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Abstract

The literature on the micro-economics of the eco-industry often assumed interiority of pollutant net emissions. In a perfectly competitive final good market vertically integrated with an upstream monopoly supply this assumption implies that an optimal tax is always greater than its associated marginal social damage. In this short note we will relax this assumption and challenge that result. The market structure generates a unique threshold on the scale of the marginal social damage, whereby for any value above the threshold an optimal tax is strictly lower and net emissions are zero.

JEL Classification: D42, D62, H23, L11, Q58

Keywords: Microeconomics, Eco-industry, Imperfect Competition, Optimal Taxation

1 Introduction

It is well known that our modern lifestyles and their associated production of goods and services have a major impact on our environment. Beginning with Pigou (1920), a first literature strain has addressed the question of fair mitigation of social costs by implementing policy tools likewise Coase (1960) and Buchanan (1969). This was the first step toward environmental awareness. As regulation alone proved to be unable to bring sustainability, opportunities arose for the private sector to profit from the demand for monitoring and waste management. This has led to the emergence of the eco-industry, which has grown continuously ever since.

Some institutions commissioned extensive studies on these eco-industries. For instance the European Commission created a large-scale program to inventory the distribution of such firms in the European Union and their global performances for more than fifteen years. In a report on their findings, Ecorys (2009), they states that:

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"However, there are groups operating on a global scale, and which are not con-
strained by the local nature of the needed resources. [...] A preliminary analysis of
the micro-economic sample indicated that the eco-industry is a well concentrated
industry with 10% of the companies responsible for almost 80% of the operating
revenue/turnover."

Then in another report Ecorys (2012) they state:

"The global market for eco-industries is estimated at roughly EUR 1.15 tril-
lion a year (2010 figures for turnover) [...] There is also broad consensus that
the global market could almost double, with the average estimate for 2020 being
around EUR 2 trillion a year".

These observations of highly profitable and concentrated market have inspired the sub-
sequent literature, now addressing questions on the eco-industry itself. Likewise David et
al. (2005)(2010)(2011), who take market power as a given and suggest the optimal design of
environmental policy within this context. Building on this seminal work, Canton et al.(2008)
and Schwartz et al. (2014) propose models that combine fair pollutant emission mitigation
and imperfect competition. The present work is mostly inspired by the work of Canton et
al. (2008), which generalizes Barnett (1980) to a scenario with a vertical production chain.
They find that an optimal tax should be less stringent to the less competitive sector for
the upstream eco-industry derives its demand from emission regulation. However, while they
suppose that net emissions are positive at the optimal solution, their model allows greater
reduction than emission if the marginal environmental damage of pollutant emission and
henceforth the policy tool are sufficiently dissuasive.

Here we stress the positivity of net emissions. Such that now the demand of pollution
abatement is kinked, revealing a shift in the production regimes. It appears, when the less
competitive sector is the upstream eco-industry, the optimal tax scheme is set higher or
lower than the marginal damage of pollution if the marginal damage itself is lower or higher
than an implicit threshold. This result is contrary to those of Canton et al. (2008). For
our finding applies when the less competitive market is the upstream one, this note focuses
on the aspects of monopoly competition from upstream. Where an eco-industry means a
firm providing an abatement good used in the production process of customers to reduce
pollutant emissions.

The remainder of the paper is outlined as follows, section 1 describes the vertically
integrated eco-industry setup. Section 2 highlights the kinked demand for pollution reduction.
The section 3 resolves the upstream monopoly outcome. Then Section 4 concentrates on
welfare analysis and policy recommendations.

2 The Vertically Integrated Eco-Industry

Consider a final good $Q$ and assume that $Q$ is produced by a large set of producers in a
perfectly competitive downstream market. To give substance to this definition suppose each
individual production generates increasingly (and convex) pollutant emission $\varepsilon (Q)$ where
each unit is taxed at rate $\tau$. To reduce this burden the producers purchase $A$ abatement goods supplied monopolistically at increasing (and convex) production cost $K(A)$ and retailed at price $P_A$. Quantified in emission unit, the consecutive reduction $\alpha(A)$, is increasing and concave such that pollution reduction is lower or equal to emission. Accounting for increasing and convex production cost $C(Q)$, the cost function of a downstream representative agent so is $C(Q) + \tau \times \max (0, \varepsilon(Q) - \alpha(A)) + P_A \times A$.

This positivity constraint on net emissions has been neglected until now by the literature. Intuitively the use of a maximum function will generate a discontinuity in the marginal profit, which would normally imply a kink. Likewise a constant marginal damage $\nu$ applies on net emission in the social benefit, thus the environmental damage is $\nu \times \max (0, \varepsilon(Q) - \alpha(A))$. The tax $\tau$ is chosen by a benevolent planner to maximize the social benefit by mitigating both environmental damage and market power of upstream firm. Finally, to close the model, demand is defined by an inverse function of the quantity $P_Q(Q)$, which is strictly decreasing.

In the following we define the decision of the agents and the market equilibrium.

### 3 The Kinked Abatement Demand

In this section we present final good market equilibrium quantity and abatement demand for any upstream supply and any tax. To do this we solve of the game’s maximization programs by backward induction. Precisely, prior to the present section the government had set a welfare maximizing tax $\tau^*$ and the upstream monopoly anticipated the demand for abatement and has accordingly chosen a profit maximizing supply quantity $A^*(\tau)$. After which on downstream market the representative agent trades competitively the quantity $Q^*(\tau, A)$, given by:

$$
\max_{A > 0} \left\{ \max_{Q > 0} \left\{ P_Q \times Q - C(Q) - \tau \times \max (0, \varepsilon(Q) - \alpha(A)) \right\} - P_A \times A \right\}
$$

such that $P_Q = P_Q(Q)$ when market clears.

To illustrate the resolution let us first suppose strictly positive net emissions at the optimal solution $\varepsilon(Q) - \alpha(A) > 0$. In this case the profit maximization is separable. When downstream market clears its unique maximizer verifies the first order conditions: (1) marginal market revenue per marginal emission equates tax and (2) inverse demand for abatement equates marginal environmental costs reduction.

$$
\xi(Q^*) := \frac{P_Q(Q^*) - C'(Q^*)}{\varepsilon'(Q^*)} = \tau
$$

$$
P_A(A^*(\tau)) = \tau \times \alpha'(A^*(\tau))
$$

This is a well known result found in David M. and Sinclair-Desgagné B. (2005) and developed in Canton J., Soubeyran A. and Stahn H. (2008) and Schwartz S., Stahn H.

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1. Another possibility could have been to consider increasing convex damage function. In such case the result would be the same and the reasoning on $\nu$ would have been on marginal damage evaluated in null emission instead.
where the end-of-pipe assumption allows us to consider separability, from now on we will refer to it as the interior production regime since it applies to a situation in which net emissions are strictly positive.

Consider now the case in which $\varepsilon (Q) - \alpha (A) \leq 0$. Obviously if an optimal $A$ could have been strictly greater than $\alpha^{-1} (\varepsilon (Q))$ then marginal profit would have been negative, a contradictory statement; which limits the range of demand for abatement such that $A = \alpha^{-1} (\varepsilon (Q))$. In this case, separability is relaxed, a solution satisfies: (3) net emission at the optimal solution is null and (4) inverse demand of abatement equates marginal market revenue of final good clean production.

\begin{align*}
Q^* &= \varepsilon^{-1} (\alpha (A^*)) \\
P_A (A^*) &= \xi (\varepsilon^{-1} (\alpha (A^*))) \alpha' (A^*)
\end{align*}

Equations describe a kink in the inverse demand of $A$ such that (1) and (3) both are true, the quantity of abatement equating for any tax level optimal pollutant emissions and reduction $A_k (\tau)$, strictly negatively correlated with the tax.

**Proposition 1** The kinked inverse demand for abatement is (2): equal to marginal environmental costs reduction if $A$ is relatively low, then (4): equal to saturated downstream net revenue, after the kink $A_k (\tau) := \alpha^{-1} (\varepsilon (\xi^{-1} (\tau)))$ equating both such that:

\begin{equation}
\forall A > 0 : P_A (A) = \max \left( 0, \min \left( \tau \times \alpha' (A), \xi (\varepsilon^{-1} (\alpha (A))) \frac{\alpha' (A)}{\varphi (A)} \right) \right)
\end{equation}

4 Taxation Regime and Monopoly Outcome

Proposition 1 allows us to compute the monopoly’s anticipation by replacing (5) into the monopoly’s program:

\begin{equation}
\max_{A > 0} \{ A \times P_A (A) - K (A) \}
\end{equation}

If a unique strictly positive maximum exists, it corresponds to one among three equilibria. Note that the inverse demand is piecewise differentiable, so we can compute its derivative. It is comprised of three parts: the first describes marginal profit in the interior regime, then an undifferentiable point on the kink quantity and finally the last part is marginal profit when the production of abatement and final good grow jointly.

For the interior regime, let us present the marginal profit at a quantity below the kink. Defined for any tax level, the unique root of this functional is $A_1 (\tau)$ such that:

\begin{equation}
\tau \alpha' (A_1 (\tau)) + \tau \alpha'' (A_1 (\tau)) \times A_1 (\tau) = K' (A_1 (\tau))
\end{equation}

Since $A_1 (\tau)$ is indeed interior if and only if $A_1 (\tau) < A_k (\tau)$, it applies only if marginal profit evaluated at full depollution is negative, a condition equivalent to positive net emission at the optimal solution. Running implicit function theorem on this equation it appears $A_1 (\tau)$ is strictly increasing with respect to $\tau$. However the function $A_k (\tau)$ is
decreasing. The analysis of the difference between the two states shows that there exists a unique $T_1$ tax level equating both. And therefore any tax level greater than $T_1$ nullifies the net emission at the optimal solution.

$$A_1 (\tau) > A_k (\tau) \iff \tau > T_1$$

(7)

Otherwise, for any abatement quantity greater than the kink, the production of final goods $Q$ saturates along the production of $A$. The inverse demand becomes $\psi (A)$, this production regime implies a new formula of marginal profit to the right of the kink, defining another unique root $A_0$, now invariant with respect to $\tau$ since there is virtually no emission:

$$\psi (A_0) + A_0 \psi' (A_0) = K' (A_0)$$

Where $A_0$ is not a global maximum if this definition of marginal profit is negative when evaluated at the kink, in other word if $A_0$ is lower than the kink:

$$A_k (\tau) := \alpha^{-1} (\varepsilon (\xi^{-1} (\tau))) > A_0 \iff \tau < T_2 := \xi (\varepsilon^{-1} (\alpha (A_0)))$$

(8)

**Proposition 2** The kink in the inverse demand is an equilibrium if marginal profit is both: positive for any quantity below (7) and negative for any quantity above (8)

$$A^* (\tau) = A_k (\tau) := \alpha^{-1} (\varepsilon (\xi^{-1} (\tau))) \iff \tau \in [T_1, T_2]$$

For any tax level lower than $T_1$, the unique profit maximizer follows from equation (6) and is $A_1 (\tau) < A_k (\tau)$. And finally for any tax greater than $T_2$ the unique profit maximizer is $A_0$. Incidentally we have identified a policy trap since markets do not respond afterward.

This way we can define market equilibrium for any positive tax level such that:

$$\begin{cases} 
A^* (\tau) = \max (A_0, \min (A_k (\tau), A_1 (\tau))) \\
Q^* (\tau) = \max (\varepsilon^{-1} (\alpha (A_0)), \xi^{-1} (\tau)) 
\end{cases}$$

5 Policy Recommendation

In this section we define a welfare criterion including the market equilibrium supply functions and derive from it an optimal tax for any damage of pollution. Say a benevolent central planner wants to maximize the social benefit of production by implementing a positive tax on emission and assume there exists a unique solution to this program:

$$\max_{\tau > 0} \{ Q^* (\tau) \}$$

$$WF (\tau) := \int_0^Q P_Q (u) \, du - C (Q^* (\tau)) - K (A^* (\tau)) - \nu \times \max (0, \varepsilon (Q^* (\tau)) - \alpha (A^* (\tau)))$$

Convex optimization allows for the use of simple intuition here; when the marginal criterion is positive the evaluated point is sub-optimal, inversely if the marginal criterion is negative the evaluated point is super-optimal. Indeed $WF (\tau)$ is piecewise differentiable, made of three relevant parts. First when $\tau < T_1$, applying (1) and (6), definitions of market equilibrium in the interior regime $Q^* = \xi^{-1} \left( \frac{\tau}{(-)} \right)$ and $A^* = A_1 \left( \frac{\tau}{(+)} \right)$ gives us the
following marginal welfare, decreasing strictly positive when evaluated at the minimum between $\nu$ and $T_1$:

$$[(\tau - \nu)] \frac{\varepsilon' (\xi^{-1}(\tau))}{\xi' (\xi^{-1}(\tau))} + [\nu - \tau (1 - \varepsilon_{\alpha'})] \times \alpha' (A_1 (\tau)) \times \frac{\partial A_1}{\partial \tau} > 0$$  \hspace{1cm} (9)

where $\varepsilon_{\alpha'} := \frac{A \times \alpha' (A)}{\alpha'' (A)}$

**Lemma 1** If optimal net emissions are strictly positive ($\nu < T_1$) then an optimal tax is greater than marginal damage ($\tau^* > \nu$)

More precisely this result also means that if marginal damage is greater than $T_1$ an optimal tax is not lower than $T_1$. Now let me show you why is it useless whatever higher marginal social cost is to continue raising the tax, that leading to a lower than marginal damage optimal tax.

When $\tau \in (T_1, T_2)$ equilibrium fixes on the kink. In which case $Q$ following (1) has the same equilibrium properties, although now $A$ saturates along $A_k$, and according to (7) in this range $A_k$ is lower than $A_1$ so that we can sign marginal profit as:

$$\frac{\partial W F (\tau)}{\partial \tau} |_{\tau \in (T_1, T_2)} := \left( \tau - \frac{K' (A^k (\tau))}{\alpha' (A^k (\tau))} \right) \times \frac{\varepsilon' (\xi^{-1}(\tau))}{\xi' (\xi^{-1}(\tau))} < 0$$  \hspace{1cm} (10)

Indeed in such case the environmental externality nullifies and the only worry of the regulator becomes consumer's surplus maximization. Otherwise there is a policy trap when $\tau > T_2 = \frac{P_a (A_0)}{\alpha' (A_0)}$ that is any tax level greater than saturated price of pollution reduction. Obviously super-optimal according to the last inequality

**Proposition 3** The market structure and the efficiency of abatement allows us to consider two kind of marginal social damages. Such that if the optimal net emission level is null then an optimal tax is strictly lower than its associated marginal damage, following inequalities (9) and (10) which applies only if the production of $Q$ is too much hurtful:

$$\forall \nu > T_1 : \tau^* = T_1 < \nu$$

Oppisingly by Lemma 1 we recover Canton, Soubeyran and Stahn (2008) conclusion that if an optimal level of emission is positive an associated optimal tax should be greater than marginal damage:

$$\forall \nu \leq T_1 : \tau^* \in [\nu, T_1]$$
6 Conclusion

To conclude, the present note explored the question of optimal taxation when the eco-industry is imperfectly competitive. Naturally one shall recommend a tax mitigating pollutant emission and the provision compression exerted by the monopoly as in Barnett (1980). Throughout this note I convinced you that public regulation of the most damaging industries should always be less of an obstacle than marginal social cost. Precisely, you were shown, that for any marginal damage of pollution greater than an implicitly defined constant called $T_1$, an optimal tax is lower than marginal damage, contradicting the optimal taxation scheme suggested by Canton et al. (2008); which is recovered for any marginal damage lower than $T_1$.

Long story short the said constant defines the edge of two production regimes. The first is referred as "interior" for the monopolistically supplied abatement level is not high enough to cover the whole emissions. In that case, an optimal marginal tax on emission is set higher than marginal damage with the intention to boost pollution reduction. It happens that the greater is the marginal social cost of pollutant the greater should be abatement joint with the lower should be the production of final good so that market equilibrium net emissions are decreasing. Given that firm-sized pollution reduction effort is lower or equal to emission then implies a kink in the abatement demand, itself optimal when the marginal social cost of emission is too large. Hence the second regime. Indeed in such a case the environmental externality nullifies and the only worry of the regulator would then be downstream production, still decreasing with respect to the tax.

As the study was focused on the kink and its implications to the optimal tax scheme, we did not investigate some natural extensions such as increasing convex marginal damage of emission, downstream and/or upstream oligopoly or a permit market for pollution rights. In the case of increasing convex social damage of emission, I foresee it could give the same result if marginal damage evaluated in 0 emission would nevertheless be greater than $T_1$, a condition we could still see fit to the most damaging productions (such as nuclear waste or contaminated water).

Considering oligopolies downstream and upstream both would only bring tedious computational work to the intuition and require that the upstream mono/oligopoly is the less competitive sector for the result to stand. Once again, as suggested in Canton et al. (2008).

Alas it could not resist in a permit market model, any positive quantity of permits naturally extends to interior regime. Since net emissions and tax are negatively correlated we would find by construction a "well-behaved" demand for permit, but in the case we are concerned with, the most damaging productions, we foresee that we would find an indetermination for the trading price of abatement and permits both when the quantity of permit supplied by state is null.

Proofs of the Market Equilibrium

In a five steps demonstration we search for existence and characterization of market equilibrium in the treated programs. First step define the convex envelop of optimal final good production and abatement
demand as a function of $\tau$ and $P_A$. Then secondly, steps 2 and 3 are respectively the demonstration of existence and characterization market equilibrium as a function of the tax. The next step, the fourth extensively define regime shifting across the kink. Finally step 5 stresses the critical role of the max function.

**step1) Downstream**

In this first step we search for existence and characterization of final good production and abatement goods demand:

$$
\max_{A > 0} \left\{ \max_{Q > 0} \left\{ Q \times P_Q - C(Q) - \tau \times \max \{0, \varepsilon(Q) - \alpha(A)\} \right\} - A \times P_A \right\}
$$

If we assume $P_Q(Q), \alpha'(A)$ strictly decreasing and $C'(Q), \varepsilon'(Q)$ strictly increasing we imply strict concavity of the program, first order conditions give the optimum. Let us suppose an interior solution (if the borders’ limits behave rightfully), then if we apply the method of Rockafellar (1997) with subgradient $\partial P_D$:

$$
\partial_{Q} P_D = \begin{cases} 
P_Q(Q) - C'(Q) & \text{if } Q < \varepsilon^{-1}(\alpha(A)) \\
[\varepsilon^{-1}(\alpha(A))] - P_A & \text{if } Q = \varepsilon^{-1}(\alpha(A)) \\
P_Q(Q) - C'(Q) - \tau \times \varepsilon'(Q) & \text{if } Q > \varepsilon^{-1}(\alpha(A)) 
\end{cases}
$$

The optimum is such that: $0 \in \partial_{Q} P_D|_{Q = Q^*(\tau, A)} \Leftrightarrow Q^*(\tau, A) = \min \left( \xi^{-1}(0), \max (\varepsilon^{-1}(\alpha(A)), \xi^{-1}(\tau)) \right)$

$$
\partial_{A} P_D = \begin{cases} 
\tau \times \alpha'(A) - P_A & \text{if } A < \alpha^{-1}(\xi^{-1}(\tau)) \\
\tau \times \alpha'(A) - P_A & \text{if } A = \alpha^{-1}(\xi^{-1}(\tau)) \\
\xi^{-1}(\alpha(A)) - P_A & \text{if } A \in (\alpha^{-1}(\xi^{-1}(\tau)), \alpha^{-1}(\xi^{-1}(0))) \\
- P_A & \text{if } A \geq \alpha^{-1}(\xi^{-1}(0)) 
\end{cases}
$$

Pose $\psi(A) = \xi^{-1}(\alpha(A)) \alpha'(A)$, this function is continuously differentiable and decreasing.

$$
0 = \partial_{A} P_D|_{A = A^{\alpha'}(P_A, \tau)} \Leftrightarrow A^{\alpha'}(P_A, \tau) = \min \left( \psi^{-1}(P_A), (\alpha')^{-1}(\frac{P_A}{A}) \right)
$$

**step2) Upstream solution existence**

$$
\max_{A > 0} \left\{ A \times \min (\tau \times \alpha'(A), \psi(A)) - K(A) \right\}
$$

Assume $K'(A)$ strictly increasing, if $\frac{A \times \psi'(A)}{\psi(A)} > -2$ and $\frac{A \times \alpha'(A)}{\alpha(A)} > -2$ both are true for any choice of $A \in \mathbb{R}_{+}$ then we ensure strict concavity of this program.

**step3) Upstream solution characterization**

Suppose the unique solution is interior and let us concentrate on $A \in (0, \alpha^{-1}(\varepsilon(\xi^{-1}(0))))$ since we have seen downstream any quantity higher would not be traded. On this space, first order conditions give the optimum if we apply subgradient $\partial_{A} P_U$:

$$
\partial_{A} P_U = \begin{cases} 
\phi_1(A) := \tau \times \alpha'(A) + A \times \tau \times \alpha''(A) - K'(A) & \text{if } A \in (0, \alpha^{-1}(\varepsilon(\xi^{-1}(\tau)))) \\
[\phi_2(\alpha^{-1}(\varepsilon(\xi^{-1}(\tau))))], \phi_1(\alpha^{-1}(\varepsilon(\xi^{-1}(\tau)))) & \text{if } A = \alpha^{-1}(\varepsilon(\xi^{-1}(\tau))) \\
\phi_2(A) := \psi(A) + A \times \psi'(A) - K'(A) & \text{if } A \in (\alpha^{-1}(\varepsilon(\xi^{-1}(\tau))), \alpha^{-1}(\varepsilon(\xi^{-1}(0)))) 
\end{cases}
$$

Let us call $A_1(\tau)$ and $A_0$ the unique value of $A \in \mathbb{R}_{+}$ such that $\phi_1(A_1(\tau)) = 0$ and $\phi_2(A_0) = 0$

$$
0 = \partial_{A} P_U|_{A = A^{\alpha'}(P_A, \tau)} \Leftrightarrow A^{\alpha'}(P_A, \tau) = \max \left( A_0 \min (\alpha^{-1}(\varepsilon(\xi^{-1}(\tau))), A_1(\tau)) \right)
$$

**step4) Regime shifting**

Finally let us remark $\frac{\partial A_1(\tau)}{\partial \tau} = -\frac{\alpha'(A) + A \times \tau \times \alpha''(A)}{\tau \times \alpha''(A) + A \times \tau \times \alpha''(A) - K'(A)} > 0$, moreover by construction

$$
A_1(0) < \alpha^{-1}(\varepsilon(\xi^{-1}(0))) \text{ which implies there exists an unique } T_1 > 0 \text{ such that}
$$
\[ A_1(T_1) = \alpha^{-1}(\varepsilon(\xi^{-1}(T_1))), \text{ and also by construction } A_1(\xi(\varepsilon^{-1}(\alpha(A_0)))) > A_0 \Leftrightarrow T_1 < \xi(\varepsilon^{-1}(\alpha(A_0))) \]

the tax level such that \( A_0 = \alpha^{-1}(\varepsilon(\xi^{-1}(\tau))) \).

**step5) Generalization**

Obviously the case without the \( \max(0, \varepsilon(Q) - \alpha(A)) \) function which enforces complementarity simply extends the solution functions when \( \tau < T_1 \) for any value of \( \tau \in \mathbb{R}_+ \).

**References**


