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Abstract

The financial industry has extensively used quantile-based risk measures relying on the Value-at-Risk (VaR). They need to be estimated from relevant historical data set. Consequently, they contain uncertainty. We propose an alternative quantile-based risk measure (the Spectral Stress VaR) to capture the uncertainty in the historical VaR approach. This one provides flexibility to the risk manager to implement prudential regulatory framework. It can be a VaR based stressed risk measure. In the end we propose a stress testing application for it.

Keywords: Historical method, Uncertainty, Value-at-Risk, Stress risk measure, Tail risk measure, Prudential financial regulation, Stress testing
JEL: G28, G32, C14

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1. Introduction

The financial industry has extensively used quantile-based risk measures based on the Value-at-Risk (VaR). In statistical terms, the VaR is a quantile reserve,

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often using the p^{th} ($p \in [0,1]$) percentile of the loss distribution. Typically the VaR is not known with certainty and needs to be estimated from sample estimators of relevant observations. Bignozzi and Tsanakas (2015) [6] point out that the observations are often very small creating statistical error, which means that the values of sample estimators can diverge substantially from the true values. Jorion (1996) [10] calls it the risk in Value-at-Risk itself. Pérignon and Smith (2010) [12] find that historical VaR is the most popular VaR method, as 73% of the banks report their VaR estimation methodologies using historical VaR.

Our paper proposes an alternative risk measure based on the historical VaR. A confidence interval (CI) is considered to integrate the uncertainty contained in the historical VaR. It is a tail risk measure at multiple confidence levels (Alexander, Baptista and Yan (2015) [2]). It provides the flexibility to the risk manager to implement a prudential regulatory framework (Basel Committee on Banking Supervision (BCBS) [3] and Acharya (2009) [1]). Additionally, it can be a VaR based stressed risk measure based on a continuous 12-month period of significant financial stress following the requirement of the Basel Committee (BCBS (2011) [5]). We propose a stress testing application for this risk measure.

Numerous papers discussed the confidence interval of the VaR. For example, Pritsker (1997) [13] computes a nonparametric CI to evaluate the accuracy of different VaR approaches. Christoffersen and Gonçalves (2005) [7] assess the precision of VaR forecast by using bootstrap prediction intervals. Jorion (1996) [10] provides the asymptotic standard error and confidence bands for sample quantile, assuming the loss distribution is known. All these approaches mainly use their CI (provided by asymptotic result or bootstrap) as a complementary tool to assess the quality of the VaR. In our work we consider another approach to build the CI (we do not assume that the loss distribution is known and we do not use simulation). We use an asymptotic result and a parametric approach.

We focus on a fat-tail distribution [1] to capture historical stress information, in order to build a stressed risk measure. Finally we use the lower (or upper) bound of CI directly as one boundary of our risk measure.

This paper is organised as follows. Section 2 describes our risk measure. Section 3 proposes a stress testing application for the risk measure. Section 4 concludes.

2. The Spectral Stress VaR measure

Consider a financial variable X (for example the return of a portfolio, the return of a risk factor or an operational loss). Assume that it is a r.v. with a cumulative distribution function (cdf) $F_{\boldsymbol{\theta}}$ ($f_{\boldsymbol{\theta}}$ is the associated probability density function (pdf) and $\boldsymbol{\theta}$ are the parameters). Let $X_1, ..., X_n$ be the historical information set of X with length n.

As in Christoffersen and Gonçalves (2005) [7], we define the historical VaR ($X_{([np]+1)}$) as the $(1-p)th$ empirical quantile of the losses data. We fit a panel of distributions using $X_1, ..., X_n$ to compute the estimators of $\boldsymbol{\theta}$, denoted $\hat{\boldsymbol{\theta}}$. Then $F_{\hat{\boldsymbol{\theta}}}$ and $f_{\hat{\boldsymbol{\theta}}}$ are the estimators of $F_{\boldsymbol{\theta}}$ and $f_{\boldsymbol{\theta}}$. Given confidence levels $0 < p < 1$ and $0 < q < 1$ [2], we build a confidence interval $CI_{p,q}$ around $X_{([np]+1)}$ (Rao (2002) [14]; Guégan, Hassani and Li (2015) [8]):

$$X_{([np]+1)} \in \left[F_{\hat{\boldsymbol{\theta}}}^{-1}(p) - z_{\frac{1+q}{2}} \sqrt{\widehat{V}}, \quad F_{\hat{\boldsymbol{\theta}}}^{-1}(p) + z_{\frac{1+q}{2}} \sqrt{\widehat{V}} \right] \tag{1}$$

where

$$\widehat{V} = \frac{p(1-p)}{[f_{\hat{\boldsymbol{\theta}}}(F_{\hat{\boldsymbol{\theta}}}^{-1}(p))]^2 n}. \tag{2}$$

[1] A fat-tailed distribution has the property that it exhibits large kurtosis or has power law decay in the tail of the distribution.

[2] p is the confidence level of historical VaR and q is the confidence level of its confidence interval.

and $z_{\frac{1+q}{2}}$ is the $\frac{1+q}{2}th$ quantile of standard Gaussian distribution. According to the expression (1), $CI_{p,q}$ depends on n, $\hat{f}$, p and q.

In practice for a sequence $p_1 < p_2 < ... < p_k$, given $\{q_i\}_{i=1,...,k}$, we compute the sequences $\{F^{-1}_{\hat{\boldsymbol{\theta}}}(p_i)\}$ and CI_{p_i,q_i} for $i = 1, ..., k$. We define an area delineated by $F^{-1}_{\hat{\boldsymbol{\theta}}}(p_i)$ and the lower (or upper) bound of CI_{p_i,q_i} for $i = 1, ..., k$. We call this area the Spectral Stress VaR measure (SSVaR). Figure 1 provides a graph of the SSVaR. The lower (green) and upper (red) curves correspond to the boundaries of CI_{p_i,q_i} for $\{p_i\}$ and $\{q_i\}$, $i = 1, ..., k$. The black curve in the middle is associated to the sequence of $\{\{F^{-1}_{\hat{\boldsymbol{\theta}}}(p_i)\}$ for $i = 1, ..., k$. The black shadow area is the SSVaR.

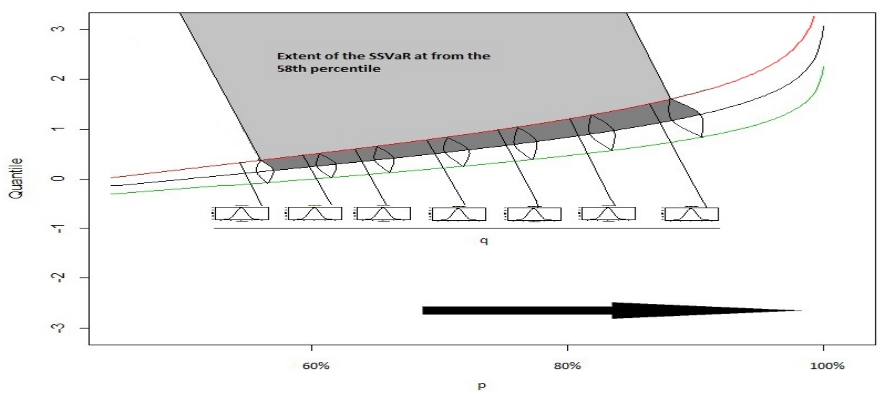

Figure 1: The lower (green) and upper (red) curves correspond to the boundaries of CI_{p_i,q_i} for $\{p_i\}$ and $\{q_i\}$, $i = 1, ..., k$. The black curve in the middle is associated to the sequence of $\{\{F^{-1}_{\hat{\boldsymbol{\theta}}}(p_i)\}$ for $i = 1, ..., k$. The black shadow area is the SSVaR. When the values of q_i change, SSVaR can shift to the grey area.

It is important to point out that when the risk manager has to work within the prudential regulatory framework, he can choose higher q_i leading to shift the SSVaR to the grey area. Also, he can shift the SSVaR to the grey area by choosing a fat-tail $\hat{f}$. In fact a fat-tail $\hat{f}$ can take more stress information from a period of significant financial turmoil than a thin tail fit. Consequently the SSVaR is a stressed risk measure in essence.

3. A stress testing application of the SSVaR

During recent crisis, some investors have suffered considerable losses due to extreme events. Consequently there has been a growing literature on stress testing. Specially, banks that use the VaR approach must have in place a rigorous stress testing program (BCBS (2005) [4]). In response, we propose a SSVaR measure applicable to the stress testing. The result of the stress testing is also a criteria to choose a reasonable $\hat{f}$ to build the SSVaR, which we can use first as an alert indicator.

To explain our purpose we consider a fictive financial institution. This one holds a Chinese market portfolio (that is, the same stock components and weights as the Shanghai Stock Exchange Composite Index (SHCOMP)). We compute the SSVaR using the daily return of SHCOMP from 29/06/2007 to 20/06/2008 (it contains 246 points and we call it Ω_1). The historical VaR of Ω_1 are computed. For the stress testing, we compute the empirical quantiles on the daily return of SHCOMP from 01/12/2014 to 09/11/2015 (it contains 241 points and we call it Ω_2). Table 1 provides the empirical statistics of the data sets. It shows these two data sets are left skewed and leptokurtic (Kurtosis > 3). The distributions which characterise these two data sets need to have these properties. In the following we build SSVaR using Ω_1, with Gaussian distribution as a benchmark and Normal-inverse Gaussian distribution (NIG, Godin (2012) [9]).

To take into account the left tail market risk, we use $0.01 \leq p_i \leq 0.1$ and fixed $q = 0.95$. We build the SSVaR for Ω_1 using Gaussian distribution [3] and NIG [4].

[3] The mean equals to -0.0017 and variance equals to 0.0007.

[4] The tail parameter parameter equals to 90.63, skewness parameter equals to -25.73,

Table 1: Empirical statistics of SHCOMP daily returns from 29/06/2007 to 20/06/2008 (Ω_1) and from 01/12/2014 to 09/11/2015 (Ω_2)

	Mean	Variance	Skewness	Kurtosis
Ω_1 ($n = 246$)	-0.0017	0.0007	-0.3796	3.7876
Ω_2 ($n = 241$)	0.0010	0.0007	-1.0509	5.0698

In Figure 2, on the left graph the dashed (blue and green) lines are the upper and lower bounds of the SSVaR corresponding to the Gaussian distribution. On the right graph the dashed (blue and green) lines are the upper and lower bounds of the SSVaR corresponding to the NIG distribution. In these two graphs, the solid (red) lines are the historical VaR and the solid-dot (brown) lines are the empirical quantiles for Ω_2.

In Figure 2, the left graph suggests that the SSVaR based on a Gaussian distribution underestimates the risk computed using Ω_1 and Ω_2, because the left part of the historical VaR and the empirical quantiles are outside the SSVaR. The right graph shows that the SSVaR built using a NIG distribution permits to control the risk more efficiently since they are almost inside the SSVaR. Additionally, ignoring the uncertainty in the historical VaR (that is, use the empirical quantiles directly as the risk measure) leads to underestimate the risk computed using Ω_2, because the left part of the empirical quantiles is lower than the historical VaR.

In practice, the SSVaR is a improvement risk measure of historical VaR. The risk manager can use it directly to allocate capital reserve to the risk of measurement uncertainty. Additionally, it can be a stressed and tail risk measure providing flexibility to the risk manager to work within the prudential regulatory framework.

location parameter equals to 0.0155 and scale parameter equals to 0.058.

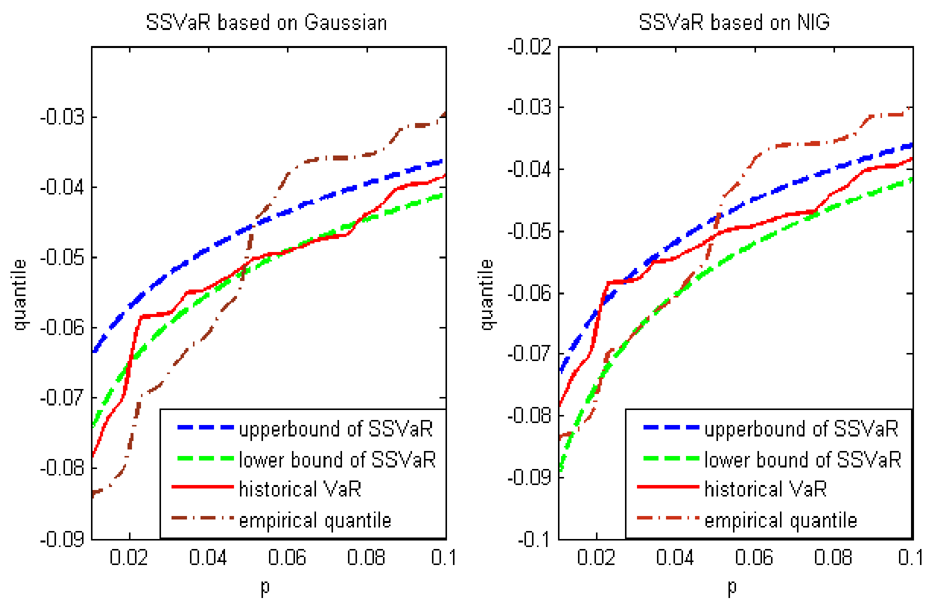

Figure 2: We use $0.01 \leq p_i \leq 0.1$ and fixed $q = 0.95$ and build the SSVaR for Ω_1 using Gaussian distribution (mean -0.0017 and variance 0.0007) and NIG (with tail parameter parameter equalling to 90.63, skewness parameter equalling to -25.73, location parameter equalling to 0.0155 and scale parameter equalling to 0.058). In Figure 2, on the left graph the dashed (blue and green) lines are the upper and lower bounds of the SSVaR corresponding to the Gaussian distribution. On the right graph the dashed (blue and green) lines are the upper and lower bounds of the SSVaR corresponding to the NIG distribution. In these two graphs, the solid (red) lines are the historical VaR and the solid-dot (brown) lines are the empirical quantiles for Ω_2.

4. Conclusion

In this article, we propose an alternative quantile-based risk measure SSVaR, to integrate the uncertainty from the historical VaR. Additionally, it is a tail risk measure. Also, it provides the flexibility to the risk manager to implement prudential regulatory framework. It can be a VaR based stressed risk measure. Additionally, We propose a stress testing application for the SSVaR, by illustrating the magnitude of the exceptions based on the empirical quantile of two data sets from SHCOMP. The results suggest that ignoring the uncertainty in the historical VaR leads to underestimate risks. Also, we observe that when the data sets are skewed and leptokurtic, risk manager needs to fit a skewed and leptokurtic distribution to build SSVaR. It leads to control the risk efficiently.

As the purpose of a forthcoming paper, some improvements of this approach could be done. Indeed, the expression (1) relies on the assumption of independence for $X_1, ..., X_n$ (Rao (2002) [14]). Nevertheless, we can extend the results in case of $\alpha - mixing$ (Leadbetter et al. (1983) [11]) data sets. Also, the SSVaR can be used directly for the operational risks which are mainly independent. For other risks we can calibrate dynamics on $X_1, ..., X_n$, like $X_t = f(X_{t-1}) + \epsilon_t$ where ϵ_t is a white noise. Then we build the SSVaR using the residuals $\{\epsilon_t\}$, and the time series modelling can be used to introduce dynamics inside the SSVaR.

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