lim\limits_{\overline{p}_i^c\to 0}\dfrac{S(\overline{p}_i^c)+\overline{K}(1+\delta)-k_r(\overline{p}_i^c)}{S(\overline{p}_i^c)+\delta+\lambda}=\dfrac{\overline{K}(1+\delta)}{\delta+\lambda}>0$.

As a consequence, $\phi(a\left(\overline{p}_i^c\right))$ starts below $\hat{p}_i^c(z)$ when the asset belongs to the riskless level class.

4. Limit of $\hat{p}^c_i(z)$ and $\phi(a(\overline{p}^c_i))$ when $\overline{p}^c_i \to \overline{p}^c_{\max}$.

$$\lim_{\overline{p}^c_i \to \overline{p}^c_{\max}} \phi(a(\overline{p}^c_i)) = \phi\left(\frac{\phi^{-1}\left(\overline{p}^c_{\max}\right) + \sqrt{\rho}\ \phi^{-1}\left(\alpha^*_B\right)}{\sqrt{(1-\rho)}}\right) > 0$$

$$\lim_{\overline{p}^c_i \to \overline{p}^c_{\max}} \hat{p}^c_i(z) = \frac{S(\overline{p}^c_{\max}) + \overline{K}(1+\delta) - k_r(\overline{p}^c_{\max})}{S(\overline{p}^c_{\max}) + \delta + \lambda} > 0$$

Consequently, there an interior solution i.f.f $\lim_{\overline{p}^c_i \to \overline{p}^c_{\max}} \hat{p}^c_i(z) > \lim_{\overline{p}^c_i \to \overline{p}^c_{\max}} \phi(a(\overline{p}^c_i))$ which is realised for $\overline{K} < \dfrac{\phi(a(\overline{p}^c_{\max}))(\delta+\lambda) - \left(1-\phi(a(\overline{p}^c_{\max}))\right) S\left(\overline{p}^c_i\right) + k_r\left(\overline{p}^c_{\max}\right)}{(1+\delta)}$.

If $\overline{K} > \dfrac{\phi(a(\overline{p}^c_{\max}))(\delta+\lambda) - \left(1-\phi(a(\overline{p}^c_{\max}))\right) S\left(\overline{p}^c_i\right) + k_r\left(\overline{p}^c_{\max}\right)}{(1+\delta)}$, the left part of equation (11) is always lower than the right part and the bank will retain the riskiest asset available $\overline{p}^c_{\max}$.

Proof of part c.

A rise in $\overline{K}$ only influence the value of $\hat{p}^c_i(z)$ with $\dfrac{\partial \hat{p}^c_i(z)}{\partial \overline{K}} > 0$. Consequently, all things equal, $\hat{p}^c_i(z)$ increases whereas $\phi(a(\overline{p}^c_i))$ is unchanged and the bank will choose a riskier asset.

A rise in α^*_B only influence the value of $\phi(a(\overline{p}^c_i))$ with $\dfrac{\partial \phi(a(\overline{p}^c_i))}{\partial \alpha^*_B} > 0$. Consequently, all things equal, $\phi(a(\overline{p}^c_i))$ increases whereas $\hat{p}^c_i(z)$ is unchanged and the bank will choose a lower risky asset.

The proof of proposition 1 is completed ■

Appendix 6. Proof of PROPOSITION 2.

Recall that the asset's class of risk retains by the bank at equilibrium is given by the following condition:

$$\phi\left(\frac{\phi^{-1}\left(\overline{p}^{c*}_i\right) + \sqrt{\rho}\ \phi^{-1}\left(\alpha^*_B\right)}{\sqrt{(1-\rho)}}\right) = \frac{\overline{K}(1+\delta) - k_r\left(\overline{p}^{c*}_i\right) + S(\overline{p}^{c*}_i)}{S(\overline{p}^{c*}_i) + \delta + \lambda} \qquad (11)$$

With $\hat{p}^c_i(z) = \dfrac{\overline{K}(1+\delta) - k_r\left(\overline{p}^{c*}_i\right) + S(\overline{p}^{c*}_i)}{S(\overline{p}^{c*}_i) + \delta + \lambda}$ and $\phi\left(a\left(\overline{p}^{c*}_i\right)\right) = \phi\left(\dfrac{\phi^{-1}\left(\overline{p}^{c*}_i\right) + \sqrt{\rho}\ \phi^{-1}\left(\alpha^*_B\right)}{\sqrt{(1-\rho)}}\right)$

We have $\frac{\partial \hat{p}_i^c(z)}{\partial c} = \underbrace{\frac{\partial \hat{p}_i^c(z)}{\partial \overline{p}_i^c}}_{-} \underbrace{\frac{\partial \overline{p}_i^c}{\partial c}}_{-} > 0$ and $\frac{\partial \phi\left(a\left(\overline{p}_i^{c^*}\right)\right)}{\partial c} = \underbrace{\frac{\partial \phi\left(a\left(\overline{p}_i^{c^*}\right)\right)}{\partial \overline{p}_i^c}}_{+} \underbrace{\frac{\partial \overline{p}_i^c}{\partial c}}_{-} < 0$.

Consequently, as the $\phi\left(a\left(\overline{p}_i^{c^*}\right)\right)$ is decreasing and $\hat{p}_i^c(z)$ is increasing with the cycle, there is an incentive for the bank to increase the asset's class of risk it finances during a boom and to reduce the asset's class of risk it finances during recession.

Moreover, if $\left|\frac{\partial \hat{p}_i^c(z)}{\partial c}\right| < \left|\frac{\partial \phi\left(a\left(\overline{p}_i^{c^*}\right)\right)}{\partial c}\right|$ the rise in the asset's class of risk financed by the bank lead to a rise in financial fragility since the adjusted bankruptcy rate of default decreases for the new financed asset ■

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