Marriage, Labor Supply, and Home Production

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Marriage, Labor Supply, and Home Production: A Longitudinal Microeconomic Analysis of Marriage, Intra-Household Bargaining and Time Use Using the BHPS, 1991-2008

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Abstract:
We extend the search-matching model of the marriage market of Shimer and Smith (2000) to allow for labor supply, home production, match-specific shocks and endogenous divorce. We study nonparametric identification using panel data on marital status, education, family values, wages, and market and non market hours, and we develop a semiparametric estimator. We estimate how much sorting results from time use specialization or homophilic preferences. We estimate how equilibrium marriage formation affects the wage elasticities of market and non market hours. We estimate individuals' willingness to pay for marriage and quantify the redistributive effect of intra-household resource sharing.

Keywords: Search-matching, sorting, assortative matching, collective labor supply, structural estimation.

JEL Classification: C78, D83, J12, J22.

1 Introduction

Lundberg and Pollak (1996) end their insightful survey on bargaining and (re)distribution in marriage by stating that "bargaining models provide an opportunity for integrating the analysis of distribution within marriage with a matching or search model of the marriage market." They make this statement after arguing that policy interventions affecting the distribution of resources within the family can have very different short-run and long-run effects because of marriage market equilibrium feedback.

This strategy was a recipe for success in macroeconomics. A series of applied search-matching papers on the macrodynamics of marriage and distribution were indeed published shortly after Lundberg and Pollak's survey.[1] Surprisingly enough the early contributions of Manser and Brown (1980), McElroy and Horney (1981), McElroy (1990) and Lundberg and Pollak (1993) did not generate the same amount of applied microeconomic work. Surely the fault can be laid to P.-A. Chiappori who, in a series of influential papers,[2] demonstrated that assuming efficiency was sufficient to deliver testable empirical restrictions.[3]

However, Lundberg and Pollak's concern resurfaces as soon as the aim of the empirical work is not only to test a theory but also to understand the effects of family and anti-poverty policies such as, for example, the Working Family Tax Credit program in the UK or the Earned Income Tax Credit in the USA. Collective models very successfully describe resource sharing within the family given a sharing rule. A matching model – that is an equilibrium model of the marriage market – is required in order to endogenize the distribution of powers in the family (Chiappori, 2012).

This is the reason behind the recent revival of interest for matching markets. Long after Becker's seminal work (Becker, 1973, 1974, 1981), Choo and Siow (2006) were the first to estimate a transferable utility model of the marriage market using the perfect-information assignment framework of Shapley and Shubik (1971).[4] Transferable utility models postulate the existence of a match surplus, function of spouses' characteristics, to be shared between spouses. A matching equilibrium is a distribution of spouses' types and a way of sharing the surplus that is such that no couple recomposition can improve

[1] See Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, and Knowles (2000), Caucutt, Guner, and Knowles (2002), Gould and Paserman (2003), Fernandez, Guner, and Knowles (2005).

[2] See Chiappori, 1988, 1992, Browning, Bourguignon, Chiappori, and Lechene, 1994, Browning and Chiappori, 1998, Chiappori, Fortin, and Lacroix, 2002. A dynamic version of the collective model was explored by Mazzocco (2004, 2007).

[3] The assumption of efficient allocations within the family has been disputed, in particular, by Del Boca and Flinn (2005, 2006, 2009).

[4] Dagsvik (2000) had previously developed a non-transferable utility model. Other applications of transferable utility models to to marriage markets include Chiappori, Iyigun, and Weiss (2008), Chiappori and Oreffice (2008), Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanié, and Weiss (2011). Hitsch, Hortacsu, and Ariely (2010) is another recent application of a non-transferable utility framework using the Gale-Shapley algorithm (Gale and Shapley, 1962).

aggregate utility.[5] Choo, Seitz, and Siow (2008a,b) are early attempts to propose an empirical collective model with endogenous marriage formation and labor supply.

In this paper we follow a different route. We extend the search-matching models of Lu and McAfee (1996) and Shimer and Smith (2000), and the empirical implementation of Wong (2003), to allow for labor supply and domestic production. Men and women differ by a vector of characteristics, which in our empirical application comprise education, the wage and an index of family values. Both spouses decide how much time to allocate to market work, leisure and to the in-house production of a public good. Home production requires some market goods expenditure as input. So spouses must also decide how much income to save for home production. They take these decisions cooperatively by divorce-threat Nash bargaining.

By saving time and money for home production spouses can transfer utility to each other. This is the first source of marriage externalities. Secondly, existing complementarities between spouses' types are embodied in domestic production's total factor productivity (TFP), which we interpret as public good quality. This is the supplement of well-being that spouses enjoy just by being together, and that is not produced by time and expenditure inputs, for example because they share the same cultural interests or family values. Individuals can thus compensate income differences by monetary and time transfers, but such transfers may not be able to compensate for the disutility of education or family values mismatch.

One advantage of the sequential search framework is that a single dimension of match heterogeneity is enough to introduce the amount of noise necessary to change the binary decision of matching, given male and female types, into a probability. Match-specific heterogeneity is subject to infrequent shocks in order to endogenize divorce. The probability of divorce at any point in time is the probability of a shock occurrence times one minus the matching probability. This modeling strategy not only allows to characterize good matches as matches that are consummated with a high probability but also as matches that last longer. In addition, this will allow us to separately identify the matching probabilities from the rates at which singles meet in the bachelor market and the rate at which the match-specific component is reset.

Another advantage of the search-matching framework is that time is a natural dimension of the model as married couples must anticipate the risk of divorce and single individuals must predict their chances of meeting the right partner. Our model thus goes a long way toward fulfilling Lundberg and Pollak's program. It is an equilibrium

[5] Choo and Siow's empirical model is a static, multinomial logit models where every individual, characterized by some discrete type $i = 1, ..., K$, is endowed with a vector of extreme-value distributed idiosyncratic tastes, one for each matching characteristic $j = 1, ..., K$. Without type-specific random utilities, matching would be perfect and few match combinations (i, j) would be observed. Decker, Lieb, McCann, and Stephens (2013) provide useful theoretical results on Choo and Siow's model. Galichon and Salanié (2012) extend it to a larger class of distributions of tastes, and Dupuy and Galichon (2014) extend it to continuous characteristics.

search-matching model. Spouses share time and income resources collectively via Nash bargaining. Contracts are dynamic spot contracts without commitment. The threat point is the intertemporal value of divorce. Divorce is endogenous, triggered by public good quality shocks.

We identify and estimate it in two steps. First we use data on the distribution of male and female exogenous types (education, family values and wages) by marital status at a given point in time, as well as the distribution of types in the flows of new marriages and divorces, to estimate meeting rates for singles (assuming that married people do not search for alternative partners), divorce rates and matching probabilities. Second, we use time use data to estimate preferences, the home production function, the Nash bargaining parameter and the distribution of the match-specific shock. Identification rests on the existence of an instrument for home production. Specifically, we exclude the family values index from the list of variables conditioning the tradeoff for private consumption and leisure. We use GMM to estimate a parametric specification. We show that public good quality (the TFP of home production) is in a one-to-one relationship with the matching probability given the other parameters. This allows us to back out public good quality nonparametrically. Lastly, we show that the Nash bargaining parameter is identified from leisure and home production specialization, and that the variance of the match-specific shock is identified by second-order moments of time uses (given types).

The collective framework takes population type distributions as given and does not relate them to the parameters of private preferences, home production and the sharing rule. Partial identification is possible if there exist environmental factors shifting the sharing rule but not individual preferences, and the sharing rule is identified up to an additive function of prices. This is true as well for bargaining models with exogenous threat points. By subsuming the collective model into an equilibrium theory of matching, we can make use of the considerable amount of information that is incorporated in matching probabilities to completely identify the sharing rule. And it is considerable indeed, as Choo and Siow (2006), Galichon and Salanié (2012) and Dupuy and Galichon (2014) show that, in a matching model with exogenous surplus, type distributions by gender and marital status are enough to identify surplus sharing.

We use data from the British Household Panel Survey, 1991-2008. The BHPS allows to match the different individuals that belong to the same household and provides individual data on time uses. We document changes in time uses and the distributions of education, wages and family values by gender and marital status. We estimate our model and discuss how it makes sense of these changes. The bargaining power coefficient is estimated 0.5, which implies that the balance of powers between spouses in the family is only function of the outside options. We show that marriage externalities responsible for sorting in the marriage market entirely result from tastes and not task specialization within the household. We estimate wage elasticities of market and non market hours for both men and women. We find similar orders of magnitude for females as in Blundell, Dias, Meghir, and Shaw (2015), and we are able to quantify the contribution of equilibrium changes in marriage probabilities to the determination of these elasticities. We find that the marriage market tends to augment the response of market hours of married individuals to male wage increases. Following Chiappori and Meghir (2014b,a) we also estimate a monetary measure of how much each spouse values his or her marriage in comparison to divorce. Remarkably, we find that within-household redistribution is considerably reducing the gender gap in welfare and within-household inequality. The correlation between male and female wages is about 30% whereas the correlation between total equivalent incomes is estimated 70%.

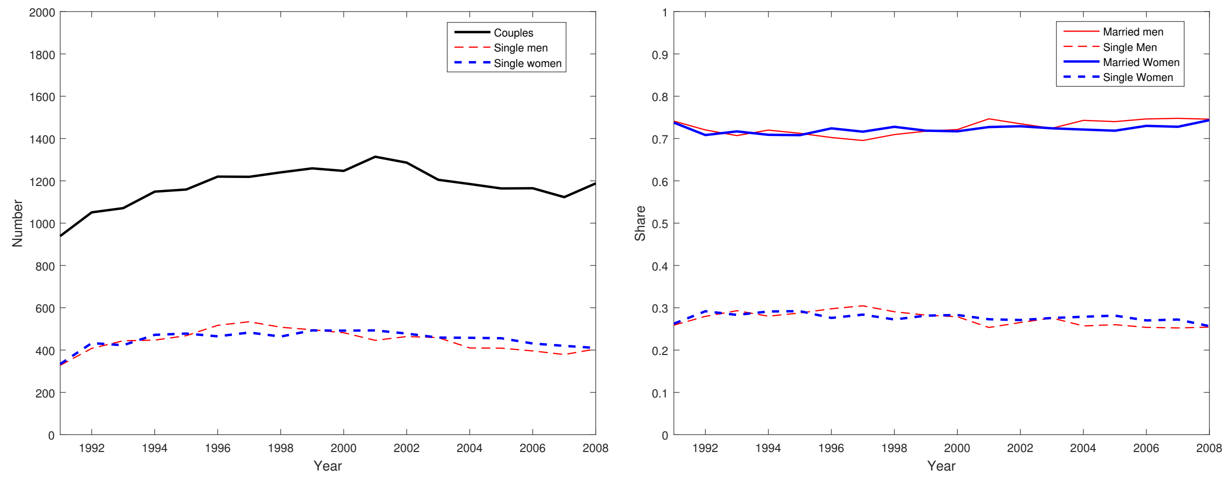
The layout of the paper is as follows. We start by describing some salient features of intrahousehold allocation of time and matching between 1991 and 2008, that the model is challenged to reproduce. Then we construct the matching model. The next section develops the estimation procedure and we then show the results. Various appendices are devoted to technical details.

2 Data

The data are drawn from the British Household Panel Survey (BHPS). We only use the original BHPS sample comprising 5,050 households and 9,092 adults interviewed at wave 1 (1991), whom we then follow yearly until 2008, even after separation from the original household.[6] The panel not only follows all individuals from the first wave (original sample members) but also all adult members of all households comprising either an original sample member, or an individual born to an original sample member. The sample therefore remains broadly representative of the British population (excluding Northern Ireland and North of the Caledonian Canal) as it changes over time. We only keep households composed of heterosexual couples (married or cohabiting) and single-person households who are between 22 and 50 years of age at the time of interview. The same individuals are re-interviewed in successive annual waves. To reduce non response biases we use the individual adjustment weights provided in the survey (specifically, Individual Respondent Weight, AXRWGHT).

We use information on usual gross pay per month for the current job, the number of hours normally worked per week (including paid and unpaid overtime hours) and the number of hours spent in a week doing housework (core non market work excluding child caring and rearing, information not provided by the survey). Hourly wage is the usual gross pay per month divided by the number of hours normally worked per month (without overtime). Wages are deflated by the Consumer Price Index and computed in

[6] This survey is a stratified clustered sampling with 250 Primary Sampling Units in England, Scotland and Wales.



(a) Sample sizes

(b) Shares of married and unmarried by gender

Fig. 1: Sample sizes and shares

2008 pounds.[7] In order to reduce the number of labor supply corners (zero market hours and missing wages) we replace current observations on wages and market hours by a moving average of past, present and future observations. Specifically, suppose that we observe wages $w_1, w_2, \ldots$ and hours $h_1, h_2, \ldots$ We replace w_t and h_t by

$$\widehat{w}_t = \frac{\sum_{\tau=-\infty}^{+\infty} w_{t+\tau}\mathbf{1}\{h_{t+\tau} \neq 0\}\phi\left(\tau/k\right)}{\sum_{\tau=-\infty}^{+\infty} \mathbf{1}\{h_{t+\tau} \neq 0\}\phi\left(\tau/k\right)}, \quad \widehat{h}_t = \frac{\sum_{\tau=-\infty}^{+\infty} h_{t+\tau}\phi\left(\tau/k\right)}{\sum_{\tau=-\infty}^{+\infty} \phi\left(\tau/k\right)},$$

where ϕ is the standard normal PDF and k is a smoothing parameter that we arbitrarily choose equal to 2 (after experimenting with other choices) yielding weights 1, 0.882, 0.607, 0.325, 0.135, 0.044, 0.011 for 0, 1, 2, 3, 4, 5, 6 years apart. Then we trim the 1% top and bottom tails of wage and time use variables. We thus obtain an unbalanced panel of 18 years (1991-2008), whose cross-sectional sizes are displayed in Figure 1. The sample sizes vary across years, but sample shares are remarkably stable.

We finally construct a Family Values Index (FVI) based on individuals' responses (1: strongly agree; 2: agree; 3: neither agree nor disagree; 4: disagree; 5: strongly disagree) to various statements about children, marriage, cohabitation and divorce. Table 1 lists the questions used to construct the index and displays the corresponding factor loadings estimated by Principal Component Analysis.[8] Given the signs of these loadings, our family values index is thus a measure of conservativeness.

These nearly twenty years between 1991 and 2008 have seen some remarkable changes in time uses by gender and marital status, as well as changes in wages, attitudes and

[7] We do not detrend wages further as we use inly cross-sectional data for estimation.

[8] The variable "Attendance at religious service" is coded the following way: 1: Once a wk or +; 2: At least once per month; 3: At least once per year; 4: Practically never; 5: Only weddings etc. After 1998, the item "Cohabiting is always wrong" becomes "Cohabiting is alright". So we change it as "6-item_answer" to maintain coherency over the years.

Tab. 1: Family Values Index

Question	Loading
Pre-school child suffers if mother works	-0.24
Family suffers if mother works full-time	-0.25
Woman and family happier if she works	0.16
Husband and wife should both contribute	0.14
Full time job makes woman independent	0.12
Husband should earn, wife stay at home	-0.21
Children need father as much as mother	-0.05
Employers should help with childcare	0.12
Single parents are as good as couples	0.17
Adult children should care for parents	-0.07
Divorce better than unhappy marriage	0.12
Attendance at religious services	-0.07
Cohabiting is always wrong	-0.16

Notes: The Family Values Index is a weighted sum of all responses. The table gives each question's weight estimated by Principal Component Analysis.

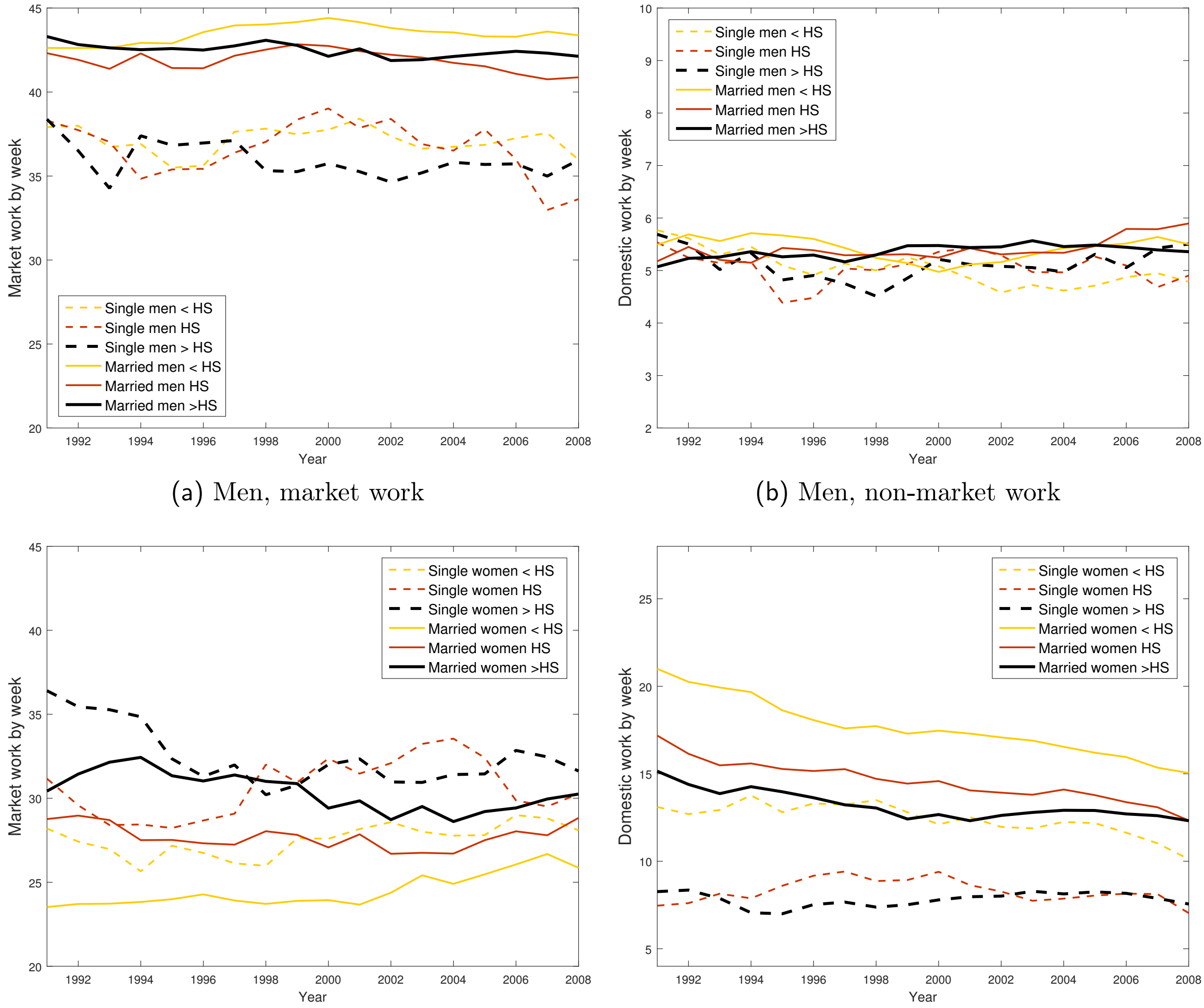
education. We now briefly describe them.[9]

Figure 2 confirms well known facts about market and non market work. Married men work more outside the home than single men (moving average including periods of unemployment), and all men, married and single, devote the same amount of time (little) to home production. Married women devote less time to market work and more time to household chores than single women. Male hours are remarkably stable over time. Education is not a key determinant for men; it is for women. Educated women work more outside the home than inside, by comparison to less educated women. Over time, female time use differentials by education and marital status are progressively reduced.

There are also some interesting composition changes observable in that period (Fig. 3). All marital groups gradually tend to follow the educational norm of single men. In 1991, about 45% of single men did not have A levels, a much lower figure than the corresponding 60% for married men and single women, and 70% for married women. By 2008 all groups had narrowed the gap to 45%. Wages do not display the same convergence pattern, as married people's wages increase both in absolute terms and with respect to the wages of singles. Men are more traditionalist than women, couples are more traditionalist than singles, and all tend to become less traditionalist over time (both because individuals' opinions change and because new individuals enter the panel).

Lastly, Figure 4 shows how market and non-market hours vary with one's wage and family values. Labor supply responses to wages are as expected. Family values strongly

[9] See Ramos (2005) and Gimenez-Nadal and Sevilla (2012) for a more detailed description.



(a) Men, market work

(b) Men, non-market work

(c) Women, market work

(d) Women, non-market work

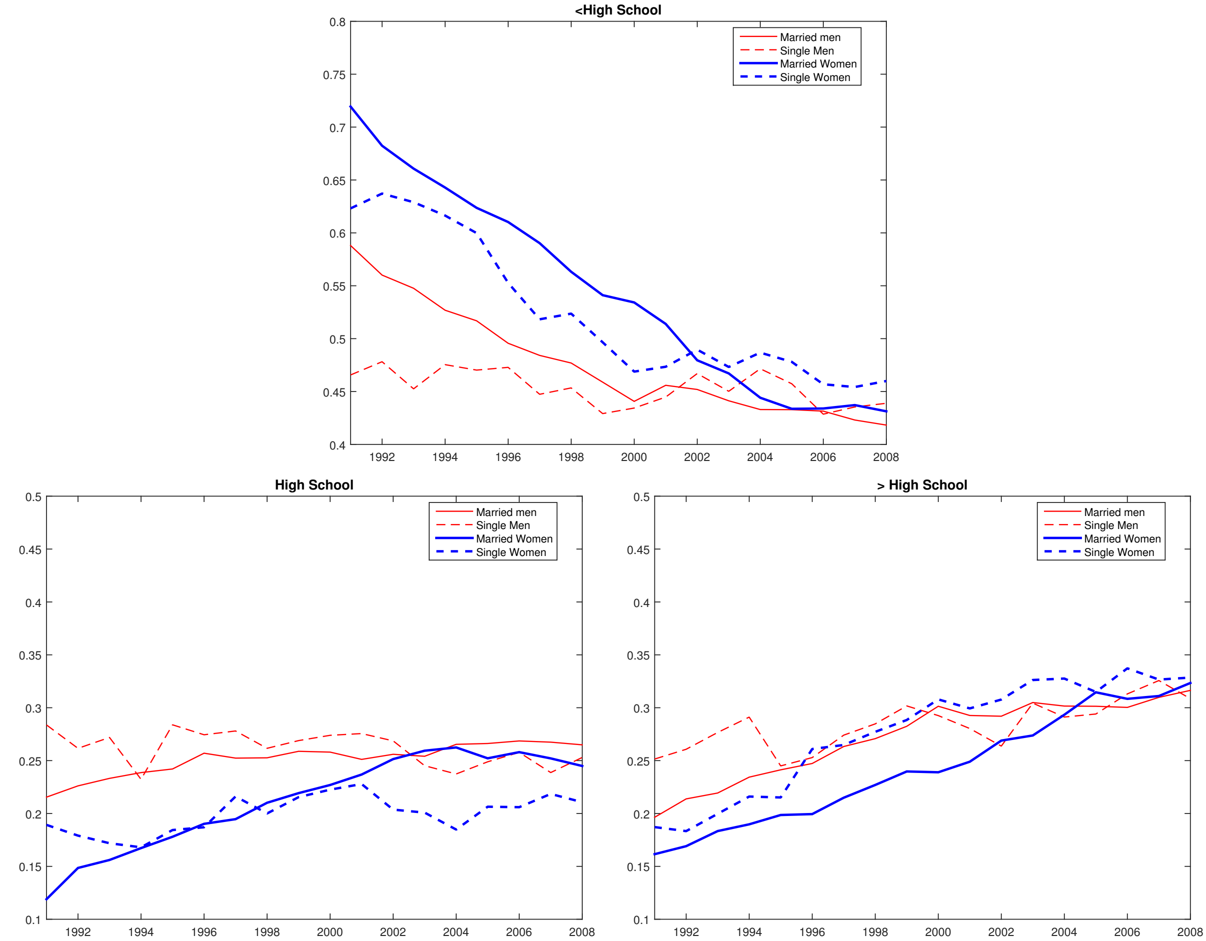
Fig. 2: Time use trends

determine the market and non-market hours of married men and women.[10]

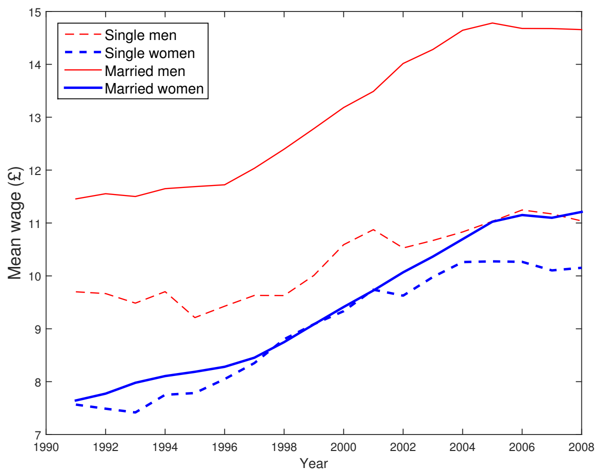
3 Measuring sorting in the marriage market

It is usual to describe sorting by the cross-sectional distribution of spouses' characteristics in the population of married couples. However, we want to think of marriage as the outcome of a double process of searching for potential partners and decision about partnering. In this section we propose a statistical model to separately estimate meeting and matching probabilities, as well as divorce rates.

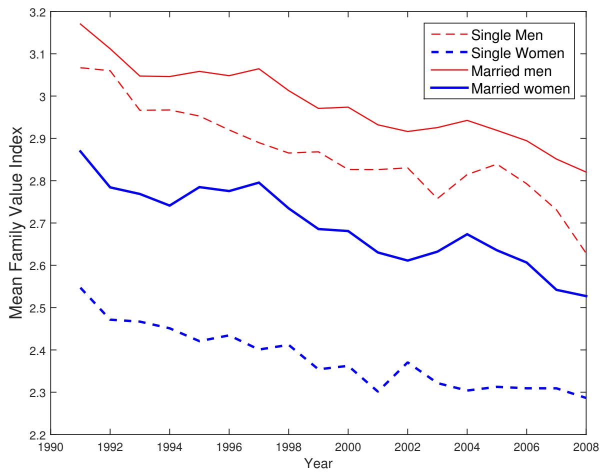
[10] See Fortin (2005) for a study of gender attitudes and labor market outcomes across OECD countries.



(a) Education

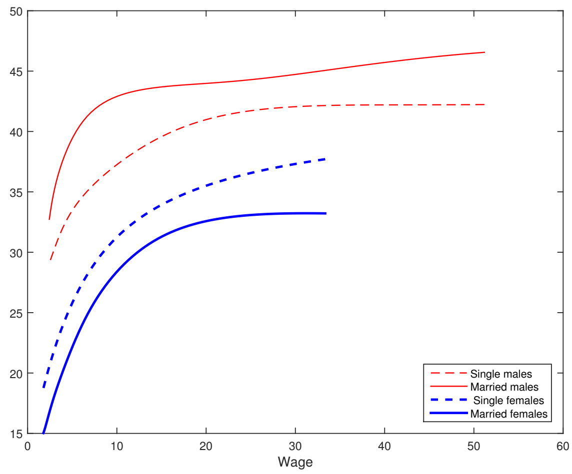


(b) Mean wages

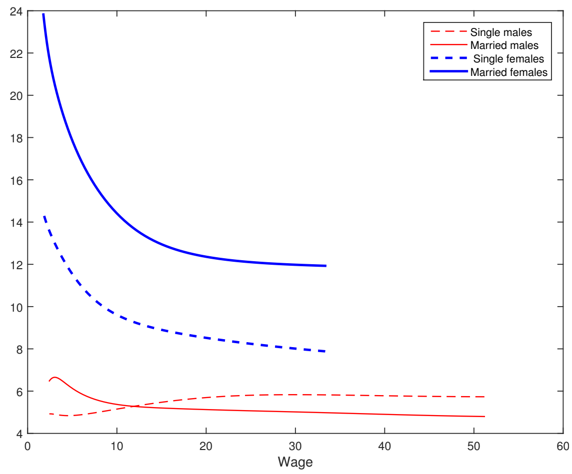


(c) Family values

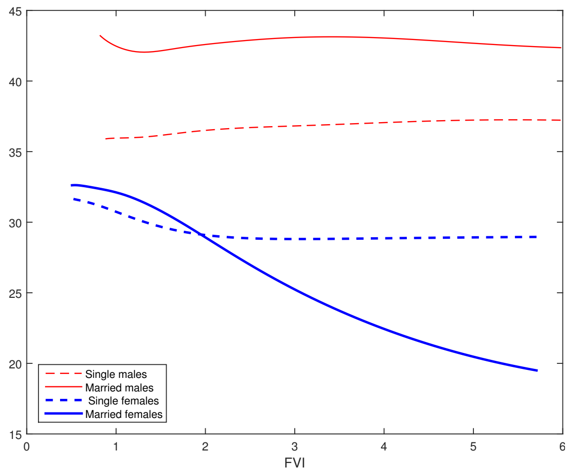
Fig. 3: Composition changes



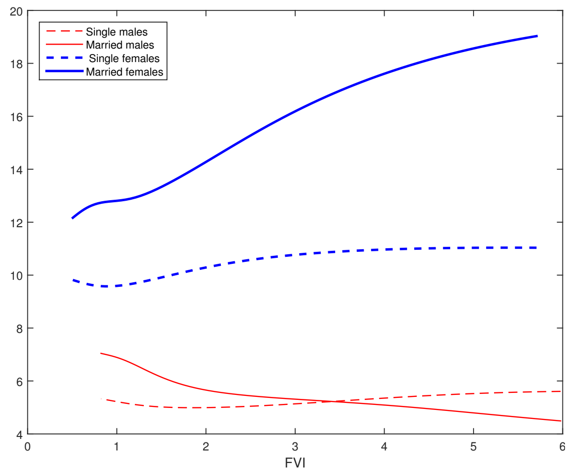
(a) Market hours by wage



(b) Non-market hours by wage



(c) Market hours by FVI



(d) Non-market hours by FVI

Fig. 4: Time use by wage and family values

3.1 A statistical model of marriage and divorce flows

Men and women are fully characterized by a vector of characteristics such as education, wages and family values that we call a *type*. Conventionally, we shall use the index i for male types and j for female types. Let $\ell_m(i)$ and $\ell_f(j)$ denote the density functions of male and female types in the whole population at a given point in time (there is an implicit indexation on time), with $L_m = \int \ell_m(i)\,\mathrm{d}i$ and $L_f = \int \ell_f(j)\,\mathrm{d}j$ denoting the total numbers of men and women in the population. Let $n_m(i), n_f(j), N_m, N_f$ be the corresponding notations for the sub-populations of single men and women. Lastly, let $m(i,j)$ and $M = \iint m(i,j)\,\mathrm{d}i\,\mathrm{d}j$ denote the density function and the total number of couples' types.

Assume that only singles search, ruling out search during marriage. Let λ_m and λ_f denote the rates at which male and female singles meet other singles per unit of time. The number of males meeting a female ($\lambda_m N_m$) is equal to the number of females meeting a male ($\lambda_f N_f$). Hence, let $\lambda = \lambda_m / N_f = \lambda_f / N_m$. As $N_m N_f$ is the number of possible matches and $\lambda N_m N_f$ is the number of meetings, λ is the rate at which potential meetings are actualized.

We can then define marriage flows as follows. Let α_{ij} denote the probability of marriage for a male of type i meeting a female of type j. The number of new marriages (or cohabitations) of type (i,j) per unit of time is

$$MF(i,j) = n_m(i)\,\lambda_m\,\frac{n_f(j)}{N_f}\,\alpha_{ij} = n_f(j)\,\lambda_f\,\frac{n_m(i)}{N_m}\,\alpha_{ij} = \lambda\,n_m(i)\,n_f(j)\,\alpha_{ij}. \tag{3.1}$$

We observe the flows $MF(i,j)$ and the stocks $n_m(i), n_f(j)$. We are thus faced with the usual inference problem: How can one disentangle λ from α_{ij}?

Our solution proceeds from linking marriage and divorce flows by a common mechanism. Assume that there is a match-specific value z that is drawn from a distribution G and revealed to singles when they meet for the first time. The probability of marriage can now be thought of as the probability that z satisfies some matching rule given types (i,j). We shall make this rule explicit later. For the moment it suffices to assume that a condition for marriage to be consummated involving (i,j,z) exists. Assume further that this match-specific component z is subject to i.i.d. shocks according to a Poisson process with parameter δ. Divorce occurs when the last draw of z ceases to satisfy the matching rule (with probability $1-\alpha_{ij}$). It follows that the flow of divorces per unit of time is

$$DF(i,j) = m(i,j)\,\delta\,(1-\alpha_{ij}). \tag{3.2}$$

In this model, matches with a higher probability of marriage also have a lower probability of divorce. Therefore, we should observe that a marriage that results from an exceptional realization of z (love at first sight) should break faster than a marriage based on solid fundamentals. A more sophisticated model may have autoregressive shocks to z. It is however not clear how this dynamics would be identified. Postel-Vinay and Turon (2010) show for example that individual wage dynamics can be perfectly fitted using a sequential auction model with i.i.d. match-specific shocks in many ways similar to this model.

3.2 Identification and estimation

Define $MR(i,j) \equiv \frac{MF(i,j)}{n_m(i)n_f(j)}$, the marriage rate (by potential match type), and $DR(i,j) \equiv \frac{DF(i,j)}{m(i,j)}$, the divorce rate. If λ and δ are not nil, then equations (3.1) and (3.2) are equivalent to the following ones:

$$\frac{MR(i,j)}{\lambda} + \frac{DR(i,j)}{\delta} = 1, \tag{3.3}$$

$$(1-\alpha_{ij})\frac{MR(i,j)}{\lambda} - \alpha_{ij}\frac{DR(i,j)}{\delta} = 0. \tag{3.4}$$

The parameters $1/\lambda$ and $1/\delta$ are the average length of time separating two meetings or two shocks to z. Under the assumption that $MR(i,j)$ and $DR(i,j)$ are not collinear, it is easy to show that minimum distance estimation yields estimates for $1/\lambda$ and $1/\delta$ equal to the regression of the constant 1 on marriage and divorce rates, and α_{ij} is then estimated as

$$\alpha_{ij} = \frac{\delta MR(i,j)}{\delta MR(i,j) + \lambda DR(i,j)}. \tag{3.5}$$

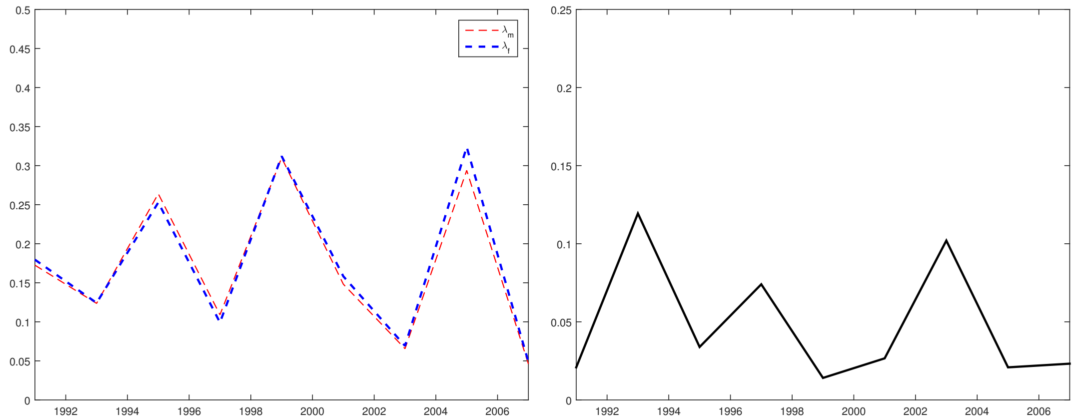
If $MF(i,j) = DF(i,j)$, this formula simplifies as

$$\alpha_{ij} = \frac{\delta m(i,j)}{\delta m(i,j) + \lambda n_m(i) n_f(j)}. \tag{3.6}$$

This is the value of α_{ij} under stationarity. In fact, strict stationarity is not necessary for the latter formula to provide a good approximation. It suffices that gross flows be large with respect to net flows. This happens to be true whenever population distributions move smoothly over time. This is therefore the formula that we use in practice, because conditional flows are too imprecisely estimated.[11]

In practice, we consider two-year periods, say 1995 and 1996. We estimate $n_m(i)$, $n_f(j)$ and $m(i,j)$ (standardized by population sizes) by kernel density estimation from the stocks of male singles, female singles and married couples in that period. We estimate $MF(i,j), DF(i,j)$ in the same way from the flows of new marriages and divorces in that period (with a larger smoothing width to account for smaller sample sizes).

[11] Estimates of $1/\lambda$ and $1/\delta$ rely on flows. There is no other way. But they are more precisely estimated than matching probabilities because the regression procedure averages over types (i,j) and smoothes out small sample errors. Yet, in practice, to contain measurement errors on flows to its minimum, we estimate $1/\lambda$ and $1/\delta$ from aggregate marriage and divorce flows by education.



(a) Estimated meeting rates λ_m, λ_f (b) Estimated δ

Fig. 5: Demographic rates (annual)

3.3 Estimation results on matching and divorce rates

The estimated values of meeting rates (λ_m, λ_f) and of the rate at which shocks to the public good occur (δ) are displayed in Figure 5. Meeting rates for singles are estimated around 20% annual, and δ around 5% annual. There is no evidence of any marked trend. So, in the estimation of the structural model, we shall set these rates equal to their 1991-2008 average estimates.

We then estimate matching probabilities. Table 2 displays a subset of the parameters of the quadratic projection of estimated matching probabilities on individual characteristics including all interaction terms, those quantifying the contributions of male-female complementarities. Quadratic effects altogether explain 76% of the variance on average across years, which indicates that the higher-order effects excluded from the projection explain about a quarter of the total variance. The relative contribution of male-female complementarities is 22% on average (the difference between the last two rows).

The parameter estimates reported in the table correspond to standardized characteristics (i.e. divided by their standard deviations). Hence they are comparable with each other. The predominant complementarities are related to education and family values. The only sizable complementarities involving wages are those where male education is interacted with female wage. There is a clear positive trend in the attractiveness of high-educated persons, and perhaps a recent reduction in homophily by family values.

Matching by education, family values and wages show very different patterns. Figure 6 displays matching probability estimates projected in the education plane. Panel (b) contains the same information as Panel (a), but transposed in order to emphasize existing symmetries. Matching probabilities are symmetric in spouses' education and there is strong evidence of homophily. Mismatch probabilities (less than high school married to some college) are below 30%, and about half of same-type marriage probabilities. Lastly, high-educated individuals (i.e. with high school education or higher) increase their

Tab. 2: Quadratic projection of matching probabilities (complementarities)

	1991	1993	1995	1997	1999	2001	2003	2005	2007	Mean
$(Educ_m > HS) * (Educ_f > HS)$	0.067	0.065	0.067	0.067	0.082	0.086	0.084	0.091	0.09	0.078
$FVI_m * FVI_f$	0.032	0.033	0.05	0.039	0.035	0.034	0.031	0.024	0.018	0.033
$(Educ_m > HS) * (Educ_f = HS)$	0.022	0.029	0.027	0.026	0.027	0.031	0.027	0.024	0.037	0.028
$(Educ_m = HS) * (Educ_f > HS)$	0.017	0.009	0.029	0.024	0.019	0.024	0.029	0.028	0.027	0.023
$(Educ_m > HS) * wage_f$	0.015	0.025	0.02	0.024	0.014	0.014	0.027	0.015	0.016	0.019
$(Educ_m = HS) * (Educ_f = HS)$	0.024	0.024	0.031	0.021	0.018	0.003	0.009	0.01	0.011	0.017
$(Educ_m = HS) * wage_f$	0.015	0.024	0.005	0.01	0.014	0.012	0.01	0.015	0.019	0.014
$FVI_m * (Educ_f = HS)$	-0.005	-0.006	-0.01	-0.008	-0.006	-0.016	-0.025	-0.017	-0.014	-0.012
$FVI_m * wage_f$	-0.001	-0.012	-0.011	-0.015	-0.009	-0.014	-0.013	-0.01	-0.008	-0.010
$wage_m * (Educ_f > HS)$	0.008	0.011	0.009	0.008	0.001	0.003	0.008	0.001	0.016	0.007
$FVI_m * (Educ_f > HS)$	-0.004	-0.004	0	-0.008	-0.007	-0.012	-0.019	-0.01	0	-0.007
$wage_m * (Educ_f = HS)$	0.009	0.01	0.005	0.006	0.007	0.009	0.005	0.002	0.007	0.007
$wage_m * wage_f$	0.003	0.01	0.009	0.007	0.01	0.009	0.002	0.005	-0.003	0.006
$wage_m * FVI_f$	0.002	0.005	0.006	0.009	0.013	0.007	0.002	0.001	0.001	0.005
$(Educ_m > HS) * FVI_f$	0.002	-0.003	0.003	0.007	0.011	0.003	0	0	0.004	0.003
$(Educ_m = HS) * FVI_f$	-0.004	-0.003	0	0.008	0.005	0.003	-0.002	-0.002	0.004	0.001
R^2 (with male-female interactions)	0.72	0.76	0.78	0.74	0.75	0.74	0.81	0.79	0.73	0.76
R^2 (without)	0.44	0.48	0.54	0.53	0.53	0.55	0.64	0.63	0.56	0.54

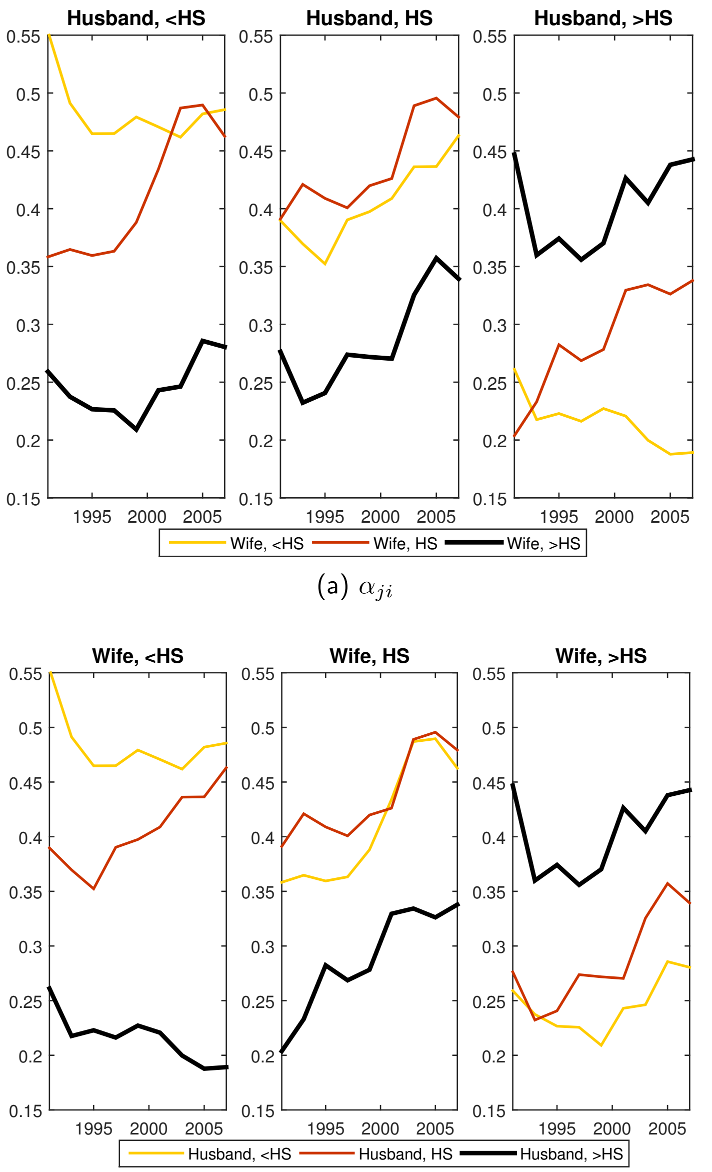
Notes: All the variables are standardized. The R2 is calculated with male*female interaction terms and without. The rows are ordered by decreasing order of the absolute mean value across years.

matching probability over the 1991-2008, which is consistent with previously documented composition changes.

Figure 7 shows matching probabilities by quartiles of family values indices. Matching probabilities are again found to be symmetric but the homophily pattern is different. Progressive individuals (index lower than first quartile) are basically indifferent about their partner's attitudes. Homophily builds up with conservatism as traditional individuals are much more likely to marry someone sharing similar ideas about the family. More traditional men are also more likely to marry less educated and lower wage women (see Tab. 2).

By comparison, there is very limited sorting by spouses' wages (Fig. 8). Matching probabilities increase a lot with male wage, and relatively little with female wage, in conformity with the quadratic regression results. Richer men are more attractive; female wage matters little. There is no evidence of the pattern documented for the US by Bertrand, Kamenica, and Pan (2015), that is "among married couples in the United States, the distribution of the share of household income earned by the wife drops sharply at 1/2 —where the wife starts to earn more than the husband." British high wage women do not seem to have a lower marriage probability.

The patterns of sorting by one spouse's wage and the other spouse's education are also interesting (Fig. 9). High-wage men marry more, but high-educated men marry less, roughly irrespective of female wage. If any, high-educated men, to a rather limited extent, are more likely to match with higher wage women. On the other hand, although



(a) α_{ji}

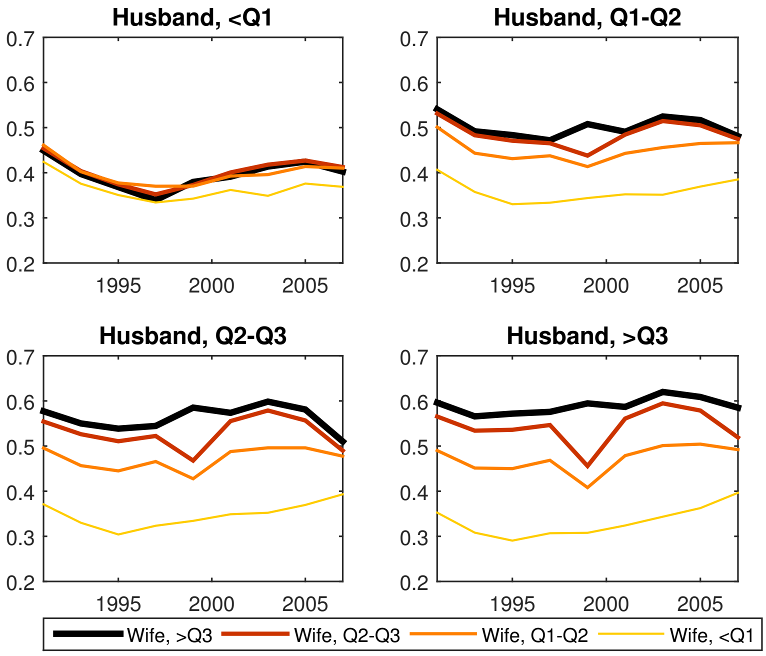
(b) α_{ij}

Fig. 6: Matching probabilities by spouses' education

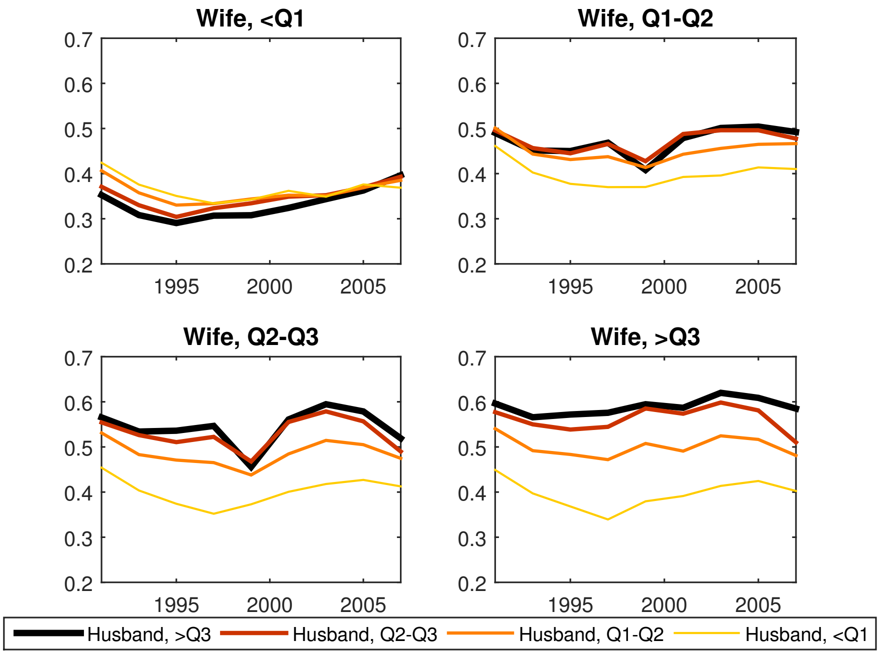
low-educated women did marry more than high-educated ones in 1991, this educational differential has disappeared by 2008. At the same time, women are much more likely to marry richer men, irrespective of education. Hence, there is limited positive sorting by female wage and male education, but none by male wage and female education.

4 The economics of marriage and intrahousehold decisions

What do these documented patterns of task specialization and sorting by education, family values and wages allow us to infer on individual preferences and the amount of redistribution at work in the household? In order to answer this question, we need to build a theory of marriage decision and of the allocation of resources within the household.

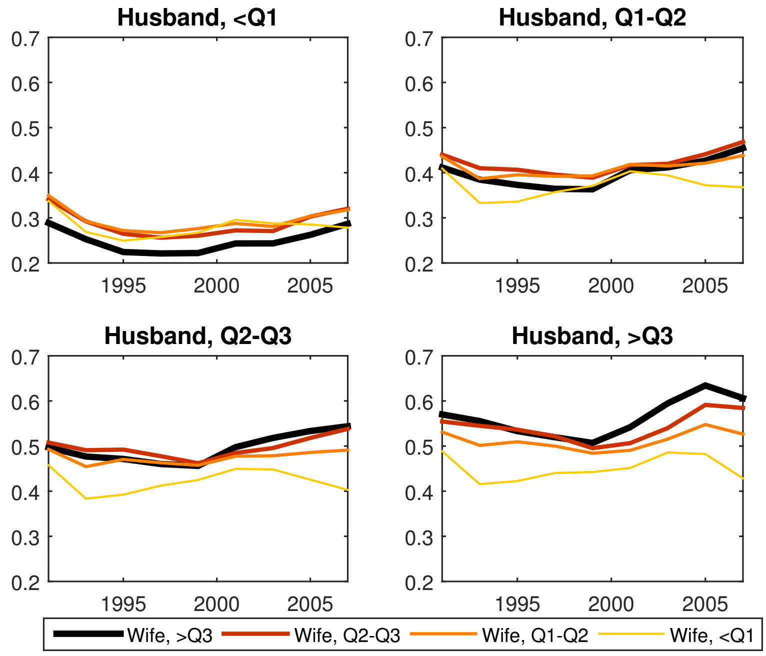


(a) α_{ji}

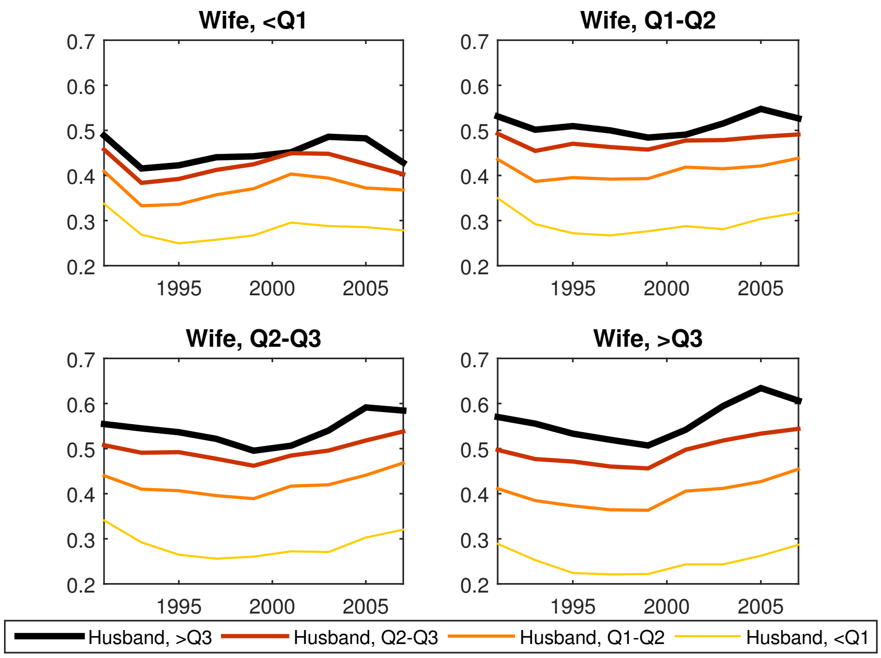


(b) α_{ij}

Fig. 7: Matching probabilities by spouses' family values indices

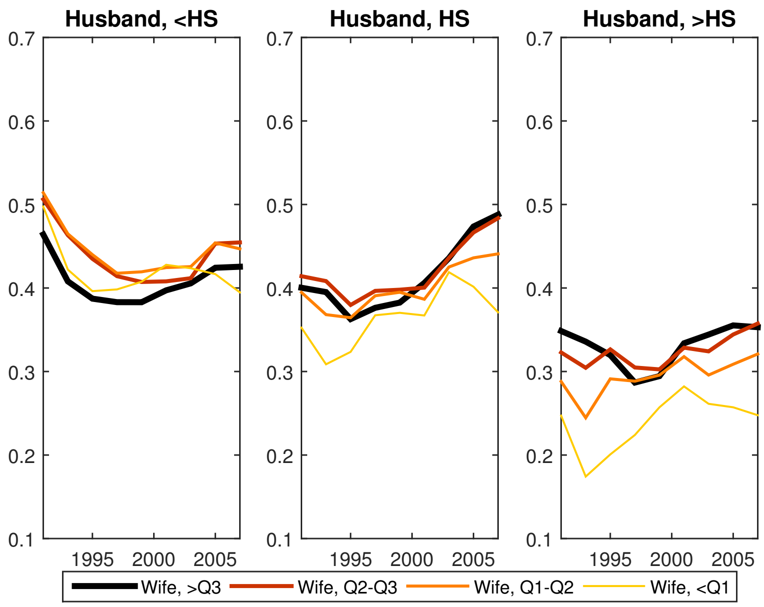


(a) α_{ji}

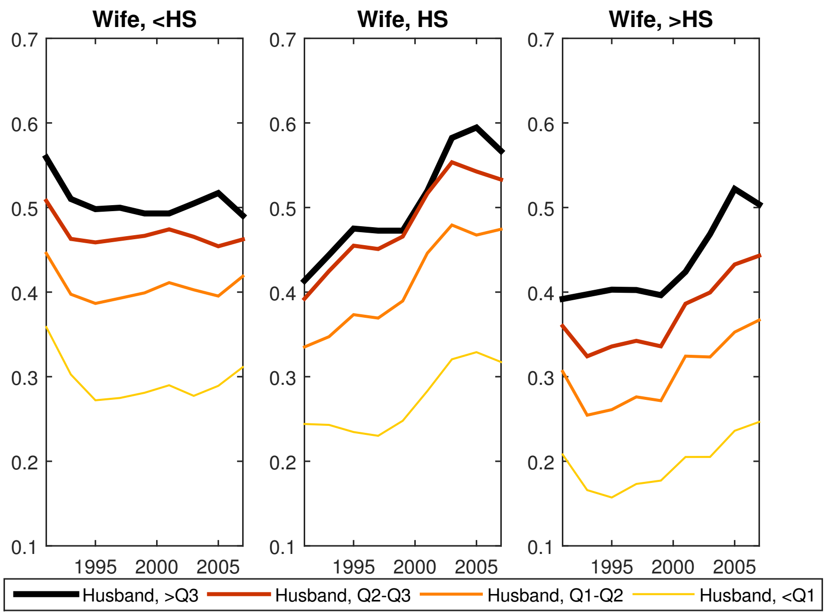


(b) α_{ij}

Fig. 8: Matching probabilities by spouses' wages



(a) Male education vs female wage



(b) Male wage vs female education

Fig. 9: Matching probabilities by wage and education

This is the aim of this section. We build a matching model on the equilibrium search-bargaining model of Shimer and Smith (2000), which we enrich with labor supply and home production decisions as in Manser and Brown (1980) and McElroy and Horney (1981).

4.1 Private preferences

Individuals draw utility from private consumption c_0, whose price is normalized to one, private leisure e, and a public good q that is produced in-house. Let $U_i(c_0, e, q)$ denote the utility function of an individual with exogenous characteristics i. For later use, we also define the conditional indirect utility function

$$\psi_i(R, q) = \max_{c_0, e} U_i(c_0, e, q) \text{ s.t. } c_0 + w_i e \leq R, \tag{4.1}$$

for a given income R and public good q, and where w_i is the wage that is associated to the individual type i. The demands for consumption and leisure follow from the indirect utility function by application of Roy's identity.

Singles have access to a household production technology that requires domestic time as single input: $q = F_i^0(d)$. For married couples, the home production inputs are the hours spent on domestic chores by both spouses (d_m, d_f) and some expenditure c.[12] This additional expenditure c may capture children's consumption although we abstract from modeling fertility. Domestic production for couples varies by spouses' types (i, j) and a match-specific scale z as $q = zF_{ij}^1(c, d_m, d_f)$. The match-specific component z is drawn from a distribution G at the time of first meeting and is infrequently updated, with independent draws from the same distribution G accruing at random times according to a Poisson process with parameter δ.

We normalize to one the total amount of time available per week to any individual. So market hours is $h \equiv 1 - e - d$, and w_i is both the wage rate and the total income available to the individual, to be shared between market work, non-market work and leisure. In this paper, we only consider interior solutions with $1 > e, d, h > 0$.

For a single of type i, income R in equation (4.1) is equal to $w_i(1 - d)$. For a married couple of male-female type (i, j), we need to allow for the possibility of transfers between spouses and for funding of the home production expenditure c. We therefore introduce intra-household "taxes" on private consumption t_m, t_f such that

$$c_{0m} + w_i e_m = w_i(1 - d_m) - t_m \equiv R_m, \quad c_{0f} + w_j e_f = w_j(1 - d_f) - t_f \equiv R_f.$$

[12] We do not observe home production expenditure. A home production function for singles of the form $q = F_i^0(c, d)$ would not be identifiable. We shall show that we can afford this extension for married couples.

The household budget is balanced if $c_{0m} + c_{0f} + c = w_i h_m + w_j h_f$, which implies that $c = t_m + t_f$.

4.2 Marriage contracts

Assuming that individuals do not make long term commitments and can walk away from the negotiation at any time, a marriage contract between a male of type i and a female of type j, for a current match-specific shock z, specifies a utility level for both spouses, u_m and u_f, and two promised continuation values, $V_m^1(z')$ and $V_f^1(z')$, for any realization z' of the next match-specific shock. Let V_i^0 and V_j^0 denote the equilibrium values of being single for type-i men, and type-j women.

Let W_m and W_f denote the present value of a marriage contract for any given choice of (u_m, u_f, V_m^1, V_f^1). The marriage contract values are related to the values of singlehood by the following option-value equation:

$$rW_m = u_m + \delta \int \left[\max\left\{V_i^0, V_m^1(z')\right\} - W_m\right] \mathrm{d}G(z'), \tag{4.2}$$

where r is the discount rate and the second term of the right-hand side is the option value of divorce after a shock to the match-specific component.

Marriage utilities u_m, u_f depend on controls c, d_m, d_f, t_m, t_f as

$$u_m = \psi_i\left[w_i(1-d_m) - t_m, q\right], \quad u_f = \psi_j\left[w_j(1-d_f) - t_f, q\right], \quad q = zF_{ij}^1(c, d_m, d_f), \tag{4.3}$$

and these controls are chosen so as to maximize the Nash bargaining criterion

$$\max_{c,d_m,d_f,t_m,t_f} \left[W_m - V_i^0\right]^\beta \left[W_f - V_j^0\right]^{1-\beta}, \tag{4.4}$$

subject to the feasibility constraint $c = t_m + t_f$.

Finally, without commitment, the promise-keeping constraint imposes that $W_m = V_m^1(z)$. Hence,

$$(r+\delta)\left[V_m^1(z) - V_i^0\right] = u_m + \delta \int \left[V_m^1(z') - V_i^0\right]^+ \mathrm{d}G(z') - rV_i^0, \tag{4.5}$$

denoting $x^+ \equiv \max\{x, 0\}$ and with a symmetric expression for $V_f^1(z)$.[13] Note that the equilibrium value of a marriage contract between spouses is a function of types i, j and z. We shall use the notation $V_m^1(i,j,z), V_f^1(i,j,z)$ whenever necessary to make precise the dependence of i, j.

[13] Notice that the solution to the bargaining problem in c, d_m, d_f, t_m, t_f is invariant to an affine change of indirect utility functions (ψ_i, ψ_j) into $(a_i + b_i\psi_i, a_j + b_j\psi_j)$. Utilities u_m, u_f and flow values $rV_m^1(z), rV_f^1(z)$ are then subject to the same affine changes.

The matching probability is the probability that the participation constraint holds at the current value of (i, j, z), that is

$$\alpha_{ij} = \Pr\{V_m^1(z) - V_i^0 \geq 0 \text{ and } V_f^1(z) - V_j^0 \geq 0\}.$$

4.3 The value of being single

The present value of singlehood follows as

$$\begin{aligned} rV_i^0 = \max_d \psi_i[w_i(1-d), F_i^0(d)] \\ + \lambda \iint \left[V_m^1(i,j,z) - V_i^0\right]^+ \mathbf{1}\{V_j^1(i,j,z) - V_j^0 > 0\} \mathrm{d}G(z)\, \widehat{n}_f(j)\, \mathrm{d}j, \end{aligned} \quad (4.6)$$

where $\widehat{n}_f(j)$ denotes singles' expectations about type distributions in the future. Single individuals have indeed to forecast their chances of meeting a partner of any type. Assuming that the economy is in a steady state easily solves the expectation formation problem. Any alternative solution would make the model tremendously more complicated.

4.4 Steady state

In steady state, flows in and out of the stocks of married couples of each type must exactly balance each other out. This means that, for all (i, j),

$$\delta\,(1 - \alpha_{ij})\, m(i,j) = \lambda n_m(i) n_f(j) \alpha_{ij}. \quad (4.7)$$

The left-hand side is the flow of divorces. The right-hand side is the flow of new (i, j)-marriages. It has three components: a single male of type i, out of the $n_m(i)$ identical ones, meets a single female with probability λN_f; this woman is of type j with probability $n_f(j)/N_f$; the marriage is consummated with probability α_{ij}.

Now, making use of the accounting restrictions,

$$\ell_m(i) = n_m(i) + \int m(i,j)\, \mathrm{d}j, \quad \ell_f(j) = n_f(j) + \int m(i,j)\, \mathrm{d}i,$$

and replacing $m(i,j)$ by its value from (4.7), i.e. $m(i,j) = \frac{\lambda}{\delta} \frac{\alpha(i,j)}{1-\alpha(i,j)} n_m(i) n_f(j)$, the equilibrium measures of singles, $n_m(i), n_f(j)$, are solutions to the following fixed-point system:

$$n_m(i) = \frac{\ell_m(i)}{1 + \frac{\lambda}{\delta} \int n_f(j) \frac{\alpha(i,j)}{1-\alpha(i,j)}\, \mathrm{d}j}, \quad n_f(j) = \frac{\ell_f(j)}{1 + \frac{\lambda}{\delta} \int n_m(i) \frac{\alpha(i,j)}{1-\alpha(i,j)}\, \mathrm{d}i}. \quad (4.8)$$

Note that λ may be an endogenous parameter, function of the aggregate numbers of singles N_m and N_f, say $\lambda(N_m, N_f)$. With quadratic returns, as in Shimer and Smith (2000), λ is constant. But a more reasonable assumption may be constant returns to scale in the meeting function $\lambda N_m N_f$, for example Cobb-Douglas. In this case, this dependence has to be incorporated into the equilibrium equations.

5 Equilibrium solution with transferable utility

We assume that the indirect utility is of the quasi-linear form

$$\psi_i(R, q) = q\frac{R - A_i}{B_i}, \tag{5.1}$$

where $A_i \equiv A_i(w_i)$ and $B_i \equiv B_i(w_i)$ are differentiable, non-decreasing and concave functions of the wage w_i and other individual characteristics such as gender and education. We also normalize the denominator as $B_i(1) = 1$.

We show in Appendix A that under this assumption the equilibrium satisfies the following two properties:

1. **Separability**. Domestic production inputs are determined independently of transfers and continuation values.

2. **Transferability**. There exists a match surplus $S_{ij}(z)$ that is shared between spouses, and matching requires positive surplus.

5.1 Separability

The first order conditions of the Nash bargaining problem with respect to domestic production are

$$\frac{1}{w_i}\frac{\partial \ln F^1_{ij}(c, d_m, d_f)}{\partial d_m} = \frac{1}{w_j}\frac{\partial \ln F^1_{ij}(c, d_m, d_f)}{\partial d_f} = \frac{\partial \ln F^1_{ij}(c, d_m, d_f)}{\partial c} = \frac{1}{X}$$

where $X \equiv w_i(1 - d_m) - A_i + w_j(1 - d_f) - A_j - c$ is the *net total private expenditure*, that is what is left of total income $w_i + w_j$ to be spent on private consumption and leisure, after spending $w_i d^1_m + w_j d^1_f + c$ on home production, above and beyond the minimal expenditures $A_i + A_j$.

These conditions deliver three functions $c(i, j), d^1_m(i, j), d^1_f(i, j)$ of observable match characteristics.[14] Let then X_{ij} denote the equilibrium value of X,

$$X_{ij} = w_i[1 - d^1_m(i, j)] - A_i + w_j[1 - d^1_f(i, j)] - A_j - c(i, j), \tag{5.2}$$

[14] These demand functions are independent of z because the production function for couples is proportional to z. This is a simplifying condition that is not essential for separability.

and let

$$Q_{ij} \equiv F_{ij}^1\left(c(i,j), d_m^1(i,j), d_f^1(i,j)\right) X_{ij}. \tag{5.3}$$

The function zQ_{ij} of (i,j,z) is the equilibrium value of the aggregate per-period welfare of the household, $u_m + u_f$.

5.2 Transferability

In Appendix A, we show that the first-order conditions of the Nash bargaining problem with respect to transfers yield the restrictions

$$(r+\delta)B_i\left[V_m^1(z) - V_i^0\right] = \beta S_{ij}(z), \quad (r+\delta)B_j\left[V_f^1(z) - V_j^0\right] = (1-\beta)S_{ij}(z), \tag{5.4}$$

and the match surplus $S_{ij}(z)$ solves

$$S_{ij}(z) = zQ_{ij} - B_i r V_i^0 - B_j r V_j^0 + \frac{\delta}{r+\delta}\overline{S}_{ij}, \tag{5.5}$$

where $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \,\mathrm{d}G(z')$ is the integrated surplus.

Let $\mathcal{G}(s) \equiv \int (z-s)^+ \,\mathrm{d}G(z) = \int_s^{+\infty} z \,\mathrm{d}G(z) - s[1-G(s)]$. The function $\mathcal{G}$ is decreasing and invertible on the support of G (with $\mathcal{G}' = -(1-G)$). We show that $\overline{S}_{ij}$ solves

$$\overline{S}_{ij} = Q_{ij}\mathcal{G}\left(\frac{B_i r V_i^0 + B_j r V_j^0 - \frac{\delta}{r+\delta}\overline{S}_{ij}}{Q_{ij}}\right), \tag{5.6}$$

and equation (4.6) becomes

$$B_i r V_i^0 = B_i \psi_i^0 + \frac{\lambda\beta}{r+\delta}\int \overline{S}_{ij} n_f(j)\,\mathrm{d}j, \quad B_j r V_j^0 = B_j \psi_j^0 + \frac{\lambda(1-\beta)}{r+\delta}\int \overline{S}_{ij} n_m(i)\,\mathrm{d}i, \tag{5.7}$$

respectively for single men and women.

A match (i,j,z) is consummated if $S_{ij}(z) \geq 0$, and the matching probability is

$$\alpha_{ij} = \Pr\{S_{ij}(z) > 0\} = 1 - G\left[\mathcal{G}^{-1}\left(\frac{\overline{S}_{ij}}{Q_{ij}}\right)\right]. \tag{5.8}$$

Lastly, equilibrium transfers, $t_m(i,j,z)$ and $t_f(i,j,z)$, are a way of sharing net total private expenditure X_{ij}:

$$w_i(1-d_m^1) - t_m - A_i = \beta_{ij}(z)X_{ij}, \quad w_j(1-d_f^1) - t_f - A_j = [1-\beta_{ij}(z)]X_{ij}, \tag{5.9}$$

where

$$\beta_{ij}(z) = \beta + \frac{1}{z}\frac{(1-\beta)B_i r V_i^0 - \beta B_j r V_j^0}{Q_{ij}}. \tag{5.10}$$

(Note that indirect utility can be negative.)

5.3 Equilibrium

The equilibrium is fully characterized by the following functions of individual types, $\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0, n_m(i), n_f(j)$. They are obtained by iterating the fixed-point operator defined by equations (4.8), (5.6), (5.7), using (5.8) for α_{ij} and with $N_m = \int n_m(i)\,\mathrm{d}i$ and $N_f = \int n_f(j)\,\mathrm{d}j$. Proving equilibrium existence is difficult (possibly by application of Schauder fixed point theorem as in Shimer and Smith, 2000) and the equilibrium is likely not unique.[15]

6 Specification and estimation

In this section we specify parametrically the utility functions and the production functions, and explain how we let them depend on individual types. Then, we develop the constructive identification/estimation procedure. Finally, we comment on goodness of fit and parameter estimates.

6.1 Parametric specification

The way parameters depend on exogenous variables (gender g_i, education Ed_i, wage w_i and family values index x_i) is specified as follows. We allow private preferences to depend on gender, education and wages, but not on the family values index. Domestic production depends on gender, education and family values but not on wages. We assume that once domestic production has been collectively chosen, the private trade-off between consumption and leisure does not depend on the family values index. We make this exclusion restriction to introduce a source of independent variation in housework hours. This is not strictly necessary for identification as we shall see in the next subsection on identification.

Preferences. The indirect utility for consumption and leisure is a smooth function of gender and wage with

$$A_i = -\frac{a_{0g}(Ed_i)}{b_g} + \frac{a_{1g}}{1-b_g} w_i + \frac{a_{2g}}{2-b_g} w_i^2, \quad \ln B_i \equiv b_g \ln w_i, \quad g_i = g \in \{m, f\}, \tag{6.1}$$

with $a_{0g}(Ed_i) = a_{0gH}$ or a_{0gL}, depending on the education indicator $Ed_i \in \{H, L\}$, where L refers to high school dropout or vocational and H to higher education (high school and

[15] In order to calculate a fixed point $x = Tx$ it is often useful, if the operator is not contracting, to use iterations of the form: $x_{n+1} = k\,Tx_n + (1-k)x_n$, with $0 < k < 1$. In our experiments, this algorithm always converged fast to a single equilibrium.

higher). The demand for leisure is thus

$$w_i e_i = a_{0g}(Ed_i) + a_{1g} w_i + a_{2g} w_i^2 + b_g R_i, \quad R_i = w_i(1-d_i) - t_i, \tag{6.2}$$

given domestic time d_i and transfer t_i ($t_i = 0$ for singles).

Domestic production. The domestic production functions are Stone-Geary:

$$[couples] \quad F_{ij}^1(c, d_m, d_f) = Z_{ij}\left(c - C_{ij}\right)^{K_c}\left(d_m - D_i^1\right)^{K_m^1}\left(d_f - D_j^1\right)^{K_f^1}, \tag{6.3}$$

$$[singles] \quad F_i^0(d) = (d - D_i^0)^{K_i^0}. \tag{6.4}$$

Note the absence of TFP parameter in front of the home production for singles. This is a normalizing constraint that is rendered necessary by the ordinal nature of preferences. Choo and Siow (2006), Galichon and Salanié (2012) make a similar assumption. The time used in home production is therefore – for singles

$$d^0 = D_i^0 + \tfrac{K_i^0}{1+K_i^0}\left(1 - D_i^0 - A_i/w_i\right), \tag{6.5}$$

and for couples,

$$d_m^1 = D_i^1 + K_m^1 X_{ij}/w_i, \quad d_f^1 = D_j^1 + K_f^1 X_{ij}/w_j, \quad c = C_{ij} + K_c X_{ij}, \tag{6.6}$$

with net total private expenditure (see equation (5.2)) being

$$X_{ij} = \frac{w_i(1-D_i^1) + w_j(1-D_j^1) - C_{ij} - A_i - A_j}{1 + K_c^1 + K_i^1 + K_j^1}. \tag{6.7}$$

The equilibrium domestic productions are, for singles,

$$F_i^0(d^0) = \left[\tfrac{K_i^0}{1+K_i^0}\left(1 - D_i^0 - A_i/w_i\right)\right]^{K_i^0}, \tag{6.8}$$

and for couples,

$$F_{ij}^1(c, d_m^1, d_f^1) = Z_{ij}\left(K_c^1\right)^{K_c^1}\left(\frac{K_i^1}{w_i}\right)^{K_i^1}\left(\frac{K_j^1}{w_j}\right)^{K_j^1} X_{ij}^{K_c^1+K_i^1+K_j^1}. \tag{6.9}$$

This specification is quite flexible. Suppose that some factor increases D_j^1. Then the wife will increase her non-market hours and her husband will reduce his input to home production. Minimal inputs (D_i^1, D_j^1) thus govern home-production specialization. At the same time, increasing D_j^1 reduces X_{ij}, hence output $F_{ij}^1(c, d_m^1, d_f^1)$. It is therefore important to allow the factors determining minimal inputs to determine public good quality Z_{ij} (home production TFP) at the same time.

Hence, we specify minimal inputs as

$$D_i^0 \equiv D_g^0(Ed_i, x_i) = \delta_{0g}^0(Ed_i) + \delta_{1g}^0 x_i, \quad K_i^0 = K_g^0, \quad g_i = g \in \{m, f\}, \tag{6.10}$$

with $\delta_{0g}^0(Ed) = \delta_{0gH}^0$ or δ_{0gL}^0, for $Ed \in \{H, L\}$, and

$$D_i^1 = D_m^1(Ed_i, x_i) = \delta_{0m}^1(Ed_i) + \delta_{1m}^1 x_i, \quad K_i^1 = K_m^1, \tag{6.11}$$

$$D_j^1 = D_f^1(Ed_j, x_j) = \delta_{0f}^1(Ed_j) + \delta_{1f}^1 x_j, \quad K_j^1 = K_f^1, \tag{6.12}$$

with $\delta_{0g}^1(Ed) = \delta_{0gH}^1$ or δ_{0gL}^1, $g \in \{m, f\}$. We also specify C_{ij} with no interactions between spouses' types:

$$C_{ij} \equiv C + c_{mH}\mathbf{1}(Ed_i = H) + c_{fH}\mathbf{1}(Ed_j = H) + c_{1m}x_i + c_{1f}x_j, \tag{6.13}$$

where $C, c_{mH}, c_{fH}, c_{1m}, c_{1f}$ are 5 scalar parameters and $\mathbf{1}(Ed_i = H)$ is equal to 1 if $Ed_i = H$ and 0 otherwise. Lastly, public good quality Z_{ij} is a general function of both spouses' characteristics:

$$Z_{ij} \equiv Z(Ed_i, w_i, x_i, Ed_j, w_j, x_j), \tag{6.14}$$

that will be estimated non-parametrically as indicated below. As already indicated, it is important that education and family values, which determine minimal inputs, also determine Z_{ij}. We also let Z_{ij} depend on wages and on interactions between male and female factors. This will allow us to estimate the source of marriage externalities that is not already accounted for by task-sharing.

Lastly, the distribution of match-specific shocks z is log-normal: $G(z) = \Phi(\ln(z)/\sigma_z)$, where Φ is the standard normal cdf and σ_z is the standard deviation of z.[16]

6.2 Identification

Although identification may hold under far less restrictive assumptions, we only discuss identification under the preceding parametric restrictions.

First consider singles' demand for leisure, which equation (6.2) tells us that it depends on education, wage and home production time as

$$w_i e^0 = a_{0g}(Ed_i) + a_{1g}w_i + a_{2g}w_i^2 + b_g w_i(1 - d^0). \tag{6.15}$$

This equation is identified if there is exogenous variation in non market time d^0, which is

[16] We then have

$$\mathcal{G}(s) = -s\Phi\left(-\frac{\ln s}{\sigma_z}\right) + e^{\frac{\sigma_z^2}{2}}\Phi\left(-\frac{\ln s}{\sigma_z} + \sigma_z\right).$$

the case if d^0 varies with family values. If non market hours d^0 do not exhibit independent variation, then $a_{1g} + b_g$ is identified but not a_{1g} and b_g separately. This is the usual identification issue with labor supply models. They are not identified unless a source of non-earned income is observed.

Next, consider married couples. The leisure equation (6.2) together with equations (5.9), (5.10) for transfers implies that

$$w_i e_m^1 = \frac{a_{1m}}{1-b_m} w_i + \frac{2a_{2m}}{2-b_m} w_i^2 + b_m \beta_{ij}(z) X_{ij}, \tag{6.16}$$

$$w_j e_f^1 = \frac{a_{1f}}{1-b_f} w_j + \frac{2a_{2f}}{2-b_f} w_j^2 + b_f [1-\beta_{ij}(z)] X_{ij}. \tag{6.17}$$

The sharing rule $\beta_{ij}(z)$ depends on structural parameters and characteristics, wages in particular, in a complicated, nonlinear way. One can remove the dependency of $\beta_{ij}(z)$ on z by averaging. However, the potential link between $\beta_{ij}(z)$ and wages makes it unlikely that these two equations suffice to identify preference parameters. If $\beta_{ij}(z)$ does not vary with wages, then it is easy to see that equations (6.15) and (6.16), (6.17) together identify preference parameters.[17]

However, adding up equations (6.16) and (6.17) yields

$$\frac{w_i}{b_m}\left(e_m^1 - \frac{a_{1m}}{1-b_m} - \frac{2a_{2m}}{2-b_m} w_i\right) + \frac{w_j}{b_f}\left(e_f^1 - \frac{a_{1f}}{1-b_f} - \frac{2a_{2f}}{2-b_f} w_j\right) = X_{ij}. \tag{6.18}$$

Then, identification of the home production function for couples **and** parameters b_m, b_f follows from the demand equations

$$d_m^1 = D_i^1 + K_m^1 X_{ij}/w_i, \quad d_f^1 = D_j^1 + K_f^1 X_{ij}/w_j.$$

As D_i^1 and D_j^1 do not depend on wages, it is easy to see that differentiating d_m^1 and d_f^1 with respect to wages w_i, w_j delivers plenty of identifying restrictions. For example,

$$\frac{\partial w_i d_m^1}{\partial w_i} = D_i^1 + K_m^1 \left[\frac{1}{b_m}\left(\frac{\partial w_i e_m^1}{\partial w_i} - \frac{a_{1m}}{1-b_m} - \frac{4a_{2m}}{2-b_m} w_i\right) + \frac{1}{b_f}\frac{\partial w_j e_f^1}{\partial w_i}\right]$$

suffices to identify K_m^1, b_m, b_f. The remaining parameters of the home-production function follow from the equilibrium value of X_{ij}:

$$X_{ij} = \frac{w_i(1-D_i^1) + w_j(1-D_j^1) - C_{ij} - A_i - A_j}{1 + K_c^1 + K_m^1 + K_f^1}.$$

Finally, we show in Appendix B that values $\overline{S}_{ij}, V_i^0, V_j^0$ are uniquely obtained from

[17] Differentiate equations (6.16), (6.17) with respect to wages and eliminate all dependency to $\beta_{ij}(z)$ and X_{ij}.

matching probabilities α_{ij} given the other parameters, i.e. those already identified, β (bargaining power parameter) and σ_z (standard deviation of the match shock z). This inversion result is similar, though different, to the argument in Choo and Siow (2006), Galichon and Salanié (2012) for static, competitive matching models. This type of inversion argument is common to many decision and equilibrium models.[18] It allows to identify β and σ_z from the first- and second-order moments of private leisure e_m^1, e_f^1 (or market time). Note that we already used $w_i e_m^1/b_m + w_j e_f^1/b_f$ to identify X_{ij} and the home production function for couples. Parameter β is thus identified by the way this sum is shared between spouses. Then σ_z is identified from the residual dispersion of leisure expenditures conditional on observed types.

6.3 Estimation strategy

Let us consider one two-year cross-section of household data on time uses, gender, wages, family values and education. We use couples of years to increase sample size. The index i now refers to an observation unit of the sample of male singles, j refers to female singles, and (i, j) refers to couples. For singles, we observe domestic time use d_i^0, labor supply h_i^0 and education $Ed_i \in \{L, H\}$, wages w_i and family values indices x_i. For couples, the corresponding time use observations are $d_{mij}^1, d_{fij}^1, h_{mij}^1$ and h_{fij}^1. Leisure is $e = 1 - d - h$.

The estimation procedure is iterative and goes through the following steps.

1. Estimate $\lambda, \delta, \alpha_{ij}$ as indicated in Section 3 from nonparametric estimates of stock densities $n_m(i), n_f(j), m(i, j)$ and corresponding flows. In practice we smooth out fluctuations in λ, δ by taking the average over all two-year cross-sections.

2. Given a value for σ_z, estimate the parameters of preferences and domestic productions, as well as bargaining power β, by two-stage GMM[19] based on the following residuals and instruments:

   (a) For single men, the residuals are

$$u_i^0 = \begin{pmatrix} d_i^0 - D_m^0(Ed_i, x_i) - \frac{K_m^0}{1+K_m^0}[1 - D_m^0(Ed_i, x_i) - A_i/w_i] \\ e_i^0 - a_{0m}(Ed_i)/w_i - a_{1m} - a_{2m}w_i - b_m(1 - d_i^0) \end{pmatrix},$$

   with a similar expression for single women. The instruments are

$$\xi_i = (1, \mathbf{1}(Ed_i = H), x_i, w_i, \mathbf{1}(Ed_i = H)/w_i)\,.$$

   This is the way the exogenous characteristics condition the residuals.

[18] See e.g. Hotz and Miller (1993), Berry, Levinsohn, and Pakes (1995), Laffont, Ossard, and Vuong (1995), and Bontemps, Robin, and Van den Berg (1999, 2000).

[19] First with metric equal to the identity matrix and second with metric equal to the diagonal of the inverse of variance-covariance matrix of moments.

(b) For couples, the residuals are

$$u_{ij}^1 = \begin{pmatrix} d_{mij}^1 - D_m^1(Ed_i, x_i) - K_m^1 X_{ij}/w_i \\ d_{fij}^1 - D_f^1(Ed_j, x_j) - K_f^1 X_{ij}/w_j \\ e_{mij}^1 - \frac{a_{1m}}{1-b_m} - \frac{2a_{2m}}{2-b_m} w_i - b_m \overline{\beta}_{ij} X_{ij}/w_i \\ e_{fij}^1 - \frac{a_{1f}}{1-b_f} - \frac{2a_{2f}}{2-b_f} w_j - b_f(1-\overline{\beta}_{ij}) X_{ij}/w_j \end{pmatrix},$$

with

$$X_{ij} = \frac{w_i[1 - D_m^1(Ed_i, x_i)] + w_j[1 - D_f^1(Ed_j, x_j)] - C_{ij} - A_i - A_j}{1 + K_c^1 + K_m^1 + K_m^1},$$
$$\overline{\beta}_{ij} = \beta + \mathbb{E}\left(\frac{1}{z}|z \geq G^{-1}(1-\alpha_{ij})\right) \frac{(1-\beta)B_i r V_i^0 - \beta B_j r V_j^0}{Q_{ij}}.$$

The instruments are $\xi_i \otimes \xi_j$.[20] The leisure residuals follow from equations (6.16), (6.17), (5.9), after taking the expectation with respect to z.[21] We back out $\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0$ and Q_{ij} from matching probabilities α_{ij}, given type densities $n_m(i), n_f(j)$ and the other parameters, by solving a fixed-point system similar to the equilibrium system in Section 5. See Appendix B.2 for details.

3. Estimate σ_z by fitting the variance-covariance matrix of market hours for couples. See Appendix B.3 for details. Then repeat steps 2 and 3 until numerical convergence. This simple iterative procedure worked in our case although there is no guaranty that it would always converge.

4. Lastly, estimate public good quality Z_{ij} from equation (5.3):

$$Z_{ij} = Q_{ij} \left[\left(K_c^1\right)^{K_c^1} \left(\frac{K_i^1}{w_i}\right)^{K_i^1} \left(\frac{K_j^1}{w_j}\right)^{K_j^1} X_{ij}^{1+K_c^1+K_i^1+K_j^1} \right]^{-1}.$$

Once the model has been estimated, an economy can be simulated by computing the equilibrium as indicated in Section 5. Specifically, for every two-year cross section, given estimated parameters and the observed distributions of male and female types in the population (i.e. $\ell_m(i), \ell_f(j)$), we use the equilibrium fixed point to calculate conditional distributions $n_m(i), n_f(j), m(i,j)$ together with values $\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0$. This is what we have to do, in particular, to evaluate the goodness of fit of the model.

Finally, individual types comprise one continuous variable, the wage, and the family values index is approximately continuous as it is constructed by aggregation of many

[20] Many interaction terms can be omitted without prejudice.

[21] Note that $\mathbb{E}\left(\frac{1}{z}|z > s\right) = e^{\frac{\sigma_z^2}{2}} \Phi\left(-\frac{\ln s}{\sigma_z} - \sigma_z\right)/\Phi\left(-\frac{\ln s}{\sigma_z}\right)$ for $z \sim \mathcal{LN}(0, \sigma_z^2)$, and that marriage is consummated if $z \geq G^{-1}(1-\alpha_{ij})$ for the marriage probability to be equal to α_{ij}.

discrete variables. Hence, functions $\overline{S}_{ij}$, $B_i r V_i^0$, $B_j r V_j^0$, $n_m(i)$, $n_f(j)$, $m(i,j)$ have to be discretized and integrals in equilibrium operators have to be approximated. We rely for that on Clenshaw-Curtis quadrature and Chebyshev polynomials (see Trefethen, 2013). These numerical techniques allow to approximate functions and to operate on these approximations (differentiation, integration, interpolation) in a very economical way as far as memory storage is concerned.[22]

6.4 Fit

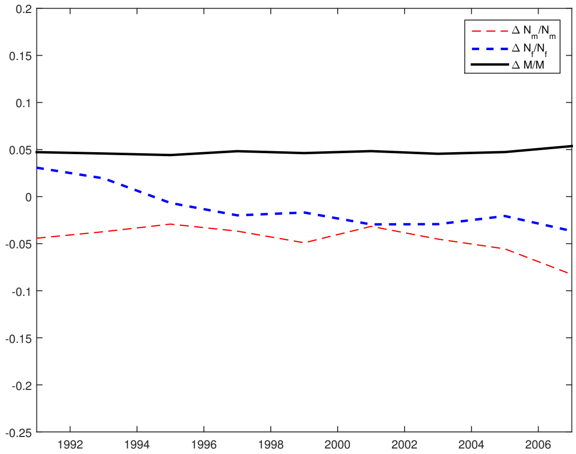
Fitting the distributions of types (i.e. education, wages and family values indices), and the functional links between market and non-market hours and types, by gender and marital status, and how they change over time, represented a significant challenge that our simple model tackled brilliantly. Figure 10 shows how the model fits the numbers of single men and women over time, and the fit of time trends for market and non-market hours. We tried the simulations both with a quadratic meeting function (λ constant) and with a linearly homogenous one. Surprisingly enough, the fit was slightly better with λ constant. An online appendix contains figures and tables for the fit of the distributions of wages and family values indices for all cross-sections, as well as conditional means of market and non market hours and matching probabilities. The fit is good for all 9 cross sections.

6.5 Parameter estimates

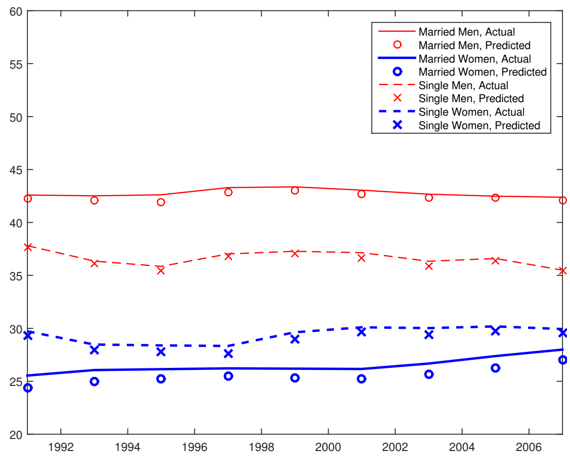
We now comment on some parameter estimates, which are all displayed in Table 3. The bargaining power parameter β is estimated close to 0.5, which is good news as it indicates that rent sharing is determined by outside options and not by the generalized Nash bargaining parameter. The (log) match-specific shock has an estimated standard error that is estimated around 1/3, which is far from negligible. This implies that it moves public good quality in a plus-or-minus-50% range with 95% probability.

We will return to the discussion of public good estimates and the effect of wages in the following sections. The most striking difference between men and women, as far as home production is concerned, is found in the parameter of the family values index conditioning the level of non-market time of couples (δ_{1m} and δ_{1f}). In the early 1990s, traditional married women devoted much more time to home production than progressive women, and the opposite was true for men. This very strong link between non-market time and the family values index was greatly attenuated by the end of the period. Moreover, family values have little impact on home production for singles. We

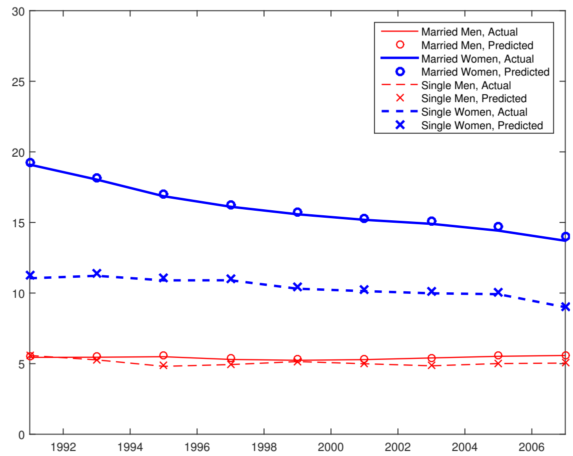
[22] For example, evaluating a smooth bounded function f on a bounded interval at 11 Chebishev points (including the two extreme bounds) is like approximating this function by a 10th-order polynomial. (See Appendix C for details.)



(a) Numbers of singles and married couples



(b) Mean market hours



(c) Mean non-market hours

Fig. 10: Fit

also find that education is discriminating for females but not for males; low-educated women are willing to spend more time in home production tasks, whatever the marriage status, and this gender-education differential is stable over time.

7 Sorting

The fit of marriage probabilities is very good because of the one-to-one relationship between marriage probability and public good quality Z_{ij}. If we project the estimated $\ln Z_{ij}$ on spouses' types and their interactions, we find very similar estimates than for the quadratic projection of matching probabilities displayed in Table 2. So we do not report them.

Table 4 shows the R2 of the regression of the actual matching probabilities (assigned to every couple in the sample)[23] on the equilibrium matching probabilities predicted using

[23] More precisely we calculate $R^2 = 1 - \mathrm{Var}(u)/\mathrm{Var}(y)$, where $u = y - \widehat{y}$.

Tab. 3: Estimated parameters

	1991	1993	1995	1997	1999	2001	2003	2005	2007
β	0.4969 (0.1795)	0.5181 (0.1381)	0.5182 (0.1176)	0.4909 (0.1247)	0.4815 (0.1437)	0.4802 (0.1251)	0.4807 (0.1317)	0.4800 (0.1055)	0.4956 (0.1062)
σ_z	0.3247 (0.1205)	0.2981 (0.1160)	0.2981 (0.0608)	0.3148 (0.0532)	0.3583 (0.0445)	0.3718 (0.0486)	0.3431 (0.0542)	0.3981 (0.0694)	0.2973 (0.0616)
Preferences									
a_{0fL}	0.7022 (0.1060)	0.7009 (0.1002)	0.7992 (0.1298)	0.7900 (0.1037)	0.7029 (0.1181)	0.8873 (0.1506)	1.0082 (0.1408)	0.7001 (0.1242)	0.7145 (0.1308)
a_{0mL}	0.7329 (0.1539)	0.9505 (0.1406)	1.4148 (0.1515)	0.8795 (0.1176)	0.8578 (0.1259)	1.3126 (0.1686)	1.5358 (0.1773)	1.0750 (0.1435)	1.0790 (0.1325)
a_{0fH}	0.7936 (0.1097)	0.8361 (0.1046)	1.0675 (0.1414)	0.9398 (0.1179)	0.7888 (0.1241)	1.0657 (0.1470)	1.1365 (0.1344)	0.7719 (0.1251)	0.8040 (0.1226)
a_{0mH}	0.8896 (0.1579)	1.2926 (0.1462)	1.6578 (0.1620)	1.2048 (0.1252)	1.0840 (0.1329)	1.5701 (0.1752)	1.7770 (0.1796)	1.3534 (0.1510)	1.5012 (0.1343)
a_{1f}	0.2403 (0.0543)	0.2528 (0.0492)	0.1798 (0.0491)	0.2035 (0.0337)	0.2436 (0.0448)	0.2074 (0.0526)	0.2331 (0.0540)	0.2832 (0.0346)	0.2481 (0.0421)
a_{1m}	0.0876 (0.0638)	0.1283 (0.0554)	0.0520 (0.0476)	0.0847 (0.0427)	0.0627 (0.0500)	0.0005 (0.0633)	0.0009 (0.0656)	0.0774 (0.0478)	0.0990 (0.0463)
a_{2f}	0.0034 (0.0019)	0.0007 (0.0017)	0.0022 (0.0020)	0.0020 (0.0013)	0.0020 (0.0013)	0.0040 (0.0015)	0.0028 (0.0013)	0.0001 (0.0010)	-0.0009 (0.0011)
a_{2m}	0.0029 (0.0015)	0.0020 (0.0012)	0.0047 (0.0013)	0.0016 (0.0009)	0.0014 (0.0009)	0.0035 (0.0012)	0.0036 (0.0012)	0.0014 (0.0009)	0.0014 (0.0008)
b_{1f}	0.1678 (0.0394)	0.1898 (0.0378)	0.2444 (0.0392)	0.2436 (0.0316)	0.2133 (0.0396)	0.1962 (0.0382)	0.1756 (0.0403)	0.1950 (0.0316)	0.2517 (0.0413)
b_{1m}	0.3057 (0.0551)	0.2502 (0.0434)	0.2518 (0.0333)	0.3051 (0.0365)	0.3441 (0.0475)	0.3392 (0.0482)	0.3270 (0.0467)	0.3197 (0.0398)	0.2985 (0.0396)
Home production, singles									
δ^0_{0fL}	0.0347 (0.0258)	0.0635 (0.0246)	0.0481 (0.0301)	0.0448 (0.0299)	0.0446 (0.0320)	0.0814 (0.0298)	0.0745 (0.0285)	0.0724 (0.0304)	0.0892 (0.0272)
δ^0_{0mL}	0.0607 (0.0178)	0.0579 (0.0137)	0.0422 (0.0141)	0.0454 (0.0128)	0.0357 (0.0180)	0.0284 (0.0133)	0.0437 (0.0131)	0.0422 (0.0146)	0.0375 (0.0141)
δ^0_{0fH}	-0.0086 (0.0232)	0.0041 (0.0228)	-0.0091 (0.0294)	-0.0098 (0.0285)	0.0013 (0.0300)	0.0368 (0.0277)	0.0317 (0.0260)	0.0322 (0.0280)	0.0580 (0.0255)
δ^0_{0mH}	0.0588 (0.0168)	0.0552 (0.0130)	0.0382 (0.0132)	0.0422 (0.0125)	0.0353 (0.0174)	0.0339 (0.0129)	0.0475 (0.0127)	0.0463 (0.0145)	0.0411 (0.0140)
δ^0_{1f}	0.0032 (0.0061)	0.0074 (0.0064)	0.0067 (0.0071)	0.0066 (0.0063)	0.0083 (0.0072)	0.0039 (0.0077)	0.0070 (0.0068)	0.0054 (0.0070)	0.0040 (0.0068)
δ^0_{1m}	0.0012 (0.0031)	0.0012 (0.0030)	0.0051 (0.0035)	0.0045 (0.0029)	0.0081 (0.0032)	0.0091 (0.0029)	0.0035 (0.0029)	0.0045 (0.0031)	0.0065 (0.0031)
K^0_f	0.0770 (0.0203)	0.0570 (0.0181)	0.0697 (0.0212)	0.0817 (0.0248)	0.0741 (0.0276)	0.0396 (0.0181)	0.0349 (0.0156)	0.0509 (0.0237)	0.0224 (0.0239)
K^0_m	0.0003 (0.0106)	0.0001 (0.0061)	0.0001 (0.0041)	0.0002 (0.0062)	0.0002 (0.0103)	0.0001 (0.0058)	0.0001 (0.0055)	0.0001 (0.0086)	0.0004 (0.0071)
Home production, couples									
δ^1_{0fL}	0.0971 (0.0117)	0.0778 (0.0109)	0.0786 (0.0096)	0.0852 (0.0080)	0.0969 (0.0082)	0.0977 (0.0078)	0.0822 (0.0070)	0.0996 (0.0077)	0.0945 (0.0080)
δ^1_{0mL}	0.0605 (0.0082)	0.0565 (0.0068)	0.0571 (0.0065)	0.0506 (0.0064)	0.0421 (0.0069)	0.0470 (0.0063)	0.0579 (0.0064)	0.0582 (0.0080)	0.0543 (0.0083)
δ^1_{0fH}	0.0485 (0.0102)	0.0296 (0.0095)	0.0398 (0.0085)	0.0471 (0.0070)	0.0530 (0.0073)	0.0537 (0.0070)	0.0450 (0.0063)	0.0673 (0.0068)	0.0633 (0.0071)
δ^1_{0mH}	0.0593 (0.0078)	0.0542 (0.0065)	0.0552 (0.0062)	0.0524 (0.0061)	0.0481 (0.0067)	0.0521 (0.0059)	0.0605 (0.0058)	0.0600 (0.0072)	0.0541 (0.0075)
δ^1_{1f}	0.0340 (0.0039)	0.0344 (0.0037)	0.0278 (0.0033)	0.0266 (0.0029)	0.0258 (0.0029)	0.0227 (0.0029)	0.0237 (0.0027)	0.0151 (0.0031)	0.0115 (0.0031)
δ^1_{1m}	-0.0103 (0.0017)	-0.0077 (0.0015)	-0.0079 (0.0015)	-0.0095 (0.0013)	-0.0080 (0.0014)	-0.0074 (0.0014)	-0.0063 (0.0014)	-0.0054 (0.0017)	-0.0033 (0.0018)
C	6.4912 (1.5952)	5.7834 (1.5556)	4.4648 (1.4147)	5.6610 (1.0576)	5.8145 (1.2019)	6.9979 (1.5299)	8.0896 (2.0303)	7.2098 (1.4034)	4.7255 (1.4898)
c_{1f}	-0.8693 (0.2851)	-0.8261 (0.3198)	-0.5089 (0.3338)	-0.8781 (0.2544)	-0.7567 (0.2471)	-0.8822 (0.2840)	-0.7016 (0.2922)	-0.8505 (0.2867)	-0.3570 (0.2876)
c_{1m}	-0.7494 (0.3041)	-0.5378 (0.3224)	-0.4846 (0.3591)	-0.5965 (0.2846)	-0.6549 (0.2738)	-1.0198 (0.3123)	-1.2674 (0.3668)	-0.9872 (0.3769)	-0.5050 (0.3469)
c_{fH}	0.5027 (0.4638)	0.2828 (0.4438)	0.4765 (0.4448)	0.3940 (0.4196)	0.2484 (0.4419)	0.4638 (0.5114)	-0.2081 (0.5811)	-0.5772 (0.5146)	-0.7786 (0.4894)
c_{mH}	-0.2923 (0.3167)	0.3457 (0.3835)	-0.2032 (0.3562)	0.2067 (0.2722)	-0.2399 (0.2653)	-0.6561 (0.2941)	-0.9349 (0.3508)	-0.4112 (0.3499)	0.5143 (0.3566)
K^1_f	0.0154 (0.0020)	0.0199 (0.0022)	0.0180 (0.0021)	0.0177 (0.0020)	0.0138 (0.0019)	0.0132 (0.0016)	0.0155 (0.0015)	0.0202 (0.0022)	0.0247 (0.0026)
K^1_m	0.0217 (0.0035)	0.0186 (0.0027)	0.0162 (0.0024)	0.0245 (0.0029)	0.0245 (0.0033)	0.0163 (0.0023)	0.0100 (0.0018)	0.0121 (0.0025)	0.0122 (0.0026)
K_c	0.0208 (0.0660)	0.0299 (0.0678)	0.0372 (0.0592)	0.0378 (0.0501)	0.0118 (0.0463)	0.0034 (0.0416)	0.0095 (0.0636)	0.0220 (0.0515)	0.0224 (0.0528)

Notes: Standard errors in parentheses.

Tab. 4: Fit of matching probabilities

Year	1991	1993	1995	1997	1999	2001	2003	2005	2007	Mean
Nonparametric estimation of Z_{ij}										
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Quadratic projection of $\ln Z_{ij}$										
R^2	0.71	0.77	0.81	0.55	0.79	0.79	0.84	0.82	0.76	0.76
Quadratic projection of $\ln Z_{ij}$ without interactions										
R^2	0.41	0.30	0.45	0.51	0.26	0.38	0.64	0.37	0.56	0.43

Fig. 11: Matching probabilities by spouses' education with no spouse complementarities in public good quality (Counterfactual for Fig. 6(a))

the nonparametric estimates of Z_{ij} (perfect because of the one-to-one binding function), the regression on the marriage probabilities predicted by the quadratic projection of the nonparametric estimates of $\ln Z_{ij}$, and finally the regression on the probabilities predicted by the same quadratic projection without male-female interaction terms. By using the quadratic approximation the fit is reduced by about 25%. Removing male-female complementarities reduces the fit further by 33%. Therefore the contribution of male-female complementarities in public good quality may account for about 43% ($= 1 - \frac{0.43}{0.76}$) of the total variance of marriage probabilities.

The next question is how much of the actual sorting that we described in Section 3 is explained by the model without interaction terms in public good quality? Put in another way, how much sorting is explained by task sharing vis-à-vis preferences? The answer is none. We just give here one example by showing the matching probabilities by husband and wife education that are predicted without spouse complementarities in Z_{ij} (Fig. 11). The same differences across female education groups can be observed for all male education groups. All homophily by education is gone. We have put in the web appendix the other sorting patterns that we commented in Section 3. There is no sorting left, neither positive (homophily) nor negative (specialization). Hence, our model predicts that one does not marry in anticipation of gains from specialization, but because birds of a feather flock together. This confirms the argument developed by Stevenson and Wolfers (2007) that "reduced market discrimination and technological advances [...] reduce the benefits from specialization of spouses in the home and market spheres, thereby decreasing the gains from marriage. However, increasing leisure time and wealth, along with the changing landscape defining sexual relations, potentially raise the gains from consumption complementarities."

8 Wage elasticities

In order to interpret preference parameter estimates we calculate wage elasticities. We increase all wages in the 1999-2000 sample by 10%, separately for men and women, and simulate changes in market and non market hours. We first run the simulations with unchanged individual expectations of the distributions of individual types by gender and marital status, and then we simulate the complete new equilibrium including distributional changes.

The results are reported in Table 5. The first column displays the actual average hours. The second column displays the average simulated hours for the estimated parameters (baseline). There are small discrepancies between actual and baseline numbers as the fit, though good, is not perfect. Then we show the changes in hours at the new equilibrium, and finally using baseline distributions instead of the equilibrium ones. In the literature, estimates of uncompensated or Marshallian elasticities vary a lot across publications (see Blundell and Macurdy, 1999, Meghir and Phillips, 2008). Our estimates of female own-wage labor supply elasticities (0.32 and 0.26 for married and single women) are similar to the estimates in the recent work of Blundell, Dias, Meghir, and Shaw (2015), which is a dynamic labor supply model for female workers. The elasticity of participation rates (extensive margin) is estimated 0.47 and the elasticity of hours worked (intensive margin) is estimated 0.22.

We also find that married and low-educated women have larger elasticities than single and high-educated women. Male elasticities are lower than female elasticities, but the gap is smaller for singles than for married individuals. Education does not seem to play a large role for men. We also calculate the elasticities of domestic hours. For married men, non market hours respond to wage changes more than market hours. For married women, the opposite is true. Single men do not reduce further the already small amount of time devoted to home chores. Single women reduce non market hours in similar proportion as they increase market hours. Cross effects (e.g. male wage on female labor supply) are much smaller. Married women, especially if non educated, yet reduce labor supply significantly after an exogenous increase in male wages.

Finally we comment on the consequences of endogenizing marriage formation on the

Tab. 5: Wage elasticities of hours – 1999-2000

	Actual	Baseline simulation	Equilibrium distributions $\frac{\Delta w_m}{w_m} = 10\%$		Equilibrium distributions $\frac{\Delta w_f}{w_f} = 10\%$		Fixed distributions $\frac{\Delta w_m}{w_m} = 10\%$		Fixed distributions $\frac{\Delta w_f}{w_f} = 10\%$	
Working hours										
Married men	43.36	43.04	43.72	1.6%	42.79	-0.6%	43.41	0.9%	42.58	-1.1%
Single men	37.28	37.04	37.75	1.9%	37.01	-0.1%	37.72	1.8%	37.00	-0.1%
Married women	26.19	25.29	24.81	-1.9%	26.10	3.2%	25.11	-0.7%	26.28	3.9%
Single women	29.63	29.01	28.97	-0.1%	29.78	2.6%	29.02	0.0%	29.76	2.6%
Domestic hours										
Married men	5.24	5.30	5.16	-2.5%	5.37	1.3%	5.17	-2.5%	5.39	1.7%
Single men	5.15	5.16	5.16	0.0%	5.17	0.1%	5.15	-0.2%	5.15	-0.2%
Married Women	15.58	15.72	15.84	0.7%	15.55	-1.1%	15.84	0.7%	15.53	-1.2%
Single Women	10.31	10.42	10.43	0.2%	10.19	-2.1%	10.40	-0.1%	10.18	-2.2%
Working hours by education level										
Married men <HS	44.28	43.91	44.59	1.5%	43.64	-0.6%	44.27	0.8%	43.43	-1.1%
Married men HS	42.79	41.57	42.37	1.9%	41.31	-0.6%	42.04	1.1%	41.10	-1.1%
Married men > HS	42.44	42.99	43.55	1.3%	42.80	-0.4%	43.27	0.7%	42.59	-0.9%
Single men <HS	37.62	37.37	38.09	1.9%	37.36	0.0%	38.05	1.8%	37.34	-0.1%
Single men HS	38.69	35.50	36.35	2.4%	35.48	-0.1%	36.33	2.4%	35.47	-0.1%
Single men >HS	35.50	37.95	38.52	1.5%	37.89	-0.2%	38.51	1.5%	37.90	-0.1%
Married women <HS	23.91	22.91	22.38	-2.3%	23.84	4.1%	22.73	-0.8%	24.07	5.0%
Married women HS	27.44	26.63	26.17	-1.7%	27.49	3.2%	26.46	-0.6%	27.68	3.9%
Married women > HS	30.15	29.49	29.09	-1.4%	30.05	1.9%	29.40	-0.3%	30.18	2.3%
Single women <HS	27.60	26.88	26.85	-0.1%	27.79	3.4%	26.88	0.0%	27.72	3.2%
Single women HS	31.66	29.93	29.89	-0.1%	30.77	2.8%	29.95	0.0%	30.73	2.7%
Single women >HS	31.41	31.85	31.83	-0.1%	32.41	1.7%	31.86	0.0%	32.43	1.8%
Domestic hours by education level										
Married men <HS	5.06	5.07	4.93	-2.8%	5.14	1.5%	4.93	-2.8%	5.16	1.8%
Married men HS	5.28	5.64	5.49	-2.7%	5.71	1.2%	5.49	-2.6%	5.73	1.6%
Married men > HS	5.47	5.36	5.25	-2.1%	5.41	0.9%	5.25	-2.0%	5.45	1.7%
Single men <HS	5.17	5.21	5.21	0.0%	5.21	0.1%	5.20	-0.1%	5.20	-0.1%
Single men HS	5.24	5.10	5.10	0.0%	5.11	0.1%	5.09	-0.2%	5.10	-0.2%
Single men >HS	5.03	5.15	5.15	0.1%	5.15	0.0%	5.14	-0.2%	5.14	-0.2%
Married women <HS	17.38	17.69	17.83	0.8%	17.49	-1.1%	17.81	0.7%	17.46	-1.3%
Married women HS	14.51	13.75	13.86	0.8%	13.57	-1.3%	13.87	0.8%	13.54	-1.5%
Married women > HS	12.54	13.05	13.16	0.8%	12.90	-1.2%	13.11	0.5%	12.87	-1.4%
Single women <HS	12.46	12.49	12.50	0.1%	12.22	-2.2%	12.48	-0.1%	12.23	-2.1%
Single women HS	9.17	8.76	8.77	0.1%	8.50	-2.9%	8.75	-0.1%	8.51	-2.9%
Single women >HS	7.66	8.20	8.21	0.1%	8.02	-2.2%	8.20	-0.1%	8.00	-2.5%

Tab. 6: Wage elasticities of matching probabilities – 1999-2000

	Actual	Baseline simulation	Equilibrium distributions			
			$\frac{\Delta w_m}{w_m} = 10\%$		$\frac{\Delta w_f}{w_f} = 10\%$	
Number of single men	978	1026	989	-3.6%	1031.00	0.5%
Number of single women	984	1001	976	-2.5%	989.00	-1.2%
Number of couples	2505	2390	2416	1.1%	2386.00	-0.2%
Matching probabilities of men by education level						
< HS	0.40	0.39	0.40	2.8%	0.39	-0.9%
HS	0.38	0.37	0.38	3.2%	0.37	-0.2%
> HS	0.28	0.28	0.29	3.7%	0.28	0.4%
Matching probabilities of women by education level						
< HS	0.39	0.38	0.39	2.6%	0.38	-0.6%
HS	0.37	0.36	0.38	3.0%	0.36	-0.8%
> HS	0.28	0.27	0.29	4.7%	0.27	0.6%

wage elasticities of hours. A difference is only visible for married couples and only for market hours. The response of male and female labor supply to an increase in male wage is stronger in the general equilibrium case. The response of male and female market hours to an increase in female wage is weaker. To understand why, we display in Table 6 the effects of a wage increase on matching probabilities. The only noticeable effect is for male wages, where a 10% wage rise increases matching probabilities by 0.3-0.4 percentage points, and there are about 3% fewer singles. A wage rise renders males more attractive in the marriage market; there are more marriages and increased specialization in paid work for men and housework for women.

9 Quantifying the individual value of marriage

In this section we develop a way of measuring marriage externalities and apply it to the estimated model.

9.1 The willingness to pay for marriage

Following Chiappori and Meghir (2014a,b), we quantify the value of marriage in the tradition of public economics as follows. Let us first extend the definition of the value for singles as

$$V_i^0(\mu) \equiv \max_d \psi_i \left[w_i(1-d) + \mu, F_i^0(d)\right] + \lambda \iint \left[V_m^1(i,j,z) - V_i^0(\mu)\right]^+ \mathrm{d}G(z)\, n_f(j)\, \mathrm{d}j,$$

for some non-earned income μ paid to singles until marriage. The Total Equivalent Income I_i (resp. I_j) is the total income $w_i + \mu_m$ (resp. $w_j + \mu_f$) that makes a married individual indifferent between remaining married at the current contract or becoming single with the income supplement μ_m (resp. μ_f).[24] Note that Chiappori and Meghir's framework is static. It makes sense, in a dynamic environment, to assume that the bachelor subsidy (μ_m, μ_f) is lost when a new marriage occurs.

For a man of type i, we thus define $\mu_m(i,j,z)$ as the solution to $V_i^0(\mu_m) = V_m^1(i,j,z)$, that is, $\mu_m(i,j,z)$ solves

$$\max_d \psi_i\left[w_i(1-d)+\mu_m, F_i^0(d)\right] = V_m^1(i,j,z) \\ - \lambda \iint \left[V_m^1(i,j,z') - V_m^1(i,j,z)\right]^+ \mathrm{d}G(z')\, n_f(j')\,\mathrm{d}j'. \quad (9.1)$$

With a similar expression for female's $\mu_f(i,j,z)$.

In our setup husband and wife save resources to finance the production of the public good q. Therefore, knowing transfers t_m, t_f is not very useful. What we want to know is how much the consumption of the public good q does contribute to individual welfare net of the costs t_m and t_f. That is exactly what the equivalent transfer aims to quantify. The μ_m, μ_f thus measure the willingness to pay for a marriage of given characteristics. Marriage is consummated if and only if both μ_m and μ_f are positive.

9.2 Measuring the gains from marriage

Figure 12 displays the dynamics of wages and willingness to pay for marriage over the period. Marriage is worth a lot, a hefty proportion of the wage, and the willingness to pay for marriage increases faster than wages. There is evidence of redistribution ($\mu_f > \mu_m$, albeit slightly, while $w_j < w_i$).

Figure 13 shows the ratio $\frac{\mu_f}{\mu_f+\mu_m}$ by spouses' education. The willingness to pay for marriage increases with education but low-educated women obtain less than the fair share, and high-educated women obtain more. Women's share increases over time for less educated women.

Finally, we display in Figure 14 mean values of μ_m, μ_f by both spouses' wages, family values and education as estimated for one representative cross-section, 1999-2000. The willingness to pay for marriage varies more by spouses' wages than by family values or education. These are interesting results. We have already established that wage complementarity does not affect public good quality, unlike education and family values, as there is less sorting on wages than there is on education and family values. This is because income is far more transferable than education or family values. In other words, education and family values are a stronger source of homophily than wages because they affect preferences in a non – at least less – transferable way.

[24] Chiappori and Meghir call it a Money Metric Welfare Index.

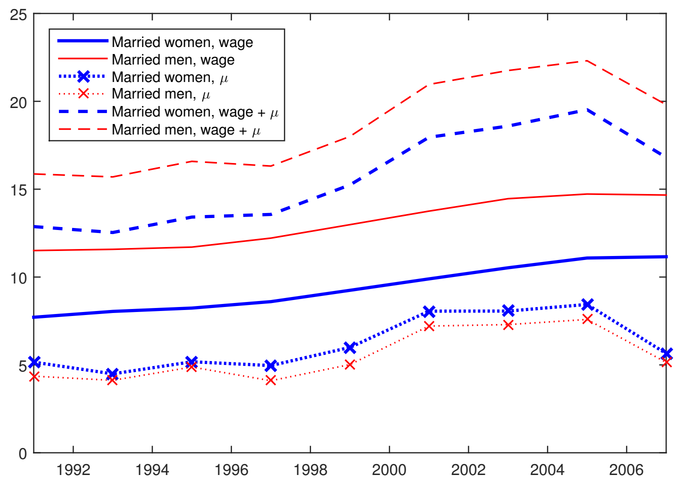


Fig. 12: Wages and willingness to pay for marriage

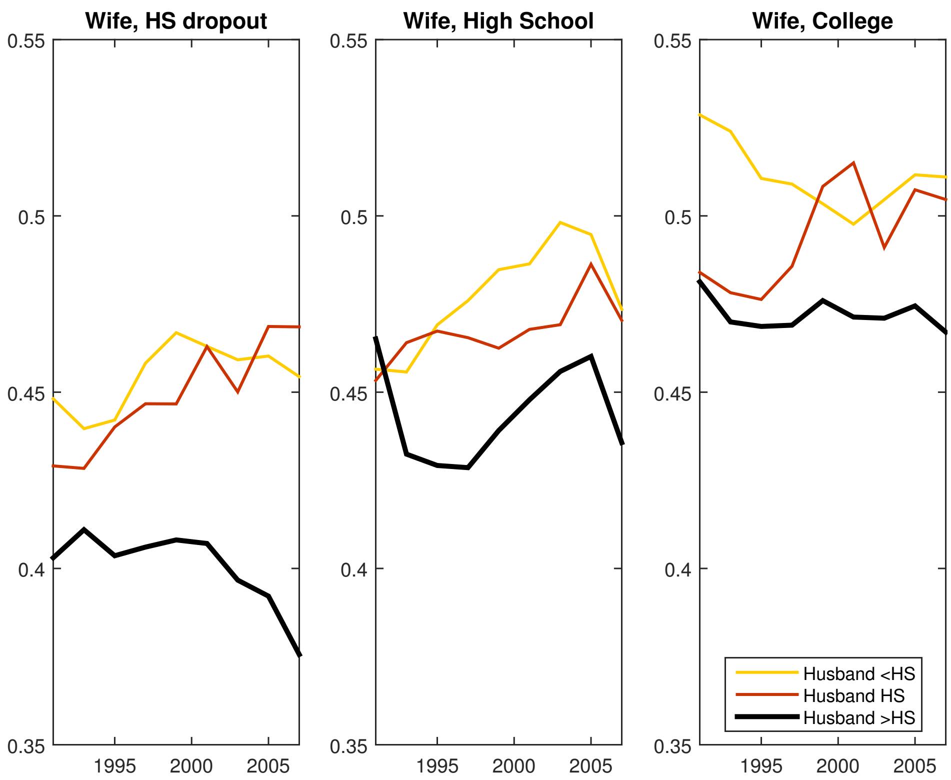


Fig. 13: Willingness to pay for marriage by education (share $\frac{\mu_f}{\mu_f+\mu_m}$)

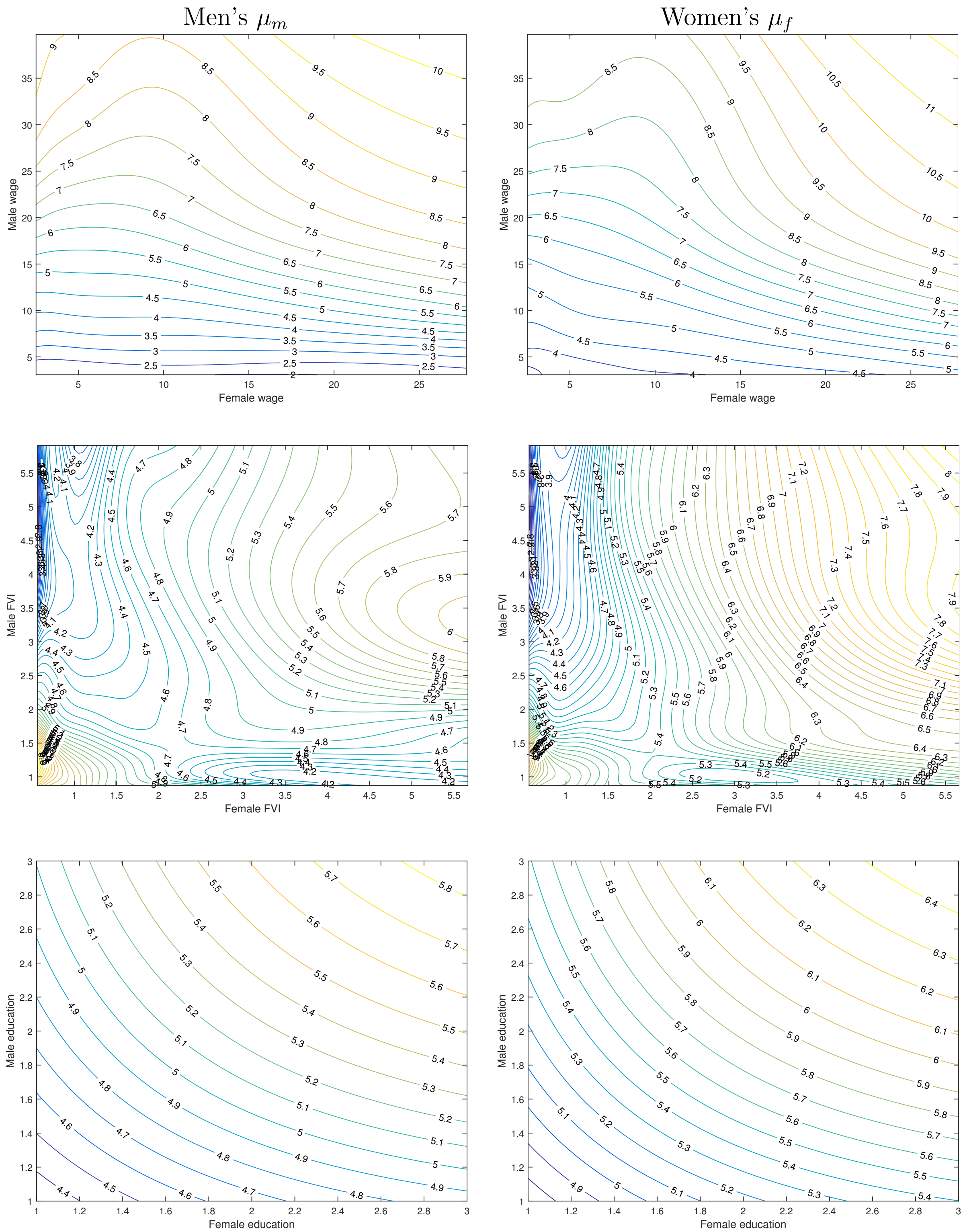


Fig. 14: Willingness to pay for marriage by both spouses' types (isolines), 1999-2000

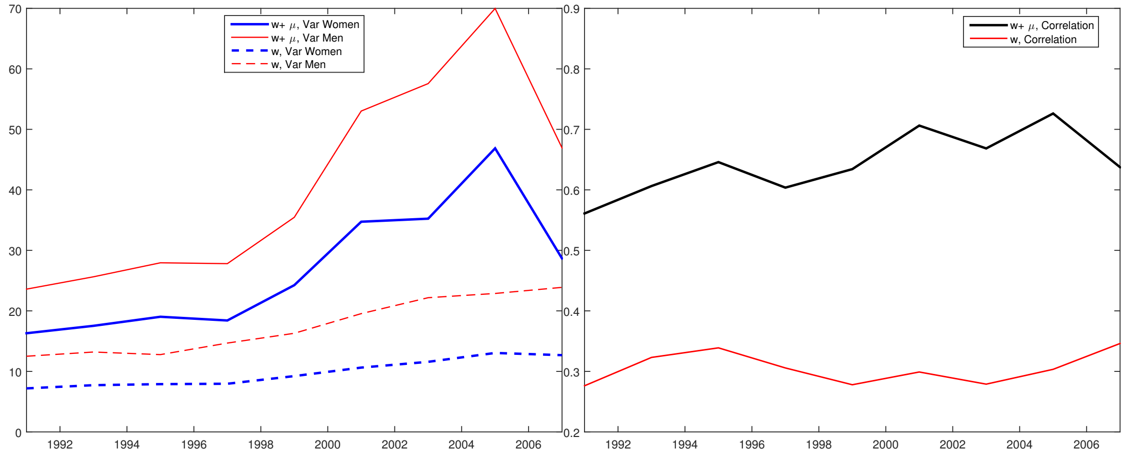


Fig. 15: Income inequality

9.3 Within and between-household welfare inequality

We now consider the total equivalent incomes $I_i \equiv w_i + \mu_m$ and $I_j \equiv w_j + \mu_f$ for male and female spouses. Figure 15 shows variances of spouses' incomes and their correlation. The variance of total equivalent income is obviously necessarily greater than the variances of wages, and it also increases faster. The correlation between spouses' total equivalent incomes is twice the correlation of wages, and it increases between 1991 and 2008 from 60% to 70%.

We then calculate the within (*intra*)-household variance

$$\mathbb{E}\mathbb{V}\left[I|i,j\right] = \mathbb{E}\left[\frac{I_i^2 + I_j^2}{2} - \left(\frac{I_i + I_j}{2}\right)^2\right] = \frac{\mathbb{V}(I_i - I_j)}{4} + \frac{(\mathbb{E}I_i - \mathbb{E}I_j)^2}{4},$$

and the between (*inter*)-household variance

$$\mathbb{V}\mathbb{E}\left[I|i,j\right] = \mathbb{V}\left(\frac{I_i + I_j}{2}\right) = \frac{\mathbb{V}(I_i + I_j)}{4}.$$

Figure 16 shows this decomposition. The between-household variance dominates the within-household variance, and this increasingly more over the years.

Finally, we compare and contrast intra- and inter-household variances for wages alone and welfare indices $(w + \mu)$ separately. The right panel of the figure shows the ratio of the between-household variance to the total variance separately for $w + \mu$ and w. The dynamics of these two ratios mimic the dynamics of correlations. The within-household income variance is considerably reduced in proportion to the between-household variance, when intra-household transfers are taken into consideration, because the total incomes of both spouses are much more alike than their ex ante wages. There is thus considerable redistribution within the household, and increasingly more over the 1991-2008 period.

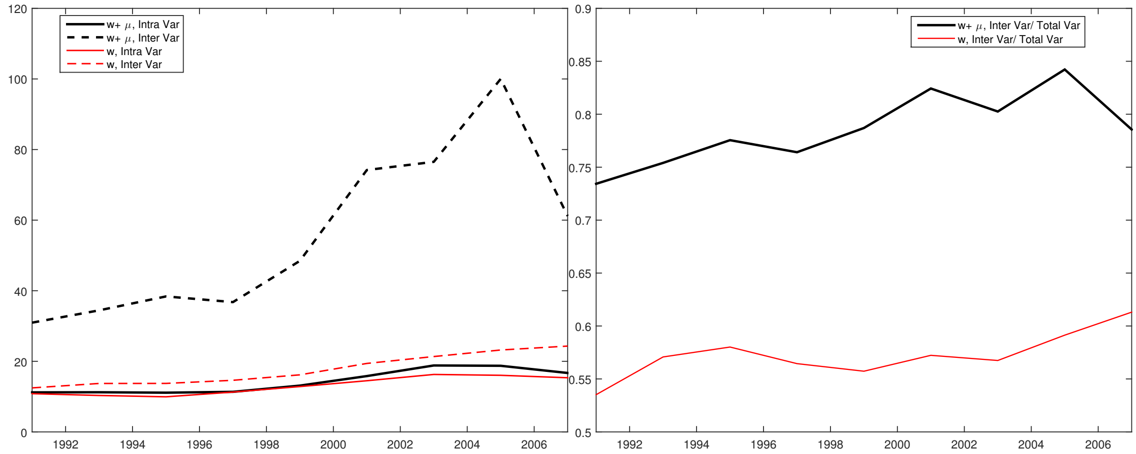


Fig. 16: Income inequality decomposition

10 Conclusion

In this paper we build a model of marriage formation and within-household allocation of resources. By endogenizing marriage formation as the result of Nash bargaining between spouses, we are able to identify the level of transfers between spouses (and not only the link to distribution factors). Identification of the levels of transfers only requires a normalization of the level of the household production function for singles as in stable matching models a la Shapley-Shubik.

Adding search frictions to the matching framework is likely not altering the fundamental properties of matching models both in terms of their theoretical properties and as far as identification is concerned (see for example the equivalence result of Adachi, 2003). The main contribution of this paper is therefore not search frictions per se, but the search-matching framework rather easily allows to put together in the same model considerations about matching and considerations about task sharing. It is then only by using both information on matching probabilities by spouses' characteristics (wages, education, family values) and on the link between time uses of both spouses and these characteristics, that we are able to identify the levels of transfers and the individual values of marriage.

We estimate that most of the observed changes in task sharing in the 1991-2008 period is due to changes in the structure of the home-production function. Most of the observed sorting in the marriage market results from homophilic preference and not from intra-household gains from time use trade. The contribution of the marriage market equilibrium to wage elasticities of paid work hours is sizable. Lastly, the willingness to pay for marriage of married individuals is huge, comparable to wages, and there is a considerable amount of redistribution at work in the household.

In future work it will be interesting to explore further the subjects of study of family economics, such as fertility and children, divorce laws, the evaluation of family tax credits, etc. In addition, our description of matching can and should be improved by introducing other individual characteristics such as people's age and children. Allowing for other types of shocks and uncertainty, such as wage and unemployment shocks is also important. More generally, we should aim at understanding better the link between marriage and labor markets. For example, women who specialize in home production may be losing human capital. These are just a few examples of the many extensions that come to mind.

Appendix

A Equilibrium solution with transferable utility

Match surplus. Spouses solve the Nash bargaining problem

$$\max_{c,d_m,d_f,t_m,t_f} \left[W_m - V_i^0\right]^\beta \left[W_f - V_j^0\right]^{1-\beta}$$

subject to $c = t_m + t_f$ and with

$$u_m = q\frac{w_i(1-d_m) - t_m - A_i}{B_i}, \quad q = zF_{ij}^1(c, d_m, d_f),$$

and

$$(r+\delta)\left[W_m - V_i^0\right] = u_m + \delta \int \left(V_m^1(z') - V_i^0\right)^+ \mathrm{d}G(z') - rV_i^0 \equiv u_m + v_m \text{ (say)}.$$

The first-order conditions for transfers t_m, t_f,

$$\frac{\beta}{(r+\delta)[W_m - V_i^0]}\frac{\partial u_m}{\partial R} = \frac{1-\beta}{(r+\delta)[W_f - V_j^0]}\frac{\partial u_f}{\partial R},$$

yield the rent sharing conditions

$$\frac{\beta}{u_m + v_m}\frac{q}{B_i} = \frac{1-\beta}{u_f + v_f}\frac{q}{B_j} \equiv \frac{q}{S_{ij}(z)} \quad \text{(say)}.$$

It follows that

$$S_{ij}(z) = B_i(u_m + v_m) + B_j(u_f + v_f) = B_i(\underline{u}_m + v_m) + B_j(\underline{u}_f + v_f) - qc,$$

using $t_m + t_f = c$ and with

$$\underline{u}_m = q\frac{w_i(1-d_m) - A_i}{B_i}.$$

Domestic production inputs. The first-order condition for c, d_m, d_f is

$$\begin{aligned}\frac{w_i d_m}{\epsilon_m} = \frac{w_j d_f}{\epsilon_f} = \frac{c}{\epsilon} &= q\left[\frac{\partial u_m/\partial q}{\partial u_m/\partial R} + \frac{\partial u_f/\partial q}{\partial u_f/\partial R}\right] \\ &= w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - c,\end{aligned}$$

for elasticities $\epsilon_m = \frac{\partial \ln F_{ij}^1(c,d_m,d_f)}{\partial \ln d_m}, \epsilon_f, \epsilon$. The optimal home-production inputs c, d_m, d_f and home-production output q are thus simple functions of match characteristics. Let X_{ij} be the equilibrium value of $w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - c$ obtained from the above first-order conditions. Because of the multiplicative nature of the dependence of home production to z – $q = zF^1_{ij}(c, d_m, d_f) - X_{ij}$ only depends on i, j and not z.

Continuation values. Making use of the promise keeping constraints, $W_m = V^1_m(z)$, we have

$$(r+\delta)\left[V^1_m(z) - V^0_i\right] = u_m + v_m = \beta \frac{S_{ij}(z)}{B_i},$$
$$(r+\delta)\left[V^1_f(z) - V^0_j\right] = u_f + v_f = (1-\beta) \frac{S_{ij}(z)}{B_j}.$$

Hence

$$v_m \equiv \delta \int \left(V^1_m(z') - V^0_i\right)^+ \mathrm{d}G(z') - rV^0_i = \frac{\delta\beta}{r+\delta}\frac{\overline{S}_{ij}}{B_i} - rV^0_i,$$
$$v_f \equiv \delta \int \left(V^1_f(z') - V^0_j\right)^+ \mathrm{d}G(z') - rV^0_j = \frac{\delta(1-\beta)}{r+\delta}\frac{\overline{S}_{ij}}{B_j} - rV^0_j,$$

for $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \,\mathrm{d}G(z')$, and

$$S_{ij}(z) = B_i(\underline{u}_m - rV^0_i) + B_j(\underline{u}_f - rV^0_j) - qc + \frac{\delta}{r+\delta}\overline{S}_{ij}.$$

Thus $S_{ij}(z)$ solves the integral equation

$$S_{ij}(z) = qX_{ij} - B_i r V^0_i - B_j r V^0_j + \frac{\delta}{r+\delta}\int S_{ij}(z')^+ \,\mathrm{d}G(z'), \tag{A.1}$$

where $qX_{ij} \equiv zQ_{ij}$ (say) is the joint utility $u_m + u_f$. Transfers follow from the above rent sharing equations as

$$q\left[w_i(1-d_m) - t_m - A_i\right] = B_i r V^0_i + \beta\left[qX_{ij} - B_i r V^0_i - B_j r V^0_j\right], \tag{A.2}$$

with a similar expression for t_f. Note that we can then write

$$w_i(1-d_m) - t_m - A_i = \beta_{ij}(z)\, X_{ij}, \quad w_j(1-d_f) - t_f - A_j = \left[1 - \beta_{ij}(z)\right] X_{ij},$$

for $\beta_{ij}(z) = \beta + \frac{1}{z}\frac{B_i r V^0_i - \beta\left[B_i r V^0_i + B_j r V^0_j\right]}{Q_{ij}}$.

Singles. It remains to work out the value for singles, i.e.

$$rV^0_i = \max_d \psi_i\left[w_i(1-d), F^0_i(d)\right] + \beta\frac{\lambda}{r+\delta}\frac{1}{B_i}\iint S_{ij}(z)^+ \mathrm{d}G(z)\, n_f(j)\,\mathrm{d}j, \tag{A.3}$$

with a similar expression for females.

Solving for values. Let $\overline{S}_{ij} \equiv \int S_{ij}(z)^+ \mathrm{d}G(z)$ denote the integrated surplus. The following fixed-point equation thus defines $\overline{S}_{ij}$:

$$\overline{S}_{ij} = Q_{ij}\mathcal{G}\left(\frac{B_i r V^0_i + B_j r V^0_j - \frac{\delta}{r+\delta}\overline{S}_{ij}}{Q_{ij}}\right), \tag{A.4}$$

with $\mathcal{G}(s) \equiv \int (z-s)^+ \,\mathrm{d}G(z) = \int_s^{+\infty} z \,\mathrm{d}G(z) - s[1-G(s)]$. Note that $\mathcal{G}'(s) = -[1-G(s)] \in (-1,0)$ for all interior point s. Hence $\mathcal{G}$ is a contracting operator.[25] Moreover,

$$B_i r V_i^0 = B_i \psi_i^0 + \frac{\lambda\beta}{r+\delta} \int \overline{S}_{ij} n_f(j) \,\mathrm{d}j, \tag{A.5}$$

and

$$B_j r V_j^0 = B_j \psi_j^0 + \frac{\lambda(1-\beta)}{r+\delta} \int \overline{S}_{ij} n_m(i) \,\mathrm{d}i. \tag{A.6}$$

These two equation define $\left(\overline{S}_{ij}, B_i r V_i^0, B_j r V_j^0\right)$ as a fixed-point of a contracting operator. Then $S_{ij}(z)$ follow.

B Identification of preferences and home production

B.1 An inversion formula for values

In this section, we show that values $\overline{S}_{ij}, V_i^0, V_j^0$ are uniquely obtained from matching probabilities α_{ij} given the other parameters.

Let $\overline{S}_{ij} \equiv \int S_{ij}(z)^+ \mathrm{d}G(z)$ denote the integrated surplus. It solves the fixed-point equation:

$$\frac{\overline{S}_{ij}}{Q_{ij}} = \mathcal{G}\left(\frac{B_i r V_i^0 + B_j r V_j^0}{Q_{ij}} - \frac{\delta}{r+\delta}\frac{\overline{S}_{ij}}{Q_{ij}}\right), \tag{B.1}$$

with $\mathcal{G}(s) \equiv \int (z-s)^+ \,\mathrm{d}G(z) = \int_s^{+\infty} z \,\mathrm{d}G(z) - s[1-G(s)]$. In addition,

$$B_i r V_i^0 = B_i \psi_i^0 + \beta \frac{\lambda}{r+\delta} \int \overline{S}_{ij} n_f(j) \,\mathrm{d}j, \quad \psi_i^0 \equiv \max_d \psi(w(1-d), F^0(x,d)),$$

with a similar expression for for $B_j r V_j^0$ with $1-\beta$ instead of β.

Next, consider the definition of the matching probability:

$$\begin{aligned}
\alpha_{ij} &= \Pr\{S_{ij}(z) > 0\} \\
&= \Pr\left\{ z Q_{ij} - B_i r V_i^0 - B_j r V_j^0 + \frac{\delta}{r+\delta}\overline{S}_{ij} > 0 \right\} \\
&= 1 - G\left(\frac{B_i r V_i^0 + B_j r V_j^0}{Q_{ij}} - \frac{\delta}{r+\delta}\frac{\overline{S}_{ij}}{Q_{ij}}\right) \\
&= 1 - G\left[\mathcal{G}^{-1}\left(\frac{\overline{S}_{ij}}{Q_{ij}}\right)\right].
\end{aligned}$$

Hence,

$$\frac{\overline{S}_{ij}}{Q_{ij}} = \mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right].$$

[25] In the particular lognormal case of $G(s) = \Phi(\ln s/\sigma_z)$ and $G^{-1}(x) = e^{\sigma_z \Phi^{-1}(x)}$, then

$$\mathcal{G}(s) = -s\Phi\left(-\frac{\ln s}{\sigma_z}\right) + e^{\frac{\sigma_z^2}{2}}\Phi\left(-\frac{\ln s}{\sigma_z} + \sigma_z\right).$$

Inverting equation (B.1),

$$\frac{B_i r V_i^0 + B_j r V_j^0}{Q_{ij}} - \frac{\delta}{r+\delta} \frac{\overline{S}_{ij}}{Q_{ij}} = \mathcal{G}^{-1}\left(\frac{\overline{S}_{ij}}{Q_{ij}}\right),$$

it follows that

$$\overline{S}_{ij} = \left(B_i r V_i^0 + B_j r V_j^0\right) \theta_{ij},$$

with

$$\theta_{ij} \equiv \frac{\overline{S}_{ij}}{Q_{ij}} \frac{Q_{ij}}{B_i r V_i^0 + B_j r V_j^0} = \frac{\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}{G^{-1}(1-\alpha_{ij}) + \frac{\delta}{r+\delta}\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}$$

This implies that $\left(B_i r V_i^0, B_j r V_j^0\right)$ is a fixed point of the operator (T_m, T_f) with

$$\begin{aligned} B_i r V_i^0 &= T_m\left(B_i r V_i^0, B_j r V_j^0\right) \\ &= B_i \psi_i^0 + \frac{\lambda}{r+\delta}\beta \int \left(B_i r V_i^0 + B_j r V_j^0\right) \theta_{ij}\, n_f(j)\, \mathrm{d}j, \end{aligned}$$

and

$$\begin{aligned} B_j r V_j^0 &= T_f\left(B_i r V_i^0, B_j r V_j^0\right) \\ &= B_j \psi_j^0 + \frac{\lambda}{r+\delta}(1-\beta) \int \left(B_i r V_i^0 + B_j r V_j^0\right) \theta_{ij}\, n_m(i)\, \mathrm{d}i. \end{aligned}$$

This is a linear system that can be easily solved for after discretizing the state space, or value function iteration. Suppose that i and j are discrete variables. Define the matrices

$$\Theta_m = -\frac{\lambda\beta}{r+\delta}\left[\theta_{ij}\, n_f(j)\right]_{ij}, \quad \Theta_f = -\frac{\lambda(1-\beta)}{r+\delta}\left[\theta_{ij}\, n_m(i)\right]_{ij}^{\top},$$

and

$$\Delta_m = 1 - \frac{\lambda\beta}{r+\delta}\operatorname{diag}\Big(\sum_j \theta_{ij}\, n_f(j)\Big)_i, \quad \Delta_f = 1 - \frac{\lambda(1-\beta)}{r+\delta}\operatorname{diag}\Big(\sum_i \theta_{ij}\, n_m(i)\Big)_j.$$

Then

$$\left[\begin{array}{c} B_i r V_i^0 \\ B_j r V_j^0 \end{array}\right] = \left(\begin{array}{cc} \Delta_m & \Theta_m \\ \Theta_f & \Delta_f \end{array}\right)^{-1} \left[\begin{array}{c} B_i \psi_i^0 \\ B_j \psi_j^0 \end{array}\right].$$

In Appendix C, we show how a similar approach can be used with a continuous state space. We use Chebyshev nodes for the discretization grid and Clenshaw-Curtis quadrature to approximate the integral.

Note lastly that the normalization of the production function for singles implies an implicit normalization of present values for singles and match surplus.

B.2 Mean transfers

The inversion formula in Appendix B.1 allows to calculate $B_i r V_i^0$ and $B_j r V_j^0$ given demographic parameters λ, δ, $n_m(i)$ and $n_f(j)$, matching probabilities α_{ij}, the bargaining power coefficient β, and preference/domestic production parameters

$$B_i \psi_i^0 \equiv \left(\frac{K_i^0}{w_i}\right)^{K_i^0} \left(\frac{w_i(1-D_i^0) - A_i}{1+K_i^0}\right)^{1+K_i^0}, \quad B_j \psi_j^0 \equiv \left(\frac{K_j^0}{w_j}\right)^{K_j^0} \left(\frac{w_j(1-D_j^0) - A_j}{1+K_j^0}\right)^{1+K_j^0}.$$

Then, with $\theta_{ij} = \frac{\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}{G^{-1}(1-\alpha_{ij})+\frac{\delta}{r+\delta}\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}$,

$$\overline{S}_{ij} = \theta_{ij}(B_i r V_i^0 + B_j r V_j^0), \quad Q_{ij} = \frac{\overline{S}_{ij}}{\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}.$$

And we can calculate transfers using equation (5.9):

$$w_i(1-d_m^1) - t_m - A_i = \beta_{ij}(z)\, X_{ij},$$

for $\beta_{ij}(z) = \beta + \frac{1}{z}\frac{B_i r V_i^0 - \beta\left[B_i r V_i^0 + B_j r V_j^0\right]}{Q_{ij}}$.

Let $\overline{t}_{mij} = \mathbb{E}(t_m|i,j,S_{ij}(z)>0)$ and $\overline{t}_{fij} = \mathbb{E}(t_f|i,j,S_{ij}(z)>0)$. As

$$S_{ij}(z) \geq 0 \Leftrightarrow z \geq \frac{B_i r V_i^0 + B_j r V_j^0}{Q_{ij}} - \frac{\delta}{r+\delta}\frac{\overline{S}_{ij}}{Q_{ij}} = \mathcal{G}^{-1}\left(\frac{\overline{S}_{ij}}{Q_{ij}}\right) = G^{-1}(1-\alpha_{ij}),$$

we have

$$\frac{w_i(1-d_m^1) - \overline{t}_{mij} - A_i}{X_{ij}} = \beta + \frac{1}{\mathbb{E}\left(\frac{1}{z}|z > G^{-1}(1-\alpha_{ij})\right)}\frac{(1-\beta)B_i r V_i^0 - \beta B_j r V_j^0}{Q_{ij}} \equiv \overline{\beta}_{ij}, \quad \text{(B.2)}$$

and

$$w_j(1-d_f^1) - \overline{t}_{fij} - A_j = (1-\overline{\beta}_{ij})X_{ij}.$$

Note finally that with z log normal $\mathcal{LN}(0,\sigma_z^2)$, the distribution of $1/z$ is also log normal $\mathcal{LN}(0,\sigma_z^2)$. Hence

$$\mathbb{E}\left(1/z|z>s\right) = e^{\frac{\sigma_z^2}{2}}\Phi\left(-\frac{\ln s}{\sigma_z} - \sigma_z\right)/\Phi\left(-\frac{\ln s}{\sigma_z}\right), \quad \text{(B.3)}$$

B.3 Variance of match-specific shock σ_z^2

Male leisure demand satisfies the equation

$$\frac{w_i(e_m^1 - A_i')}{\eta_i} = w_i(1-d_m^1) - t_m - A_i$$

with $\eta_i = B_i'/B_i$ is the income elasticity of leisure. Hence

$$\frac{w_i(e_m^1 - A_i')}{\eta_i X_{ij}} = \beta + \frac{1}{z}\frac{(1-\beta)B_i r V_i^0 + \beta B_j r V_j^0}{Q_{ij}}.$$

For the parametric specification,

$$A_i' = \frac{a_{1m}}{1-b_m}w_i - 2\frac{a_{2m}}{2-b_m}w_i^2, \quad \eta_i = b_m.$$

Moreover,

$$X_{ij} = \frac{w_i(e_m^1 - A_i')}{\eta_i} + \frac{w_j(e_f^1 - A_j')}{\eta_j}.$$

Given that the first order moments of couples' leisure are already fitted, we can base the estimation of σ_z on fitting the second-order moments of

$$Y_i \equiv \frac{w_i(e_m^1 - A_i')}{\eta_i X_{ij}} - \beta, \quad Y_j = \frac{w_j(e_f^1 - A_j')}{\eta_j X_{ij}} - \beta.$$

With z log normal $\mathcal{LN}(0, \sigma_z^2)$,

$$\mathbb{E}\left(1/z^2 | z > s\right) = e^{2\sigma_z^2}\Phi\left(-\frac{\ln s}{\sigma_z} - 2\sigma_z\right)/\Phi\left(-\frac{\ln s}{\sigma_z}\right).$$

C Computational details

This appendix shortly describes the numerical tools used in estimation.

First, we discretize continuous functions on a compact domain using Chebyshev grids.[26] For example, let $[\underline{x}, \overline{x}]$ denote the support of male wages, we construct a grid of $n+1$ points as

$$x_j = \frac{\overline{x} + \underline{x}}{2} + \frac{\overline{x} - \underline{x}}{2} \cos\frac{j\pi}{n}, \; j = 0, ..., n.$$

Second, to estimate wage densities $m(x,y)/N$, $u_m(x)/U_m$ and $u_f(y)/U_f$ on those grids we use kernel density estimators with a lot of smoothing. This is important as, for instance, we divide m by $u_m u_f$ to calculate α.

Third, many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals:

$$\int_{\underline{x}}^{\overline{x}} f(x)\,\mathrm{d}x \simeq \frac{\overline{x} - \underline{x}}{2} \sum_{j=0}^{n} w_j f(x_j),$$

where the weights w_j can be easily computed using Fast Fourier Transform (FFT). The following MATLAB code can be used to implement CC quadrature (Waldvogel, 2006):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-l;
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).

Fourth, we need to solve functional fixed point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed point operator T involves integrals, we simply iterate the finite dimensional operator $\widehat{T}$ obtained by replacing the integrals by their approximations at grid points. For example, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1 + \rho \int_{\underline{x}}^{\overline{x}} u(y)\alpha(x,y)\,\mathrm{d}y}$$

becomes

$$\boldsymbol{u} = [u(x_j)]_{j=0,\dots,n} = \widehat{T}(\mathbf{u}) = \left[\frac{\ell(x_j)}{1 + \rho\sum_{k=0}^{n} w_k u(x_k)\alpha(x_j, x_k)}\right]_{j=0,\dots,n}.$$

[26] It can be shown that the error associated to a polynomial approximation (of any order) of an unknown function at any point x is proportional to $\prod_{j=0}^{n}(x - x_j)$. The Chebyshev points are the $\{x_j\}_{j=0,\dots n}$ minimizing this quantity.

It was sometimes necessary to "shrink" steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0, 1]$. A stepsize $\theta < 1$ may help if T is not everywhere strictly contracting.

Fifth, the fact that CC quadrature relies on Chebyshev polynomials of the first kind also allows to interpolate functions very easily between points $y_0 = f(x_0), ..., y_n = f(x_n)$ using Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^{n} Y_k \cdot T_k(x), \tag{C.1}$$

where Y_k are the OLS estimates of the regression of $y = (y_0, ..., y_n)$ on Chebyshev polynomials

$$T_k(x) = \cos\left(k \arccos\left(\frac{x - \frac{\overline{x}+\underline{x}}{2}}{\frac{\overline{x}-\underline{x}}{2}}\right)\right),$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, ..., y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
X = real(fft(X'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))...
*Y(1:n+1,1:n+1)...
*cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin)));
```

The fact that the grid $(x_0, ..., x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon).

Another advantage of DCT is that, having calculated $Y_0, ..., Y_n$, then polynomial projections of $y = (y_0, ..., y_n)$ of any order $p \le n$ are obtained by stopping the summation in (C.1) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},$$

and

$$\int \cos(k \arccos x)\, \mathrm{d}x = \begin{cases} x & \text{if } k = 0, \\ \frac{x^2}{2} & \text{if } k = 1, \\ \frac{\cos(k+1)x}{2(k+1)} - \frac{\cos(k-1)x}{2(k-1)} & \text{if } k \ge 2. \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation:

$$\int_{\underline{x}}^{x} \mathbf{1}\{t \leq x\} f(x)\, \mathrm{d}x \simeq \sum_{k=0}^{n} w_k \mathbf{1}\{t \leq x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

We implemented these procedures with numbers of grid points such as $n = 50, 100, 500$ on a laptop without running into any memory or computing time difficulty.

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