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To cite this version:
Andrea Bonilla. Initiative for Infrastructure Integration in South America: Way toward Regional Convergence. 2015. halshs-01189319

HAL Id: halshs-01189319
https://halshs.archives-ouvertes.fr/halshs-01189319
Submitted on 1 Sep 2015

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Andrea Bonilla Bolaños

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Keywords:
catch up policy, convergence, economic integration, infrastructure integration, IIRSA, South America, steady state

JEL codes:
C61, F11, F42, O54
Initiative for Infrastructure Integration in South America: Way toward Regional Convergence

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1 Introduction

The onset of the European sovereign debt crisis in mid-2009 renewed interest in the role of fiscal and political unification in economic integration. Numerous authors have highlighted the major role of the fiscal and political aspects of integration, confronting the concept of an economic union with a political (and/or fiscal) one (Evers, 2012, Issing, 2011, Lane, 2012, Sapir, 2011). In contrast, little attention has been given to the role of physical (or infrastructure) integration in economic integration. Considering that all three aspects—economic integration, political and fiscal integration, and physical integration—are part of a whole concept, namely regional integration (ECLAC, 2009), it is possible that not only political and fiscal aspects but also physical ones influence economic integration.1

A general consensus is that a nation’s public infrastructure network serves as a basis for economic activities. Various studies have revealed that public infrastructure positively affects a country’s productivity (i.e., Canning, 1999, Cohen and Morrison Paul, 2004, Rioja, 1999) and economic growth (i.e., Canning and Pedroni, 2008, Pradhan et al., 2014, Sahoo and Dash,

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1“Physical integration” refers to the coordinated provision of infrastructure and its services (e.g., construction of communication channels by connecting transportation, energy, and telecommunications networks at a regional level between two or more neighboring nations’ governments, resulting in further regional economic interdependence (ECLAC, 2009, Gramlich, 1994, World-Bank, 1994).
At an international level, these country effects are likely to spill over across national borders. For instance, a government’s investment in expanding communication and transportation networks potentially benefits neighboring commercial partner countries by, for example, facilitating trade.\footnote{The role of infrastructure in determining transportation costs, and thus trade volumes, is well reported by empirical literature (i.e., Francois and Manchin, 2013, Limao and Venables, 2001, Shepherd and Wilson, 2006).} Such externalities are known as “infrastructure spillovers” (Bougheas et al., 1999, 2003).\footnote{For evidence on infrastructure spillovers, see Canning (1999), Cohen and Morrison Paul (2003, 2004), Del Bo and Florio (2008), Pereira and Andraz (2010).} Hence, there is good reason to believe that physical integration projects boost economic integration. This study, thus, proposes to incorporate the topic of infrastructure investment into the economic integration debate by investigating whether common infrastructure investments increase long-run real output convergence and trade integration among countries.

For the purpose of this analysis, the South American region serves as a suitable case study. Since 2000, South America’s nations have engaged in a process of regional integration, for which physical integration is a major pillar. The so-called “Initiative for the Integration of Regional Infrastructure in South America (IIRSA)” or Cosiplan proposes the coordination of various transportation, communication, and energy infrastructure projects involving two or more countries.\footnote{In 2008, the formation of the Union of South American Nations (Unasur) provided IIRSA with a new institutional framework and asserted its continuity. Later in 2011, the South American Infrastructure and Planning Council (Cosiplan) assumed this coordination role and defined the Strategic Action Plan, 2012–2022. The physical integration project in South America continues to function even today and provides an appealing set of dynamics to be examined in line with the objectives of this study.} According to the 2014 IIRSA Portfolio, this initiative is composed of 579 structural infrastructure projects—with an estimated investment of US $163,324.5 millions—of which 18.3% (106) have already been completed, 30.9% (179) are in the execution stage, 27.1% (157) are in the pre-execution phase, and 23.7% (137) are being profiled.\footnote{It is worth mentioning that IIRSA is Unasur’s most advanced project. The initiative has been cataloged as a process of “silent” integration because, despite the difficulties in political and economic integration (e.g., political frictions from Paraguay’s expulsion from the bloc owing to the 2012 coup d’etat), physical integration continues to work (ECLAC, 2009).}

### Table 1: IIRSA’s Integration and Development Hubs (EIDs)

<table>
<thead>
<tr>
<th>Hub</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andean Hub</td>
<td>Guianese Shield Hub</td>
</tr>
<tr>
<td>Peru, Brazil-Bolivia Hub</td>
<td>Amazon Hub</td>
</tr>
<tr>
<td>Paraguay-Parana Waterway Hub</td>
<td>Central Inter-oceanic Hub</td>
</tr>
<tr>
<td>Capricorn Hub Southern</td>
<td>Mercosur-Chile Hub</td>
</tr>
<tr>
<td>Southern Andean Hub</td>
<td>Southern Hub</td>
</tr>
</tbody>
</table>

Source: Bid-Intal (2011). IIRSA’s achievements and challenges 10 years after its inception: 2000-2010

In addition to the implied investment and consequent economic impacts, perhaps, the most attractive characteristic of this initiative is its proposal of a new division of territorial spaces.
National borders are no longer the reference. As shown in Table 1, the South American continent is divided by Integration and Development Hubs (EIDs), an unprecedented territorial conception that distinguishes 10 strategic South American sub-regions. Certainly, numerous dynamics emerge from such a design. This study pays exclusive attention to the potential long-run output convergence and commercial integration effects of this initiative. More specifically, the following questions are addressed:

(i) What are the potential gains or losses (in terms of output convergence and trade integration) of raising publicly provided transportation infrastructure?

(ii) Does a coordinated increase of public investment in transportation infrastructure, as undertaken by Unasur through the IIRSA-Cosiplan initiative, improve gains or generate losses?

A micro-foundation, two-country general equilibrium model is constructed to study these issues. The role of infrastructure stock is twofold: it is a publicly provided input in intermediate goods production and it determines the bilateral cost of trade. Infrastructure is exclusively restricted to transportation (e.g., roads and highways, airports, and harbors) and transportation cost is assumed to depend on the public stock of infrastructure (the two countries are neighboring trade partners). Thus, this study focuses on implications for regional integration of the coordinated and uncoordinated public provisions of transportation infrastructure.

<table>
<thead>
<tr>
<th>No. of Projects</th>
<th>% Projects</th>
<th>Estimated Investment (US $ million)</th>
<th>% Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>159</td>
<td>30.8</td>
<td>26,784.47</td>
</tr>
<tr>
<td>Bolivia</td>
<td>45</td>
<td>8.7</td>
<td>3,866.29</td>
</tr>
<tr>
<td>Brazil</td>
<td>95</td>
<td>18.4</td>
<td>44,135.36</td>
</tr>
<tr>
<td>Chile</td>
<td>58</td>
<td>11.2</td>
<td>12,816.20</td>
</tr>
<tr>
<td>Colombia</td>
<td>28</td>
<td>5.4</td>
<td>4,790.93</td>
</tr>
<tr>
<td>Ecuador</td>
<td>34</td>
<td>6.6</td>
<td>1,385.82</td>
</tr>
<tr>
<td>Guyana</td>
<td>7</td>
<td>1.4</td>
<td>911.90</td>
</tr>
<tr>
<td>Paraguay</td>
<td>55</td>
<td>10.7</td>
<td>7,145.89</td>
</tr>
<tr>
<td>Peru</td>
<td>68</td>
<td>13.2</td>
<td>11,309.59</td>
</tr>
<tr>
<td>Suriname</td>
<td>6</td>
<td>1.2</td>
<td>3,831.90</td>
</tr>
<tr>
<td>Uruguay</td>
<td>32</td>
<td>6.2</td>
<td>3,218.06</td>
</tr>
<tr>
<td>Venezuela</td>
<td>13</td>
<td>2.5</td>
<td>722.75</td>
</tr>
<tr>
<td>Total</td>
<td>579</td>
<td>100</td>
<td>120,919.17</td>
</tr>
</tbody>
</table>

Source: COSIPLAN Portfolio Projects Database (April 2015; www.iirsa.org/proyectos)

Table 2: Portfolio summary by country (transportation projects)

To solve the model, I adopt data from Argentina and Brazil because of their consequent investment share in the IIRSA-Cosiplan portfolio. As described in Table 2, Argentina and Brazil account for the 24.7% and 40.7% of the total estimated investment in transportation projects, a majority of which are publicly financed (i.e., 99% in Argentina and 77% in Brazil; see Table 3).

The model is extended to a number of experiments and the results reveal that increasing public investment in infrastructure provides an impetus to commercial integration but does not

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6 An EID is a multinational territorial space with specific natural resources, human settlements, production areas, and logistics services. Transportation, energy, and communications infrastructure serve as its links because they facilitate the flow of people, goods and services, and information within the territorial space as well as to and from the rest of the world.
necessarily generate output convergence. In addition, the model shows that the only way for the two countries to achieve output convergence (in a win-win economic growth scenario) is to coordinate their increments on public infrastructure, as proposed by IIRSA-Cosiplan.

The remainder of this paper is organized as follows. Section 2 provides a detailed explanation of the model. Section 3 discusses the calibration and presents the benchmark steady state. Section 4 presents a number of policy experiments and the results. Section 5 concludes.

2 Model

The model comprises two countries: Home \((i = 1)\) and Foreign \((i = 2)\). Each country includes three sectors: (1) a tradable intermediate goods sector that uses domestic physical capital and labor (2) a non-tradable final goods sector that uses both Home and Foreign intermediate goods as inputs, and (3) a transportation services sector that uses labor. Neither capital nor labor is internationally mobile.\(^7\)

In each country, a representative firm is exclusive to each sector and operates in a competitive market. Trade is costly and assumed to depend on transportation cost, which in turn is contingent on the levels of public (transportation) infrastructure in both countries. Each country’s government determines the level of public transportation infrastructure and collects taxes to finance its provision. In each economy, the infinitely lived representative household supplies labor, rents capital, and gains utility from consuming a final domestic good. Finally, their markets are complete, that is, households have access to state-contingent securities.

2.1 Firms

As mentioned, each economy encompasses three sectors: intermediate goods sector, final goods sector, and transportation service sector.

2.1.1 Intermediate Goods Sector

The representative intermediate firm in country \(i\) specializes in the production of one tradable intermediate good, \(z_i\), using labor \(n_{i,t}\), private capital \(k_{i,t}\), and public capital \(k_{g,i,t}\), according to

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7 Even if Argentina and Brazil are members of the Mercosur, an incomplete common market, labor and capital mobility between them remains low. For instance, according to the capital openness index (Chinn and Ito, 2011), Argentina is the second less financially open Unasur country after Venezuela and ranks 106th among a set of 166 countries. After the collapse of convertibility (the 2001–2002 crisis), Argentina imposed strict control over capital outflows (Frenkel and Rapetti, 2010), which continues even today. Thus, the non-international mobility of factors is not completely unrealistic.
the following Cobb-Douglas technology:

\[ z_{i,t} \equiv F(k_{i,t}, n_{i,t}, k_{g}^{i,t}) = a_{i,t} (k_{i,t})^\alpha k_{g}^{i,t} n_{i,t}^{1-\alpha}, \] (1)

where \( a_{i,t} \) is total factor productivity (TFP) and \( \alpha \) and \( \gamma \) denote private and public capital shares in output. The production function has constant returns to scale in the two private inputs, \( n_{i,t} \) and \( k_{i,t} \), and increasing returns in all three factors.

Firms operate in a perfectly competitive environment. Therefore, as detailed in Appendix A.1, profit maximization implies that factors will be optimally allocated according to

\[ \alpha w_{i,t} n_{i,t} = (1 - \alpha) \kappa_{i,t} k_{i,t}, \quad \text{or} \quad \frac{w_{i,t}}{\kappa_{i,t}} = \left( \frac{1 - \alpha}{\alpha} \right) \frac{k_{i,t}}{n_{i,t}}, \] (2)

where \( w_{i,t} \) and \( \kappa_{i,t} \) denote the wage and rental rate of private capital in country \( i \) in terms of the intermediate goods produced in country 1, which is selected as the numeraire good for both countries. Note that \( \left( \frac{1 - \alpha}{\alpha} \right) \frac{k_{i,t}}{n_{i,t}} \) is the marginal rate of technical substitution and \( \frac{w_{i,t}}{\kappa_{i,t}} \) the isocost slope.

Optimal pricing under perfect competition implies that the production price equates marginal cost, that is,

\[ \alpha^{\alpha} (1 - \alpha)^{1-\alpha} a_{i,t} (k_{i,t})^\gamma = \kappa_{i,t} w_{i,t}^{1-\alpha}. \] (3)

The standard condition that factor prices equal their respective marginal products can be obtained from the combination of (2) and (3); hence,

\[ \alpha \frac{z_{i,t}}{k_{i,t}} = \kappa_{i,t}, \] (4)

\[ (1 - \alpha) \frac{z_{i,t}}{n_{i,t}} = w_{i,t}. \] (5)

### 2.1.2 Trade Cost and Transportation Infrastructure

Intermediate goods trade is costly and involves an iceberg-type trade cost, \( \tau_t \): for each good shipped, \( 1/(1 + \tau_t) \) goods arrive at the destination. Following Mun and Nakagawa (2010), it is assumed that the only cost incurred in trade is transportation; thus, the presence of \( \tau_t \) means that a certain fraction of the traded good disappears in transportation. Furthermore, this transportation cost is assumed to depend on the public stock of infrastructure: changes in this cost signify those in infrastructure. Hence, \( \frac{d\tau_t}{k_{g}^{1,t}} < 0 \) and \( \frac{d\tau_t}{k_{g}^{2,t}} > 0 \), such that an improvement in transportation infrastructure reduces the bilateral trade cost. Formally,

\[ \tau_t = \frac{\chi}{(k_{g}^{1,t})^\eta (k_{g}^{2,t})^{1-\eta}}, \] (6)

where \( \eta \) is the size of country 1 relative to country 2 and \( \chi \) is an adjustment parameter. Moreover, it is supposed that production and consumption in each country occur in a single location, which is defined as the market (i.e., intermediate goods are transported between markets in the two countries), and the importer country incurs the trade cost. In this case, by denoting \( q_t \) as the price of \( z_{2,t} \) in terms of the numeraire, \( z_{1,t} \), it follows that the price of one unit of imported intermediate good, \( z_{2,t} \), in country 1 is \( q_t (1 + \tau_t) \).

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8Public capital is provided without user charges (no congestion), and thus, public capital is assumed to be a pure public good.

9A similar reasoning is used by Martin and Rogers (1995) to examine the impact of public infrastructure on industrial location.
2.1.3 Final Goods Sector

Here, firms buy intermediate goods produced by both countries to produce a final good, which can be consumed or invested. The final good, \( y_i \), is produced according to the following constant elasticity of substitution (CES) technology function:

\[
y_{i,t} = \left[ \varphi_i^\mu (z_{i,t})^{\mu-1} + (1 - \varphi_i)^\mu (z_{ij,t})^{\mu-1} \right]^{\frac{1}{\mu}}, \text{ for } i, j = 1, 2 \text{ and } i \neq j, \tag{7}
\]

where \( \mu \geq 1 \) measures the substitutability between domestic and foreign goods. \( \varphi_i \in [0,1] \) governs the importance of the Home-produced (country 1) intermediate goods in the final goods production composite \( y_i \).\(^{10} \) \( z_{ii} \) and \( z_{ij} \) denote the units of Home- and Foreign-produced intermediate goods used as inputs in country \( i = 1, 2 \)'s final goods production. Note that \( z_{12} \) (21) is an imported input used to produce \( y_1 \) (22); thus, firms producing final goods face trade costs when importing inputs. Accordingly, they choose the quantity of Home and Foreign input to maximize their profits:

\[
\max_{z_{11,t}, z_{12,t}} \quad p_{1,t} y_{1,t} - [z_{11,t} + q_t (1 + \tau_t) z_{12,t}] \text{ for country 1}
\]

\[
\max_{z_{22,t}, z_{21,t}} \quad p_{2,t} y_{2,t} - [z_{22,t} + q_t^{-1} (1 + \tau_t) z_{21,t}] \text{ for country 2}
\]

subject to (7), where \( p_{i,t} \) is the relative price of the final good \( i \) in terms of the numeraire. As detailed in Appendix A.2, the optimal demands for \( z_{ii} \), and \( z_{ij} \) in country \( i \) are

\[
z_{11,t} = \varphi_1 p_{1,t}^{\mu} y_{1,t} ; \quad z_{12,t} = (1 - \varphi_1) \left[ \frac{p_{1,t}}{q_t (1 + \tau_t)} \right]^\mu y_{1,t} \tag{8}
\]

\[
z_{21,t} = (1 - \varphi_2) \left[ \frac{p_{2,t} q_t}{(1 + \tau_t)} \right]^\mu y_{2,t} ; \quad z_{22,t} = \varphi_2 p_{2,t}^{\mu} y_{2,t} \tag{9}
\]

and the price indexes of the final goods respectively are

\[
p_{1,t} = \left[ \varphi_1 + (1 - \varphi_1) \left[ q_t (1 + \tau_t) \right]^{1-\mu} \right]^{\frac{1}{1-\mu}}, \tag{10}
\]

\[
p_{2,t} = \left[ \varphi_2 + (1 - \varphi_2) \left[ \frac{(1 + \tau_t)}{q_t} \right]^{1-\mu} \right]^{\frac{1}{1-\mu}}. \tag{11}
\]

Accordingly, the real exchange rate can be defined as the relative price of final goods, that is,

\[
\frac{p_{2,t}}{p_{1,t}} = \left[ \varphi_2 + (1 - \varphi_2) \left[ q_t^{-1} (1 + \tau_t) \right]^{1-\mu} \right]^{\frac{1}{1-\mu}} \varphi_1 + (1 - \varphi_1) \left[ q_t (1 + \tau_t) \right]^{1-\mu} \right]^{\frac{1}{1-\mu}}.
\]

2.2 Representative Household

Each economy is inhabited by an infinitely lived representative household, which maximizes the expected intertemporal utility function:

\[
E_t \left\{ \sum_{t=0}^{\infty} \beta^t U(c_{i,s}, 1 - n_{i,s}) \right\}, \quad i = 1, 2, \tag{12}
\]

\(^{10}\varphi_i \) depends on the relative size of the Home economy (country 1), \( \eta \), and a trade openness measure, \( \theta_i \). Precisely, \( \varphi_1 = 1 - (1 - \eta) \theta \) and \( \varphi_2 = \eta \theta \).
where $\beta \in [0, 1]$ is the subjective discount factor and

$$U(c_{i,t}, 1 - n_{i,t}) = \ln (c_{i,t}) + \gamma_i \ln (1 - n_{i,t}), \quad i = 1, 2 \quad (13)$$

represents a household’s preferences over (state-contingent) domestic final goods consumption, $c_{it}$, and hours supplied to the labor market, $n_{i,t}$. A household’s time endowment is normalized to one, so that utility is obtained from leisure $(1 - n_{i,t})$ and consumption. Moreover, capital stock, $k_{i,t}$, is accumulated (to be rented to the local firm producing final goods) on the basis of the following law of motion:

$$x_{i,t} = k_{i,t+1} - (1 - \delta) k_{i,t}, \quad (14)$$

where $x_{i,t}$ denotes investment in domestic capital and $\delta \in [0, 1]$ is the depreciation rate. As in Mendoza (1991), capital accumulation is subject to quadratic adjustment costs:

$$AC_{i,t} = \frac{\phi}{2} (k_{i,t+1} - k_{i,t})^2. \quad (15)$$

In period $t \geq 0$, the representative household of country $i$ is assumed to have access to a complete contingent claims market. Let $r_{t,t+s}$ denote the stochastic discount factor, such that $E_t r_{t,t+s} d_{i,t+s}$ is the price in period $t$ of a random payment $d_{i,t+s}$ in terms of the final good 1 in period $t+s$. In addition, the representative household pays taxes on its total income at the rate of $\tau_{i,t}$. Hence, household $i$’s period-by-period budget constraint in terms of the numeraire is

$$p_{i,t} (c_{i,t} + x_{i,t} + AC_{i,t}) + E_{t} r_{t,t+s} d_{i,t+s} \leq (w_{i,t} n_{i,t} + \kappa_{i,t} k_{i,t}) (1 - \rho_{i,t}) + d_{1,t}. \quad (16)$$

The representative household $i$ chooses $c_{i,t}$, $n_{i,t}$, $k_{i,t+1}$, and $d_{i,t+1}$ to maximize (12) subject to (??), (14), and (15) and the following no-ponzi game constraint:

$$\lim_{s \to \infty} E_{t} r_{t,t+s} d_{i,t+s} \geq 0.$$

As detailed in Appendix A.3, the optimality conditions for the household problem can be summarized by

$$\frac{p_{i,t} c_{i,t}}{1 - n_{i,t}} = \frac{w_{i,t} \rho_{i,t}}{\gamma_i}, \quad (17)$$

$$1 + \phi (k_{i,t+1} - k_{i,t}) = \beta E_{t} \frac{c_{i,t}}{c_{i,t+1}} \left[ 1 - \delta + \phi (k_{i,t+2} - k_{i,t+1}) + \frac{\kappa_{i,t+1}}{p_{i,t+1}} (1 - \rho_{i,t+1}) \right], \quad (18)$$

$$\beta^{-1} E_{t} r_{t,t+1} = E_{t} \frac{p_{i,t} c_{i,t}}{p_{i,t+1} c_{i,t+1}}, \quad (19)$$

where (17) is the labor supply function that captures the intra-temporal trade-off between consumption and labor hours, (18) is the inter-temporal Euler equation that governs the accumulation of domestic capital, and $1 + \phi (k_{i,t+1} - k_{i,t})$ for $i = 1, 2$ corresponds to Tobin’s $Q$ standard representation. Moreover, the combination of (19) for $i = 1, 2$ results in the complete market risk-sharing condition, that is,

$$\theta p_{t,2} c_{2,t} = p_{1,t} c_{1,t}, \quad (20)$$

where $\theta$ is a constant that depends on the initial cross-country distribution of wealth.
2.3 Government

It is assumed that the entire revenue from taxing the representative agent’s income is used used for public infrastructure investment, such that the government’s budget constraint in country $i$ in terms of final good 1 is

$$p_{i,t}x_{1,t} = \rho_{i,t} (w_{i,t}n_{i,t} + \kappa_{i,t}k_{i,t}),$$

(21)

where $\delta_g$ is the infrastructure depreciation rate and $x_{g,t} = k_{g,t+1} - (1 - \delta_g)k_{g,t}$ is public investment, which is assumed to be the only public expenditure. It is convenient to interpret $\rho_{i,t}$ as a share of each country’s output that is allocated to infrastructure investments.

2.4 Market Clearings

The equilibrium in the state-contingent assets market is given by

$$d_{1,t} + d_{2,t} = 0.$$

The market clearing conditions in the intermediate goods markets are given by

$$\eta z_{1,t} = \eta z_{11,t} + z_{21,t},$$

(22)

$$z_{2,t} = z_{22,t} + \eta z_{12,t}.$$  

(23)

The final goods markets clear when the total purchases of final goods equal the total production of final goods, that is,

$$c_{1,t} + x_{1,t} + x_{g,1,t} + AC_{1,t} = y_{1,t},$$

(24)

$$c_{2,t} + x_{2,t} + x_{g,2,t} + AC_{2,t} = y_{2,t}.$$  

(25)

The relationship between the current account and trade balance is denoted by a combination of budget constraints, zero-profit conditions, and market clearing conditions:

$$E_t r_{t,t+1} d_{1,t+1} - d_{1,t} = z_{21,t} - (1 + \tau_t) q_t z_{12,t}.$$  

(26)

$$E_t r_{t,t+1} d_{2,t+1} - d_{2,t} = z_{12,t} - (1 + \tau_t) q_t^{-1} z_{21,t}.$$  

(27)

2.5 Equilibrium

An equilibrium is a set of prices and quantities that, for all $t > 0$, solves the household’s optimization problem, given budget constraints, borrowing constraints, and the capital law of motion, and the firms’ optimization problem, given the production functions, as well as satisfies all market clearings. Appendix A.4 provides a summary of the equilibrium conditions.

3 Quantitative Evaluation of the Model

3.1 Calibration

The quantitative analysis of the model focuses on Argentina (Home country, i.e., $i = 1$) and Brazil (Foreign country, i.e., $i = 2$). The model is parameterized at a quarterly frequency. In addition to country size, $n$, and home bias, $\varphi_i$, the two principal asymmetry sources in the model are government taxation, $\rho_i$, and TFP, $a_i$. The two sources are included to account for technological differences and publicly provided capital stock specificities in the analysis of individual output dynamics.
match the features of the concerned economies’ macroeconomic data for 2000–2011, using the Penn World Table (PWT) 8.0, IMF’s International Financial Statistics (IFS) and Government Finance Statistics (GFS), and the World Bank databases.\footnote{The period of 2000–2011 covers almost the entire lifetime of IIRSA. Remember that IIRSA was initiated in September 2000.} Table 4 resumes the retained parametrization.

**Common parameters.** The discount factor, $\beta$, is set to its conventional value of 0.99, implying an annual steady-state real interest rate of 4\%. Following Hulten (1996), Canning and Fay (1993), and Easterly and Rebelo (1993)’s average estimates for developing countries, the public infrastructure share in the production of intermediate goods, $\gamma$, is set to 0.10. The depreciation rate of private capital, $\delta$, is set to a standard value of 0.10 per year, corresponding to 0.025 per quarter. As for the public depreciation rate, the World Bank has estimated it to be about twice as high as $\delta$; thus, $\delta_g$ is set to 0.05. As is standard in literature, the elasticity of substitution between domestic and foreign goods is set to $\mu = 1.5$ (Backus et al., 1993) and capital adjustment costs are set to $\phi = 0.025$ (Mendoza, 1991). The capital share in intermediate goods production, $\alpha$, is set to 0.54 according to Elias (1992)’s estimation for seven Latin American countries, including Argentina and Brazil. The adjustment parameter, $\chi$, is set to match the benchmark steady-state transportation cost of $\tau = 0.16$, which is set using Hummels (1999)’s estimations for average expenditure on freight and insurance as a proportion of manufacturing imports’ value for Argentina (15.5\%) and Brazil (17.7\%). Thus, $\tau = 0.16$ is the Argentina–Brazil average of Hummels (1999)’s estimations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Argentina</th>
<th>Brazil</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP, $a_i$</td>
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<td>1.2488</td>
</tr>
<tr>
<td>Households’ income tax rate, $\tau_i$</td>
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</tr>
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<td>Home bias, $\varphi_i$</td>
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<tr>
<td>Weight on leisure in preferences, $\gamma_{n,i}$</td>
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<td>3.889</td>
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<tr>
<td>Discount factor, $\beta$</td>
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<td></td>
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<tr>
<td>Private capital depreciation rate, $\delta$</td>
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<tr>
<td>Public capital depreciation rate, $\delta_g$</td>
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<tr>
<td>Public infrastructure share in intermediate goods production, $\gamma$</td>
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<tr>
<td>Capital share in intermediate goods production, $\alpha$</td>
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<tr>
<td>Intermediate goods substitutability, $\mu$</td>
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<tr>
<td>Capital adjustment costs, $\phi$</td>
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<td></td>
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<tr>
<td>Adjustment parameter, $\chi$</td>
<td>0.462</td>
<td></td>
</tr>
</tbody>
</table>

Source: Original data from PWT 8.0, IFS, GFS, and World Bank databases.

Table 4: Calibrated parameters (Home country: Argentina).

**Country-specific parameters.** TFP, $a_i$, is set using PWT 8.0’s information for Argentina and Brazil (2000–2011 average). Accordingly, Argentina’s and Brazil’s TFP are 0.546 and 0.438, respectively. Foreign TFP is normalized to one (what matters is relative TFP), so that $a_1 = 1.2488$ and $a_2 = 1$.\footnote{PWT’s version 8.0 provides two types of TFP information: first is a relative measure of TFP levels across countries (relative to the United States, which is normalized to one) and the second type is conceived to compare TFP growth over time. For the purpose of this study, the first measure of TFP is retained.} As in Rioja (2003), the model assumes that tax income is used to finance public expenditure on infrastructure; thus, it is pertinent to calibrate $\rho_i$ as a fraction of GDP devoted to infrastructure investment in each country. Using data from IMF’s GFS database, the income taxation rates are set to $\rho_1 = 0.0348$ and $\rho_2 = 0.0149$.\footnote{The values are an average of the “Transport, Cash Expenses of the Budg. Cen. Govt.” item in IMF’s GFS database for 1995–2001.} The preference for Home-made intermediate goods in the final goods production composite is
computed considering the size of Argentina relative to Brazil, \( \eta \), and the trade openness of the domestic country (Argentina), \( \varphi \), on the basis of \( \varphi_1 = 1 - (1 - \eta) \varphi \) and \( \varphi_2 = \eta \varphi \). Using World Bank data, \( \eta \) is set to 0.2073 and \( \varphi \) corresponds to the Argentinian imports–GDP ratio, that is, \( \varphi = 0.139 \). Finally, the parameter governing the weight of leisure in preferences, \( \gamma_{n,i} \), is chosen such that, in the steady state, the hours worked are 30% of the daily available time; that is, for Argentina, \( \gamma_{n,1} = 3.664 \) and for Brazil, \( \gamma_{n,1} = 3.889 \).

### 3.2 Benchmark Steady State

The implied steady state is summarized in Table 5. As, according to the calibration, both Argentina’s productivity, \( a_1 \), and public investment, \( \rho_1 \), are superior, its public capital stock and real GDP are significantly higher than those in Brazil. The benchmark, thus, reflects a situation wherein Argentina is the wealthier country (e.g., higher consumption and real wages).

<table>
<thead>
<tr>
<th></th>
<th>Argentina ((i = 1))</th>
<th>Brazil ((i = 2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, ( y_i )</td>
<td>0.6614</td>
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<tr>
<td>Consumption, ( c_i )</td>
<td>0.4922</td>
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<td>Hours worked, ( n_i )</td>
<td>0.3000</td>
<td>0.3000</td>
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<tr>
<td>Private capital stock, ( k_i )</td>
<td>5.8704</td>
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<td>Capital rent rate ((\times100)), ( \kappa_i )</td>
<td>3.7300</td>
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<td>Public capital stock, ( k_{g_i} )</td>
<td>0.4492</td>
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<tr>
<td>Real wage, ( w_i )</td>
<td>2.7007</td>
<td>1.5457</td>
</tr>
<tr>
<td>Final good’s price index ( p_i )</td>
<td>1.0236</td>
<td>1.0089</td>
</tr>
<tr>
<td>Country ( i )’s imports (in ( y_i ) %)</td>
<td>8.6694</td>
<td>1.1062</td>
</tr>
<tr>
<td>Output gap, ( y_{gap} = y_1 - y_2 )</td>
<td></td>
<td>0.3577</td>
</tr>
</tbody>
</table>

*Source:* Benchmark steady-state values; the values correspond to the benchmark parameters detailed in Table 4.

Table 5: Steady-state benchmark values.

This benchmark is consistent with reality. According to the World Bank data, Argentina’s GDP PPP per capita (units) is 18,917.284, while that of Brazil is 12,525.674, respectively corresponding to 12.5% and 8.3% of the total South American GDP. Remember that the model accounts for size differences between the involved countries; thus, a per capita comparison is pertinent.\(^{15}\)

All experiments reported hereinafter reference this benchmark as a depart point.

### 4 Policy Experiments

This section analyzes the macroeconomic effects of increasing the share of output allocated to investment in transportation infrastructure \( \rho_i \). The focus is on the long-run evolution of output convergence, measured as the gap between both countries’ real GDP \( (y_{gap} = |y_1 - y_2|) \), and bilateral trade integration, measured by the standard openness index \( (\iota_i = (z_{1i} + z_{2i})/y_i) \).\(^{16}\)

\(^{15}\)Even if Argentina’s superiority is well accounted for in the benchmark, it does not perfectly match with reality. According to the model, Argentina’s output is 2.18 times Brazil’s output, while according to data, Argentina’s output is only 1.51 times that of Brazil. This imperfection does not change the essence of the conclusions but the quantification of the effects.

\(^{16}\)It is pertinent to use the standard openness index as a proxy measure of commercial integration in view of the retained two-country framework.
The experiments’ objective is twofold. First, they investigate the potential gain or loss (in terms of output convergence and trade integration) from raising publicly provided transportation infrastructure unilaterally (only one of the two country partners increases its GDP share of public investment) and bilaterally (both country partners raise its GDP share of public investment without caring (or knowing) what the other country does). Second is to elucidate whether a coordinated increase of public investment in transportation infrastructure, as undertaken by Unasur through Cosiplan, potentially improves such gains or losses.

To do so, a number of experiments are conducted and vary the size of $\rho_1$ and $\rho_2$ between their benchmark value ($\rho_1 = 0.035, \rho_2 = 0.015$) and 0.1. Each experiment re-solves the model using the new values of $\rho_1$ and $\rho_2$ and calculates the resulting net steady-state change in the variables of interest.

Before presenting the experiments’ results, it is worth recalling that the general equilibrium effects of a change in public investment through $\rho_i$ (a taxation change) are manifold. For instance, because governments invest the entire tax revenue in public infrastructure, a higher taxation rate, $\rho_i$, means higher public capital stock, $k_{g,t}$ (Eq. (21)). Then, higher public capital stock pushes up the production of domestic intermediate goods, $z_{i,t}$ (Eq. (1)), thus increasing the supply of inputs to final goods production which, in turn, could eventually increase output, $y_{i,t}$ (Eq. (7)).

On the other hand, expanded public capital stock reduces the cost of trade, $\tau_t$ (Eq. (6)), thus modifying the demand of foreign intermediate goods, $z_{ij,t}$ (Eqs. (8) and (9)) and domestic output, $y_{i,t}$. The improved availability of foreign inputs could improve the production of domestic final goods. However, less transportation cost also means higher demand for domestically produced intermediate goods abroad, and thus, less availability of domestic inputs in the local market and possibly, production of domestic final goods. The effect is, therefore, ambiguous. What is less ambiguous, however (i.e., because of the model’s construction), is that higher investment in public infrastructure is expected to directly increase the volume of exchanged merchandise between the two countries, and thus, increase commercial integration; this is true for Cosiplan because of the infrastructure projects’ nature and implementation objectives, as presented in Section 1.

Thus, the resulting effect on $y_{i,t}$ is a combination of several dynamics, without demand-side effects generated from a taxation increase in purchasing power, ergo consumption. This section focuses not on disentangling these multiple effects, even if conscientious of its importance, but on the interaction of these effects between the two countries by analyzing output gap (economic convergence) and commercial integration.

4.1 Unilateral policy

The first set of experiments comprises progressive increases in public investment in only one of the two countries. Remember that the principal sources of heterogeneity between both economies are government taxation (thus, public capital stock) and TFP. In the benchmark stage, Argentina is superior in both aspects. Hence, the Brazilian government’s attempt to increase public investment can be seen as a “catch-up” policy. Figure 1 graphically describes the effects of such a unilateral policy and displays the percentage change in the bilateral output gap from the benchmark for the several values of Brazil’s public investment, $\rho_2$. More specifically, Figure 1 presents the resulting effects on $y_1$, $y_2$, and $y_{gap}$ after successive increases of $\rho_2$: $\rho_2 \in [0.015, 0.1]$, assuming that Argentina does not implement any policy, notably that $\overline{\rho}_1$. According to Figure 1, the Brazilian policy positively affects its real GDP (the green curve is above the zero axis) and negatively affects Argentina’s output (the blue curve is below the zero axis). Indeed, the output net effect appears to favor Brazil’s catch up policy: output gap significantly

17Hereinafter, public investment refers to that in transportation infrastructure in GDP percent.
Figure 1: Brazil’s increased public investment in transportation infrastructure (GDP percent): $\rho_2$ and $\Delta \rho_1$

reduces (red line). However, this catch up causes an output loss in Argentina.$^{18}$ Thus, the result is a win-lose catch up.

Table 6 exposes the quantitative effects of the unilateral Brazilian policy. As expected, the cost of trade decreases ($\% \Delta \tau < 0$); however, the amount of exchanged merchandise increases unilaterally ($\Delta z_{12} < 0$ and $\% \Delta z_{21} > 0$). More specifically, Brazilian imports of Argentinian goods, $z_{21}$, increase, while Argentinian imports of Brazilian goods, $z_{12}$, decrease. As Argentina is worst off after the Brazilian policy ($\% \Delta y_1 < 0$) because of the home bias, its imports are reduced. The consequent effect on commercial integration is negative for Brazil, where aggregated exports and imports as a share of output, $(z_{12} + z_{21})/y_i$, decreases with the increase in taxation for the financing of public infrastructure ($\% \Delta \tau_2 < 0$).

Interestingly, Argentina’s capital stock is reduced because of the Brazilian policy. This is because, at the steady state, public capital stock is determined by not only $\rho_i$ but also $p_i$ (and $z_i$) (Eq. (71)). In turn, $p_i$ is determined by $\tau$ (Eqs. (65) and (66)). Thus, the one-sided improvement in Brazilian public capital stock reduces that of Argentina in the long run.$^{19}$

In sum, a unilateral catching-up policy is not as positive as expected, particularly in terms of economic convergence and commercial integration. In addition, further mechanisms need to be explored.

In continuing the experiment, the same logic is applied to Argentina. This time, Argentina

$^{18}$ A further version of the model is envisaged to account for individual welfare, such that the gains and/or losses of this experiment can be measured in welfare terms.

$^{19}$ As described in Appendix A.5, at the steady state, public capital stock is determined by $\rho_i$, $p_i$, and $z_i$: $k_i^g = (\rho_i/\delta_g p_i) z_i$. $p_i$, in turn, is determined by $\tau$: $\dot{p}_i = [\lambda + (1 - \lambda) (1 + \tau)]^{1/(1 - \mu)}$. Thus, capital stock in one country can be modified by the change in that of the other through $\tau_2 = \chi / [(k_{12}^g)^{\delta_g} (k_{21}^g)^{1 - \delta_g}]$.

$^{20}$ The transmission mechanism underlying the interaction of public capital stocks in the two neighboring countries will be pursued in future research.
Table 6: Unilateral increase of public investment in transportation infrastructure, $\Delta \rho_2 > 0$, in country $i = 2$

| $\rho_1$ | $\% \Delta k^1_2$ | $\% \Delta k^2_2$ | $\% \Delta \tau$ | $\% \Delta y_1$ | $\% \Delta y_2$ | $\% \Delta y_{gap}$ | $\% \Delta z_{12}$ | $\% \Delta z_{21}$ | $\% \Delta \iota_1$ | $\% \Delta \iota_2$
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</table>

unilaterally increases its taxation rate to finance new infrastructure projects. Figure 2 reports the percent change in $y_1$, $y_2$, and $y_{gap}$ after successive increases of $\rho_1$: $\rho_1 \in [0.035, 0.1]$, assuming that $\rho_2$. According to the figure, Argentina increases its real GDP ($\% \Delta y_1$) and Brazil suffers a diminishing real GDP. Considering the benchmark situation, this causes a considerable increase in the output gap (red line in Figure 2). Thus, the result is again a win-lose situation, but without an output catch up.

Table 7 presents some quantitative effects. As in the previous experiment, the cost of trade unambiguously declines; however, the opposite holds true for import demand. In this case, Argentinian imports of Brazilian goods increase, $\% \Delta z_{12} > 0$, while Brazilian imports of Argentinian goods decrease, $\% \Delta z_{21} < 0$. Interestingly, both countries appear to increase their commercial openness index ($\% \Delta \iota_1 > 0$ and $\% \Delta \iota_2 > 0$). Thus, the resulting net effect on commercial integration is positive.

In sum, a unilateral Argentinian policy is positive for commercial integration, but not for economic convergence between the two analyzed economies.

4.2 Coordinated and Uncoordinated Policies

One could also assume that both partner countries’ governments decide to increase investment in infrastructure, which is more likely in the long run. To analyze the effects of such a bilateral policy, two experiments are performed. First, it is supposed that Argentinian and Brazilian authorities decide to progressively increase their investment in public infrastructure by equal amounts: $\Delta \rho_1 = \Delta \rho_2 = 0.005$. This is a type of uncoordinated policy; that is, irrespective of what the partner country does, the government maintains their decision of a progressive increase of $\rho_i$. Second, the Cosiplan projects portfolio is considered when setting taxes; thus, $\Delta \rho_1$ and $\Delta \rho_2$ change coordinates in time, so that the required public investment to complete the regional infrastructure projects is attained.
Figure 2: Unilateral increase of public investment in transportation infrastructure (GDP percent): $\Delta \rho_1$, $\bar{\rho}_2$

<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$% \Delta k_1^1$</th>
<th>$% \Delta k_2^1$</th>
<th>$% \Delta \tau$</th>
<th>$% \Delta y_1$</th>
<th>$% \Delta y_2$</th>
<th>$% \Delta y_{gap}$</th>
<th>$% \Delta z_{12}$</th>
<th>$% \Delta z_{21}$</th>
<th>$% \Delta \iota_1$</th>
<th>$% \Delta \iota_2$</th>
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<td>-0.304</td>
<td>3.269</td>
<td>-0.14</td>
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<td>3.978</td>
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<td>-0.122</td>
<td>5.151</td>
<td>3.927</td>
<td>-0.77</td>
<td>0.622</td>
<td>3.839</td>
</tr>
<tr>
<td>0.065</td>
<td>11.340</td>
<td>0.051</td>
<td>-0.321</td>
<td>2.904</td>
<td>-0.107</td>
<td>4.767</td>
<td>3.820</td>
<td>-0.812</td>
<td>0.692</td>
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<td>4.447</td>
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<td>-0.338</td>
<td>2.639</td>
<td>-0.082</td>
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<td>0.820</td>
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<td>2.352</td>
<td>-0.052</td>
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<td>3.526</td>
<td>-1.008</td>
<td>0.991</td>
<td>3.419</td>
</tr>
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<td>0.095</td>
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<td>0.133</td>
<td>-0.369</td>
<td>2.276</td>
<td>-0.043</td>
<td>3.399</td>
<td>3.497</td>
<td>-1.044</td>
<td>1.043</td>
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</tr>
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<td>-0.377</td>
<td>2.208</td>
<td>-0.035</td>
<td>3.257</td>
<td>3.473</td>
<td>-1.079</td>
<td>1.093</td>
<td>3.361</td>
</tr>
</tbody>
</table>

Table 7: Unilateral increase of public investment in transportation infrastructure. $\Delta \rho_1 > 0$. in country $i = 1$
4.2.1 Uncoordinated Policy

Figure 3 describes the percent change of $y_1$, $y_2$, and $y_{gap}$ for different combinations of $\rho_1$ and $\rho_2$. Table 8 reports the corresponding quantitative effects. As described in Table 8, the experiment comprises successive increases of $\rho_i$ by 0.005, departing from the benchmark value. The net effect on real GDP after an improvement in public investment is positive for both countries (win-win scenario in terms of output growth). However, this effect is larger for Brazil, the country departing from a lower public investment rate, because of the usual diminishing returns of the model’s micro-foundations. The result is a positive net effect on output gap for low values of $\rho_i$ and a negative net effect thereafter. Note that a negative change in $y_{gap}$ is positive in terms of economic convergence.

![Figure 3: Increase of public investment in transportation infrastructure in both countries (GDP percent): $\Delta \rho_1 = \Delta \rho_2 = 0.005$](image)

<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$% \Delta k_1^{p}$</th>
<th>$% \Delta k_2^{p}$</th>
<th>$% \Delta \tau$</th>
<th>$% \Delta y_1$</th>
<th>$% \Delta y_2$</th>
<th>$% \Delta y_{gap}$</th>
<th>$% \Delta z_{12}$</th>
<th>$% \Delta z_{21}$</th>
<th>$% \Delta t_1$</th>
<th>$% \Delta t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.040</td>
<td>0.020</td>
<td>25.175</td>
<td>75.056</td>
<td>-7.833</td>
<td>4.248</td>
<td>11.218</td>
<td>-0.643</td>
<td>9.411</td>
<td>13.132</td>
<td>5.135</td>
<td>-1.454</td>
</tr>
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<td>0.025</td>
<td>15.98</td>
<td>43.241</td>
<td>-2.063</td>
<td>2.974</td>
<td>7.552</td>
<td>-0.621</td>
<td>4.022</td>
<td>8.216</td>
<td>1.232</td>
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<td>0.030</td>
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<td>6.141</td>
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<td>3.836</td>
<td>6.827</td>
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<td>-2.017</td>
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<td>5.257</td>
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<td>5.954</td>
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<td>12.09</td>
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<td>5.350</td>
<td>0.894</td>
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<tr>
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<td>0.045</td>
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<td>19.053</td>
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<td>2.589</td>
<td>4.201</td>
<td>1.012</td>
<td>3.353</td>
<td>4.904</td>
<td>0.833</td>
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<tr>
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<td>10.431</td>
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<td>3.22</td>
<td>4.559</td>
<td>0.783</td>
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<td>0.741</td>
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</tr>
</tbody>
</table>

Table 8: Increase of public investment in transportation infrastructure in both countries: $\Delta \rho_1 = \Delta \rho_2 = 0.005$

On the other hand, the simultaneous increase in public investment implies a positive net effect on public capital stock in both countries with a subsequent reduction in trade cost. Thus, overseas demands for both Brazilian and Argentinian intermediate products increase
(%Δz_{12} > 0 and %Δz_{21} > 0), implying increased international commerce between both nations. Nevertheless, the net effect on trade integration is different for the two countries. Argentina’s trade volume to GDP ratio, \((z_{12} + z_{21}) / y_i\), increases, while that of Brazil diminishes. Brazil’s larger net GDP improvement appears to smooth its openness index relative to Argentina.

On the whole, a simultaneous non-coordinated increase of public investment in infrastructure individually increases real output in both countries, but it does not reduce the output gap between them. Moreover, despite the increase in the volume of exchanged merchandise, the standard measure of trade openness does not evolve in the same direction in the two countries.

4.2.2 Coordinated Policy

This last experiment is based on the Cosiplan projects portfolio and is conducted in a setting with a combination of \(\rho_1\) and \(\rho_2\). As described in Table 2, Argentina and Brazil’s estimated investments in regional transportation projects is 24.7% and 40.7% of the total Cosiplan portfolio. Accordingly, Argentina’s relative estimated investment is 0.061 with respect to Brazil. Thus, the setting of the \(\rho_1\) and \(\rho_2\) values is such that each increment of 0.01 in \(\rho_2\) is equivalent to that of 0.0061 in \(\rho_1\). Table 9 reports the retained \(\rho_1 - \rho_2\) combinations and the resulting net steady-state effects on the selected variables. Figure 4 displays the percent change in \(y_1\), \(y_2\), and \(y_{\text{gap}}\).

![Figure 4: Coordinated increase of public investment in transportation infrastructure by both countries (GDP percent): \(\Delta \rho_1 = 0.0061\) and \(\Delta \rho_2 = 0.01\).](image)

According to Figure 4, similar to the precedent case, the resulting net effect on real output is positive for both countries (blue and green lines are above the zero axis): win-win scenario in terms of output growth. The main difference is the decline in the output gap (red line below the zero axis), implying a positive net effect on economic convergence. Table 9 quantifies these effects and reveals that both countries are unambiguously better off in terms of real output, after the coordinated bilateral public investment—that is, %\(\Delta y_1 > 0\) and %\(\Delta y_2 > 0\)—and in terms of output convergence, %\(\Delta y_{\text{gap}} < 0\). Thus, a coordinated policy does make a difference. Unasur’s initiative of building regional infrastructure as part of a coordinated investment plan
<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$% \Delta k^1$</th>
<th>$% \Delta k^2$</th>
<th>$% \Delta r$</th>
<th>$% \Delta y_1$</th>
<th>$% \Delta y_2$</th>
<th>$% \Delta y_{gap}$</th>
<th>$% \Delta z_{12}$</th>
<th>$% \Delta z_{21}$</th>
<th>$% \Delta y_1$</th>
<th>$% \Delta y_2$</th>
</tr>
</thead>
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<tr>
<td>0.035</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>0.035</td>
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<td>3.328</td>
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<td>3.704</td>
<td>9.45</td>
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<td>0.065</td>
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<td>-1.971</td>
</tr>
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<td>-1.988</td>
</tr>
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<td>1.632</td>
<td>7.109</td>
<td>-0.314</td>
<td>-1.896</td>
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</table>

Table 9: Coordinated increase of public investment in transportation infrastructure in both countries: $\Delta \rho_1 = 0.0061$ and $\Delta \rho_2 = 0.01$

is a potential convergence source. This finding supports, in the case of the South America, Figuières et al. (2013)'s conclusion on the long-run convergence of a balance growth paths in the case of the coordinated provision of public infrastructure.

The results for commercial integration are more or less the same as those in the previous case of non-coordination. A coordinated provision of public infrastructure between Argentina and Brazil, two neighboring commercial partner countries, is advantageous for commercial integration.

### 4.3 Output Convergence: Possibilities

Thus far, as summarized in Table 10, the results revealed that higher investment in public transportation in Argentina and Brazil, either uncoordinated or coordinated, benefits commercial integration; that is, the amount of exchanged merchandise unambiguously increases. The output convergence results depend on the type of policy; however, the effect on output convergence is only attained when the increase is coordinated by both governments.

<table>
<thead>
<tr>
<th>Trade integration</th>
<th>Output convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina (AR)</td>
<td>positive: $% \Delta z_{12} &gt; 0$ and $% \Delta z_{21} &lt; 0$ negative: $% \Delta y_{gap} &gt; 0$</td>
</tr>
<tr>
<td>Brazil (BR)</td>
<td>positive: $% \Delta z_{12} &lt; 0$ and $% \Delta z_{21} &gt; 0$ positive: $% \Delta y_{gap} &lt; 0$</td>
</tr>
<tr>
<td>AR–BR uncoordinated</td>
<td>positive: $% \Delta z_{12} &gt; 0$ and $% \Delta z_{21} &gt; 0$ negative: $% \Delta y_{gap} &gt; 0$</td>
</tr>
<tr>
<td>AR–BR coordinated</td>
<td>positive: $% \Delta z_{12} &gt; 0$ and $% \Delta z_{21} &gt; 0$ positive: $% \Delta y_{gap} &lt; 0$</td>
</tr>
</tbody>
</table>

Table 10: Summary of results

Figure 5 further generalizes the output convergence results and graphically describes the effects on output gap between Brazil and Argentina for all combinations of $\rho_1$ and $\rho_2$ between their benchmark value ($\rho_1 = 0.035, \rho_2 = 0.015$) and 0.1; that is, $\rho_1 \in (0.0035, 0.1), \rho_2 \in (0.0015, 0.1)$.

According to Figure 5, the output gap between Argentina and Brazil reaches its lowest value when Brazil, the less productive country (lowest TPF), invests 10% of its real GDP in transportation infrastructure, while Argentina remains at the benchmark. By contrast, the largest output gap between Argentina and Brazil is attained when Argentina, the more productive of the both (highest TPF), increases $\rho_1$ to 0.1. Thus, a coordinated policy does not permit the attainment of the lowest output gap; however, it is the only policy that provides a win-win scenario of output convergence.
Figure 5: *Net steady-state effect of a real output gap for several $\rho_1$ and $\rho_2$ combinations: $\rho_1 \in (0.0035, 0.1), \rho_2 \in (0.0015, 0.1)$.*

## 5 Conclusion

This study adopts a two-country general equilibrium framework to analyze commercial integration and output convergence effects when neighboring partner countries increase public investment in transportation infrastructure. The main findings are as follows.

First, Argentina and Brazil’s government decision to unilaterally increase their share of GDP allocated to infrastructure investment generates a win-lose situation in terms of economic growth, which does not necessarily imply output convergence. Only if Brazil—whose public investment and productivity is lower—(solely) improves its public investment share of output, does a catch-up situation emerge, namely, Brazil’s output grows, while that of Argentina decreases (win-lose-catch up). The contrary holds true when Argentina solely improves its public investment share of output: a win-lose divergence situation.

Second, increasing public investment in the two neighboring partner countries certainly provided an impetus to commercial integration between them, but did not necessarily generate output converge. In the case of Argentina and Brazil, only a coordinated public investment, as proposed by Unasur’s integration initiative: IIRSA-Cosiplan, can improve both commercial integration and output convergence.

Third, the lowest output gap is not attained by a coordinated public investment but by a unilateral catch-up policy. Nevertheless, coordination is the only way to achieve output convergence in a win-win economic growth scenario. This is a key point to consider when addressing regional policy issues and makes it easier to adopt a policy-generated win-win scenario in regional political negotiations.

It is worth mentioning that the main objective of IIRSA is to mitigate the biggest obstacles in physical integration (e.g., bottlenecks and missing links); however, as revealed by this study, this “physical integration” plan has a number of potential spillovers that are pro commercial integration and output convergence. Unasur, perhaps, has underestimated the projects’ effects on regional integration. To further investigate these effects, this study can be extended in
a number of directions. For instance, the results could be transposed to any pair of South American nations, given the nature of Unasur’s initiative, allowing a multi-country framework to provide interesting interactions to the effect of such an integration. Moreover, a welfare analysis is required before affirming that a potential long-run output convergence can generate regional development (convergence of living standards). Finally, the theoretical framework herein can be improved by endogenously determining the home bias parameter, which is crucial for the commercial channel.
References


Francois, J. and M. Manchin (2013). Institutions, infrastructure, and trade. World Development 46(0), 165–175.


A Appendix

A.1 Intermediate Sector Optimal Pricing

The profits derived from the optimal pricing for a competitive firm that produces intermediate goods and uses two private inputs, capital \((k_{i,t})\) and labor \((n_{i,t})\), is given by

\[
\Pi_t(z_{i,t}) = z_{i,t} - c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t}),
\]

where \(c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t})\) is the cost function. The firm selects its production level, \(z_{i,t}\), to maximize profits:

\[
\max \quad z_{i,t} - c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t}),
\]

which implies

\[
\Pi_{z_{i,t}} = 1 - c_{z_{i,t}}(z_{i,t}, \kappa_{i,t}, w_{i,t}) = 0,
\]

or, equivalently, the producer price equals marginal cost at \(z_{i,t}\) optimal.

Given the production function \((1)\), the cost function of the representative firm is defined as

\[
c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t}) = \min \ w_{i,t}n_{i,t} + \kappa_{i,t}k_{i,t} \quad \text{s.t.} \quad a_{i,t} \left( k_{i,t}^\alpha \right) \gamma \ k_{i,t}^{1-\alpha} \ k_{i,t} \geq z_{i,t}.
\]

Defining \(L = w_{i,t}n_{i,t} + \kappa_{i,t}k_{i,t} - \lambda [a_{i,t} \left( k_{i,t}^\alpha \right) \gamma \ k_{i,t}^{1-\alpha} - z_{i,t}]\) and solving it using the Lagrange method, gives us

\[
L_{k_{i,t}} = \kappa_{i,t} - \lambda \alpha a_{i,t} \left( k_{i,t}^\alpha \right) \gamma \ k_{i,t}^{1-\alpha} = 0,
\]

\[
L_{n_{i,t}} = w_{i,t} - \lambda (1-\alpha)a_{i,t} \left( k_{i,t}^\alpha \right) \gamma \ k_{i,t}^{1-\alpha} = 0,
\]

\[
L_{\lambda,t} = a_{i,t} \left( k_{i,t}^\gamma \right) k_{i,t}^{1-\alpha} - z_{i,t} = 0.
\]

The combination of \((29)\) and \((30)\) produces the optimal allocation of inputs:

\[
\alpha w_{i,t}n_{i,t} = (1-\alpha)\kappa_{i,t}k_{i,t}, \quad \text{or} \quad \frac{w_{i,t}}{\kappa_{i,t}} = \left( \frac{1-\alpha}{\alpha} \right) \frac{k_{i,t}}{n_{i,t}},
\]

where \(\left( \frac{1-\alpha}{\alpha} \right) \frac{k_{i,t}}{n_{i,t}}\) is the marginal rate of technical substitution and \(\frac{w_{i,t}}{\kappa_{i,t}}\) the isocost slope. Note that in rearranging the terms of the standard relationships and asserting them in a competitive environment, the price of production factors equals their marginal products:

\[
k_{i,t} = \left( \frac{\alpha}{1-\alpha} \right) \frac{w_{i,t}}{\kappa_{i,t}} n_{i,t}, \quad n_{i,t} = \left( \frac{1-\alpha}{\alpha} \right) \frac{\kappa_{i,t}}{w_{i,t}} k_{i,t}.
\]

Plug each one of the former optimal relationship into \((31)\) to obtain the conditional factor demands:

\[
a_{i,t} \left( k_{i,t}^\gamma \right) \left[ \frac{\alpha}{1-\alpha} \right] \frac{w_{i,t}}{\kappa_{i,t}} n_{i,t} = z_{i,t},
\]

\[
a_{i,t} \left( k_{i,t}^\gamma \right) \left[ \frac{\alpha}{1-\alpha} \right] \frac{w_{i,t}^\alpha}{\kappa_{i,t}^\alpha} n_{i,t}^{1-\alpha} = z_{i,t},
\]

\[
a_{i,t} \left( k_{i,t}^\gamma \right) \left[ \frac{\alpha}{1-\alpha} \right] \frac{w_{i,t}^\alpha}{\kappa_{i,t}^\alpha} = z_{i,t},
\]

\[
\left[ \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^\gamma \right) \alpha w_{i,t}} \right]^\alpha.
\]
\[
a_{i,t} \left( k_{i,t}^g \right)^\gamma k_{i,t} \left[ \left(\frac{1-\alpha}{\alpha}\right) \frac{\kappa_{i,t}}{w_{i,t}} \right]^{1-\alpha} = z_{i,t},
\]

\[
a_{i,t} \left( k_{i,t}^g \right)^\gamma k_{i,t} \left[ \left(\frac{1-\alpha}{\alpha}\right) \frac{\kappa_{i,t}}{w_{i,t}} \right]^{1-\alpha} = z_{i,t},
\]

\[
k_{i,t}(z_{i,t}) = \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \left[ \frac{\alpha w_{i,t}}{(1-\alpha) \kappa_{i,t}} \right]^{1-\alpha}, \quad n_{i,t}(z_{i,t}) = \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \left[ \frac{(1-\alpha) \kappa_{i,t}}{\alpha w_{i,t}} \right]^\alpha.
\]

The cost function \( c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t}) \) is then constructed by plugging the conditional factor demands into:

\[
c_{i,t}(z_{i,t}, \kappa_{i,t}, w_{i,t}) = w_{i,t} n_{i,t}(z_{i,t}) + \kappa_{i,t} k_{i,t}(z_{i,t})
\]

\[
= w_{i,t} \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \left[ \frac{1 - \alpha}{\alpha} \frac{\kappa_{i,t}}{w_{i,t}} \right]^{\alpha} + \kappa_{i,t} \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \left[ \frac{\alpha w_{i,t}}{(1-\alpha) \kappa_{i,t}} \right]^{1-\alpha},
\]

\[
= \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \kappa_{i,t}^{\alpha} w_{i,t}^{1-\alpha} \left[ \frac{\alpha - \alpha (1 - \alpha) + \alpha^{1-\alpha} (1 - \alpha)^{-1}}{\alpha} \right],
\]

\[
= \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \kappa_{i,t}^{\alpha} w_{i,t}^{1-\alpha} \left[ \alpha^{1-\alpha} (1 - \alpha)^{\alpha-\alpha} \alpha^{-\alpha} \right] \left[ 1 + \alpha (1 - \alpha)^{-1} \right],
\]

\[
= \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \kappa_{i,t}^{\alpha} w_{i,t}^{1-\alpha} \left( 1 - \alpha \right)^{\alpha-\alpha} \alpha^{-\alpha} \left( 1 - \alpha \right)^{-1},
\]

\[
= \frac{z_{i,t}}{a_{i,t} \left( k_{i,t}^g \right)^\gamma} \kappa_{i,t}^{\alpha} w_{i,t}^{1-\alpha} \left( 1 - \alpha \right)^{\alpha-1} \alpha^{-\alpha},
\]

which implies that the analytic expression of (28) is:

\[
1 = \frac{\kappa_{i,t}^{\alpha} w_{i,t}^{1-\alpha}}{\alpha^{\alpha} (1 - \alpha)^{1-\alpha} a_{i,t} \left( k_{i,t}^g \right)^\gamma}.
\]

Thus, the optimal pricing of a perfectly competitive firm producing intermediate goods 3 is derived.

### A.2 Optimal Demand of Intermediate Goods and Final Goods Price Index

The firm producing final goods in country 1 maximizes its profit:

\[
\max_{z_{11,t}, z_{12,t}} \Pi = p_{1,t} \left[ \frac{1}{\varphi_{1}^{\frac{1}{\mu}} (z_{11,t})^{\frac{\mu-1}{\mu}}} + (1 - \varphi_{1})^{\frac{1}{\mu}} (z_{12,t})^{\frac{\mu-1}{\mu}} \right]^{\frac{1}{\mu-1}} - z_{11,t} - q_t (1 + \tau_t) z_{12,t}.
\]

The associated first-order conditions are as follows:

\[
\Pi_{z_{11,t}} = p_{1,t} \left[ \frac{1}{\varphi_{1}^{\frac{1}{\mu}} (z_{11,t})^{\frac{\mu-1}{\mu}}} + (1 - \varphi_{1})^{\frac{1}{\mu}} (z_{12,t})^{\frac{\mu-1}{\mu}} \right]^{\frac{1}{\mu-1}} \varphi_{1}^{\frac{1}{\mu}} (z_{11,t})^{\frac{\mu-1}{\mu}} - 1 = 0,
\]

\[
\Pi_{z_{12,t}} = p_{1,t} \left[ \frac{1}{\varphi_{1}^{\frac{1}{\mu}} (z_{11,t})^{\frac{\mu-1}{\mu}}} + (1 - \varphi_{1})^{\frac{1}{\mu}} (z_{12,t})^{\frac{\mu-1}{\mu}} \right]^{\frac{1}{\mu-1}} (1 - \varphi_{1})^{\frac{1}{\mu}} (z_{12,t})^{-\frac{1}{\mu}} - q_t (1 + \tau_t) = 0.
\]
As $y_{1,t} = \left[ \frac{1}{\varphi_1} \left( z_{11,t} \right)^{-\frac{1}{\mu}} + (1 - \varphi_1)^\frac{1}{\mu} \left( z_{12,t} \right)^{-\frac{1}{\mu}} \right]^{\frac{\mu - 1}{\mu}}$, (33) and (34) become

$$p_{1,t}y_{1,t} \varphi_1^{-\frac{1}{\mu}} \left( z_{11,t} \right)^{-\frac{1}{\mu}} - 1 = 0,$$
$$p_{1,t}y_{1,t} (1 - \varphi_1)^\frac{1}{\mu} \left( z_{12,t} \right)^{-\frac{1}{\mu}} - q_t (1 + \tau_t) = 0.$$

Thus, the optimal demands for intermediate goods in country 1 are as follows:

$$z_{11,t} = \varphi_1 p_{1,t} y_{1,t}, \quad (35)$$
$$z_{12,t} = (1 - \varphi_1) \left[ \frac{p_{1,t}}{q_t (1 + \tau_t)} \right] y_{1,t}. \quad (36)$$

The final goods’ price index can be derived by plugging (35) and (36) into the standard zero-profit condition for the firm producing final goods:

$$p_{1,t}y_{1,t} = z_{11,t} + q_t (1 + \tau_t) z_{12,t},$$

which yields

$$p_{1,t} = \left[ \varphi_1 + (1 - \varphi_1) q_t^{1-\mu} (1 + \tau_t)^{1-\mu} \right]^{\frac{1}{1-\mu}}. \quad (37)$$

Analogously, the solution to the maximization problem of the firm producing final goods in country 2 is as follows:

$$\max_{z_{22,t},\ldots,z_{21,t}} : p_{2,t} \left[ \frac{1}{\varphi_2^\frac{\mu}{\mu-1}} \left( z_{22,t} \right)^{\frac{\mu-1}{\mu}} + (1 - \varphi_2)^\frac{1}{\mu} \left( z_{21,t} \right)^{\frac{1}{\mu}} \right]^{\frac{\mu-1}{\mu}} - z_{22,t} - q_t^{-1} (1 + \tau_t) z_{21,t},$$

which results in the following optimal demands for intermediate goods and the price index of final goods:

$$z_{22,t} = \varphi_2 p_{2,t} y_{2,t}, \quad (38)$$
$$z_{21,t} = (1 - \varphi_2) \left[ \frac{p_{2,t} q_t}{(1 + \tau_t)} \right] y_{2,t}, \quad (39)$$
$$p_{2,t} = \left[ \varphi_2 + (1 - \varphi_2) \left[ \frac{(1 + \tau_t)}{q_t} \right]^{1-\mu} \right]^{\frac{1}{1-\mu}}. \quad (40)$$

### A.3 Representative Household Problem

The representative household $i$ choose $c_{i,t}$, $n_{i,t}$, $k_{i,t+1}$, and $d_{i,t+1}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln (c_{i,t}) + \gamma_{n,i} \ln (1 - n_{i,t}) \}$$

subject to

$$p_{i,t} c_{i,t} + p_{i,t} \left[ k_{i,t+1} - (1 - \delta) k_{i,t} + \frac{\phi}{2} (k_{i,t+1} - k_{i,t})^2 \right] + E_t r_{t+1} d_{i,t+1} \leq (w_{i,t} n_{i,t} + \kappa_{i,t} k_{i,t}) (1 - \rho_{i,t}) + d_{i,t},$$

and the no-ponzi game constraint of the form $\lim_{s \to -\infty} E_t^r r_{t+s} d_{i,t+s} \geq 0$. Solving the household problem using Lagrange’s techniques implies the maximization of the following associated Lagrangian function:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \{ \ln (c_{i,t}) + \gamma_{n,i} \ln (1 - n_{i,t}) \} - \lambda_{i,t} \left[ p_{i,t} \left[ c_{i,t} + k_{i,t+1} - (1 - \delta) k_{i,t} + \frac{\phi}{2} (k_{i,t+1} - k_{i,t})^2 \right] + E_t r_{t+1} d_{i,t+1} \right] - \lambda_{i,t} \left[ - (w_{i,t} n_{i,t} + \kappa_{i,t} k_{i,t}) (1 - \rho_{i,t}) - d_{i,t} \right].$$
The resulting first order-order conditions are as follows:

\[ \beta^t \frac{1}{c_{i,t}} - p_{i,t} \lambda_{i,t} = 0 \]

\[-\beta^t \gamma_{n,i} \frac{1}{1 - n_{i,t}} + \lambda_{i,t} w_{i,t} (1 - \rho_{i,t}) = 0 \]

\[-\lambda_{i,t} \{ p_{i,t} [1 + \phi (k_{i,t+1} - k_{i,t})] \} - \lambda_{i,t+1} \{ p_{i,t+1} [-1 + \delta - \phi (k_{i,t+2} - k_{i,t+1})] - \kappa_{i,t+1} (1 - \rho_{i,t+1}) \} = 0 \]

\[-\lambda_{i,t} E_t r_{t,t+1} + \lambda_{i,t+1} = 0. \]

Eliminating the Lagrange multiplier using \( \lambda_{i,t} = \frac{\beta^t}{p_{i,t} c_{i,t}} \) yields

\[ \frac{\beta^t \gamma_{n,i}}{1 - n_{i,t}} + \frac{\beta^t}{p_{i,t} c_{i,t}} w_{i,t} (1 - \rho_{i,t}) \]

\[ = \frac{\beta^t \gamma_{n,i}}{1 - n_{i,t}} \{ p_{i,t+1} [-1 + \delta - \phi (k_{i,t+2} - k_{i,t+1})] - \kappa_{i,t+1} (1 - \rho_{i,t+1}) \} \]

\[ = \frac{\beta^t}{p_{i,t} c_{i,t}} p_{i,t+1} \gamma_{n,i} \frac{1}{1 - n_{i,t}} \{ 1 + \delta - \phi (k_{i,t+2} - k_{i,t+1}) + \kappa_{i,t+1} (1 - \rho_{i,t+1}) \} \]

\[ = \frac{\beta^t}{p_{i,t} c_{i,t}} r_{t,t+1}. \]

The first order-conditions can, thus, be summarized as

\[ \frac{p_{i,t} c_{i,t}}{1 - n_{i,t}} = \frac{w_{i,t} (1 - \rho_{i,t})}{\gamma_{n,i}} \]

\[ , 1 + \phi (k_{i,t+1} - k_{i,t}) = \beta E_t \frac{c_{i,t+1}}{c_{i,t+1}} \left[ 1 - \delta + \phi (k_{i,t+2} - k_{i,t+1}) + \kappa_{i,t+1} (1 - \rho_{i,t+1}) \right] \]

\[ , \beta^{-1} E_t r_{t,t+1} = \frac{E_t}{p_{i,t} c_{i,t+1}} = \frac{E_t}{p_{i,t} c_{i,t+1}}. \]

Equation (41) is the labor supply function that captures the intra-temporal trade-off between consumption and time spent working. Equation (42) is the inter-temporal Euler equation that governs the assumption of domestic capital. Finally, equation (43) will be applied later to derive the risk-sharing condition under the assumption of complete markets.

To obtain the risk-sharing condition under the assumption of complete markets, combine (43) for \( i = 1, 2 \) to obtain \( \frac{p_{1,t} c_{1,t}}{p_{2,t} c_{2,t}} = \frac{p_{1,t+1} c_{1,t+1}}{p_{2,t+1} c_{2,t+1}} \), which if iterated one period forward, yields \( \frac{p_{1,t+1} c_{1,t+1}}{p_{2,t+1} c_{2,t+1}} = \frac{p_{1,t+2} c_{1,t+2}}{p_{2,t+2} c_{2,t+2}} \). Now, plug the latter to the first so that

\[ \frac{p_{1,t} c_{1,t}}{p_{2,t} c_{2,t}} = \frac{p_{1,t+2} c_{1,t+2}}{p_{2,t+2} c_{2,t+2}}. \]

After repetitive substitutions, the general formulation for time \( t + k \) yields

\[ \frac{p_{1,t} c_{1,t}}{p_{2,t} c_{2,t}} = \frac{p_{1,t+k} c_{1,t+k}}{p_{2,t+k} c_{2,t+k}}, \]

which at date 0 is

\[ \frac{p_{1,0} c_{1,0}}{p_{2,0} c_{2,0}} = \frac{p_{1,t} c_{1,t}}{p_{2,t} c_{2,t}}. \]

Finally, let \( \theta = \frac{p_{1,0} c_{1,0}}{p_{2,0} c_{2,0}} \) be a constant that depends on the initial cross-country distribution of wealth to obtain the following risk-sharing condition:

\[ \theta p_{2,t} c_{2,t} = p_{1,t} c_{1,t}. \]

**A.4 Summary of Equilibrium Conditions**

**Labor supply:**

\[ \gamma_{n,1} \frac{p_{1,t} c_{1,t}}{1 - n_{1,t}} = w_{1,t} (1 - \rho_{1,t}), \]

\[ \gamma_{n,2} \frac{p_{2,t} c_{2,t}}{1 - n_{2,t}} = w_{2,t} (1 - \rho_{2,t}). \]

**Accumulation of physical capital:**

\[ 1 + \phi (k_{1,t+1} - k_{1,t}) = \beta E_t \frac{c_{1,t}}{c_{1,t+1}} \left[ 1 - \delta + \phi (k_{1,t+2} - k_{1,t+1}) + \kappa_{1,t+1} (1 - \rho_{1,t+1}) \right], \]
$1 + \phi (k_{2,t+1} - k_{2,t}) = \beta E_t \frac{c_{2,t}}{c_{2,t+1}} \left[ 1 - \delta + \phi (k_{2,t+2} - k_{2,t+1}) + \frac{k_{2,t+1}}{p_{2,t+1}} (1 - \rho_{2,t+1}) \right]. \quad (48)$

Complete markets' risk-sharing condition:

$\theta p_{2,t} c_{2,t} = p_{1,t} c_{1,t}. \quad (49)$

Intermediate goods production functions:

$z_{1,t} = a_{1,t} \left( k_{1,t}^\alpha \right)^\gamma k_{1,t}^{1-\alpha} \theta_1, \quad (50)$

$z_{2,t} = a_{2,t} \left( k_{2,t}^\alpha \right)^\gamma k_{2,t}^{1-\alpha} \theta_2. \quad (51)$

Intermediate sector factors' use:

$\alpha w_{1,t} n_{1,t} = (1 - \alpha) \kappa_{1,t} k_{1,t} \theta_3, \quad (52)$

$\alpha w_{2,t} n_{2,t} = (1 - \alpha) \kappa_{2,t} k_{2,t} \theta_4. \quad (53)$

Intermediate goods pricing:

$a^\alpha (1 - \alpha)^{1-\alpha} a_{1,t} \left( k_{1,t}^\alpha \right)^\gamma = \kappa_{1,t}^{\alpha} w_{1,t}^{1-\alpha}, \quad (54)$

$a^\alpha (1 - \alpha)^{1-\alpha} a_{2,t} \left( k_{2,t}^\alpha \right)^\gamma = \kappa_{2,t}^{\alpha} w_{2,t}^{1-\alpha}. \quad (55)$

Transportation cost:

$\tau_t = \frac{\chi}{(k_{1,t}^\theta)(k_{2,t}^\theta)}^{1-\eta} \theta_5. \quad (56)$

Final goods price indexes:

$p_{1,t} = \left[ \varphi_1 + (1 - \varphi_1) q_1^{1-\mu} (1 + \tau_1)^{1-\mu} \right]^{i_1}. \quad (57)$

$p_{2,t} = \left[ \varphi_2 + (1 - \varphi_2) \left( \frac{1 + \tau_1}{q_1} \right) \right]^{i_2}. \quad (58)$

Intermediate goods market clearing:

$\eta z_{1,t} = \eta \varphi_1 p_{1,t}^\mu y_{1,t} + (1 - \varphi_2) \left[ \frac{p_{2,t} q_t}{(1 + \tau_1)^{1-\mu}} \right]^{i_2} \theta_6, \quad (59)$

$z_{2,t} = \varphi_2 p_{2,t}^\mu y_{2,t} + \eta (1 - \varphi_1) \left[ \frac{p_{1,t}}{q_t (1 + \tau_1)^{1-\mu}} \right]^{i_1} y_{1,t}. \quad (60)$

Accumulation of public capital:

$k_{1,t+1} = (1 - \delta) k_{1,t} \theta_7, \quad (61)$

$k_{2,t+1} = (1 - \delta) k_{2,t} \theta_8. \quad (62)$

Final goods market clearing:

$c_{1,t} + k_{1,t+1} - (1 - \delta) k_{1,t} + k_{1,t+1}^{\alpha} - (1 - \delta) k_{1,t}^{\alpha} + \phi \frac{k_{1,t+1} - k_{1,t}}{2} = y_{1,t}, \quad (63)$

$c_{2,t} + k_{2,t+1} - (1 - \delta) k_{2,t} + k_{2,t+1}^{\alpha} - (1 - \delta) k_{2,t}^{\alpha} + \phi \frac{k_{2,t+1} - k_{2,t}}{2} = y_{2,t}. \quad (64)$

Hence, the system contains 20 endogenous variables: $p_{1,t}, c_{1,t}, n_{1,t}, w_{1,t}, p_{2,t}, c_{2,t}, n_{2,t}, w_{2,t}, k_{1,t}, k_{1,t}, k_{2,t}, k_{2,t}, \kappa_{2,t}, z_{1,t}, z_{2,t}, k_{1,t}^\theta, k_{2,t}^\theta, \tau_1, q_1, y_{1,t},$ and $y_{2,t}$; four exogenous variables: $a_1, a_2, p_1,$ and $p_2$; and 20 equations.

Note that, at the intermediate firm optimum $\alpha \frac{z_{1,t}}{n_{1,t}} = \kappa_{1,t}$ and $(1 - \alpha) \frac{z_{1,t}}{n_{1,t}} = w_{1,t}$, so $w_{1,t} n_{1,t} + \kappa_{1,t} k_{1,t} = z_{1,t}$. 21
A.5 Steady State

At the steady state, the model is static (all variables are constant in time). Thus, Tobin’s Q representation—\(1 + \phi(k_i - k_i)\) for \(i = 1, 2\)—equals one, and the adjustment cost \(\frac{\phi}{2}(k_i - k_i)^2\) disappears. Next, let us set \(q = 1\), so that the final goods’ price indexes can be recovered from

\[
\bar{p}_1 = \left[ \varphi_1 + (1 - \varphi_1)(1 + \tau)^{1 - \mu} \right]^{\frac{\gamma}{2}}, \tag{65}
\]

\[
\bar{p}_2 = \left[ \varphi_2 + (1 - \varphi_2)(1 + \tau)^{1 - \mu} \right]^{\frac{\gamma}{2}}. \tag{66}
\]

Now, use the dynamics of physical capital accumulation, equations (47) and (48), to obtain:

\[
\beta^{-1} - 1 + \delta \left(1 - \rho_1\right)\bar{p}_1 = \bar{k}_1,
\]

\[
\beta^{-1} - 1 + \delta \left(1 - \rho_2\right)\bar{p}_2 = \bar{k}_2.
\]

Once \(\bar{k}_1\) and \(\bar{k}_2\) are known,\(^{22}\) use (54), (55), (52), and (53), combined with (61) and (62), to find \(\bar{z}_1\) and \(\bar{z}_2\), as follows. First, use (52) and (53) to express private capital as a function of hours worked (which are set to 0.3 at the steady state); that is,

\[
k_1 = \frac{\alpha w_1 n_1}{(1 - \alpha)\bar{k}_1}, \tag{67}
\]

\[
k_2 = \frac{\alpha w_2 n_2}{(1 - \alpha)\bar{k}_2}. \tag{68}
\]

Moreover, according to (54) and (55), real wages are

\[
(1 - \alpha) \left[ \frac{\alpha}{\bar{k}_1} \right]^{\frac{\gamma}{1 - \alpha}} a_1^{\frac{\alpha}{1 - \alpha}} (k_1^g)^{\frac{\alpha}{1 - \alpha}} = w_1, \tag{69}
\]

\[
(1 - \alpha) \left[ \frac{\alpha}{\bar{k}_2} \right]^{\frac{\gamma}{1 - \alpha}} a_2^{\frac{\alpha}{1 - \alpha}} (k_2^g)^{\frac{\alpha}{1 - \alpha}} = w_2. \tag{70}
\]

Now, insert both expressions, (69) ((70)) and (67) ((68)), into the intermediate good production function, (50) ((51)), to obtain

\[
z_1 = a_1 (k_1^g)^{\gamma} k_1^g n_1^{1 - \alpha},
\]

\[
z_1 = a_1 (k_1^g)^{\gamma} \frac{\alpha}{(1 - \alpha)\bar{k}_1} (w_1)^\alpha \bar{n}_1,
\]

\[
z_1 = a_1 (k_1^g)^{\gamma} \frac{\alpha}{(1 - \alpha)\bar{k}_1} (1 - \alpha)^\alpha \left[ \frac{\alpha}{\bar{k}_1} \right]^{\frac{\gamma}{1 - \alpha}} a_1^{\frac{\alpha}{1 - \alpha}} (k_1^g)^{\frac{\gamma}{1 - \alpha}} \bar{n}_1,
\]

\[
z_1 = a_1^{\frac{\gamma}{1 - \alpha}} (k_1^g)^{\frac{\gamma}{1 - \alpha}} \left[ \frac{\alpha}{\bar{k}_1} \right]^{\frac{\gamma}{1 - \alpha}} \bar{n}_1.
\]

From (61) ((62)), it is known that

\[
k_1^g = \frac{\bar{p}_1}{\delta_g p_1} z_1; \quad k_2^g = \frac{\bar{p}_2}{\delta_g p_2} z_2. \tag{71}
\]

Accordingly, \(\tilde{z}_1\) (\(\tilde{z}_2\)) is given by

\[
\tilde{z}_1 = \left\{ a_1 \left( \frac{\bar{p}_1}{\delta_g p_1} \right)^{\gamma} \left( \frac{\alpha}{\bar{k}_1} \right)^{\alpha} \bar{n}_1 \right\}^{\frac{1 - \alpha}{1 - \gamma}} ; \quad \tilde{z}_2 = \left\{ a_2 \left( \frac{\bar{p}_2}{\delta_g p_2} \right)^{\gamma} \left( \frac{\alpha}{\bar{k}_2} \right)^{\alpha} \bar{n}_2 \right\}^{\frac{1 - \alpha}{1 - \gamma}}.
\]

Once \(\tilde{z}_1\) and \(\tilde{z}_2\) are known, public capital is

\[
\bar{k}_1^g = \frac{\bar{p}_1}{\delta_g p_1} \tilde{z}_1; \quad \bar{k}_2^g = \frac{\bar{p}_2}{\delta_g p_2} \tilde{z}_2.
\]

\(^{22}\)The bar refers to the steady-state values.
and, as \( \tau \) is set to 0.16; the trade cost equation allows us to retrieve \( \chi \) from

\[
(\bar{k}_1^g)^\eta (\bar{k}_2^g)^{1-\eta} = \chi.
\]

Furthermore, combining (69) and (70) with (67) and (68) (and using \( z_i = \bar{a}_i (k_i^g)\gamma k_i^a \bar{n}_i^{1-\alpha} \)) permits the recovery of the steady-state value for private capital and wages from

\[
\bar{k}_1 = \alpha \frac{\bar{z}_1}{\bar{k}_1}; \quad \bar{k}_2 = \alpha \frac{\bar{z}_2}{\bar{k}_2},
\]

\[
\bar{w}_1 = (1 - \alpha) \frac{\bar{z}_1}{\bar{n}_1}; \quad \bar{w}_2 = (1 - \alpha) \frac{\bar{z}_2}{\bar{n}_2}.
\]

It follows that \( \bar{y}_1 \) and \( \bar{y}_2 \) can be recovered from the intermediate goods market’s clearing conditions; that is,

\[
\eta \bar{z}_1 = \eta \varphi_1 \bar{p}_1^m y_1 + (1 - \varphi_2) \bar{p}_2^m (1 + \bar{\tau})^{-\mu} y_2, \quad (72)
\]

\[
\bar{z}_2 = \varphi_2 \bar{p}_2^m y_2 + \eta (1 - \varphi_1) \bar{p}_1^m (1 + \bar{\tau})^{-\mu} y_1. \quad (73)
\]

To do so, plug (73) of the form

\[
\left[ \bar{z}_2 - \eta (1 - \varphi_1) \bar{p}_1^m (1 + \bar{\tau})^{-\mu} y_1 \right] \varphi_2^{-1} \bar{p}_2^{-\mu} = y_2
\]

into (72) to get

\[
\bar{y}_1 = \left[ \eta \bar{z}_1 - \frac{(1 - \varphi_2)}{\varphi_2} (1 + \bar{\tau})^{-\mu} \bar{z}_2 \right] \left[ \varphi_1 - \frac{(1 - \varphi_2)(1 - \varphi_1)}{\varphi_2} (1 + \bar{\tau})^{-2\mu} \right]^{-1} \frac{1}{\eta \bar{p}_1^m}.
\]

Thus,

\[
\bar{y}_2 = \frac{1}{\varphi_2 \bar{p}_2^m} \left[ \bar{z}_2 - \eta (1 - \varphi_1) \bar{p}_1^m (1 + \bar{\tau})^{-\mu} \bar{y}_1 \right].
\]

Finally, \( c_1 \) and \( c_2 \) are recovered from the final goods market’s clearing conditions:

\[
c_1 + \delta \bar{k}_1 + \delta_g \bar{k}_1^g = \bar{y}_1,
\]

\[
c_2 + \delta \bar{k}_2 + \delta_g \bar{k}_2^g = \bar{y}_2.
\]

Note that as \( n_1 \) and \( n_2 \) are calibrated at the steady state, \( \gamma_{n,1} \) and \( \gamma_{n,2} \) can be retrieved from the labor supply equations, (45) and (46); that is,

\[
\gamma_{n,1} = \frac{\bar{w}_1 (1 - \bar{n}_1) (1 - \bar{\rho}_1)}{\bar{p}_1 \bar{c}_1},
\]

\[
\gamma_{n,2} = \frac{\bar{w}_2 (1 - \bar{n}_2) (1 - \bar{\rho}_2)}{\bar{p}_2 \bar{c}_2}.
\]

Similarly, \( \theta \) is set using the risk-sharing condition (49):

\[
\theta = \frac{\bar{p}_1 \bar{c}_1}{\bar{p}_2 \bar{c}_2}.
\]

Hence, the steady-state value of all variables is known.