Geometry of algebraic curves (1750-1850) : mathematicians and historians' readings of Gabriel Cramer's *Introduction à l'analyse des lignes courbes algébriques* (1750)

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1750-1850 : rupture(s) et continuité(s) en géométrie
26 juin 2015, Nancy

Plan

Gabriel Cramer (1704-1752)

Introducing the *Introduction*

Differential analysis of the reception of the treatise (1750-1900) : encyclopedists, historians, mathematicians

Conclusion and perspectives

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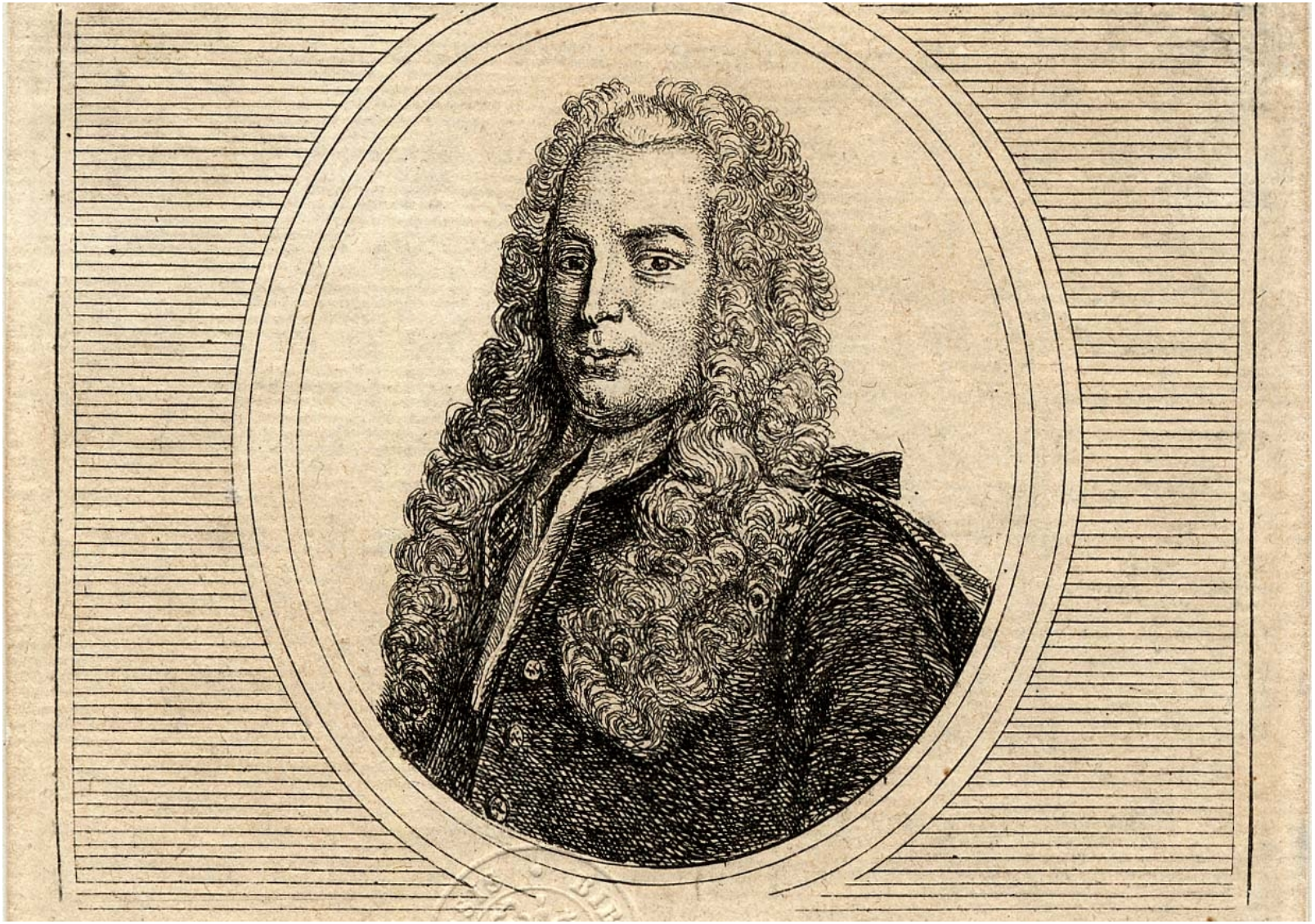

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A
L'ANALYSE
DES
LIGNES COURBES
ALGÉBRIQUES.
Par
GABRIEL CRAMER,
Professeur de Philosophie & de Mathématiques; des Académies & Sociétés Royales de Londres, de Berlin, de Montpellier, de Lyon, & de l'Académie de l'Institut de Bologne.

BIBLIOTHÈQUE NATIONALE RENNES

17195

A GENEVE,
Chez les FRERES CRAMER & CL. PHILIBERT.
MDCCL.

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The case of Euler's *Introductio*

Introductio analysin infinitorum (vol.2, Euler, 1748) has the same object and goals ... reached by different ways and means.

> *"C'est trop d'honneur que Vous faites à mon traité sur la meme matiere, en le mettant en parallel avec le Votre, car maintenant je viens de reconnoitre, que je n'ai pas pris toutes les précautions necessaires pour bien developper la nature des branches qui s'etendent à l'infini, et que suivant ma methode je me pourrois souvent tromper meme sur l'existence de ces branches. [...] A cause de ce defaut et encore d'autres dont mon ouvrage est defiguré, je Vous felicite d'avoir enrichi le public d'un ouvrage accompli sur cette matiere, qui etant delivré de tout defaut, explique la theorie des lignes courbes aussi solidement, que clairement."*

Letter from Euler to Cramer, 15 oct 1750

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A systematic study of the algebraic curves (with the only help of algebra) for the classification of the curves of the first five orders, based on infinite branches and singular points :

- Generalities on algebraic curves and their equations (chap I-V)
- Diameters and centers (chap VI)
- Method of series, analytical triangle (chap VII)
- Infinite branches (chap VIII)
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- Infinite branches (chap VIII)
- General classification of the lines of the first five orders (chap IX)
- Multiple and singular points (chap X and XIII)
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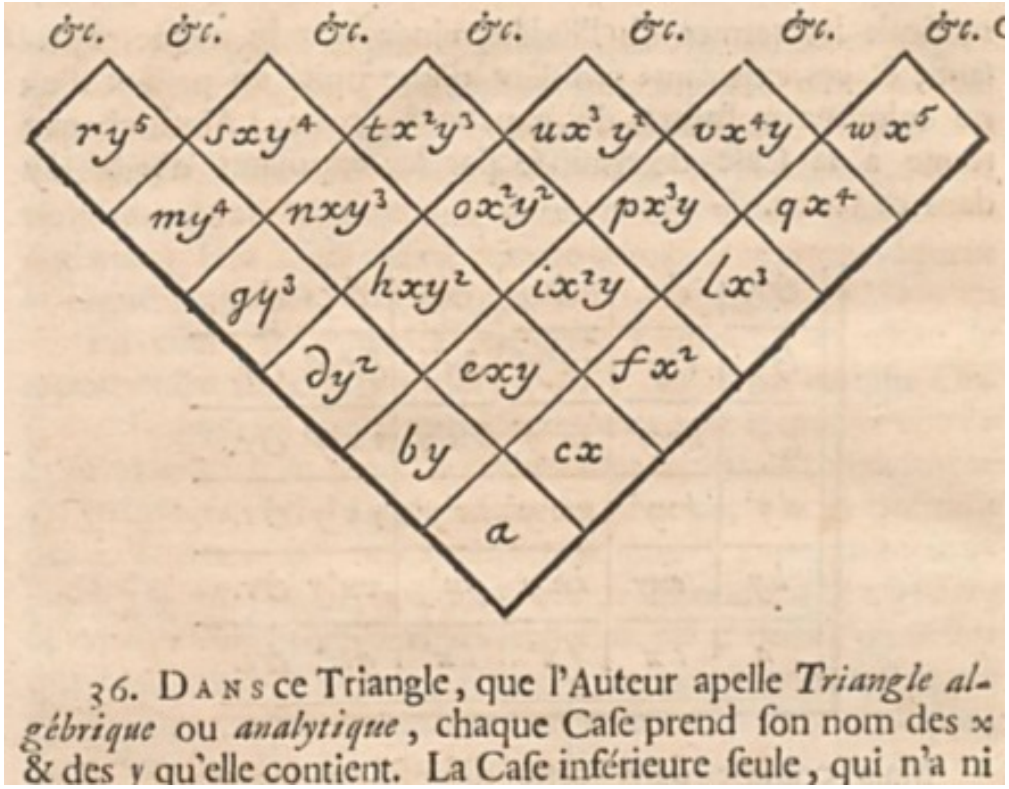
A favored tool : the **analytical triangle**



The analytical triangle is used to identify the overriding terms of the equation and calculate series developments.

A favored tool : the **analytical triangle**

« On formera le Triangle avec des points disposés en quinconce, & on en changera en une étoile, ou en une petite croix, chaque point qui tient la place d'un des termes de l'équation.[...] Si l'on suppose x ou y infiniment petite, on cherchera, avec la Règle, quelles sont les Cases pleines par le centre desquelles peut passer une Droite, sans laisser au-dessous d'elle aucune Case pleine. Cette Droite, ou ces Droites, car il peut y en avoir plus d'une, se nommeront des Déterminatrices inférieures, parce qu'elles déterminent les plus grands termes de l'équation. »

Introduction, pp 165-166

An example from the *Introduction*

Let's consider the quartic curve defined by the following equation :

$$x^4 - ax^2y - axy^2 - \frac{1}{4}aayy = 0 \tag{1}$$

If we represent the different terms of this equation on the analytical triangle, we obtain a single "déterminatrice inférieure" (for a study at the origin) :

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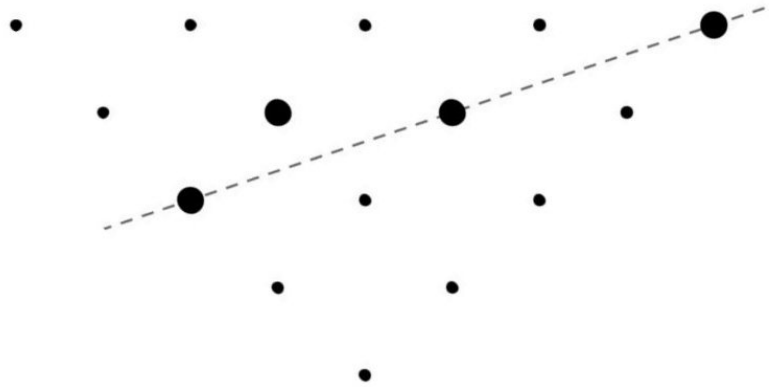

An example from the *Introduction*

This "déterminatrice" provides the following equation

$$x^4 - ax^2y + \frac{1}{4}aayy = 0 \tag{2}$$

which can be resolved by

$$y = \frac{2xx}{a}$$

Therefore, the first term of the development is $\frac{2xx}{a}$.
To calculate the next term of this development, let's substitute $\left(\frac{2xx}{a} + u\right)$ to y in the equation (2).
We obtain this new equation :

$$-4\frac{x^5}{a} - 4x^3u - axu^2 + \frac{1}{4}a^2u^2 = 0. \tag{3}$$

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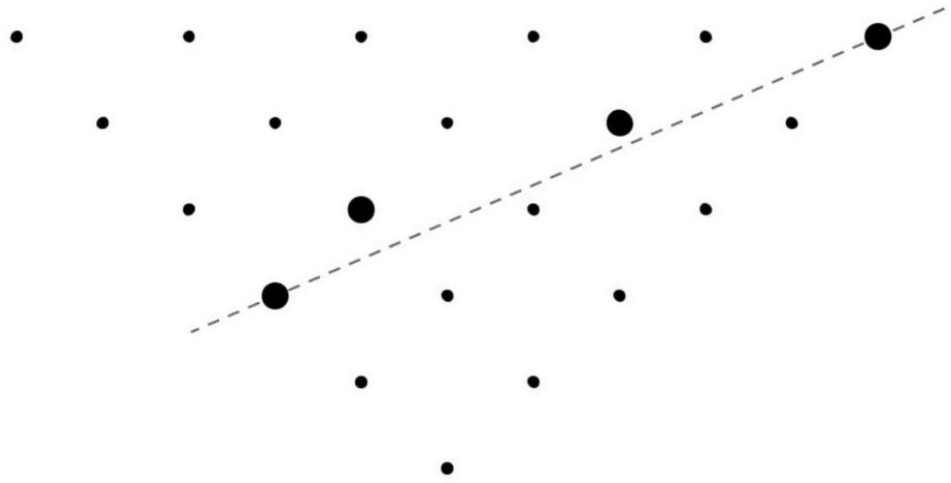

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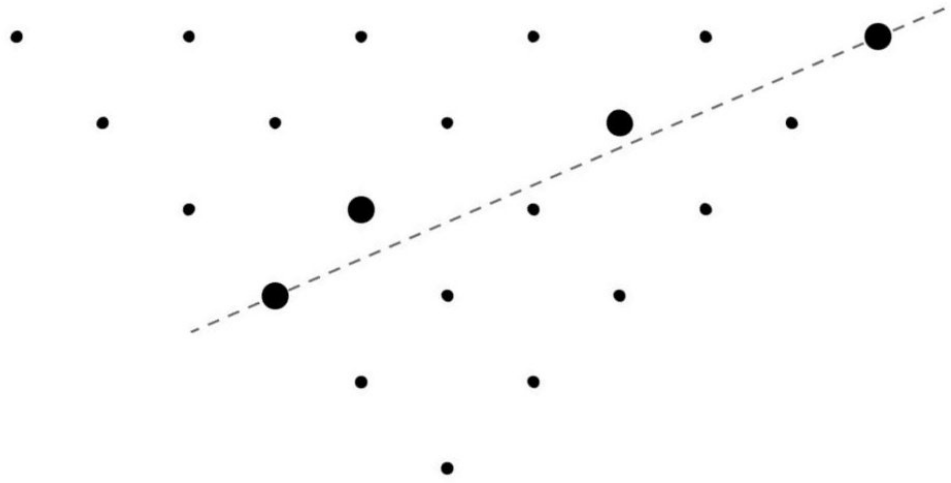

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An example from the *Introduction*

This equation, for $x > 0$, has two solutions given by :

$$u = \pm \frac{4x^{\frac{5}{2}}}{a^{\frac{3}{2}}} = \pm \frac{4xx\sqrt{ax}}{aa}.$$

The development is thus divided in two distinct series from the second term :

$$y = \frac{2xx}{a} \pm \frac{4xx\sqrt{ax}}{aa} + \frac{8x^3}{a^3} + \dots.$$

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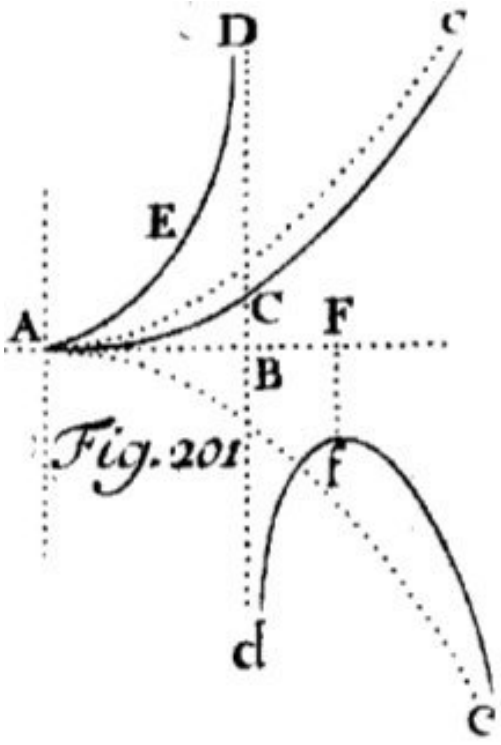
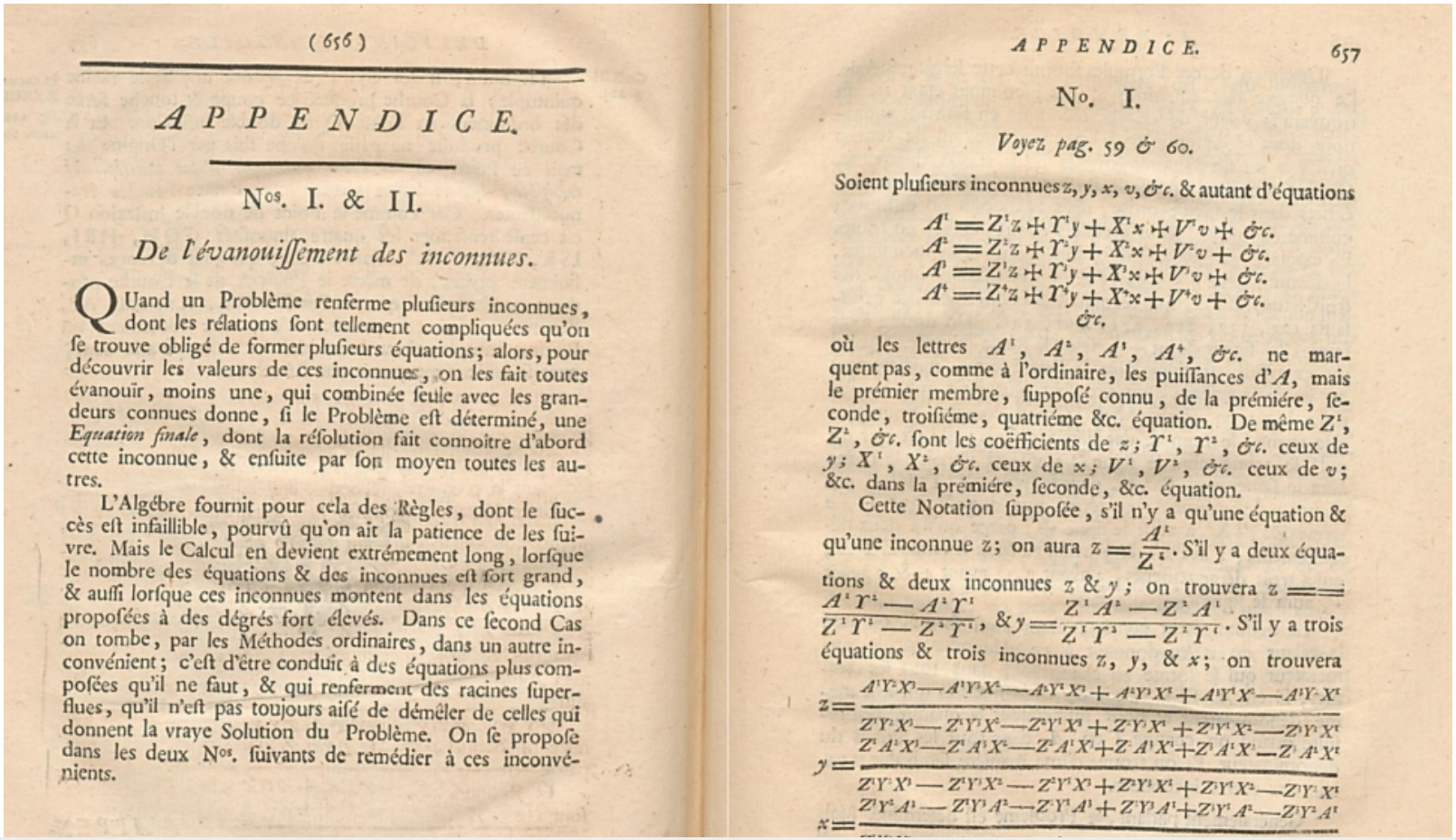
An example from the *Introduction*

Finally we get this figure, with a beak cusp at the origin (*Introduction*, plate XXVI, p 600, fig. 201)



The appendices present two important results, the first being known today as **Cramer's rule** for the resolution of a system of linear equations (Appendix I, p 656-659) :



(656)

APPENDICE.

Nos. I. & II.

De l'évanouissement des inconnues.

QUand un Problème renferme plusieurs inconnues, dont les rélations sont tellement compliquées qu'on se trouve obligé de former plusieurs équations; alors, pour découvrir les valeurs de ces inconnues, on les fait toutes évanouïr, moins une, qui combinée seule avec les grandeurs connues donne, si le Problème est déterminé, une *Equation finale*, dont la résolution fait connoître d'abord cette inconnue, & ensuite par son moyen toutes les autres.

L'Algébre fournit pour cela des Règles, dont le succès est infaillible, pourvû qu'on ait la patience de les suivre. Mais le Calcul en devient extrémement long, lorsque le nombre des équations & des inconnues est fort grand, & aussi lorsque ces inconnues montent dans les équations proposées à des dégrés fort élevés. Dans ce second Cas on tombe, par les Méthodes ordinaires, dans un autre inconvénient; c'est d'être conduit à des équations plus composées qu'il ne faut, & qui renferment des racines superflues, qu'il n'est pas toujours aisé de démêler de celles qui donnent la vraye Solution du Problème. On se propose dans les deux Nos. suivants de remédier à ces inconvénients.

APPENDICE. 657

No. I.

Voyez pag. 59 & 60.

Soient plusieurs inconnues z, y, x, v, &c. & autant d'équations

$$A^1 = Z^1 z + Y^1 y + X^1 x + V^1 v + \&c.$$
$$A^2 = Z^2 z + Y^2 y + X^2 x + V^2 v + \&c.$$
$$A^3 = Z^3 z + Y^3 y + X^3 x + V^3 v + \&c.$$
$$A^4 = Z^4 z + Y^4 y + X^4 x + V^4 v + \&c.$$
$$\&c.$$

où les lettres A^1, A^2, A^3, A^4, &c. ne marquent pas, comme à l'ordinaire, les puissances d'A, mais le prémier membre, supposé connu, de la prémiére, seconde, troisiéme, quatriéme &c. équation. De même Z^1, Z^2, &c. sont les coëfficients de z; Y^1, Y^2, &c. ceux de y; X^1, X^2, &c. ceux de x; V^1, V^2, &c. ceux de v; &c. dans la prémiére, seconde, &c. équation.

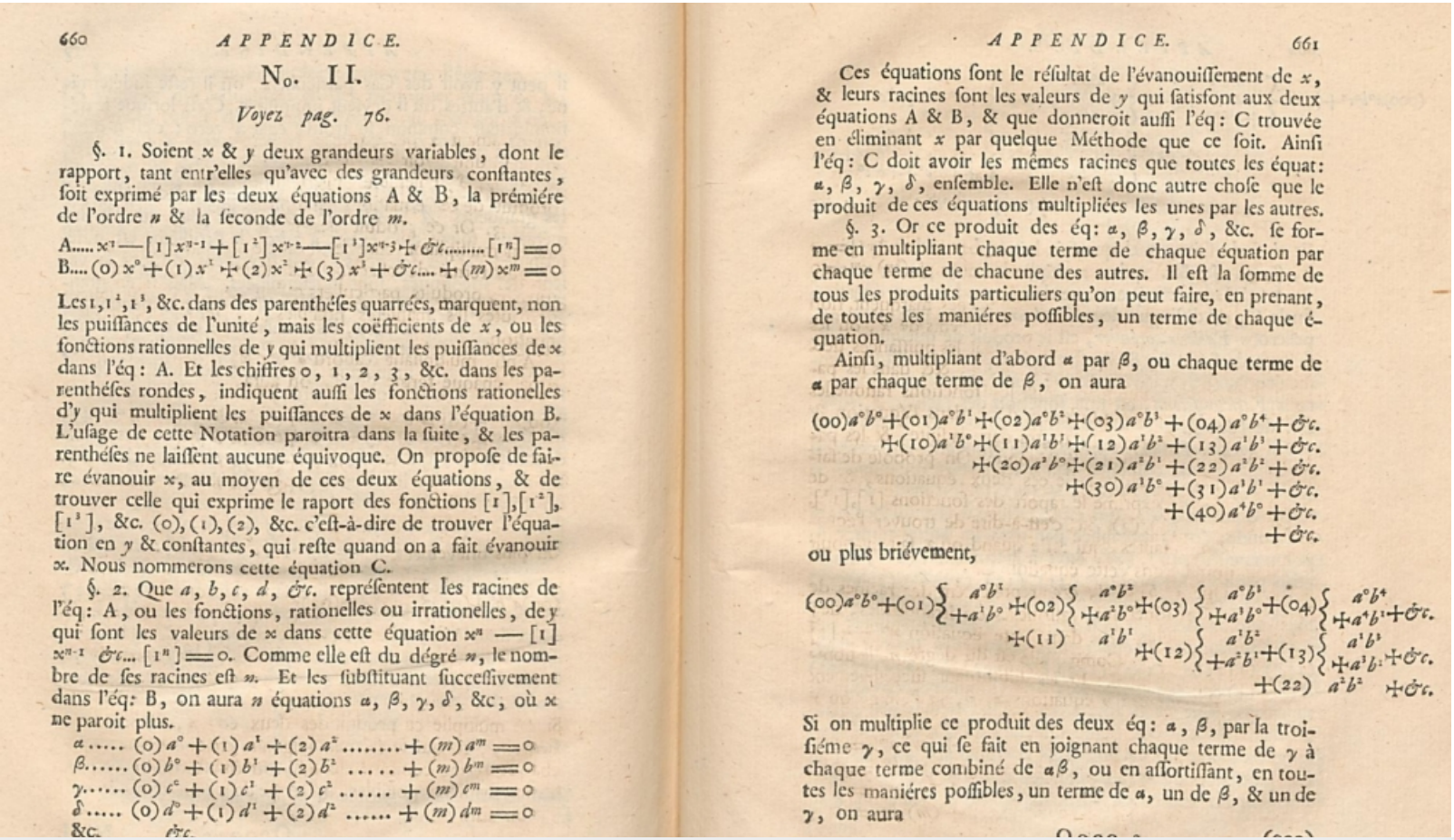
Cette Notation supposée, s'il n'y a qu'une équation & qu'une inconnue z; on aura $z = \frac{A^1}{Z^1}$. S'il y a deux équations & deux inconnues z & y; on trouvera $z = \frac{A^1 Y^2 - A^2 Y^1}{Z^1 Y^2 - Z^2 Y^1}$, & $y = \frac{Z^1 A^2 - Z^2 A^1}{Z^1 Y^2 - Z^2 Y^1}$. S'il y a trois équations & trois inconnues z, y, & x; on trouvera

$$z = \frac{A^1Y^2X^3 - A^1Y^3X^2 - A^2Y^1X^3 + A^2Y^3X^1 + A^3Y^1X^2 - A^3Y^2X^1}{Z^1Y^2X^3 - Z^1Y^3X^2 - Z^2Y^1X^3 + Z^2Y^3X^1 + Z^3Y^1X^2 - Z^3Y^2X^1}$$

$$y = \frac{Z^1A^2X^3 - Z^1A^3X^2 - Z^2A^1X^3 + Z^2A^3X^1 + Z^3A^1X^2 - Z^3A^2X^1}{Z^1Y^2X^3 - Z^1Y^3X^2 - Z^2Y^1X^3 + Z^2Y^3X^1 + Z^3Y^1X^2 - Z^3Y^2X^1}$$

$$x = \frac{Z^1Y^2A^3 - Z^1Y^3A^2 - Z^2Y^1A^3 + Z^2Y^3A^1 + Z^3Y^1A^2 - Z^3Y^2A^1}{}$$

... and the second as **Bézout's theorem**, which affirms that the number of intersections of two curves is equal to the product of their degrees (Appendix II, p 660-676)



660 APPENDICE.

No. II.

Voyez pag. 76.

§. 1. Soient x & y deux grandeurs variables, dont le rapport, tant entr'elles qu'avec des grandeurs constantes, soit exprimé par les deux équations A & B, la prémiére de l'ordre n & la seconde de l'ordre m.

$$A \ldots\ldots x^n - [1]x^{n-1} + [1^2]x^{n-2} - [1^3]x^{n-3} + \&c. \ldots\ldots [1^n] = 0$$
$$B \ldots\ldots (0)x^0 + (1)x^1 + (2)x^2 + (3)x^3 + \&c. \ldots + (m)x^m = 0$$

Les $1, 1^2, 1^3$, &c. dans des parenthéses quarrées, marquent, non les puissances de l'unité, mais les coëfficients de x, ou les fonctions rationnelles de y qui multiplient les puissances de x dans l'éq: A. Et les chiffres 0, 1, 2, 3, &c. dans les parenthéses rondes, indiquent aussi les fonctions rationelles d'y qui multiplient les puissances de x dans l'équation B. L'usage de cette Notation paroitra dans la suite, & les parenthéses ne laissent aucune équivoque. On propose de faire évanouir x, au moyen de ces deux équations, & de trouver celle qui exprime le raport des fonctions $[1]$, $[1^2]$, $[1^3]$, &c. (0), (1), (2), &c. c'est-à-dire de trouver l'équation en y & constantes, qui reste quand on a fait évanouir x. Nous nommerons cette équation C.

§. 2. Que a, b, c, d, &c. représentent les racines de l'éq: A, ou les fonctions, rationelles ou irrationelles, de y qui sont les valeurs de x dans cette équation $x^n - [1]x^{n-1} \&c. \ldots [1^n] = 0$. Comme elle est du dégré n, le nombre de ses racines est n. Et les substituant successivement dans l'éq: B, on aura n équations $\alpha, \beta, \gamma, \delta$, &c, où x ne paroit plus.

$$\alpha \ldots\ldots (0)a^0 + (1)a^1 + (2)a^2 \ldots\ldots + (m)a^m = 0$$
$$\beta \ldots\ldots (0)b^0 + (1)b^1 + (2)b^2 \ldots\ldots + (m)b^m = 0$$
$$\gamma \ldots\ldots (0)c^0 + (1)c^1 + (2)c^2 \ldots\ldots + (m)c^m = 0$$
$$\delta \ldots\ldots (0)d^0 + (1)d^1 + (2)d^2 \ldots\ldots + (m)d^m = 0$$
&c. &c.

APPENDICE. 661

Ces équations sont le résultat de l'évanouissement de x, & leurs racines sont les valeurs de y qui satisfont aux deux équations A & B, & que donneroit aussi l'éq: C trouvée en éliminant x par quelque Méthode que ce soit. Ainsi l'éq: C doit avoir les mêmes racines que toutes les équat: $\alpha, \beta, \gamma, \delta$, ensemble. Elle n'est donc autre chose que le produit de ces équations multipliées les unes par les autres.

§. 3. Or ce produit des éq: $\alpha, \beta, \gamma, \delta$, &c. se forme en multipliant chaque terme de chaque équation par chaque terme de chacune des autres. Il est la somme de tous les produits particuliers qu'on peut faire, en prenant, de toutes les maniéres possibles, un terme de chaque équation.

Ainsi, multipliant d'abord α par β, ou chaque terme de α par chaque terme de β, on aura

$$\begin{aligned}
&(00)a^0b^0 + (01)a^0b^1 + (02)a^0b^2 + (03)a^0b^3 + (04)a^0b^4 + \&c.\\
&\quad + (10)a^1b^0 + (11)a^1b^1 + (12)a^1b^2 + (13)a^1b^3 + \&c.\\
&\quad\quad + (20)a^2b^0 + (21)a^2b^1 + (22)a^2b^2 + \&c.\\
&\quad\quad\quad + (30)a^3b^0 + (31)a^3b^1 + \&c.\\
&\quad\quad\quad\quad + (40)a^4b^0 + \&c.\\
&\quad\quad\quad\quad\quad + \&c.
\end{aligned}$$

ou plus briévement,

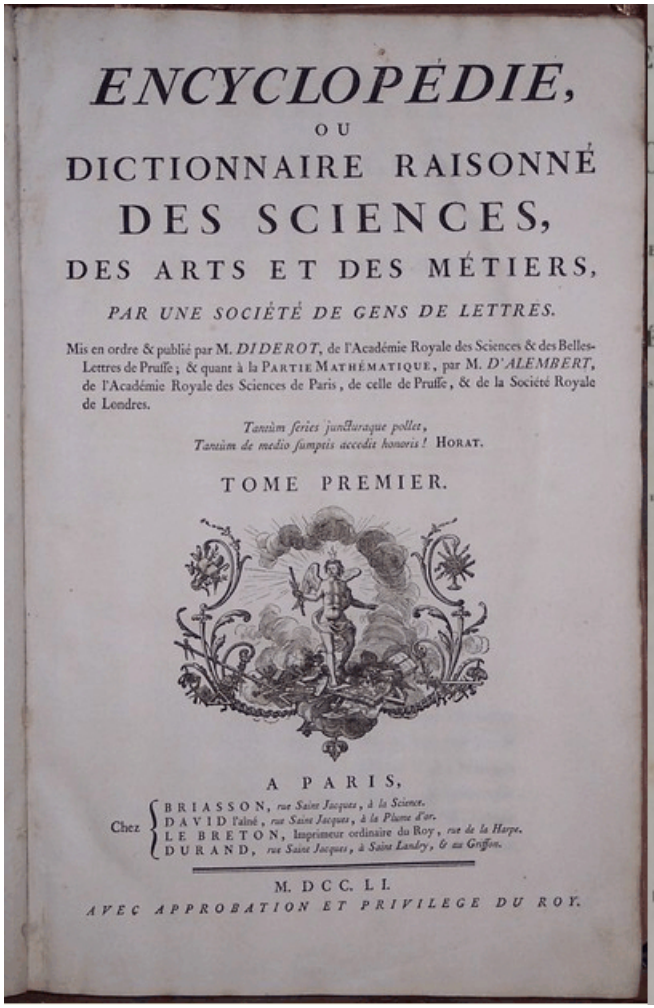
$$\begin{aligned}
&(00)a^0b^0 + (01)\left\{\begin{matrix} a^0b^1 \\ + a^1b^0 \end{matrix}\right. + (02)\left\{\begin{matrix} a^0b^2 \\ + a^2b^0 \end{matrix}\right. + (03)\left\{\begin{matrix} a^0b^3 \\ + a^3b^0 \end{matrix}\right. + (04)\left\{\begin{matrix} a^0b^4 \\ + a^4b^1 \end{matrix}\right. + \&c.\\
&\quad + (11)\ a^1b^1 \quad + (12)\left\{\begin{matrix} a^1b^2 \\ + a^2b^1 \end{matrix}\right. + (13)\left\{\begin{matrix} a^1b^3 \\ + a^3b^1 \end{matrix}\right. + \&c.\\
&\quad\quad\quad\quad + (22)\ a^2b^2 \quad + \&c.
\end{aligned}$$

Si on multiplie ce produit des deux éq: α, β, par la troisiéme γ, ce qui se fait en joignant chaque terme de γ à chaque terme combiné de $\alpha\beta$, ou en assortissant, en toutes les maniéres possibles, un terme de α, un de β, & un de γ, on aura

We must also mention the so-known **Cramer's paradox**. Cramer establishes that a curve of degree n is fully determined by $\frac{n(n+3)}{2}$ points : for example, a cubic is defined by $\frac{3(3+3)}{2} = 9$ given points. But it is also known that two different cubic curves intersect in $3 \times 3 = 9$ points...

48. Une autre contradiction apparente entre les §§. 46 & 38, eſt celle-ci. Puiſqu'une Ligne de l'ordre m ne peut rencontrer une Ligne de l'Ordre n, qu'en mn points, une Ligne de l'Ordre v ne rencontrera une autre Ligne du même ordre qu'en vv points. Si donc vv eſt égal ou plus grand que le nombre $\frac{1}{2}vv + \frac{1}{2}v$, qui eſt celui des points qui déterminent une Ligne de l'ordre v, on pourra faire paſſer plus d'une Ligne de l'ordre v par $\frac{1}{2}vv + \frac{1}{2}v$ points; ce qui ſemble contraire au §. 38 †. Ainſi deux Lignes

Introduction pp 78-79



ENCYCLOPÉDIE,
OU
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DES SCIENCES,
DES ARTS ET DES MÉTIERS,
PAR UNE SOCIÉTÉ DE GENS DE LETTRES.

Mis en ordre & publié par M. DIDEROT, de l'Académie Royale des Sciences & des Belles-Lettres de Prusse; & quant à la PARTIE MATHÉMATIQUE, par M. D'ALEMBERT, de l'Académie Royale des Sciences de Paris, de celle de Prusse, & de la Société Royale de Londres.

Tantùm series juncturaque pollet,
Tantùm de medio sumptis accedit honoris! HORAT.

TOME PREMIER.

A PARIS,
Chez BRIASSON, rue Saint Jacques, à la Science.
DAVID l'aîné, rue Saint Jacques, à la Plume d'or.
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DURAND, rue Saint Jacques, à Saint Landry, & au Griffon.

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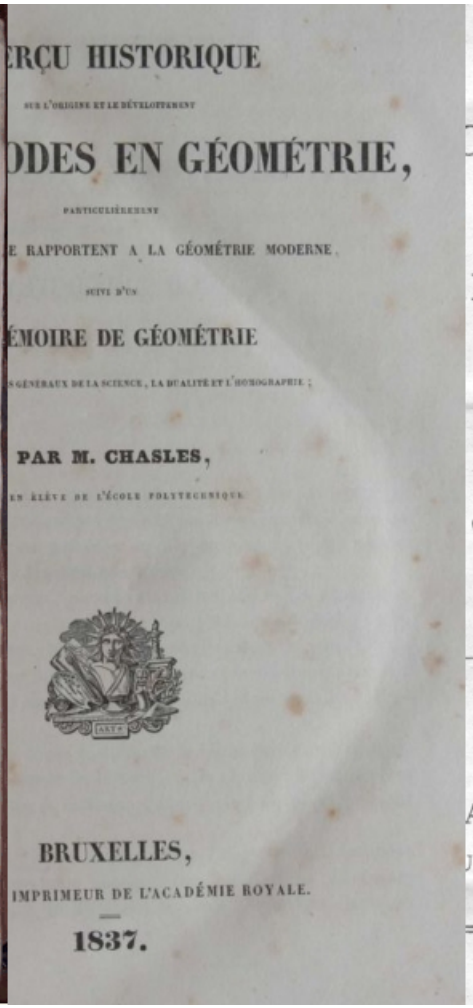

RCU HISTORIQUE
SUR L'ORIGINE ET LE DÉVELOPPEMENT
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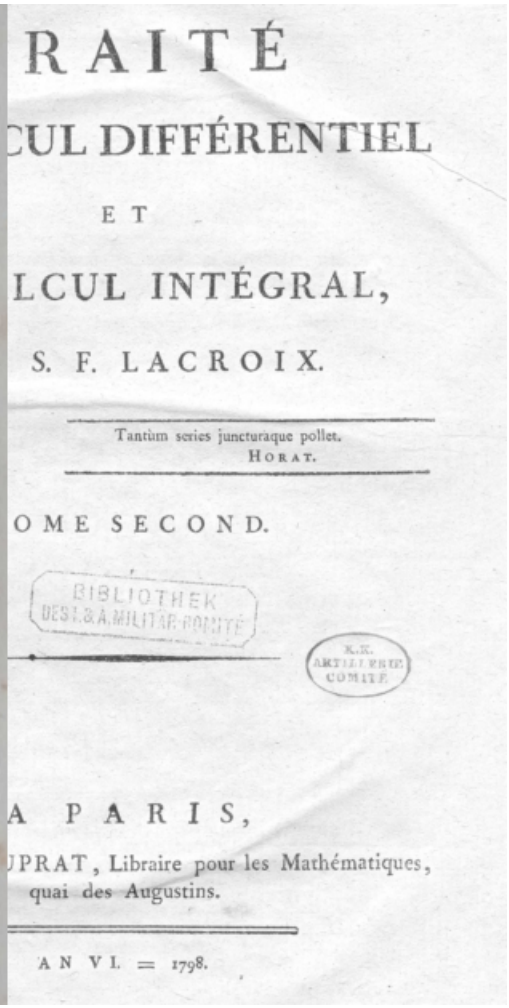
PARTICULIÈREMENT
E RAPPORTENT A LA GÉOMÉTRIE MODERNE,
SUIVI D'UN
ÉMOIRE DE GÉOMÉTRIE
S GÉNÉRAUX DE LA SCIENCE, LA DUALITÉ ET L'HOMOGRAPHIE;
PAR M. CHASLES,
EN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

BRUXELLES,
IMPRIMEUR DE L'ACADÉMIE ROYALE.
1837.



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ET
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S. F. LACROIX.

Tantùm series juncturaque pollet.
HORAT.

OME SECOND.

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AN VI. = 1798.

Reading and reception : a dynamical analysis

- through different periods of time (from 1750 to 1900),
- through different geographical spaces (France, Germany, England...),
- through different types of readers : encyclopedists, historians, mathematicians
- through different mathematical fields or practices (teachers, geometers, algebrists...)

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At the turn of the XIXth century :

- success of the differential methods in the study of curves (Lacroix, Cauchy),
- return of the synthetic methods in geometry (Poncelet, Steiner)

Is Cramer's treatise quickly turning obsolete ? Or is it still read ? By who, and what for ? What kind of posterity for Cramer's *Analyse des courbes* ?

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Is Cramer's treatise quickly turning obsolete ? Or is it still read ? By who, and what for ? What kind of posterity for Cramer's *Analyse des courbes* ?

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Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers **(d'Alembert, Paris, 1751)**

D'Alembert reads carefully the Cramer's treatise and discusses several points in three letters written between september 1750 and january 1751. He finally says :

> *"A l'egard de votre ouvrage sur les courbes je persiste toujours dans l'idée que j'en ay, et je viens d'ajouter à l'article courbe de l'Encyclopedie le jugement que j'en porte."*

Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers **(d'Alembert, Paris, 1751)**

Article COURBE :

> *"Les meilleurs ouvrages dans lesquels on puisse s'instruire de la théorie des courbes, sont 1° l'enumeratio linearum tertii ordinis de M. Newton, d'où une partie de cet article COURBE est tiré ; 2° l'ouvrage de M Stirling sur le même sujet, & Geometria organica de M. Maclaurin, dont nous avons parlé ; 3° les usages de l'analyse de Descartes par M. l'abbé de Gua, déjà cités ; [...]"*

Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers **(d'Alembert, Paris, 1751)**

"[...] ouvrage original & plein d'excellentes choses, mais qu'il faut lire avec précaution ; ***4º l'introduction à l'Analyse des Lignes Courbes, par M. Cramer, ouvrage très-complet, très-clair et très-instructif, & dans lequel on trouve d'ailleurs plusieurs méthodes nouvelles****; 5 l'ouvrage de M. Euler, qui a pour titre, introductio in analys. infinitorum, Lausan. 1748."*

Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers **(d'Alembert, Paris, 1751)**

Article Branche :

"On trouvera une théorie très-complette des branches infinies des courbes dans le huitieme chapitre de l'Introduction à l'analyse des lignes courbes par M. Cramer. Il y donne la méthode de déterminer les différentes branches d'une courbe, & leurs asymptotes droites ou courbes. Comme cette théorie nous conduiroit trop loin, nous renvoyons là-dessus à son ouvrage."

***Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers* (d'Alembert, Paris, 1751)**

Article ASYMPTOTE :

> *"Voyez [...] l'Introduction à l'analyse des Lignes courbes, par M. Cramer, p. 344. art. 147. Ce dernier ouvrage contient une excellente théorie des asymptotes des courbes géométriques & de leurs branches, chap. viii."*

Article CENTRE :

> *"M. Cramer, dans son Introduction à l'analyse des lignes courbes, donne une méthode très-exacte pour déterminer les centres généraux."*

Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers **(d'Alembert, Paris, 1751)**

Article PARALLÉLOGRAMME :

> *"On peut voir dans les usages de l'analyse de Descartes de M. l'abbé de Gua, & dans l'introduction à l'analyse des lignes courbes de M. Crammer, la démonstration, les différens usages, & les applications de cette regle, suivant les cas qui peuvent se présenter ; il suffit ici d'en donner l'esprit. Il est bon d'observer que MM. de Gua & Crammer transforment le parallélogramme en un triangle qu'ils appellent analytique, ce qui ne change rien au fond."*

Encyclopédie ou dictionnaire universel raisonné des connaissances humaines (de Felice, Yverdon, 1772) *Encyclopédie Méthodique, Mathématiques* (Pancoucke, Paris, 1784)

These encyclopaedias repeat verbatim the text of the article CURVE of the *Encyclopédie*. However :

- the *Encyclopédie d'Yverdon* adds two pages on the method of the analytical triangle,
- in the *Encyclopédie méthodique*, article APPROXIMATION : Newton's parallelogram method is "*expliquée de la manière la plus claire & la plus détaillée dans un excellent Ouvrage de M. Cramer, qui a pour titre : Introduction à l'Analyse des Lignes courbes Algébriques, Gen. 1750.*"

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Mathematical and philosophical dictionary **(Hutton, London, 1795)**

Article CURVE :

> *"The theory of curves forms a considerable branch of the mathematical sciences. Those who are curious of advancing beyond the knowledge of the circle and the conic sections, and to consider geometrical curves of a higher nature, and in a general view, will do well to study Cramer's Introduction à l'Analyse des Lignes Courbes Algebraiques, which the learned and ingenious author composed for the use of beginners."*

Mathematical and philosophical dictionary **(Hutton, London, 1795)**

Article ASYMPTOTE :

> *"See [...] Cramer's Introduction à l'analyse des lignes courbes, art 147 & seq., for an excellent theory of asymptotes of geometrical curves and their branches."*

Article [NEWTON OR ANALYTICAL] PARALLELOGRAM :

> *"And especially Cramer's Analyses des Lignes Courbes, p 148 – This author observes, that this invention, which is the true foundation of the method of series, was but imperfectly understood, and not valued as it deserved, for a long time."*

Dictionnaire des sciences mathématiques pures et appliquées (Par une société d'anciens élèves de l'École Polytechnique, dir. Montferrier, Paris, 1835)

Cramer and his *Introduction* are still cited in several articles :

- as main reference in the articles ASYMPTOTE, BRANCHE DE COURBE, CENTRE, PARALLÉLOGRAMME DE NEWTON, POINTS SINGULIERS.
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There is also an article entitled CRAMER (GABRIEL) :

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Encyclopaedia Metropolitana (Pure sciences, vol I, Londres, 1845)

Cramer's treatise remains a reference book cited at the end of the article ANALYTICAL GEOMETRY, in company with Maclaurin and Euler :

> *"For farther information on the subject of Curves and of Surfaces, the reader is referred to the following works : [...] Cramer's Introduction à l'Analyse des Lignes Courbes Algébriques ; Euler's Introductio in Analysin Infinitorum, tom. ii [...]."*

Historian readings of Cramer's *Analyse des courbes* :

- *Histoire des mathématiques* (Montucla, Paris, 1802)
- *Histoire générale des mathématiques* (Bossut, Paris, 1802)
- *Traité des propriétés projectives des figures* (Poncelet, Paris, 1823)
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Histoire des mathématiques (Montucla et Lalande, Paris, 1802)

"Il manquoit cependant encore jusqu'en 1750 un livre sur ce sujet, qui réunît à la profondeur de la doctrine, les développements nécessaires pour le rendre accessible à tous les géomètres. C'est ce que M Cramer a exécuté avec le plus grand succès, par son ouvrage trop modestement intitulé Introduction à l'Analyse des Lignes Courbes Algébriques qui parut à Genève en 1750, in 4° , ouvrage d'ailleurs original en plusieurs points, et dans lequel au mérite du fond s'ajoute celui de la forme, je veux dire une clarté et une méthode tout-à-fait satisfaisantes. On ne sauroit par toutes ces raisons trop le conseiller à tous ceux qui désirent approfondir cette théorie."

Histoire des mathématiques (Montucla et Lalande, Paris, 1802)

Beyond this laudatory words, the authors strongly rely on the analytical methods implemented by Cramer to explain how to study algebraic curves (pp 73-85).

Histoire générale des mathématiques (Bossut, Paris, 1802)

> *"Je ne dois pas oublier de citer avec distinction Cramer parmi les bienfaiteurs de la nouvelle Géométrie. Son Introduction à l'Analise des lignes courbes algébriques, est le traité le plus complet qui existe sur cette matière. L'auteur ne laisse rien à désirer sur la théorie des branches infinies des courbes, sur leurs points multiples, et en général sur tous les symptômes qui servent à les caractériser".*

Traité des propriétés projectives des figures (Poncelet, Vol 2, Paris, 1866)

In a text written in 1823, Poncelet gives an interesting and original historian point of view on Cramer's treatise :

> *"L'introduction à l'Analyse des lignes courbes par Cramer, qu'on doit considérer comme renfermant à peu près tout ce qui était alors connu sur le sujet n'est en effet que le développement des principes posés par Newton et Stirling pour la classification des lignes courbes [...]. On y enseigne, il est vrai, à déterminer, pour chaque point de la courbe, la tangente et la normale, le cercle et le rayon de courbure, ainsi que les grandeurs ou les lieux qui en dépendent, tels que les développées, etc."*

Traité des propriétés projectives des figures (Poncelet, Vol 2, Paris, 1866)

"[...] mais ces recherches, ces solutions de problèmes infiniment utiles aux arts qui se fondent sur le dessin linéaire, ne sont là présentées ou résolues qu'à l'aide des équations des courbes et par le calcul des abscisses et ordonnées ; on n'opère presque jamais sur les lignes, sauf dans quelques cas très-simples ; en un mot, c'est de l'Analyse algébrique, admirable, il est vrai, par son universalité, mais non de la Géométrie telle que le réclament les divers besoins des arts. C'est encore ainsi que, de nos jours (1823), on traite volontiers, dans les ouvrages consacrés à l'instruction, la théorie des lignes courbes."

***Aperçu historique sur l'origine et le développement des méthodes en géométrie* (Chasles, Bruxelles, 1837)**

Chasles gives, in turn, his opinion in the part of his *Aperçu* entitled "Histoire de la géométrie", after having mentioned Maclaurin, De Gua and Euler :

> *"Cramer donna, sous le titre : Introduction à l'analyse des lignes courbes algébriques (in-4° , 1750), un traité spécial, le plus complet, et encore aujourd'hui le plus estimé, sur cette vaste et importante branche de la Géométrie."*

But he eventually adds :

> *"Ce sont là, je crois, les derniers perfectionnemens notables que la science des courbes dut à la Géométrie des anciens et à l'analyse de Descartes."*

On the Singular Points of Curves, and on Newton's Method of Coordinated Exponents **(De Morgan, Transactions of the Cambridge Philosophical Society, London, 1856)**

In this article De Morgan questions the methods for studying the singular points of curves :

> *"Two methods have been adopted of treating the inquiry into the singular points of curves. The first, of which the fullest development ever given is in the well-known and highly valued, but (as I shall show) little read work of Cramer Analyse des Lignes Courbes Algébriques, Geneva, 1750 [...]"*

On the Singular Points of Curves, and on Newton's Method of Coordinated Exponents **(De Morgan, Transactions of the Cambridge Philosophical Society, London, 1856)**

A few pages later, he seems to exhume the Newton's parallelogram method from Cramer's works to rehabilitate it :

> *"But, remarkable as it may be, it is still more remarkable that Newton's parallelogram, or method of co-ordinated exponents, as I shall call it, has still more completely fallen into oblivion. [...] Lastly, Cramer has made it the leading method of his well-known work on curve lines."*

Mathematician readings of Cramer's *Analyse des courbes* :

Was the *Introduction* read by mathematicians (geometers) between 1750 and 1850 ? For what purposes ? What place does it occupy in the curriculum of students in mathematics ? Let's take a look :

- in France, from Bezout to Puiseux,
- in Germany, in company of Möbius, Plücker and Clebsch,
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***Cours de mathématiques à l'usage des gardes du pavillon et de la Marine* (4th part, Bezout, Paris, 1770)**

Two references to Cramer's *Introduction* in the fourth part of the famous *Cours de mathématiques*, "*contenant les principes généraux de la Méchanique, précédés des Principes de Calcul qui servent d'introduction aux Sciences Physico-mathématiques*" :

- in a paragraph dealing with singular points (cusps) of curves,
- and at the end of a part concerning approximations for integration.

The treatise is also mentioned twice in the third part of the *Cours*, containing the application of Algebra to Geometry.

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Leçons de mathématiques données à l'École Normale, en 1795, par M. Laplace **(Paris, 1800)**

In these lessons, Laplace prescribes the reading of the *Analyse des courbes* to his students :

> *"Les bornes de cette leçon ne me permettent pas d'insister davantage sur la théorie des courbes. On trouvera tous les détails que l'on peut désirer à cet égard, dans le second volume de l'Introduction à l'analyse des infiniment petits, par Euler, et dans l'ouvrage de Cramer sur la Théorie des Courbes. Ces deux ouvrages, quoiqu'excellens chacun dans son genre, ne dispensent pas de lire les deux ouvrages originaux qui leur ont donné naissance [...] : je veux parler de la géométrie de Descartes, et du traité de Newton, intitulé Énumération des lignes du troisième ordre."*

***Cours de mathématiques : Géométrie et application de l'algèbre à la géométrie* (Bossut, Paris, 1800)**

So does Charles Bossut :

> *"De notre temps, Euler et Cramer ont traité dans toute son étendue la théorie des courbes algébriques, l'un dans son livre intitulé Introductio analysin infinitorum, et l'autre dans son Analyse des courbes algébriques. Ces deux excellents ouvrages doivent être lus par tous ceux qui veulent pénétrer dans la géométrie transcendante."*

Traité du calcul différentiel et du calcul intégral (Lacroix, 2nd edition, Paris, 1810)

Lacroix gives his references for the chapter *Théorie des lignes courbes* : Descartes, Newton, Stirling, Maclaurin, De Gua, Euler, Cramer et Dionis du Séjour et Goudin. However, he puts Cramer's analytical methods in perspective, compared to powerful differential methods :

> *"Les Géomètres ont imaginé différens moyens pour distinguer parmi les termes d'une équation, ceux qui sont les plus grands. Newton inventa le parallélogramme analytique que de Gua réduisit ensuite à un triangle ; Taylor a employé une construction géométrique ; mais M. Lagrange a donné un procédé analytique très-simple et très-commode, que nous allons faire connaître [...]"*

Traité du calcul différentiel et du calcul intégral (Lacroix, 2nd edition, Paris, 1810)

A few lines later, he adds :

> *"[...] La méthode ci-dessus peut sembler incomplète, à quelques égards, puisqu'on n'y entre dans aucun détail sur le nombre des développemens qu'on doit tirer d'une équation, d'après son degré [...] ; mais ce détail aurait peu d'intérêt, puisqu'on emploie rarement la méthode dont il s'agit ; et d'ailleurs on peut consulter le chapitre VII de l'Introduction à l'Analyse des lignes courbes, par Cramer, qui mettra sur la voie de ces recherches."*

And after Lacroix ?

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- **Chasles** and **Poncelet**, though having expressed their historian point of views, don't seem to use or mention any result or method from Cramer's book.
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A farther look in german mathematics, beyond 1850 :

- **Clebsch** shows himself as an attentive reader of Cramer's treatise, in his *Leçons sur la géométrie* (vol 2 : *Courbes algébriques en général et courbes du troisième ordre*, Paris, 1880) and makes good use of Cramer's methods to distinguish the overriding terms of an equation of a curve.
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In Great Britain, Cramer's book is also mentioned, but his methods, and the work of classification itself seem pointless :

Peacock, in his *Collection of examples of the applications of the differential and integral calculus* (London, 1820) writes about Cramer's *Introduction* :

> *"This enumeration, however, though not destitute of interest, is of little real utility, for the properties of every curve are most readily deduced by direct methods from the particular equations."*

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Lardner, in his *Treatise on algebraic geometry* (London, 1831) adds :

> *"Those who are desirous of further information concerning the general properties of algebraic curves are referred to Cramer, Int. a l'Analyse de Lignes Courbes. Euler's Analysis Infinitorum. Stirling on Newton's enumeration of lines of the third order. De Gua, Usage de l'Analyse, & c. We conceived that entering further on the subject in the present treatise would be swelling the volume without offering any adequate advantage."*

A glance beyond 1850 :

- **Salmon**, *A Treatise on the Higher Plane Curves* (Dublin 1852) : cites Cramer's *Introduction* a few times (Paradox, singular points) and borrows a lot of examples from it.
- **Frost**, *An elementary treatise on curve tracing* (London, 1872) : cites Cramer in the preface and devotes a whole chapter to the analytical triangle, in order to determine infinite branches, or to sketch curves in the vicinity of the origin.
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Conclusion

- Between 1750 and 1850 Cramer's *Introduction* is strongly and constantly referenced and prescribed by encyclopedists (d'Alembert, Montferrier, later Molk...)
- But at the turn of the XIXth century, its importance and its modernity are questioned by mathematicians (Lacroix, Plücker), including the ones talking as historians (Chasles, Poncelet)
- We can agree with De Morgan when he says that Cramer's treatise has been finally poorly read in the first half of the XIXth century.
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Perspectives

Now this work (recollection of references and mentions) has to be sharpened :

- by studying other sources (mathematical journals, correspondences, textbooks, library catalogs, etc),
- by examining the circulation of the knowledge and methods exposed in Cramer's book ("joining the dots"),
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Thank you for your attention. Any questions ?