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Abstract

In this paper, we study an eco-industry providing an environmental service to a competitive polluting sector. We show that even if this eco-industry is highly concentrated, a standard environmental policy based on a Pigouvian tax or a pollution permit market reaches the first-best outcome, challenging the Tinbergen rule. To illustrate this point, we first consider an upstream monopoly selling eco-services to a representative polluting firm. We progressively extend our result to heterogeneous downstream polluters and heterogeneous upstream Cournot competitors. Finally, we underline some limits of this result. It does not hold under the assumption of abatement goods or downstream market power. In this last case, we obtain Barnett's result.

Key words: Environmental regulation, Eco-industry, Imperfect Competition, Abatement services
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1. Introduction

The so-called Environmental Goods and Services Sector (EGSS hereafter) and its link to environmental regulation is nowadays clearly recognized. This sector "*includes the provision of environmental technologies, goods and services for every kind of use, i.e. intermediate and final consumption as well as gross capital formation*" (Eurostats' handbook on EGSS [13]). Even if some methodological problems remain (see the UNEP report, [35]), the EGSS is largely documented by most of the statistical institutes.[4] They agree upon the fact that the size of the EGSS remains relatively moderate: the share of the EGSS gross value-added in the GDP is around 2.0% in both Europe and the US. But they also point out the notable growth rate of this sector, its capacity to generate new job opportunities and its export performance. For instance, estimates for the European Union show an increase of EGSS output per unit of GDP of 50 % between 2000 and 2011, while employment grew at around 40%.

Two other observations concerning the EGSS seem to be important. First, it is widely acknowledged that this sector is controlled by worldwide firms like CH2M Hill, Veolia Environmental Services, Vivendi Environment or Suez Environnement. The Ecorys report [11] on the European EGSS even mentions that 10% of the companies account for almost 80% of the operating revenue. Secondly, a large share of EGSS activity is dedicated to the provision of environmental services. Environmental services represent more than 40% of this sector's activity (see Sainclair Desgagné [33] table 2) and are largely involved in international trade.[5]

These two last observations motivate our paper. The question is quite simple: should we regulate a polluting industry in the same way when these firms supply abatement goods to an imperfectly competitive eco-industry as when they supply abatement services? This raises a second question: what is the difference between abatement goods and abatement services? If we follow the Eurostats' handbook on EGSS [11]: "*Services are outputs produced to order and which cannot be traded separately from their production. Services are not separate entities over which ownership rights can be established*". In the context of end-of-pipe abatement, this means that the polluter buys "pollution reduction" without concerning itself about the way this task is performed: it simply outsources this activity. This is, for instance, the case for waste management, water sewerage and treatment, remediation and clean-up activities. In other words, since services are *not separate entities*, they cannot be used as an input in the polluter's production process, unlike environmental goods such as filters, scrubbers or incinerators, over which the polluter keeps some control. This simple observation fundamentally modifies the polluter's purchasing behavior. When

[4] Several empirical studies have recently sought to quantify the EGSS. For instance, the Canadian statistical institute [34] conducts a biennial survey of the EGGS (http: //www.statcan.gc.ca /eng /survey /business /1209). In Europe, Eurostats has initiated a study over 28 member states (see http: //ec.europa.eu /eurostat /statistics-explained /index.php /Environmental_goods_and_services_sector) based on a methodology described in Eutostats [13]. In 2010, the US department of commerce published a survey called "Measuring the Green Economy" [36].

[5] See for instance the report of the US international trade commision [37].

he buys abatement services, i.e. pollution reduction, he only makes a trade-off between the price of this service and the cost of non-compliance with an environmental act. While for environmental goods, he must also take into account the marginal rate of pollution abatement, since these goods are viewed as inputs that reduce emissions.

If the eco-industry is imperfectly competitive, the difference between abatement goods and abatement services also affects the expected demand for abatement and therefore the strategic behaviors of the members of this industry. For instance, under a Pigouvian tax, the purchasing behavior for abatement services is only motivated by the difference between the environmental tax and the price of the abatement service. Thus, the regulator implicitly controls the market for abatement services by setting the level of the Pigouvian tax. By exploiting these particular features, we show that the regulator can obtain the first-best outcome simply by setting a Pigovian tax equal to the marginal damage. In other words, the first-best outcome can be reached with only one economic policy tool, although there are two market failures in this economy: market power on the market for abatement services and pollution. This result challenges the Tinbergen rule and suggests that the environmental agency must distinguish between the regulation of goods and the regulation of services.

To the best of our knowledge, this distinction has not been introduced into the eco-industry literature. If we invoke the previous Eurostats' definition, most of the contributions on these vertical structures are concerned with either environmental technologies and R&D or the provision of abatement goods.

The first branch considers the incentives provided by environmental policy instruments for the adoption and development of advanced abatement technology (see Requate [27] for an overview). Not all of these contributions explicitly introduce an EGSS, since this requires innovation to be a private good. The studies which go in this direction often consider an innovative firm investing in R&D to obtain a patent over a pollution-reducing new technology. Within this framework, the performance of taxes and tradeable permits are compared under various settings. Denicolo [10] and Requate [28] make these comparisons under different timing and commitment regimes. A threat of imitation is introduced by Fischer et al. [14] while Perino [25] studies green horizontal innovation, where new technologies reduce pollution of one type while causing a new type of damage. More recently, Perino [26] focuses on the second-best policies for all combinations of emission intensity and marginal abatement costs.

The second branch, which is closer to our contribution, takes as given the existence of imperfect competition in the eco-industry selling abatement goods to a polluting sector and explores the second-best regulation policy under alternative instruments. Greaker [16] and Greaker and Rosendahl [17] introduce emission standards. David and Sinclair-Desgagné [7] and Canton et al. [6] explore the case of the Pigouvian tax, the former with imperfect competition upstream and the latter with imperfect competition both upstream and downstream, while Schwartz and Stahn [32] introduce tradable pollution rights. Endres and Friehe [12] examine the impact of environmental liability laws. Some other papers like David et al. [9] or Canton et al. [5] introduce the entry or merger of firms in the eco-industry.

Our contribution is also very close to Nimubona and Sinclair-Desgagné [24], who initiate a discussion about an internal abatement effort and an external procurement of abatement facilities. However, they do not depart from the Katsoulacos and Xepapadeas [20] formulation of an end-of-pipe pollution which is common to almost all papers on the eco-industry, according to which emissions depend on the levels of production and abatement. Most of these papers also assume that abatement facilities have decreasing returns. In other words, they function as an input and not as a service, which would have constant returns.[6] This is why the first-best outcome can be obtained with only one instrument, contrary to David and Sinclair-Desgagné [8], who introduce both Pigouvian tax and subsidy.

The intuition behind our main result is quite simple. If a Pigouvian tax is imposed, a polluter purchasing an *eco-service* either (i) prefers to pay the tax if the price of the abatement service is higher, (ii) decides, in the opposite case, to fully abate the pollution issued from its equilibrium production level, or (iii) is indifferent between the two if the price and the tax are equal. This implies that the demand for eco-services becomes perfectly elastic over a range of quantities which depends on the tax level, so that any monopoly selling these services loses - at least partially - its market power. If the regulator is able to set a tax level with the property that the monopoly solution belongs to this range of quantities, he clearly destroys upstream monopoly power and has the opportunity, if the downstream polluting market is competitive, to reach a first-best allocation. Owing to the structure of the demand for eco-services, this situation occurs when the monopoly has an incentive to set the highest price for which the demand is positive, *i.e.* the tax level, and to supply, due to marginal cost concerns, a quantity of services lower than the one corresponding to full pollution abatement, so that we end up in a situation in which the price equates the tax level and the marginal cost. If the efficient abatement level does not require full abatement, it remains for the regulator to set the Pigouvian tax at the optimal marginal damage in order to obtain the first best.

This argument clearly holds with homogeneous competitive polluters, an eco-service monopoly and an optimal abatement level that does not require full abatement. That is why we start with this benchmark case. We then show that our argument can easily be extended in order to (i) include the "boundary" solutions corresponding to efficient full pollution abatement,[7] (ii) take into account regulation by a pollution permit market, and (iii) consider polluters who are heterogeneous with regard to their production costs and emissions. The extension of our analysis to Cournot competition in the eco-industry is less obvious, which is why we have dedicated a whole section to this study. As both main assumptions nevertheless remain in this new case - upstream eco-services and downstream

[6] Our end-of-pipe emission reduction technology can therefore be viewed as a particular case of the Katsoulacos and Xepapadeas [20] emission function in which the abatement good has constant returns to scale. To the best of our knowledge, this case has not been explored, probably for technical reasons: standard differential calculus does not really apply and corner solutions emerge.

[7] The case of efficient full downstream pollution abatement is particularly conceivable if the upstream eco-industry is also polluting, as in Sans et al. [30].

perfect competition - our result still holds. We finally exhibit some limits of our analysis, taking into account first abatement goods and then downstream imperfect competition. This last section enables us firstly, to underline the fact that the existence of eco-services is crucial, and secondly, to extend Barnett's results [2] on Pigouvian taxation to eco-service industries.

The structure of the article is as follows. Section 2 describes the model. Section 3 presents the simplest case: downstream homogeneous polluters and an upstream monopoly. Section 4 introduces some straightforward extensions: downstream full abatement, pollution permit market and heterogeneous downstream firms. Section 5 is dedicated to Cournot competition in the eco-industry. Section 6 challenges both main assumptions: environmental services and downstream perfect competition. Finally, some concluding remarks are given in Section 7 and technical proofs are relegated to an appendix.

2. A basic model of environmental services

We first present the main assumptions of the model and then we characterize the first-best allocation.

2.1. The main assumptions

We consider a standard *polluting industry* first characterized by a representative firm which produces a quantity Q at a given cost $c(Q)$ and latter by heterogeneous firms. This cost is increasing and convex (i.e., $c'(Q) > 0$ and $c''(Q) > 0$), inaction is allowed (i.e., $c(0) = 0$), $c'(0) = 0$ and $\lim_{q\to+\infty} c'(q) = +\infty$. This activity is polluting. Emissions are given by $\varepsilon(Q)$, an increasing and convex function (i.e., $\varepsilon'(Q) > 0$ and $\varepsilon''(Q) > 0$) which satisfies $\varepsilon(0) = 0$, $\varepsilon'(0) = 0$ and $\lim_{q\to+\infty} \varepsilon'(q) = +\infty$. This dirty firm can buy environmental services to reduce its "end-of-pipe" pollution. In doing so, a part A of its emissions is abated by a specialized external firm and the remaining pollution is $E = \max\{\varepsilon(Q) - A, 0\}$.

The *eco-services* are supplied on a non-competitive market at price p_A. We initially assume that these services are provided by a monopoly. This firm is characterized by an increasing and convex cost function and inaction is allowed (i.e., $\kappa'(a) > 0$, $\kappa''(a) > 0$ and $\kappa(0) = 0$). We also assume that $\kappa'(0) = 0$ to ensure that the eco-service market is activated when an environmental policy is implemented.[8]

The environmental damage induced by the remaining emissions E is measured by a standard damage function $D(E)$. As usual, this function is increasing and convex (i.e., $D'(E) > 0$ and $D''(E) > 0$) and without emission there is no damage (i.e., $D(0) = 0$). We

[8] A discussion about the emergence of an eco-industry related to the fact that $\kappa'(0) > 0$ can be found in Canton et al. [6].

also set $D'(0) = 0$. This last assumption is essentially made for convenience: it ensures that full abatement never occurs at an efficient allocation.[9]

Finally, to close the model, we introduce an *inverse demand function* for the polluting goods $P(Q)$. This function is decreasing (i.e., $P'(Q) < 0$) and verifies that $\lim_{Q \to 0} P(Q) = +\infty$ and $\lim_{Q \to +\infty} P(Q) = 0$.

2.2. The first-best allocation

Under these assumptions, a first-best allocation is given by:

$$\left(Q^{opt}, A^{opt}\right) \in \arg\max_{Q,A \geq 0} \int_0^Q P\left(q\right) dq - c\left(Q\right) - D\left(\max\left\{\varepsilon\left(Q\right) - A, 0\right\}\right) - \kappa(A) \tag{1}$$

This is typically a non-smooth optimization problem, but remember that we have assumed that $D(0) = 0$ and $D'(0) = 0$. The first equality ensures that the optimal level of abatement cannot be larger than emissions because abatement is costly, hence $\varepsilon\left(Q^{opt}\right) - A^{opt} \geq 0$, while the second combined with the positivity of the marginal cost of abatement ensures that this inequality holds strictly. Consequently, the first-best allocation is characterized by the usual first order conditions:

$$P(Q^{opt}) - c'\left(Q^{opt}\right) - D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right).\varepsilon'\left(Q^{opt}\right) = 0 \tag{2}$$

$$D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \tag{3}$$

Let us now introduce the function $\beta(Q) = \frac{P(Q) - c'(Q)}{\varepsilon'(Q)}$ defined on $[0, Q_{\max}]$ where $Q_{\max}$ stands for the optimal level of production without environmental damage (i.e., $P(Q_{\max}) = c'\left(Q_{\max}\right)$). This function measures, for each $Q \leq Q_{\max}$, the marginal benefit from an additional unit of pollution. Therefore an optimal allocation has the property that the marginal benefit of pollution is equal to (i) the marginal damage and (ii) the marginal cost of abating an additional unit of pollution:

$$\beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) = \kappa'(A^{opt}) \tag{4}$$

For later use, let us also notice this marginal benefit is decreasing and $\beta(Q_{\max}) = 0$ so that $\beta^{-1} : [0, +\infty] \to [0, Q_{\max}]$ is defined.

3. Upstream monopoly power and first-best regulation

Let us first show that a policy maker reaches the efficient allocation with a standard Pigouvian tax scheme even if the provider of environmental services has a monopoly power. To illustrate this point, we proceed in three steps. We first introduce a Pigouvian tax and

[9] If the marginal damage at zero is high enough and/or the marginal abatement cost is not too excessive, the "end-of-pipe" pollution assumption, i.e., $E = \max\left\{\varepsilon(Q) - A, 0\right\}$, can lead to an efficient allocation requiring full abatement (see Sans et al. [30] for a discussion). In Section 4.1, we extend our result to this case, but this requires additional discussions which are not central to our main argument.

compute the inverse demand for abatement services under a downstream market clearing assumption. This brings us, in a second step, to the characterization of the behavior of the upstream monopolist whatever the Pigouvian tax is. It remains, in the last step, to show that a Pigouvian tax equal to the marginal damage regulates both environmental and market power inefficiencies.

3.1. The (inverse) demand for abatement services

The competitive dirty firm chooses its production supply and its demand for the abatement good by solving:

$$\max_{Q\geq 0}\left\{ p_Q \times Q - c\left(Q\right) - \underbrace{\min_{A\geq 0}\left\{p_A \times A + \tau \times \max\left\{\varepsilon\left(Q\right) - A, 0\right\}\right\}}_{=C_A(p_A,\tau,Q)} \right\} \tag{5}$$

An inspection of the cost minimization part of this program shows that the conditional demand for abatement services never exceeds $\varepsilon\left(Q\right)$ and that the objective function is linear in A on $[0, \varepsilon\left(Q\right)]$. Both properties imply that the conditional demand for abatement services is either 0 or $\varepsilon(Q)$ when $p_A > \tau$ or $p_A < \tau$ respectively, and any quantity within $[0, \varepsilon\left(Q\right)]$ if $p_A = \tau$. Hence, the abatement cost is given by $C_A(p_A, \tau, Q) = \min\left\{p_A, \tau\right\} \times \varepsilon(Q)$. The optimal product supply therefore solves the following FOC:

$$p_Q - c'\left(Q\right) - \min\left\{p_A, \tau\right\} \times \varepsilon'\left(Q\right) \leq 0 \text{ (with equality if } Q > 0) \tag{6}$$

If we now introduce the market clearing condition for the final good, we can replace p_Q by $P\left(Q\right)$, and, using the above definition of $\beta\left(Q\right)$, i.e., the marginal benefit of an additional unit of pollution, this quantity is given by:

$$\frac{P(Q) - c'(Q)}{\varepsilon'(Q)} = \min\left\{p_A, \tau\right\} \Rightarrow Q\left(p_A, \tau\right) = \beta^{-1}\left(\min\left\{p_A, \tau\right\}\right) \tag{7}$$

and the demand for abatement services becomes:

$$A^d\left(p_A, \tau\right) = \begin{cases} 0 & \text{if } p_A > \tau \\ \left[0, \varepsilon\left(\beta^{-1}\left(\tau\right)\right)\right] & \text{if } p_A = \tau \\ \varepsilon\left(\beta^{-1}\left(p_A\right)\right) & \text{if } p_A < \tau \end{cases} \tag{8}$$

The last two equations (Eqs (7) and (8)) stress the consequence of introducing abatement services as opposed to abatement goods. In the first case, the dirty firm simply delegates its abatement activity to another firm, while in the second case, the firm buys additional inputs which enter the production process and help to reduce pollution in a more or less efficient way. This means that the dirty firm considers that the marginal pollution abatement of an environmental service is constant (even equal to one). Its purchasing is therefore simply motivated by the difference between the price of the service and the Pigouvian tax. If the price is higher than the Pigouvian tax, it is optimal to pay the tax and to adjust the output level due to this additional cost. In the opposite case, the firm totally abates its emissions but adjusts the production level, as previously, since the abatement price now enters the global marginal production cost.

By introducing abatement services instead of abatement goods, we therefore obtain a particular abatement demand curve (see Eq (8)) characterized by a flat part when the price is equal to the Pigouvian tax and a decreasing part for prices lower than this tax and corresponding to full abatement. When this price goes to 0, we even observe, by our above definition of β, that the equilibrium production level will be equal to $Q_{\max}$, the production level without regulation. In other words, this demand has an upper bound given by $A^d(0,\tau) = \varepsilon(Q_{\max})$ which is independent of τ, so that the associated inverse demand function is:

$$P_A(A,\tau) = \max\left\{\min\left\{\tau, \beta\left(\varepsilon^{-1}(A)\right)\right\}, 0\right\} \tag{9}$$

3.2. The monopoly provision of environmental services

We now address the question of how a monopoly behaves in the face of such an inverse demand curve. Since the demand is bounded from above by $\varepsilon(Q_{\max})$ which is reached at a zero price, its production choice can be restricted to $A \in [0, \varepsilon(Q_{\max})]$ and its optimal decision solves:

$$\max_{A \in [0,\varepsilon(Q_{\max})]} \{\min\{\tau, p(A)\} \times A - \kappa(A)\} \tag{10}$$

where $p(A) = \beta(\varepsilon^{-1}(A))$ is the inverse demand curve corresponding to full pollution abatement that occurs if the Pigouvian tax is larger than this price.

Since we maximize a continuous function on a compact set, existence is not a real issue. But the characterization of this solution requires some additional concavity properties. So let us assume, as usual for a monopoly, that $e_p(A) = \frac{p'A}{p}$, the elasticity of $p(A)$ is decreasing and $\lim_{A \to \varepsilon(Q_{\max})} e_p(A)$ is larger than -1.[10]

The inverse demand curve $\min\{\tau, p(A)\}$ nevertheless exhibits a flat part since the dirty firm only reacts to prices lower than the Pigouvian tax. We are therefore dealing with a non-smooth optimization problem leading to several regimes delineated by thresholds which are related to the levels of the Pigouvian tax.

To get some intuitions (see Figure 1), let us first introduce the production level A_m. This corresponds to the monopoly solution under full pollution abatement behavior by the dirty firm, i.e., the quantity that equates the marginal cost to the marginal revenue computed with $p(A)$. The price associated with this full abatement case is therefore given by $p(A_m) = \frac{1}{1+e_p(A_m)}\kappa'(A_m)$, where $\frac{1}{1+e_p(A_m)}$ stands for the standard margin taken by a

[10] Of course, the reader may object that these assumptions are not set on the primary data, especially given that $p(A) = \beta\left(\varepsilon^{-1}(A)\right)$. Other sufficient conditions can be introduced, such as $2e_\varepsilon + e_{\beta'} - e_{\varepsilon'} > 0$ and $e_\varepsilon + e_\beta > 0$, where e denotes the elasticity.

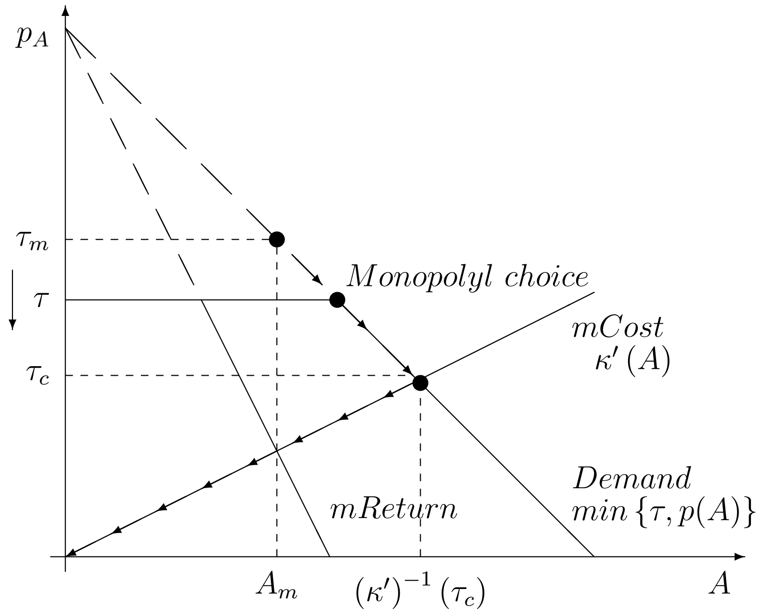

Figure 1: The monopoly solution as τ decreases

monopolist. But this situation only occurs if the Pigouvian tax is higher than this price, i.e., $\tau \geq p(A_m)$, otherwise the dirty firm pays the tax instead of abating pollution. This means that there exists a tax rate t_m implicitly given by:

$$\tau_m = \frac{1}{1+e_p(p^{-1}(\tau_m))}\kappa'(p^{-1}(\tau_m)) \tag{11}$$

for which $p(A_m) = \tau_m$, and we can conclude that, for $\tau > \tau_m$, the monopoly always provides A_m units of abatement services.

If the tax rate is lower than τ_m, the monopoly is unable to reach this optimal outcome simply because the monopoly price associated with full abatement is not reachable. In this case, this firm has an incentive to choose the solution that leads to the highest price $p = \tau$ at the production level $A = p^{-1}(\tau)$, i.e., to remain at the kink in the demand function. But this behavior is only optimal for prices $p = \tau$ which are larger than the marginal production cost, i.e., $\kappa'(p^{-1}(\tau))$. If this is not the case, the firm will adjusts its behavior to equate the tax rate with the marginal cost. This means that there exists another threshold $\tau_c < \tau_m$ with the property that $\forall \tau < \tau_c$, the monopoly adopts, in some sense, a competitive behavior. This new threshold is given by:

$$\tau_c = \kappa'(p^{-1}(\tau_c)) \tag{12}$$

From this discussion, we conclude that:

Lemma 1. *Under our assumptions, (i) the monopoly problem (Eq 10) has a unique solution for each tax rate, (ii) there exist two unique thresholds τ_c and τ_m which solve Eq*

(12) and Eq (11) respectively, (iii) the monopoly provision of abatement services is, for any tax τ, a continuous function given by:

$$A^m(\tau) = \begin{cases} (\kappa')^{-1}(\tau) & \text{if } \tau < \tau_c \\ p^{-1}(\tau) = \varepsilon\left(\beta^{-1}(\tau)\right) & \text{if } \tau \in [\tau_c, \tau_m] \\ A_m = p^{-1}(\tau_m) = \varepsilon\left(\beta^{-1}(\tau_m)\right) & \text{if } \tau > \tau_m \end{cases} \tag{13}$$

(iv) the price of these services is $P_A^m(\tau) = \min\{\tau, \tau_m\}$, and (v) from Eq (7), the production of the dirty good is:

$$Q^m(\tau) = \beta^{-1}(\min\{\tau, \tau_m\}) \tag{14}$$

3.3. *The efficient regulation of emissions*

The previous lemma has an interesting consequence: for any tax rate lower than τ_c, the monopoly behaves like a competitive firm. This firm equates its marginal cost with the tax rate, which is nothing other than the price of the abatement services. Since the polluting firm also behaves competitively, the regulator should be able to implement the first-best allocation, by selecting, as in a competitive case, a Pigouvian tax equal to the marginal damage of pollution, i.e., by setting $\tau^{opt} = D'(\varepsilon(Q^{opt}) - A^{opt})$.

This point is obvious as long as $\tau^{opt} < \tau_c$. In this case, we know, from Eq (13), that the monopoly provision of environmental services verifies $\kappa'(A^m(\tau^{opt})) = \tau^{opt}$, while Eq (14) says that $\beta(Q^m(\tau^{opt})) = \tau^{opt}$, and since β and κ' are both monotonic, we conclude, by identification with Eq (4), that $Q^m(\tau^{opt}) = Q^{opt}$ and $A^m(\tau^{opt}) = A^{opt}$, i.e., that the first-best allocation is reached.

It therefore remains to verify that $\tau^{opt} < \tau_c$. Intuitively, τ_c is by definition (see Figure 1) (i) the highest tax level at which the monopoly behaves competitively and (ii) the lowest rate at which full abatement occurs, so that any tax $\tau < \tau_c$ induces perfect competition and partial abatement. Since there is also partial abatement at the optimum, τ^{opt} should be one of them. If this is not the case, *i.e.* $\tau^{opt} \geq \tau_c$, we know from the definition of the threshold τ_c (see Eq (12)) that:

$$\tau^{opt} \geq \kappa'(p^{-1}\left(\tau^{opt}\right)) \Leftrightarrow (\kappa')^{-1}\left(\tau^{opt}\right) \geq \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) \text{ since } p(A) = \beta\left(\varepsilon^{-1}(A)\right) \tag{15}$$

Moreover, by Eq (4), which characterizes the first-best solution, $(\kappa')^{-1}(\tau^{opt}) = A^{opt}$ and $\beta^{-1}(\tau^{opt}) = Q^{opt}$, so that $A^{opt} \geq \varepsilon(Q^{opt})$. But from our discussion about the efficient allocation, we know that the assumptions $D(0) = D'(0) = 0$ ensure that there is always a residual pollution at the optimum, i.e., that $\varepsilon(Q^{opt}) > A^{opt}$. We can therefore say:

Proposition 1. *Even if an upstream monopoly controls the price of the environmental services while the downstream commodity market remains competitive, the regulator reaches the first-best by setting the Pigouvian tax at the marginal damage of the emissions (evaluated at the first-best), i.e., by setting $\tau^{opt} = D'(\varepsilon(Q^{opt}) - A^{opt})$.*

4. Some straightforward extensions

The previous result suggests that the existence of a residual pollution at the optimal outcome is a crucial assumption. However, as we will see, this is only a simplifying assumption: an identical result can be obtained for $D'(0) \neq 0$. In this section, we can also examine whether the result is maintained when the regulator uses a different incentive-based mechanism such as tradable pollution permits. The answer is again yes as long as this new market is competitive. Finally, we relax the representative polluting firm assumption and investigate heterogeneous polluters.

4.1. Efficient regulation and full abatement

To illustrate this point, let us return to the construction of the efficient outcome and relax $D'(0) = 0$ [11]. This outcome solves the optimization program (Eq (1)) introduced in Section 2.2. But if we only assume that $D(0) = 0$, we can only argue that $\varepsilon(Q) - A \geq 0$, (i.e., without a strict inequality). The interior first-order optimality conditions given by Eqs (2) and (3) must therefore be amended. If λ denotes the associated Lagrangian multiplier, the new FOC become:

$$\begin{cases} P(Q^{opt}) - c'(Q^{opt}) - (D'(\varepsilon(Q^{opt}) - A^{opt}) - \lambda)\,\varepsilon'(Q^{opt}) = 0 \\ D'(\varepsilon(Q^{opt}) - A^{opt}) - \kappa'(A^{opt}) - \lambda = 0 \\ \lambda(\varepsilon(Q^{opt}) - A^{opt}) = 0 \text{ and } \lambda \geq 0 \end{cases} \tag{16}$$

If the constraint is not binding, we are, of course, back in the case of partial abatement, analyzed above. So let us concentrate on the case in which $\lambda > 0$. In this situation, the first and second conditions of system (16) suggest that an efficient allocation has the property that the marginal benefit $\beta(Q^{opt})$ of an additional unit of pollution must be equal to the marginal abatement cost. But to achieve full abatement, this marginal benefit only needs to be smaller than the marginal damage of the first unit of pollution. This situation essentially occurs if $D'(0)$ is high enough. In this case, the efficient allocation verifies:

$$\begin{cases} E^{opt} = \varepsilon(Q^{opt}) - A^{opt} = 0 \\ \beta(Q^{opt}) = \kappa'(A^{opt}) < D'(0) \end{cases} \tag{17}$$

instead of the interior condition introduced in Eq. (4).

Let us now return to the monopoly case. Since the marginal damage never enters the definition of the different behaviors, the monopoly outcome depicted in Lemma 1 remains unchanged. This means that we simply have to ensure that the regulator is able to obtain the first-best solution when it is optimal to abate all the pollution (i.e. for $\lambda > 0$).

So let us assume that he sets the Pigouvian tax at $\tau^{opt} = \tau^c$ given by Eq (12). From Lemma 1, the equilibrium abatement and production levels are $A^m(\tau^c) = \varepsilon(\beta^{-1}(\tau^c))$ and $Q^m(\tau^c) = \beta^{-1}(\tau^c)$, so that the first optimality condition of Eq (17) is satisfied. It

[11] This situation is, for instance, met when the damage function is linear. Full abatement is required when the damage coefficient is large enough.

then remains for us to use the definition of τ^c to verify the second condition. This is $\tau_c = \kappa'(p^{-1}(\tau_c))$, so that $\beta(Q^m(\tau^c)) = \kappa'(A^m(\tau_c))$. We can therefore note:

Proposition 2. *Assume that the marginal damage of the first unit of pollution is sufficiently large for full abatement to become the efficient outcome. If the regulator sets the Pigouvian tax at $\tau^{opt} = \tau^c$ given by Eq (12), he again obtains the first-best outcome.*

4.2. Pollution permit market

Let us now verify that our result also holds if the regulator implements a pollution permit market instead of a Pigouvian tax. To illustrate this point, let us return to the monopoly case depicted in Section 3 and introduce a competitive market of pollution rights. The regulator sets the pollution cap E. Without loss of generality, we assume that pollution permits are sold by means of an auction.[12] One right corresponds to one unit of emission and the competitive price of these rights is denoted by p_E.

At the agent level, the competitive permit price operates like a Pigouvian tax. Under our assumptions, the results obtained in Section 3 concerning the inverse demand and the supply of abatement services by the monopoly extend to this case: it simply remains for us to replace the Pigouvian tax τ by the price p_E of the emission rights. Thus, our result is maintained if there exists a pollution cap E^{opt} with the property that the equilibrium price of the pollution rights is equal to the optimal level of the Pigouvian tax introduced in Proposition 1. This nevertheless leaves two questions open: (i) what is the value of this pollution cap?, and (ii), more crucially, is $p_E = \tau^{opt}$ the unique equilibrium of the pollution permit market when this cap is set? Otherwise there may be several equilibria, some of them being inefficient.

To answer these two questions, let us observe that in our new setting, the quantities introduced in Eqs (13) and (14) of Lemma 1 describe the equilibrium allocation conditional on each pollution permit price p_E. So if we want to look at the global equilibrium of this vertical structure with tradable rights, we must clear the permit market. The demand for pollution rights is given by:

$$E^D(p_E) = \varepsilon\left(Q^m\left(p_E\right)\right) - A^m\left(p_E\right) \tag{18}$$

and we observe that for any price $p_E \geq \tau_c$ there is full abatement, hence:

$$E^D(p_E) = \begin{cases} \varepsilon\left(\beta^{-1}\left(p_E\right)\right) - \left(\kappa'\right)^{-1}\left(p_E\right) \text{ if } p_E < \tau_c \\ 0 \text{ if } p_E \geq \tau_c \end{cases} \tag{19}$$

Now let us recall from our earlier discussion that $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) < \tau_c$. Hence, the optimal pollution cap must be:

$$E^{opt} = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) - \left(\kappa'\right)^{-1}\left(\tau^{opt}\right) \tag{20}$$

[12] For simplicity, we do not introduce the initial distribution of pollution permits explicitly. Following Montgomery [22], the competitive equilibrium of a pollution permit market is obtained irrespective of the initial distribution of permits.

Moreover, to ensure that $p_E = \tau^{opt}$ is the unique equilibrium, let us observe that the demand for tradable rights is decreasing for all $p_E < \tau_c$:

$$\frac{dE^D(p_E)}{dp_E} = \frac{\varepsilon'\left(\beta^{-1}\left(p_E\right)\right)}{\beta'\left(\beta^{-1}\left(p_E\right)\right)} - \left(\kappa''\left(\left(\kappa'\right)^{-1}\left(p_E\right)\right)\right)^{-1} < 0 \tag{21}$$

since under our assumptions, $\kappa"$, $\varepsilon' > 0$ and $\beta' < 0$. We can therefore say:

Proposition 3. *If pollution is regulated by a market of pollution rights, the regulator also achieves an efficient allocation by choosing the optimal pollution cap E^{opt} given by Eq. (20).*

4.3. Heterogeneous polluters

Finally, it is also interesting to verify whether this result extends to heterogeneous polluters. So let us introduce m polluting firms, indexed by j, with different cost and emission functions, $c_j(q)$ and $\varepsilon_j(q)$, each of them satisfying the assumptions introduced in Section 2. All the other assumptions are maintained, especially those concerning the marginal damage at 0, so that an efficient allocation is now given by:

$$\forall j \quad P(\textstyle\sum_{j=1}^{m} q_j^{opt}) - c_j'\left(q_j^{opt}\right) - D'\left(\sum_{j=1}^{m}\varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right).\varepsilon_j'\left(q_j^{opt}\right) = 0 \tag{22a}$$

$$D'\left(\textstyle\sum_{j=1}^{m}\varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \tag{22b}$$

The intuition behind this extension is quite simple. Even if the polluting firms are heterogeneous in costs and/or emissions, they invariably choose their level of abatement by comparing the price p_A with the Pigouvian tax τ. One can therefore expect the *aggregate* demand for abatement goods to behave in the same way: no abatement if $p_A > \tau$, full abatement denoted $A_f(p_A)$ if $p_A < \tau$ and any situation between the two if $p_A = \tau$. Moreover, if the demand on the domain corresponding to full abatement is again decreasing and bounded from above, the inverse demand has the same structure as that obtained in Section 3.1. So, with similar assumptions on its elasticity, the properties of the monopoly outcome provided in Lemma 1 should extend to the case of heterogeneous polluters.

The main weakness of this argument is that the computation of the aggregate level of abatement corresponding to full pollution reduction ($A_f(p_A)$) and, more generally, the construction of the market-clearing production levels for all τ and p_A, are now trickier. In fact - as in Section 3.1 - it is easy to compute the individual conditional demand for abatement services and the cost function related to this activity. But to compute the market-clearing production level, we now face a system of m equations, since for each firm, the price of the polluting good is equal to the full marginal cost including abatement cost. In other words, these individual production levels solve:

$$\forall j = 1, \ldots, m \quad P\left(\textstyle\sum_{j=1}^{m} q_j\right) = c_j'(q_j) + \min\left\{p_A, \tau\right\} \times \varepsilon_j'(q_j) \tag{23}$$

instead of the single equation given by Eq. (7). Nevertheless, it can be shown that:

Lemma 2. *Under our assumptions on demand, costs and emissions, the system of Eqs. (23) admits a unique solution* $(q_j(k))_{j=1}^m$ *for each constant* $k = \min\{p_A, \tau\} \geq 0$*. Moreover,* $A_f(p_A) = \sum_{j=1}^m \varepsilon_j(q_j(p_A))$ *- the total quantity of abatement good which induces full pollution reduction - is decreasing (for all* $p_A \leq \tau$*) and bounded from above by* $A_{\max} = \sum_{j=1}^m \varepsilon_j(q_j(0))$*.*

It finally remains to verify that the Pigouvian tax $\tau^{opt} = D'\left(\sum_{j=1}^m \varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right)$ (i) is lower than the highest tax τ_c^h that induces competitive behavior by the abatement producer and (ii) can achieve the first-best outcome.[13] The first part is obvious. If $\tau^{opt} < \tau_c^h$, the eco-service firm equates its marginal cost with the Pigouvian tax, i.e., $\tau^{opt} = k'(A)$ so that the second efficiency condition (Eq. (22b)) is satisfied. Since, in this case, the price of the abatement good is τ^{opt}, the set of Eqs 23 describing the equilibrium production levels corresponds exactly to the first efficiency condition (Eqs. (22a)).

If $\tau^{opt} \geq \tau_c^h$, this implies, by the definition of τ_c^h, that the tax τ^{opt} is higher than the marginal abatement cost which induces full abatement at price $p_A = \tau^{opt}$, i.e., $\tau^{opt} > k'(A_f(\tau^{opt}))$. This again implies that one reduces, at the optimum, more pollution than the existing level, which is impossible. We can therefore state that:

Proposition 4. *Even if the polluting sector is composed of heterogeneous firms, especially in terms of their emissions, the regulator can neutralize the monopoly power on the abatement service market and obtain the first-best solution by setting the tax rate at the marginal damage.*

5. Cournot competition in the eco-industry

Let us now restore the representative polluting firm assumption, but now introduce Cournot competition in the eco-service industry. There are now n heterogeneous firm indexed by i in the eco-industry, each characterized by a cost function $\kappa_i(a)$. Seeing that all the other assumptions are maintained, an efficient allocation now verifies:

$$\forall i = 1\ldots, n \quad \beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - \textstyle\sum_{i=1}^n a_i^{opt}\right) = \kappa_i\left(a_i^{opt}\right) \tag{24}$$

Could we again implement the first-best allocation by setting the Pigouvian tax at $\tau^{opt} = D'(\varepsilon(Q^{opt}) - \sum_{i=1}^n a^{opt})$? To answer this question, we first study the best reply of these Cournot players, since the behavior of the polluting firm remains unchanged by construction.

[13] This threshold τ_c^h is now defined $\tau_c^h = \kappa'\left(A_f(\tau_c^h)\right)$ and using a similar argument to that used in the proof of Lemma 1, we can show that it exists and is unique.

5.1. The best reply of a firm

The main difficulty appears at that stage. The different thresholds which characterize the behavior of a monopoly now come out during the computation of the best response and are linked to the behavior of the opponents. So instead of dealing with a piecewise continuous monopoly solution, we have, even if the intuition is maintained, to manage piecewise continuous best responses.

If we denote by $A_{-i} = \sum_{j=1,j\neq i}^{n} a_j^{opt}$ the aggregated abatement supply of the opponents, this best response is given by:

$$BR_i(A_{-i}, \tau) \in \arg\max_{a_i} \{\min\{\tau, p(a_i + A_{-i})\} \times a_i - \kappa_i(a_i)\} \tag{25}$$

where $p(A) = \beta(\varepsilon^{-1}(A))$ stands for the inverse demand corresponding to full abatement behavior of the polluting industry. To gain some intuition about $BR_i(A_{-i}, \tau)$, we introduce $br(A_{-i})$, the best response obtained when the demand always corresponds to full abatement behavior, i.e., with $p(a)$. This is defined by:

$$br(A_{-i}) = \max_{a_i \in [0, \varepsilon(Q_{\max})]} p(a_i + A_{-i}) \times a_i - \kappa_i(a_i) \tag{26}$$

and solves:

$$p'(a_i + A_{-i}) \times a_i + p(a_i + A_{-i}) - \kappa'(a_i) = 0 \tag{27}$$

We know that (i) $br_i(\varepsilon(Q_{\max})) = 0$, since $p(\varepsilon(Q_{\max})) = 0$, (ii) $br_i(0) = A_m < \varepsilon(Q_{\max})$ the monopoly solution and (iii) br_i is decreasing as long as $e(p') > -1$, the elasticity of p' is larger than -1.

We now try to understand, at least graphically - see Figure 2 - what happens when the constraint on the price, i.e. $p(a_i + A_{-i}) \leq \tau$, begins to matter.

In Figure 2, we consider the unconstrained best response $br_i(A_{-i})$ and the $-45°$ line given by $a_i + A_{-i} = p^{-1}(\tau)$. Since $p(a)$ is decreasing, this linear constraint provides, for each A_{-i}, the minimum production level of firm i ensuring that the price is lower than τ, or in other words, the minimal production level that preserves market power. So, as long as the best reply $br_i(A_{-i})$ lies above this line, the firm is able to exert market power, i.e., his best reply is $BR_i(A_{-i}, \tau) = br_i(A_{-i})$. This remains true until $br_i(A_{-i})$ cuts this line. This intersection occurs when the A_{-i} is equal to $M_i(\tau)$ which is defined by:

$$p'(p^{-1}(\tau)) \times \left(p^{-1}(\tau) - M_i(\tau)\right) + \tau - \kappa_i'\left(p^{-1}(\tau) - M_i(\tau)\right) = 0 \tag{28}$$

If $A_{-i} \leq M_i(\tau)$, firm i is, as in the monopoly case, unable to manipulate the price. This is why it becomes optimal to produce the quantity that keeps the price at τ. In other words, the best reply is $BR_i(A_{-i}, \tau) = p^{-1}(\tau) - A_{-i}$. However, as A_{-i} decreases, firm i increases its market share while the price remains constant at τ. It is therefore possible for the marginal production cost to be higher than the price. This situation occurs for all $A_{-i} < m_i(\tau)$ with $m_i(\tau)$ solution to:

$$\kappa_i'\left(p^{-1}(\tau) - m_i(\tau)\right) = \tau \Leftrightarrow m_i(\tau) = p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) \tag{29}$$

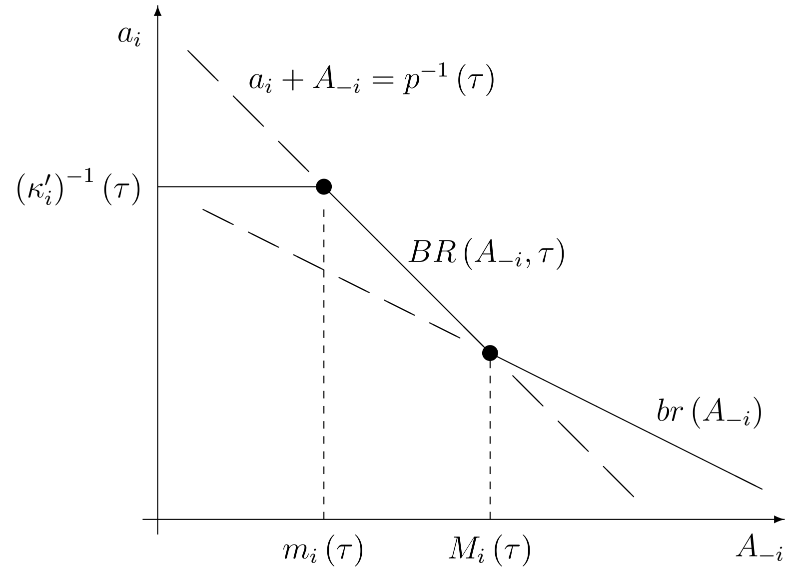

Figure 2: The best response of Firm i

In this last case, the best reply depicts a competitive behavior and is given by $BR_i(A_{-i}, \tau) = (\kappa_i')^{-1}(\tau)$. Moreover, by construction, it is immediate that $m_i(\tau) \leq M_i(\tau)$.

This explanation nevertheless requires that (i) the line $a_i + A_{-i} = p^{-1}(\tau)$ crosses $br_i(A_{-i})$ at a unique positive point and (ii) $p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) \geq 0$, otherwise some of these cases are vacuous. A formal construction of the best response is provided in the appendix. We show that the case depicted in Figure 1 only occurs when the rate τ is lower than τ_c^i given by $\tau_c^i = \kappa_i'(p^{-1}(\tau_c^i))$, i.e., the upper bound of the tax rates at which firm i adopts a competitive behavior. But it is essentially this set of taxes in which we are interested. This is why we only spell out the characterization of the best reply for $\tau \leq \tau_c^i$.

Lemma 3. *For any tax rate $\tau < \tau_c^i$ and any $A_{-i} \in [0, \varepsilon^{-1}(Q_{\max})]$, the best response of an eco-service firm is given by:*

$$BR(A_{-i}, \tau) = \begin{cases} (\kappa_i')^{-1}(\tau) \text{ if } A_{-i} < m_i(\tau) \\ p^{-1}(\tau) - A_{-i} \text{ if } m_i(\tau) \leq A_{-i} \leq M_i(\tau) \\ br_i(A_{-i}) \text{ if } A_{-i} \geq M_i(\tau) \end{cases} \tag{30}$$

Moreover, this best response is continuous and, since $e(p') > -1$, it is also non-increasing with A_{-i}.

5.2. Cournot equilibrium and efficient taxation

From the previous Lemma, we essentially learn that competitive behavior is part of the best response of firm i when $\tau \leq \tau_c^i$. So we will now concentrate on taxes smaller than $\tau_c^{\min} = \min_{i=1,\dots,n} \{\tau_c^i\}$. In this case, it can be shown that:

Lemma 4. *For any tax $\tau < \tau_c^{\min}$, the unique equilibrium supply of eco-services is, for each firm, $a_i^C(\tau) = (\kappa_i')^{-1}(\tau)$, while the price of these services is $P_A^C(\tau) = \tau$. From Eq (7), we observe that the production of the dirty good is $Q^C(\tau) = \beta^{-1}(\tau)$.*

The existence part of this result is obvious. Since $\tau < \tau_c^{\min}$, then $\forall i$, $\tau < \kappa_i'(p^{-1}(\tau))$ by the definition of τ_c^i or equivalently $\forall i$, $(\kappa_i')^{-1}(\tau) < p^{-1}(\tau)$. We can therefore say that:

$$\forall i \quad A_{-i}^c = \sum_{j=1, j\neq i}^{n} \left(\kappa_j'\right)^{-1}(\tau) < p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) = m_i(\tau) \tag{31}$$

This means from Eq. (30) that playing $a_i^C = (\kappa_i')^{-1}(\tau)$ is a best response for each firm. Concerning uniqueness, let us first observe that the best response is bounded from above by $(\kappa_i')^{-1}(\tau)$. So if there exists another equilibrium, say b^C, there must be at least one firm i_0 such that $b_{i_0}^C < \left(\kappa_{i_0}'\right)^{-1}(\tau)$ and, due to the upper bound, $B_{-i_0}^c \leq \sum_{j=1, j\neq i_0}^{n} \left(\kappa_j'\right)^{-1}(\tau)$. But this leads to a contradiction, since for $\tau < \tau_c^{\min}$, we have as before that $B_{-i_0}^c < m_{i_0}(\tau)$, so that $b_{i_0}^C = \left(\kappa_{i_0}'\right)^{-1}(\tau)$ should be the best response.

Let us now assume that the regulator sets $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a_i^{opt}\right)$. If $\tau^{opt} \leq \tau_c^{\min}$, the Cournot equilibrium meets the first-best conditions given by Eq. (24) since, by Lemma 4, we have:

$$\forall i = 1\ldots, n \quad \beta\left(Q^C(\tau^{opt})\right) = \tau^{opt} = \kappa_i'\left(a_i^C(\tau^{opt})\right) \tag{32}$$

It remains for us to verify that $\tau^{opt} \leq \tau_c^{\min}$. If this is not the case, there must exist at least one agent, say i_0, for which $\tau^{opt} > \kappa_{i_0}'(p^{-1}(\tau^{opt}))$. But this implies, for our characterization of an optimal allocation (Eq. (24)), that:

$$a_{i_0}^{opt} = \left(\kappa_{i_0}'\right)^{-1}(\tau^{opt}) > p^{-1}\left(\tau^{opt}\right) = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) = \varepsilon(Q^{opt}) \tag{33}$$

so that $\sum_{i=1}^{n} a_{i_0}^{opt} > \varepsilon(Q^{opt})$, since all the $a_i^{opt} \geq 0$. In other words there is, at the optimum, more abatement than emissions, which is a contradiction. We can therefore say:

Proposition 5. *If there is Cournot competition in an eco-service industry and pure competition in the polluting sector, the first-best allocation can be reached by setting the tax rate to the marginal damage as usual.*

6. Two main limits of the result

We have extended our leading case of Section 3 to various settings. Both main assumptions nevertheless remained: an upstream non competitive market of eco-services and a downstream competition polluting industry. In this section, we show that both assumptions are crucial. We first introduce a counter example showing that our result cannot be extended to abatement goods. In a second step we introduce downstream monopoly power. Due to this new market imperfection, the first-best allocation cannot be implemented. In this case, we show that the optimal Pigouvian tax must be lower than the marginal damage, thus finding Barnett's result [2].

6.1. Abatement goods versus abatement services

Our results only apply for environmental services, whereas most of the literature considers environmental goods. In this latter case and under an "end-of-pipe" pollution assumption (see Katsoulacos and Xepapadeas [20]), the emissions $\varepsilon(Q, A)$ are for the dirty firm negatively correlated with the use of abatement goods. The marginal productivity of abatement goods $(-\partial_A \varepsilon(Q, A))$ therefore matters in the abatement choice, contrary to abatement services for which the purchase decision is only based on the difference between the environmental tax and the price. As we will show, this strongly reduces the control that the regulator has over the equilibrium and limits implementation of the first-best allocation.

As the case of abatement goods is largely documented in the literature under alternative sets of assumptions, we simply illustrate our purpose by a (counter) example to highlight what changes compared with eco-services.

Example 1. *We consider (i) quadratic cost functions, i.e., $c(Q) = \frac{1}{2}Q^2$ and $\kappa(A) = \frac{1}{2}A^2$, (ii) a linear demand $P = 1 - Q$, (iii) a linear damage function $D(E) = 0.2E$ and (iv) an emission function $\varepsilon(Q, A) = \max\left\{Q - \sqrt{A}, 0\right\}$ which is now "non-linear" in abatement.*

If we construct the inverse demand for abatement goods as in Section 3.1, we observe, after some computation, that the conditional demand for abatement goods and the cost associated with this activity are given by:

$$A(p_A, \tau, Q) = \min\left\{Q^2, \left(\tfrac{\tau}{2p_A}\right)^2\right\} \qquad C_A(p_A, \tau, Q) = \begin{cases} p_A Q^2 \text{ if } Q \leq \frac{\tau}{2p_A} \\ \tau Q - \frac{\tau^2}{2p_A} \text{ if } Q > \frac{\tau}{2p_A} \end{cases} \tag{34}$$

As expected, the conditional demand does not move from full abatement (here Q^2) to no abatement, since $\left(\frac{\tau}{2p_A}\right)^2$ is a demand for partial pollution reduction. Moreover, the marginal cost $\partial_Q C_A$ associated with this activity is no longer linear in quantities, since $\partial_Q C_A = \min\{2p_A Q, \tau\}$. This drastically modifies the computation of the dirty good market clearing condition, which is given by:

$$\begin{aligned} P(Q) = \frac{dc}{dQ} + \frac{\partial C_A}{\partial Q} &\Leftrightarrow 1 - Q = Q + \min\{2p_A Q, \tau\} \\ &\Leftrightarrow Q(p_A, \tau) = \begin{cases} \frac{1}{2(1+p_A)} \text{ if } p_A \leq \tau(1 + p_A) \\ \frac{1-\tau}{2} \text{ else} \end{cases} \end{aligned} \tag{35}$$

It follows that the demand for abatement goods consistent with market clearing and the inverse demand curve are:

$$A(p_a, \tau) = \min\left\{\left(\tfrac{1}{2(1+p_A)}\right)^2, \left(\tfrac{\tau}{2p_A}\right)^2\right\} \qquad P_A(A, \tau) = \begin{cases} \frac{\tau}{2\sqrt{A}} \text{ if } A < \left(\frac{1-\tau}{2}\right)^2 \\ \frac{1}{2\sqrt{A}} - 1 \text{ if } A \geq \left(\frac{1-\tau}{2}\right)^2 \end{cases} \tag{36}$$

Since $\frac{1}{2\sqrt{A}} - 1$ stands for $p(A)$, the inverse demand under full abatement, this inverse demand can be written as $P_A(A,\tau) = \min\left\{p(A), \frac{\tau}{2\sqrt{A}}\right\}$. Clearly, this expression differs from $\min\{p(A), \tau\}$. The monopoly is now able to exert market power on the whole range of its inverse demand. The flat part disappears but a kink remains. This is why the monopoly solution nevertheless leads to three different outcomes: (i) the monopoly solution under partial abatement for small taxes, (ii) a solution which sticks in the kink of the inverse demand curve, and (iii) the monopoly strategy under full abatement for high taxes. It can be shown, in this example, that:

$$A^m(\tau) \approx \begin{cases} \left(\frac{\tau}{4}\right)^{2/3} \text{ for } \tau < 0.2291 \\ \left(\frac{1-\tau}{2}\right)^2 \text{ for } \tau \in [0.2291, 0.5265] \\ 0.0561 \text{ for } \tau > 0.5265 \end{cases} \quad \text{and } Q^m(\tau) \approx \begin{cases} \left(\frac{1-\tau}{2}\right) \text{ for } \tau \leq 0.5265 \\ 0.2368 \text{ for } \tau > 0.5265 \end{cases} \tag{37}$$

Moreover, a simple computation shows that $A^{opt} = 0.2$ and $Q^{opt} = 0.4$. As $A^m(\tau)$ is bounded from above by 0.1485, the first-best outcome is unreachable simply because the monopoly can now take a margin in the case of full and partial pollution abatement.

6.2. Downstream market power and Barnett's result

As we will see, our result is also limited by the number of market failures and/or imperfections that the regulator controls. We have shown that one instrument, the Pigouvian tax, can regulate the environmental externality in the dirty sector and the imperfect competition problem in the eco-service industry. However, this result will fail if a new market imperfection is introduced. In this case, the regulator can only implement a second-best policy. To illustrate this problem, let us return to the basic case of Section 3, and introduce monopoly power in the dirty sector instead of pure competition.[14]

We will obtain the same results as Barnett [2] under this new framework, since we will show that the second-best Pigouvian tax must be lower than the marginal damage. This result therefore contrasts with Canton et al. [6]. To fit with both papers, we assume a linear damage function $D(E) = vE$.

Let us first quickly revisit Section 3 in order to see what changes. Since the dirty firm remains competitive on the abatement market, its conditional demand and the abatement cost $C_A(p_A, \tau, Q) = \min\{p_A, \tau\} \cdot \varepsilon(Q)$ both remain unchanged. As this firm now has monopoly power on the output market, its output choice therefore satisfies:

$$Q^m \in \arg\max_Q P(Q) \times Q - c(Q) - \min\{p_A, \tau\} \times \varepsilon(Q) \tag{38}$$

If we assume, as usual for a monopoly, that the elasticity $e_P = \frac{QdP}{PdQ}$ of the inverse demand curve belongs to $[-1, 0)$ and is decreasing, the solution of this program can be summarized

[14]This is clearly a simplifying assumption. Similar results can be obtained by introducing Cournot competition on the downstream polluting good market. We introduce monopoly power on the output market in order to remain as close as possible to Barnett's paper [2].

by the following FOC:

$$P'(Q) \times Q + P(Q) - c'(Q) - \min\{p_A, \tau\} \times \varepsilon'(Q) \leq 0 \text{ (with equality if } Q > 0) \quad (39)$$

This equation is quite similar to the FOC under competition (see Eq. 6). So, let us now introduce:

$$\beta_m(Q) = \frac{P'(Q) \times Q + P(Q) - c'(Q)}{\varepsilon'(Q)} \quad (40)$$

instead of the marginal benefit for pollution $\beta(Q)$. This function shares similar properties with $\beta(Q)$: (i) it is positive up to $\bar{Q}_m$, which is now the monopoly production level without environmental regulation, and (ii) it is decreasing on $\left[0, \bar{Q}_m\right]$ under our restriction on the elasticity e_P.[15] It follows that $Q = \beta_m^{-1}(\min\{p_A, \tau\})$ and the rest of the argument of Section 3.1 and 3.2 can be extended to this case, as long as we replace the function β by β_m and $p(A)$ by $p_m(A) = \beta_m(\varepsilon^{-1}(A))$. Hence:

Lemma 5. *If the elasticity of $p_m(A)$ is decreasing and belongs to $(-1, 0)$, the equilibrium quantities with upstream and downstream monopoly power are given by:*

$$A^*(\tau) = \begin{cases} (\kappa')^{-1}(\tau) & \text{for } \tau < \tau_c \\ \varepsilon(Q^*(\tau)) & \text{for } \tau \geq \tau_c \end{cases} \quad \text{and } Q^*(\tau) = \beta_m^{-1}(\min\{\tau, \tau_m'\}) \quad (41)$$

with τ_c defined as in Eq. (12) and τ_m' given by $\tau_m' = \frac{1}{1+e_{p_m}(p_m^{-1}(\tau_m'))}\kappa'(p^{-1}(\tau_m'))$. Moreover, the price of the abatement services is $P_A^* = \min\{\tau, \tau_m'\}$.

Now, to find the second-best Pigouvian tax, we have to solve:

$$\max_{\tau} \underbrace{\int_0^{Q^*(\tau)} P(q)dq - c(Q^*(\tau)) - \kappa(A^*(\tau)) - D(\max\{\varepsilon(Q^*(\tau)) - A^*(\tau), 0\})}_{=SB(\tau)} \quad (42)$$

As the equilibrium quantities $A^*(\tau)$ and $Q^*(\tau)$ are constant $\forall \tau \geq \tau_m'$ (see Eq. 41), $\forall \tau \geq \tau_m'$, $SB(\tau)$ is also constant. We can therefore restrict our attention to tax rates $\tau \in [0, \tau_m']$. So let us now consider a tax $\tau \in (\tau_c, \tau_m']$. In this case, the monopoly supply of abatement services totally removes pollution. It follows that $\forall \tau \in (\tau^c, \tau^m)$:

$$\frac{dSB(\tau)}{d\tau} = \left(P(Q^*(\tau)) - c'(Q^*(\tau)) - \kappa'(\varepsilon(Q^*(\tau)))\,\varepsilon'(Q^*(\tau))\right)\frac{dQ^*}{d\tau} \quad (43)$$

Since the FOC of the polluting firm (see Eq. (39)) is satisfied at equilibrium and $P_A^* = \min\{\tau, \tau_m'\}$, Eq. (43) becomes:

$$\frac{dSB(\tau)}{d\tau} = \left(-P'(Q^*(\tau))\,Q^*(\tau) + (\tau - \kappa'(\varepsilon(Q^*(\tau))))\,\varepsilon'(Q^*(\tau))\right)\frac{dQ^*}{d\tau} \quad (44)$$

[15] This follows from computation and the fact that (i) $P''(Q)Q + 2P'(Q) = P'(Q)(1 + e_P) + P\frac{de_P}{dQ}$ and (ii) $\xi_m(Q) \geq 0$ on $\left[0, \bar{Q}_m\right]$.

Moreover, $\frac{dQ^*(\tau)}{d\tau} = 1/\left(\beta'_m\left(\beta_m^{-1}(\tau)\right)\right) < 0$, since β_m is decreasing and by the definition of τ_c (see Eq. (12)), and we know that $\forall \tau \in (\tau^c, \tau^m)$, $\tau > \kappa'(A^*(\tau))$. We can therefore assert that $\forall \tau \in (\tau^c, \tau^m)$, $\frac{dSB(\tau)}{d\tau} < 0$.

Following these developments a second-best solution necessarily belongs to $[0, \tau_c]$. If this solution is an interior one, we can write:

$$\frac{dSB(\tau)}{d\tau} = \left(P(Q^*(\tau)) - c'(Q^*(\tau)) - v\varepsilon'(Q^*(\tau))\right)\frac{dQ^*}{d\tau} - \left(\kappa'(A^*(\tau)) - v\right)\frac{dA^*}{d\tau} = 0 \quad (45)$$

By using the FOC of the polluting firm (see Eq. (39)) again, we have:

$$\left(-P'(Q^*(\tau))\,Q^*(\tau) + (\tau - v)\,\varepsilon'(Q^*(\tau))\right)\frac{dQ^*}{d\tau} - (\tau - v)\frac{dA^*}{d\tau} = 0 \quad (46)$$

which implies that:

$$\tau^{sb} - v = \frac{P'(Q^*(\tau))\,Q^*(\tau)\frac{dQ^*}{d\tau}}{\varepsilon'(Q^*(\tau))\frac{dQ^*}{d\tau} - \frac{dA^*}{d\tau}} < 0 \quad (47)$$

since $\forall \tau \in [0, \tau_c]$, $\frac{dA^*}{d\tau} = 1/k''\left((k')^{-1}(\tau)\right) > 0$ and $\frac{dQ^*(\tau)}{d\tau} = 1/\left(\beta'_m\left(\beta_m^{-1}(\tau)\right)\right) < 0$. We can therefore state:

Proposition 6. *If there is monopoly power on the final good and on the abatement service market, the second-best taxation rule neutralizes the market power on the abatement service market (since $\tau^{sb} \leq \tau_c$), but remains lower than the marginal damage to limit the reduction of the production of the final good induced by the monopoly power.*

7. Concluding remarks

The EGSS is highly concentrated. The economic literature has mainly analyzed the design of environmental regulation while taking this feature into account. However, no study has yet analyzed the extent to which distinguishing between abatement goods and abatement services matters for environmental regulation. That was the topic of this paper. We found a very interesting result for policy makers. Whereas there are two market failures in our economy - market power on the abatement service market and pollution generated by downstream firms - the regulator can reach the first-best outcome with only one tool: environmental regulation. This result challenges the Tinbergen rule.

Abatement services introduce a flat part in the inverse demand curve since the polluting firm only makes a trade-off between the price of the abatement services and the Pigovian tax to comply with the environmental regulation. He is indifferent between both choices if the price of the abatement service equals the Pigovian tax. In this context, an accurate setting of the Pigovian tax can lead the monopoly to choose the first-best level of production. We have shown that if this tax is set so that it is equal to the marginal damage, the economy reaches the first-best outcome.

We then extended our model to chech the robustness of the result. We first set assumptions such as that total abatement is allowed; we then considered a pollution permit market instead the Pigovian tax, and thirdly, we studied heterogenous polluters. We finally assumed that instead of a monopoly, the eco-industry was characterized by Cournot competition. We finished by underlining some limits of our result. It no longer holds if we consider abatement goods instead of services or if we add another market failure in the output market.

If we essentially explore the case of upstream market power as a limit to our result, other additional market imperfections could also be considered. If a pollution permit market is organized, a polluting firm may exert a dominant position on this market, e.g., simple manipulation (see Hahn [19] and Westskog [39], or manipulating the costs of its opponents on the output market (what is called exclusionary manipulation - see Misiolek and Elder [21], Sartzetakis [31] or Von der Fehr [38]). Some other externalities, such as a polluting eco-industry (see Sans et al. [30]) may also modify our result. In this case, the Pigouvian tax modifies not only the demand for the abatement firms but also the production costs of the abater.

In this article, we also restrict our attention to a benevolent regulator controlling a closed economy. However, it is well-known that lobbies influence the definition of environmental policy (Aidt [1]), and abatement services are often exchanged on an international market. Canton [4] studies the role of lobbies in the case of an eco-industry providing environmental goods. In an open economy, each firm is subject to national environmental regulations. In this case, environmental policies can be used in a strategic way (see for instance Barrett [3] or Hamilton and Requate [18]). Nimubona [23] studies the effect of reductions in trade barriers on the eco-industry sector that were agreed in the Doha Round of the WTO.

Further research is needed to investigate how taking these new features into account might challenge or modify our results.

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A. Proof of Lemma 1

We need to solve:

$$\max_{A\in[0,\varepsilon(Q_{\max})]} \underbrace{\left[\min\left\{\tau, p\left(A\right)\right\} A - \kappa\left(A\right)\right]}_{\pi(A,\tau)} \text{ where } p\left(A\right) = \beta\left(\varepsilon^{-1}\left(A\right)\right)$$

Step 1: *Existence of a unique solution*

Since we maximize $\pi(A,\tau)$ over a compact set, it remains to verify that $\pi(A,\tau)$ is strictly concave in A. Moreover, $\kappa\left(A\right)$ being strictly convex, we only need to check that $\min\left\{\tau A, p\left(A\right)A\right\}$ is concave. But let us first observe that, under the assumption that $e_p(\varepsilon\left(Q_{\max}\right)) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, $p\left(A\right)A$ is concave since :

$$\frac{d^2}{\left(dA\right)^2}\left(p\left(A\right).A\right) = p'(A)\left(e_p(A)+1\right) + p(A)\frac{de_p(A)}{da} \leq 0$$

It follows that $\forall\lambda \in [0,1]$ and $\forall A_1, A_2 \in [0, \varepsilon\left(Q_{\max}\right)]$

$$\begin{aligned}
&\min\left\{\tau\left(\lambda A_1 + (1-\lambda) A_2\right), p\left(\left(\lambda A_1 + (1-\lambda) A_2\right)\right)\left(\lambda A_1 + (1-\lambda) A_2\right)\right\} \\
&\geq \min\left\{\lambda\tau A_1 + (1-\lambda)\tau A_2, \lambda p\left(A_1\right) A_1 + (1-\lambda) p\left(A_2\right) A_2\right\} \text{ (concavity of } p(A)A) \\
&\geq \lambda \min\left\{\tau A_1, p\left(A_1\right) A_1\right\} + (1-\lambda)\min\left\{\tau A_2, p\left(A_2\right) A_2\right\} \text{ (concavity of the } \min\left\{x,y\right\}
\end{aligned}$$

Step 2: *Construction of the thresholds*

This program is not smooth but nevertheless concave. This means (see Rockafellar [29]) that an optimum is reached iff $0 \in \partial_A \pi$ where $\partial_A \pi$ denotes the sub-derivative of $\pi(A,\tau)$ with respect to A. By computation, we get:

$$\partial_A \pi = \begin{cases} \tau - \kappa'(A) & \text{if } A < p^{-1}(\tau) \\ \left[\underbrace{\tau + p'(p^{-1}(\tau))p^{-1}(\tau) - \kappa'\left(p^{-1}(\tau)\right)}_{:=\phi_m(\tau)}, \underbrace{\tau - \kappa'\left(p^{-1}(\tau)\right)}_{:=\phi_c(\tau)} \right] & \text{if } A = p^{-1}(\tau) \\ p(A) + p'(A)A - \kappa'(A) & \text{if } A > p^{-1}(\tau) \end{cases} \tag{48}$$

Now observe that $\phi_m(\tau) = 0$ and $\phi_c(\tau) = 0$ implicitly defines the two thresholds τ_c and τ_m introduced in Eqs (11) and (12). It remains to verify that these thresholds exist, are unique and that $\tau_c < \tau_m$. These results directly follow from the next observations:

(i) ϕ_c and ϕ_m are both increasing. More precisely, $\phi_c'(\tau) = 1 - \frac{\kappa''\left(p^{-1}(\tau)\right)}{p'(p^{-1}(\tau))} > 0$, and

$$\begin{aligned} \phi_m'(\tau) &= \frac{d}{d\tau}\left((p(A) + p'(A)A) - \kappa(A)|_{A=p^{-1}(\tau)} \right) \\ &= \underbrace{\left(\frac{d^2}{(dA)^2}\left(p(A)A\right) - \kappa''(A) \right)}_{<0 \text{ (second order condition)}} \Bigg|_{A=p^{-1}(\tau)} \times \frac{1}{p'(p^{-1}(\tau))} > 0 \end{aligned}$$

(ii) $\lim_{\tau\to 0}\phi_c(\tau) < 0$ and $\lim_{\tau\to 0}\phi_m(\tau) < 0$. Let us remember that $p(\varepsilon(Q_{\max})) = 0$, it follows that $\lim_{\tau\to 0}\phi_c(\tau) = -\kappa'(\varepsilon(Q_{\max})) < 0$ and $\lim_{\tau\to 0}\phi_m(\tau) = p'(\varepsilon(Q_{\max}))\varepsilon(Q_{\max}) - \kappa'(\varepsilon(Q_{\max})) < 0$

(iii) $\lim_{\tau\to\tau_m}\phi_c(\tau) > 0$ and $\lim_{\tau\to+\infty}\phi_m(\tau) > 0$. From the implicit definition of τ_m, we observe that $\lim_{\tau\to\tau_m}\phi_c(\tau) = -p'(p^{-1}(\tau_m))p^{-1}(\tau_m) > 0$. Concerning the second limit, we note:

$$\lim_{\tau\to+\infty}\phi_m(\tau) = \lim_{\tau\to+\infty}\tau\left(1 + e_p(A)|_{A=p^{-1}(\tau)}\right) - \lim_{\tau\to+\infty}\kappa'\left(p^{-1}(\tau)\right)$$

The second term of the r.h.s. is clearly bounded since $p^{-1}(\tau) \in [0, \varepsilon(Q_{\max})]$. If we now remember that $e_p(A)$ is decreasing and $e_p(\varepsilon(Q_{\max})) > -1$, we have $\lim_{\tau\to+\infty}\phi_m(\tau) = +\infty$.

Step 3: The optimal provision of abatement services

Let us come back to the subdifferential given by Eq (48). With similar arguments as in (i) of Step 2, it can now be argued that the first and the last equation of Eqs (48) are both decreasing function. Let us also note that (i) $\lim_{A\to 0}(\tau - \kappa'(A)) = \tau \geq 0$ and (ii)

$$lim_{A\to\varepsilon(Q_{\max})}\left(p(A) + p'(A)A - \kappa'(A)\right) = lim_{A\to\varepsilon(Q_{\max})}\left(p(A)(1 + e_p(A)) - \kappa'(A)\right) = -\kappa'(\varepsilon(Q_{\max}))$$

since by construction $p(\varepsilon(Q_{\max})) = 0$ and $e_p(A)$ bounded. From these observations and the fact that at a maximum $0 \in \partial_A\pi$, we can immediately say that:

(i) if $\phi_c(\tau) < 0$ or, equivalently, $\tau < \tau_c$, the zero of $\partial_A\pi$ is given by $\tau - \kappa'(A) = 0$, so that $A = (\kappa')^{-1}(\tau)$

(ii) if $\phi_m(\tau) \leq 0$ and $\phi_c(\tau) \geq 0$, or $\tau \in [\tau_c, \tau_m]$, the zero is obtained for $A = p^{-1}(\tau)$

(iii) if $\phi_m(\tau) > 0$, *i.e.* $\tau > \tau_m$, the optimal provision solves the last equation and this is nothing else than the standard monopoly solution associated to $p(A)$ (i.e. without the kink introduced by the min function).

B. Proof of Lemma 2

Step 1: Existence of a solution

Let us denote by $Q = \sum_{j=1}^{m} q_j$ and let take $\min\{\tau, p_A\}$ as given. We observe that (i) the r.h.s. of each equation of Eqs (23) is increasing in q_j since $c_j^{''}, \varepsilon_j^{''} > 0$ (ii) the range of these functions is $[0, +\infty)$ since $c_j'(0) = \varepsilon_j'(0) = 0$ and both functions goes to $+\infty$ as $q_j \to +\infty$. We can therefore reverse the function given by the r.h.s. and say that $q_j = \phi_j(Q)$. Moreover, we also observe that (i) $\lim_{Q\to 0} \phi_j(Q) = +\infty$ since $\lim_{Q\to 0} P(Q) = +\infty$ so that the equality (23) requires that $q_j \to +\infty$, and (ii) $\lim_{Q\to+\infty} \phi_j(Q) = 0$ since $\lim_{Q\to+\infty} P(Q) = 0$ and therefore $q_j \to 0$ to maintain the equality.

Let us now aggregate over the q_j. We obtain $Q = \sum_{j=1}^{m} \phi_j(Q)$. So if there exists a solution in Q to this equation our existence problem is solved. It remains to observe that (i) $\Phi(Q) = Q - \sum_{j=1}^{m} \phi_j(Q)$ is continuous (ii) $\lim_{Q\to 0} \Phi(Q) = -\infty$ and (iii) $\lim_{Q\to+\infty} \Phi_j(Q) = +\infty$.

Step 2: Uniqueness of the solution

Let us set $K = \min\{\tau, p_A\}$ and *write the system* (23) as:

$$\Psi\left((q_j)_{j=1}^m, K\right) = \left(c_j'(q_j) + K.\varepsilon_j'(q_j)\right)_{j=1}^m - P\left(\sum_{j=1}^{m} q_j\right) e \text{ with } e' = (1, \ldots, 1)$$

By computation, we observe that $\partial_{(q_j)_{j=1}^m} \Psi = D - P'\left(\sum_{j=1}^{m} q_j\right) e \cdot e'$ where D is a diagonal matrix whose generic term is $c_j^{''}(q_j) + K\varepsilon_j^{''}(q_j)$. This symmetric matrix is clearly positive definite since $c_j^{''}$, $\varepsilon_j^{''} > 0$ and $P' < 0$. It follows from Gale-Nikaido (1965 Theorem 6) that the solution $(q_j(K))_{j=1}^m$ of $\Psi\left((q_j)_{j=1}^m, K\right) = 0$ is unique for every K.

Step 3: $A_f(K) = \sum_{j=1}^{m} \varepsilon_j(q_j(K))$ is decreasing

Let us first observe from the implicit theorem applied to $\Psi\left((q_j)_{j=1}^m, K\right) = 0$ that $\frac{\partial (q_j)_{j=1}^m}{\partial K} = -\left(\partial_{(q_j)_{j=1}^m} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^m$. It follows that:

$$\frac{dA_f}{dK} = \left(\left(\varepsilon_j'(q_j)\right)_{j=1}^m\right)' \cdot \frac{\partial (q_j)_{j=1}^m}{\partial K} = -\underbrace{\left(\left(\varepsilon_j'(q_j)\right)_{j=1}^m\right)' \cdot \left(\partial_{(q_j)_{j=1}^m} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^m}_{>0} < 0$$

since the inverse of a positive definite matrix remains positive definite.

C. Proof of Lemma 3

We need to solve for all $A_{-i} \in [0, \varepsilon(Q_{\max})]$

$$\max_{a_i \in [0, \varepsilon(Q_{\max}) - A_{-i}]} \underbrace{\left[\min\{\tau, p(a_i + A_{-i})\}(a_i + A_{-i}) - \kappa_i(a_i)\right]}_{\pi_i(a_i, A_{-i}, \tau)} \text{ where } p(A) = \beta\left(\varepsilon^{-1}(A)\right)$$

Step 1: $\pi_i(a_i, A_{-i}, \tau)$ is strictly concave in a_{-i}

Under the assumption that $e_p(\varepsilon(Q_{\max})) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, we have for $a_i > 0$:

$$\begin{aligned} 0 &> \tfrac{a_i}{A}\left(p'(A)\left(e_p(A) + 1\right) + p(A)\frac{de_p(A)}{da}\right) = P''(A)a_i + \tfrac{2a_i}{A} P'(A) \\ &> P''(A)a_i + 2P'(A) = \tfrac{\partial^2}{(\partial a_i)^2}\left[p(a_i + A_{-i})\, a_i\right] \end{aligned}$$

We can therefore use the similar argument as in Step 1 of Lemma 1 in order to show that $\pi_i(a_i, A_{-i}, \tau)$ is strictly concave in a_i. We simply need to decompose A in $(a_i + A_{-i})$ and take a convex combination of two a_i.

Step 2: the subdifferential and the thresholds

Let us now compute the sub-derivate of $\pi_i(a_i, A_{-i}, \tau)$ with respect to a_i. For $A_{-i} < p^{-1}(\tau)$, we obtain:

$$\partial_{a_i}\pi = \begin{cases} \tau - \kappa_i'(a_i) & \text{if } a_i < p^{-1}(\tau) - A_{-i} \\ \left[\varphi_m^i(\tau, A_{-i}), \varphi_c^i(\tau, A_{-i})\right] & \text{if } a_i = p^{-1}(\tau) - A_{-i} \\ \underbrace{p(a_i + A_{-i}) + p'(a_i + A_{-i})\, a_i - \kappa_i'(a_i)}_{:=\psi_i(a_i, A_{-i})} & \text{if } a_i > p^{-1}(\tau) - A_{-i} \end{cases} \tag{49}$$

$$\text{with } \begin{cases} \varphi_m^i(\tau, A_{-i}) = \tau + p'(p^{-1}(\tau))\left(p^{-1}(\tau) - A_{-i}\right) - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right) \\ \varphi_c^i(\tau, A_{-i}) = \tau - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right) \end{cases}$$

If $A_{-i} \geq p^{-1}(\tau)$, the first and even the second line (if $A_{-i} > p^{-1}(\tau)$) are simply vacuous.

So let us for the moment assume that $A_{-i} < p^{-1}(\tau)$ and let us introduce the thresholds $m_i(\tau)$ and $M_i(\tau)$ given by $\varphi_c^i(\tau, m_i(\tau)) = 0$ and $\varphi_m^i(\tau, M_i(t)) = 0$. Concerning $m_i(\tau)$, we observe that (i) $\partial_{A_{-i}}\varphi_c^i(\tau, A_{-i}) = \kappa_i''\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\lim_{A_{-i}\to p^{-1}(\tau)} \varphi_c(\tau, A_{-i}) = \tau > 0$ and (iii) $\lim_{A_{-i}\to 0} \varphi_c^i(\tau, A_{-i}) = \tau - \kappa_i'\left(p^{-1}(\tau)\right) = \phi_c^i(\tau)$ this last function being the same as in Eq. (48) but now indexed by agent i. So by using the Step 2 of the proof of Lemma 1, we know that $\left(\phi_c^i\right)' > 0$ and that there exists a unique τ_c^i such that $\phi_c^i(\tau_c^i) = 0$. We can therefore say that:

$$\begin{cases} \forall\tau \leq \tau_c^i, \;\; \exists m_i(\tau) \in \left[0, p^{-1}(\tau)\right], \; \varphi_c(\tau, m_i(\tau)) = 0 \\ \forall\tau > \tau_c^i, \;\; \varphi_c(\tau, A_{-i}) > 0 \text{ for all } A_{-i} \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Concerning $M_i(\tau)$, we now observe that (i) $\partial_{A_{-i}}\varphi_m(\tau, A_{-i}) = \kappa''\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\lim_{A_{-i}\to p^{-1}(\tau)} \varphi_m(\tau, A_{-i}) = \tau > 0$ and (iii) $\lim_{A_{-i}\to 0} \varphi_m(\tau, A_{-i}) = \phi_m^i(\tau)$ this last function being again the same as in Eq (48)). Using again Step 2 of the proof of Lemma 1, we have:

$$\begin{cases} \forall\tau \leq \tau_m^i, \;\; \exists M_i(\tau) \in \left[0, p^{-1}(\tau)\right], \; \varphi_m(\tau, M_i(\tau)) = 0 \\ \forall\tau > \tau_m^i, \; \varphi_m^i(\tau, A_{-i}) > 0 \text{ for all } A_i \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Finally since $\varphi_m^i(\tau, A_{-i}) < \varphi_c^i(\tau, A_{-i})$ and both are increasing we can say that for $\tau \leq \tau_c^i$, $m_i(\tau) < M_i(\tau)$.

<u>*Step 3*</u>: *The unconstraint best response*

Let us concentrate on the last equation of Eq. (49). If we compute the associated best response (without taking care to $a_i > p^{-1}(\tau) - A_{-i}$) we obtain a standard best response $br_i(A_{-i})$ which corresponds to a Cournot game in which the inverse demand is $p(A)$. This function exists for all $A_{-i} \in \left[0, \varepsilon^{-1}(Q_{\max})\right]$, since (i) $\psi_i(a_i, A_{-i})$ is decreasing in a_i (see Step 1 and remember that $\kappa'' > 0$), (ii) $\lim_{a_i\to 0}\psi_i(a_i, A_{-i}) = p(A_{-i})\left(1 + e_p(A_{-i})\right) > 0$ since $e_p > -1$ by assumption and (iii) $\lim_{a_i\to(\varepsilon^{-1}(Q_{\max}) - A_{-i})}\psi_i(a_i, A_{-i}) = -\kappa_i'\left(\varepsilon^{-1}(Q_{\max}) - A_{-i}\right) < 0$ for $A_{-i} < \varepsilon^{-1}(Q_{\max})$ while for $A_{-i} = \varepsilon^{-1}(Q_{\max})$ the best response is $a_i = 0$.

<u>*Step 4*</u>: *The best response*

Three cases must be distinguished.

<u>*Case 1*</u>: $\tau > \tau_m^i$

In this case we have $\varphi_c^i(\tau, A_{-i}) > \varphi_m^i(\tau, A_{-i}) > 0$. If we now keep in mind that $(\tau - \kappa'(a_i))$ is decreasing and converges to $\varphi_c^i(\tau, A_{-i})$ as $a_i \to \left(p^{-1}(\tau) - A_{-i}\right)$, $\partial_{a_i}\pi$ only admits a zero in third case of Eq (49). In other words the best response is $BR_i(A_{-i}, \tau) = br_i(A_{-i})$ defined in Step 3.

<u>*Case 2*</u>: $\tau \in \left(\tau_c^i; \tau_m^i\right]$

Here we know that $\varphi_c^i(\tau, A_{-i}) > 0$ and therefore $\tau - \kappa_i'(a_i) > 0$ for all $a_i < p^{-1}(\tau) - A_{-i}$, but $\exists M_i(\tau) \in \left[0, p^{-1}(\tau)\right]$, $\varphi_m^i(\tau, M_i(\tau)) = 0$. This means that the best response is given by:

$$BR_i(A_{-i}, \tau) = \begin{cases} p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \leq M_i(\tau) \\ br_i(A_{-i}) \text{ else} \end{cases}$$

<u>*Case 3*</u>: $\tau \leq \tau_c^i$

In this case both thresholds matter so that the best response is given by:

$$BR_i(A_{-i},\tau) = \begin{cases} (\kappa_i')^{-1}(\tau) \text{ for all } A_{-i} < m_i(\tau) \\ p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \in [m_i(\tau), M_i(\tau)] \\ br_i(A_i) \text{ else} \end{cases}$$