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« A Regional Analysis of Markets Uncertainty Spillover »

Kamel Malik BENSAFTA
A Regional Analysis of Markets Uncertainty Spillover

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Abstract: This study aims to describe the transmission of uncertainty between the stock markets of four aggregate regions: North America, Europe non Euro-zone, Asia and the euro area. We use a non-linear VAR model with innovations following a Multivariate GARCH with variance regime change. The interest of the model with regime change is to correct the estimation bias caused by the overestimation of the shocks persistence. We apply the non-linear VAR model with regime change in daily MSCI data aggregated from four regions over the period from June 2005 to October 2013. This period included the crisis episodes in 2007 and 2011. Our results indicate the importance of taking into account changes in variance in measuring the persistence of volatility shocks. They also show the high exposure of European and Asian markets to the uncertainties of North American markets. The transmission in time of crisis is higher compared to the quiet period. This result confirms the contagious nature of the crises of 2007 and 2011 and supports the thesis of the contingency theory to crisis.

Keywords: MGARCH, Structural breaks, subprime crisis, volatility transmission.

JEL classification: C5, E5, F3, G1
Analyse Régionale de la Transmission de l’Incertitude des Marchés.


Mots clés : MGARCH, Ruptures Structurelles, Crise des Subprimes, transmission de volatilité.

Classification JEL : C5, E5, F3, G1
1. Introduction

A number of empirical studies look at the effects of contagion that were observed following the American stock exchange crash of October 1987 and the frequent crises of the emergent countries during 1990’s (King and Wadhawani, 1990 and Edwards, 1998, Forbes and Rigobon, 2002; McAleer and Nam, 2005). Studying spillover mechanisms and market correlations leads to two conclusions: first, market co-movement is important in any strategy of international diversification (King and al, 1994). This international diversification is based on a weak cross-correlation of geographically spaced markets. Secondly, in the current context of financial globalization, mean and variance spillover reinforce market interdependencies and reduce dramatically any benefit of diversification strategies.

International transmission mechanisms were established between stock markets and well demonstrated by studying volatility (Ng, 2000 and Granger et al, 2000). The volatility transmission translates the exogenous part of market turbulence linked to other market-uncertainties (Engle et al, 1990). According to this definition, the dependence in variance is a sign of market imperfections and allows risks and returns predictability. It appears that those markets are increasingly dependent in variance (Hamao et al, 1990, Koutmos et al, 1995), since there is more information in market volatility than market prices (Kyle, 1985).

The GARCH models are widely used for modelling the volatility of financial series. Under GARCH process, shocks to volatility persist according to ARMA process of squared innovations. Empirical findings show strong persistence of high frequency financial series and this is usually near unity. However, Lamoureux and Lstrapes (1990) show that misspecification in conditional variance processes explain higher persistence measurement. They found that time varying coefficients may exhibit persistence and they proposed time variation of unconditional variance. When taking into account structural change in unconditional variance, they obtained mode reduces persistence value. Theoretically it is hard to detect such structural change but there exist various methods to detect structural change empirically such as regime switching models (Susmel, 2000). In this paper we consider a method based on the CUSUM test, namely the Iterated Cumulative Sum of Squared ICSS algorithm developed by Inclan and Tiao (1994) and Sanso et al. (2004).

In this paper, the model of market volatility is a multivariate GARCH process. Short-run Mean and variance spillovers are based on Granger Causality and variance Causality. First, we look for bias estimates in volatility spillover estimates in the standard BEKK model (Engle and Kroner, 1995) compared to the BEKK model with a Structural Break in Variance subsequently called BEKK-BSV (Bensafta and Semedo, 2011). In the last section, we provide ways to extend this work.

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2 Engle et al (1990) described volatility spillover as “meteor showers”.

3 Shock persistence is the measurement of cumulative effects of shocks on volatility. For a GJR-GARCH (p, q) process, persistence is equal to $\varphi_x = \sum_{i=1}^{p} \alpha_i + \frac{1}{2} \gamma_i + \sum_{j=1}^{q} \beta_{ij}$. The measurement of unconditional variance is $(h_t) = \frac{\omega}{1-\varphi}$. A higher unconditional variance leads to highly persistence estimates.
2. The Econometric model

Volatility is the fluctuation of the value of an unspecified asset around a central tendency; it can also be measured as variation asset return over time. Negative shocks have more impact on volatility than positive ones of the same magnitude say asymmetric effect (Engle and NG, 1993, Koutmos and Booth, 1995). These interactions give useful information about market dynamics, such as the value of financial assets, market indexes, and the foreign exchange rate. Since Engle (1982) first developed the ARCH process, generalized by Bollerslev (1986), GARCH models have proved their ability to capture properties of high frequency financial series such as volatility clustering and thick tails. Volatility clustering indicates a persistence of volatility shocks. For reasons of parsimony and simplicity, the GARCH(1,1) model could suffice. Non-linear univariate GARCH models meet the objective of estimating volatility and the management characteristics of financial series such as volatility clustering. However, univariate models contain some important biases: omitted variable bias, overestimating the persistence of volatility shocks, and quadratic specification of the conditional variance masks the asymmetric effects of shocks. Moreover, they do not show volatility transmission. Multivariate GARCH models such as BEKK-GARCH and Vech meet the latter objective, but still overestimate persistence. This bias is particularly responsible for the overvaluation of the conditional correlations between markets. Because of this bias, these standard models underestimate the proportion of exogenous volatility related to external events.

Ewing and Malik (2005), and Bensafta and Semedo (2009, 2011) used a MGARCH-BEKK model with structural breaks in variances to study the cross-market volatility transmission and contagion. Ng (2000) and Bensafta and Semedo’s model (2009, 2011), confirms the leading place of the U.S. market worldwide. The introduction of structural break in variance corrects the overestimation of the persistence of volatility shocks (Lamoureux and Lastrapes, 1990). This correction shows that crises are not always contagious, but markets should be cautious of crises originating in the USA, supporting the results of "No Contagion only Interdependence" from Forbes and Rigobon (2002).

In this paper, we used a quadrivariate MGARCH model with structural changes in the variance. We used a quadrivariate version to introduce regional as well as overall effects of the American aggregate stock markets. Intuitively, the introduction of the structural change in variance should improve the standard BEKK models in terms of log-likelihood, while being closer to the real model.

3.1. Econometric model of market volatility

The market volatility model used in this study is a MGARCH. First of all, we produced a BEKK-diagonal asymmetric model (Engle and Kroner, 1995). This standard model was then compared with our model: a BEKK model increased by structural change in variance. Because the data are daily, we included the possibility of holiday’s effects and weekends effects. We also included central bank disclosures, the conduct of monetary policy, and surprise effects. In this multivariate model, mean transmission is measured by the VAR coefficients of conditional mean equations.

Let \( r_t = (r_{1t} \ldots r_{Mt})^{tr} \) denote the vector of logarithmic returns of MSCI indices, and \( u_t = (u_{1t} \ldots u_{Mt})^{tr} \) the vector of dynamic VAR(n1) residual, such that:

---

4 Edwards (1998) used a GARCH model to investigate the national and regional impact of interest rate spread during the Tequila crisis. Park and Song (2000) applied a similar GARCH model to test volatility spillover across Asian markets during the Asian flu epidemic.
\[ \Phi(L)(r_t - \mu) = u_t \]  
(1)

Where \( \Phi(L) \) is the function with lags in the VAR \( (n_1) \) process. Mean equations are augmented with exogenous regressors such as monetary policy variables. This general case is:

\[ \Phi(L)(r_t - \mu) = C(L)G(X_t) + u_t \]  
(2)

Where \( G(X_t) \) is exogenous variables vector and \( C(L) \) the exogenous coefficient matrix. The mean cross-market transmission is described by \( \Phi^k \) coefficients of the VAR process. Suppose that \( u_t \) is a vector of non-autocorrelated VAR residual, and:

\[ u_t = H_t^{-1/2} \varepsilon_t \]  
(3)

Where \( \varepsilon_t \) is an N-dimensional vector of white noise elements, such that \( \varepsilon_t \sim \text{i.i.d}(0, I) \) and \( H_t \) is the conditional variance-covariance matrix of \( u_t \). \( H_t \) is symmetric and positive-definite. Clearly, the \( u_t \) have a conditional distribution, given \( \psi(t-1)'s \) information set at time \( t-1 \). The conditional distribution is:

\[ u_t/ \psi_{t-1} \sim (0, H_t) \]  
(4)

\( H_t \) is a MGARCH process. Several specifications for the matrix \( H_t \) exist such as the BEKK and BEKK diagonal \( (\text{Engle and Kroner, 1995}) \), and the Dynamic Conditional Correlation \( \text{(DCC)} \) \( (\text{Tse and Tsui, 2002, and Engle and Shepphard, 2001}) \). Bauwens et al. (2003) provided an extensive literature review of the MGARCH model \( (\text{see appendix A. for details}) \). The BEKK-diagonal-asymmetrical model with transmission in variance and structural break is defined as follows:

\[ H_t = \frac{\text{Structural break}}{(C_B)^{tr} (C_B)} + \sum_{i=1}^{p} A_i u_{t-i} u_{t-i}^t A_i + \sum_{i=1}^{q} B_i H_{t-i} B_i + \sum_{i=1}^{p} G_i u_{t-i} u_{t-i}^t D_{Nt-i} D_{Nt-i} G_i \]  
(4)

\[ + \sum_{i=1}^{p} T_i u_{t-i} u_{t-i}^t T_i + \sum_{i=1}^{q} Z_i H_{t-i} Z_i \]  
Volatility transmission

\[ + \sum_{i=1}^{5} (\delta_i)^{tr} D_i \delta_i \]  
Day of the week effects

\( C_B, A_i, B_j \text{ and } G_j \) are coefficients matrices in conditional variance-covariance equations of standard model. \( u_t \) is innovation vector, \( D_{M_{t-i}} \) is M dimension vector of dummy variables \( (d_{it}) \) where:

\[ (d_{it}) = \begin{cases} 1 & \text{when } u_{it} < 0 \\ 0, & \text{else} \end{cases} \]

\( C_B \) is a constant coefficient matrix in conditional variance-covariance equations. In the diagonal, asymmetric model with structural changes in variance and variance spillover \( \text{(BEKK-BSV model)} \)

\(^5\Phi(L) = I_m - \Phi^1 L \cdots - \Phi^{n_1} L^{n_1}, \text{ where } n_1 \text{ is VAR process order defined by sequential LR test (see details in appendix A.).} \)

\(^6\text{The DCC models of Engle et al. (2001) and Tse et al. (2002) have the advantage of a two-step estimation. However, these templates provide a linear structure to the correlation dynamics and impose a similar dynamic conditional correlation. In addition, these models do not permit variance spillover.} \)

\(^7\text{M is the number of markets} \)
thereafter), the \((C_B)^{\prime\prime} (C_B)\) take into account structural changes in variance. Each diagonal element of \(C_B\) is defined as follows:

\[
\{C_{B_{jj}}\} = \omega_{0j} + \sum_{i=1}^{NR_j} \omega_{ji} S^i_{jt}
\]  

(5)

\(S^i_{jt}\) are dummy variables for variances regimes detected by ICSS-heteroskedastic algorithm (see appendix A for details). * is an element-by-element matrix product, \(u_t\) the innovations vector and \(p\) and \(q\) the GARCH process order.

\(T_i\) and \(Z_i\) are coefficient matrix of volatility transmission and \(p\) and \(q\) are GARCH process orders. \(T_i\) is a coefficient matrix of elements \((tcv^i)\) for shock to volatility transmission between markets and \(Z_i\) a coefficient matrix of element \((tv^i)\) for volatility transmission between markets. The volatility transmission regressors are defined as follows:

\[
T_i'u_{t-i}u'_{t-i}T_i = \begin{bmatrix}
(\Sigma^M_{j=2}(tcv^i_{j,1})u_{j,t-i})^2 & \cdots & \cdots \\
\vdots & \ddots & \vdots \\
(\Sigma^M_{j=2}(tcv^i_{j,1})u_{j,t-i})(\Sigma^M_{j=1}(tcv^i_{j,M})u_{j,t-i}) & \cdots & (\Sigma^{M-1}_{j=1}(tcv^i_{j,M})u_{j,t-i})^2
\end{bmatrix}
\]

and

\[
Z_i'H_{t-i}Z_i = \begin{bmatrix}
\Sigma^M_{j=2}(tv^i_{j,1})^2h_{jj,t-i} & \cdots & \cdots \\
\vdots & \ddots & \vdots \\
\vdots & \cdots & \Sigma^{M-1}_{j=1}(tv^i_{j,M})^2h_{jj,t-i}
\end{bmatrix}
\]

Volatility spillover named \textit{Volatility Causality} is measured by the sum \(\Sigma^p_{i=1}(tcv^i)^2 + \Sigma^q_{i=1}(tv^i)^2\). The last regressor \(\Sigma^5_{i=1}(\delta_i)D_i\delta_i\) permits « day of the week effects » and « holiday effects » in variance. \(D_i\) is a diagonal matrix whose element \(\{D_{ji}\} = D_{it}\) are:

\[
D_{it} = \{D_{1t}; D_{2t}; D_{3t}; D_{4t}; D_{5t}\}
\]  

(6)

Where \(D_{1t} = 1\) (resp \(D_{2t} = 1, D_{3t} = 1, D_{4t} = 1\) and \(D_{5t} = 1\)) for Monday (resp, Tuesday, Wednesday, Friday and holiday day) and \(D_{1t} = 0\) (resp \(D_{2t} = 0, D_{3t} = 0, D_{4t} = 0\) and \(D_{5t} = 0\)) otherwise. This effect may be present in daily data frequencies (Solnik and Bousquet, 1990, Barone, 1990, Agrawal and Tandon, 1994). \(\delta_i\) is a vector of coefficients to be estimated.

3.3. Transmission in means and variances: Estimation and tests

3.3.1. Estimation

The standard model and the BEKK-BSV model are estimated in two steps: first, we obtain the VAR residual, and secondly we estimate the conditional variance-covariance matrix parameters. This two-step estimation is possible because of the block-diagonal character of the variance-covariance matrix. Errors are conditionally normal and the likelihood function of all distribution is the sum of log-likelihood of each element. Let \(f\) denote the joint conditional distribution and \(L_N\) its log-likelihood function:
\[
\log(L_{N_b}) = \sum_{t=1}^{N_b} \log(f_t(\Omega_{t-1}; \theta))
\]

\[
\log(f_t(\Omega_{t-1}; \theta)) = -(M/2)\log 2\pi - 0.5 \log \left( \det(H_t, \theta) \right) - (u_t)^{tr}(H_t, \theta)^{-1} u_t/2 \tag{7}
\]

\[
\log(L_{N_b}) = -0.5 \sum_{t=1}^{N_b} \left[ M \log(2\pi) + \log \left( \det(H_t, \theta) \right) + (u_t)^{tr}(H_t, \theta)^{-1} u_t \right]
\]

Where \( N_b \) is the number of observations, \( M \) the number of markets, \( \theta = \text{vech}(A, B, C_B, G, f, \delta, \zeta, \xi, g) \) the vector parameters to be estimated, and \( u_t \) the normally distributed vector of innovation. Products \( u_{it}u_{jt} \) are second order correlated and the joint distribution of \( u_{1t}; u_{2t}; \ldots; u_{Mt} \) may not be normal. For this reason, \( \theta \) is estimated by the Quasi-Maximum likelihood method (QML) of Bollerslev and Wooldridge (1992). Optimization is obtained with BHHH algorithm (Berndt et al., 1974) which is very suitable for non-linear maximizations (Engle and Kroner, 1995).

### 3.3.2. Tests

Mean spillover can be assessed using the Granger Causality test (Lütkepohl, 2005). All tests of variance effects (volatility transmission, variance regime are performed by the mean of likelihood Ratio (LR) test and/or Wald test. Mean transmission from market «i» to market «j» is measured by \( \Phi_{ij}^1 \ldots \Phi_{ij}^{n1} \) VAR coefficients. No transmission in mean is a Granger-Causality test called NMT. Variance transmission is linked to \( (tcv_{ij}^k) \) and \( (tv_{ij}^k) \) coefficients. There are no variance transmission (NVT thereafter) from market «i» to market «j» when all \( (tcv_{ij}^k) \) and \( (tv_{ij}^k) \) coefficients are zero. NVT is a Wald-test of No-Causality in variance. Finally, we test the equality of volatility transmission during calm period and crisis period. This is Wald test of No strict contagion (see Appendix. C). Next we discuss the main empirical results.

### 3. Empirical results

#### 3.1. Data descriptive

The data cover America, aggregate Europe, aggregate EMU and aggregate Asia markets. MSCI standard capitalization indices are used since they are better adapted than usual market indices. Indeed they include the mid-cap and the large-cap companies’ capitalization. Daily observations from June, 2005 to October 4, 2013 are used.

Table I: Descriptive statistics of markets returns.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
<th>N. Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMERICA</td>
<td>0.018</td>
<td>10.626</td>
<td>-9.708</td>
<td>1.289</td>
<td>-0.316</td>
<td>12.371</td>
<td>10876.920</td>
<td>2959</td>
</tr>
<tr>
<td>EUROPE Non-EMU</td>
<td>0.017</td>
<td>10.885</td>
<td>-9.907</td>
<td>1.420</td>
<td>-0.110</td>
<td>10.650</td>
<td>7221.861</td>
<td>2959</td>
</tr>
<tr>
<td>EMU</td>
<td>0.012</td>
<td>10.869</td>
<td>-10.443</td>
<td>1.653</td>
<td>-0.086</td>
<td>8.164</td>
<td>3291.764</td>
<td>2959</td>
</tr>
<tr>
<td>ASIA</td>
<td>0.017</td>
<td>9.691</td>
<td>-8.848</td>
<td>1.258</td>
<td>-0.330</td>
<td>8.129</td>
<td>3297.742</td>
<td>2959</td>
</tr>
</tbody>
</table>

*** Significant at 1%. JB Jarque-Bera statistic.
Descriptive statistics show the usual characteristics of financial data: asymmetry, excess kurtosis and non-normality (Table I). MSCI indices returns are weak. The EMU markets are more volatile than others according to standard deviation. The asymmetry is most pronounced in Asian and American markets than the EMU and European markets. Excess kurtosis shows that extreme values are more frequent than predicted by normality. The Jarque-Bera statistic confirms the non-normality of data.

### 3.2. Conditional variance model

The Variance regimes detected by the ICSS-H algorithm are given in table II and represented in figure 1. The number of regimes is 11 for Europe, 10 for America and 8 for the EMU and Asian markets. Start and end dates of regimes are not equal although there are some coincidences during the multiple stage of subprime crisis period between July 2007 till December 2009:

<table>
<thead>
<tr>
<th>Regime</th>
<th>Start</th>
<th>End</th>
<th>Variance</th>
<th>Regime</th>
<th>Start</th>
<th>End</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>03/06/2005</td>
<td>20/07/2007</td>
<td>0.649</td>
<td>1</td>
<td>03/06/2005</td>
<td>20/07/2007</td>
<td>0.832</td>
</tr>
<tr>
<td>4</td>
<td>01/12/2008</td>
<td>15/05/2009</td>
<td>2.301^C</td>
<td>4</td>
<td>08/12/2008</td>
<td>14/07/2009</td>
<td>2.155^C</td>
</tr>
<tr>
<td>5</td>
<td>18/05/2009</td>
<td>06/11/2009</td>
<td>1.248</td>
<td>5</td>
<td>15/07/2009</td>
<td>16/12/2009</td>
<td>1.443</td>
</tr>
<tr>
<td>7</td>
<td>26/04/2010</td>
<td>31/08/2010</td>
<td>1.496</td>
<td>7</td>
<td>26/04/2010</td>
<td>10/08/2010</td>
<td>1.844</td>
</tr>
<tr>
<td>8</td>
<td>01/09/2010</td>
<td>29/07/2011</td>
<td>0.795</td>
<td>8</td>
<td>11/08/2010</td>
<td>01/08/2011</td>
<td>1.080</td>
</tr>
<tr>
<td>9</td>
<td>01/08/2011</td>
<td>19/12/2011</td>
<td>2.039^D</td>
<td>9</td>
<td>02/08/2011</td>
<td>14/12/2011</td>
<td>2.227^D</td>
</tr>
<tr>
<td>10</td>
<td>20/12/2011</td>
<td>01/10/2013</td>
<td>0.755</td>
<td>10</td>
<td>02/08/2012</td>
<td>01/10/2013</td>
<td>1.237</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime</th>
<th>Start</th>
<th>End</th>
<th>Variance</th>
<th>Regime</th>
<th>Start</th>
<th>End</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>03/06/2005</td>
<td>20/07/2007</td>
<td>0.911</td>
<td>1</td>
<td>03/06/2005</td>
<td>10/05/2006</td>
<td>0.909</td>
</tr>
<tr>
<td>3</td>
<td>03/09/2008</td>
<td>05/12/2008</td>
<td>4.528^B</td>
<td>3</td>
<td>08/08/2006</td>
<td>25/07/2007</td>
<td>0.829</td>
</tr>
<tr>
<td>4</td>
<td>08/12/2008</td>
<td>14/07/2009</td>
<td>2.384^C</td>
<td>4</td>
<td>26/07/2007</td>
<td>04/09/2008</td>
<td>1.507^A</td>
</tr>
<tr>
<td>5</td>
<td>15/07/2009</td>
<td>06/07/2011</td>
<td>1.627</td>
<td>5</td>
<td>05/09/2008</td>
<td>18/05/2009</td>
<td>2.606^B,C</td>
</tr>
<tr>
<td>7</td>
<td>30/11/2011</td>
<td>02/08/2012</td>
<td>1.793</td>
<td>7</td>
<td>29/07/2011</td>
<td>20/12/2011</td>
<td>1.608^D</td>
</tr>
<tr>
<td>8</td>
<td>03/08/2012</td>
<td>01/10/2013</td>
<td>1.178</td>
<td>8</td>
<td>21/12/2011</td>
<td>01/10/2013</td>
<td>0.878</td>
</tr>
</tbody>
</table>


- 09/2008 – 12/2008: the most turbulent regime and the most volatile during the second phase of the economic and financial crisis.
- 12/2008 – 07/2009: a third phase of the crisis which is less violent than the two previous phases.
Return to a quite calm period from July 2009 to June 2011.


Return to a quite calm period from July 2011 to nowadays.

These regimes clearly indicate a structural break in the variance during long-run bull and bear market volatility. These distinctions produce a more accurate market volatility model and permit a better comprehension of mean and volatility transmission and interdependences between stocks markets.

### 3.3. Mean and Prices Causality

Prices and asset returns spillover between stock exchanges markets. It is also shown that market prices are often transmitted unilaterally from US market to stock markets around the world. We produce measures of the mean transmission coefficient by the mean of linear VAR model.

Coefficients estimates show that recent information had more impact on returns than older information. The bigger impact came from the American market. One day lagged American returns explain nearly 62% of European (Non EMU) prices, 60% of EMU prices and 48% of Asian markets prices. Markets returns are also explained by own lagged returns. Prices are also transmitted from European (Non EMU) markets to Asian and EMU markets (table III, panel A).

Looking for Granger Causality between markets confirms the largest impact of American markets on all other markets. Granger Causality test of non-transmission is accepted from Asian market to all other markets (table III, panel B).

Long-Run Mean Spillover (LRMS hereafter) is measured by $\Phi = (I_4 - \sum_{i=1}^{5} \Phi^i)^{-1}$. One can see that American market LRMS estimates are more important than others LRMS. At long-run more than 50% of European (Non EMU) and Asian prices are exogenous and determined by American prices. It is about 49% in the case of EMUs markets (table III, panel C).

These findings are confirmed with Cumulative Impulse-Response Functions (CIRF hereafter). Figure 2 plots CIRF for linear VAR. CIRF of the American aggregate market are much important than others. There is no accumulate responses of American market to others market. American prices seem to be exogenous (figure2, panel A); whether Asian and European (Non EMU) prices are highly affected by American prices (figure 2, panel B and C).

EMU aggregate markets are highly responsive to American market and quite responsive to European market. The long-run effect of American market is more important than the endogenous prices of EMU theme self (figure 2, panel D).

Concerning the “Days of the Week” effects and according to our results, there are no such effects.

Concerning the causality during crisis, one can see a higher prices transmission during subprime crisis compared to transmission during the entire period (table IV, panels A and B). American aggregate prices are still affecting all others market (table IV, panel C). According to Wald test of coefficient equality, mean transmission coefficients are significantly different during the crisis than those of the entire period (table IV, panel D).

### 3.4. Conditional Variances-Covariance’s Estimates

Table V and VI provides measures for $H_t$ coefficients estimates, volatility spillover and various measures such as persistence, half-life, long-run volatility spillover and model diagnosis for both standard VAR-BEKK and BEKK-BSV.
First, all asymmetric coefficients are significant and confirm the asymmetric behavior of market’s volatility according to positive and negative shocks. The American market is the most asymmetric one (table V panel A and table VI panel A).

Second, in the case of the standard BEKK model, volatility persistence is close to unity (>0.94) particularly for American markets. Results show lesser persistence estimates with BEKK-BSV. Half-life of volatility is two times shorter than that estimated by the standard model and varies from 1 day for the Asian market to 8 days for the American market. Volatility is much more persistent in the American markets than in the Asian markets. Hence, the standard model without multiple regimes variances tends to overestimate the persistence (Lamoureux and Lastrapes, 1994). This overestimation leads to a bias in market spillover estimates and market interdependence estimates (table V panel A and table VI panel A).

Third, the omegas estimates in the conditional variance equation are higher during the crisis periods: regimes 2, 3 and 4 for the American market, European (Non EMU) markets and EMU markets. Those regimes coincide with the multiple steps of the sub-prime crisis between July 2007 and July 2009 (table VI panel A)\(^8\). A very high volatility regime in the American market from

### 3.5. Volatility spillover and Variance Causality

Volatility spillover is given for both models (tables V and VI). The comparison between the standard model and the BEKK-BSV model shows important differences: the standard model underestimates the intensity of European market exposure to American market turbulence. Results confirm the unilateral spillover from American markets to all others regions (table VI panel B).

Overall, the share of the volatility of the American market in the European market volatility is about 5%, it is about 13% to Asian market and 4.5% to EMU market. Volatility spillover estimates from American market to European market obtained with standard model is two times less than the one obtained by structural break model. Feedback from European and Asian markets are low (less than 1%) and non significant. There is no sensitivity of the European and EMU markets to the turbulence of the Asian market and vice-versa. Within Europe, there is no volatility spillover between EMU and non-EMU market. Concerning days of the week and holiday’s effects we did not find any significant results\(^9\).

The LR test confirms that the model with structural breaks is better than the standard model (table VI panel B). Figure 2 shows the volatilities estimated by BEKK-BSV models for the four aggregate markets between 2005 and 2013. Figures indicate clearly high volatility regimes during Sub-prime crisis period between July 2007 and April 2009 and during economic crisis period from 2010 to 2011\(^10\).

### 3.6. Volatility spillover during crisis period

Are the transmission phenomena different during crises? We identify crisis period from 23 July 2007 to 14 July 2009. This sub-period of turbulence is determined by successive more volatile regimes obtained by ICSS-H algorithm\(^11\).

---

\(^8\) High volatility regimes are indicated by bold script.

\(^9\) These results are not presented in the paper but can be provide upon request.

\(^10\) Unconditional variance measurements confirm the presence of high and low variance regimes (table 4). The measurement of unconditional variance is \(E(h_t) = \frac{s^2}{1-\phi}\). Estimates are available upon request.

\(^11\) See Table 1 for regimes detected by ICC-H algorithm.
There are three main results:

First of all, transmission between markets during crisis period is higher than transmission outside the crisis. The transmission from the American market to European markets almost doubled in intensity during the crisis: from 5% to 15% for European market, from 13% to 29% for Asian market and from 4% to 13% for EMU market (table VI pane, B).

Second, there are also new transmissions during subprime crisis: for example, during crisis period, 26% of European (Non EMU) market is transmitted to the Asian markets (table VI pane, B). These changes during the crisis undoubtedly indicate shift contagion (Forbes and Rigobon, 2002).

Concerning the stability of transmission during crisis, Wald test of no-strict contagion during subprime crisis indicate that volatility transmission from American market to European markets was significantly different from the one during calm period. This results support the shift contagion theory. There is also contagion from European non-EMU market to EMU market (table VI panel C).

4. Concluding remark

In this empirical paper, we demonstrate that linear VAR is misspecified because of the significance of second order autocorrelations. This is a serious bias in mean spillover estimates if we consider as here the links between the American, the European and the Asian markets.

This paper highlights the importance of structural breaks in variance. The standard BEKK model tends to overestimate the persistence of volatility shocks, while also overestimating the endogenous part of volatility in market uncertainty. By introducing multiple regimes of variance, we reduced the persistence and improved the model in terms of maximum log-likelihood. The Granger Causality test revealed transmission from the American to European markets in means and variances. Moreover, it appears that it takes a week for information to filter through to all the markets. We have shown that the standard BEKK model underestimates the transmission variance, including the impact of the American market on European markets. The model with structural breaks in variance confirms unilateral transmission from the American market to European and Asian markets.

This study of the transmission of uncertainty Markets has confirmed the key role of the market in North America region. It appears that almost half the prices of European markets and Asian markets are determined by the U.S. market. This price transmission is valid in quiet times and in times of crisis. Having regard to the transmission of volatility, it also appears that the uncertainty of the U.S. market is transmitted unilaterally to other regions. This transmission is very important in the case of the Asian markets. The market impact is also important in European markets outside the euro area and in the markets of the euro area. In times of crisis, such as the subprime crisis between July 2007 and July 2009, the transmission of American uncertainty to other regions has increased significantly. This episode of crisis that shook the world of finance seems to be contagious. Our results showed the appearance of new transmission channels during the crisis and support the contingent crises, for which the transmission mechanism during the crisis (or just after) is fundamentally different from that which prevailed before the crisis theory. The latter causes a structural change such that the shocks are propagated through a channel that does not exist in the periods of financial stability. The regional analysis of aggregated
markets showed the vulnerability of markets in the euro area face shocks to U.S. and European shocks outside the euro zone.

What was the outcome of this crisis at the end of 2009? While financial stability was restored, there was a subsequent downturn. In Europe, governments reacted differently. Overall, it is clear that the costs of inaction during a crisis are high. Action in times of crisis should be thorough, creative and aggressive, based on principle of separation and robust control to influence financial stability on the one hand, and real-estate stability on the other. Finally, we must prepare for the possibility of financial and monetary crises, especially with the intense movement of capitals a result of globalization. Crises can spread rapidly and have negative spillovers on the procurement of goods and services and on labor markets, requiring greater coordination of fiscal and monetary policies.

References


Figure 1: Variance regimes (May, 2005 – October, 2013).

Variance regimes are obtained by ICSS-H algorithm applied to MSCI yields (mid-cap and large-cap) for America, Europe, Asia and EMU. Shading areas indicate crisis periods.
Figure 2: Cumulative Impulse-Response-Function CIRF.

CIRF are from linear VAR(5) for 10 periods. Legend: y1 (America), y2 (Europe exclude EMU), y3 (Asia) and y4 (EMU).
Figure 3: American, European, Asian and EMU market’s Volatility estimates

### Appendix B

Table III: Price return transmission, Mean Causality and Long-Run Mean Spillover.

#### Panel A: Mean Causality (Entire period 2006-2013)

<table>
<thead>
<tr>
<th></th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>America</strong> (-1)</td>
<td>-0.103 ***</td>
<td>0.626 ***</td>
<td>0.484 ***</td>
<td>0.609 ***</td>
<td>Asia (-1)</td>
<td>0.003</td>
<td>0.017</td>
<td>-0.332 ***</td>
</tr>
<tr>
<td>(-2)</td>
<td>-0.011</td>
<td>0.252 ***</td>
<td>0.190 ***</td>
<td>0.231 ***</td>
<td>(-2)</td>
<td>-0.004</td>
<td>-0.006</td>
<td>-0.146 ***</td>
</tr>
<tr>
<td>(-3)</td>
<td>0.056</td>
<td>0.150 ***</td>
<td>0.131 ***</td>
<td>0.138 ***</td>
<td>(-3)</td>
<td>-0.002</td>
<td>-0.023</td>
<td>-0.079 ***</td>
</tr>
<tr>
<td>(-4)</td>
<td>-0.013</td>
<td>0.061</td>
<td>0.028</td>
<td>0.047</td>
<td>(-4)</td>
<td>-0.007</td>
<td>-0.006</td>
<td>-0.056 **</td>
</tr>
<tr>
<td>(-5)</td>
<td>-0.016</td>
<td>0.095 ***</td>
<td>-0.033</td>
<td>0.048</td>
<td>(-5)</td>
<td>0.015</td>
<td>0.003</td>
<td>-0.020</td>
</tr>
<tr>
<td><strong>Europe</strong> (-1)</td>
<td>-0.104 *</td>
<td>-0.371 ***</td>
<td>0.098 **</td>
<td>-0.395 ***</td>
<td><strong>EMU</strong> (-1)</td>
<td>0.114 **</td>
<td>-0.053</td>
<td>0.080 **</td>
</tr>
<tr>
<td>(-2)</td>
<td>-0.141 **</td>
<td>-0.193 ***</td>
<td>-0.006</td>
<td>-0.158 **</td>
<td>(-2)</td>
<td>0.068</td>
<td>-0.019</td>
<td>0.055</td>
</tr>
<tr>
<td>(-3)</td>
<td>-0.044</td>
<td>-0.116 *</td>
<td>0.006</td>
<td>-0.039</td>
<td>(-3)</td>
<td>0.012</td>
<td>-0.017</td>
<td>0.021</td>
</tr>
<tr>
<td>(-4)</td>
<td>0.009</td>
<td>0.016</td>
<td>0.082 *</td>
<td>0.061</td>
<td>(-4)</td>
<td>0.025</td>
<td>-0.020</td>
<td>-0.029</td>
</tr>
<tr>
<td>(-5)</td>
<td>-0.027</td>
<td>-0.122 **</td>
<td>-0.018</td>
<td>-0.104</td>
<td>(-5)</td>
<td>-0.027</td>
<td>0.001</td>
<td>0.096 **</td>
</tr>
</tbody>
</table>

#### Panel B: Granger Causality tests (entire period) b

<table>
<thead>
<tr>
<th></th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>America</strong></td>
<td>16.737 ***</td>
<td>408.012 ***</td>
<td>461.665 ***</td>
<td>268.993 ***</td>
<td>America</td>
<td>0.858</td>
<td>0.535</td>
<td>0.541</td>
</tr>
<tr>
<td><strong>Europe</strong></td>
<td>7.842</td>
<td>47.046 ***</td>
<td>7.668</td>
<td>33.951 ***</td>
<td>Europe</td>
<td>-0.204</td>
<td>0.452</td>
<td>-0.114</td>
</tr>
<tr>
<td><strong>Asia</strong></td>
<td>0.522</td>
<td>0.917</td>
<td>207.021 ***</td>
<td>0.911</td>
<td>Asia</td>
<td>0.003</td>
<td>-0.003</td>
<td>0.613</td>
</tr>
<tr>
<td><strong>EMU</strong></td>
<td>7.811</td>
<td>1.434</td>
<td>13.676 **</td>
<td>2.594</td>
<td>EMU</td>
<td>0.160</td>
<td>0.046</td>
<td>0.216</td>
</tr>
</tbody>
</table>

(\(..\)) Delay. \***\, ** and * Significant coefficients at 1%, 5% and 10%. \(a\) Long-run mean spillover is measured as \(\Phi = (I_m - \Phi_1 \cdots - \Phi_5)^{-1}.\) \(b\) Under null hypothesis of non Granger Causality test statistic follow \(\chi^2_{(5)}.\) \(c\) Under null hypothesis of Equal transmission test statistic follow \(\chi^2_{(5)}.\)
Table IV: Price return transmission, Mean Causality and Long-Run Mean Spillover during Crisis periods.

<table>
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<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>-0.136 ***</td>
<td>0.677 ***</td>
<td>0.451 ***</td>
</tr>
<tr>
<td>(-2)</td>
<td>-0.013</td>
<td>0.286 ***</td>
<td>0.165 ***</td>
</tr>
<tr>
<td>(-3)</td>
<td>0.150 **</td>
<td>0.218 ***</td>
<td>0.178 ***</td>
</tr>
<tr>
<td>(-4)</td>
<td>-0.054</td>
<td>0.039</td>
<td>0.039</td>
</tr>
<tr>
<td>(-5)</td>
<td>0.014</td>
<td>0.144 ***</td>
<td>-0.006</td>
</tr>
</tbody>
</table>

Panel B: EMU Debt Crisis (2010-2011)  
<table>
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<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>-0.162</td>
<td>-0.320 ***</td>
<td>0.117 *</td>
</tr>
<tr>
<td>(-2)</td>
<td>-0.205 **</td>
<td>-0.096</td>
<td>-0.021</td>
</tr>
<tr>
<td>(-3)</td>
<td>-0.177 *</td>
<td>-0.203 **</td>
<td>-0.079</td>
</tr>
<tr>
<td>(-4)</td>
<td>-0.114</td>
<td>0.039</td>
<td>0.096</td>
</tr>
<tr>
<td>(-5)</td>
<td>-0.035</td>
<td>-0.213 **</td>
<td>-0.167 **</td>
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Panel C: Granger Causality tests (crisis period)  
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<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-1)</td>
<td>0.170</td>
<td>* -0.154 *</td>
<td>0.138 **</td>
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<tr>
<td>(-2)</td>
<td>0.051</td>
<td>-0.175 **</td>
<td>0.102 *</td>
</tr>
<tr>
<td>(-3)</td>
<td>0.153 *</td>
<td>0.054</td>
<td>0.110 *</td>
</tr>
<tr>
<td>(-4)</td>
<td>0.194 **</td>
<td>0.030</td>
<td>-0.003</td>
</tr>
<tr>
<td>(-5)</td>
<td>-0.088</td>
<td>0.019</td>
<td>0.288 ***</td>
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Panel D: Equality of transmission  
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<th>Asia</th>
<th>EMU</th>
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</thead>
<tbody>
<tr>
<td>America</td>
<td>18.762 ***</td>
<td>19.487 ***</td>
<td>55.659 ***</td>
</tr>
<tr>
<td>Europe</td>
<td>2.094</td>
<td>8.308</td>
<td>4.406</td>
</tr>
<tr>
<td>Asia</td>
<td>18.609 ***</td>
<td>6.036</td>
<td>9.900 *</td>
</tr>
<tr>
<td>EMU</td>
<td>3.972</td>
<td>2.480</td>
<td>3.935</td>
</tr>
</tbody>
</table>

(. .) Delay. ***, ** and * Significant coefficients at 1%, 5% and 10%. * Long-run mean spillover is measured as \(\Phi = (I_m - \Phi_1 - \cdots - \Phi_5)^{-1}\).  
(a) Under null hypothesis of non Granger Causality test statistic follow \(\chi^2_5\).  
(b) Under null hypothesis of Equal transmission test statistic follow \(\chi^2_5\).
Table V: Conditional variances coefficients estimates, Variance spillover and Variance Causality estimates for BEKK model

### Panel A: Coefficients estimates

<table>
<thead>
<tr>
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<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega^2$</td>
<td>0.140 ***</td>
<td>0.112 ***</td>
<td>0.055 **</td>
<td>0.123 ***</td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>0.000 ***</td>
<td>0.009 ***</td>
<td>0.090 ***</td>
<td>0.014 ***</td>
</tr>
<tr>
<td>$\gamma^2$</td>
<td>0.111 ***</td>
<td>0.067 ***</td>
<td>0.000 ***</td>
<td>0.073 ***</td>
</tr>
<tr>
<td>$\beta^2$</td>
<td>0.903 ***</td>
<td>0.906 ***</td>
<td>0.726 ***</td>
<td>0.914 ***</td>
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</tbody>
</table>

### Panel B: Persistence and Half-life

<table>
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<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
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</thead>
<tbody>
<tr>
<td>Persistence</td>
<td>0.959</td>
<td>0.948</td>
<td>0.816</td>
<td>0.964</td>
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</table>

### Panel C: Variance spillover

<table>
<thead>
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<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(tcv^2 + tv^2)_{\text{AMERICA},i}$</td>
<td>--</td>
<td>3.72% ***</td>
<td>8.51% ***</td>
<td>2.70% ***</td>
</tr>
<tr>
<td>$(tcv^2 + tv^2)_{\text{EUROPE},i}$</td>
<td>3.82% **</td>
<td>--</td>
<td>1.78%</td>
<td>0.05%</td>
</tr>
<tr>
<td>$(tcv^2 + tv^2)_{\text{ASIA},i}$</td>
<td>0.20%</td>
<td>0.14%</td>
<td>--</td>
<td>0.09%</td>
</tr>
<tr>
<td>$(tcv^2 + tv^2)_{\text{EMU},i}$</td>
<td>0.63%</td>
<td>0.04%</td>
<td>0.16%</td>
<td>--</td>
</tr>
</tbody>
</table>

Log-likelihood: -5943.683
AIC: 5.604

* (...) Variance Causality Wald test significance. ***.*, ** and * Significant coefficients at 1%, 5% and 10%.*
Table VI: Conditional variances coefficients estimates, Variance spillover and Variance Causality estimates.

<table>
<thead>
<tr>
<th>Panel A: Coefficients estimates</th>
<th>America</th>
<th>Regime period</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0^2$</td>
<td>0.3963</td>
<td>03/06/2005</td>
<td>0.5849</td>
<td>0.5017</td>
<td>0.7788</td>
</tr>
<tr>
<td>Regime 2</td>
<td>1.2886</td>
<td>23/07/2007</td>
<td>0.8339</td>
<td>0.8337</td>
<td>0.7800</td>
</tr>
<tr>
<td>Regime 3</td>
<td>7.9544</td>
<td>12/09/2008</td>
<td>1.5817</td>
<td>0.2535</td>
<td>0.7237</td>
</tr>
<tr>
<td>Regime 4</td>
<td>1.0863</td>
<td>01/12/2008</td>
<td>0.3690</td>
<td>0.3994</td>
<td>1.0130</td>
</tr>
<tr>
<td>Regime 5</td>
<td>7.9544</td>
<td>12/09/2008</td>
<td>1.5817</td>
<td>0.2535</td>
<td>0.7237</td>
</tr>
<tr>
<td>Regime 6</td>
<td>1.0863</td>
<td>01/12/2008</td>
<td>0.3690</td>
<td>0.3994</td>
<td>1.0130</td>
</tr>
<tr>
<td>Regime 7</td>
<td>1.3017</td>
<td>01/08/2011</td>
<td>0.5273</td>
<td>0.2768</td>
<td>0.8251</td>
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<tr>
<td>Regime 8</td>
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<td>01/09/2010</td>
<td>0.5858</td>
<td>0.1501</td>
<td>1.2551</td>
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<tr>
<td>Regime 9</td>
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<td>01/09/2010</td>
<td>0.5858</td>
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<td>1.2551</td>
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<tr>
<td>Regime 10</td>
<td>0.3991</td>
<td>01/09/2010</td>
<td>0.5858</td>
<td>0.1501</td>
<td>1.2551</td>
</tr>
</tbody>
</table>

$\alpha^2$: 0.004 *** 0.005 *** 0.027 ***
$\beta^2$: 0.106 *** 0.115 *** 0.072 ***
Persistence: 0.922 0.483 0.926
Half-life: 8.482 6.765 9.052

Panel B: Variance spillover

<table>
<thead>
<tr>
<th>Entire Period</th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
<th>Crisis Period</th>
<th>America</th>
<th>Europe</th>
<th>Asia</th>
<th>EMU</th>
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<tr>
<td>$\sum_{t=1}^{N^i}{N^R_i} = 1$</td>
<td>33</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Log-likelihood</td>
<td>-5843.18</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>AIC</td>
<td>5.540</td>
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<td>LR test</td>
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</tr>
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</table>

Appendix C

Detecting a Structural Break in Variance

We use an ICSS algorithm based on the CUSUM test to detect the structural change in variance. Following Inclan and Tiao (1994), the variance of a given series shows a structural change due to an exogenous shock. These changes mean a permanent decline in the trend which continues until the appearance of a new significant shock. This analysis supposes a stationary variance between two points of structural change. Let $M$ series of independent and normally distributed observations: $x_{i,t}$ ($i = 1, M$). The non-conditional variance of each one of them is $\sigma^2_{i,t}$ and $NR_i$ the number of break point in the variance. On the whole sample of $N$ observations, we have:

$$\sigma^2_{i,t} = \begin{cases} 
\sigma^2_{i,0} & 1 < t < c_{i,1} - 1 \\
\sigma^2_{i,1} & c_{i,1} < t < c_{i,2} - 1 \\
\vdots \\
\sigma^2_{i,NR_i} & c_{i,NR_i} < t < N 
\end{cases}$$

Where the $c_{i,j}$ ($j = 1 \ldots NR_i$) are the dates of break in variance. To estimate the number of changes of variance tendency, the cumulative sum of the square residuals $C_k = \sum_{t=1}^{N} u_{i,t}^2$ is calculated. Inclan and Tiao (1994) define the statistics $D_k = \frac{C_k}{C_N} - \frac{k}{N}$ with $D_0 = D_N = 0$. When there is no change in variance tendency in the sample, $D_k$ oscillates around zero. Otherwise, when break points exist, $D_k$ is strictly different from zero. Under the null assumption of homogeneous variance $H_0: \text{Var}(x_{i,t}) = \sigma^2_i$ (constant), the $D_k$ statistic converges in distribution towards a standard Brownian motion. The null assumption $H_0$ of non structural break in variance is rejected when $k^* = \max_k \left( \frac{N/2}{D_k} \right)$ is outside the critical interval $\pm 1.358$. Then $k^*$ is a break point at 95%. However, this original version of the ICSS algorithm is defined for a homogeneous variance and does not consider the heteroskedastic nature of the financial series. Sans et al. (2004) make a modification in $D_k$ statistics by taking into account the fourth moment, namely the ICSS-H algorithm. They replace $D_k$ by $G_k = \delta_4^{-1/2} \left( \frac{C_k}{C_N} - \frac{k}{N} \right)$, where $\delta_4$ is a consistent estimator of the fourth order moment\footnote{$\delta_4$ is obtained from the non parametric estimator : $\delta_4 = N^{-1} \sum_{t=1}^{N} (\tau_{t}^2 - \bar{\tau}^2)^2 + 2N^{-1} \sum_{t=1}^{N} w(l, m) \sum_{s=1}^{t-1} (\tau_{t-s}^2 - \bar{\tau}^2)(\tau_{t-s}^2 - \bar{\tau}^2)$, where $w(l, m)$ is a Bartlett window. The $\delta_4$ estimates depend on the choice of $m$ parameter with the Newey-West method. It’s usually equal to 2.}. The null assumption $H_0$ is rejected when $k^*_S = \max_k \left( G_k / \sqrt{N} \right)$ is outside the critical interval $\pm 1.405$. The $k^*_S$ point is a break point in variance. The ICSS-H algorithm detects $NR_i + 1$ regimes of variance for each series. The structural breaks are located by the dummy variables $S_{j,t}^i$. For each series $x_{i,t}$

$$S_{j,t}^i = \begin{cases} 
1 \text{ if } c_{i,j-1} < t < c_{i,j} - 1 \\
0, \text{ else} 
\end{cases} \quad (j \text{ from } 1 \text{ to } NR_i)$$

For each series $x_{i,t}$, there are $NR_i$ break points in variance and $NR_i + 1$ régimes of variances.
No Transmission tests and No Strict contagion test

No Mean Transmission test

Mean transmission from market « i » to market « j » is measured by $\Phi_{ij}^1 \ldots \Phi_{ij}^{n_1}$ VAR. No transmission in mean concern the null hypothesis of all $\Phi_{ij}^k$ coefficients. It is a Granger-Causality test called NMT:

Null hypothesis of no mean transmission from i to j $H_{0,i\rightarrow j}^{NMT}$

\[
\begin{align*}
H_{0,i\rightarrow j}^{NMT} : & \quad \Phi_{ij}^1 = \Phi_{ij}^2 = \cdots = \Phi_{ij}^{n_1} = 0 \\
H_{1,i\rightarrow j}^{NMT} : & \quad \text{non } H_{0,i\rightarrow j}^{NMT}
\end{align*}
\]

Under $H_{0,i\rightarrow j}^{NMT}$, test statistic follow a Fisher $F(n_1, N - 2n_1 - 1)$. (See Lütkepohl, 2005, p103).

No Variance Transmission test

Variance transmission is linked to $(tcv_{ij}^k)$ and $(tv_{ij}^k)$ coefficients. There no variance transmission NVT from market « i » to market « j » when all $(tcv_{ij}^k)$ and $(tv_{ij}^k)$ coefficients are zero. NVT is a Wald-test of No-Causality in variance:

Null hypothesis of no variance transmission from i to j $H_{0,i\rightarrow j}^{NVT}$

\[
\begin{align*}
H_{0,i\rightarrow j}^{NVT} : & \quad (tcv_{ij}^1)^2 = 0, \ldots , (tcv_{ij}^p)^2 = 0, (tv_{ij}^1)^2 = 0, \ldots , (tv_{ij}^q)^2 = 0 \\
H_{1,i\rightarrow j}^{NVT} : & \quad \text{non } H_{0,i\rightarrow j}^{NVT}
\end{align*}
\]

Under $H_{0,i\rightarrow j}^{NVT}$, Wald statistic follow a $\chi^2(p + q)$.

No Strict Contagion test

There is strict contagion when transmission during crisis period is significantly different then transmission during calm period. Let $TPC = \sum_{i=1}^{p} T_i^* u_{t_i} \cdot i_t \cdot T_i^* + \sum_{i=1}^{q} Z_i^* H_{t_i} \cdot Z_i^*$ be and new regressor for variance transmission during crisis period $[t_1, t_2]$. Where $T_i$ is a coefficient matrix of cshck to volatility transmission during crisis and $Z_i$ is a coefficient matrix of shock to volatility transmission during crisis:

\[
T_i^* = \left[ \begin{array}{ccc} 0 & \cdots & (tcv_{i,1}^1)^* \\
\vdots & \ddots & \vdots \\
(tcv_{i,M}^1)^* & \cdots & 0 \end{array} \right] \text{ and } Z_i^* = \left[ \begin{array}{ccc} 0 & \cdots & (tv_{i,1}^1)^* \\
\vdots & \ddots & \vdots \\
(tv_{i,M}^1)^* & \cdots & 0 \end{array} \right]
\]

There is Strict contagion when $T_i^*$ and $Z_i^*$ coefficients are significant and significantly different from $T_i$ and $Z_i$ coefficients.

Null hypothesis of non-strict contagion NSC from market « i » to market « j » $H_{0,i\rightarrow j}^{NSC}$

\[
\begin{align*}
H_{0,i\rightarrow j}^{NSC} : & \quad (tcv_{ij}^1)^*, \cdots , (tcv_{ij}^p)^*, (tv_{ij}^1)^*, \cdots , (tv_{ij}^q)^* \\
H_{1,i\rightarrow j}^{NSC} : & \quad \text{non } H_{0,i\rightarrow j}^{NSC}
\end{align*}
\]

Under $H_{0,i\rightarrow j}^{NSC}$, Wald statistic follow a $\chi^2(2p + 2q)$. 

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