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Predicting US banks bankruptcy: logit versus Canonical **Discriminant analysis**

Zeineb AFFES, Rania HENTATI-KAFFEL

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Predicting US banks bankruptcy: logit versus Canonical Discriminant analysis

Zeineb Affes^{*} Rania Hentati-Kaffel[‡]

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Abstract

Using a large panel of US banks over the period 2008-2013, this paper proposes an early-warning framework to identify bank leading to bankruptcy. We conduct a comparative analysis based on both Canonical Discriminant Analysis and Logit models to examine and to determine the most accurate of these models. Moreover, we analyze and improve suitability of models by comparing different optimal cut-off score (ROC curve vs theoretical value). The main conclusions are: i) Results vary with cut-off value of score ii) the logistic regression using 0.5 as critical cut-off value outperforms DA model with an average of correct classification equal to 96.22%. However, it produces the highest error type 1 rate 42.67% iii) ROC curve validation improves the quality of the model by minimizing the error of misclassification of bankrupt banks: only 4.42% in average and exhibiting 0% in both 2012 and 2013. Also, it emphasizes better prediction of failure of banks because it delivers in mean the highest error type II 8.43 %.

Keywords: Bankruptcy prediction, Canonical Discriminant Analysis, Logistic regression, CAMELS, ROC curve, Early-warning system

1 Introduction

The financial crisis of 2007 is considered as the first real crisis of excess financial complexity. It illustrates the degree of the existing inter-connectivity between banks and

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financial institutions and highlighted the phenomenon of contagion that might exist in the interbank market. Since then, a swarming literature has been developed on the subject of quantification, prediction and control of systemic risk.

One of the methods proposed to prevent contagion of bank failures is to assess the bank failure rate. This approach helps to establish an early warning model of bank difficulties. Thus, interactions that may exist between solvency and refinancing risk can identify the banks which have the most difficulties to refinance and therefore be perceived as risky by the other institutions. This stigma will limit part counterparty risk and warn the financial authorities of a liquidity risk in case of default of these banks.

The existing financial literature engorges methods and models which aim to identify institutions whose financial situation appears alarming and deserves attention by supervisors. In this study, we propose to estimate and test the effectiveness of forecasting models of bank failures in the United States. The specificity of our study lies in the fact that it takes into account several financial ratios (solvency ratios, quality of assets, cash or liquidity ...) and is based on a very large sample of US banks (large and small bank) and from 2008 to 2013. The results of our study confirm that early warning system of banking difficulties including CAMEL financial variables is of great efficacy.

The empirical literature distinguishes two methods: parametric and non-parametric validation.Beaver (1966) was the pioneer in using a statistical model for predicting bankruptcy. The approach is to select from thirty financial ratios those which are the most effective indicators of financial failures. The study concludes that the (cash flow/-Total debt) ratio is the best forecasting indicator.

Altman (1968) tested Multiple Discriminant Analysis (MDA) to analyze 70 companies, first by identifying the best five significant explanatory variables from a list of 22 ratios and then by applying the (MDA) to calculate a Z-Altman score for each company. This score was almost accurate in predicting bankruptcy one year ahead. This model was then subsequently improved in Altman and Narayanan (1997) by proposing the zeta model that includes seven variables and classified correctly 96% of companies one year before bankruptcy and 70% five years before bankruptcy.

Since then, the use of discriminant analysis has continued to grow through the different published studies (Bilderbeek (1979); Ohlson (1980); Altman (1984);Zopounidis and Dimitras (1993)...). The vast majority of studies achieved after 1980 used the logit models to overcome the drawbacks of the DA method (Zavgren (1985); Lau (1987); Tennyson et al. (1990)...) The logit analysis fits linear logistic regression model by the method of maximum likelihood. The dependent variable(the probability of default) gets the value "1" for bankrupted banks and "0" for the healthy banks.

The second main approach has been developed to fix the constraints of traditional statistical model. Non parametric methods such TRA (Trait Recognition Analysis) also called Trait Recognition Models (TRM) has no prior assumption about variables to predict. It is associated to neural network models and allows the information exploration when interactions between the independent variables are nonlinear (Kolari et al. (2002)). In this sense, since (Frydman et al., 1985) work, decision tree has become a popular data mining technique and commonly used for classification and prediction.

Many other studies have applied this same technique in commercial US banks and have shown that it performs better than the Probit model (Marais et al. (1984)). Messier Jr and Hansen (1988) show that inductive algorithm is better than the DA.

Numerous comparative studies were carried out (Keasey and Watson (1991); Dimitras et al. (1996)); Altman and Narayanan (1997); Wong et al. (1997); Adya and Collopy (1998); O'leary (1998);Zhang et al. (1998); Vellido et al. (1999); Coakley and Brown (2000); Aziz and Dar (2004); Balcaen and Ooghe (2006); Balcaen et al. (2004); Kumar and Ravi (2007)). However, the supremacy of one method over another remains subject to various controversies because of the heterogeneity of the data used for validation (database, number of points in the data, sample selection, validation methods for forecasting, the number and the nature of explanatory variables tested in the model (financial, qualitative ...).

However, recent studies have shown the superiority of neural networks over other techniques (Du Jardin (2010); Jo et al. (1997); Tsai and Wu (2008)). In particular, Du Jardin (2010) results, based on over 200 previous articles, showed that neural network based model leads to better results in terms of failure rate prediction.

The aim of this paper is twofold: descriptive and predictive. In the financial literature, the analytical part is often not addressed. Nevertheless, we believe that this is the cornerstone of a better interpretation of the results. Thus, we proceed by describing and analyzing key financial ratios of the active and non-active banks for the entire period from 2008 to 2013.

In this paper we combined three parametric models (Canonical Discriminant Analysis and Logit) with the descriptive Principal Component Analysis model (PCA) to construct an early warning system (EWS).

First, (PCA) reduced the size of data (dimension below 10) and insure an uncorrelated blend of variables framework. Then, factor scores were estimated for each bank. These scores were used to estimate (CDA) and Logit models.

One among the important results of this paper is to have compared several methods to calculate the theoretical value of the probability of default that will serve as threshold to split the bank universe into two set : failed or healthy.

The paper consists of four sections. After the introduction, an overview of the existing literature concerning the bank failure prediction is given. Section two describes used data, the methodology and (PCA) results. Section three provides the empirical analysis and the study results. Finally section four contains concluding remarks.

2 Description of the Methodology and the variables

This section focuses on the data gathered for the estimation of our models. We begin by describing data collection and variables selection process. Next, we present the financial and economic ratios followed by descriptive statistics and correlation analysis.

2.1 Data description

We proceed to the constitution of our database of US banks from mainly two sources: "BankScope" and FDIC. Our database covers the period 2008-2013. Statistics shows that the period from 2008 to 2013 is marked by a wave of bank failures in the United States: more than 450 bank failures and FDIC estimated losses to more than US 85 billion dollar.

After data reprocessing, the sample banks contains two categories: active banks and non-active banks. Non-active banks are those which have been declared as bankrupted by the Federal Deposit Insurance Corporation (FDIC). The information on the identity and the bank's balance sheet data are obtained from the FDIC website. Indeed, all US banks must report their financial statements in the Uniform Bank Performance Report. Some treatments have been applied to our sample to allow homogeneity between banks. Indeed, a bank that has been declared bankrupted in the first quarter of the year "N" will be reclassified and considered as bankrupted in late "N-1"

For banks declared bankrupt by the FDIC after 01/04/N and for which there is no information for the current year, they will be considered as inactive for the year "N". For banks that will make bankruptcy at date later and which data are available in 31/12/N, they will be considered as active for "N". Financial variables of active banks were retrieved from the database "BankScope". Data were available for only 928 banks each year in the period 2008-2013. After processing and verifying data availability of the financial statements required in our study, the number of banks was reduced to 411 failed banks over the entire period 2008-2013 and 836 active banks each year. Table 1 gives more details on our database.

2.2 Variables: review of the literature

Federal regulators developed the numerical CAMEL rating system in the early 1970s to help structuring their examination process. This rating is based on the capital adequacy, asset quality, management quality, earnings ability, and liquidity position ratios. Capital adequacy evaluates the quality of a bank's capital. Asset quality measures the level of risk of a bank's assets. This is reflects the quality and the diversity of the credit risk and the ability of the bank to repay issued loans. Management quality is a measure of the quality of a bank's officers and the efficiency of its management structure. Earnings ability reflects the performance of banks and the stability of its earnings stream. Liquidity measures the ability of banks to meet unforeseen and unexpected deposit outflow in the short time. In February 1997, a sixth component sensitivity to market risk was added to the CAMEL rating system.

A very abundant literature tried to identify the most significant variables of the financial health of banks. According to Sinkey (1975), the quality of bank assets is the most significant ratio. Assets composition, loans characteristics, capital adequacy, source and use of income, efficiency and profitability are also discriminant variables. Poor asset quality and low capital ratios were the two characteristics of banks most consistently associated with banking problems during the 1970s (Sinkey (1978)). Avery et al. (1984), Barth et al. (1985) and Benston (1985) conclude that the proxies of loans portfolio composition and quality, capital ratio and the source of income are significant. Thomson (1991) demonstrate that the probability that a bank will fail is a function of variables related to its solvency, including capital adequacy, asset quality, management quality, and the relative liquidity of the portfolio. Martin (1977) found that the capital asset ratio, and the loans portfolio's composition to total assets ratio have a high level of significance. Pantalone et al. (1987) proposed a model including most of CAMEL proxies: profitability, management's efficiency, leverage, diversification and economic environment. Their results confirm the main cause of default was bad credit risk management. The model of Barr et al. (1994) include CAMEL proxies and efficiency scores as management's quality proxies and a proxy of the economic conditions. The six variables selected for their failure-prediction models are equity/total loans (C), non performing loans/total assets (A), DEA efficiency score (M), net income/ total assets (E) and large dollar deposits/ total assets(L).

Our main objective in this study is to provide an accurate bank failure model based on the significant fragility factors. In line with the literature, we maintain the most commonly used financial ratios which can forecast potential failures (Beaver (1966), Altman (1968)), Thomson (1991), Kolari et al. (1996), Jagtiani et al. (2003), Dabos and Sosa-Escudero (2004), and Lanine and Vander Vennet (2006)).

3 Principal Component Analysis

3.1 Variable's Statistics description

We include in our analysis four categories of variables: (1) two measures of capital adequacy. These latter indicates the measurement of the financial strength of a bank and determines the capacity of the bank in terms of meeting time liabilities and other risk such as credit risk, market risk, operational risk and others. The most popular proxy for capturing capital adequacy in previous literature is total equity divided by either total assets or total loans. (2) assets quality measures are considered in data construction. These variables have a crucial role in the assessment of the current condition and financial capacity in the future. We employ four variables related to asset quality (NPLTA, NPLGL, LLRTA, and LLRGL). We note that for NPLTA and NPLGL variables we use the proxy loans not accruing plus loans over 90 days late/ total assets (non-performing loans/total assets). (3) Bank profitability which is assessed through two ratios. The first ratio is the net profit as a share of total assets. As for the second measure, it is the net profit as a share of total shareholders' equity. Both measures are positively related to the financial performance of the bank and negatively related to the failure (Hassan Al-Tamimi and Charif (2011)). (4) The liquidity level of the bank is assessed through employing three ratios. The first one is total liquid assets to total assets. This indicates the ability of the bank to cover its liabilities. The second ratio which was used to estimate liquidity is total liquid assets as a share of total deposits. This ratio depicts the capacity of the bank to cover unanticipated deposit withdrawal. The ratio

Categories CAMEL	Variables	Definition
Capital Adequacy	EQTA	Total Equity/Total Assets
	EQTL	Total Equity/Total Loans
Assets Quality	NPLTA	Non Performing Loans/Total Assets
	NPLGL	Non Performing Loans/Gross Loans
	LLRTA	Loan Loss Reserves/Total Assets
	LLRGL	Loan Loss Reserves/Gross Loans
Earnings Ability	ROA	Net Income/Total Assets
	ROE	Net Income/Total Equity
Liquidity	TLTD	Total loans /Total customer Deposits
	TDTA	Total Customer Deposits/Total Assets

of liquid assets to short term liabilities is the last ratio to determine the liquidity. The explanatory variables are shown below:

Table (2) presents the means of the ten financial ratios for the two groups (Non Failed Bank (NFB) and Failed Bank (FB)), and significance tests for the equality of group means for each ratio.

First, according to capital adequacy ratios which are measures of how much capital is used to support the banks' risk assets. (EQTA) ratios for (FB) are on average very low. A low ratio means a significant leverage of these banks. This makes banks less resistant to shocks. Thus, the higher (EQTA) value is; the lower the probability of default will be. As banks trend toward failure, their equity position is likely to decrease, thus a negative relationship is expected between total loans and failure. The same conclusions emanate from the (EQTL) ratio analysis.

According to the asset quality ratio, we note that (NPLTA) ratios for (FB) are very low and disparate for the period spanning between 2008 and 2013. The immediate consequence of large amount of non-performing loans(NPL) is bank failure. In fact, according to our data, the economic environment has pushed up (NPL) thus the ratios (NPLTA), (NPLGL), (LLRTA) and (LLRGL) decrease. Banks with a high (NPL)amount tend to carry out internal consolidation to improve the asset quality rather than distributing creditand will be obliged to raise provision for loan loss. For example, (NPLGL) ratio fell by 3.92% from 13.35% to 9.31% for (FB) and by 1.17% for (NFB). We note that low value of loan portfolio signals the potential existence of an important vulnerability in the financial system (17.75% in 2008 and 17.83% in 2010). (LLRTA) and (LLRGL) provide an useful indication for analysts because they indicate a bank's sense of how stable its lending base is. The higher the ratio, the poorer the quality of the loan portfolio will be (3.59% for (LLRTA) ratio and 5.17% for (LLRGL) ratio for the (FB) in 2011).

Finally (TLTD) and (TDTA) liquidity ratios are often used by policy makers to assess the lending practices of banks and get some statistics. If the ratio is too high, it means that banks might not have enough liquidity to cover any unforeseen fund requirements; if the ratio is too low, banks may not be earning as much as they could be. Table (2) exhibits in average for(FB) high values (for example 91.13% and 82.63% for 2008 and 2009). These high ratios reflect the fact that they are relying on borrowed funds.

Table (3) is used to analyze the correlation coefficients between the different explanatory variables and the dependent variable (probability of default). We note the significance at 1% and 5% of all the variables that we have retained in our study. Note also that most of the coefficients have the expected signs. For example, a negative correlation is confirmed for (EQTA) and (EQTL) for all years. Indeed, an increase in the value of the two ratios has a negative effect on the bank's survival.

Table(4), presents the correlation matrix of ratios. Here, it can be seen that most of the ratios shows correlation to each other. When scrutinizing the correlation's matrix pairwise, we can distinguish the following aspects. There is a strong correlation (over 90%) between the pairs of variables (NPLTA) / (NPLGL) and (LLRTA)/(LLRGL). This result is generalized for all years and reflects a strong link between them and that one of the variablescan be replaced by another. Also, the Asset quality component (AQ) which groups (NPLTA), (NPLGL), (LLRTA) and (LLRGL) variables is negatively correlated with the return on assets (ROA). The ratio (EQTA) variable, which is a proxy of capital adequacy, and profitability of assets are negatively correlated with proxies of asset quality. There is a positive correlation between asset quality and profitability of the bank's assets. The same interpretation is still valid for years 2010 and 2011.

3.2 PCA

In this section we present the results of variables selection under the Principal Component Analysis (PCA). The aim is to extract the most important information from the data and to compress the data dimension by keeping only the most important ratios to explain the changes in financial conditions of banks.

Several tests are provided as following:

- i) Bartlett's test to validate the assumption of equality of variances. In this sense, if the test statistic is larger than the critical value, we reject the null hypotheses at the 5% significant level (Table 5). Thus, the sample correlation matrix did not come from a population where the correlation matrix is an identity matrix.
- ii) Kaiser-Meyer-Olkin (KMO) to test if the variables have enough in common to warrant a factor analysis. In this test (KMO) retain only components with eigenvalues greater than one. Eigenvalues, also called characteristic roots are presented in Table (5).

In addition to this tests, we perform (PCA) by analysing Factor Loading which are correlation coefficients between the financial variables and factors. Finally, we determine the (PCA) scores.

Before getting to the description of (PCA), we first analyse the correlation matrix. Then after centering and standardizing each ten variables, we determine the optimal number of principal component analysis. The starting point is the correlation matrix. Table (4) presents the degree of dependence between the initial ten variables. It can be easily seen that variables are correlated. This means that the information they convey have some degree of redundancy. To perform this finding of correlation, we present in Table (5) Bartlett's test of Sphericity. Bartlett test compares the correlation matrix with a matrix of zero correlation . A zero p-value is obtained over all the period from 2008 to 2013. Thus, we perform a valid factor analysis.

Table (6) describes the estimated factors and their eigenvalue. In 2008, we retain the first three factors. These factors explain 71.93% of the total variation of the financial conditions of banks. The first factor is the most important dimension to explain the changes in financial conditions of banks. It explains 41.54% of the total variance of the selected financial ratios. Factors F2 and F3 respectively explain 16.15% and 14.23% of the total variance.

Under the same decision rule of (KMO) measure and based on the results of the Eigenvalue's factors of 2009. These four factors account for 81.76% of the total variation of the financial conditions of banks. The first factor explains 46.81% of the total variance of financial ratios. Factors F2, F3 and F4 respectively explain 13.23%, 11.56% and 10.15% of the total variance.

In 2010, the first three factors explained 71.4% of the total variation. The first factor explains 46.6% of the total variance. Factors F2 and F3 respectively explain 13.9% and 10.89% of the total variance. The choice of four factors is validated for 2011 and account for 81.68% of total variation. Finally, for the years 2012 and 2013, the first four selected factors explain almost 77% of the total variation.

We follow (PCA) by considering and evaluating Factor's loadings (see Table 7). In our case, the contribution ratios in the main components vary between 0 and 1 in absolute values. If a variable contributes more than 0.5 in a specific factor, it will be considered as the main indicator. However, if its contribution is below this threshold, the variable will be considered as a secondary indicator.

For 2008, variables that explain better the first factor F1 are (NPLTA), (NPLGL), (LLRTA), (LLRGL) and (ROA). F1 refers to both assets quality and return on assets components. The component loadings tell us how much of the variation in a variable is explained by the component. For example asset quality loading values are negatives. Thus, an increase in the value of these ratios will result in a lower score factor F1. So, the increase in these ratios will decline the asset quality. This implies, subsequently, an increase in the probability of default of the bank. The (ROA) ratio has a positive loading, which means that an increase in its value will increase the F1 score. We find the same results for 2009, 2010 and 2011 for F1 factor. For 2012 and 2013, F1 groups only variables of asset quality. For 2013, all the ratios have a positives loadings, which means that an increase will increase the score of the factor F1.

F2 groups ratios of Capital Adequacy (EQTA) and (EQTL) only for 2008, 2012 and 2013. Loadings for these two variables are positives. An increase in the value of these two ratios will increase the value of the score of the Capital Adequacy factor and reduce the probability of default.

In 2009 this factor is composed by the ratios (TLTD) and (TDTA). Loadings are negatives and an increase of the value will accentuate the probability of default.

In 2010 and 2011, we retain also the second factor F2 which includes liquidity components. (TLTD) ratio has a positive loading, which means that an increase in its value will increase the D-score of the liquidity factor. (TLTD) ratio is considered as a good proxy of short term viability and a low value means that there is no optimal reallocation of resources.

For 2009, 2010 and 2011 the factor F3 is composed by capital adequacy variables and (TDTA) variable.

For the other years studied, a fourth factor (F4) is considered and it brings the ratio (ROE) and the two ratios of asset quality.

Finally, we determine the factor score coefficient matrix for each bank. According to the Table (8) which describes the factor score coefficients, we calculate factor scores for each bank using the formula below:

$$F_{bi} = \sum u_{ij} z_{bj} \tag{1}$$

Where:

- F_{bi} : the estimated factor i for bank b
- z_{bj} : the standardized value of the jth ratios for a bank b
- u_{ij} : the factor score coefficient for the ith factor and the jth ratios
- These scores (F_{ai}) were used as independent variables in estimating the discriminant and the Logit model.

4 Empirical results

4.1 Canonical Discriminant Analysis

In this section, we provide Canonical Discriminant Analysis (CDA) to conduct and recuperate an early warning system indication of failed banks. In this sense, we propose to describe the relationships among the two groups of banks (bankrupt or not) based on a set of discriminating variables.

The canonical discriminant function is expressed as follow:

$$D_{bi} = b_0 + \beta_1 F_{b1} + \beta_2 F_{b2} + \dots + \beta_i F_{bi}$$
⁽²⁾

Where:

- D_{bi} : the value (score) on the canonical discriminant function for bank b.
- F_{bi} : represents factors validated in (PCA) section.

Each sampling group of bank has a single composite canonical score, and the group centroids indicate the most typical location of a bank from a particular group. Discriminant analysis assumes the normality of the underlying structure of the data for each group. The proposed procedure is as follow:

- 1. Estimate the D-score of each bank via the equation ??
- 2. Calculate the cut-off score.
- 3. Classify banks according to the optimal cut-off.

We recall that the optimal cut-off score is approximately zero. This is the weighted average of scores for bankrupt banks and active banks. Thus, the decision rule applied in the separation of sample studied is: a bank with a D-score less than zero will be considered as a bankrupt bank. However a bank with a D-score greater than zero will be classified in the group of healthy banks.

Eigenvalue's for 2008, 2009 and 2013 are respectively 0.2389, 0.4626 and 0.4775 (see Table 9). This result shows that the discriminant function does not allow easy identification of status between banks. Statistics of Wilk's lambda which correspond to the total variance in the discriminant scores not explained by differences among the groups confirm this result. Wilk's values for 2008, 2009 and 2013 are respectively 80.72%, 68.37% and 67.68% of the variance are not explained by group differences.

For 2012, a high eigenvalue (0.8013) shows that the discriminant function differentiates the two groups of banks. The model explains only 44.49% (Square canonical R) of the variance between the two classes (FB, NFB). Wilk's lambda greater in average than 70% shows that the most of total variability is attributable to differences between the means of D-scores of the groups. The square canonical correlation coefficient testifies the weak association between the discriminant scores and the set of independent variables (among 35%).

Finally, the linear combination of the factors scores provide for each bank a D-score, according to the estimated canonical discriminant model below :

$$D_{score2008} = 0.9550F_1 + 0.3071F_2 + 0.2059F_3$$
(3)

$$D_{score2009} = 0.8921F_1 - 0.2337F_2 - 0.6026F_3$$

$$D_{score2010} = 0.9217F_1 + 0.2594F_2 - 0.5791F_3$$

$$D_{score2011} = 0.9536F_1 + 0.2449F_2 - 0.5198F_3$$

$$D_{score2012} = 0.8876F_1 + 0.5917F_2 + 0.5595F_4$$

$$D_{score2013} = -0.8190F_1 + 0.3770F_2 + 0.4593F_3 + 0.5229F_4$$

Correlations between predictor variables and standardized canonical discriminant function are given in the table 10.

Over all there are no surprises in score's factors. In 2008 for example, F1, F2 and F3 are positively related to the D-score's bank. Clearly, good asset quality and a high level

of equity, improve profitability. In addition, a sufficient level of liquidity help bank to be able to perform its score and its ranking so it promotes in the non-failed bank group. In 2009, according to the structure of the matrix correlation (see Table 10) we retain only 3 Factors. F2 and F3 are negatively related to the score. This means that the rise of these later will reduce the bank's score. Indeed, a low level of liquidity coupled with a low level of funds reduces the bank's score.

4.2 Logit Regression

In this section, we propose the validation of the logit model which is considered as one of the most commonly applied parametric failure prediction models in both the academic literature as well as in the banking regulation and supervision. Logit model is based on a binomial regression and is based on the estimation of the probability of failure P(Z). This probability is defined as a linear function of a vector of covariates Z_i and a vector of regression coefficients β_i :

$$Z_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n \tag{4}$$

In this study, the logistic regression model used, the dependent variable Y_i Which takes a value of 1 (Y=1) when a failure occurred in a predefined period following the date at which the financial statement data are determined. If not, Y_i , takes on a value of 0 (Y=0) when no failure occurred.

The relationship between the dependent variable and the predictor variables is expressed as follows:

$$\begin{cases} P(Y=1) = P(Z) = \frac{1}{(1+\exp^{(-Z)})} \\ P(Y=0) = 1 - P(Z) = \frac{1}{(1+\exp^{(Z)})} \end{cases}$$
(5)

In our analysis we consider the factors determined from the (PCA) as explanatory variables. After estimating the coefficients of the Logit model, we obtain the score of each bank Z_a :

$$Z_a = \beta_0 + \sum_{i=1}^n \beta_i F_i \tag{6}$$

Subsequently, we determine the probability of default of each bank:

The estimated probability of default allows the reallocation of each bank to a specific risk class. Subsequently, a threshold P^* is set to enable segregation between banks and the allocation of these to one of two classes. If the estimated default probability is greater than P^* then the bank will be considered bankrupted, conversely, if the estimated default probability is lower than P^* , it will be considered active. Most previous work considers a bank as faulty if its default probability is greater than or equal to 0.5.

$Z_{score2008}$	=	$-6.5238 - 0.245F_1 - 3.0669F_2$	(7)
$Z_{score2009}$	=	$-4.0591 - 0.6651F_1 + 2.1396F_3$	
$Z_{score2010}$	=	$-4.9435 - 0.8357F_1 + 2.7505F_3$	
$Z_{score2011}$	=	$-7.7326 - 1.1374F_1 + 3.6381F_3$	
$Z_{score2012}$	=	$-7.5883 - 0.7508F_1 - 2.1392F_2 - 1.4157F_3 - 0.6151F_4$	
$Z_{score2013}$	=	$-8.2816 + 0.4643F_1 - 3.1646F_2 - 0.3756F_3$	

For 2008, only F1 and F2 are significant with a value of R^2 equal to 50.72%. For 2009, the level of R^2 is relatively low (47%) and only F1 and F3 are significant. In general, we found the same results for 2010 and 2011. For 2012, the model is very satisfactory with R^2 value close to 75%. All factors are significant. For the last year, the quality of the regression is good. All of F1, F2 and F3 are significant at the 5% level.(Table 11, Table 12)

For 2008 and 2012, all the significant factors are negatively related to the score of the bank. This means that an improvement in the asset quality, a better profitability, a high level of equity and a sufficient level of liquidity will increase the score of the bank and reduce the probability of default.

For 2009, 2010 and 2011, the factor F3 was positively related to the score. In fact, a low level of equity increases the probability of default of the bank. For 2013, factor F1 is positively related to the score, meaning that the rise of this factor (bad asset quality) will penalize the bank with a high probability of default. To sum up, all variables (ratios) have the expected signs.

4.3 Evaluation of the models

To evaluate the prediction performance of DA and Logit models, we consider type I/II error rates. They can be measured by a confusion matrix shown in Table 13. This matrix summarizes the correct and incorrect classifications that models produced for our dataset. Rows and columns of the confusion matrix correspond to the true and predicted classes respectively. The error type I is the error of not rejecting a null hypothesis when the alternative hypothesis is the true state of nature. This latter concern the prediction error of the classifier which incorrectly classifies the bankrupted bank into non-bankrupted bank. Thus, error type II presents the rate of prediction errors of a classifier to incorrectly classify the non-bankruptcy bank into bankrupted bank. As consequence, a natural criterion for judging the performance of a classifier is the probability for making a misclassification error.

We consider an early warning model as good when it delivers a low probability of committing error Type I and avoid classifying a failed bank in to the group of nonbankrupted banks.

For the prediction accuracy of the (CDA) model we proceed by two approaches to select the best cut-off score.

In the first one, we calculate the cut-off score.

In literature and according to Canbas et al. (2005), the default cut-off value in two class classifiers is approximately equal to zero and computed by the equation below:

$$Cut - Off = \frac{(N_1 D_1 + N_0 D_0)}{(N_1 + N_0)}$$
(8)

Where

- N₁ :number of bankrupted bank
- D₁ :average score for bankrupted bank
- N₀ :number of non-bankrupted bank
- D₀ :average score for non-bankrupted bank

But, if the two classes are asymmetric and unequal in term of size, the optimal cutting score for a discriminant function is the weighted average of the group centroids Hair et al. (2010). The formula for calculating the critical score between two groups is:

$$Cut - Off = \frac{(N_A Z_B + N_B Z_A)}{(N_A + N_B)} \tag{9}$$

Where Z_A and Z_B are the centroids for group A and B and N_A and N_B are the number of banks in each group. This formula is adopted in our paper for the (CDA) analysis. The second methodology to select the optimal cutting score is based on the

Receiver operating characteristics (ROC curve) graphs.

After, we classify bank in failed or healthy group according to the comparison between D-score and the cut-off score:

- if D-score > cut-off, the bank is classified to the non bankrupt group
- if D-score < cut-off, the bank is classified to the bankrupted group

From the results in tables 14 and 17, we can observe that the average correct classification rate is about 90%. For example, in 2008, the number of miss-classified bankrupt bank is 10 (error type I is equal to 27.03%). 79 healthy banks were classified in the FB group (type II error 6.55%). Looking more closely at our database, we found that among these 79 banks predicted to failing banks, 75 banks will actually fail during the years 2009, 2010 and 2011.

In 2009, type II error is equal to 11.31%. By scrutinizing the state of these banks that are considered by the (CDA) model as bankrupted, we find that they go bankruptcy in the years following. The model is quite severe in its classification and penalizes some

banks even before they were to fail. The results of the discriminant analysis for the year 2012 were significant. Indeed, the model was able to correctly predict 97.62% of banks. 10 failed banks were classified in the group of NFB (30.30% type I error) and 11 healthy banks were allocated to the FB group (1.29% type II error). Among active banks classified bankrupt, actually seven banks will go bankrupt in the year 2013.

In 2013, the discriminant function deliver a good rate of 98.82% ranking. In other words, the model has failed to correctly classify banks 1.18%. Indeed, five failed banks were predicted as active banks (33.33% type I error) and 5 active banks were allocated to the FB group (type II error 0.6%).

To sum up, with the canonical discriminant analysis we obtained good prediction rates of about 92.85%, 87.36%, 89.99%, 93.52%, 97.62% and 98.82% for respectively 2008, 2009, 2010, 2011, 2012 and 2013 years. We also noted that the type II error rate is relatively high (6.55%, 11.31%, 9.2%, 5.54%, 1.29% and 0.6% for the years 2008, 2009, 2010, 2011, 2012 and 2013). This means that some non-failed banks were predicted as failed banks. Therefore, and after analyzing our database, we can conclude that discriminant analysis predicts the failure of banks in the years ahead.

Finally according to the best cut-off obtained via the ROC curve methods, we obtain results in term of accuracy (classification rate), Type I, type II error, sensitivity and specificity which are presented in table 17.

The optimal critical point corresponds to the value which minimizes both the error of the type I (bankrupted banks classified in the group of the non-failed banks) and the error of the type II (active banks classified like failing). It is also the value which makes it possible to maximize sensitivity (correctly classified failing banks) and specificity (active banks correctly assigned to the group of the non-failing banks).

The application of the research of the critical point in the (CDA) model gives the following results: in 2008, according to figures 1 and 2, the optimal threshold is reached at the value of -1.4306. The discriminating analysis succeeded in classifying 91.72% of the banks in the adequate groups. Indeed, 86.49% of the defaulted banks and 91.88% of the non-failed banks were correctly classified. Among, the 37 failing banks, 32 banks were assigned to the group of the banks in bankruptcy. On the other hand, 5 banks in bankruptcies were classified in the group of the non-failed banks (error type I : 13.51%). Moreover, 98 active banks were declared like failing banks (error type II was 8.12%).

In 2009, the model made it possible to classify 90.83% of the banks correctly. 77.86% from the failing banks and 92.4% of the non-failed banks were correctly classified. According to the confusion matrix, we notice that 29 failing banks were assigned to the group of the healthy banks (error type I was 22.14%) and 82 active banks were classified as failing banks (error type II of 7.60%).

For the year 2010, the discriminating analysis succeeded in classifying 95.37% of the banks correctly. Indeed, 84.43% of the failing banks and 96.76% of the non-failed banks were assigned to their adequate group. Among the 122 failing banks, 19 banks were classified as non-failed banks (error type I 15.57%). 3.24% of the healthy banks were classified as failing banks.

In 2011, the results show an improvement in the rate of good classification (96.87%).

Indeed, 91.78% of the failing banks and 97.29% of the non-failed banks were correctly classified. Conversely, 6 banks of the failed group were predicted as healthy banks (error type I was 8.22%).

In 2012, the best critical point took the value of -1.8593 and made it possible to classify 99.77% of the banks correctly. Indeed, the discriminating model made it possible to classify all the healthy banks in the group of the non-failed banks (specificity 100%). Yet, 2 banks among the 33 failing banks were classified in the group of the non-failed banks (error type I 6.06%).

The same results are observed for the year 2013. With a cut-off of -0.9216, the rate of correct classification is of 99.76% and all of the non-failed banks were correctly classified (specificity 100%).

We can conclude that the Canonical Discriminant Analysis classify correctly on average 95.7% of banks in the appropriate groups.

In 2008, the Discriminant Analysis was able to classify 91.72% of banks in the appropriate groups. The results in the following years show an increase in the accuracy of the model. We notice that the rate of the correct classification was improved passing from 91.72% in 2008 to 99.76% in 2013. Based on the confusion matrix, we note that the rate of misclassification of the active banks tend to decrease over the years (from 8.12% in 2008 to 2.71% in 2011). In addition to that, for the period spanning between 2012-2013 the model classify correctly all active banks (error type II : 0.00%). The obtained results indicate that the discriminant analysis does not correctly classified bankrupt banks. Indeed, we found that the error Type I rates are relatively high. For example in 2009, 22.14% failed banks were allocated to active banks group.

The model seems to be very effective for the years 2012 and 2013. The banks were able to clean up their balance sheet and to recover after the crisis (years 2008 and 2009.)

For the prediction accuracy of the Logit model, we Firstly compared the probability of default obtained from the scoring function with the theoretical threshold (probability of default =0.5). After, we used the ROC curve to find the best cut-off point which minimizes the overall error (sum of error type I and error type II).

According to the results in Tables 15 and 17, we observe that the Logistics regression allows to have a satisfactory overall result in terms of correct classification rate (97.83%, 92.56%, 94.35%, 96.24%, 97.85% and 98.47% for the years 2008, 2009, 2010, 2011, 2012 and 2013). The Logit model permits to classify correctly 99.75%, 97.59%, 97.60%, 97.62%, 99.06% and 99.52% of the non-failed banks over the period 2008-2013. In this sense, the model misclassify only 0.33%, 2.41%, 2.4%, 2.38%, 0.94% and 0.48% of the non-failed banks (error type II). In the other hand, we obtained a higher error rate type I (62.16%, 48.85%, 38.15%, 20.55%, 33.33% and 60%) which means that the model was not able to classify correctly the bankrupt banks.

The research of the critical point by the ROC curve in the Logit model gives the following results (Table 16):

In 2008, the best critical point corresponds to the value of 0.0402 (cf. figures 1 & 2) the model of the logistic regression makes it possible to obtain a rate of good ranking of 92.36%. Thus, 7.64% of the banks were not correctly classified. Indeed, 4 failing banks

were assigned to the classes of the non-failed banks (error type I was 10.81%). On the other hand, 91 healthy banks were predicted as failing (error type II was 7.54%).

The results of year 2009 show a rate of bad classification of about 16.12% which means that 83.88% of the banks were correctly classified. The confusion matrix also shows that 12 failing banks were assigned to the group of healthy banks (error type I 9.16%). However, 183 healthy banks were declared like failing banks (error type II 16.96%).

For the year 2010, the Logistic regression did not succeed in classifying 10.10% of the banks correctly. Indeed, 8 failing banks were assigned to the group of healthy banks (error type I = 6.56%) and 101 healthy banks were classified as failing banks (error type II 10.55%). The Logit model made it possible to correctly classify 93.44% from failing banks and 89.45% of non-failed banks.

For the year 2011, 100% of failing banks were correctly classified. According to the confusion matrix, we noticed that 70 active banks were assigned to the group of failing banks (error type II 7.92%).

The same results are observed in 2012 and 2013 in terms of sensitivity (100%) which means that the Logit model classify correctly all the failed banks (error type I 0%)

As a conclusion, one can say that the results of the logistic regression are overall satisfactory in terms of % of error type I. (0% per 2011, 2012 and 2013 - % of failed banks correctly predicted). We also validated that the error rate of the type I is larger with the discriminating analysis than with the Logit model over the whole period of analysis. Using the matrices of confusion, we could also check that the discriminating analysis does not manage to detect the banks which are really in bankruptcy. On the other hand, with the Logit model, the error of classification of the failing banks in the group of the active banks is not very high.

It is clear that for the years from 2011 to 2013, the Logit model classifies the whole of the failing banks correctly (sensitivity 100%). The discriminating analysis makes it possible to have weaker error rates of the type II (8.12%; 7.60%; 3.24%; 2.71%; 0.00% and 0.00% against 7.54%; 16.96%; 10.55%; 7.92%; 6.46% and 3.23% for the Logit model over the period 2008-2013). This shows the supremacy of the logistic regression in term of forecast of failure. For example, for the year 2009, among the 183 banks predicted like failing, 163 banks really will default in the following years.

Lastly, for better apprehending the impact of the choice of the cut-off on the classification and the forecast in the two models applied, we propose to compare results (average of values obtained on the totality of the period of 2008 to 2013 for the whole set of parameters of the obtained matrices of confusions (Table 18)).

For that purpose, we compared the results of the probability of default and the D-score derived from the score function with the following thresholds:

- The theoretical threshold $(P^*=0.5; \text{ critical cut-off for the DA})$
- The critical point obtained by the ROC curve

For the Logistic analysis, the results show the supremacy of the latter to generate better results in term of rate of good classification on average about 96.22% with a

theoretical cut-off of 0.5. However, by calculating the probability of default with the critical limit of the ROC curve, the Logistic regression makes it possible to reduce the error type I (4.42% (ROC) against 42.67% (theoretical cut-off)). Furthermore, by using the theoretical score, the Logit model permits to classify on average 98.51% of the non-failed banks. On the other hand, by using the critical score of the ROC curve, we obtained success rates of classification of the non-failed banks on average 91.22%. For type II error, which informs us about the predictive power of the model to detect defaulted banks, we demonstrate the supremacy of the Logit model using the optimal cut-off of the ROC curve (average error type II: 1,49% (theoretical cut-off) against 8,78% (ROC)).

The comparison of the overall results of the discriminant analysis show that the model using the optimal cut-off of the ROC curve abtain more accurate results in term of average correct classification (95.72% against 93.36% (theoretical cut-off)).

We also observed that the Discriminant Analysis with a cut-off of the ROC curve reduce on average the error type I (24.74% vs 13.14%) and the error type II (5.75% vs 3.61%).

For error type II, which informs us about the predictive power of the model to detect defaulted banks, and for error type I which provides information on the capacity of the model to recognize the failing banks, we demonstrate the supremacy of the Logit model using the optimal cut-off of the ROC curve.

5 Conclusion

This paper shows how accurately U.S bank failures can be predicted with Logit and Canonical Discriminant Analysis models by utilizing CAMEL's Variables.

First, Principal Component Analysis (PCA) was performed to compress the data dimension by keeping only the most important ratio combinations.

We compared different cut-off point formulas to provide and evaluate classification accordingly.

Our results confirm, first that the more accurate the theoretical critical probability of default value are, the more accurate the sensitivity of the model. In this sense comparative results over the entire period prove that correct classification was improved with the ROC curve cut-off value for both Logit and DA model. The first finding proves that sensitivity of classification is improved and in average Logit model outperforms DA (95.58% vs 86.86%).

The second finding concerns the supremacy of ROC curve validation concerning the quality of the model by minimizing the error of misclassification of bankrupt banks: only 4.42% in average and 0% in both 2012 and 2013.

Third, in term of correct classification (both for failed and non-failed banks) we prove that DA is better by using theoretical probability of default 0.5 (96.22% against 93.36%).

Finally, models were used to provide early warning signals. Moreover, The combination of the two models allows a better information about the future prospect of banks. Indeed, ROC curve validation emphasizes better prediction of bank failure because it delivers, in average, the highest error type II 8.78%. This means that the model classifies some solvable banks in bankrupt group. So, we can conclude that the Logit was able to predict the failure of banks. Thus, it gives good signal about banks, which would failed one or two year later.

Overall, the study reveals also that our choice resulting from combinations of ten financial ratios which represent Capital adequacy, Assets quality, Earnings ability and Liquidity are obvious determinants of predicting bankruptcy.

Our results can be used for several purposes. For instance, regulators and banks can predict problems in order to avoid financial distress which can lead to bankruptcy. This improves banking supervision to establish supervisory guidelines. In fact, our methodological framework helps to construct an Early Warning System that can be used by supervisory authorities to detect banks that present significant and serious risks.

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Year	Number of failed banks	Number of non-failed bank
2013	15	836
2012	33	851
2011	73	884
2010	122	957
2009	131	1079
2008	37	1207

Table 1: Data analysis

Variable	Years	NAB	AB	Variable	Years	NAB	AB
EQTA	2008	3.74%	10.33%	EQTL	2008	5.43%	16.57%
	2009	1.48%	9.72%		2009	2.27%	16.07%
	2010	1.32%	10.04%		2010	1.91%	17.11%
	2011	1.04%	10.76%		2011	1.60%	18.94%
	2012	0.51%	11.13%		2012	0.86%	20.32%
	2013	1.13%	11.34%		2013	1.87%	19.83%
Variable	Years	NAB	AB	Variable	Years	NAB	AB
NPLTA	2008	13.35%	2.64%	NPLGL	2008	17.75%	3.58%
	2009	11.82%	3.49%		2009	16.27%	4.92%
	2010	12.95%	3.12%		2010	17.83%	4.57%
	2011	11.97%	2.54%		2011	17.25%	3.87%
	2012	10.23%	1.94%		2012	16.38%	3.04%
	2013	9.39%	1.47%		2013	14.01%	2.32%
Variable	Years	NAB	AB	Variable	Years	NAB	AB
LLRTA	2008	3.27%	1.23%	LLRGL	2008	4.39%	1.71%
	2009	3.43%	1.47%		2009	4.73%	2.11%
	2010	3.46%	1.46%		2010	4.75%	2.17%
	2011	3.59%	1.32%		2011	5.17%	2.02%
	2012	3.03%	1.17%		2012	4.75%	1.85%
	2013	2.96%	1.06%		2013	4.39%	1.68%
Variable	Years	NAB	AB	Variable	Years	NAB	AB
ROA	2008	-5.53%	-0.50%	ROE	2008	-155.24%	-9.42%
	2009	-6.27%	-0.90%		2009	1266.86%	15.53%
	2010	-3.51%	-0.30%		2010	-306.91%	-21.03%
	2011	-2.84%	0.21%		2011	6.11%	-5.23%
	2012	-2.90%	0.60%		2012	-1016.60%	4.68%
	2013	-1.91%	0.70%		2013	-1076.54%	7.01%
Variable	Years	NAB	AB	Variable	Years	NAB	AB
TLTD	2008	83.08%	91.13%	TDTA	2008	0.8673	78.47%
	2009	75.80%	82.63%		2009	91.17%	80.93%
	2010	75.51%	80.13%		2010	91.84%	81.67%
	2011	70.94%	77.79%		2011	92.73%	81.43%
	2012	64.77%	76.51%		2012	94.71%	81.59%
	2013	66.43%	77.73%		2013	94.65%	81.43%

Table 2: Means of the ten variables

Variables	2008	2009	2010	2011	2012	2013
EQTA	-0,2210*	-0,5056*	-0,5486*	-0,5260*	-0,4347*	-0,2795*
\mathbf{EQTL}	-0,1058*	-0,2764*	-0,3043*	$-0,2774^{*}$	-0,1712*	-0,1311*
NPLTA	$0,4146^{*}$	$0,\!4729^*$	$0,5974^{*}$	0,5944*	$0,5487^{*}$	$0,\!4556^*$
NPLGL	$0,4122^{*}$	$0,\!4713^*$	$0,5861^{*}$	0,5856*	$0,5740^{*}$	$0,4469^{*}$
LLRTA	$0,3227^{*}$	$0,4278^{*}$	$0,4628^{*}$	0,5214*	$0,4456^{*}$	$0,3777^{*}$
LLRGL	$0,3116^{*}$	$0,4202^{*}$	$0,4454^{*}$	0,5162*	$0,4472^{*}$	$0,3347^{*}$
ROA	-0,3339*	$-0,4925^{*}$	-0,4020*	$-0,4382^{*}$	-0,4872*	$-0,3162^{*}$
ROE	-0,2811*	0,0420	-0,1586*	0,0087	-0,2885*	-0,2697
TLTD	-0,0586**	-0,1068*	-0,0587***	-0,0822**	-0,1042*	-0,0672**
TDTA	0,1129*	$0,2683^{*}$	$0,2686^{*}$	$0,2483^{*}$	$0,2094^{*}$	$0,1457^{*}$

Table 3: Correlation coefficients between the explanatory variables and the dependent variable

*Significant at 1%, **Significant at 5%, ***Significant at 10%

Table 4: Correlation Matrix

_	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
	EQTA	1	0,6373	-0,2858	-0,2835	-0,2413	-0,1933	0,1594	0,1659	0,0397	-0,2745
	EQTL	0,6373	1	-0,1906	-0,1427	-0,2028	-0,0521	0,0898	0,0837	-0,3372	-0,1846
	NPLTA	-0,2858	-0,1906	1	0,9784	0,6925	0,6330	-0,5540	-0,3109	-0,0305	0,1964
2008	NPLGL	-0,2835	-0,1427	0,9784	1	0,6644	0,6526	-0,5578	-0,3255	-0,0983	0,1748
2008	LLRTA	-0,2413	-0,2028	0,6925	0,6644	1	0,9161	-0,6006	-0,3037	0,0713	0,1419
	DOA	-0,1935	-0,0521	0,0330	0,0520	0,9101	0 5567	-0,5567	-0,3000	-0,0981	0,1047
	ROF	0,1594	0,0898	-0,3340	-0,0078	-0,0000	-0,5507	0 4447	0,4447	0,0049	-0,1431
	TITD	0,1059	0,0837	-0,3109	-0,3233	-0,3037	-0,3000	0,4447	0.0720	0,0720	-0,1015
	TDTA	0,0397	0.1846	0.1964	0.1748	0.1410	0.1047	0,0049	0,0720	0 4065	-0,4005
	ibin	-0,2140	-0,1040	0,1004	0,1140	0,1410	0,1041	-0,1401	-0,1010	-0,4000	1
	Variables	ЕОТА	EOTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	ТДТА
-	EOTA	1	0.6472	-0.5584	-0.5505	-0.5019	-0.4653	0.5951	0.0497	0 1005	-0.4105
	EOTL	0.6472	1	-0.3528	-0.3311	-0.3462	-0.2631	0.3496	0.0274	-0.2926	-0.2744
	NPLTA	-0,5584	-0,3528	1	0,9795	0,6452	0,5984	-0,6503	-0,1281	-0,0430	0,2840
0000	NPLGL	-0,5505	-0,3311	0,9795	1	0,6096	0,5986	-0,6525	-0,0908	-0,1180	0,2663
2009	LLRTA	-0,5019	-0,3462	0,6452	0,6096	1	0,9643	-0,7071	-0,0790	0,0015	0,2305
	LLRGL	-0,4653	-0,2631	0,5984	0,5986	0,9643	1	-0,7099	-0,0541	-0,1353	0,1945
	ROA)	0,5951	0,3496	-0,6503	-0,6525	-0,7071	-0,7099	1	0,0511	0,1502	-0,3255
	ROE	0,0497	0,0274	-0,1281	-0,0908	-0,0790	-0,0541	0,0511	1	-0,0143	-0,0366
	TLTD	0,1005	-0,2926	-0,0430	-0,1180	0,0015	-0,1353	0,1502	-0,0143	1	-0,2911
	TDTA	-0,4105	-0,2744	0,2840	0,2663	0,2305	0,1945	-0,3255	-0,0366	-0,2911	1
	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
-	EQTA	1	0,6178	-0,5878	-0,5850	-0,5066	-0,4792	0,5649	0,1448	0,0956	-0,4045
	EQTL	0,6178	1	-0,3720	-0,3512	-0,3673	-0.2883	0,3288	0,0809	-0.2476	-0,2489
	NPLTA	-0,5878	-0,3720	1	0,9801	0,6528	0,6161	-0,6309	-0,1195	-0,0016	0,2629
0010	NPLGL	-0,5850	-0,3512	0,9801	1	0,6190	0,6200	-0,6341	-0,1278	-0,0694	0,2521
2010	LLRTA	-0,5066	-0,3673	0,6528	0,6190	1	0,9618	-0,6362	-0,1029	0,1740	0,1809
	LLRGL	-0,4792	-0,2883	0,6161	0,6200	0,9618	1	-0,6384	-0,1062	0,0203	0,1663
	ROA	0,5649	0,3288	-0,6309	-0,6341	-0,6362	-0,6384	1	0,1964	0,1234	-0,2862
	ROE	0,1448	0,0809	-0,1195	-0,1278	-0,1029	-0,1062	0,1964	1	0,0121	-0,0343
	TLTD	0,0956	-0,2476	-0,0016	-0,0694	0,1740	0,0203	0,1234	0,0121	1	-0,3190
	TDTA	-0,4045	-0,2489	0,2629	0,2521	0,1809	0,1663	-0,2862	-0,0343	-0,3190	1
-	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
	EQTA	1	0,6263	-0,5236	-0,5037	-0,4773	-0,4396	0,5324	0,0246	0,1257	-0,3987
	EQTL	0,6263	1	-0,3293	-0,2830	-0,3629	-0,2773	0,2924	0,0149	-0,3046	-0,2261
	NPLIA	-0,5236	-0,3293	1	0,9774	0,6936	0,6604	-0,6104	-0,0334	0,0101	0,1618
2011	LIDTA	-0,5057	-0,2830	0,9774	0.6559	0,0558	0,0000	-0,6197	-0,0509	-0,0735	0,1347
2011	LIRCL	-0,4773	0.2773	0,0930	0,0558	0.0545	1	-0,5868	0,0184	0,1071	0,1275
	BOA	0 5324	0.2924	-0.6104	-0.6197	-0 5868	-0 5956	-0,5550	0.0795	0.1514	-0.2320
	ROE	0.0246	0.0149	-0.0334	-0.0569	0.0184	-0.0220	0.0795	1	0.0411	-0.0002
	TLTD	0.1257	-0.3046	0.0101	-0,0735	0.1671	-0.0256	0.1514	0.0411	1	-0,3200
	TDTA	-0,3987	-0,2261	0,1618	0,1547	0,1273	0,1226	-0,2320	-0,0002	-0,3200	1
	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
-	EQTA	1	0,4540	-0,3820	-0,3588	-0,3220	-0,2156	0,4467	0,1307	0,0802	-0,3505
	EQTL	0,4540	1	-0,2166	-0.1608	-0,2718	-0,1071	0.1589	0,0512	-0,3626	-0.1386
	NPLTA	-0,3820	-0,2166	1	0,9713	0,5909	0,5393	-0,5006	-0,2203	0,0394	0,0867
0010	NPLGL	-0,3588	-0,1608	0,9713	1	0,5562	0,5792	-0,5077	-0,1991	-0,0835	0,0962
2012	LLRTA	-0,3220	-0,2718	0,5909	0,5562	1	0,9025	-0,3145	-0,0848	0,2033	0,0635
	LLRGL	-0,2156	-0,1071	0,5393	0,5792	0,9025	1	-0,3294	-0,0701	-0,1029	0,0609
	ROA	0,4467	0,1589	-0,5006	-0,5077	-0,3145	-0,3294	1	0,2335	0,2156	-0,2600
	ROE	0,1307	0,0512	-0,2203	-0,1991	-0,0848	-0,0701	0,2335	1	0,0156	-0,0790
	TLTD	0,0802	-0,3626	0,0394	-0,0835	0,2033	-0,1029	0,2156	0,0156	1	-0,3036
	TDTA	-0,3505	-0,1386	0,0867	0,0962	0,0635	0,0609	-0,2600	-0,0790	-0,3036	1
	** • • •	DOT	DOT	NETC	NET CT		1156-	B C :	B 0 =		mr = :
-	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
	EQTA	1	0,5373	-0,1598	-0,1294	-0,1388	0,0319	0,2990	0,0792	0,0785	-0,3017
	EQTL	0,5373	1	-0,1307	-0,0227	-0,2133	0,0547	0,1075	0,0364	-0,3447	-0,1530
	NDICI	-0,1090	-0,1307	0.0697	1,3007	0,5225	0,4300	-0,2011	0.0552	0.0550	0.0160
2013	LLBTA	-0,1294	-0.2133	0.5225	0 4837	1	0,4697	-0.0847	-0.1284	0.2403	0,0109
-010	LLRGI	0.0310	0.0547	0.4388	0.4897	0.8613	1	-0.1258	-0.1209	-0.1129	0.0194
	ROA	0,2990	0,1075	-0.2611	-0.2659	-0.0847	-0.1258	1	0,1254	0.2132	-0,1488
	ROE	0,0792	0,0364	-0,0514	-0,05935	-0,1284	-0,1292	0,1254	1	0,0264	-0,0422
	TLTD	0,0785	-0,3447	0,0690	-0,0552	0,2403	-0,1129	0,2132	0,0264	1	-0,3224
	TDTA	-0,3017	-0,1530	0,0109	0,0169	0,0090	0,0194	-0,1488	-0,0422	-0,3224	1

Table 5: Results of Bartlett's test of sphericity and KMO

2008		2009		2010	
Bartlett's test		Bartlett's test		Bartlett's test	
CHISQ	10377, 26	CHISQ	12095,3	CHISQ	10597,54
d.f.	45	d.f.	45	d.f.	45
p-value	0	p-value	0	p-value	0
KMO	$0,\!6299962$	KMO	0,6490987	KMO	$0,\!6334672$
		•			

2011		2012		2013	
Bartlett's test		Bartlett's test		Bartlett's test	
CHISQ	$9277,\!847$	CHISQ	$7054,\!398$	CHISQ	6082,105
d.f.	45	d.f.	45	d.f.	45
p-value	0	p-value	0	p-value	0
KMO	0,5907141	KMO	0,5393469	KMO	$0,\!4434393$

2008				2009			
Factors	Eigenvalues	Variance%	Cumulative%	Factors	Eigenvalues	Variance%	Cumulative%
$\mathbf{F1}$	4,1545	41,5445	41,54	F1	4,6813	46,8132	46,8132
$\mathbf{F2}$	1,6153	16,1530	57,70	$\mathbf{F2}$	1,3230	13,2304	60,0436
F3	1,4230	14,2295	71,93	F3	1,1560	11,5604	71,6041
$\mathbf{F4}$	0,8856	8,8557	80,78	$\mathbf{F4}$	1,0152	10,1520	81,7560
$\mathbf{F5}$	0,6227	6,2267	87,01	$\mathbf{F5}$	0,7237	7,2366	88,9926
$\mathbf{F6}$	0,5534	5,5344	92,54	$\mathbf{F6}$	0,5204	5,2039	94,1965
$\mathbf{F7}$	0,4367	4,3670	96,91	$\mathbf{F7}$	0,3178	3,1784	97,3749
$\mathbf{F8}$	0,2354	2,3542	99,27	$\mathbf{F8}$	0,2248	2,2481	99,6230
F9	0,0607	0,6074	99,87	F9	0,0303	0,3026	99,9256
F10	0,0127	0,1273	100	F10	0,0074	0,0744	100
2010 Factors	Eigenvalues	Variance%	Cumulative%	2011 Factors	Eigenvalues	Variance%	Cumulative%
F1	4,6600	46,5998	46,5998	F1	4,4960	44,9600	44,9600
$\mathbf{F2}$	1,3906	13,9055	60,5053	$\mathbf{F2}$	1,4375	14,3750	59,3350
F3	1,0895	10,8955	71,4008	$\mathbf{F3}$	1,2317	12,3170	$71,\!6520$
$\mathbf{F4}$	0,9841	9,8413	81,2421	$\mathbf{F4}$	1,0028	10,0285	$81,\!6805$
$\mathbf{F5}$	$0,\!6920$	6,9195	88,1616	$\mathbf{F5}$	0,6253	6,2527	87,9332
$\mathbf{F6}$	0,5175	5,1747	93,3363	$\mathbf{F6}$	0,5291	5,2910	93,2242
$\mathbf{F7}$	0,3652	$3,\!6521$	96,9884	$\mathbf{F7}$	0,4074	4,0738	$97,\!2980$
$\mathbf{F8}$	0,2623	2,6230	99,6115	$\mathbf{F8}$	0,2322	2,3218	$99,\!6199$
$\mathbf{F9}$	0,0321	0,3207	99,9322	$\mathbf{F9}$	0,0303	0,3027	99,9226
$\mathbf{F10}$	0,0068	0,0678	100	$\mathbf{F10}$	0,0077	0,0774	100
2012			~ ~ ~ ~ ~	2013		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	~ ~ ~ ~ ~
Factors	Eigenvalues	Variance%	Cumulative%	Factors	Eigenvalues	Variance%	Cumulative%
F1	3,7893	37,8931	37,8931	F1	3,0532	30,5320	30,5320
$\mathbf{F2}$	1,5727	15,7268	$53,\!6200$	$\mathbf{F2}$	1,7585	17,5846	48,1165
F3	1,3784	13,7844	67,4044	F3	1,5399	15,3995	63,5160
$\mathbf{F4}$	1,0208	10,2080	$77,\!6124$	$\mathbf{F4}$	1,0564	10,5643	74,0803
$\mathbf{F5}$	0,7640	$7,\!6401$	85,2525	$\mathbf{F5}$	0,9499	9,4987	83,5790
$\mathbf{F6}$	0,5656	5,6558	90,9083	F6	0,7128	7,1283	90,7073
$\mathbf{F7}$	0,4723	4,7225	$95,\!6308$	$\mathbf{F7}$	0,5387	5,3870	96,0943
F8	0,3821	3,8213	99,4521	F8	0,3223	3,2235	99,3178
F9	0,0433	0,4327	99,8848	F9	0,0548	0,5483	99,8661
$\mathbf{F10}$	0,0115	0,1152	100	F10	0,0134	0,1339	100

Table 6: Eigenvalues of the factors

variables	L I	F Z	ГJ	variable	s	F I	F 2	гз	F 4	
EQTA		0,83633		EQTA				-0,73457	7	
EQTL		0,92147		EQTL				-0.83003	3	
NPLTA	-0.86968	· ′		NPLTA	-0	.9166		, í		
NPLGL	-0.87377			NPLGI	-0.	90976				
LLRTA	-0.88724			LLBTA	-0.	81544				
LLRCL	-0.87416			LLRCI		819/1				
BOA	0.75857			BOA	· -0,	70376				
POF	0,10001			POF	0,	19510			0.0840	0
			0.87206	TITD			0.01291		-0,9840.	
			0,87590				-0,91381	0.04020		
IDIA			-0,77505	IDIA			0,56919	0,64032		
	1	I	1							
2010				2011					-	
Variables	F1	F2	F3	Variabl	es	F1	F2	F3	F4	
EQTA	0,50475		-0,69426	EQTA	L			-0,7426	51	
\mathbf{EQTL}			-0,88276	EQTI				-0,8597	77	
NPLTA	-0,83952			NPLTA	A -(),88879				
NPLGL	-0,83936			NPLG	L -(),89173				
LLRTA	-0,88932				A -(),87795				
LLRGL	-0,90309			LLRG	L -(),89148				
ROA	0.76912			ROA	0	,72363				
ROE	,			ROE		,			0.9930	2
TLTD		0.90043		TLTD)		0.9146		- ,	
TDTA		-0.63782	0.51363	TDTA			-0.6108'	7 0.5814	4	
12111		0,00102	0,01000		-		0,0100	. 0,0011		
	I	I	1	I	I		I	I	I	
2012	1	I	1		20	19	1		1	I
Variables	F1	F2	F3	F4	Varia	bles	F 1	F2	F3	F4
FOTA	F1	0 72074	10	1.4	FO		r I	0 76275	10	1.4
FOTI		0,12914	0.00004		EQ FO			0,70275		
	0.91110		0,00004				0.04051	0,90457		
NPLIA	-0,01119				NPI		0,94051			
NPLGL	-0,8227				NPI		0,94447			0.00000
LLRTA	-0,88558						0,63648			-0,62689
LLRGL	-0,90438				LLR	GL	0,61686			-0,65394
ROA		0,50323			RC)A			0,55196	
ROE				0,8728	RC	DE				0,69764
TLTD			-0,81233		TL	ГD			0,80717	
TDTA		-0,80568			TD	TA			-0,65367	

Table 7: Factors Loading

2009

2008

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2008	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
	F 1	-0,0869	-0,1276	-0,4344	-0,4411	-0,4699	-0,4796	0,4039	0,2331	-0,0227	0,0961
	F2	0,6399	0,7241	-0,0240	0,0119	0,0136	0,1262	-0,0851	-0,0104	-0,2164	-0,2480
	F3	0,1490	-0,1450	0,0110	-0,0210	0,1361	0,0505	0,0035	0,0706	0,7317	-0,6433
2000	Variables	FOTA	FOTI	NDITA	NPLCL	LIRTA	LLRCL	BOA	ROF	TITD	
2000	F1	0.0526	0.2239	0.4204	0.4235	0.5799	0.6164	-0.3893	-0.0163	-0.0040	-0.3311
	F2	-0.0430	0.3255	-0.0292	0.0187	-0.1126	-0.0210	-0.0751	-0.0139	-0.8006	0.4998
	F3	-0.5576	-0.7362	-0.0173	-0.0399	-0.1957	-0.2846	-0.0061	-0.0402	0.1278	0.6121
	F4	0.0101	0.0272	0.1389	0.1435	-0,1050	-0.1077	0.0501	0,9680	0.0074	-0.0233
		- ,	- ,	-,	-,	-,	-,	- ,	- ,	- /	- ,
		1									
2010	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
	FI	-0,0363	-0,3190	-0,4363	-0,4489	-0,5479	-0,6048	0,4048	0,0705	0,0652	0,2344
	F2	0,1129	-0,2369	-0,0300	-0,0746	0,2042	0,1056	0,1272	0,0386	0,7782	-0,5135
	гз	-0,5282	-0,8556	-0,0326	-0,0658	-0,1490	-0,2640	0,0599	-0,0225	0,2636	0,4544
		I									
2011	Variables	EQTA	EQTL	NPLTA	NPLGL	LLRTA	LLRGL	ROA	ROE	TLTD	TDTA
2011	Variables F1	EQTA -0,0386	EQTL -0,2362	NPLTA -0,4921	NPLGL -0,5125	LLRTA -0,4902	LLRGL -0,5353	ROA 0,3481	ROE -0,0307	TLTD 0,0579	TDTA 0,2420
2011	Variables F1 F2	EQTA -0,0386 0,1132	EQTL -0,2362 -0,3093	NPLTA -0,4921 0,0069	NPLGL -0,5125 -0,0516	LLRTA -0,4902 0,1679	LLRGL -0,5353 0,0415	ROA 0,3481 0,1566	ROE -0,0307 -0,0311	TLTD 0,0579 0,7752	TDTA 0,2420 -0,4906
2011	Variables F1 F2 F3	EQTA -0,0386 0,1132 -0,5623	EQTL -0,2362 -0,3093 -0,7718	NPLTA -0,4921 0,0069 -0,0942	NPLGL -0,5125 -0,0516 -0,1448	LLRTA -0,4902 0,1679 -0,0720	LLRGL -0,5353 0,0415 -0,1745	ROA 0,3481 0,1566 -0,0108	ROE -0,0307 -0,0311 -0,0071	TLTD 0,0579 0,7752 0,1923	TDTA 0,2420 -0,4906 0,5092
2011	Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322	NPLTA -0,4921 0,0069 -0,0942 -0,0130	NPLGL -0,5125 -0,0516 -0,1448 -0,0369	LLRTA -0,4902 0,1679 -0,0720 0,0882	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718	ROE -0,0307 -0,0311 -0,0071 0,9895	TLTD 0,0579 0,7752 0,1923 0,0064	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322	NPLTA -0,4921 0,0069 -0,0942 -0,0130	NPLGL -0,5125 -0,0516 -0,1448 -0,0369	LLRTA -0,4902 0,1679 -0,0720 0,0882	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718	ROE -0,0307 -0,0311 -0,0071 0,9895	TLTD 0,0579 0,7752 0,1923 0,0064	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322	NPLTA -0,4921 0,0069 -0,0942 -0,0130	NPLGL -0,5125 -0,0516 -0,1448 -0,0369	LLRTA -0,4902 0,1679 -0,0720 0,0882	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718	ROE -0,0307 -0,0311 -0,0071 0,9895	TLTD 0,0579 0,7752 0,1923 0,0064	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322	NPLTA -0,4921 0,0069 -0,0942 -0,0130	NPLGL -0,5125 -0,0516 -0,1448 -0,0369	LLRTA -0,4902 0,1679 -0,0720 0,0882	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718	ROE -0,0307 -0,0311 -0,0071 0,9895	TLTD 0,0579 0,7752 0,1923 0,0064	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322	NPLTA -0,4921 0,0069 -0,0942 -0,0130	NPLGL -0,5125 -0,0516 -0,1448 -0,0369	LLRTA -0,4902 0,1679 -0,0720 0,0882	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718	ROE -0,0307 -0,0311 -0,0071 0,9895	TLTD 0,0579 0,7752 0,1923 0,0064	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F3 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA	NPLGL -0,5125 -0,0516 -0,148 -0,0369 NPLGL	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA	LLRGL -0,5353 0,0415 -0,1745 0,0537	ROA 0,3481 0,1566 -0,0108 0,0718 ROA	ROE -0,0307 -0,0311 -0,0071 0,9895 ROE	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA
2011	Variables F1 F2 F3 F4 Variables F1	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692	EQTL -0.2362 -0.3093 -0.7718 0.0322 EQTL -0.1301	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA -0,4080	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927	ROE -0,0307 -0,0311 -0,0071 0,9895 ROE -0,2015	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170	TDTA 0,2420 -0,4906 0,5092 0,0766
2011	Variables F1 F2 F3 F4 Variables F1 F2	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159	NPLTA -0,4921 0,0069 -0,0130 NPLTA -0,4080 0,0823	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851	ROE -0,0307 -0,0071 0,09895 0,9895 ROE -0,2015 -0,1409	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001
2011	Variables F1 F3 F3 F4 Variables F1 F2 F3	EQTA -0,0386 0,1132 -0,5623 0,0079 -0,0692 0,5575 0,2440	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684	NPLTA -0,4921 0,0069 -0,0130 NPLTA -0,4080 0,0823 -0,0024	NPLGL -0,5125 -0,0516 -0,0369 NPLGL -0,4339 -0,6662 0,0942	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056
2011	Variables F1 F3 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531	NPLTA -0,4921 0,0069 -0,0942 -0,0130 -0,0130 -0,0130 -0,024 -0,2383	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678
2011	Variables F1 F2 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA -0,4080 0,0823 -0,0024 -0,2383	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678
2011	Variables F1 F2 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 -0,0079 -0,0692 0,5575 0,2440 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531	NPLTA -0,4921 0,0069 -0,942 -0,0130 NPLTA -0,4080 -0,0823 -0,0024 -0,2383	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 -0,6439 0,0662 0,0942 -0,2075	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627	LLRGL -0.5353 0.0415 -0.1745 0.0537 LLRGL -0.6438 0.0884 0.1364 0.2978	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1678
2011	Variables F1 F3 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 -0,0692 0,5575 0,2440 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531	NPLTA -0,4921 0,0069 -0,942 -0,0130 NPLTA -0,4080 0,0823 -0,024 -0,2383	NPLGL -0,5125 -0,0516 -0,0369 NPLGL -0,4339 -0,6662 0,0942 -0,2075	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678
2011 2012	Variables F1 F3 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA -0,4080 0,0823 -0,0024 -0,2383	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,2015 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1678
2011 2012 2013	Variables F1 F2 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065 -0,0065	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531 EQTL	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA -0,4080 0,0823 -0,0024 -0,2383 NPLTA	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075 NPLGL	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627 LLRTA	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978 LLBGL	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,2015 -0,2015 -0,1409 0,0254 0,8788	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,1075 0,1678
2011 2012 2013	Variables F1 F2 F3 F4 Variables F1 F2 F3 F4 Variables F1	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065 EQTA 0,0159	EQTL -0.2362 -0.3093 -0.7718 0.0322 EQTL -0.1301 0.3159 0.6684 -0.0531 EQTL EQTL 0.0627	NPLTA -0,4921 0,0069 -0,0942 -0,0130 NPLTA -0,0480 -0,0823 -0,0024 -0,2383 NPLTA 0,6599	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075 NPLGL 0,6654	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627 LLRTA 0,2252	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978 LLRGL 0,2222	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558 ROA -0,1987	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,9895 -0,2015 -0,1409 0,0254 0,8788 ROE 0,3370	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708 TLTD 0,0179	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678
2011 2012 2013	Variables F1 F3 F3 F4 Variables F1 F2 F3 F4 Variables F1 F2	EQTA -0,0386 0,1132 -0,5623 0,0079 -0,0692 0,5575 0,2440 -0,0065 - EQTA 0,0159 0,5612	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531 EQTL 0,0627 0,7038	NPLTA -0,4921 0,0069 -0,942 -0,0130 NPLTA -0,4080 0,0823 -0,0243 -0,2383 NPLTA 0,6599 -0,0295	NPLGL -0,5125 -0,0516 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075 NPLGL 0,6654 0,0654	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627 LLRTA 0,2252 -0,0760	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978 LLRGL 0,2222 0,1973	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2558 ROA -0,1147 0,2558 ROA	ROE -0,0307 -0,0311 -0,0071 0,9895 BOE -0,2015 -0,1409 0,0254 0,8788 BOE 0,3370 0,0247	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,0444 -0,6739 -0,0708 TLTD 0,0179 -0,3548	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678 TDTA -0,1515 -0,1515
2011 2012 2013	Variables F1 F3 F3 F4 Variables F1 F3 F4 Variables F1 F2 F3 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065 EQTA -0,0065 EQTA	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531 EQTL 0,0627 0,7038 -0,1420	NPLTA -0,4921 0,0069 -0,942 -0,0130 NPLTA -0,4080 -0,0823 -0,02383 NPLTA 0,6599 -0,6295 -0,0295 -0,0001	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,942 -0,2075 NPLGL 0,6654 0,6654 0,6651 -0,0705	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627 LLRTA 0,2252 -0,0760 0,2065	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978 LLRGL 0,2222 0,1973 0,0173	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558 ROA -0,1987 0,1104 0,4148	ROE -0,0307 -0,0311 -0,0071 0,9895 ROE -0,2015 -0,1409 0,0254 0,8788 ROE 0,03370 0,0247 0,139	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708 TLTD 0,0179 -0,3548 0,6595	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1678 TDTA -0,1515 -0,1556 -0,5012
2011 2012 2013	Variables F1 F2 F3 F4 Variables F1 F2 F3 F4 Variables F1 F2 F3 F4	EQTA -0,0386 0,1132 -0,5623 0,0079 EQTA -0,0692 0,5575 0,2440 -0,0065 EQTA 0,0159 0,5612 0,2636 -0,0322	EQTL -0,2362 -0,3093 -0,7718 0,0322 EQTL -0,1301 0,3159 0,6684 -0,0531 EQTL 0,0627 0,7038 -0,1420 0,0140	NPLTA -0,4921 0,0069 -0,0130 -0,0130 NPLTA -0,4080 0,0823 -0,0024 -0,2383 NPLTA 0,6599 -0,0001 0,2538	NPLGL -0,5125 -0,0516 -0,1448 -0,0369 NPLGL -0,4339 0,0662 0,0942 -0,2075 NPLGL 0,6654 0,0651 -0,0705 0,2403	LLRTA -0,4902 0,1679 -0,0720 0,0882 LLRTA -0,5819 0,0878 -0,1177 0,2627 LLRTA 0,2252 -0,0760 0,2065 -0,4624	LLRGL -0,5353 0,0415 -0,1745 0,0537 LLRGL -0,6438 0,0884 0,1364 0,2978 LLRGL 0,2222 0,1973 0,0173 -0,5034	ROA 0,3481 0,1566 -0,0108 0,0718 ROA 0,0927 0,2851 -0,1147 0,2558 ROA -0,1987 0,1104 0,4148 -0,1230	ROE -0,0307 -0,0311 -0,0071 0,9895 -0,2015 -0,1409 0,0254 0,8788 ROE 0,3370 0,0247 0,1139 0,7183	TLTD 0,0579 0,7752 0,1923 0,0064 TLTD 0,0170 0,3444 -0,6739 -0,0708 TLTD 0,0179 -0,3548 0,6595 0,0195	TDTA 0,2420 -0,4906 0,5092 0,0766 TDTA 0,1703 -0,7001 0,1056 0,1678 TDTA -0,1515 -0,1515 -0,1515 -0,5012 -0,5012 -0,512

 Table 8: Factor scores coefficient matrix

Table 9: Statistics of the estimated CDA model

	2008	2009	2010	2011	2012	2013
Eigenvalue	0,2389	0,4626	0,6091	0,71	0,8013	$0,\!4775$
Proportion	1	1	1	1	1	1
Canonical R	$0,\!4391$	0,5624	$0,\!6152$	$0,\!6444$	$0,\!667$	0,5685
Wilks Lambda	0,8072	$0,\!6837$	$0,\!6215$	$0,\!5848$	$0,\!5551$	$0,\!6768$
CHI-2	265,7519	458,5101	$511,\!6064$	$511,\!2647$	$517,\!9045$	$330,\!6099$
d.f.	3	4	3	4	4	4
p-value	0	0	0	0	0	0
Sq Canonical corr.	0,1928	0,3163	$0,\!3785$	$0,\!4152$	$0,\!4449$	0,3232

		$\mathbf{F1}$	$\mathbf{F2}$	$\mathbf{F3}$	$\mathbf{F4}$
2008	Total	0,9425	0,2781	0,1856	
	Within	$0,\!9301$	$0,\!2517$	$0,\!1673$	
2009	Total	0,8211	-0,1974	-0,5231	-0,0521
	Within	0,7654	-0,1643	-0,4525	-0,0431
2010	Total	0,854	0,2061	-0,4777	
	Within	0,7913	$0,\!1638$	-0,3939	
2011	Total	0,8903	$0,\!1887$	-0,4123	-0,0416
	Within	$0,\!8312$	$0,\!1454$	-0,3271	-0,0318
2012	Total	0,7712	0,4636	0,0223	$0,\!4357$
	Within	$0,\!67$	0,3632	0,0166	$0,\!3393$
2013	Total	-0,7434	0,3153	0,3875	0,4447
	Within	-0,6749	0,2637	0,3268	0,3781

Table 10: Factor Structure Matrix - Correlations

2008	Attribute	Coef.	Std-dev	Wald	Signif
	constant	-6,5238*	0,5864	123,7532	0
	F1	$-0,6245^{*}$	0,0764	66,7775	0
	F2	-3,0669*	0,4689	42,7782	0
	F3	-0,2908	0,1599	$3,\!3061$	0,069
2009	Attribute	Coef.	Std-dev	Wald	Signif
	constant	-4,0591*	0,2657	233,4688	0
	F1	-0,6651*	0,0636	$109,\!4318$	0
	F2	-0,1361	0,1798	0,5729	0,4491
	F3	2,1396*	0,2257	$89,\!8791$	0
	F4	0,0076	0,2265	0,0011	0,9732
2010	Attribute	Coef.	Std-dev	Wald	Signif
	constant	-4,9435*	0,3825	167,0195	0
	F1	-0,8357*	0,0773	117,0012	0
	F2	-0,0998	0,0815	$1,\!4984$	0,2209
	F3	2,7505*	$0,\!2744$	100,4928	0
2011	Attribute	Coef.	Std-dev	Wald	Signif
	Constant	-7,7326*	0,9651	64,1892	0
	F1	-1,1374*	0,1498	$57,\!622$	0
	F2	0,3166	0,1637	3,7415	0,0531
	F3	$3,\!6381^*$	0,5187	$49,\!1882$	0
	F4	0,0173	0,1452	0,0143	0,9049
2012	Attribute	Coef.	Std-dev	Wald	Signif
	constant	-7,5883*	1,1153	$46,\!2949$	0
	F1	-0,7508*	0,1334	$31,\!6892$	0
	F2	-2,1392*	0,4461	$22,\!9925$	0
	F3	-1,4157**	0,6393	4,9046	0,0268
	F4	-0,6151*	0,2013	9,3375	0,0022
2013	Attribute	Coef.	Std-dev	Wald	Signif
	$\operatorname{constant}$	-8,2816*	1,3358	$38,\!4348$	0
	F1	$0,4643^{*}$	0,1192	$15,\!18$	0,0001
	F2	-3,1646*	0,7601	$17,\!3321$	0
	F3	-0,3756**	0,1696	4,9044	0,0268
	F4	-0,2193	0,1621	$1,\!831$	$0,\!176$

Table 11: Significance tests of factors

*Significant at 1%, **Significant at 5%

Model	2008	2009	2010	2011	2012	2013
Chi-2 test	168,9029	390,3093	441,7951	$359{,}6493$	208,2682	107,2245
d.f.	3	4	3	4	4	4
P(>Chi-2)	0	0	0	0	0	0
McFadden's R	0,5072	$0,\!4704$	0,5802	$0,\!697$	0,7392	0,7106

Table 12: Statistical tests of Logit models

Table 13: Confusion Matrix

		PREDICTED class			
		Bankruptcy Y=1	Non Bankruptcy Y=0		
		bankrupt banks	bankrupt banks		
	Bankruptcy Y=1	correctly classified	incorrectly classified		
	ap :		(error type I)		
Actual class		non-bankrupted banks	non-bankrupt banks		
netual class	Non Bankruptcy Y=0	classified as bankrupted ones	correctly classified		
	1 0	(error type II)			

2008		Bankrupt	Non-bankrupt	Sum
	Bankrupt	27	10	37
	Non-bankrupt	79	1128	1207
	\mathbf{Sum}	106	1138	1244
2009		Bankrupt	Non-bankrupt	Sum
	Bankrupt	100	31	131
	Non-bankrupt	122	957	1079
	\mathbf{Sum}	222	988	1210
2010		Bankrupt	Non-bankrupt	Sum
	Bankrupt	102	20	122
	Nonbankrupt	88	869	957
	\mathbf{Sum}	190	889	1079
2011		Bankrupt	Non-bankrupt	Sum
	Bankrupt	60	13	73
	Non-bankrupt	49	835	884
	\mathbf{Sum}	109	848	957
2012		Bankrupt	Non-bankrupt	Sum
	Bankrupt	23	10	33
	Non-bankrupt	11	840	851
	\mathbf{Sum}	34	850	884
2013		Bankrupt	Non-bankrupt	Sum
	Bankrupt	10	5	15
	Non-bankrupt	5	831	836
	\mathbf{Sum}	15	836	851

Table 14: Results of DA confusion matrix

Table 15: Results of Logit

Logit model	2008	2009	2010	2011	2012	2013
Failed banks correctly predicted	14	67	84	58	22	6
Non-failed banks correctly predicted	1203	1053	934	863	843	832
Error type I	23	64	38	15	11	9
Error type II	4	26	23	21	8	4
Incorrectly predicted total	27	90	61	36	19	13
Correctly predicted total	1217	1120	1018	921	865	838
% of failed banks correctly predicted	37,84%	$51,\!15\%$	$68,\!85\%$	$79,\!45\%$	$66,\!67\%$	$40,\!00\%$
% of non-failed banks correctly predicted	99,67%	$97,\!59\%$	$97,\!60\%$	$97,\!62\%$	99,06%	99,52%
% of total incorrectly predicted	2,17%	$7,\!44\%$	$5{,}65\%$	3,76%	$2,\!15\%$	1,53%
% of total correctly predicted	97,83%	92,56%	$94,\!35\%$	$96,\!24\%$	$97,\!85\%$	$98,\!47\%$

Logit model ROC curve	2008	2009	2010	2011	2012	2013
Failed banks correctly predicted	33	119	114	73	33	15
Non-failed banks correctly predicted	1116	896	856	814	796	809
Error type I	4	12	8	0	0	0
Error type II	91	183	101	70	55	27
Incorrectly predicted total	95	195	109	70	55	27
Correctly predicted total	1149	1015	970	887	829	824
% of failed banks correctly predicted	89,19%	$90,\!84\%$	$93,\!44\%$	$100,\!00\%$	$100,\!00\%$	$100,\!00\%$
% of error type I	10,81%	$9{,}16\%$	$6{,}56\%$	$0,\!00\%$	$0,\!00\%$	$0,\!00\%$
% of non-failed banks correctly predicted	92,46%	$83,\!04\%$	$89,\!45\%$	$92,\!08\%$	$93{,}54\%$	96,77%
% of error type II	7,54%	$16{,}96\%$	$10,\!55\%$	$7,\!92\%$	$6{,}46\%$	$3{,}23\%$
% of total incorrectly predicted	7,64%	$16,\!12\%$	$10,\!10\%$	$7{,}31\%$	$6{,}22\%$	$3,\!17\%$
% of total correctly predicted	92,36%	$83,\!88\%$	89,90%	$92,\!69\%$	$93,\!78\%$	$96,\!83\%$
Canonical Discriminant Analysis	2008	2009	2010	2011	2012	2013
Canonical Discriminant Analysis Failed banks correctly predicted	2008 32	2009 102	2010 103	2011 67	2012 31	2013 13
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted	2008 32 1109	2009 102 997	2010 103 926	2011 67 860	2012 31 851	2013 13 836
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I	2008 32 1109 5	2009 102 997 29	2010 103 926 19	2011 67 860 6	2012 31 851 2	2013 13 836 2
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II	2008 32 1109 5 98	2009 102 997 29 82	2010 103 926 19 31	2011 67 860 6 24	2012 31 851 2 0	2013 13 836 2 0
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total	2008 32 1109 5 98 103	2009 102 997 29 82 111	2010 103 926 19 31 50	2011 67 860 6 24 30	2012 31 851 2 0 2	2013 13 836 2 0 2
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total	$\begin{array}{r} \textbf{2008} \\ 32 \\ 1109 \\ 5 \\ 98 \\ 103 \\ 1141 \end{array}$	2009 102 997 29 82 111 1099	2010 103 926 19 31 50 1029	2011 67 860 6 24 30 927	2012 31 851 2 0 2 882	2013 13 836 2 0 2 849
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted	$\begin{array}{r} \textbf{2008} \\ \hline 32 \\ 1109 \\ 5 \\ 98 \\ 103 \\ 1141 \\ 86,49\% \end{array}$	2009 102 997 29 82 111 1099 77,86%	2010 103 926 19 31 50 1029 84,43%	2011 67 860 6 24 30 927 91,78%	2012 31 851 2 0 2 882 93,94%	2013 13 836 2 0 2 849 86,67%
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted % of error type I	$\begin{array}{r} \textbf{2008} \\ \hline \textbf{32} \\ \textbf{1109} \\ \textbf{5} \\ \textbf{98} \\ \textbf{103} \\ \textbf{1141} \\ \textbf{86,49\%} \\ \textbf{13,51\%} \end{array}$	2009 102 997 29 82 111 1099 77,86% 22,14%	$\begin{array}{r} \textbf{2010} \\ 103 \\ 926 \\ 19 \\ 31 \\ 50 \\ 1029 \\ 84,43\% \\ 15,57\% \end{array}$	$\begin{array}{r} \textbf{2011} \\ \hline 67 \\ 860 \\ 6 \\ 24 \\ 30 \\ 927 \\ 91,78\% \\ 8,22\% \end{array}$	$\begin{array}{c} \textbf{2012} \\ 31 \\ 851 \\ 2 \\ 0 \\ 2 \\ 882 \\ 93,94\% \\ 6,06\% \end{array}$	$\begin{array}{c} \textbf{2013} \\ \hline 13 \\ 836 \\ 2 \\ 0 \\ 2 \\ 849 \\ 86,67\% \\ 13,33\% \end{array}$
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted % of error type I % of non-failed banks correctly predicted	$\begin{array}{r} \textbf{2008} \\ \hline \textbf{32} \\ \textbf{1109} \\ \textbf{5} \\ \textbf{98} \\ \textbf{103} \\ \textbf{1141} \\ \textbf{86,49\%} \\ \textbf{13,51\%} \\ \textbf{91,88\%} \end{array}$	$\begin{array}{r} \textbf{2009} \\ 102 \\ 997 \\ 29 \\ 82 \\ 111 \\ 1099 \\ 77,86\% \\ 22,14\% \\ 92,40\% \end{array}$	$\begin{array}{r} \textbf{2010} \\ 103 \\ 926 \\ 19 \\ 31 \\ 50 \\ 1029 \\ 84,43\% \\ 15,57\% \\ 96,76\% \end{array}$	2011 67 860 6 24 30 927 91,78% 8,22% 97,29%	$\begin{array}{c} \textbf{2012} \\ 31 \\ 851 \\ 2 \\ 0 \\ 2 \\ 882 \\ 93,94\% \\ 6,06\% \\ 100,00\% \end{array}$	$\begin{array}{c} \textbf{2013} \\ 13 \\ 836 \\ 2 \\ 0 \\ 2 \\ 849 \\ 86,67\% \\ 13,33\% \\ 100,00\% \end{array}$
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted % of error type I % of non-failed banks correctly predicted % of error type II	$\begin{array}{r} \textbf{2008} \\ \hline \textbf{32} \\ \textbf{1109} \\ \textbf{5} \\ \textbf{98} \\ \textbf{103} \\ \textbf{1141} \\ \textbf{86,49\%} \\ \textbf{13,51\%} \\ \textbf{91,88\%} \\ \textbf{8,12\%} \end{array}$	$\begin{array}{r} \textbf{2009} \\ 102 \\ 997 \\ 29 \\ 82 \\ 111 \\ 1099 \\ 77,86\% \\ 22,14\% \\ 92,40\% \\ 7,60\% \end{array}$	$\begin{array}{r} \textbf{2010} \\ \hline 103 \\ 926 \\ 19 \\ 31 \\ 50 \\ 1029 \\ 84,43\% \\ 15,57\% \\ 96,76\% \\ 3,24\% \end{array}$	2011 67 860 6 24 30 927 91,78% 8,22% 97,29% 2,71%	$\begin{array}{c} \textbf{2012} \\ 31 \\ 851 \\ 2 \\ 0 \\ 2 \\ 882 \\ 93,94\% \\ 6,06\% \\ 100,00\% \\ 0,00\% \end{array}$	$\begin{array}{c} \textbf{2013} \\ 13 \\ 836 \\ 2 \\ 0 \\ 2 \\ 849 \\ 86,67\% \\ 13,33\% \\ 100,00\% \\ 0,00\% \end{array}$
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted % of error type I % of non-failed banks correctly predicted % of error type II % of total incorrectly predicted	$\begin{array}{r} \textbf{2008} \\ \hline \textbf{32} \\ \textbf{1109} \\ \textbf{5} \\ \textbf{98} \\ \textbf{103} \\ \textbf{1141} \\ \textbf{86,49\%} \\ \textbf{13,51\%} \\ \textbf{91,88\%} \\ \textbf{8,12\%} \\ \textbf{8,28\%} \end{array}$	$\begin{array}{r} \textbf{2009} \\ 102 \\ 997 \\ 29 \\ 82 \\ 111 \\ 1099 \\ 77,86\% \\ 22,14\% \\ 92,40\% \\ 7,60\% \\ 9,17\% \end{array}$	$\begin{array}{r} \textbf{2010} \\ 103 \\ 926 \\ 19 \\ 31 \\ 50 \\ 1029 \\ 84,43\% \\ 15,57\% \\ 96,76\% \\ 3,24\% \\ 4,63\% \end{array}$	$\begin{array}{r} \textbf{2011} \\ \hline 67 \\ 860 \\ 6 \\ 24 \\ 30 \\ 927 \\ 91,78\% \\ 8,22\% \\ 97,29\% \\ 2,71\% \\ 3,13\% \end{array}$	$\begin{array}{c} \textbf{2012} \\ 31 \\ 851 \\ 2 \\ 0 \\ 2 \\ 882 \\ 93,94\% \\ 6,06\% \\ 100,00\% \\ 0,00\% \\ 0,23\% \end{array}$	$\begin{array}{c} \textbf{2013} \\ \hline 13 \\ 836 \\ 2 \\ 0 \\ 2 \\ 849 \\ 86,67\% \\ 13,33\% \\ 100,00\% \\ 0,00\% \\ 0,00\% \\ 0,24\% \end{array}$
Canonical Discriminant Analysis Failed banks correctly predicted Non-failed banks correctly predicted Error type I Error type II Incorrectly predicted total Correctly predicted total % of failed banks correctly predicted % of error type I % of non-failed banks correctly predicted % of error type II % of total incorrectly predicted % of total correctly predicted	$\begin{array}{r} \textbf{2008} \\ \hline \textbf{32} \\ \textbf{1109} \\ \textbf{5} \\ \textbf{98} \\ \textbf{103} \\ \textbf{1141} \\ \textbf{86,49\%} \\ \textbf{13,51\%} \\ \textbf{91,88\%} \\ \textbf{8,12\%} \\ \textbf{8,28\%} \\ \textbf{91,72\%} \end{array}$	$\begin{array}{r} \textbf{2009} \\ 102 \\ 997 \\ 29 \\ 82 \\ 111 \\ 1099 \\ 77,86\% \\ 22,14\% \\ 92,40\% \\ 7,60\% \\ 9,17\% \\ 90,83\% \end{array}$	$\begin{array}{r} \textbf{2010} \\ 103 \\ 926 \\ 19 \\ 31 \\ 50 \\ 1029 \\ 84,43\% \\ 15,57\% \\ 96,76\% \\ 3,24\% \\ 4,63\% \\ 95,37\% \end{array}$	$\begin{array}{r} \textbf{2011} \\ \hline 67 \\ 860 \\ 6 \\ 24 \\ 30 \\ 927 \\ 91,78\% \\ 8,22\% \\ 97,29\% \\ 2,71\% \\ 3,13\% \\ 96,87\% \end{array}$	$\begin{array}{c} \textbf{2012} \\ 31 \\ 851 \\ 2 \\ 0 \\ 2 \\ 882 \\ 93,94\% \\ 6,06\% \\ 100,00\% \\ 0,00\% \\ 0,23\% \\ 99,77\% \end{array}$	$\begin{array}{c} \textbf{2013} \\ 13 \\ 836 \\ 2 \\ 0 \\ 2 \\ 849 \\ 86,67\% \\ 13,33\% \\ 100,00\% \\ 0,00\% \\ 0,24\% \\ 99,76\% \end{array}$

Table 16: classification results with ROC curve

Table 17: Comparaison

Logit model	2008		2009		2010	
	cut-off	ROC	cut-off	ROC	cut-off	ROC
	0,5	0,0402	0,5	0,0818	0,5	$0,\!1139$
Sensitivity	$37,\!84\%$	$89,\!19\%$	51,15%	$90,\!84\%$	68,85%	$93,\!44\%$
Error type I	$62,\!16\%$	$10{,}81\%$	48,85%	$9,\!16\%$	$31,\!15\%$	$6{,}56\%$
Specificity	$99,\!67\%$	$92{,}46\%$	97,59%	$83,\!04\%$	$97,\!60\%$	$89,\!45\%$
Error type II	$0,\!33\%$	$7{,}54\%$	2,41%	$16{,}96\%$	2,40%	$10,\!55\%$
Error rate	$2,\!17\%$	$7{,}64\%$	7,44%	$16,\!12\%$	$5,\!65\%$	$10,\!10\%$
Correct classification	$97,\!83\%$	$92,\!36\%$	92,56%	$83,\!88\%$	$94,\!35\%$	89,90%

Logit model	2011		2012		2013	
	cut-off	ROC	cut-off	ROC	cut-off	ROC
	$0,\!5$	0,0314	0,5	0,0077	$_{0,5}$	0,0272
Sensitivity	$79,\!45\%$	$100,\!00\%$	$66,\!67\%$	$100,\!00\%$	$40,\!00\%$	$100,\!00\%$
Error type I	$20,\!55\%$	$0,\!00\%$	$33{,}33\%$	$0,\!00\%$	$60,\!00\%$	$0,\!00\%$
Specificity	$97,\!62\%$	$92{,}08\%$	99,06%	$93{,}54\%$	99,52%	96,77%
Error type II	$2,\!38\%$	$7{,}92\%$	0,94%	$6,\!46\%$	$0,\!48\%$	$3{,}23\%$
Error rate	3,76%	$7{,}31\%$	$2,\!15\%$	$6{,}22\%$	$1,\!53\%$	$3,\!17\%$
Correct classification	$96,\!24\%$	$92{,}69\%$	$97,\!85\%$	$93,\!78\%$	$98,\!47\%$	$96,\!83\%$
Error type II Error rate Correct classification	2,38% 3,76% 96,24%	7,92% 7,31% 92,69%	0,94% 2,15% 97,85%	6,46% 6,22% 93,78%	0,48% 1,53% 98,47%	$3,23\% \ 3,17\% \ 96,83\%$

Canonical DA	2008		2009		2010	
	cut-off	ROC	cut-off	ROC	cut-off	ROC
	-2,7039	$-1,\!4306$	-1,7135	-0,3301	-1,9055	-0,779
Sensitivity	$72,\!97\%$	$86,\!49\%$	76,34%	$77,\!86\%$	$83,\!61\%$	$84,\!43\%$
Error type I	$27,\!03\%$	$13{,}51\%$	$23,\!66\%$	$22,\!14\%$	$16,\!39\%$	$15{,}57\%$
Specificity	$93,\!45\%$	$91,\!88\%$	88,69%	$92,\!40\%$	$90,\!80\%$	96,76%
Error type II	$6{,}55\%$	$8,\!12\%$	11,31%	$7{,}60\%$	9,20%	$3,\!24\%$
Error rate	$7,\!15\%$	$8,\!28\%$	$12,\!64\%$	$9{,}17\%$	10,01%	$4,\!63\%$
Correct classification	$92,\!85\%$	91,72%	87,36%	$90,\!83\%$	89,99%	$95{,}37\%$

2011		2012		2013	
cut-off	ROC	cut-off	ROC	cut-off	ROC
-2,6872	-1,8846	-4,3646	-1,8593	-5,0601	-0,9216
$82,\!19\%$	$91,\!78\%$	69,70%	$93,\!94\%$	$66,\!67\%$	$86{,}67\%$
$17,\!81\%$	$8,\!22\%$	$30,\!30\%$	$6{,}06\%$	$33{,}33\%$	$13{,}33\%$
$94,\!46\%$	$97,\!29\%$	98,71%	$100,\!00\%$	$99,\!40\%$	$100,\!00\%$
5,54%	$2,71\%_{5}$	$1,\!29\%$	$0,\!00\%$	$0,\!60\%$	$0,\!00\%$
$6,\!48\%$	$3,\!13\%$	$2,\!38\%$	$0{,}23\%$	$1,\!18\%$	$0,\!24\%$
$93{,}52\%$	$96,\!87\%$	$97,\!62\%$	99,77%	$98,\!82\%$	99,76%
	$\begin{array}{c} \textbf{20} \\ \hline \text{cut-off} \\ \hline -2,6872 \\ 82,19\% \\ 17,81\% \\ 94,46\% \\ 5,54\% \\ 6,48\% \\ 93,52\% \end{array}$	2011 cut-off ROC -2,6872 -1,8846 82,19% 91,78% 17,81% 8,22% 94,46% 97,29% 5,54% 2,71% 6,48% 3,13% 93,52% 96,87%	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

 Table 18: Average Results

	LOGIT			DA	
	cut-off	ROC		cut-off	ROC
Sensitivity	$57,\!33\%$	$95{,}58\%$	Sensitivity	$75,\!25\%$	$86,\!86\%$
Error type I	$42,\!67\%$	$4,\!42\%$	Error type I	24,75%	$13,\!14\%$
Specificity	98,51%	$91,\!22\%$	Specificity	$94,\!25\%$	$96,\!39\%$
Error type II	$1,\!49\%$	8,78%	Error type II	5,75%	$3{,}61\%$
Error rate	$3{,}78\%$	$8,\!43\%$	Error rate	$6{,}64\%$	$4,\!28\%$
Correct classification	$96,\!22\%$	$91,\!57\%$	Correct classification	$93,\!36\%$	95,72%



Figure 1: ROC curve CDA vs Logit regression

curve1.png



curve2.png



Figure 2: Best cut off point that maximizes the sensitivity and the specificity

cut-off1.png



cut-off2.png



Figure 3: Error type I and II : Logit vs CDA

