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Matthieu Fontaine, Shahid Rahman

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Fiction, Creation and Fictionality
An Overview

Matthieu Fontaine and Shahid Rahman
(Université Lille3, UFR Philosophie, UMR 8163)

Abstract
The philosophical reflection on non-existence is an issue that has been tackled at the very start of philosophy and constitutes since the publication in 1905 of Russell's "On Denoting" one of the most thorny and heated debates in analytic philosophy. However the fierce debates on the semantics of proper names and definite descriptions which took off after the publication of Strawson's 'On Referring' in 1950 did not trigger a systematic study of the semantics of fiction. In fact, the systematic development of a link that articulates the approaches to fiction of logic; philosophy and literature had to wait until the work of John Woods, who published in 1974 the book *Logic of Fiction*: *: A Philosophical Sounding of Deviant Logic*. One of the most exciting challenges of Woods' book relates to the interaction between the internalist or inside-the-story (mainly pragmatist) and externalist or outside-the-story (mainly semantic) points of view. For that purpose Woods formulated as first a fictionality operator to be read as "according to the story …" in relation to the logical scope of which issues on internalism and externalism could be studied. The discussions on fiction that followed Woods' book not only seem not to fade away but even give rise to new and vigorous research impulses. Relevant fact for our paper is that in the phenomenological tradition too, the study of fiction has a central role to play. Indeed, one of the most controversial issues in intentionality is the problem of the existence-independence; i.e. the purported fact that intentional acts need not be directed at any existent object. Influenced by the work of the prominent student of Husserl, Roman Ingarden (1893-1970), Amie Thomasson develops the phenomenological concept of ontological dependence in order to explain how we can perform inter- and transfictional-reference - for example in the context of literary interpretation. The main claim of this paper is that a bi-dimensional multimodal reconstruction of Thomasson's-Ingarden's theory on fictional characters which takes seriously the fact that fictions are creations opens the door to the articulation between the internalist and the externalist approaches. We will motivate some changes on the artifactual approach – including an appropriate semantics for the fictionality operator that, we hope, will awaken the interest of theoreticians of literature. The paper could be also seen as an overview of how different concepts of intentionality might yield different formal semantics for fictionality. We will provide a dialogical framework that is a modal extension of a certain proof system developed by Matthieu Fontaine and Juan Redmond. The dialogical framework develops the inferential counterpart to the the bi-dimensional semantics introduced by Rahman and Tulenheimo in arecent paper.

Key words: fiction, non-existents, ontological dependence, intentionality, fictional character

Introduction

It is an interesting point that one of the most influential papers at the very start of the development of analytic philosophy, namely Bertrand Russell's "On Denoting" (1905), explores issues linked with non-existence and the so-called *ontological import of propositions* that were lively subjects of discussion between the end of the 19th and the start of the 20th centuries. [1] Russell's choice is clever: he was keen to show how the new instruments of logic might offer an original approach to some venerable metaphysical and epistemic problems such as the problem of judgements of existence. Actually, the paper gives Russell the opportunity to stress the main contribution of the "new logic": the notion of "quantifier" that could now bring an unexpected twist to Kant's remark that "existence is not a real predicate". "On Denoting" displayed the method of logical analysis of language that prompted important researches on the formal semantics of natural language including issues such as reference and the meaning of empty names. However the fierce debates on the semantics of proper names and definite descriptions which took off after the publication of Strawson's 'On Referring' in 1950 did not give rise to a systematic study of the semantics of fiction. In fact, the systematic development of a link that articulates the approaches to fiction of logic; philosophy and

[1] See Appendix AI.

literature had to wait until the work of John Woods, who published in 1974 the book *Logic of Fiction*: *: A Philosophical Sounding of Deviant Logic*. The discussions on fiction that followed Woods' book not only seem not to fade away but even trigger new and vigorous research impulses, particularly in the context of the interaction between the formal semantics approach and researches coming from the (mainly pragmatist) perspective of literature theories. In fact; one of the most exciting challenges of Woods' book relates to the link between the pragmatic and the semantic level with the internalist and externalist points of view. In other words, Woods' challenge is about the interaction between the notion of fiction as activity (creation) and fiction as product (creature) and the relation of this pair with the "inside-talk" and "outside-talk" of and about the corresponding fictional work. For that purpose Woods formulated as first a fictionality operator[2] to be read as "according to the story …" in relation to the logical scope of which issues on internalism and externalism could be studied. Some of the new research paths make use of a predicate of existence that is combined in various and different ways with quantifiers. This device has been used by some authors to take up anew some issues of Meinongianism in the context of a formal semantics.

Relevant fact for our paper is that in the phenomenological tradition too, the study of fiction has a central role to play. Indeed, one of the most controversial issues in intentionality is the problem of the existence-independence; i.e. the purported fact that intentional acts need not be directed at any existent object. Influenced by the work of the prominent student of Husserl, Roman Ingarden (1893-1970), Amie Thomasson develops the phenomenological concept of ontological dependence in order to explain how we can perform inter- and transfictional-reference - for example in the context of literary interpretation.

The main aim of the paper, centred on literature, is to build a bridge between the mainly semantic and the pragmatic point of view. The claim is that an appropriate formal reconstruction of Thomasson's-Ingarden's theory which takes seriously the fact that fictions are creations opens the door to an articulation between the approaches mentioned above. Actually, the formal reconstruction will allow some gaps to be filled on Thomasson's "artifactual" version of Ingarden's theory related to the combination of the notion of ontological dependence with a fictionality operator. We will thus motivate some changes on the artifactual approach that, we hope, will awaken the interest of theoreticians of literature – up to now quite sceptical towards formal approaches. The problem is that the formal semantics perspective and the literary one start from quite different points of view. These points of departure, we think, might explain why the logical approaches to fiction had not only not impressed theoreticians of literature but had no real impact on literary studies. Indeed, researchers coming from literary studies such as Margaret MacDonald, Jean-Marie Schäffer and Gerard Genette adopt as a base of their reflection, at least fat the start, the point of view of sitting inside the fictional discourse.[3]

Now, external points of view on fiction are also unavoidable, and they involve necessarily some considerations on formal semantics. Indeed, in the context of a semantics of fiction we would like to know the truth-functional or, at least, the assertability conditions, of sentences like "Emma Zunz is a character created by Borges", "Y(a)vh(e), God of War, was worshipped

[2] Woods fictionality operator was set in a modal framework 1978 by David Lewis

[3] One additional exciting question is why fiction? in the sense of why do we need fiction? in the context of the development of our cognitive skills. The relevant literature relates fiction and playing in animals, children and adults. Unfortunately this issue will not be tackled in this paper though it has some links with the issue of emotions triggered by fictions that will be briefly discussed later on.Cf. Schäffer 1999, Steen and Owen 2001, Mitchel 2002

by the Ugaritics", "Flaubert admired Don Quijote". If we are prepared to accept that such kinds of sentences express true (or assertable) propositions, we should be able to explain how it is that they come to be true despite the fact that the singular terms "Quijote", "Meursault" and "Zunz" do not denote anything or do not denote anything real.

The paper could be also seen as an overview of how different concepts of intentionality might yield different formal semantics for fictionality. We would like to point out that the long introductory overview also pursues a systematic aim, namely the formulation of the internalist and externalist approach to be articulated in the last part of the article. We will provide a dialogical framework that is a modal extension of a certain proof system developed by Matthieu Fontaine and Juan Redmond. The dialogical framework develops the inferential counterpart to the the bi-dimensional semantics introduced by Rahman and Tulenheimo in a recent paper.[4]

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[4] Cf. Rahman/Tulenheimo 2009b, 2009c and 2009d.

I. The internalist point of view

One of the most important precedents of the internalist perspective is the work of Margaret MacDonald. Indeed in a paper of 1954 MacDonald anticipated many tenets of the "pragmatist" perspective developed later on by John Searle, Greg Curry, Kendall Walton, Gerard Genette and Jean Marie-Schäffer. MacDonald's main point is that in fictional texts the purely fictional characters are rather few. She defends the thesis that the Napoleon of Tolstoy is a different Napoleon from the "real Napoleon" and similarly, that the Russia of Tolstoy has not been introduced to give some information about Russia but to give a frame to the development of the characters of *War and Peace*. Furthermore, MacDonald claims that in a fictional work there are no assertions conveying information about real facts, persons or objects.

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I.1 Paratext, intention and pretended assertions

MacDonald's analysis has been developed further by Searle, Genette and Schäffer. Searle thinks that there are two kinds of elements in a fiction, those that are fictional and those that are not.[5] Genette's position seems in this respect an extreme form of MacDonald's approach. According to Genette, the point is to create a framework to situate and create the fiction as a whole in. According to Gennette, though there are denotational elements the result is not so: the fictional whole is more fictional than its parts. The fictional work as a whole does not refer to any extra-textual reality and those elements borrowed from reality are transformed into fiction[6]. Once the fictional work is finished and considered in its entirety, there is a kind of non-permeability from the fictional to the real context. Genette calls this feature *intransitivity*.

The main common motivation of the different brands of the pragmatist and internalist approach is that, from the "inside" of the text there is no way to know if that text is or not a fiction. When we read we might not know if the subject of the text as a whole is or not extra-textual[7]. Sainsbury formulates the internalist argument in the following way: a reader could grasp the content of the text independently of knowing whether the work is fact or fiction - a documentary might be mistaken for an ordinary drama-movie, or vice versa. According to Sainsbury, this shows that there is no distinctive species of meaning, "fictional meaning", distinct from everyday meaning, and it shows that what it is to be fictional cannot be an intrinsic property of a text or film[8].
Since this is the case, so the argument, we need some contextual information, the title, the publicity text, a certain kind of preface or other device that signals to the potential reader that we are indeed in the case of fiction. This extra information is usually called *the paratext*. Searle and Genette conceive the paratextual information as the manifestation of the intention of the author. More generally, Searle considers that this intention is displayed by means of a certain kind of illocutionary act – that is, those acts that are performed to express a different one – without the intention of lying but of producing a "suspension of belief"[9]. These kinds of

[5] Genette 1991, 36-37.
[6] Genette 1991, 89-93. This seems to be closely linked with Roland Barthes' (1968, 88-89) reading of Flaubert's accurate and "vain" description of a barometer in "Un cœur simple". According to Barthes, Flaubert's text presents us with a new approach to what is to be *likelihood* (vraisemblable) in literature: what seem to be denotational elements are introduced to produce the effect of "referential illusion".
[7] See the thorough study of Genette 1991.
[8] Sainsbury 2009, chapter 1.
[9] The expression "voluntary suspension of disbelief", has been picked up from Samuel Stanley Coleridge (1772-1834): *In this idea originated the plan of the 'Lyrical Ballads'; in which it was agreed, that my endeavours*

speech acts are called "pretense". Furthermore, fictions are produced, according to Searle, by means of the performances of *pretended assertions*.

- Notice that within this kind of approaches understanding a fictional text does not necessarily assume that we know if the singular terms involved in the text denote: we might only *pretend* that they denote. This presents a complementary argument of the internalists against the standard truth-functional semantics approach to fiction. Actually, internalists take the point of view of truth *in* the story: *in* the story, Joseph Cartaphilus is immortal, in the story, Rostov leaves in Russia. Now, if the reader has the contextual information that Rostov is a fictional character of Tolstoy, the reader *pretends* that Rostov leaves in Russia; but this is not for real. The contextual information of the paratext allows internalists to produce some assertions beyond the purely inside-point-of-view. However, this commits the internalist to concede that these assertions are not real assertions but pretended ones and that the truth involved is a pseudo-truth. There is a way to make sense of *pretended truth*, as we will suggest at the end of chapter I.3, but the resulting semantics might motivate a kind of *fictional semantics* after all, namely, a supervaluational one.

Now, whereas pragmatists such as Searle and Genette emphasize the paratextual feature of fictions some others, such as David Lewis, Greg Currie and Kendall Walton emphasize the reader perspective. Greg Currie (1990) and Kendall Walton (1990) are rather sceptical in relation to the intention of the author particularly so because of myths. Indeed, it could be sensibly argued that myths are not propounded as fiction but as truths. With the evolution of time these very myths could be read as fictions. According to Walton, it is the active reader's perspective that makes of text a fiction. One different way to express the critics of Currie and Walton is that Searles' theory might assume a naïve and static notion of author. On the contrary Currie and Walton display a more dynamic notion of author that we might call *author-reader*. Anyway, as rightly pointed out by Sainsbury, the notion of *pretense* does not help us very much to know if we are not in the case of fiction. In fact, pretending to assert is certainly not a sufficient condition for characterizing a fiction: I can illustrate an incorrect reasoning by pretending to assert some claims; or take the case of me practicing a talk where I pretend to assert something you, who are watching my practice, are absolutely unconvinced of the contents. Fiction should succeed in getting the audience make-believe. Let us recall once more the remarks of MacDonald, Barthes, and Genette in relation the (in principle) "denotational" elements in fictional work. Those elements are there to help the reader to build a convincing framework by the means of which he (the reader) can succeed to make-believe in the seemingly non denotational elements of fiction.[10].

I.2 Creation as *make-believe* and the reader's perspective

The way out from the objections to the pretense-approach without leaving the pragmatist approach is to add to the notion of pretense the notion of *make-believe*. The point of Currie and Walton is that there is a dynamic approach to the notion of fiction resulting from the

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should be directed to persons and characters supernatural, or at least romantic, yet so as to transfer from our inward nature a human interest and a semblance of truth sufficient to procure for these shadows of imagination that willing suspension of disbelief for the moment, which constitutes poetic faith. Coleridge, 1817 (1985) Chapter XIV.

[10] Sainsbury 2009, chapter 1.2.

complicity of author (or story-teller) and reader (audience).[11] An author is not a liar; the liar is trying to get the audience to believe the things he says, whereas the author (story-teller) of a fiction is trying to get the reader (audience) to *make-believe* the things he says. Thus, when does the author succeed in making believe that, according to a given story, *p* is the case? Well, when the reader, in the context of that story, is willing to suspend his incredulity in relation to *p*. The creation of a literary work results when the intended audience is willing to play the game of make-believe involved.

What really differentiates Walton's approach to those of Currie and Searle is what triggers the game of make-believe or pretense. Indeed, for Walton, the pretense proper of fiction is prompted by the presence of a particular real-world object or fact, in which case this object is referred to as a *prop*[12]. A real-world object becomes a prop due to the imposition of a rule or *principle of generation*, prescribing what is to be imagined as a function of the presence of the object. If someone imagines something because he is encouraged to do so by the presence of a prop he is engaged in a game of make-believe. Fictions - including linguistic fictions, representational paintings and sculptures - serve as props in games of make-believe. A text constitutes a fiction when, roughly, there is a rule in force that we are to make-believe that there are objects and/or facts such that the words of that text refer to and describe those objects and/or facts. (Thus we make-believe that 'Anna Karenina' is a genuine proper name that directly refers to a Russian woman of whom the story is telling us, and so on.). The fictionality of a proposition consists in there being a prescription from the prop-facts to the pretended proposition, in a given cultural context, that participants imagine it to be true. Children may play a game where bicycles are horses and in the context of such a game all that is needed, for the prescription to make-believe that horses are in a corral to be in force, is the fact that the bicycles are in the garage.[13] According to Walton, we only make-believe that through Tolstoy's sentences about Anna Karenina we get genuine reference, predication and truth. Here the sentences of the text will play the role of the prop.

As already mentioned, props generate fictional truths in such games. (Thus in a game in which the rule is that stumps are to be imagined to be bears, a stump by its presence generates the fictional truth that a bear is present.) Some rules of generation are ad hoc, for instance when a group of children spontaneously imposes the rule that stumps are bears and play the game 'catch the bear'. Other rules are publicly agreed on by a large linguistic community and hence relatively stable. Games based on public rules are 'authorized'; games involving ad hoc rules are 'unauthorized'. By definition, a prop triggers a literary fiction if it is a prop in an authorised game. Borges' *Tlön Uqbar* is a literary fiction because everybody who understands Spanish is invited to imagine its content, and this has been so since the work came into existence.

In Walton's account, the fact that words constitute a fiction thus depends on their being a prop in a game of make-believe, not on the fact that an author intends the audience to make-believe that the propositional content expressed by the words is true. Hence, for Walton, fiction has nothing special to do with communicative acts of intentional agents; naturally occurring cracks in a rock can spell out a story. Walton thinks that this is really the point; and we could really dispense with the idea of pretense prompted by the intention of the author and go directly to the games of make-believe.

I.3 Objections

[11] Lewis 1978, 270.
[12] Walton 1990, 11.
[13] Walton 2000, 72.

Walton's approach requires a very active participation in both the production and the reception of fiction. This comes from the strong analogy that Walton draws between playing and games of make-believe. According to Walton, a make-believe game of fiction is the same kind of game involved in a children's game where, for example, a piece of wood plays the role of an aeroplane. One child, in proposing this game to another child, is proposing to make-believe to this other player that it is an aeroplane. Such games involve a notion of fictional truth: a big piece of wood is a big aeroplane and a small piece of wood a small aeroplane and pieces of wood fly, but it is false that those pieces of wood are at the same time, say, bears. Now, there are some objections to the play analogy.

- It looks as though there is no place for bad or good fiction. If a text does not manage to make that the audience suspends their incredulity, then the audience will take the text as telling falsities and stop the game of make-believe. Nevertheless, on our view, despite the fact that bad fiction might not manage to induce suspension of incredulity it is still fiction.

- Though this approach is quite effective in explaining the passage from myths to fictions, one might think that the reader perspective is too salient in this approach.

The following objections are based in the idea that while games involve the role of imagination they are not exactly the same as games of making believe.

- Some have pointed out that in a game such as the one with the stump mentioned above the similarity between the stump and the bear is not a mayor issue whereas in literature the game of making believe is more successful the more accurate it is.[14]

- According to Sainsbury, children games can be extended (adding for example stones as lions to the original play), but fictions are somehow closed. Sainsbury complains that Walton's approach is too active; he (Sainbury) thinks that the reader has a more passive role than that of a player.[15]

The first objection does not seem to be very harmful, it only signalises that in literature the props must fulfil some conditions. In our last chapter, we will come back to the purported closeness of fiction mentioned in the second objection where we differentiate between a compatibility and an interpretation operator. A different kind of objection relates to Walton's approach to emotions triggered by games of make-believe.

- Since, according to Walton, games of make-believe trigger a suspension of incredulity they also trigger a kind of a suspension of real emotions. The point is that, according to many philosophers, at least some emotions require corresponding beliefs. For example we can fear of something if we really believe that it is dangerous. Now, in reading Poe's "Black Cat" some reader might have some emotions. But the reader is playing a make-believe game. He is not afraid but plays the game of being afraid. Thus, Walton concludes, the emotions are not real emotions. This might be supported by the fact that though the spectator of a movie might be afraid of the "murderer" of that movie he does

[14] Carroll 1991.
[15] Sainsbury 2009, chapter 1.4

not run away and calls the police.[16] Notice that this feature of the Walton's theory is common to the internalism: from the inside of the story, the story is true, but what makes of the story a fiction is that there is a game of make-believe that the story is true.

The objection of Sainsbury is based on the general objection to the too active role of the audience assumed by Walton. Emotions come despite us in a similar way to the perception of illusions: I cannot avoid seeing the stick breaking when I submerge half of it in the water. I can not avoid seeing it despite the fact that I know that it is not that the case. Furthermore, knowing that it is not broken helps me in correcting my use of it in the water. In a similar case, though I suspend my incredulity and I know that the murderer in the screen is not for real I am really scared. My knowledge helps me not to go for the police. The objection to Walton version of games of make-believe is that they assume too much control of the audience.

- Amie Thomasson formulates some other kind of objections form the *external* point of view. Indeed, Thomasson remarks that is willing to concede that in many cases Walton's approach is fruitful. Games for make-believe perform quite well at explaining cases that involve the internal point of view such as discourse *within* the fictional text (that is the make-believe game that the story is true) and the comment of some readers about the contents of the text (e.g. what Orestes said to Electra in Sartre's "Flies"). However, Thomasson points out that we sometimes need to step outside of the game of make-believe to assert for example that "Samsa is a fiction created by Kafka".[17] In general, it looks as if the theory of Walton does not help for trans-fictional discourse and for issues linked with identity.

Once more this is due to the pragmatist approach of internalists: if the reader steps out of the game of make believe of the story then the assertions of the story are not true. But how do we explain the switch from inside to outside with only one theory of meaning? The answer of Currie and Walton is that the swtich is a pragmatic move and means of this pragmatic move we realize that the purported fictional truths are not.

One further question of Thomasson is the following: how do we know that we are in the presence of discourse that requires that we play a game of make-believe? Walton considers the same question himself, and offers this answer:

> *How do we know whether to look for an implied unofficial game [of make-believe] at all, rather than taking a given statement to be ordinary? There is no easy recipe…. There is, I suppose, an initial presumption that statements concerning fiction are to be regarded as ordinary in the absence of good reasons to construe them otherwise… Beyond that, a principle of charity is operative. Understanding an utterance in a way that would make it an absurd or blatantly false or trivial or stupid thing to say is to be avoided if an alternative is available…*[18]

In other words, the idea is to start taking the text or story literally and then later on perhaps, because for example some incompatibilities of the content with our background knowledge or because of some kind of paratextual information, switch to a game of make-believe game. However, notice that this dynamics, at least as presented by Walton might bring some complications to the argument of Genette and Sainsbury of the independence of the content

[16] Sainsbury (2009, chapter 1.4) points out that this goes back to Radford 1975.
[17] Cf. Thomasson 1999, 97-100 and 2003. paragraph 1. Similar observations can be found in Woods 2003n 217-219.
[18] Walton 1990, 409-10.

from its qualification as fiction. One should perhaps rather say that the procedure by which something is qualified as fiction is the result of content and some background knowledge. Genette's argument is quite different; he seems to think that it is the intention of the author (not the audience) that is decisive. This might also cause difficultiesin the case of myths and fables: was the intention of Aesop to talk about animals or humans? One might argue that Genette's perspective might drive him to the conclusion that Aesop's fables are not fictions because we discovered some documents that Aesop was talking of his neighbours. However the title of the book is "fables". What should we take as *the* decisive paratext? That is, which of the two is the manifestation of his "real intention"?

It is important to notice that in general pragmatists are ready to defend that fictional characters are empty names. They share this view with irrealists in relation to fiction.
This brings them some difficulties in the case of externalist assertions, like the assertion that Kafka has created Samsa.

> *There may be an unofficial game in which one who says ["Gregor Samsa" is a (purely fictional) character] fictionally speaks the truth, a game in which it is fictional that there are two kinds of people: "Real" people and "fictional characters".*[19]

Thus, in such cases, Walton assumes a second kind of (non official-)game of make-believe where we play as if there were objects such as Samsa and then we assert in this game that Kafka created this "thing". But is there such a thing as Samsa? No: this is only a game we were just playing.

In general, one might say that the absolute control that the games of make-believe require goes beyond the purely internal point of view: the player always knows that he is playing. Thus, it is like being inside the scope of the fictionality operator but with a perspective outside of it. Unfortunately, the truth-conditions, or better, the logic of the pretended truth of propositions in the scope of a fictionality operator have not been yet described, we only know that they are quasi assertions expressing quasi-truths. True, pragmatists might not see that as a problem: according to their view; the difference between quasi truth and truth is a pragmatic not a semantic one. However, adequate truth-conditions might be accomplished by the means of supervaluations and what I would call *superinterpretations* (see appendix AII.2). Superinterpretations might be compatible within the *content theory of intention* (to be discussed in the last main paragraph). Superinterpretations seem to provide an instrument to achieve a uniform semantics to tackle issues of truth inside and truth outside the fictional text. Unfortunately, superinterpretation will not be sufficient if we are looking to describe a uniform semantics. A uniform semantic requires also to spell out the semantics of the expression " according to the story" by the explicit formulation of a fictionality operator such as the one introduced by Woods mentioned above. Moreover, the semantics of such an operator must be able to give account of the so-called incompleteness of fictional texts. Let us assume that we are playing a game of make-believe on Agamemnon. Should the reader assume that he, Agamemnon, wears underpants? Should we assume that he was of a certain blood type? I think we can assume, from our background knowledge, that Agamemnon did not wear underpants and that he was of a certain blood type. In general inferences and implicatures in relation to the background knowledge of the reader must be taken into account if the truth conditions of the fictionality operator are to be spelled out. This could be linked

[19] Walton 1999, 423.

with Genette's reformulation of the traditional discussion on the requirement likelihood (vraisemblable) and might allow "proper" truth or assertion conditions to slip in:

> *Ce qui* [...] *définit le vraisemblable, c'est le principe formel de la norme, c'est-à-dire l'existence d'un rapport d'implication entre la conduite particulière attribuée à tel personnage, et telle maxime générale implicite et reçue*. [20]

David Lewis, as we will see in the next paragraph, gives a new twist to the approach to likehood: Fictional texts are, according to Lewis, closed in relation to logical and pragmatic inferences. That is, the fictional text includes the expected implicatures (such as presuppositions) and inferences that could be drawn from some background knowledge and the text. If in the fiction it is said that a given character such as *Aureliano Buendia* is a colonel, we expect him, if it is not stated otherwise explicitly in the text, to know about weapons. More generally, Lewis tried to provide a formal semantics for the fictionality operator compatible to the make belief approach and able to describe the truth-conditions of internal assertions and inferences.

I.4 Truth *in* fiction is truth: the modal interpretation of fictionality operators

I.4.1 Truth conditions for the fictionality operator

One influential internalist point of view, developed by David Lewis in the framework of modal logic, claims that sentences within the scope of a fictionality operator express truths rather than *pretended* truths. However, these true assertions happen in non-actual worlds. According to this approach, pretended truth amounts to the fact that the story-teller invites the audience to make a trip into a non-actual world where the voluntary suspension of incredulity will take place. The actual world is excluded because in the actual world stories are told as stories and not as known facts. Furthermore the point is that this corresponds to the standard semantics of believe: If X believes φ, φ does not have to be true in the actual world. Worlds are here conceived as variations of the actual world and

- sentences within a fictionality operator will be considered to be true iff they express true propositions in all such worlds that (i) these sentences are told there as known facts and (ii) they differ from the actual world no more than required for the story to be enacted .

Notice that the second condition produces the selection of a subset and it relates to Lewis' approach to likelihood. Indeed, recall that according to his approach fictional texts are closed in relation to logical and pragmatic inferences that are compatible with the story told and the background knowledge assumed of the intended audience (we might add). That the truth conditions are set over a plurality of worlds should tackle the problem of the incompleteness of fictional texts. There will be worlds where Aureliano Buendia loves mangos and ones where he does not. Now, so far as I can remember, Marquez does not tell us what the true answer is. However, according to the story *Cien años de soledad* the disjunction *Buendia loved mangos or not* holds.[21] As internalist, Lewis does not seem to be very interested in problems of reference he seems to think that reference problems are an issue that should be separated of the problem of truth in fiction. Truth conditions of a story are related to context, to its environment, not about some constitutive properties of the objects inside the story –

[20] Genette 1969; 74
[21] Cf. Lewis 1978.

besides the property of being non-actual. He might have a point there but, as we will discuss later on, a theory on fictionality from the internalist point of view should be compatible with the external point of view and will require that we be more specific about the constitutive features of the individuals occurring in a fictional work. True, Lewis' work on the truth-conditions of sentences in the scope of fictionality operator can be seen as an attempt to combine the internal and the external point of view. Let us take a literary text on Sarah Palin that describes some properties that she does not have in the real world. Then, if we assume *rigid designation*[22] and we do not want to produce a contradiction, the fictional character Palin must be exported to a different world that realizes the properties the fiction describes. In fact Lewis does not assume rigid designation but *counterparts* of individuals in the actual world: in the world that realizes the fiction of Palin, we do not have Palin but a counterpart of her and this counterpart is as real as Palin. Indeed, Lewis adds to his theory a "modal realism", that is, he claims that non-actual possible worlds and the non-actual fictional characters inside some of them are real. Possible worlds are, thus, real worlds, in which real possibilities are made actual, and many of them are very much like our world except for not being actual. Modal-realism is then combined with the claim that truth relative to those non-actual possible worlds is truth and not a kind of truth-surrogate.

Non-actuality is the response to sceptical questions such as where are the possible men? Can we have a cup of tea with them? Quine's challenges of identity criteria for fictions condensed in the famous "null entity without identity" are not really the end of a theory on fiction but the start. Indeed, most of us will concede that Jonathans Swift's *Gulliver* and Jorge Luis Borges' *Brody*, are fictional characters and that it makes sense to assert that the fictional character *Gulliver* is not the fictional character *Brody* and we might even compare them. We might also concede that identification criteria in such cases are particularly difficult but this is not the same as conceding that the assertions mentioned above are senseless.

I.4.2 Objections

1) How to handle tautologies? What to do with contradictions in fictions? Let us start with the latter on contradictions: in a postscript to his paper on fiction Lewis (1978) considers separating the text in consistent fragments such that as a whole the fiction will contain, say φ and $\neg\varphi$ (some fragments will contain the negative sentence and some others the the positive), though the fiction will not contain $\varphi \wedge \neg\varphi$. The point is to use some kind of paraconsistent conjunction (close to S. Jaskowski's *discussive logic*). Now, the problem is that there are some contradictions that can not be separated in consistent fragments. Think of the story where a mathematician succeeded in squaring the circle. More generally, it looks as if we are willing to accept contradictions then; we need a kind of non-normal world, where not only facts but also logics can be different. We will discuss this issue in the next paragraph. In fact, Lewis considers the case of fictions not reducible to consistent fragments, but he dismisses them as uninteresting. In relation to tautologies, Lewis assumes that fiction is closed under logical implication; thus every fiction also contains tautologies and all logical validities. The

[22] In the context of a semantics for first-order modal logic we say that a name (i.e. a individual constant) is *rigid* iff the interpretation of the name is the same for every world of the frame. If the constant k is an abbreviation for the name "Barak Obama" the interpretation of k will be always the person Barak Obama. "President of the United States" is not a rigid designation because, though in the actual world it designate the same person as the name "Barak Obama" there are alternatives to the actual world where the interpretation of "President of the USA" might be someone other than Barak Obama.

way out, if we have the feeling that we should get rid of deductive closure, is once more to assume non-normal worlds.
2) Lewis's approach to the semantics of the fictionality operator assumes quite a large set of worlds that add to the explicit part of the story implications and the background knowledge of the audience for which the story has been intended. The problem with this is that the audience can change; thus we should distinguish between the "original" worlds constituting the story and the others. Certainly we should add that relevant pieces of information should be added. However the latter might be difficult to specify.
3) Iteration: in the same postscript mentioned above, Lewis concedes that he can not handle iterations. Certainly, one can iterate fictionality operators but how do we distinguish the truth of one level from the other level? Are they not, so to speak, simply truths in different worlds? How should we make it clear that the truth of the second level is dependent on the truth of the first one, if both are truths in different worlds? In our view, the problem is the counterfactual approach. What we need is to say that the fictions of one level are dependent on the fictions of the others. According to our view; as we will discuss below, Thomasson's use of the notion of ontological dependence offers a clear way out.
4) Lewis concedes at the very start of the paper that he actually has nothing to say about the external point of view and would perhaps follow some kind of free logic. That is, Lewis approach can not handle sentences such as "Cthulhu does not exist", "Cthulhu has been created by H. P. Lovecraft". Thus, wee take it, that Lewis would endorse the make-believe approach.

II. The externalist point of view

As mentioned above, the internalist point of view is not really concerned with truth (or assertion) conditions ,- with the exception of Lewis' approach; but if we come to sentences such as "Georges W. Bush admires Merseault", we must have a theory of reference that explains our theory about the truth-(or assertion) conditions of such sentences. Indeed, as bluntly expressed by Amie Thomasson (we will come to her work below):

> *If we deny that there are fictional entities (and so deny that we ever refer to them), we must explain how we can have true statements involving non-referring terms. If we accept that there are fictional entities, we must explain how we can refer to non-existent objects*.[23]

From the point of view of the semantics of non-existence two standard main rivals, namely *irrealists* and *realists*, deal with the ontological features of fictions. The irrealists, mostly based on the classical tradition of Frege, Russell and Quine, see fictions as pure signs. More precisely, fictions can be named or predicated away but they refer to no object of the domain. The other rival position,considers that fictions are some precise subset of the domain: fictions are entities. They subdivide in "neo-Meinongnians" and "artifactualists"

The irrealists are also subdivided into two main[24] subgroups:

- "(negative) free-logic-irrealists" and

[23] Thomasson 2009, forthcoming.
[24] There is a third group of irrealists, a kind of free logic based on the idea of Frege, that propositions in which names occur without denotation are neither true nor false. We will leave them aside because, in oue view, it is not very helpful for understanding the externalist point of view: why should "Flaubert admired Don Quijote" express a proposition without truth-value? Having said that, such a position could be defended with the combination of neuter free logic with Bencinvenga's superinterpretation approach (see appendix II).

- "descriptivists".

Both streams of irrealists are based on Frege's distinction between sense and denotation and the claim that there can be sense without denotation, both claim too that atomic propositions about with singular terms that name fictional characters are false.
The approach of the descriptivists consists in the standard classical (Russell-Quine) view that names of fictional characters are analyzed away with the help of definite descriptions. Definite descriptions are then understood as the sense of proper names, and quantifiers are conceived as a kind of second-order predicate. The outcome of the position, under the assumption of compositionality of reference, is that with the exception of negative existentials such as "Gregor Samsa does not exist" (=: the predicate of "being Gregor Samsa" is empty) all other types of sentence where names of fictional characters occur express false propositions. We will not comment further on this much known formulation of Quine [25].
Negative free logicians allow some sentences to express some complex propositions and block, as we will discuss below, existential generalization and universal instantiation. Moreover, irrealists such as Mark Sainsbury add to the negative free logic approach a theory of names that considers them as having sense though this sense does not reduce to definite descriptions. Let us discuss this theory before going to the realists.

II.1 Reference without referents

Mark Sainsbury's approach is a combination of

- logic of presuppositions
- names have sense that do not reduce to definite descriptions and
- negative free logic

II.1.1 The logic of presuppositions

The logic of presuppositions is the basis of Sainsbury's attack on what he calls the literalists. Literalists would claim that "Holmes is a detective" expresses a genuine truth. Literalists would go so far to concede that truth in fiction might be understood as a species of truth, namely a presupposition-relative or fictional-operator relative truth. However a species of truth is genuine truth. According to Sainsbury the sentence "Holmes is a detective" does not express a true proposition, but a proposition that is true under the presupposition of the existence of Holmes. Similarly, "Woody Allen admires Kagemusha" is true under the presupposition that Woody Allen and Kagemusha exist. The main point of Sainsbury is that in fictions we might have contradictions and this means that, the true propositions in fiction can not be literally true – this position of Sainsbury precludes paraconsistent approaches to truth in fiction.

> *In sum, literalists (taken literally) are committed to contradictions in three possible ways.*
> 1. *There may be a single fiction according to which p, and according to which not-p.*
> 2. *There may be a fiction according to which p and another according to which not-p.*
> 3. *There may be a fiction according to which p when, in reality, not-p.*
>
> *In all these cases, the literalist is committed to believing both p and not-p.*[26]

Fidelity to the story and not to truth is, according to Sainsbury, what sentences in fiction are about. The presuppositional approach would explain the beautiful examples of John Woods.

[25] Cf. Quine [1953]
[26] Sainsbury 2009, chapter 2.2.

If, in one of Conan Doyle's stories, we read the sentence "Holmes had tea with Gladstone" we seem to think that this sentence expresses a true proposition, but if we read somewhere that "Gladstone had tea with Holmes" we will take it as expressing something false.[27] In the presuppositional approach the asymmetry is explained by the fact that the heads of the sentences trigger different presuppositions: Holmes triggers the presupposition that we are in one of the Holmes-stories and thus that Holmes and Gladstone exist. The second sentence triggers the presupposition that we are talking of the real Gladstone, in the actual world, and thus that Holmes does not exist. Now, I think that the example does not show necessarily that literalists are wrong, but that the semantics of fiction must be dynamic; the second sentences triggers an ontological revision: the reader thinks first that we are talking about the real Gladstone and later on revises it and places the whole sentence under the scope of a fictionality operator. Fidelity, following Sainsbury, relates to assertion-conditions not to truth-conditions: during a driving test the answer: "I am applying my breaks and coming to a complete halt" to the question "What are you doing now?" in the context of constructed situation; is not true but assertible.

- **II.1.2** Names have sense that do not reduce to definite descriptions
  In the context of negative free logic

While presuppositions speak about the assertion-conditions inside of the fictionality operator Frege's claim that sense without reference is possible furnishes the basis of Sainsbury's irrealistic approach to names involving fictions outside the fictionality operator. Names involving fictions have, according to Sainsbury, sense but no reference. The point is that this claim is being shared also by Russell and Quine. However, as mentioned above, Sainsbury maintains at the same time that the sense of names does not reduce to definite descriptions. This conception leads Sainsbury to formulate the following axiom that builds the core of the theory on names developed in his book *Reference without referents*:

For all x ("k" refers to x iff x=k)

Take Holmes: For all x ("Holmes" refers to x iff x=Holmes)

Now, according to Sainsbury, there is nothing to which Holmes refers, so both parts of the biconditional are false and the biconditional is true. Now, unfortunately, in classical logic, for all x ("Holmes" refers to x iff x=Holmes) entails, by existential generalization, the existence of Holmes or more precisely that there is something that is equal to Holmes.
At this point Sainsbury introduces negative free logic. A free logic is a logic where singular terms might not refer at all. The consequence is that existential generalisation and universal instantiation are only valid under the restriction that the singular term introduced refer.[28]

[27] Woods 1974, 41-2.

[28] One standard classification of free logics distinguishes between positive, negative and neuter free logic: Positive free logic, allows singular terms to refer to non-real objects. The domain might contain real and non real elements. The result is that the identity axiom holds in any such logic extended with equality. That is, there might be identity of non-existent objects. Furthermore in positive free logic we might introduce two pairs of quantifiers: ontologically committed quantifiers and ontologically not committed quantifiers.

- Negative free logic, allows constants not refer at all. The identity axiom holds under the same restriction as existential generalisation and universal instantiation. Atomic formulae in which constant occur that do not refer are false.
- Neuter free logic, is a negative free logic that allows formulae to be neither false nor true. That is if a formula contains a constant that does not refer then the whole formula is neither true nor false. While

Sainsbury studies intentionality and intentional operators. However his irrealist approach is committed to take the creation of fictional characters as *objectless intentional acts*. We will come to discuss this issue in our last paragraph on the artifactual theory.

Objections:

1) The solution then, is to tackle the truth-conditions of propositions outside the fictionality operator in the framework of negative free logic. Hence "Holmes is a detective" and "Holmes is ballet dancer" are both false, though it is true that "Holmes does not exist". Furthermore, "Holmes is a detective but he does not exist" is quite delicate. In principle it is false because the left part of the conjunction is. If we rewrite it as "According to the story Holmes is a detective but Holmes does not exist" the formulation seems to ignore the anaphora. The problem is to understand how to get the reference of the anaphoric "he". The solution is to explain that anaphora might not always preserve reference, as in "He drank the whole bottle and smashed it to the floor".[29]
2) Another issue to be tackled is identification: since identity fails for non existents how to identify them. Here, Sainsbury assumes rigid designation though the name might not have a bearer.
3) According to the presuppositional analysis of Sainsbury « Joseph Cartaphilus speaks many languages" is true under the presupposition that .there is such a person. Now, let us take "X is A and not A" is true (?). It seems quite hopeless to find the right presuppositional analysis. The dialectic, according to Sainsbury, is this: there are apparent truths involving fictional names. A natural understanding of these is a realist one. An irrealist has to say either that these are not really truths or that the names in the context do not need to refer to contribute to a truth. With a contradiction, this dialectic does not get started, since, according to the irrealist analysis, no contradictions are apparent truths. So the irrealist treatment of contradictions leaves things just as they were. [30] Still, we would like to know how does this analysis yield that it is true that "According to the story A is not A".

II.2 Modal Meinongianism

II.2.1 A first approach: Denizens of worlds with constant domains

Let me briefly describe this position: Take a structure of possible worlds, where a possible world describes an alternative way to how our real world could have been. For reasons of technical simplicity related to identity and to what has been called the Barcan Formulae[31] it is assumed by many that the domain is constant: that is that the set of individuals of any world is the same. Now, this sounds counterintuitive and terrifying: why should Bush exist in all

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positive and negative free logic do not induce changes to classical propositional logic (without equality), neuter free logic does: Ak∨~Ak is not generally valid.
For some formal details of the corresponding semantics see Appendix AII.

[29] This example has been provided by Sainsbury in a personal e-mail.

[30] The answer to my example has been provided by Sainsbury in a personal e-mail.

[31] Barcan formula: $\Diamond\exists xAx \rightarrow \exists x\Diamond Ax$ and the converse Barcan formula : $\exists x\Diamond Ax \rightarrow \Diamond\exists xAx$. The first assumes a decreasing domain, the latter an increasing domain. The validity of both characterizes locally constant domains : that is the domain of two worlds related by the accessibility relation is constant.

possible alternatives to this world? Is he eternal? Thus, logicians, usually innocent minds, introduce a predicate of existence: $E!x$. The predicate is usually taken as primitive but one could also use Hintikka's definition: $E!x =: \exists y(x=y)$.
All objects are part of the domain but some exist and some not. This is called modal-Meinongianism – that is, roughly, the metaphysical doctrine (in the tradition of analytic philosophy) inspired by the work of the Austrian philosopher Alexius Meinong (1853-1920). When I speak of Meinongians am talking about the interpretation of Meinong in a modal framework, in particular am talking of the work of Graham Priest and Edward Zalta and I will not discuss if their interpretation is or is not faithful to the original work of Alexius Meinong. Most of these new reconstructions of Meinong combine the predicate of existence with the theory of rigid designation of Kripke to conceive a logic of fictions. As you know, rigid designation is the theory that singular terms refer in all worlds to the same object: that is Bush will in all counterfactual circumstance designate Bush though he might not be more the president of USA. If we assume rigid designation a basic characterization is possible: Fictions are those that are in some worlds inexistent. Most of these approaches are not epistemic but metaphysical: that is, it is not about if I know if something is fiction or it is not a fiction, but rather if it is or it is not. In relation to identity: the problem is solved quite straightforwardly: an object is equal to itself despite not having the property of existence (it can be seen as displaying the semantics for some kind positive free logic without empty domains).
In relation to quantifiers Meinongians make use of two sets of quantifiers: one set is to be interpreted as ontologically neutral quantifiers, say ∇ (something, *x*, is such that …) Δ (all *x* are such that).[32] In other words the scope of its domain is all the objects of the universe at stake: existents and non existents. In this way we can speak of "Some dragons spit fire" without committing to its existence. This type of quantifiers is also called *possibilist* quantifiers. The second set of quantifiers range over those objects that have also the property of existence in the world at stake – there have an *actualist* feature: they range of the subset of existents of the domain of a given world:

All those existents *x* such …: $\forall x\, \alpha[x] =: \Delta x(E!x \rightarrow \alpha[x])$

Some existent *x* is such …: $\exists x\, \alpha[x] =: \nabla x(E!x \wedge \alpha[x])$.

Here some use of these double quantifications in natural language:

"There is something which has been sought by many, namely the site of Atlantis, but it does not exist"

"Holmes is a detective but he does not exist "
(There is a detective called Holmes but he does not exist)

More generally, modal Meinongians distinguish the *Sein* of objects – their existential status – from their *Sosein*, their having – certain – features or properties. And modal Meinongians claim that an object can have a set of properties even if it doesn't exist. This is the so-called *Principle of Independence:* Pegasus, Ulysses, and Joseph Cartaphilus can be said to have properties without that the assertions involved become false.

Sometimes, recall Quine's *On what there is*, Meinongianism is presented as stressing the difference between *there is* and *exists* . The paper presents us with two philosophers, McX

[32] Priest 2005, 13.

and Wyman. Wyman is taken to be Meinong.[33] He is not. According to Wyman, all terms denote; the objects that are denoted all have being; but some of these exist (are actual) and the rest merely *subsist*. This is rather Russell in the *Principles* than Meinong. In fact, Meinong and the modal Meinongians we are talking about such as Richard Routley and Graham Priest, understand the non ontologically loaded quantifiers as expressing *Sosein,* i.e. ; as having such and such properties.[34] Interesting is the fact that the difference between *there is* and *exist* in natural language is often related to what is called *restricted quantification.* When I say, *there is no more chalk,* I usually mean, there is no more chalk in, say, the classroom, but if I say *there exists no more chalk.* It looks as if I am talking of the fact that chalk disappeared in general. We will come to this later since it is sometimes used to study the truth conditions within the fictionality operator.

As rightly stressed by Francesco Berto (2009), endorsement of Meinongianism pushes one towards the thesis that *any* singular term denotes an object, existent or not. One should add that is what constant domains is all about. This holds in particular for (definite and indefinite) descriptions, that is, noun phrases of the form "a/the object with such-and-such properties". We therefore have what we may call, following Parsons (1980), and by analogy with naïve set theory, an "Unrestricted Comprehension Principle" for objects:

(UCP) For any condition $\alpha[x]$ with free variable x, some object satisfies $\alpha[x]$.

$\nabla x\, \alpha[x]$, for every $\alpha[x]$.

Actually, Russell's (1905a and) famous criticisms of Meinong addressed the UCP as applied to definite descriptions. The reformulation is straightforward:

Any definite description $\iota x \alpha[x]$ designates an object satisfying the description.

The idea of the UCP is that we specify an object via a given set of properties, such as *is a horse, is ridden by Don Quijote, has a philosophical discussion with Sancho Panza's donkey, ...* . Take $\alpha[x]$ to be the conjunction of the relevant predicates expressing all of the relevant properties; then, according to the UCP, an object is described by $\alpha[x]$, namely $\alpha[r]$. Cervantes called it *Rocinante.*

It is very important to notice that UCP is the only way, modal Meinongians (up to now)[35] tackle the issue of creation. Indeed, since Meinongians usually assume a constant domain objects are always there, though they do not exist; and they thus can not be said to have been created. At the very end they are non-existents, the creation of a fictional character will not bring it into existence (in the strong ontologically loaded sense). Thus, the only available solution is to leave the ontology as it is and take the UCP as a kind of procedure by means of which the author, who picked up some given objects of the domain, describe them in such a way that they will constitute the content of a fictional text. Creation is thus to ascribe some properties to a given object of the domain. But why did we select this one and not the other one? Well, perhaps its original Sosein was more adequate for the application of the UCP the author had in mind. Here we are at the start of the objections and they mostly relate to the UCP. Let me mention the two most famous ones due to Russell

[33] See Appendix AI.
[34] Cf. Priest 2005, 108.
[35] There is a very recent paper of Priest introducing increasing domains in the framework of modal Meinongianism.

1) Can we deploy the UCP to describe objects with contradictory properties? So far nothing prevents using UCP in this way
2) Can we deploy the UCP to produce some kind of ontological argument for whatever? Indeed nothing prevents us either to do so if we combine it with the fact that existence is taken to be a property? Take the properties of *being a Cyclops*, *having one eye*, *being son of Poseidon* and *being existent*. If we apply UCP we have that an object, called Polyphem, has all the properties mentioned above including that of existence.

In order to solve the problem, different restrictions to the application of UCP were deployed. The point is to find out how to restrict the class of sets of predicates to be used to characterize objects. Meinong's student Ernst Mally (18979-1944) suggested distinguishing between, what has been later called, *nuclear* from *non-nuclear properties*. Nuclear properties are those that allow safe uses of the CP (not any more unrestrictedly). When applied to fictional characters, this device is deployed to assert that they have all those nuclear properties that the relevant story attributes to them. The problem is that to give an exhaustive list of predicates expressing nuclear properties. Parsons (1980) comes to the following list:

- Nuclear predicates: "is blue", "is tall", "is golden", "is a mountain", …

- Extranuclear predicates:
  Ontological: "exists", "is mythical", "is fictional", …
  Modal: "is possible", "is impossible", …
  Intentional: "is thought about by Meinong", "is worshipped by someone", …
  Technical: "is complete", "is consistent", …

Unfortunately the lists are difficult to establish and what should we do with the property of being a round square? Priest suggest a solution that starts with Lewis's non-actualism

II.2.2 UCP and non-actualism to the rescue: Denizens of non actual possible and impossible worlds

UCP strikes back: impossible worlds and the liberty of creation:

Priest solution is based on Lewis' idea that fictions are denizens of non-actual worlds. If this is the case; then the UCP can be deployed in its original unrestricted form:

- Given *any* condition $\alpha[x]$some object is described by it. However, it has its characterizing properties, not necessarily at *this* world, but at *others* – at the worlds that make the characterization true.

Now our counterexample:
Being a Cyclops, having one eye, being son of Poseidon and *being existent,*

has lost its teeth. Indeed, the example is no more a counterexample, for we need not assume that an object so characterized, that is, an existent Cyclops, has its characterizing properties at the actual world. But Polyphem is existents at the worlds at which the Homer's story is enacted.

- It is important to point out that from the point of Meinong this reformulation of the CP might be seen as a too strong weakening of the original notion of *Sosein*. According to Meinong's *Sosein* an object might have properties or not and this is not contextually dependent or parameter-bounded.

Still, we have the problem of contradictions. The answer is that an object might have its characterizing properties at non-normal worlds.
For short, a non-normal world is a world where not only facts (atomic propositions) could be different to the facts of actual world, but worlds where the logics could be different. This definition is logic-relative: given some logic L, an impossible world is one in which the set of truths is not one that holds in any acceptable interpretation of L.[36] Just as there are worlds that realize the way that things are conceived to be when that conception is logically possible there are worlds that realize how things are conceived to be when that conception is impossible. In those worlds, in some sense, everything is possible, including contradictions, but nothing is necessary. For our purposes let us extend the standard basic modal logic by providing two sets of worlds: a set of worlds containing two subsets, namely normal and non-normal worlds. The complement of the subset of normal world is the set of non-normal worlds. The accessibility relation will be defined only for the normal worlds and the non modal connectives will have the standard truth-conditions. For the modal connectives, we will have that every necessary formula will be false and every possible formula true. Logical validity is defined as truth preservation at normal worlds. Per definition, the actual world will be considered to be normal. Thus; though contradictions might be possible at non-normal worlds that might enact the fiction at stake, the contradiction will not be the true at the actual world.

One of the consequences of this approach is a kind of analogue of Descartes position on eternal truths. Indeed, in this framework no necessity will necessarily necessary even not at the normal world. Indeed; take a formula, say φ; if it is necessarily necessary at the actual world it should be true that it is necessary at any (accessible) world. Now, if this world is non-normal then it will be false that φ is necessary, since necessities are false at those worlds. Thus, it is not universally true that φ is necessarily necessary at the actual world. This feature of non-normal worlds provides also an answer to the tautology-problem of Lewis approach mentioned above. Logical truths will not be necessary in the worlds enacting fictional texts.

There is still a problem related to closure under entailment: If φ is a logical truth that entails some logical truth ψ then if the reader believes the former, does he also believe the latter? Well, though we might expect (or might not) expect this to happen, the author might wait him with a surprise. Standard non-normal logics will not prevent that. A stronger extension of the non-normal framework is needed. At this point we can make use of Priest's open worlds - worlds not closed under entailment. The idea is to introduce sets of « isolated » non-normal worlds, such that though I know φ and I even might know that φ entails ψ , there might still be an open world where φ holds but ψ not.[37]

Let me finish this paragraph with the following remark: the idea of introducing non-actual non-normal worlds opens the way to rationally understand if not creation as such but the liberty of rational creation. Still, creations are explained away. In the case of non-fictions the framework allows to understand creation as the passage from existence to non existence, the case of fiction is harder.

[36] See Rahman 2002.
[37] Priest 2005, 20-25.

In appendix AII.3 we describe a semantics of first order modal logic able to display a positive free logic without empty domains. The varying domain frames, could be seen as furnishing the basis of a modal Meinongianism framed on normal worlds. The extension to non-normal worlds should be straightforward. The advantage of varying domains is that it might be seen as implementing a kind of "creationism" within Meinongianism. Now, on our view, this will not be fully accomplished until the created elements of each world are taken seriously as creations in the sense Thomasson (see chapter III above).[38]

III The Artifactual Theory

III.1 Fictions and Intentionality

As we have already seen in the formal approaches to the semantics of non-existents – such as in first-order modal logic – a predicate of existence is combined in various and different ways with Quantifiers. However, the phenomenological tradition has another device to deal with non-existence, namely intentionality and more precisely the notion of ontological dependence of Brentano and Husserl. Influenced by the work of Roman Ingarden (1893-1970), a student Husserl's, Amie Thomasson develops the concept of ontological dependence in order to explain how we can refer to non-existent objects for example in the context of literary interpretation. Let us first present the general framework of intentionality as understood by Thomasson and compare it with the positions mentioned above. I take the choice to follow the interpretation of Thomasson of intentionality in general and of Ingarden in particular, since at the very end the aim is to offer a semantics for her artifactual theory of fiction - by the way she has an excellent on Ingarden in the *Stanford Encyclopaedia of Philosophy*.

The task of a theory of intentionality, as duly stated by Thomasson, is to offer an analysis of the directedness of our thoughts and experiences towards those objects in the world that they are about. John Searle, Barry Smith and Ronald McIntyre reconstructed Husserl's *content theory of intentionality.* This reconstruction, developed within the spirit of analytic philosophy, is at the base of Thomasson's approach to intentionality.

The content theory of intentionality
According to the *content theory*, ordinary intentional relations to our veridical perceptions are constituted by three basic parts:

- The conscious act, the object and the content. The conscious act is the particular perceiving, thinking, whishing … that occurs at a particular place and time.

- The object of the intentional relation is the thing the conscious act is about, normally, just a physical individual or state of affairs.

- The content of an act opens the access in a similar way to Frege's senses, to the object of the intentional act. The content is dependent upon the subject's conception or angle

[38] In fact, the semantics described appendix III and the correspondent dialogues of appendix IV, can be understood as displaying a basic semantics for the artifactual theory. Objects outside the domain of a world will play the role of dependent objects. Fontaine, Redmond and Rahman developed dialogues that can be shown to be sound and complete to this semantics (Fontaine/Redmond/Rahman 2009). We will not furnish this proofs here but the proof given below should give an idea on how to implement the necessary adaptations.

of perception of the object and thus each content specifies a particular way to conceive the object at stake.[39]

In the content theory framework salient distinctive features of intentionality are[40]:

- Existence independence: the purported fact that intentional acts need not to be directed at any existent object – notice how close it is to Frege's conception of senses without denotations.

- Conception dependence: the fact that one and the same object may be acceded by two different contents, and thus by two different intentional acts –recall once more how according to Frege, two senses might pick up the same denotation.

- Context sensitivity: the dual of conception dependence, that is, the fact that two internally indistinguishable intentional acts (: acts with the same content) might pick up different objects in different contexts.

Objections:
The main objections against the content approach target the cooperation of the three principles mentioned above.

Let us assume that, X has thoughts about, say, *Mahatma*, and the *creator of the passive resistance movement*.
Furthermore, let us assume that X has also thoughts about *Orestes' sister* and *Iphigenia's sister*.

In the first case, because of Conception independence, we know that both different intentional acts pick up the same object, namely the individual *Mohandas Karamchand Gandhi*, since an object is there. Now, How can we be sure that in the second pair of acts X is thinking about *Electra* (we take it that she is a fiction) and not about *Quetzalcoatl*? There is, certainly content, but because of Existence independence, there is no object.

The logical consequence of this approach is the Fregean inspired Negative Free Logic point of view: every proposition about fictions – with the exception of negative existential claims – is false. Frege proposed even that all fictions denote the same object: the empty class.

Now, content theorists, proposed some ways out of the dilemma. The unification of the acts involving fiction could be explained by postulating that the experience of these acts is as *if they were of the same object*. There is a phenomenological individuation of the object *for consciousness* though there is no external object. The assumption of an *if-object* for consciousness, seems to be compatible with the approach of make-believe Walton and the superinterpretations theory of Bencivenga. Smith and McIntyre define the phenomenological individuation by means of a background of beliefs about

- Principles of individuation for the kind of the individual given. A human being, an animal and so forth. In our case, Electra, a woman
- Identity-relevant properties for that individual: in our case, the property of inducing Orestes to kill Aegisthos.

[39] Cf. Thomasson 1999, 76.
[40] Cf. Smith/Woodruff/McIntyre 1982, 10-18.

These might be used to distinguish those acts about *Electra* from those about *Quetzalcoatl*: the content about humans is certainly of a different kind of that about feathered snake-Gods. However, as pointed out by Thomasson, in many cases the individuation procedure described above will not work. Fictions do often violate the categorical principle of individuation of our background knowledge: humans are not insects. How can we explain that the contents of our thoughts about *Samsa* are also about the insect into which *Samsa* has been transformed?
Worse, let us assume that we have the same content and that the phenomenological individuation conditions are satisfied, how should we distinguish instances of context sensitivity? That is how should we distinguish Cervantes' *Don Quijote* from, the *Don Quijote* written by someone else, like in the short story *Menard, author of the Quijote* by Jorge Luis Borges?

The intentional object theory of intentionality

Thomasson's approach to intentionality has its roots in the work of Kazimierz Twardoswski (1866-1938)[41] according to whom:

> *We must discern, not just a twofold, but a threefold aspect of every presentation: The act, the content and the object.*[42]

This view of Twardoswski is also followed by modal Meinongians such as Zalta and gives the ground for their endorsement to the thesis that every singular term has a denotation. The following point is less akin to the Meinongians and is crucial for Thomasson's approach:

- *The intentional object theory of intentionality explains, the phenomenon that the objects of our intentional acts "need not exist" in part by rewriting this claim. The objects or our intentional acts need not be physical, spatiotemporal, or ideal entities, and they need not exist independently of intentional acts. This is because one term (the object term) may depend in a variety of ways on the other term (the intentional act) and may even (in the case of creative acts of fictionalizing or hallucinating) be brought into existence by that very intentional act.*43

Since this approach assumes an object, the objections to the content theory cannot be raised against this conception of intentionality.

III.2 Fictions as creations

The key of Thomasson's approach to fictions lays in acknowledging fictions a full ontological status. According to her view, fictional objects are inhabitants of domains of worlds just like non-fictional ones. On one hand, they are creations or more precisely, artifacts like chairs, buildings and on the other hand, they abstract creations such as marriages, universities and theories. Fictional objects are bounded to the everyday world by dependencies on books, readers and authors
In her book, *Fiction and Metaphysics* Thomasson displays several types of ontological dependence, we will take up only two main kinds, namely *historical* and *constant*

[41] Twardowski was the father of the famous Lvov-Warsaw School, that produced people that changed the history of logic such as, among others Stanislaw Lesniewski (1886-1939), Ian Lukasiewicz (1878-1956), Alfred Tarski (1903-1983) and Stanislav Jaskowski (1906-1965).
[42] Twardowski 1977, 8.
[43] Thomasson 1990, 90.

dependence, and both have their roots in Ingarden. Ingarden distinguishes, among other, between the following sorts of dependence:

- *Contingency*: the dependence of a separate entity on another in order to *remain in* existence. Corresponds to Thomasson's constant ontological dependence.
- *Derivation*: the dependence of an entity on another in order to *come into* existence. Corresponds to Thomasson's historical ontological dependence.

According to Ingarden, a fictional character is created by an author who constructs sentences about it – fictions have thus a derivative dependence on their creators, but it is maintained in its existence thereafter not by the imagination of individuals, but by the words and sentences. Thomasson, as already mentioned, develops these notions of Ingarden and combines it with the idea of *rigid* and *generic* dependence:

> *We can begin by distinguishing between constant dependence, a relation such that one entity requires that the other entity exists at every time at which it exists, from historical dependence, or dependence for coming into existence, a relation such that one entity requires that the entity exist at some time prior to or coincident with every time at which exists.*
>
> (Thomasson 1999, p. 31)

The point is that the fictional character Holmes is ontologically historically dependent on Conan Doyle and that Holmes as an artifact or creation can survive even after Conan Doyle's death (as a real person, i.e. as an independent object). Moreover, the ontological dependence is in this example a *rigid* one: Holmes depends historically on a fixed object, namely Conan Doyle. Now, after Conan Doyle's death Holmes survives as an artifact because it is ontologically sustained by copies of the texts of Conan Doyle. In fact, while the historical dependence relates to the creation act, the role of the constant ontological dependence is to assure that the artifact Holmes, once created by Conan Doyle, is still here despite that his creator is not. In other words, the constant ontological dependence assures that artifacts are denizens of our world. Furthermore, if also the object(s) on which Holmes constantly depends disappear, also Holmes will disappear or at least be inaccessible. Important for these kinds of examples is to allow the constant ontological dependence relation to be *generic*, that is, Holmes is not constantly dependent on one particular copy of the texts, but at each time he is constantly dependent on one of the copies (or memories). The historical dependence relation is transitive and asymmetric. Reflexive cases of the relation of constant dependence can be used to define independent objects (see definition 6 below).
Interesting is that ontological dependence is to be thought as being bi-dimensional, that is, in a frame of worlds and moments of time with their respective relations. Indeed, Thomasson writes

> *Assuming that an author's creative acts and literary works about the character are also jointly sufficient for the fictional character, the character is present in all and only those worlds containing all of its requisite supporting entities. If any of these conditions is lacking, then the world does not contain the character, If Doyle does not exist in some world, then Holmes is similarly absent. If there is a world in which Doyle's work were never translated at all and all of the speakers of English were killed off,…, then Sherlock Holmes also ceases to exist in that world,…*
>
> (Thomasson 1999, 39).

If historical dependence allows the creations to survive the creator, then the situation described in the quote above is only possible if we are talking in a bi-dimensional framework of world and time. Conan Doyle must be present in the same world where Holmes is present, but not necessarily at the same time.

III.3 Dependence[44]

- **Historical Dependence**

The *Eigenart* of fictional objects (and any other artifacts for that matter) becomes, according to this approach, clear in connection with a multitude of worlds. They are ontologically dependent objects. Any such object requires for its existence the maker of this object (while the converse requirement does not prevail). For instance, Sherlock Holmes exists only in those worlds in which Conan Doyle does, while there are possible worlds with Conan Doyle but without Sherlock Holmes.
X requires Y if in every world in which X exists, also Y exists. X depends on Y if X requires Y but Y does not require X. Note that under this definition, both Holmes and Watson depend on Conan Doyle. What is more, supposing that according to the oeuvre of Conan Doyle, Holmes and Watson are without exception co-existent, we must conclude that Holmes requires Watson and that Watson requires Holmes. Just because of their symmetrically requiring each other, we avoid the undesirable conclusion that one of the two characters depended on the other. Observe also that by this definition, any object requires itself, but no object depends on itself. Now, actually we should add a temporal aspect, it is surely the case that in no world may Holmes's occurrence precede Conan Doyle's occurrence the temporal aspect, yet it is surely the case that in no world may Holmes's occurrence precede Conan Doyle's occurrence.

Notice that the approach is ontological rather than epistemological. We might not know who the creator of the table I am writing on is, but I acknowledge that someone must have done it.

The first two definitions below should capture what Thomasson calls "historical rigid designation" in a bi-dimensional framework. In other words frames, will constituted by a set W of worlds (situations), a set T of moments of Time, and two relations, namely, the standard accessibility relation R, defined on W and the relation "earlier than" $\prec$ defined on T. The valuation function will be defined on pairs <w,t>.

Definition 1. (*Historically requires*) Object X **historically requires** object Y at w,t, if for all $t' \geq t$ for which $X \in D^{t'}w$, we have that there is at least one time $t'' \leq t'$ such that $Y \in D^{t''}w$.

Definition 2. (*Historically depends*) X **historically depends** on object Y at w,t, if X historically requires Y at t, but Y does not historically require X at t. When this is the case and the interpretation of k_i (k_j) at w,t is X (respectively Y), we say that the sentence $\Re k_i,k_j$ holds at w,t, for short: $w,t \vDash \Re k_i,k_j$.

- **Constant Dependence**

As mentioned above, this kind of relation is crucial for the "existence" and "death" of the fictional characters as depending on the copies of the correspondent works. However, certainly some copy is responsible for this ontological dependence and not all of them.

[44] Most of the definitions of ontological dependence below have been developed with Tero Tulenheimo. Correia (2005) discusses similar definitions though he separates the temporal from the modal formulation.

Moreover, the generic feature explains the abstract character of fictions and more generally of the literary work. Let us once more quote Thomasson:

> *A literary work is only generically dependent on some copy (or memory) of it. So although it may appear in various token copies, it cannot be identified with any of them because it may survive the destruction of any copy, provided there are more. Nor can it be classified as a scattered object where all of its copies are, because the work itself does not undergo any change in size, weight, or location if some of its copies are destroyed or moved.*
>
> *But copies of the text are the closest concrete entities on which fictional characters constantly depend. … Because they are not constantly dependent on any particular spatiotemporal entity, there is no reason to associate them with the spatiotemporal location of any of their supporting entities.*
> (Thomasson 1999, 36-37).

Definition 3. (*Constantly requires*) Object X **constantly requires** object Y at w if for all t' for which $X \in D^{t'}w$, we have that $Y \in D^{t'}w$.

Definition 4. (*Constantly depends*) X **constantly depends** on object Y at w, if X constantly requires Y at w, but Y does not constantly require X at w.

Definition 5. (*Constantly generically depends*) Let X be an object existing at w and let Γ be a set of objects existing at w, not all of which need exist simultaneously. If Γ is a set of objects, X **constantly generically depends on** Γ at w if for all t' for which $X \in D^{t'}w$, there is some $Z \in \Gamma$ such that $Z \in D^{t'}w$.

Definition 6. (*Independence)* X is ontologically **independent** if it constantly requires nothing else but itself.

III.4 The creation of fictional works

III.4.1 Text, composition and literary work

In the preceding paragraphs we defined the different kinds of ontological dependencies in relation to objects, but in Thomasson's theory the whole work should be considered as an artefact. The point is to provide the semantic counterpart to the introduction of an operator of fiction that should allow the evaluation of sentences such as "According to the story, Holmes is a detective". The truth-conditions for the fictionality operator deployed by Thomasson are still lacking. Notice that we can not adapt whatever semantics for it – as she sometimes does. It has to be one that is compatible with the artifactual theory. The present reflections should provide the basis for such a semantics. In fact the semantics though it is compatible with the artifactual theory it is independent of it. But before we go to the formal semantics let us introduce Thomasson's decomposition of a literary work. According to our author, a literary work can be divided in three components:

- *text*: the sequence of symbols in a language or languages.
- *composition*: the text as created by the author. That was explicitly has been written We call *content all* what is logically compatible with the composition.[45]

[45] Actually this notion is too broad as it stands. Indeed the notion will allow to include tautologies as part of the content. Now, this is a problem other theories have too and one could adapt one of their solutions though we will not do this here.

- *literary work*: the interpretation or the reader perspective with their background and is logically compatible with the content. We call interpretation those statements that logically compatible with composition.[46]

For each of these we can speak of the abstract type or token. The type texts are those that are responsible for the constant ontological dependence. Compositions tokens are separated by the authors upon whom they depend: different authors different compositions. Two interpretations tokens of the same composition might deeply differ: think in mythology.[47]

III.4.2 The fictionality operator

Our proposal for the semantics of the fictionality operator is to interpret both the "content" and the interpretation of the fictional work as displayed by a plurality of worlds. For reasons of simplicity of exposition we will leave the temporal aspect by side and speak of worlds instead of pairs <w,t>.

The content: *We take the construction according to the fiction,* φ *holds* (: $\mathcal{F}\varphi$) to behave formally as a modality. What this means is that we take the story to specify (relative to the actual world — or if that for some reason does not suffice, relative to a number of other worlds as well) the totality of all the worlds that are compatible *with all that the fiction says*. That φ holds according to the fiction then means that φ holds in *all* the worlds compatible with the fiction. That is, the content consists in the explicit sentences of the work plus its logical implication –like in Lewis in this first approach we will leave out the complications of contradictions and open worlds deployed by Graham Priest[48].

The interpretation and the reader's perspective: We may also be interested in statements that are true only in *some* world compatible with the fiction. Here we introduce the reader's perspective. For example, presumably Conan Doyle's oeuvre leaves it perfectly open whether Watson's grandfather's cousin's dog was a German shepherd. In fact it leaves open even the existence of a cousin to Watson's grandfather, let alone the existence of the former's dog. However, there is presumably also nothing that precludes the possibility that Watson's grandfather had a cousin who furthermore had a dog, which might even have been a German shepherd. The latter is compatible with the story while surely not necessitated by it. While *according to the fiction,* φ *holds* (: $\mathcal{F}\varphi$) expresses a universal modality, φ *is compatible with the fiction.*(: $\langle\mathcal{F}\varphi\rangle$) is an existential statement.' We may even read $\langle\mathcal{F}\varphi\rangle$ as *the fiction admits an interpretation according to which* φ

Each of the worlds displaying the content and the interpretation, will be conceived with a domain that contains dependent and independent objects

Impermeability: Many, scholars of theory of literature, perhaps even most of them, think that the "real" elements occurring in fictional works are not in fact real but rather creations based on same characters of the real ones: the Napoleon of *War and Peace* is not the real Napoleon.

[46] Not to be confused with *fictional work* that we use to speak of the fiction as a whole (text, composition; interpretation).
[47] Thomasson 1999, 64-66.
[48] Priest [2005]

Fictions, to use the language applied to Genette, are "impermeable in relation to element of reality". Let us call this position the *strong impermeable point of view*.

In our setting; the case of strong impermebeality can be formulated by considering that every assertion inside the scope of a fictionality operator involves objects that outside the scope of the operator are dependent ones.

The problem with impermebeality is that satyrics is not possible: the object of satyrics is not the real one.

A way out is a less strong version of impermeability that we might call *weak impermebeality*. According to this theory, the Napoleon inside the scope of the operator and Napoleon outside from it might be related by the reader because of some background knowledge. Such an approach would really need of a totally different formulation of the semantics of quantifiers, namely Hintikka's world-lines conception. We can not work it out here but will be mentioned very briefly at the end.

Permeability: Others, from what we might call *the permeable point of view*, assume that assertions inside the scope of a fictionality operator might involves objects that outside the scope of the operator are independent ones. From this point of view the fiction adds "invented" statements about reals (i.e. objects that are independent outside the fictionality operator).

The reference world: The reference world is the world where the evaluation is performed outside the fictionality operator. It contains all objects, including the corresponding fictional characters and the objects upon they ontologically. True statements in relation to a given w about objects that are elements of D^w might be false at the reference world: *Holmes is a detective* is false at the actual-reference world. Indeed, *Holmes is a detective* according to the story; not in the actual world

Let us here sketch the dialogical semantics of the fictionality operator from the permeability point of view. The adaptation to the case of strong impermeability is straightforward.

For a general and formal introduction to dialogues, both classical and modal, see appendix III.

III.4.2.1 The fictionality operator from the permeability point of view

From the point of view of dialogues the semantics of the $\mathcal{F}$ operator results from its uses in argumentative practices. In this respect, dialogical logic allows us to capture the meaning of this operator in terms of challenges and defences, but also and particularly in terms of choices. This point on choices is crucial if we want to properly understand the difference between the operators $<\mathcal{F}>$ and $[\mathcal{F}]$. Indeed, formulae in the scope of $<\mathcal{F}>$ relate to the reader-interpretation of the work. Therefore, in a dialogue, the player X who defends a formula like $<\mathcal{F}>\varphi$ is allowed to choose the context that is compatible with the fiction at stake and in which he will have to justify φ. On the other hand, if X defends a formula like $[\mathcal{F}]\varphi$, then he claims that φ holds in every context compatible with what the fiction says whatever the interpretation is. Therefore, his opponent Y is allowed to choose the context compatible

with the fiction in which X will have to justify φ. More formally, we add to the usual dialogical rules the following ones:

Particle rules for the fictionality operator

Assertion	Attack	Defence
X - ! – [$\mathcal{F}$]φ – w_i	Y - ? - w_j Y chooses a context $j \neq i$	X - ! – φ – w_j
X - ! – <$\mathcal{F}$>φ – w_i	Y - ? - $\mathcal{F}$	X - ! – φ – w_j X chooses a context $j \neq i$

Structural rules for the fictionality operator

(RS-$\mathcal{F}$) Only O is allowed to introduce a fictional context.

Let φ for *Holmes is a detective*. If, we translate it as *According to the fiction, Holmes is a detective* by [$\mathcal{F}$]φ : it means that in every context compatible with what the fiction says, it the case that φ.

In the dialogues below, we will consider that the opponent concedes different possible interpretations at the start of the dialogue (Σ_0, Σ_1 and Σ_2). Σ_0 is a concession about accessible contexts. Σ_1 and Σ_2 are concessions about what is the case in the contexts compatible with what the fiction says.

In the following example, where the thesis is [$\mathcal{F}$]φ, we just assume that we have two interpretations of the fiction that justify w_j and w_k. What has been explicitly stated in the text is conceded in all contexts compatible with the fiction. What changes is the interpretation from the reader's perspective. In the dialogue above, we capture here is the idea that the proponent defends the thesis that φ holds in every context compatible with the fiction. Therefore, he must be able to justify φ whatever the context that the opponent chooses.

		Opponent			**Proponent**		
	Σ_0	**R : {<w_i,w_j> , <w_i,w_k>}**					
w_j	Σ_1	**φ ; ψ**					
w_k	Σ_2	**φ ; δ**					
					[$\mathcal{F}$]φ	0	w_i
w_i	1	? – w_j	0		φ	2	w_j

Notice that in order to give a complete semantics of the fictionality operator, we should clarify which inferences are allowed within the scope of this operator and define an appropriate notion of deductive closure. For the sake of simplicity we will assume a classical logic.

Conversely, the claim that *Holmes is of blood type* O^+ (ψ) is not explicitly stated in the Conan Doyle's composition. In fact, all that we know is that in the fiction Holmes has blood. If he has blood, then we must concede (following Genette) that it is *vraisemblable* that he is either of blood type O^+ or of another type. But ψ must not hold at every interpreative context

Thus, a player committed to defend the stronger thesis [$\mathcal{F}$]ψ will loose if there is context where it holds that *Holmes is of blood type* O^- (δ) :

		Opponent			Proponent		
	Σ_0	**R : {<w_i,w_j> , <w_i,w_k>}**					
w_j	Σ_1	**φ ; ψ**					
w_k	Σ_2	**φ ; δ**					
					$[\mathcal{F}]\psi$	0	w_i
w_i	1	$? - w_k$	0				

Dually ψ will hold if occurs in the scope of $<\mathcal{F}>$. Indeed, in this case, the proponent chooses a context which satisfies the interpretation at stake:

		Opponent			Proponent		
	Σ_0	**R : {<w_i,w_j> , <w_i,w_k>}**					
w_j	Σ_1	**φ ; ψ**					
w_k	Σ_2	**φ ; δ**					
					$<\mathcal{F}>\psi$	0	w_i
w_i	1	$? - \mathcal{F}$	0		ψ	2	w_k

Thus, in this framework the difference between *content* and *interpretation* is formulated with help of the specification of who of the players has the choice.

III.4.2.1 Ontological dependence relationships

In order to implement the ontological dependence relationships in dialogues, we develop some ideas contained in *To be is to be chosen* (*être*, *c'est être choisi*) by M. Fontaine, J. Redmond & S. Rahman [2009][49] and combine it with the dialogic fo the fictionalituy operator mentioned above. This idea consists to understand existence claims as the selection of choice functions. Those function are grounded on the dialogical *introduction* rule. According to this rule, the ontological commitment of the individual constants played during a game have to be frist conceded by the opponent: The opponent concedes existence by the introduction individual constants while defending an existential quantifier or challenging universal quantifier. In what follows, we rely on the idea that the introduction of an individual constant k commits to defend the assertion that k relates to an object that is only ontologicallly dependent of itself. By this means, we restrict the range of the quantifiers to ontological independent objects.

- Let us discuss the dialogical point a bit longer. Dialogical logic understands the meaning of names and propositions as interacting (*handeln*) with them in an argument. This allows a very simple formulation of a logic for fiction that results from restricting the introduction of singular terms in the context of quantification to a formal use of them. That is, the Proponent is allowed to use a constant iff this constant has been explicitly chosen by the Opponent. In fact this is one of the most powerful applications of what has been considered by researchers such as Johan van Benthem, the main contribution of dialogical and game theoretical semantics to the notion of quantifiers, namely: the meaning of a quantifier is determined by a choice that is an own move in a

[49] « Etre et être choisi », Fontaine, Redmond, Rahman [2009]

game. If this move is explicitly introduced as conditioning the choices of the Proponent; then a free logic results where commitment to existence in an argument amounts to the existence commitments of its premises established by explicit choices involving the singular terms at stake. The logic described below develops these ideas a step further: instead of restricting the choices, the choices are seen as explicit moves that can be challenged. This produces some dynamics: player X might chose an individual constant by means of a moven move n and then, after this choice has been perfomerd, player Y might challenge the move n asking X to stay for the ontological commitment triggered by his choice. The model-theoretical counterpart of this kind of ontological committed choices are assertions about the ontological independence of the objects referred by the individual constants involved in those assertions. In a GTS framework dialogical choice moves will be understood as choice functions restricted to a special domain, containing ontologically independent objects.

Thus, we will understand a statement like *Conan Doyle exists* as *Conan Doyle exists as an independent individual*, whereas a statement like *Holmes does not exist* will be rendered as *Holmes exists as a dependent object (abstract artefact).*

Notice that for issues on existence only constant dependence is required. Dialogically, we add the following rules:

Rules

(D1) A singular term k_i played by X is said to be *introduced* iff. :

a- X asserts the formula $\phi[x/k_i]$ at w_i while defending a formula of the form $\exists x\phi$, and k_i has not been used previously, or

b- X choses k_i by means of the move $< ?\text{-}x/k_i >$ at w_i (we will call these moves *choice-moves*), while challenging a formula of the form $\forall x\phi$ and k_i has not been used previously.

(D2) An individual constant is said to be *symbolic* if it has not been challenged by means of moves specified by the rules **R$\mathfrak{R}$-0 - R$\mathfrak{R}$-2.**

(D3) A dialogue is said to be *symbolically finished* iff there is no move available according to the standard dialogical rules.

(D4) We call a *symbolic sub-dialogue* a sub-dialogue in which the ontological commitment of the individual constants has not been specified by application of the rules **R$\mathfrak{R}$-0 - R$\mathfrak{R}$-2**.

(D5) We call a given sub-dialogue *ontological* if the ontological commitment of the individual constants is determined by application of the rules **R$\mathfrak{R}$-0 - R$\mathfrak{R}$-2**.

(R$\mathfrak{R}$-0) Ontological sub-dialogue starting rule: when a dialogue is symbolically finished;, X is allowed to launch an ontological subdialogue by challenging with help of rules **R$\mathfrak{R}$-1 - R$\mathfrak{R}$-2** an atomic formula of Y that closes a branch of the dialogue.

Remark 1: The restriction on atomic formulae that close branches is related to the dynamics of the procedure. During a dialogue, the proponent might use constants without justifying until the last move. The last move is the relevant one and it is precisely in this move where the constants involved (if any occur) must be justified. The rules have beed designed in such a way that if the constants involved result from a quantification rule, then their independent ontological status must be expliclty justified. This device yields a logic that intersects with the one produced by superinterpretations (Appendix I). Indeed, in both logics, the one described herewith and the one that results from superinterpretations, the set of valid formulae involving no individual constant but bounded variables will coincide the corresponding set in classical logic.

Extension of the formal rule (RS-3): The proponent cannot assert $\Re k_i k_j$ if the opponent has not introduced it before.

(R$\Re$-1) When X has played in a context $w_i \in W$ an atomic formula where k_i occurs symbolically, Y is allowed to attack this formula by asking X to chose a k_j on which k_i at context w_i:

Assertion	Attack	Defence
X - ! - $\varphi[x/k_i]$ - w_i	Y - ? - $\Re$ - k_i - w_i	X - ! - $\Re k_i k_j$ - w_i X choses a k_j possibly different from k_i

(R$\Re$-2)

(a) When X has played an atomic formula of the form $\Phi[x/k_i]$ at a given context w_i, and k_i has been chosen at, say, $w_j \in W$ by X, , then
Y is allowed to attack this formula by asking X to justify the reflexive ontological relationship at w_j:

Assertion	Attack	Defence
. X - ! – $\Phi[x/k_i]$ – w_i (where k_i has been chosen before at w_j)	Y - ? - $\Re k_i k_i$ – w_j	X - ! - $\Re k_i k_i$ – w_i

(b) When X played a choice-move of the form $[x/k_i]$ at a given context w_i, then
Y is allowed to attack this formula by asking X to justify the reflexive ontological relationship at w_i:

Assertion	Attack	Defence
. X - ? – $\Phi[x/k_i]$ – w_i	Y - ? - $\Re k_i k_i$ – w_i	X - ! - $\Re k_i k_i$ – w_i

(c) When X has played an atomic formula of the form $\Phi[x/k_i]$ at a given context w_i, and k_i has not been chosen at $w_i \in W$ by X then,
Y is allowed to consider the involved constant to be symbolical and apply **(R$\Re$-1).**

Remark 2: In order to avoid infinite games we assume that constants occurring in assertions of the form - $\Re k_i k_i$ can not be challenged.

Remark 3: For the sake of simplicity we avoid to consider cases of "empty names". Model-theoretically speaking, every names denotes something, either ontologically dependent or independent.

- **Notice** that if X asserts and atomic formula $\Phi[x/k_i]$ and ki has not been chosen by X, then **R$\Re$-2** does not apply.

Variable Domains

We assume here a dialogic that yields a logic that in a modeltheoretical framework has a varable domain (for a proof see Appendix III). However, the variabilityof the domains is related to the domain of independent objects, not to the whole domain of independent and dependent objects. Accordingly, the set of dialogical rules described above do not render neither the Barcan formulae nor their converse as valid. We can therefore claim that according to the fiction Holmes exists (as an independent object), but not outside the fiction. This is not exactly Thomasson's idea, who has two forms of existence: as independent and dependent object. More generally, both kind of objects exist. According to Thomasson, at the actual world Holmes exists (as a fictional character, i.e. as a dependent object) but he is not a detective nor lives in Baker Street. Moreover, in the original setting of Thomasson; the disproof of the Barcan formula given below will not hold. However it could be re-constructed the help of the Meinongnian device of two sets of quantifiers (ontologically committed and not ontologically committed). In the present paper, we made the choice to reserve existence only for ontological independent objects. Let us see how to invalidate the Barcan formula in relation to the fictionality operator.

		Opponent			**Proponent**		
					$<\mathcal{F}>\exists xPx \rightarrow \exists x<\mathcal{F}>Px$	0	@
@	1	$<\mathcal{F}>\exists xPx$	0		$\exists x<\mathcal{F}>Px$	2	@
@	3	$?\exists$	2		$<\mathcal{F}>Pk_1$	8	@
w_1	5	$\exists xPx$		1	$?<\mathcal{F}>$	4	@
w_1	7	Pk_1		5	$?\exists$	6	w_1
@	9	$?<\mathcal{F}>$	8		Pk_1	10	w_1
@	11	$? - \Re k_1 k_1$	10				
w_1	13	$\Re k_1 k_1$		7	$? - \Re k_1 k_1$	12	w_1

(notice that we highlighted the ontological subdialogue with red letters)

This dialogue shows that the opponent can concede that according to the fiction there exists an **independent individual** of whom P (is a winged-horse) can be asserted, without allowing the proponent to defend the assertion that there is an ontologically independent individual at the actual context but that is a winged-horse according to the fiction. The point is that Pegasus

can well be an existent winged-horse according to the fiction (see opponent's defensive move 13) , but at the actual world there is no such an independent object (see opponents challenge in move 11).

Similarly the following version of particularisation fails:

		Opponent			**Proponent**		
					$[\mathcal{F}]Pk_1 \rightarrow \exists x[\mathcal{F}]Px$	0	@
@	1	$[\mathcal{F}]Pk_1$	0		$\exists x[\mathcal{F}]Px$	2	@
@	3	$?\exists$	2		$[\mathcal{F}]Pk_1$	4	@
@	5	$? -_{w1}$	4		Pk_1	8	w_1
w_1	7	Pk_1		1	$? -_{w1}$	6	@
@	9	$? - \Re k_1 k_1$	8				
w_1	11	$\Re k_1 k_2$		7	$? - \Re - k_i$	10	w_1

Let us conclude with a dialogue that shows the difference between internal and external discourses in relation to permeability.

- Let Vxy stand for *x is the city where y lives*.
- Let $\exists x(\neg Vxh \wedge [\mathcal{F}]Vxh)$ stand for *There is a city (at the actual world) that is not the city where Holmes lives, but that, according to the fiction, is the city where Holmes lives.*

We obtain the following dialogue:

		Opponent			**Proponent**		
@	Σ_0	$\neg Vlh ; \Re ll ; \Re hk_1$					
w_1	Σ_1	$Vlh ; \Re ll ; \Re hh$					
					$\exists x(\neg Vxh \wedge [\mathcal{F}]Vxh)$	0	@
@	1	$?\exists$	0		$\neg Vlh \wedge [\mathcal{F}]Vlh$	2	@
@	3	$?\wedge 1$	2		$\neg Vlh$	4	@
@	5	$?\wedge 2$	2		$[\mathcal{F}]Vlh$	6	@
@	7	$? - w_1$	6		Vlh	8	w_1
@	9	$? - \Re ll$	8		$\Re ll$	10	@
w_1	11	$? - \Re l - k_i$	8		$\Re hh$	12	w_1
w_1	13	$? - \Re l - k_i$			$\Re 11$	13	w_1

In move 9 the opponent asks (by **(R$\Re$-2a)** the proponent to justify at the actual world the ontological commitment engaged by the introduction of h at move 2.

In move 11 the opponent asks **(R$\Re$-1)** for the ontological status of the constant h (occurring in the thesis and nowhere introduced)

In move 13 the opponent asks **(R$\mathfrak{R}$-2c)** for the ontological status of h at w_1. This move makes sense since l might be independent at the actual world but dependent at the fictional world. In our case the concession Σ_1 already assert the independence of l at w.

In the case of impermeability the set of concessions Σ_1 would not contain $\mathfrak{R}$ll – i.e, l could not be the name of an object that at the actual world is independent.

III.4.4 Examples

Let us now return to some example sentences.

1. *Sherlock Holmes is fictional*. This sentence has the simple form $P(k_1)$, where 'k_1' stands for Sherlock Holmes., and 'P' is interpreted so as to be true of all fictional (dependent) objects at that world. The sentence expresses a true proposition at the actual world.

Notice that to establish that the sentence expresses a truth *at the actual world* is the artifactual way to state that Sherlock Holmes does not exist: that is Holmes is not an independent object. Indeed, Thomasson argues in a forthcoming paper (*Fiction, Existence* and *Reference*), that negative existential claims should be understood as claims involving one determined ontological category: Holmes does not exist (*as independent object*)
.
Note also that this sentence is compatible with the following further sentence: *According to the story, S.H. is not fictional*.

Recall the last dialogue where Holmes is fictional at the actual world but ontological independent at the fictional world. Only at the actual world, that is, only when we are outside of the scope of the fictionality operator can we establish that Sherlock Holmes is a fiction.

2. *Conan Doyle does not exist*. Consider this statement as a statement made by the story: according to the story, Conan Doyle does not exist. (It does not matter that the actual stories by C.D. do not make such an explicit statement; suppose they do.) This statement has the form $\mathcal{F}\neg(\exists x{=}k_2)$ and it is rendered true precisely in the case that the k_2 names a dependent object "C.D."

Notice that in the case of strong impermeability this will always be true.

3. *Watson's grandfather's cousin's dog was a German shepherd*. This statement is not true according to the story, because it is perfectly compatible with the story that this sentence is false (or even meaningless, thanks to the lack of satisfaction of the relevant presuppositions related to genitives). However, it is compatible with the fiction. As long as the fiction does not lay it down that Watson's grandfather had no cousin — or as long as there was a cousin but the cousin had no dog — the story leaves open the 'interpretation' or 'realization' in which the sentence is true.

Let us see what we think we have accomplished: namely the articulation between the externalist and the internalist point of view. Externalist points of view are given at the actual world. It is there where « categorial » claims are asserted : *Samsa is a fiction, Poe is the author of* ***The Golden Bug*** and so forth. Internalist points of view involve the worlds that interpret the fictionality operator. The articulation is the actual world where the fictionality

operator starts –iterations might start placing the fictionality operator at one of the worlds triggered by the semantics of the fictionality operator situated at the actual world. Once more, negative existential claims can be placed either internally or externally (at the actual world). Negative existential claims at the actual world can be seen, from the point of view of the artifactual theory, as claims involving the ontological category of the object at stake: they involve claims about ontological dependence. Let us see in a table the usual problematic cases and how they are solved in our setting[50]:

Problem sentence	**Ontological Dependence Approach**
Holmes does not exist (at the actual world)	Holmes does not exist as an independent object. This is true at the actual world.
There are fictional characters that are dragons	According to the story there are individuals such that they are dragons…
Holmes lived in Baker Street	False: The dependent object Holmes does not live in Baker Street (at the actual world).
According to the fiction, Holmes lived in Baker Street	True in all worlds compatible with the fiction
It is compatible with the fiction *The Adventures of Sherlock Holmes,* that the King of France is dead	It is true: In at least one world v we might have that the King of France is not part of its domain of quantification: .
Anna Karenina is more intelligent than Emma Bovary	It is compatible with each of the fictions *Anna Karenina* and *Emma Bovary*, that Anna Karenina is more intelligent than Emma Bovary.
Laura thought about (knows about/loves/admires) Pegasus	i. Laura thinks about the dependent object Pegasus ii. It is compatible with the fiction that Pegasus is thought (known/loved/admired) by Laura

[50] The second and the last three cases of the table are variations on examples by Sainsbury 2009.

Problem sentence	Ontological Dependence Approach
Some characters in the Classical Dictionary are mythological, but most of them really existed.	According to the Classical Dictionary, there are many characters, some are ontologically independent objects but most are ontologically dependent objects

The strong impermebeality approach, as mentioned above, might sound too strong and precludes satyrics.

The weaker is semantically more complicated. We think that here we should abandon the rigid designation theory assumed by Thomasson and go over to the notion of world lines. In fact, instead of assuming Kripke's rigid designation theory we think that from the ontological and epistemic perspective it is natural to Thomasson's and Ingarden's theory of identity to cast it in the framework of Hintikka's notion of world-lines, where an individual, is understood as a (partial) function that might pick up one object of the domain of w called the *manifestation* or *aspect* of the individual at w, (e.g. the ontological independent object that "manifests" or is an "aspect" of the individual who lived at t) and a different object at a different scenario. The interesting point is that in this setting, we might conceive that two different individuals might share some scenarios. E.g. the upon Marquez dependent object (creation of Marquez) whose manifestation is called Simon Bolivar– might be also a manifestation of the "real" ontologically independent individual that is called Bolivar at the actual world. However, both individuals are different since they do not share the same manifestations in all scenarios. We will leave this issue for a future paper..

APPENDIX I[51]

Hugh MacColl and Russell' Meinong

AI.1 Introduction

The most influential approach to the logic of non-existents is certainly the one stemming from the Frege-Russell tradition. The main idea is relatively simple and yet somehow disappointing, to reason with fictions is to reason with propositions which are either (trivially) true, because with them, on Russell's view, we deny the existence of these very fictions, or otherwise they are (according to Russell) false or (according to Frege) lack truth-value in the same trivial way. One of the most important early dissidents to that tradition was Hugh MacColl. It is in regard to the notions of existence and arguments involving fictions that MacColl's work shows a deep difference from the formal work of his contemporaries Indeed, MacColl was the first to attempt to implement in a formal system the idea that to introduce fictions in the context of logic amounts to providing a many sorted language. Interesting is the relation between Bertrand Russell's critics to Alexius Meinong's work and Russell's discussions with MacColl on existence. Recent scholars of Meinong such as Rudolph Haller and Johan Marek and modal Meinongians such as Graham Priest, Richard Routley and Edward Zalta make the point that Russell's Meinong is not Meinong.
An interesting historical question, is to study how Russell's critics of Meinong could have been influenced by his discussion with MacColl. Notice that the main papers on this subject by Russell, Meinong and MacColl, where published between 1901 and 1905. We can not discuss this here thoroughly but I will nevertheless point out some issues for a future deeper research. MacColl's work on non-existents resulted from his reaction to one lively subject of discussion of the 19th century, namely the *existential import of propositions*. This topic was related to the traditional question about the ontological engagement or not of the copula that links subject and predicate in a judgement. Franz Brentano published 1874 his theory on the existential import of the copula and on how to define away the alleged predicate of existence.[52] J. S. Mill, after some discussions, acknowledged by February 1873 in a letter to Brentano that he has been convinced, despite his early arguments of his *System of Logic*. However, the most of the British traditional logicians did not follow Brentano and the opposition between them and the "Booleans", who also charged the copula with existential import, triggered a host of papers on that subject.[53] The early Russell of the *Principles* and Hugh MacColl defended the idea that there is a *real* and a *symbolic* existence, that seems to be close to Russell's use of *subsistence*. MacColl's example, probably borrowed from Mill, targeted the meaning of the copula "is" in expressions such as "is not existent.[54]

AI.2 MacColl's Logic of Non-Existence

MacColl's logic of non-existence is based on a two-fold ontology and one domain of quantification, namely:

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[51] The present appendix is based on the first 5 pages of Rahman 2009a.
[52] Cf. Brentano 1874, chapter 7.
[53] J.P.N Land's paper 1876 "Brentano's Logical Innovations" spelled out the position of the traditionalists and triggered in *Mind* the discussions on the existential import of propositions. .
[54] *Take, for example, the proposition, "**Non-existences are non-existent**". This is a self-evident truism; can we affirm that it implies the existence of its subject **non-existences**?* […] *In pure logic the subject, being always a statement, **must** exist – that is, it **must** exist as a **statement**.* MacColl 1902, 356.

- the class of existents, MacColl, calls them *reals*

> *"Let* e_1, e_2, e_3, *etc. (up to any number of individuals mentioned in our argument or investigation) denote our universe of real existences.* [55]

> […] *these are the class of individuals that, in the given circumstances, have a* ***rea****l existence.*[56]

- The class of non-existents

> *Let* 0_1, 0_2, 0_3, *etc., denote our universe of non-existences, that is to say, of unrealities, such as centaurs, nectar, ambrosia, fairies, with self-contradictions, such as round squares, square circles, flat spheres, etc., including, I fear, the non-Euclidean geometry of four dimensions and other hyperspatial geometries.* [57]

> […] *the class of individuals that, in the given circumstances, have* ***not*** *real existence.* […] *It does not exist* ***really****, though (like everything else named), it exists* ***symbolically****.*[58]

> *In no case, however, in fixing the limits of the class e, must the* ***context****, or given circumstances be overlooked* [59]

- And the domain of quantification, the *Universe of Discourse*, containing the two precedent classes:

> *Finally, let* S_1, S_2, S_3, *etc., denote our Symbolic Universe, or "Universe of Discourse," composed of all things real or unreal that are named or expressed by words or other symbols in our argument or investigation* [...].[60]

As expected, individuals, that are elements of the Universe of Discourse, might be element of the first two classes:

> *We may sum up briefly as follows: Firstly, when any symbol* A *denotes an* individual*; then any intelligible statement* ϕ(A)*, containing the symbol* A*, implies that the individual represented by* A *has a* symbolic *existence; but whether the statement* ϕ(A) *implies that the individual represented by* A *has* real *existence depends upon the context.* [61]

and predicates might be interpreted by the means of classes containing reals, unreals or both of them.

> *Secondly, when any symbol* A *denotes a* class*; then any intelligible statement* ϕ(A)*, containing the symbol* A *implies that the whole class* A *has a* symbolic *existence; but whether the statement* ϕ (A) *implies that the class* A *is* wholly real, *or* wholly unreal, *or* partly real and partly unreal, *depends upon the context.*[62]

> *When the members* A_1, A_2, &*c of any class* A *wholly of realities or wholly of unrealities, the class is*

[55] MacColl 1905a, 74
[56] MacColl 1906, 42
[57] MacColl 1905a, 74.
[58] MacColl 1906, 42
[59] MacColl 1906, 43
[60] MacColl 1905a, 7.
[61] MacColl 1905a, 77.
[62] MacColl 1906, 77

> *said to be a* ***pure*** *class, when A contains at least one reality and also at least one unreality, it is called a* ***mixed*** *class.* [63]

(Notice that MacColl actually speaks of the existence of the class. I think that we should understand it as talking about the existence of the elements of the class. See below his rejection to interpret *hunger* independently of a hungry person)

The partition of the universe of discourse into existents and non-existents, might lead the modern reader to think in the anachronistic setting of a free logic with outer and inner domains. However, the description of the symbolic universe sounds puzzling. On one hand it sounds as we might do logic in such a universe abstracting away whether objects are or not existent On the other hand, MacColl, while replying in 1905 to Russell[64] and to Arthur Thomas Shearman, insists that the distinction between existent and non-existents within the symbolic universe is crucial for his logic:

> *The explanation from my point of view is, that the confusion is solely on their side* [Shearman's and other symbolists' side] *and that it arises from the fact that they (like myself formerly) make no symbolic distinction between realities and unrealities* […]. *With them 'existence' means simply existence in the Universe of Discourse, whether the individuals composing that universe be real or unreal.* […].*Once anything (real or unreal) is spoken of, it must, from that fact alone, belong to the symbolic universe S, though not necessarily to the universe of realities* ***e****.* [65]

With some hindsight, some readers might think that according to the last quote above, MacColl is thinking on two kinds of existential quantification or at least of two kinds of existential predication, one that has as scope the whole symbolic universe and the other, when the classification between reals and not reals within the universe has been established, that applies to reals. In this sense, individuals might have a "symbolic" existence and a "real" existence.
Perhaps, there is some room to think dynamically about the interaction between the symbolic and the real existence. The real existence might come into play once the precise constitution of the universe of discourse has been *spoken out*. Juan Redmond and Mathieu Fontaine are developing a dialogic that renders justice to this dynamics from an epistemic point of view: symbolic existence will be assumed so long as we do not know about the ontological constitution of our universe of discourse. Do not fear we will not discuss this approach here.

A different source of puzzles might relate to ontological questions. What are those objects that are non-existent? Did MacColl come to a conception close to some kind of Meinongianism? Some arguments in favour of a positive answer are the following:

1) MacColl's claim of two kinds of existence mentioned above. In fact, MacColl's notion of existence seems to be closer to that of the early Russell than to the one of Meinong. Meinong had also three ontological domains: the existents, non-existents and subsistents. However Meinong's concept of subsistents only applied to abstract objects while MacColl's *symbolic existence* and Russell's version of *subsistence*

[63] MacColl 1906, 43
[64] *This sense of existence [the meaning in which we enquire whether God exists] lies wholly outside Symbolic Logic, which does not care a pin whether its entities exist in this sense or not.* Russell 1905, 401.
[65] MacColl 1905b, p. 579.

included existents and non-existents. Compare, e.g., once more MacColl's remarks of 1902 and 1906

*Take, for example, the proposition, "**Non-existences are non-existent**". This is a self-evident truism; can we affirm that it implies the existence of its subject **non-existences**?* […] *In pure logic the subject, being always a statement, **must** exist – that is, it **must** exist as a **statement**.*[66]

It [the class of non-existents…] *does not exist **really**, though (like everything else named), it exists **symbolically**.*[67]

with the Russell of the ***Principles***:

*Whatever may be an object of thought, or can occur in a true proposition, or can be counted as one, I call **term*** […]. *Every term has being, i.e. is in some sense. A man, a moment, a number, a class, a relation, a chimera, or anything else that can be mentioned is sure to be a term.*[68]

MacColl and Russell make the point that everything named must have some kind of being. This point of theirs might be seen as an ontologically charged reading of Aristotle's remark:

Even non-existents ca be signified by a name[69]

2) MacColl's two notions of existence (the *real* and the *symbolic* existence) seem to have been conceived as predicates. Indeed; in MacColl's notation existence, when applied to an individual or to (the members of a) class, is signalised by an exponential. Now; in general, letting by side the many changes and hesitations of his notational system, exponentials are used in principle to express a predicative role. In fact, the basic expressions of MacColl's formal language are expressions of the form

H^B

where H is the domain and B a predicate. He gives the following example:

> H: the domains of horses
> B: brown
> H^B: The horse is brown: all of the elements of H (horses) are brown.

Similar applies to the use of the predicates of *symbolic, real existence and non-existence*:

> $H^{\boldsymbol{e}}$: The horse is real or has a real existence: all of the elements of H (horses) are *really* existent.
> $H^{\boldsymbol{0}}$: The horse is an unreality: all of the elements of H (horses) are not *really* existent.
> $H^{\boldsymbol{S}}$: The horse has a symbolic existence: all of the elements of H (horses) are *symbolically* existent.
>
> Certainly, while in this context to introduce a predicate of existence for "reals" might be a sensible idea, to introduce symbolic existence as a third predicate,

[66] MacColl 1902, 356.
[67] MacColl 1906, 42
[68] Russell 1903, 43.
[69] Aristotle, *Posterior Analytics* 92b29-30.

for symbolic existence is not. At least if the latter should render formally the semantics of a copula without ontological engagement. Symbolic existence should be understood as a perspective where the difference between existence and non-existence has not been (yet) drawn.

3) MacColl assumes a logic of equality for terms that refers to existens and non existent objects.

4) More generally, recall that according to Meinong we should distinguish the *Sein* of objects – their existential status – from their *Sosein*, their having – certain – features or properties. Thus, Meinongians claim that an object can have a set of properties even if it does not exist. This is the so-called *Principle of Independence:* Pegasus, Ulysses, and Joseph Cartaphilus can be said to have properties without that the propositions involved become false. MacColl's *mixed classes* could be seen as assuming the principle of independence

To state this clearly, it is doubtful that MacColl ever read Meinong's work. However, while reading MacColl it is tempting to understand Russell's version of Meinong's notion of "subsistence" as an adaptation of MacColl's *symbolic existence* to the Meinongian framework. Nevertheless, in the overall context of MacColl's philosophy in relation to which he explicitly acknowledged sympathies for Poincarés conventionalism and Peirce's pragmatism we might contest considering him as guilty of Meinongianism. At least not of the kind where non-existents are some kind of independent entities that are part of our universe since the creation of the universe. Indeed, in his texts he explicitly defends the idea that thoughts and abstract notions and are not to be considered as independent of the thinker who is thinking them:

> *There can be no hunger without a hungry person or animal; there can be no hardness without some hard-substance* […]. *Similarly, I cannot conceive of a thought apart from a thinker or a feeling or sensation without a soul or feeler.*[70]

In this context, it sounds plausible that we might extend this conception of abstract objects and thoughts as ontologically dependent objects to the understanding of fictions in the way developed in part III of the present paper.[71] Actually, this is what we will try to show, that is, how to put all the pieces of MacColl's ontology to work together into one semantic frame for modal logic. In doing so, we will be guilty of a further anachronism: we will assume a kind of modal semantics of the sort that has been made popular after the work of Jaakko Hintikka and Saul Kripke. It is worth mentioning that Stephen Read showed that the modal system **T** is due to MacColl. Modal logic is thus in fact part of the achievements of MacColl though certainly he did not deployed or even conceived a model theoretical semantics. Nevertheless, let us make the point that he stubbornly makes the point that the classification between reals and not reals have to be relativized to given *circumstance*s.

[70] MacColl 1910, pp. 349-350.
[71] For the compatibility of the semantics developed here with MacColl's view see Rahman 2009a.

APPENDIX II[72]

Free Logics

The aim of this appendix is to present a short overview of free logics from the modeltheoretical point of view. For alternative non model-theoretical approaches see Rahman/Rückert/Fischmann (1997), Rahman (2001) and Fontaine/Redmond/Rahman (2009).

One of the first formal developments towards the logic of non-existents is the one of free logics. Free logic is shorthand for logic free of *existence assumptions*. There are two existence assumptions built into classical logic, namely

- The domain of quantifiers is not empty. Thus, the following holds:

  $\exists x(Axv\neg Ax)$

  $\exists x(x=k)$.

- Every term denotes. This assumption renders the following valid (in classical logic)
  Infer $\phi[k]$ from $\forall x\phi$

  Infer $\exists x\phi$ from $\phi[k]$

  Infer $\exists y\neg\exists x(x=y)$ from $\neg\exists x(x=k)$.

  .

Not every free logic rejects the first existence assumption. Those that do are known as *universally free logics*. Existential generalization and universal specification hold under the restriction that the terms involved exist. Usually, the assumption of existence is made explicit by the means of a first-order existence-predicate (E!).

Karel Lambert [(960), who penned the expression *free logic*, distinguishes three types of free logics, negative, positive and neuter. Ermanno Bencivenga (1986) added a new type based on supevaluations. Let us start with the description of the first three.

AII.1 Negative, positive and neuter free logics

Negative free logic, allows constants not refer at all. The identity axiom holds under the same restriction as existential generalisation and universal instantiation. Atomic formulae in which constant occur that do not refer are false.

Formulae φ are evaluated in modelsM = (D,I) relative to variable assignments γ and the interpretation function I, that, when applied to constant it is a *partial* function, i.e. to each constant k and to some members of D, I assigns a member of D. In other words, I(k) may not be defined, but if it is, then $I(k) \in D$. While I, when applied to constant is a partial function,

[72] Part of the content of this appendix has been extracted from : Fontaine/Redmond/Rahman 2009, though the semantics have been adapted to the objectual interpretation of quantifiers. .

assignments are total functions. In other words, if x is free variable the value of $\gamma(x)$ is an element of D.

The relevant truth conditions are

(i) $V_M(Pk_1,\ldots,k_n) = 1$ iff. $I(k_1), \ldots, I(k_n)$ are defined and $\langle I(k_1), \ldots, I(k_n)\rangle \in I(P)$.

(ii) $V_M(k_i = k_j) = 1$ iff. $I(k_i)$ et $I(k_j)$ are defined and $I(k_i)$ yields the same element that $I(k_j)$.

(iii) $V_M(E!k_i) = 1$ iff $I(k_i)$ is defined.

The rest of the truth-conditions are standard.

Thus, if one the function I is undefined for at least one ki, the following are all false:

$k_1 = k_1$
$k_1 = k_2$
Pk_1

Neuter free logic, is a negative free logic that allows formulae to be neither false nor true. That is if a formula contains a constant that does not refer then the whole formula is neither true nor false. While positive and negative free logic do not induce changes to classical propositional logic (without equality), neuter free logic does: $Ak \vee \neg Ak$ is not generally valid. That is, interpretations are also defined as partial functions though, different to the case of negative free logic; a valuation might yield truth-value gaps:

The relevant truth conditions are

(i) $V_M(Pk_1,\ldots,k_n) = 1$ if. $I(k_1), \ldots, I(k_n)$ are defined and $\langle I(k_1), \ldots, I(k_n)\rangle \in I(P)$
otherwise $V_M(Pk_1,\ldots,k_n)$ is undefined

(ii) $V_M(k_i = k_j) = 1$ if $I(k_i)$ et $I(k_j)$ are defined and $I(k_i)$ yields the same element that $I(k_j)$
otherwise $V_M(k_i = k_j)$ is undefined.

(iii) $V_M(E!k_i) = 1$ iff $I(k_i)$ is defined.

Positive free logic, allows singular terms to refer to non-real objects. The domain might contain real and non real elements. The result is that the identity axiom holds in any such logic extended with equality. That is, there might be identity of non-existent objects. Furthermore in positive free logic we might introduce two pairs of quantifiers: ontologically committed quantifiers and ontologically not committed quantifiers.

The most important semantics for positive logic partitions the domain D in two, namely an *outer* domain and an *inner* domain. The inner domain is as in classical logic (except that we may allow it to be empty – in the case the logic ought to be universally free): it contains real existing objects. The outer domain consists of the references of terms designating non-existents such as Pegasus, King Lear, Martin Fierro. Every term refers (either in the outer or in the inner domain). More precisely a model is a triple $\langle D_I, D_O, I\rangle$ where D_I is the internal domain, D_O the external domain and I is the interpretation function defined over the whole domain, that is over the union of inner with the outer domain. The ontological-charged quantifiers range only over the inner domain D_I. A second pair of quantifiers, non-ontologically charged, can also be defined: their range is $D_I \cup D_O$.

Thus, I is defined as follows:

(i) For any constant k I(k) k is an element of $D= D_I \cup D_O$.

(ii) For any predicate of n-places P, ***including identity***, I(P) is the set of n-tuples of members of $D= D_I \cup D_O$.

- An assignment γ in a constant (varying) domain first-order model ***M*** is a mapping that assigns to each free variable *x* some member γ(*x*) of $D= D_I \cup D_O$

- Let γ and γ**'** be two assignments. We say that γ**'** is an ***x*-variant*** of γ if both assignments agree on all variables except possibly the variable *x* **and** γ'(x) is a member of $\mathbf{D_I}$.

The point is that free variables that can find an assignment somewhere in D (exactly like interpretations) but the ontologically charged quantifiers will be defined with the help of a special kind of *variants*, restricted to the inner domain, in such a way that the quantifiers only range over D_I.

Thus, we have:

(i) $V_{M,}\gamma(\forall x\phi) = 1$ iff for every ***x*-variant*** γ' of γ $V_{M,}\gamma'(\phi[x/\delta]) = 1$, where $\delta \in D_I$.

(ii) $V_{M,}\gamma(\exists x\phi) = 1$ iff for some ***x*-variant*** γ' of γ $V_{M,}\gamma'(\phi[x/\delta]) = 1$, where $\delta \in D_I$

The introduction of a second pair of non-ontologically charged quantifiers is strighforward, we need only to define assginement variants that range over $D= D_I \cup D_O$ and not only over D_I

In appendix AII.3 we will describe a semantics of positive logic with inner and outer domains in the context of first order modal logic.

AII.2 Supervaluations and Superinterpretations

AII.2.1 Supervaluations

From the point of fiction irrealists irrealists claim that names of fictions are empty, they do not refer at all. In this sense, they have the feeling that that positive free logic cheats. When irrealists say a name is empty, that it refers to nothing, they do not mean that it refers, but to something which does not exist; they mean that it does not refer at all. However, the gap values of the neuter free logic might make it difficult to define a notion of logical consequence involved. The classical account of consequence says that one proposition is a consequence of others provided no interpretation leads from the truth of the latter to the falsity of the former. But this definition will allow inferences that are rejected in free logics. Indeed, take existential generalization: If ϕ[k] is true then ∃xϕ must be true, if ϕ[k] lacks a truth-value (i.e. when k is empty), ∃xϕ lacks it too.The inference did not lead from truth to falsity. Thus, according to this notion of consequence, existential generalization is valid. This is not right. To excluded unwanted inferences such as the one mentioned above, we need to rule out the move from lack of value to falsity (or lack of value) as invalid. That is, it seems that we ought to say that one propostion is a consequence of others if no interpretation leads from propositions none of which are false to one which is false of lacks a value. Unfortunadely, as pointed out by Stephen Read (1995), this revised criterion invalidates

inferences which we wish to class as valid.[73] Consider a model where Bk_2 is false (i.e. k_2 refers but it does not satify B) and $Ak_1 \wedge \neg Ak_1$ lacks a value (since k_1 does not refer). Thus the inference from $Ak_1 \wedge \neg Ak_1$ to Bk_2 leads from lack of value to falsity, and so will be invalid.[74].

One way out, for irrealists, is to make use of Bas van Frassens' (1966) method of supervaluations. Moreover, this approach seems to fit very well with the make-believe theory on fiction.

According to the theory of supervaluations models consist of *partial valuations*, *classical extensions* and *supervaluations*.

- Partial valuations allow truth-value gaps such as in some three-valued logics In other words an assignement of truth to some propositions, falsity to others and not value to the rest.

- Consider all ways of extending this partial valuation to a total valuation by arbitrarily assigning values (consistent with the truth-conditions - if a given proposition is arbitrarily made true, then any disjunction containing this proposition will be made true too) to those propositions that the partial function yields a lack of value. Call these, the *classical extensions* of the original partial valuation.

- A *supervaluation* is defined as follows :
  A proposition is true according to the supervaluation if it is true in all classical extensions, false according to the supervaluation ; if it is false in all classical extensions and has no (super-)value if it takes different values in different classical extensions.

Logical consequence: A proposition is a logical consequence of other propositions if there is no partial valuation every classical extension of which makes all the premises true and the conclusion false

Validity: A proposition is valid according to supervaluation if there is no partial valuation the classical extension of which renders that proposition false.

Take some instance of the principle of non-contradiction where we assume that some empty name occurs, and a partial valuation as described in the matrix below by the lines 1 to 4.

1	ϕ	$\neg\phi$	$\neg(\phi \wedge \neg\phi)$
2	1	0	1
3	#	#	#
4	0	1	1
5	**1**	**0**	**1**
6	**0**	**1**	**1**

The first of the two possible classical extensions (line 5) assign true to ϕ and accordingy false to its negation while the second extension (line 6) assigns the dual values. According to both of these extensions $\neg(\phi \wedge \neg\phi)$ is true, and thus so is its supervalue. Non-contradiction is thus valid according to supervaluation.

[73] Read 1995, 139.
[74] Read 1995, 139

One way to read supevaluation is to read classical extension as *if-valuations.* In other words, some propostions are neither true nor false, e.g. those involving fictional terms, but we do- by means of classical extensions, as if they were true or false. This reading might provide the semantics of the make-believe approach. However, the framework is still incomplete. Supervaluations are only efficient at the propositional level. What about quantifiers? What about $k_i = k_i$. Is it true ; false of lacks of a value? If we consider equalities to be atomic propositions then identity will lack a supervalue. Bencinvenga accomplished the task to extend the supervaluational framework to first order free logic by combining outer domains with supervaluations.

AII.2.2 Superinterpretation: Comibining outer domains with supervaluations

Superinterpretation : Bencivenga (1986) does not consider all classical extensions. Instead he considers all ways of assigning a denotation to the empty terms, and the total valuations which will result from that. However, the denotations of the empty terms are chosen not from the domain of the partial valuation, but from arbitrary extensions of that domain. Woodruff (1971, 1984) developed the view that extensions of denotations should be thought as extended interpretations that find their values in an added outer domain. We will follow Woodruff's account. .

- A *free extension* of a partial valuation comprises an extension of the domain by the addition of (non-empty) outer domain, together with an extension of the interpretation of the predicate letters to the outer domain and the resulting total valuation resulting from these extensions.[75]

- Supervaluation, consequence and validity are defined as before but by substituting *classical extension* with *free extension.*

It is important to point out that the approach of Bencivenga still yields a free logic and not a classical logic. Let us see the details for the rejection universal specification within this framework:

Consider U' to be the result of a free extension of model U in relation to the constant k. In U we might have :

$V_U(\forall xPx) = 1$ and, if we assume that k is an empty name : $V_U(Pk_1) = \#$

In U' we will have free extensions such as

$V_{U'}(\forall xPx) = 0$ (or $V_{U'}(\forall xPx) = 1$) et $V_{U'}(Pk_1) = 0$ (or $V_{U'}(Pk_1) = 1$) – recall that in U' while the interpretation of the constants have values in the outer domain, quantifiers range over the inner domain.

What values should we then chose, those of U' or of U? The idea is that, if the U-valuation does not yield lack of value, then that U-valuations should be given priority. Thus, in our case we will retain $V_U(\forall xPx) = 1$, but switch to U' for the consequent. Now; for the consequent

[75] Cf. Read 1995, 138-139

we have as one of the possible free extensions $V_{U'}(Pk_1) = 0$. Hence, universal specification fails to hold.[76].

In other words, the semantics of superinterpretation starts with an interpretation function defined as in neuter freelogic but later on, the truth-value gaps produced by the partial function are filled up in such a way that the result is a positive logic.

More generally, according to superinterpretations, we do as if empty names denote some non existent. Empty names are empty and the correspondent propositions might lack truth-value but if we pretend that they refer some non-existent object of a pretended outer domain, then we might consider those propositions to be true.

AII.3 Free logics in a modal context

The aim of this appendix to develop a semantics of first order modal logic able to display a positive free logic without empty domains with and without constant domains.[77] The varying domain frames, could be seen as furnishing the basis of a modal Meinongianism framed on normal worlds. The extension to non-normal worlds should be straightforward. The advantage of varying domains is that it might be seen as implementing a kind of "creationism" within Meinongianism. Now, on our view, this will not be fully accomplished until the created elements of each world are taken seriously as creations in the sense Thomasson (see last part of the paper above).

Varying and constant domains

Let us have the frame <*W*, R, *D*> for first-order modal logic

The domain could be constant or varying

Constant Domains:

- We say that the domain of the frame <*W*, R, *D*> is *globally constant* if *D* is a non-empty set, called the domain of the frame over which quantifiers can range, no matter at what world

But we could also have the following weaker version:

- We say that the domain of the frame <*W*, R, *D*> is *locally constant* if *D* is a non-empty set, such that for for *w*, *w'*∈ *W* such that wRw', then $D(w) = D(w')$.

Barcan and converse Barcan-formulae are valid in locally constant domain models. Thus the simultaneous validity of the Barcan- and converse-Barcan formulae characterize locally constant domain models.

More generally, a sentence is valid in all locally constant domain models iff this sentence is valid in all globally constant models

[76] Cf. Woodruff 1971
[77] Cf. Fitting/Mendelson 1998 and Garson 2006

Varying Domains:

- We say that the domain of the frame <*W*, R, *D*> is *globally varying* if *D* is a non-empty set, such that for any *w*, *w'*∈ *W*, then it is not always the case that *D*(*w*) = *D*(*w'*).

In such domains neither the Barcan ($\Diamond\exists xAx \rightarrow \exists x\Diamond Ax$)- nor the converse Barcan ($\exists x\Diamond Ax \rightarrow \Diamond\exists xAx$)- formulae are valid.

Here too some weaker versions are available:

- We say that the domain of the frame <*W*, R, *D*> is *monotonic (or increasing)* if *D* is a non-empty set, such that for any *w*, *w'*∈ *W*, such that wRw, then $D(w) \subseteq D(w')$.

The converse Barcan-formulae, such as $\exists x\Diamond Ax \rightarrow \Diamond\exists xAx$, characterize such a domain.

- We say that the domain of the frame <*W*, R, *D*> is *anti-monotonic (or decreasing)* if *D* is a non-empty set, such that for any *w*, *w'*∈ *W*, such that wRw, then $D(w') \subseteq D(w)$ (Note that the order of inclusion has been reversed)

The Barcan-formulae, such as $\Diamond\exists xAx \rightarrow \exists x\Diamond Ax$, characterize such a domain

For varying domains we have one first important decision to take: should we allow to have singular terms and free variables in our language that may or may not be in the domain of some possible world? If we do not allow this to happen then varying domains become very difficult to handle and quite a big amount of ad hoc clauses have to be implemented (See Gamut vol. II). We will take rather the second choice and allow to have singular terms in our language that may or may not be in the domain of some possible world. In this case two main approaches are possible:

- the positive free logic one (terms need not to designate in every world of the frame but in at least one world) and
- the deflacionists (neuter and negative free logics): things which do not exist can not be referred to or mentioned, no statement can be about them

The argument in favour of the positive free logic approach comes from the fact that if we allow to have singular terms in our language that may or may not be in the domain of some possible world, what should we say of the truth of a formula *Pk* in world *w* where the value assigned to *k* is not an element of the domain of *w*?. In such a set up we have three reasonably choices

1) allways take *Pk* to be false at *w*.
2) leave the truth of *Pk* undetermined
3) make no special restrictions: in particular *Pk* could be true though k does not exist in the world at stake:

The first option is only sensible if it applies to atomic formulae: notice that we do not want to say that the negation of *Pk* is false if *k* does not exist at *w* (because *Pk* is false too). But Kripke (1963, p. 85, footnote 1) has observed that if we take solution 1 imposing this

requirement on atomic sentences leads to a modal logic without uniform substitution. The real options are positions two or three. We will follow position 3. It amounts to the rejection of the classically valid formulae:

$\forall xAx \rightarrow Ak$

$Ak \rightarrow \exists xAx$

The point is that the value assigned to *k* might lie beyond the values assigned to the variables of the quantifiers at the world at stake, say *w*: the value assigned to *k* might be an element of the domain of *w'* and not of *w*.

Inner and outer domains in the modal setting:
In other words: the idea of outer domains is implemented as distinguishing between the interpretations that yields values in the domain of the world at stake, say *w*, (the domain of *w* corresponds then to to the inner domain) or values in the domain of a different world *v* (the domain of *v* corresponds to the outer domain of *w)* .

Definition 1: Domain of the frame:
The domain of the frame D_F is the union of the domains of all the possible worlds of the domain

Definition 2: Interpretation in constant and varying domains:
The interpretation *i* is an *interpretation* in a constant (varying) domain frame ***F***: <*W*, R, *D*> if *i* assigns, to each n-place relation symbol R of the language, and to each possible world *w* of *W* some n-place relation on the domain of the frame D_F.

Definition 3: Model in constant and varying domains:
A constant (varying) domain first-order model is a structure ***M*** =: <*W*, R, *D, i*> where <*W*, R, *D, i*> is the correspondent first-order frame and *i* the appropriate interpretation

Definition 4: Assignment in constant and varying domains:
An assignment γin a constant (varying) domain first-order model ***M*** is a mapping that assigns to each free variable *x* some member $\gamma(x)$ of D_F**.**

Definition 5: Variant in constant domains:
Let γ and γ' be two assignments. We say that say that γ' is an *x*-variant of γ in constant domain if both assignments agree on all variables (including k-terms) except possibly the variable *x*.

Definition 6: Variant in varying domains:
Let γ and γ' be two assignments. We say that say that γ' is an *x**-variant of γ if both assignments agree on all variables (including k-terms) except possibly the variable *x* (where x is not a k-term) and γ'(x) is a member of D^w

- The point is that in varying domain a *variant at a world *w* can not assign objects that are beyond the domain of *w*, if quantifers should range only over the domain of the world at stake. Furthermore, notice that this restriction does not apply free variables that can find an assignment somewhere in the domain of the frame.

Definition 7: Truth in a model with varying domains

The point that the variants defining the truh in the model for quantifiers in varying domains will assign objects of the domain of the world at stake. The relevant definitions are those of positive logic (see appendix AII.2.1).

Thus, we have:

(i) $V_{M,}\, w, \gamma\, (\forall x \phi) = 1$ iff for every ***x*-variant** γ' of $\gamma\, V_{M,}\, w, \gamma'(\phi[x/\delta]) = 1$, where $\delta \in D^w$

(ii) $V_{M,}\, w, \gamma\, (\exists x \phi) = 1$ iff for some ***x*-variant** γ' of $\gamma\, V_{M,}\, w, \gamma'(\phi[x/\delta]) = 1$, where $\delta \in D^w$

Final remark: We have chosen to describe positive free logic for varying domains. In fact, to implement positive free logic in a constand domain setting is straightforward: it is sufficient to introduce a predicate of existence and distinguish with the help of this predicate the inner and outer domain of each world.

APPENDIX III[78]

Dialogues: for first order logic and for modal positive free logic

Formal dialogues

Let us see what is at stake in dialogical logic by reconstructing in dialogical terms the notion of validity in First-order logic.[79] We first define a language $\mathsf{L}[\tau]$; this language will basically be obtained from First-order logic (of vocabulary τ) by adding certain metalogical symbols.

We introduce special *force symbols ?* and *!*.

An *expression* of $\mathsf{L}[\tau]$ is either a formula of **FO**$[\tau]$, or one of the following strings:

L, R, $\vee$, $\forall x_i/k_j$ or $\exists x_i/k_j$

where x_i is any variable and k_j any constant. We refer to the latter type of expressions as *attack markers*.

In addition to expressions and force symbols, for $\mathsf{L}[\tau]$ we have available *labels* **O** and **P**, standing for the players (*Proponent*, *Opponent*) of dialogues.

Every expression *e* of $\mathsf{L}[\tau]$ can be augmented with labels **P** or **O** on the one hand, and by the force symbols *?* and *!* on the other, so as to yield the strings

P-!-*e*, **O**-!-*e*, **P**-?-*e*, **O**-?-*e*

These strings are said to be (*dialogically*) *signed expressions*. Their role is to signify that in the course of a dialogue, the move corresponding to the expression *e* is to be made by **P** or **O**, respectively, and that the move is made as a defence (*!*) or an attack (*?*). We will use **X** and **Y** as variables for **P** and **O**, always assuming X≠**Y**.

Particle rules

An *argumentation form* or *particle rule* is an abstract description of the way a formula, according to its outmost form, can be criticized, and how to answer the critique. It is abstract in the sense that this description can be carried out without reference to a specified context. In dialogical logic, these rules are said to state the *local semantics*, for they show how the game runs locally: what is at stake is only the critique and the answer corresponding to a given logical constant, rather than the whole context where the logical constant is embedded.[80] The particle rules fix the dialogical semantics of the logical constants of $\mathsf{L}[\tau]$ in the following way:

Assertion	Attack	Defence

[78] The present appendix is based on the first 6 pages of Rahman 2009b.

[79] This version is esentially from Rahman/Tulenheimo [2006]. For somewhat different accounts, see Rahman \Keiff 2005, Fontaine/Redmond 2008, and Keiff 2009.)

[80] There can be no particle rule corresponding to atomic formulae. But it is possible to add a set of *Opponent*'s initial concessions to the particle rules. This is done in `material dialogues' (see Rahman/Tulenheimo [2006].

X-!-A∨B	**Y**-?-∨	**X**-!-A *or* **X**-!-B (the defender chooses)
X-!-A∧B	**Y**-?∧1 *or* **Y**-?∧2 (the challenger chooses)	**X**-!-A *respectively* **X**-!-B
X-!-A→B	**Y**-!-A	**X**-!-B
X-!-¬A	**Y**-!-A	⊗ No defence possible. Only a counterattack is available
X-!-∀*x*A	**Y**-?-∀*x*/*k* For any *k* available to **Y**	**X**-!-A[*x*/*k*] For any *k* chosen earlier by **Y**
X-!-∃*x*A	**Y**-?-∃	**X**-!-A[*x*/*k*] For any *k* available to **Y**

In the diagram, A[*x*/*k*] stands for the result of substituting the constant *k* for every occurrence of the variable *x* in the formula A.

A more thorough way to stress the sense in which the particle rules determine local semantics is to see these rules as defining the notion of *state* of a (structurally not yet determined) game.

DEFINITION [*State of a dialogue*]:
Let A be a formula of **FO**[τ], and let a countable set $\{k_0, k_1, \ldots\}$ of individual constants be fixed. A *state of the dialogue* $\mathcal{D}(A)$ corresponding to the formula A is a quintuple

<B, X, Υ, *e*, σ> such that:

- B is a (proper or improper) subformula of A.
- X,-Υ-*e* is a dialogically signed expression. Thus, X is either **O** or **P** and , $\Upsilon \in \{?, !\}$, and $e \in \mathcal{L}[\tau]$.
- σ : Free[*B*] ξ $\{k_0, k_1, \ldots\}$ is a function mapping the free variables of B to individual constants.

➢ *The component e is either a formula of* **FO**[τ*], or an attack marker. We stipulate that in the former case, always e:*=B

Given a force Υ,, let us write Υ', for the opposite force, i.e. let
$\Upsilon' \in \{?, !\} \setminus \Upsilon$.

Each state <B, X, Υ, *e*, σ> has an associated *role* assignment, indicating which player occupies the role of *challenger* and which the role of defender. In fact, the role assignment is a function ρ : {**P**, **O**} ξ {?, !} such that ρ(**X**) = $\Upsilon$and ρ(**Y**) = Υ''.

We say that the state $<B_2, X_2, \Upsilon_2, e_2, \sigma_2>$ is *reachable* from state $<B_1, X_1, \Upsilon_1, e_1, \sigma_1>$ if it is a result of X_1 making a move in accordance with the appropriate particle rule in the role Υ_1. If the role is that of *challenger* (Υ_1=?), the player states an attack, whereas if the role is that of *defender* (Υ_1=!), the player poses a defence.

Let us take a closer look at the transitions from one state to another. Particle rules determine which state S_2 of a dialogue is reachable from a given other state S_1. Notice that the player who defends need not be the same at both states. In order for state S_2 to be reachable from state S_1=<B, X, Υ, *e*, σ>, it must satisfy the following.

- *Particle rule for negation*: If B=*e*, Υ= ! and B is of the form ¬C, then

  S_2=<C, Y, !, C, σ>

  So if **P** is *defender* of ¬C at S_1, then **O** is *defender* of C at S_2, and **P** will challenge (counterattack) C; and dually, if **P** is *challenger* of ¬C at S_1.

Notice that here state S_2 involves the claim that C can be defended; however, this claim has been asserted in the course of an attack, and the whole move from S_1 to S_2 counts as an attack on the initial negated formula, i.e. an attack on C. Actually this follows from the fact that at S_2, the roles of the players are inverted as compared with S_1. Counterattack may yield from S_2 a further state, S_3= <C, X, ?, *, σ>, where C is the formula considered, and the attack pertains to the relevant logical constant of C, for which * is a suitable attack marker determined by the logical form of C.

- *Particle rule for conjunction*:} If B=*e*, Υ= ! and B is of the form C∧D, then

  S_2=<C, X, !, C, σ> or S_2=<D, X, !, D, σ>

  according to the choice of the *challenger* between the attacks ?-L and ?-R. (Here the challenger is Y: Y's role is ? here.)

- *Particle rule for disjunction*: If B=*e*, Υ= ! and B is of the form C∨D, then

  S_2=<C, X, !, C, σ> or S_2=<D, X, !, D, σ>

according to the choice of the *defender*, reacting to the attack ?- ∨ of the challenger. (Here the defender is X: X's role is ! here.)

- *Particle rule for conditional*: If B=*e*, Υ= ! and B is of the form C→D, then

  S_2=<C, Y, !, C, σ>

and, further, state

S_3=<D, X, !, D, σ>

is reachable from S_2. So if **P** is the *defender* of C→D at S_1, and hence **O** is the *defender* of C at S_2, it is **P** who will be the *defender* of D at S_3.

To attack a conditional amounts to being prepared to defend its antecedent, and so it should be noticed that the defence of C at state S_2 counts as an attack. If **P** is the *defender* of C→D at S_1, then at state S_3 reachable from S_2, either **P** may defend D, or else **P** may counterattack C, thus yielding a further state, S_4=<C, X, ?, *, σ>, where C is the formula considered, and the attack pertains to the relevant logical constant of C, for which * is a suitable attack marker determined by the logical form of C.

- *Particle rule for universal quantifier*: If B=*e*, Υ= ! and B is of the form $\forall xDx$, then

  S_2=<*Dx*, X, !, *Dx*, $\sigma[x/k_i]$>

where k_i is the constant chosen by the *challenger* (who here is Y) as a response to the attack ?-$\forall x/k_i$.

As usual, the notation '$\sigma[x/k_i]$' stands for the function that is otherwise like σ, but maps the variable x to k_i.Hence if σ is already defined on x, $\sigma[x/k_i]$ is the result of reinterpreting x by k_i otherwise it is the result of extending σ by the pair (x, k_i)

- *Particle rule for existential quantifier*: If B=*e*, Υ= ! and B is of the form $\exists xDx$, then

  S_2=<*Dx*, X, !, *Dx*, $\sigma[x/k_i]$>

where k_i is the constant chosen by the *defender* (who here is X) reacting to the attack ?-$\exists x$ of the *challenger* (Y).

Structural rules

As we analyze dialogues, we will make use of the following notions: *dialogue, dialogical game,* and *play of a dialogue.* It is very important to keep them conceptually distinct. Dialogical games are sequences of dialogically signed expressions, i.e. expressions of the language L[τ] equipped with a pair of labels, **P**-!, **O**-!, **P**-?, or **O**-?. The labels carry information about how the dialogue proceeds. Dialogical games are a special case of plays: all dialogical games are plays, but not all plays are dialogical games. However, all plays *are* sequences of dialogical games. Finally, dialogues are simply sets of plays.

A complete dialogue is determined by game rules. They specify how dialogical games in particular, and plays of dialogues in general, are generated from the thesis of the dialogue. Particle rules are among the game rules, but in addition to them there are so-called *structural rules*, which serve to specify the general organization of the dialogue.

Different types of dialogues have different kinds of structural rules. *When the issue is to test validity* - as it is for the dialogues considered in the present paper - a dialogue can be thought of as a tree, whose (maximal) branches are (finished) plays relevant for establishing the validity of the thesis. The structural rules will be chosen so that *Proponent* succeeds in defending the thesis against all allowed critique of *Opponent* if, and only if, the thesis is valid in the standard sense of the term (`true in every model'). In dialogical logic the existence of such a winning strategy for *Proponent* is typically taken as the definition of validity; however, this dialogical definition indeed captures the standard notion (see the discussion in connection with the definition of validity below).

Each split into two branches - into two plays - in a dialogue tree should be considered as the outcome of a propositional choice made by *Opponent*. Any choice by **O** in defending a disjunction, attacking a conjunction, and reacting to an attack against a conditional, gives rise to a new branch: a new play. By contrast, *Proponent*'s choices do not generate new plays; and neither do *Opponent*'s choices for quantifiers (defending an existential quantifier, attacking a universal quantifier).

The participants **P** and **O** of the dialogues that we are here interested in - the dialogues used for characterizing validity - are of course idealized agents. If real-life agents took their place, it might happen that one of the players was cognitively restricted to the point of following a strategy which would make him lose against some, or even every sequence of moves by the opponent - even if a winning strategy would be available to him. The idealized agents of the dialogues are not hence restricted: their `having a strategy' means simply that there exists, by combinatorial criteria, a certain kind of function; it does not mean that the agent possesses a strategy in any cognitive sense.

Plays of a dialogue are sequences of dialogically signed expressions, and they share their first member, the *thesis* of the dialogue. In particular, plays can always be analyzed into dialogical games: any play is of the form $\Delta_1 \ldots \Delta_n$, where the Δ_1 are dialogical games (i := 1, … ,n). The members of plays other than the thesis are termed *moves*. A move is either an attack or a defence.
The particle rules stipulate exactly which moves are to be counted as attacks. Exactly those moves X-γ-*e* whose expression component *e* is a first-order formula, are said to have *propositional content*. Recall that in the case of conditional and negation some moves with propositional content count as attacks. (In the actual design of a dialogue there usually is a notational device to differentiate between those moves with propositional content that are attacks and those that are not.)

We move on to introduce a number of structural rules for dialogues designed for the language L[τ]. We will write $\boldsymbol{D}$(A) for the dialogue about A, i.e. the dialogue whose thesis is A. Further, we will write Δ[n] for the member of the sequence Δ with the position n. Let A be a first-order sentence of vocabulary τ. We have the following structural rules (SR-0) to (SR-6) regulating plays τ in $\Delta \in \boldsymbol{D}$(A), i.e. members of the dialogue $\boldsymbol{D}$(A)

(**SR-0**) (**Starting rule**)

a) The dialogically signed expression <**P**-!-A> belongs to the dialogue $\boldsymbol{D}$(A): the thesis A stated by *Proponent* is itself a play in the dialogue about A.

b) If Δ is any play in the dialogue $\boldsymbol{D}$(A), then the thesis A has position 0 in Δ. If $\Delta \in \boldsymbol{D}$(A), then

   Δ[0]= <**P**-!-A>.

c) At even positions **P** makes a move, and at odd positions it is **O** who moves. That is, each Δ[2n] is of the form <**P**-γ-B> for some $\gamma \in$ {?, !} and B∈ Sub(A); and each Δ[2n+1] is similarly of the form <**O**-γ-B>. Every move after Δ[0] is a reaction to an earlier move made by the other player, and is subject to the particle rules and the other structural rules.

(**SR-1.I**) (**Intuitionistic round closing rule**).
Whenever player X has a turn to move, he may attack any (complex) formula asserted by his opponent, Y, or he may defend himself against the *last not already defended attack* (i.e. the attack by Y with the greatest associated natural number such that X has not yet responded to that attack).

A player may postpone defending himself as long as he can perform attacks. Only the latest attack that has not yet received a response may be answered: If it is X's turn to move at position n, and positions l and m both involve an unanswered attack (l<m<n), then player X may *not* at position n defend himself against the attack of position l.

(**SR-1.C**) (**Classical round closing rule**)
Whenever player X has turn to move, he may attack any (complex) formula asserted by his opponent, Y, or he may defend himself against any attack, including those which have already been defended. That is, here even redoing earlier defences is allowed.

(**SR-2**) (**Branching rule for plays**)
If in a play $\Delta \in \boldsymbol{D}$(A) it is **O**'s turn to make a propositional choice, that is, to defend a disjunction, attack a conjunction, or react to an attack against a conditional, then Δ extends into two plays $\Delta_1, \Delta_2 \in \boldsymbol{D}$(A),[81]

$\Delta_1 = \Delta^{\frown}\alpha$ and $\Delta_2 = \Delta^{\frown}\beta$

differing in the chosen disjunct, conjunct *resp.* reaction, α vs. β. More precisely: Let {n≤max{m : Δ[m]}.

- If Δ[n]= <**O**-!-B∨C> and Δ[max] = <**P**-?-∨>, then

  α := <**O**-!-B> and β := <**O**-!-C>.

- If Δ[n]= Δ[max]=<**P**-!-B∧C>, then

  α := <**O**-!-L> and β := <**O**-!-R>.

- If Δ[n]= <**O**-!-B→C> and Δ[max] = <**P**-?-B>, then

  α := <**O**-?-*> and β := <**O**-!-C>.

  where * is an attack marker corresponding to the logical form of the formula B.

No moves other than propositional moves made by **O** will trigger branching.

(**SR-3**) (**Shifting rule**)
When playing a dialogue $\boldsymbol{D}$(A), **O** is allowed to switch between 'alternative' plays Δ, $\Delta' \in \boldsymbol{D}$(A). More exactly, if **O** loses a play Δ, and Δ involves a propositional choice made by **O**, then **O** is allowed to continue by switching to another play - existing by the *Branching rule*

[81]) If $\bar{a}=(a_0, \ldots, a_n)$ is a finite sequence and a_{n+1} is an object, $\bar{a}^{\frown}a_{n+1}$ is by definition the sequence $(a_0, \ldots, a_n, a_{n+1})$. If $\bar{a}=\bar{a}_1{}^{\frown}\bar{a}_2$, then, $\bar{a}_1$ is said to be an initial segment of $\bar{a}$, and, if the sequence $\bar{a}_2$ is not empty, then we say that $\bar{a}_1$ is the initial proper segment of $\bar{a}$.

(SR-2). Concretely this means that the sequence $\Delta^\frown\Delta'$ will, then, be a play, i.e. an element of $\boldsymbol{D}$(A).

It is precisely the *Shifting rule* that introduces plays which are not plain dialogical games. (Dialogical games are a special case of plays: they are identified with unit sequences of dialogical games.)
As an example of applying the *Shifting rule*, consider a dialogue $\boldsymbol{D}$(A) proceeding from the hypotheses (or *initial concessions* of *O*) B, ¬C, with the thesis A := B∧C. If **O** decides to attack the left conjunct, the result will be the play

(<**P**-!-B∧C>, <**O**-?-L>, <**P**-!-B>)

and **O** will lose (because he has already conceded B). But then, by the *Shifting rule*, **O** may decide to do have another try. This time he wishes to choose the right conjunct. The result is the play

(<**P**-!-B∧C>, <**O**-?-L>, <**P**-!-B>, <**P**-!-B∧C>, <**O**-?-R>, <**P**-!-C>)

Observe that this play consists of two dialogical games, namely
(<**P**-!-B∧C>, <**O**-?-L>, <**P**-!-B>) and <**P**-!-B∧C>, <**O**-?-R>, <**P**-!-C>

By contrast, this play is not itself a dialogical game.

(**SR-4**) (**Winning rule for plays**)
A play $\Delta\in\boldsymbol{D}$(A) is *closed*, if $\Delta=(\Delta_1, \ldots, \Delta_n)$, where the Δ_i are dialogical games, and in the most recent dialogical game Δ_n there appears the same positive literal in two positions, one stated by X and the other one by Y. That is, Δ is closed if for some k, m<ω and some positive literal $\ell\in$ Sub(A)∪{A}, we have:

$$\Delta_n[k]=\ell=\Delta_n[m]$$

where k<m and furthermore, k is odd if, and only if m is even or equal to zero. If this condition is not satisfied, Δ is *open*.

If a play is closed, the player who stated the thesis (that is, **P**) *wins* the play; otherwise he loses it. A play is *finished*, if it is either closed, or else such that no further move is allowed by the particle rules or (other) structural rules. If a play is finished and open, **O** wins the play. Observe that whenever a play $\Delta\in\boldsymbol{D}$(A) is finished, there is no further play $\Delta'\in\boldsymbol{D}$(A) such that Δ is an initial segment of Δ'.

(**SR-5**) (**Formal use of atomic formulae**)
P cannot introduce positive literals: any positive literal must be stated by **O** first. Positive literals cannot be attacked.

In the following, when introducing material dialogues we will consider too, when speaking of First-order logic, intuitionistic dialogues with additional hypotheses introduced as *initial concessions* by **O**, such as:

$$\forall x_1 \ldots \forall x_n(Rx_1 \ldots x_n \vee \neg Rx_1 \ldots x_n)$$

where *R* is a relation symbol of a fixed vocabulary τ. That is, the relevant hypotheses are instances of (a universal closure of) *tertium non datur*. In the presence of such hypotheses, we may use a more general formulation of the rule (**SR-5**):

(**SR-5***)
P cannot introduce literals: any literal (positive or not) must be stated by **O** first. Positive literals cannot be attacked.

Before we can state the structural rule (SR-6), or the *No delaying tactics rule*}, we need some definitions.

DEFINITION [**Strict repetition of an attack / a defence**]

a) *We speak of a **strict repetition of an attack**, if a move is being attacked although the same move has already been challenged with the same attack before. (Notice that even though choosing the same constant is a strict repetition, the choice of ?-L and ?-R are in this context different attacks.)*

   In the case of moves where a universal quantifier has been attacked with a new constant, the following type of move must be added to the list of strict repetitions:

   1. ***A universal quantifier move is being attacked using a new constant, although the same move has already been attacked before with a constant which was new at the time of that attack.***

   2. ***A universal quantifier move is being attacked using a constant that is not new, although the same move has already been attacked before with the same constant.***

b) *We speak of a **strict repetition of a defence**, if a challenging move (attack) m_1, which has already been defended with the defensive move (defence) m_2 before, is being defended against the challenge m_1 once more with the same defensive move. (Notice that the left part and the right part of a disjunction are in this context two different defences.)*

c) *In the case of moves where an existential quantifier has been defended with a new constant, the following type of move must be added to the list of strict repetitions:*

   1. ***An attack on an existential quantifier is being defended using a new constant, although the same quantifier has already been defended before with a constant which was new at the time.***

   2. ***An attack on an existential quantifier is being defended using a constant that is not new, although the same quantifier has already been defended before with the same constant***

- *Notice that according to these definitions, neither a new defence of an existential quantifier, nor a new attack on a universal quantifier, represents a strict repetition, if it uses a constant that is not new but is however different from the one used in the first defence (or in the first attack) that was new.*

(**SR-6**) (`**No delaying tactics' rule**) This rule has two variants, classical and intuitionistic, depending on whether the dialogue is played with the classical structural rule (SR-1.C), or with the intuitionistic structural rule (SR-1.I).

Classical: No ***strict*** repetitions are allowed.

Intuitionistic: If **O** has introduced a new atomic formula which can now be used by **P**, then **P** may perform a repetition of an attack. No other ***strict*** repetitions are allowed.

DEFINITION [**Validity**] *A first-order sentence A is said to be* ***dialogically valid*** *in the classical (intuitionistic) sense, if all plays belonging to the classical (resp. intuitionistic) dialogue* $\mathcal{D}(A)$ *are closed.*

It is possible to prove that the dialogical definition of validity coincides with the standard definition, both in the classical and in the intuitionistic case. First formulations of the proof were developed in the PhD-Thesis by Kuno Lorenz (reprinted in Lorenzen/Lorenz 1978), Haas (1980) and Felscher (1985) proved the equivalence for intuitionistic First-order logic (by proving the correspondence between intuitionistic dialogues and intuitionistic sequent calculi); while Stegmüller (1964) established the equivalence in the case of classical First-order logic. Rahman (1994: 88-107), who stressed the idea that dialogues for validity could be seen as a proof-theoretical frame to build tableaux systems, proved directly the equivalence between the two types of dialogues and the corresponding semantic tableaux, from which the result extends to the corresponding sequent calculi.

Philosophical remarks: propositions as games.
Particle rules determine dynamically how to extend a set of expressions from an initial assertion. In the game perspective, one of the more important features of these rules is that they determine, whenever there is a choice to be made, who will choose. This is what can be called the pragmatic dimension of the dialogical semantics for the logical constants. Indeed, the particle rules can be seen as a proto-semantics, i.e. a game scheme for a not yet determined game which when completed with the appropriate structural rules will render the game semantics, which in turn will build the notion of validity.
Actually by means of the particle rules games have been assigned to sentences (that is, to formulæ). But sentences are not games, so what is the nature of that assignment? The games associated to sentences are meant to be *propositions* (i.e. the constructions grasped by the (logical) language speakers). What is connected by logical connectives are not sentences but propositions. Moreover, in the dialogic, logical operators do not form sentences from simpler sentences, but games from simpler games. To explain a complex game, given the explanation of the simpler games (out) of which it is formed, is to add a rule which tells how to form new games from games already known: if we have the games *A* and *B*, the conjunction rule shows how we can form the game $A \wedge B$ in order to assert this conjunction.
Now, particle rules have another important function: they not only set the basis of the semantics, and signalise how it could be related to the world of games – which is an outdoor world if the games are assigned to prime formulæ, but they also show how to perform the relation between sentences and propositions. Sentences are related to propositions by means of assertions, the content of which are propositions. Assertions are propositions endowed with a theory of force, which places logic in the realm of linguistic actions. The forces performing this connection between sentences and propositions are precisely the attack (?) and the defence (!). An attack is a demand for an assertion to be uttered. A defence is a response (to

an attack) by acting so that you may utter the assertion (e.g. that *A*). Actually the assertion force is also assumed: utter the assertion that *A* only if you know how to win the game *A*.
Certainly the "know" introduces an epistemic moment, typical of assertions made by means of judgements. But it does not presuppose in principle the quality of knowledge required. The constructivist moment is only required if the epistemic notion is connected to a tight conception of what means that the player X *knows that there exists* a winning game or strategy for *A*.

Let us take examples of dialogues, classical and intuitionistic.

EXAMPLE: Consider the classical dialogue $\boldsymbol{D}(p\vee\neg p)$. Its thesis is $p\vee\neg p$, where p is an atomic sentence. In Figure 1, a dialogical game from dialogue $\boldsymbol{D}(p\vee\neg p)$ is described. This dialogical game is won by **P**:

O			**P**		
				$p\vee\neg p$	0
1	?-$\vee$	0		$\neg p$	2
3	P	2		—	
[1]	[?-$\vee$]	[0]		p	4

II.1.3.f1. Classical rules, **P** wins.

The outer columns indicate the position of the move inside the dialogical game, while the inner columns state the position of the earlier move which is being attacked. The defence is written on the same line with the corresponding attack: an attack together with the corresponding defence constitutes a so-called *closed round*. The sign '— ' indicates that there is no possible defence against an attack on a negation.

In the dialogical game of the example, **P** wins because after **O**'s last attack in move 3, **P** is allowed - according to the classical rule SR-1.C - to defend (once more) himself against **O**'s attack made in move 1, which was certainly *not the last attack* of **O**, and so the game in question is closed. **P** states his new defence in move 4. (Actually **O** does not repeat his attack of move 1: what we have written between square brackets simply serves to remind of the attack against which **P** is re-acting.)

- In fact the described dialogical game is the only finished play of the dialogue $\boldsymbol{D}(p\vee\neg p)$: **O** could not prolong the play any further by making different moves. Hence not only does **P** win the described particular dialogical game - in fact he has a winning strategy in the dialogue, i.e. he is able to win no matter what **O** does. In other words, the sentence $p\vee\neg p$ is dialogically valid in the classical sense (cf. Definition *Def:validity*).

Here is an example concerning Peirce's Law and which requires to consider two plays:

EXAMPLE:
In the version of strategy dialogues what actually happens is that **O** generates two dialogical games one defending and the other counterattacking. Both dwill be closed and thus won by **P**.

O			**P**		
				$((p\to q)\to p)\to p$	0
1	$(p\to q)\to p$	0		p	4
			1	$p\to q$	2
2	p	2			

II.1.3.f2. Classical rules, **P** wins.

O			**P**		
				$((p\to q)\to p)\to p$	0
1	$(p\to q)\to p$	0		p	4
3	p		1	$p\to q$	2

II.1.3.f3 Classical rules, **P** wins.

Actually this produces a play with two dialogical games. Let us label each dialogical game with a roman letter and put all in only one graphic. In the graphic below we splitted the play in two showing the dialogical games produced - simpler would be to eliminate the outer columns and add the label directly to the formulae, but this notation will make it easier to show the relation to (the branches produced by a correspondent) sequent calculi.
The expression between the signs '<' and '>' signalise that the Opponent has decided in the choice I.2 not to counterattack the expession inside those signs but defend himself. This expression is then not at stake in the play I.2 and can be considered as an *attack marker* rather than a formula
Because of the tree-like structure of the proof we will assume that the thesis (move 0),t he first challenge of **O** (move 1), *which occur before the splitting takes place*, and the answer (move 4) are shared by both dialgical games and will neither repeat them:

O			**P**		
				$((p\to q)\to p)\to p$	0
1	$(p\to q)\to p$	0		p	I.4 | II.4
I.3 |	p |		1	$<p\to q>$ | $p\to q$	I.2 | II.2
| II.3	| p	II.2			

II.1.3.f4 Classical rules, **P** wins.

Let us consider now the intuitionistic variant of the dialogue of the first example.

EXAMPLE: In figure below, a dialogical game from the intuitionistic dialogue $\boldsymbol{D}(p\vee\neg p)$ is described. This game is won by **O**:

O			**P**		
				$p\vee\neg p$	0
1	?-$\vee$	0		$\neg p$	2
3	p	2		—	

II.1.3.f5. Intuitionistic rules, **O**wins.

It is **O** who wins the dialogical game of the example: the game is open, and no further move is possible following the intuitionistic structural rules. In particular remaking an earlier move (i.e., answering to an attack which was *not the last one* - as in the above example of a classical dialogue - is not possible.

In fact **O** has trivially a winning strategy in the intuitionistic dialogue $\boldsymbol{D}(p\vee\neg p)$: **P** cannot prevent, by making different moves, **O** from generating precisely the described play won by **O**.

- Observe, in particular, that the sentence $p\vee\neg p$ is not dialogically valid in the intuitionistic sense. (This does not mean, of course, that thereby the sentence $\neg(p\vee\neg p)$ would be intuitionistically valid!)

The following example shows the fail of double negation in intuitionistic logic

EXAMPLE
$\boldsymbol{D}\neg\neg p\to p$

O			**P**		
				$\neg\neg p\to p$	0
1	$\neg\neg p$	0			
	—		1	$\neg p$	2
3	p	2		—	

II.1.3.f6. Intuitionistic rules, **O** wins.

O wins because **P** is not allowed to use the atomic formula stated by **O** at move 3 to defend the challenge of move 1. Indeed, move *3 is the last attack* of **O** and **P** must answer now to this attack. Unfortunately, by the particle rule of negation, there is no defence to challenged negation. Only counterattacks are possible. But p is an atomic formula which cannot be counterattacked!

To come back to a success story for **P** let us see a more trickier case. Namely, an intuitionistic dialogue for $\boldsymbol{D}(\neg\neg(p\vee\neg p))$ where **P** should have a winning strategy. Indeed, the double negation of any valid classical formula is valid intuitionically too!

EXAMPLE
$\boldsymbol{D}(\neg\neg(p\vee\neg p))$

O			**P**		
				$\neg\neg(p\vee\neg p)$	0
1	$\neg(p\vee\neg p)$	0		—	
	—		1	$p\vee\neg p$	2
3	?-$\vee$	2		$\neg p$	4
5	p	4		—	
	—		1	$p\vee\neg p$	6
7	?-$\vee$	6		p	8

II.1.3.f7. Intuitionistic rules, **P**wins.

The tricky point is move 6 where **P** is allowed to repeat the attack on the first move of **O** because since move 1, **O** introduced a new atomic formula (see SR-6)S. Indeed at move 5 **O** introduced the positive literal *p* and this can be now used to defend the new occurrence of the disjunction.

The way to build a winning strategy for dialogues for first-order logic is not really different from the propositional case: Here the Proponent will try to wait so long as he can before choosing a value for the variables. More precisely, he will wait until the Opponent has chosen first the value for the variables at stake and later on he will simply *copy-cat* them.Let us show examples of dialogues for first order logic:

EXAMPLE
$\boldsymbol{D}((\forall x((Ax\vee Bx)\wedge\neg Ax)))\rightarrow\forall x\,(\neg\neg Bx\vee Cx))$

O			**P**		
				$(\forall x((Ax\vee Bx)\wedge\neg Ax)))\rightarrow\forall x\,(\neg\neg Bx\vee Cx)$	0
1	$\forall x((Ax\vee Bx)\wedge\neg Ax))$	0		?-$\forall x(\neg\neg Bx\vee Cx)$	2
3	?-$\forall x/k$	2		$\neg\neg B_k\vee C_k$	4
5	?-$\vee$	4			
7	$(A_k\vee B_k)\wedge\neg A_k$		1	?-$\forall x/k$	6
9	$A_k\vee B$	6	5	?-L	8
11	A_k		9	?-$_\vee$	10
13	$\neg A_k$		7	?-R	12
	—		13	Ak	14

II.1.3.f7. Classical rules, **P** wins.

Validity on frames: the dialogical approach to modal logic

Introduction to modal dialogic

Modal dialogic is a systematic account of an explicit notion of context, in the sense that the latter is introduced by an explicit label. Modal moves are hence dialogical expressions with a supplementary label, indicating the context in which the move has been made. The usual modal operators are then defined in the following way:

□, ◊	*Attack*	*Defence*
X □*A* **i** (□*A* has been stated by player **X** at context **i**)	**Y** ? $_{j\,(iR^{D}j)}$ **i** (at the context **i** the challenger **Y** attacks by choosing a dialogically accessible context **j**)	**X** *A* **j** (the defender claims that *A* holds at the label **j**)
X ◊*A* **i** (◊*A* has been stated by player **X** at context **i**)	**Y**?◊ **i** (at the context i the challenger **Y** attacks asking **X** to choose a **j** where *A* holds)	**X***A* **j**$_{(iR^{D}j)}$ (the defender chooses the context **j** such that **j** is dialogically accessible from **i**)

V.3.t1

[add states of the game for modal dialogic]

In modal dialogic the frame conditions implemented as special structural rules which allow the Proponent to increase his choice possibilities while challenging a necessity operator or defending a possibility operator.

DEFINITION
If at **i** the Opponent while challenging a necessity operator of defending a possibility operator chooses a new label **j** such that **i** is a proper initial segment of **j** we say that the Opponent has *introduced* **j** and *conceded* that the label **j** is *dialogically accessible* from the label **i** (for short **$iR^{D}j$):**

□, ◊	*Attack*	*Defence*
P □*A* **i** (□*A* has been stated by **P** at context i)	**O** ? $_{j\,(iR^{D}j)}$ **i** (at the context **i O** ***introduces*** j and ***concede***s $iR^{D}j$))	**P** *A* j
O ◊*A* **i** (◊*A* has been stated by **O** at context **i**)	**P**?◊ i (at the context **i P** asks **O** to choose a **j** where *A* holds)	**O***A* j $_{(iR^{D}j)}$ (at the context **i O** ***introduces*** **j** and ***concede***s $iR^{D}j$)

V.3.t2

MODAL FORMAL RULE
At label **i** the Proponent may choose a label **j** such that **$iR^{D}j$** iff **j** has been introduced by the Opponent before or this choice has been allowed by the appropriate modal structural rule.

□, ◊	*Attack*	*Defence*
O □*A* **i** (□*A* has been stated by **O** at context i)	**P** ? $_{j\,(iR^{D}j)}$ **i** at the context i **P** ***chooses*** a **j** $iR^{D}j$ such that **j** has been introduced by **O** or has been allowed by the appropriate structural rule)	**O** *A* j
P ◊*A* **i** (◊*A* has been stated by **P** at context **i**)	**O**?◊ **i** (at the context **i O** asks **P** to choose a **j** where *A* holds)	**P***A* j $_{(iR^{D}j)}$ (at the context i **P** ***chooses*** a **j** $iR^{D}j$ such that **j** has been introduced by **O** or has been allowed by the appropriate structural rule)

V.3.t3

LOGIC	**MODAL STRUCTURAL RULES**
K	No conditions
D	The Proponent may choose a label i though it has NOT been chosen by the Opponent before.
T	Assume that **P** is at i. **P** may then choose i.
B	Assume that **P** is at i.n. **P** may then choose i.n and **P** may then choose also i (i.n is the immediate extension of i).
K4	Assume that **P** is at i. **P** may then choose a j such that i is an initial segment of j.
S4	Assume that **P** is at i. **P** may then choose i and he may choose j such that i is an initial segment of j . For short an formulating both conditions at once: **P** may choose a j such that i is an initial segment (proper or otherwise) of j.
S4.3	For the reflexive and transitive cases take the structural rules for **S4**-frames. For the linear condition: Assume that **P** is at i and also assume that **O** conceded that $iR^{D}j$ and $iR^{D}k$. **P** may then ask **O** to choose between conceding either $jR^{D}k$ or $kR^{D}j$.
K5	Assume that **P** is at i.j **P** may then choose at that context i.k and i.j.
S5.	Take the structural rules for the reflexive-, symmetric- and transitive-frames. For short: Assume that **P** is at i. Then **P** can choose any (already introduced) label j (even j=i).

Dialogues for first order modal logic: varying and constant domains

Dialogues for Constant Domains:

Dialogues for constant domains are quite simple to develop: One just uses classical first-order logic without restrictions on the Proponent -choices of constants.

Actually, for reasons that will be clear below, we will assume that dialogues for first-order logic are defined with the help of two disjoint sets of free variables called the set ***K*** of k-terms, and the set ***P*** called parameters, that will never be bounded by quantifiers. The particle rules are defined in the union of both sets, but strategically, while proving validity, we will assume that the opponent will always chose (when he can), new elements of ***P*** and the proponent will make his choices in the union of both sets.

Dialogues for Varying Domains:

For dialogues corresponding to globally varying domain semantics within the free logic approach there is one change needed: we introduce a whole family of lists of parameters, one for each label, rather than a single list of parameters. More specifically, we assume that to each context-label i there is associated an infinite list of parameters, in such a way that different context-labels never have the same parameter associated with them.

Thus, we write *p*i.*i* to indicate that *p* is a parameter associated with *wi.i*

Actualist Structural Rule for globally varying domains: the free logic approach

- Choices for quantifiers stated at *wi* by any player have to be chosen from the set of ***P***i of parameters associated to *wi*.

Note that the quantifiers are understood as actualist: That is, the range of their variables does neither go beyond the context *w* where these quantifiers have been stated nor does it extend to the set ***K***. . Thus, neither

$\forall xAx \rightarrow Ak$, nor

$Ak \rightarrow \exists xAx$

will be valid. Indeed any dialogue starting with them will force the Proponent to choose a parameter and thus he will not be able to produce the required atomic formula *Ak*.

Exercise:
Prove as an exercise that neither the Barcan- nor the converse Barcan-formulae are valid

Actualist Structural Rule for monotonic varying domains: the free logic approach

- Choices for quantifiers stated at *wi.i..* have to be chosen from the set of ***P****i.i...* or ***P****i...* where i is an initial fragment of i (that is, *wi..* R *wi.i...*).

Exercise:

Prove that

the converse Barcan-formulae are valid
the Barcan-fromulae are non valid
the free logic formulae are non valid

Take the following formulation

> The Proponent might choose pi at *wi* if, pi has been **already asserted** (by means of a challenge to a universal quantifier or of a defence to an existential quantifier) at *wi* or if it is completely new in the dialogue.

Does this correspond to one of the logics described above? Why?

Actualist Structural Rule for anti-monotonic varying domains: the free logic approach

- Choices for quantifiers stated at *wi*... have to be chosen from the set of ***P****i.i*... or ***P****i*... where i is an initial fragment of i.*i* (that is, *wi*.. R *wi.i*...).
  (Note that here that this is about the choices at *wi*.. not at *wi.i* as in the monotonic case)

Exercises

Prove that
the converse Barcan-formulae are non-valid
the Barcan-fromulae are valid
the free logic formulae are non valid
give a formulation for monotonic and anti-monotonic varying domains without free logic.

Soundness of first-order positive free logic K

When we say in metalogic that we prove that a given proof system is "sound" we mean that with this system we cannot prove any formula it should not. For example; if our dialogical proof system for **K** were not sound then we would be able to prove some formula, such as $\Box A \rightarrow A$, which is beyond of the frame validity characterizing **K**. More precisely, to say that our dialogical proof system is sound means: if a formula φ has a dialogical proof for **K**; then this formula is valid in the logic **K** as described by the model theoretical characterisation of **K**-validity.

To prove this we need some previous work:

- **Dialogical tree**: For the sake of simplicity we will assume a tree-like rewriting of the dialogues. That is, instead of the notation

O			**P**		
				iX	0
1	*iV*			*i.nZ*	4
3	*i.nY*			*i.W*	2

We will write

0 i**P***X*
1 i**O***V*
2 i**P***W*
3 i.n**O***Y*
4 i.n**P***Z*

- If the application of the appropriate rule forces **P** to state an atomic formula in the dialogue and he can not, then we will anyway write them down but between square brackets.
  E.g. if the formula to be extended is i(**P***pvq*), and **O** did not assert before neither i*p* nor i*q*, then we will extend the branch in the following way

  i[**P***p*]
  i[**P***q*]

  The square brackets should indicate that though **P** can not play them in the dialogue, they will obtain a valuation in the model (see below) , namely

  $v_{f(i)} \neg p = 1$ and $v_{f(i)} \neg q = 1$

  If the Proponent answers with a box such as [**P***a*], he can play a further move. Indeed, as mentioned before, the box means the Proponent can not really play that move in the standard dialogue.

  n *i**O*□ *b*
  n+1 *i**P*□ *a*
  n+2o *i.**O?** (n+1) i.1*□ (the Opponent challenges line n+1 choosing *i.1)*
  n+3 *[i.1**P**a]*
  since the move *n*+3 is a box the Proponent may move
  n+4 *i.**P?** (n) i1.1*□
  n+5 *i.1**O**b*

  If the branch is closed with an atomic formula, say *a*, that has been played by the Opponent after the move [**P***a*], then we will assume a further move where the Proponent opens the box, though this moves will not have a new move-number: it is the move closing the branch.

  Furthermore in the cases of the connectives where the Proponent has a choice (defending a disjunction and when challenging a conjunction), both choices will be played without delay one after the other

Notice that dialogical trees are not Beth(Smullyan)-Tableaux though the resulting proof might be indistinguishable from those Tableaux . Indeed the semantics of a logical constant in a Tableau-System is defined by the **T** (**O**)-rules and the **F** (**P**)-rules, but the semantics of a logical constant in a dialogue is primarily defined by the so-called particle rules. Particle-rules should be player independent: the difference between **T** (**O**)-rules and the **F** (**P**)-rules is a

result of the strategical level. This makes it that we could formulate tonk-particles for Tableaux but not for dialogues. See exercise 2 at the end of the soundness proof.

Definition 7 [Satisfiable in varying domain models]:

Let us consider a set ***S*** of signed (and labelled) formulae, where members of ***S*** may contain parameters and k-terms. We say that ***S*** is satisfiable in the varying domain model ***M***=: <*W*, R, *D, i*> with respect to an assignment ***g*** if there is a mapping ***f*** assigning to each label *i* of a formula in the branch some possible world ***f*** (*i*) in *W* such that:

1. If i and *i*.n both occur as labels in ***S***, then ***f***(*i*)R ***f***(*i*.n) in ***M***.
2. If i**X***A* is in S, (where **X** signalises that the formula is **O**- or **P**-signed), then $v_{f(i)*g}(A)=1$ in ***M***. In words *A* is true at the world ***f***(*i*) of the model ***M*** with respect to the assignment ***g*** – where:
   $v_{f(i)*g}(\mathbf{O}A)= v_{f(i)*g}(A) =1$,
   $v_{f(i)*g}(\mathbf{P}A)= v_{f(i)*g} \neg A =1$ and
   $v_{f(i)*g}([\mathbf{P}A])= v_{f(i)*g} \neg A =1$ (where *A* is atomic – see the comment in the chapter on the propositional case)
3. If the parameter *p*i occurs in ***S***, then $g_{f(i)}(pi) \in D(f(i))$
4. If a free variable *x* (including a k-term) **other** than a parameter occurs in ***S***, then $g_{f(i)}(x) \in D_F$

The rest carries over directly from the propositional case:

Since ***S*** has been restricted to a set of formulae the following kind of moves (of any player) will not be mapped into the model:

? $\vee$i,

?-$\wedge$i,

?-$\forall x/k$,

?$\exists$

- We say that a branch of a dialogue (produced by the shifting rule) is satisfiable if the set of labelled signed formulae on it is satisfiable in some model with respect to some valuation.
- We say that a dialogue (dialogical game) is satisfiable if some branch of it is satisfiable

Soundness lemma Q1 (SL-Q1):

A closed dialogue for varying domains (a dialogue won by **P**) is not satisfiable

PROOF:

- Suppose that we had a dialogue that was both closed and satisfiable.
- Since it is satisfiable, some branch of it is. Let S be the set of formulae on that branch and let it be satisfiable in the model ***M*** by means of the mapping ***f***.

- Since the dialogue is closed (won by **P**) then for some labelled atomic formula *A* we must have i **O***A* and i **P***A*. But then both $v_{f(i)}(A)=1$ and $v_{f(i)} \neg A=1$ must be the case in ***M*** but this is not possible.

Soundness lemma Q2 (SL-Q2):

If (a section of) a dialogue for varying domains is satisfiable and a branch (produced by the shifting rule) of that (section of) dialogue is extended by appropriate particle rules, the result is another satisfiable (section of) a dialogue.

PROOF:

Let ***D*** be a (section of a) satisfiable dialogue and let **B** be the branch that is extended.

The steps to be considered are exactly the same steps as before though we must add the quantifier cases.
T
Assume that ***S*** on **B** is satisfiable in the varying domain model ***M***=: <*W*, R, *D*, *i*> with respect to an assignment ***g*** and the mapping ***f***

Let us take the case of i**O**$\exists x\phi(x)$. If we apply the correspondent rule we will produce the branch **B1** containing the formulae:

i**O** $\phi(pi)$ ([i**O** $\phi(pi)$)]),

where *pi* is a parameter that is new to the branch

We must show that **B1**- that consists of the formulae of ***S*** and i**O**$\phi(pi)$ is still satisfiable

Since **B** is by hypothesis satisfiable in ***M*** with the mapping ***f*** and assignment ***g***, and i**O**$\exists x\phi(x)$. is on **B**, by definition **IX-7** we have

$v_{f(i)*g}(\mathbf{O}\exists x\phi(x)) = v_{f(i)*g}(\exists x\phi(x)) = 1$

Thus for some *x*-variant ***g'*** of ***g*** we have

$v_{f(i)*g'}(\phi(x)) = 1$

Define a new assignment ***g''*** in the following way: for each variable (including parameters),
g'' is the same as ***g*** in all variables except *x*.
g'' (*pi*)= ***g'***(*x*).

Notice that since ***g'*** is an *x*-variant at ***f***(*i*), then ***g'***(*x*) and ***g''***(*pi*) is in *D*(***f***(*i*).

Now, ***g''*** and ***g*** agree on all variables except *pi* and pi is new to the branch **B**, hence it does not occur in any labelled formula on **B**.

Since the set of labelled formulae on **B** was satisfiable in the model with respect to the assignment ***g***, this is also the case if we use the assignment ***g''*** (once more: **g** and g**''** differ only on pi but *pi* is not on the branch!) (see lemma IX.1)

Since ***g*** is an *x*-variant, ***g*** and ***g'*** agree on all variables except for x;
Recall that ***g''*** and ***g*** agree on all variables except *pi*
Therefore, ***g''*** and ***g'*** agree on variables except for *x* and *pi*

But *pi* does not occur neither in $\phi(x)$ nor in the branch (because it was new).
Thus, ***g''*** and ***g'*** agree on all the free variables of $\phi(x)$ except for *x* and by definition of ***g''*** (*pi*)= ***g'***(*x*)

Now we can use the lemma IX.2 that says that one variable can be substituted by another provided the valuations are adjusted adequately

$$\boldsymbol{M}, v_{f(i)*}\boldsymbol{g'}(\phi x) = 1 \Leftrightarrow \boldsymbol{M}, v_{f(i)*}\boldsymbol{g''}(\phi pi) = 1$$

Since we know that $\boldsymbol{M}, v_{f(i)*}\boldsymbol{g'}(x) = 1$ is the case, then we have $\boldsymbol{M}, v_{f(i)*}\boldsymbol{g''}(pi) = 1$
And so we finished with the existential case

I leave the universal case for the reader

Soundness theorem:

If **P** has has winning strategy for *A* using the **K**-structural rules, *A* is valid.

PROOF: Assume that **P** has a winning strategy for *A* using the **K**-structural rules, but *A* is not **K**–valid. We show that from this a contradiction follows.

Since **P** has a winning strategy for *A* using the **K**-structural rules there is a closed dialogue ***D*** that starts with 0**P***A.* Thus, the first section of ***D*** is $\boldsymbol{D_0}$ that consists in the thesis 0**P***A.* The following sections of ***D*** are constructed by extending $\boldsymbol{D_0}$**.**

Since *A* is not **K**–valid, there is a world wi in some model *M* at which *A* is not true. Let , ***f***(1)=wi, using this model and mapping {0**P***A* } is satisfiable. Thus $\boldsymbol{D_0}$ is satisfiable, since the set of formulae on its only branch is satisfiable

Since $\boldsymbol{D_0}$ is satisfiable by lemma SL-Q2 so is any dialogue ***D*** we get that starts with $\boldsymbol{D_0}$ and results by extending $\boldsymbol{D_0}$.
It follows that ***D*** is satisfiable.

D is closed by hypothesis, and this is impossible by SL-Q1.

Quod erat demonstrandum

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