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Oil and Unemployment in a New-Keynesian Model

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Abstract

The effects of oil shocks in inflation and growth have been widely discussed in the literature, however few have focused on the impact of oil price increases on unemployment. In order to shed some light on this problem, this paper develops a medium scale Dynamic Stochastic General Equilibrium model (DSGE) that allows for oil utilization in production and consumption as in Acurio-Vásconez (2015); unemployment as in Mortensen & Pissarides (1994); and staggered nominal wage contracting as in Gertler & Trigari (2009). It then analyzes the effects of oil price increases on the economy. The model recovers most of the well-known stylized facts observed after the oil shock in the 2000s'. A sensitivity analysis shows that the reduction of the bargaining power of households to negotiate wage contracts reduces the impact of an oil shock in unemployment, without affecting negatively GDP. However, it also shows that the reduction of bargaining power, together with wage flexibility strongly reduces the increase in unemployment after an oil shock, but causes a decrease in real wages, which reduces household income and affects GDP.

JEL Codes: D58, E24, E32, Q43

Keywords: New-Keynesian Model, DSGE, oil, CES, Match & Search models, Unemployment.

1 Introduction

In the two recession periods that occurred in the United States in the 1970s', oil prices reached a secular peak just prior to an economic contraction. However in the last oil shock of the 2000s', it seems that this typical characteristic vanished. Despite the large increase in oil prices from the beginning of 2002 until mid-2008, the effects on inflation were less striking than the ones observed in the 1970s' and the effects in growth as well as in unemployment were just visible in the aftermath of the sub-prime crisis.

The study of oil shocks and its macroeconomic effects is not a new subject. The literature has already studied the various transmission channels through which oil price increases may have an impact on economic activity: growth, inflation, and unemployment.

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If we focus on unemployment, in his seminal work, Hamilton (1983) raised doubts about the proposition that the correlation between oil shocks and future levels of unemployment is a coincidence. Later on, Hamilton (1988) constructed a General Equilibrium Model of Unemployment and the Business Cycle and showed that energy price increases could be the source of fluctuations in aggregate employment and could exert large effects on real output. Rotemberg & Woodford (1996) estimated the responses of private sector output and real wages to oil price increases over the period 1947-1980 and found that private output and real wages do indeed decline following a rise in oil prices. A one percent increase in oil prices results in a reduction in output of about 0.25 percent after five-seven quarters and a decline in real wages, with a maximum decline of about 0.10 percent occurring only in the second year. Those authors constructed as well a one-sector stochastic growth model and showed that imperfectly competitive models can explain the estimated effect of oil price increases on output and real wages to a much greater extent than can a stochastic growth model that assumes a perfectly competitive product market. Carruth et al. (1998) developed an efficiency-wage model and show that a simple framework based of only two prices, real price of oil and real rate of interest, is able to explain the main postwar movements in the rate of U.S. joblessness. Davis & Haltiwanger (2001) studied the effects of oil price shocks on the creation and destruction of U.S. manufacturing jobs from 1972 to 1988 using a VAR approach and showed that oil shocks account for 20-25 percent of the variability in employment growth. In a recent work, Löschel & Oberndorfer (2009) analyzed oil price impacts on unemployment for Germany within the framework of a vector autoregression (VAR) and found as well a positive relationship between oil shocks and unemployment increases.

Nevertheless it seems that the interest in oil shocks and unemployment has decreased since the mid-1990s, which is understandable given the relative steady unemployment rate between 1995 and 2006, with the exception of 2001. However the rate of unemployment rose by 5 per cent between 2007 and 2010, which is comparable with the 4 percent unemployment increase some quarters after the oil shocks in the 1970s'. While the influence of the subprime crisis of 2008 on the high unemployment rate observed in 2010 cannot be denied, we should also not dismiss the effects of oil price shocks and postulate that what happened in 2008 was entirely due to a market anomaly. The objective of this paper is to address this issue by laying the grounds of a Dynamic Stochastic General Equilibrium model (DSGE), which could be used as a tool for policy analysis.

In this particular field of DSGE modelization with oil, Blanchard & Galí (2007), Blanchard & Riggi (2013), and the models developed in Acurio-Vásconez et al. (2015) and Acurio-Vásconez (2015), among others, have constructed DSGE models that introduce oil in consumption and production. However, none of these models allows for unemployment. They all understand labor as hours worked and assume full employment. On the other hand, the recent literature related to unemployment is based on Gertler & Trigari (2009) (GT thereafter) and Gertler et al. (2008). These models use the unemployment dynamics introduced by Mortensen & Pissarides (1994) and add staggered multi-period wage contracting, features that have proven to better fit unemployment and wage dynamics.

With this in mind, this paper constructs a calibrated DSGE model with these three elements (unemployment, staggered multi-period wage contracting, and oil) and then analyzes the impact of an oil shock. The model relies on the unemployment dynamics as in Mortensen & Pissarides (1994), with staggered multi-period wage contracting as in Gertler & Trigari (2009). It also assumes a small open economy where oil is imported from a foreign country at an exogenous real price and used in consumption and production. Furthermore, as in Acurio-Vásconez (2015), the model allows for oil imperfect substitutability. Finally the model also includes staggered good prices as in Gertler et al. (2008) and Blanchard & Galí (2010).

The economy consists of three sectors: Households, Firms, and Government. The oil and capital markets have exogenous prices. The intermediate firm market will be considered as perfectly competitive. However, the retailers' market is monopolistic, thus as in Calvo (1983), just a fraction of firms are able to renegotiate prices. In contrast to models without unemployment, labor in this paper will be traded in a process that exhibits search externalities for individual households and vacancy openness for firms.

I assume that there exists a representative household with a continuum of members of measure unity, who put their income in a pool and lets the head of the family self-insure their consumption path against unemployment risk. A fraction of them work for the intermediate firms and earn a salary. The remaining part searches for a job and receives unemployment "benefits". Besides, the family has a diversified ownership stake in firms, which pay out profits, pays lump-sum taxes, consumes final domestic goods and oil, invests in government bonds, for which it receives a nominal interest rate and invests in capital, which is rented to firms at a real rental rate of capital.

There are two kinds of firms. Intermediate good producers and Final good firms (or retailers). I assume that all intermediate good firms are price-takers and use labor, oil and capital to produce their goods that are sold to the retailers. Each of them also posts vacancies in order to attract new workers for the next period. Posting vacancies has a quadratic cost. The representative intermediate firm maximizes its profit by choosing quantities of oil, capital, hours of work, and vacancies. In addition to that, a fraction of the intermediate firms can bargain with the households in order to fix a new wage. This negotiation will be done in a Nash bargaining framework. The retailers on the other hand are monopolistic firms and a fraction of them is able to re-optimize its price at each period.

Finally, there is a Government sector that has exogenous spending and a Central Bank that sets the nominal short-term interest rate.

Under the baseline calibration, the model recovers most of the well-known stylized facts after an oil price shock in the 2000s': the absent of recession, coupled with a low increase in domestic inflation, a low price elasticity of oil demand, and in this particular model, an increase in unemployment. Here, as in Acurio-Vásconez (2015), capital and labor are not perfectly substitutes for oil, then an oil price shock induces the reduction in hiring cost which in turn decreases the quantity of vacancies that the firm posts in order to attract new workers. This provokes the increase in unemployment. A sensitivity analysis shows that the reduction of the bargaining power of households to negotiate wage contracts could diminish the raise in unemployment after an oil shock without affecting negatively GDP. However, it also shows that the reduction of bargaining power, together with wage flexibility strongly reduces the increase in unemployment after an oil shock, but causes a decrease in real wages, which reduces household income and affects GDP.

The rest of the paper is constructed as follows. Section 2 describes the basic characteristics of the model. Section 3 analyzes the impulse response functions and performs a sensitivity analysis. Section 4 concludes.

2 Model

In the same way as in Acurio-Vásconez (2015), I consider an oil-imported small open economy with CES functions in production and consumption. Along with this framework this paper introduces unemployment as in Mortensen & Pissarides (1994); hiring costs and staggered wages are introduced following Gertler & Trigari (2009) (for now on GT). Then, within the framework of the model developed in Acurio-Vásconez (2015), this model is a variation of the Mortensen-Pissarides Search and Matching model, which as in GT allows for staggered multi-period wage contracting.

Households and firms interact in three markets, the exchange of goods, oil, and capital markets. Capital, oil and intermediate firm markets will be considered as perfectly competitive and the retailers (or final good firms) will interact in a monopolistic market.

2.1 Households

I assume that there exists a representative household with a continuum of members of measure unity. Denote $L_t \in (0,1)$ the share of family members currently employed at time t.[1] In order to introduce complete consumption insurance, I will assume as in Merz (1995), that all the family members put their income in a pool and let the head of the family optimally choose per capita consumption and asset holdings. Under this assumption, each family member self-insures its consumption path against unemployment risk.

Each member of the family who is working at firm i, works $T_t(i)$ hours, and earns a nominal wage, $W_t(i)$. I adopt the hypothesis of full participation as in most of the new Keynesian models with unemployment, meaning that I assume that the remaining fraction of family members that are not working, $1-L_t$, are searching for a job and receive unemployment benefits, b, financed through lump-sum taxes.

Households can consume two different types of goods: a domestic good, $C_{q,t}$, at nominal price P_q that is produced inside the country[2] and oil, $C_{e,t}$, which comes from a foreign country, at nominal price P_e. The consumption flow of household j is defined as in Acurio-Vásconez (2015) by:

$$C_t := \left((1-x_c)^{1-\sigma_c} C_{q,t}^{\sigma_c} + x_c^{1-\sigma_c} C_{e,t}^{\sigma_c}\right)^{\frac{1}{\sigma_c}} \tag{1}$$

where $C_{q,t} := \left(\int_0^1 C_{q,t}(j)^{\frac{\epsilon_p-1}{\epsilon_p}}\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$ is a Dixit-Stiglitz aggregator, where $j \in [0,1]$ indexes the type of good; x_c represents the share of oil consumption out of total consumption[3] and $\sigma_c = \frac{\eta_c-1}{\eta_c}$ where η_c is the elasticity of substitution between domestic goods and oil consumption.

The optimal allocation expenditure among these different goods, subject to the budget constraint $P_{c,t}C_t = P_{q,t}C_{q,t} + P_{e,t}C_{e,t}$, where $P_{c,t}$ stands for the CPI price index[4], gives the following consumption demand functions and the equation for the CPI index (Cf. Acurio-Vásconez (2015)):

$$C_{q,t}(j) = (1-x_c)\left(\frac{P_{q,t}}{P_{c,t}}\right)^{\frac{1}{\sigma_c-1}} C_t(j), \quad C_{e,t}(j) = x_c\left(\frac{P_{e,t}}{P_{c,t}}\right)^{\frac{1}{\sigma_c-1}} C_t(j) \tag{2}$$

$$P_{c,t} = \left((1-x_c)P_{q,t}^{\frac{\sigma_c}{\sigma_c-1}} + x_c P_{e,t}^{\frac{\sigma_c}{\sigma_c-1}}\right)^{\frac{\sigma_c-1}{\sigma_c}} \tag{3}$$

[1] Then employment is different from hours worked

[2] P_q could be interpreted as being the CPI without gasoline and other energy goods.

[3] Cf. Appendix Acurio-Vásconez (2015) for details in the calculation.

[4] Defined as the minimum expenditure required to buy one unit of C_t.

Moreover, the family has a diversified ownership stake in firms, which pay out profits, Π_t, pays lump-sum taxes, Tax_t, consumes, C_t, invests in government bonds, B_t, for which it receives a nominal interest rate, i_t, and invest in capital, I_t, which is rented to firms at a real rental rate of capital, r_t^k. Then conditional on $L_t(i)$ and $T_t(i)$ it seeks to maximize the following lifetime discounted utility function:

$$\mathcal{W}_t = U(C_t) - \int_0^1 V(T_t(i))L_t(i)di + \beta\mathbb{E}_t\mathcal{W}_{t+1}$$

where $\beta \in (0,1)$ is the discount factor and

$$U(C_t) = ln(C_t) \quad \text{and} \quad V(T_t(i)) = \frac{T_t(i)^{1+\widetilde{w}}}{1+\widetilde{w}}$$

It does so under the following budget constraint:

$$\begin{aligned} &P_{c,t}C_t + P_{k,t}I_t + B_t \leq \\ &\quad \leq (1+i_{t-1})B_{t-1} + \int_0^1 W_t(i)L_t(i)di + (1-L_t)P_{q,t}b + \Pi_t + r_t^k P_{k,t}K_t + \text{Tax}_t \end{aligned} \tag{4}$$

The time path considered here is a month, then note that, with this construction, W_t represents a monthly wage.[5] I assume that the dynamics of capital accumulation follows:

$$I_t := K_{t+1} - (1-\delta)K_t$$

where $\delta \in (0,1)$ is the depreciation rate.

The first order conditions with respect to C_t, B_t and K_{t+1} are:[6]

$$C_t : U'(C_t) = \lambda_t P_{c,t} \tag{5}$$

$$B_t : \lambda_t = \beta\mathbb{E}_t\left[(1+i_t)\lambda_{t+1}\right] \tag{6}$$

$$K_{t+1} : \lambda_t P_{k,t} = \beta\mathbb{E}_t\left[\lambda_{t+1}\left(r_{t+1}^k + 1 - \delta\right)P_{k,t+1}\right] \tag{7}$$

Following Thomas (2008) and GT, hours per worker will be determined by firm and worker in a privately efficient way. They will maximize the joint surplus of their employment relationship.

2.2 Matching, Vacancies, and Unemployment

Assume that each intermediate firm indexed by $i \in [0,1]$ employs $L_t(i)$ workers at time t. It also post $V_t(i)$ vacancies in order to attract new workers for the next period of operation. Before production starts and following Mortensen & Pissarides (1994), assume that ρL_{t-1} jobs that are not matched are destroyed. The parameter ρ represents then the rate of destruction or separation rate, which will be assumed constant.[7] Workers that have lost their jobs at time $t-1$ start searching immediately and can still be hired at time t for the

[5] I adopt the same assumption as in Thomas (2008).

[6] $\lambda_t(j)$ being the Lagrangian multiplier. Cf. Acurio-Vásconez et al. (2015) for details on the derivation

[7] As pointed out by GT within this framework, fluctuations in unemployment come from the cyclical variation in hiring and not from fluctuations in the separation rate, a fact corroborated in U.S. labor market. Cf. Hall (2005) and Shimer (2005).

next period.[8] The total number of employed workers and vacancies are $L_t := \int_0^1 L_t(i)di$ and $V_t := \int_0^1 V_t(i)di$ respectively. Accordingly, the law of motion of aggregate employment is then:

$$L_t = (1-\rho)L_{t-1} + M_{t-1} \tag{8}$$

where M_t represents the number of job aggregate matches that are made at time t. Then the total number of unemployed workers, who are searching for a job, S_t, is given by:

$$\begin{aligned} S_t &:= 1 - L_t \\ &= \underbrace{1 - L_{t-1}}_{(a)} + \underbrace{\rho L_{t-1}}_{(b)} - \underbrace{M_t}_{(c)} \end{aligned}$$

Then unemployment is composed by (a) unemployed workers at time t, plus (b) workers who have lost their jobs at the end of time t-1, minus (c) the matches at time t that could include workers that lost their jobs at the end of time t-1.

Remark that the distinction between being unemployed and not being in the labor force is ignored with this assumption. Then full participation is guaranteed (i.e at all times, all individuals are either employed or looking for a job).

Define now:

$$\widetilde{q}_t := \frac{M_t}{V_t}, \quad \theta_t := \frac{V_t}{S_t} \quad and \quad z_t := \frac{M_t}{S_t}$$

then $\widetilde{q}_t$ represents the probability a firm fills a vacancy in period t, which could be interpreted as the probability of transition from unfilled vacancy to a filled one; θ_t is a measure of the tightness of the labor market and z_t represents the probability an unemployed worker find a job, i.e the probability of transition from unemployment to employment. I assume that the probabilities $\widetilde{q}_t$ and z_t are taken as given by both firms and workers.

2.3 Intermediate Goods Firms

Assume that there exists a continuum of perfectly competitive intermediate goods producers indexed by $i \in [0,1]$ that produce an homogeneous good $Q_t(i)$ through a nested CES production function involving oil, capital, and labor as in Acurio-Vásconez (2015) defined by:

$$Q_t(i) := \left(x_p(A_{E,t}E_t(i))^{\sigma_p} + (1-x_p)(A_{LK,t}(K_t(i)^{\alpha}\left(T_t(i)L_t(i)\right)^{1-\alpha}))^{\sigma_p}\right)^{1/\sigma_p}$$

where $E_t(i)$ is the quantity of oil used, $K_t(i)$ is the capital rented, $L_t(i)$ is the amount of labor rented by the intermediate firm i and, $T_t(i)$ is the total of hours provided by each worker at firm i. $A_{E,t}$ and $A_{LK,t}$ represent respectively a measure of oil productivity and the total factor productivity (TFP), which measures the productivity of the combination of labor and capital. The "share" of capital in the composite factor is measured by $\alpha \in [0,1]$ and $\sigma_p = \frac{\eta_p - 1}{\eta_p}$, with η_p being the elasticity of substitution between the utilization of oil and the composite factor (of capital and labor). Finally, x_p is a distribution parameter. As remarked in Cantore & Levine (2012) one can rewrite this equation as:

$$Q_t(i) := \left(\alpha_e\left(\frac{A_{E,t}E_t(i)}{A_E E}\right)^{\sigma_p} + (1-\alpha_e)\left(\frac{A_{LK,t}K_t(i)^{\alpha}\left(T_t(i)L_t(i)\right)^{1-\alpha}}{A_{LK}K^{\alpha}\left(TL\right)^{1-\alpha}}\right)^{\sigma_p}\right)^{1/\sigma_p} \tag{9}$$

[8] Under this assumption workers that have been hired at time t become productive at time $t+1$. This assumption is well accepted whenever the time period is one month.

where variables without a time subscript represent the steady state of the equally named variable. As proved in Acurio-Vásconez (2015), with this normalization α_e is the oil's output elasticity at steady state. I assume that both technologies shocks are $AR(1)$ processes:

$$ln(A_{E,t}) = \rho_{ae} ln(A_{E,t-1}) + e_{ae}, \quad ln(A_{LK,t}) = \rho_{alk} ln(A_{LK,t-1}) + e_{alk}$$
$$e_{ae} \sim \mathcal{N}(0, \sigma_{ae}^2), \quad e_{alk} \sim \mathcal{N}(0, \sigma_{alk}^2)$$

For simplicity I assume that capital and oil are perfectly mobile across firms and that there is a competitive market in oil and capital. The intermediate firm sells this good $Q_t(i)$ to the final good firm at the perfectly competitive real price $P_q^r := \frac{P_{q,t}(i)}{P_{q,t}}$ where $P_{q,t}$ is the aggregate final good price.

As mentioned before, each firm enters the market with $L_{t-1}(i)$ quantity of employees, and each follows the same law of motion defined in (8):

$$L_{t+1} := (1-\rho)L_t + M_t$$

where $M_t := \int_0^1 M_t(i)di$ denotes the aggregate matches in period t. Assume that the aggregate matching process follows:

$$M_t := \psi S_t^{\gamma} V_t^{1-\gamma}$$

where ψ is a scale parameter that captures the efficiency of the search and matching process.

Because firms post vacancies, there are hiring costs that introduce employment adjustments. These adjustments introduce time frictions in the process in which households and firms exchange labor and give sense to the bargaining problem between firms and workers. Accordingly, denote $X_t(i)$ the hiring rate, i.e, the percent change in the firm's workforce from t to $t+1$, as the ratio of new hires, $\widetilde{q}_t V_t(i)$, to the existing workforce $L_t(i)$, then:

$$X_t(i) := \frac{\widetilde{q}_t V_t(i)}{L_t(i)} \tag{10}$$

Note that by the law of large numbers, $X_t(i)$ is known with certainty at time t, since $\widetilde{q}_t$ is known with certainty at time t. Then the total workforce could be expressed as the sum of the surviving workers $(1-\rho)L_t$ and the new hires $\widetilde{q}_t V_t$, thus one has:

$$L_{t+1}(i) := (1-\rho + X_t(i))L_t(i) \tag{11}$$

As explained before, remark that this time assumption reflects the fact that new hires start to work one period after being hired.[9]

As discussed in GT and Thomas (2008), in order to introduce staggered wages rigidities, it is necessary to consider convex vacancy cost.[10] Following GT, I assume quadratic hiring cost (in real terms) by:

$$\frac{\kappa}{2} X_t^2(i) L_t(i)$$

where κ is the cost of adjustment parameter, which I suppose constant across time.

[9] This assumption has been used in numerous papers as GT, Thomas (2008), Groshenny (2013), among others.

[10] As explained in GT, the staggered wage setting introduces dispersion of wages across firms in equilibrium, convex hiring cost ensures that, at equilibrium, every firm posts vacancies, so that the equilibrium is deterministic

Firms maximize profits. In order to study this problem, I will divide it into three stages: (1) Each firm i takes all prices, including the nominal current wage $W_t(i)$, the demand $Q_t(i)$, the probability of filling a vacancy $\widetilde{q}_t$ and its existing employment stock as given and then it chooses quantities of oil, $E_t(i)$, capital, $K_t(i)$ and the hiring rate $X_t(i)$, by posting vacancies, in order to maximize its real profit; (2) Firms and workers determine the quantity of hours by maximizing the joint surplus of their employment relationship and; (3) The firm will bargain, if it can, with the households, in order to fix the wage, solving a Nash bargaining problem.

Let us denote $\Lambda_{t,t+1} := \frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is the Lagrangian multiplier associated to the household problem. Denote $F_t(i)$ the present real value of firm i, i.e:

$$F_t\left(i\right) := P^r_{q,t}Q_t(i) - W_{r,t}(i)L_t(i) - r^k_t S_{k,t}K_t(i) - S_{e,t}E_t(i) - \frac{\kappa}{2}X^2_t(i)L_t(i) + \beta\mathbb{E}_t\left[\Lambda_{t,t+1}F_{t+1}\right] \tag{12}$$

where $W_{r,t}(i) := \frac{W_t(i)}{P_{q,t}}$ is the real wage paid by the firm i, $S_{k,t} := \frac{P_{k,t}}{P_{q,t}}$ the real price of capital and $S_{e,t} = \frac{P_{e,t}}{P_{q,t}}$, the real price of oil.[11] The real prices of oil and capital are considered exogenous and given by:

$$ln(S_{e,t}) := \rho_{se}ln(S_{e,t-1}) + e_{e,t}, \quad ln(S_{k,t}) := \rho_{sk}ln(S_{k,t-1}) + e_{k,t}$$

where $e_{e,t} \sim \mathcal{N}(0, \sigma^2_e)$ and $e_{k,t} \sim \mathcal{N}(0, \sigma^2_k)$ are Gaussian white noises.

The first order conditions with respect to $E_t(i)$ and $K_t(i)$ give:

$$E_t(i): \quad S_{e,t} = P^r_{q,t}x_p A^{\sigma_p}_{E,t}\left(\frac{E_t(i)}{Q_t(i)}\right)^{\sigma_p - 1} \tag{13}$$

$$K_t(i): \quad r^k_t S_{k,t} = P^r_{q,t}\alpha(1-x_p)A^{\sigma_p}_{LK,t}\left(\frac{K_t(i)}{Q_t(i)}\right)^{\alpha\sigma_p - 1}\left(\frac{L_t(i)}{Q_t(i)}\right)^{(1-\alpha)\sigma_p} \tag{14}$$

Then the first-order condition with respect to $K_t(i)$ and $E_t(i)$ equalizes the marginal revenue product of capital and oil to their respective real price.

Using these equations one can derive the following expression:

$$r^k_t S_{k,t} = \alpha P^r_{q,t}\frac{Q_t(i)}{K_t(i)} - \alpha S_{e,t}\frac{E_t(i)}{K_t(i)} \tag{15}$$

Given constant returns to scale in production and perfect capital and oil mobility, all firms choose the same capital-output ratio and the same oil-output ratio. This implies that the marginal product of labor is defined by $\Gamma_t(i) := \frac{\partial Q_t(i)}{\partial \bar{L}_t(i)}$, where $\bar{L}_t(i) = L_t(i)T_t(i)$ is equalized across firms as well. Then one has $\Gamma_t(i) = \Gamma_t$, for all i.

Let $J_t(i)$ be the real value of the firm for adding another worker at time t after adjustment costs are sunk. Remark that $J_t(i)$ could be interpreted as being the firm surplus and it is equal to the partial derivative of $F_t(i)$ with respect to $L_t(i)$ holding $X_t(i)$ fixed. Then $J_t(i)$ is given by:

$$J_t(i) = P^r_{q,t}T_t(i)\Gamma_t - W_{r,t}(i) - \frac{\kappa}{2}X^2_t(i) + \beta(1-\rho+X_t(i))\mathbb{E}_t\Lambda_{t,t+1}J_{t+1}(i) \tag{16}$$

[11] Note that the intermediate firm acts in a perfectly competitive market, so it cannot impose the price. The staggered prices will be included in the final good firm process.

Remark that firms choose $L_t(i)$ by setting $X_t(i)$, or equivalently, by choosing $V_t(i)$. Then the first order condition for vacancy posting equates the marginal cost of adding a worker with the discounted marginal benefit. Then one has:

$$\kappa X_t(i) = \beta \mathbb{E}_t \Lambda_{t,t+1} J_{t+1}(i) \tag{17}$$

Using equation (16), one can derive the following relationship for the hiring rate:

$$X_t(i) = \frac{\beta}{\kappa} \mathbb{E}_t \Lambda_{t,t+1} \left[P^r_{q,t} \Gamma_{t+1} T_{t+1}(i) - W_{r,t+1}(i) + \frac{\kappa}{2} X^2_{t+1}(i) + (1-\rho)\kappa X_{t+1}(i) \right] \tag{18}$$

which is a forward looking difference equation for the hiring rate. The hiring rate depends thus on a discounted steam of the firm's expected surplus from the marginal worker which is the real marginal product of labor times hours worked minus the real wage (the net earning at the margin), plus the saving on adjustment cost $\frac{\kappa}{2} X^2_{t+1}(i)$.

Define $D_t(i)$ the real value to a worker working at firm i and define U_t the value of total unemployment at time t. Remark that these two values are defined after hiring decisions at time t have been made. One then has:

$$D_t(i) := W_{r,t}(i) - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left((1-\rho) D_{t+1}(i) + \rho U_{t+1} \right) \tag{19}$$

The value $D_t(i)$ depends on the real salary specific to firm i, plus the disutility in consumption units, plus the discounted value generated if it remains employed or not in the subsequent period.

Let us define:

$$D_{x,t} := \int_0^1 D_t(i) \frac{X_{t-1}(i) L_{t-1(i)}}{X_{t-1} L_{t-1}} di \tag{20}$$

the average value of employment conditional on being a new worker at time t. Then the subscript x is for the average taken from new workers, i.e those who have been hired at period $t-1$. Then the real value of unemployment, denoted U_t, is given by:

$$U_t := b + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(z_t D_{x,t+1} + (1-z_t) U_{t+1} \right)$$

then, as in the case of value of a worker at firm i, U_t is defined by the unemployment compensation plus the discounted value restraint to the probability that the worker will find a job or not in the subsequent period.

Note that the value of finding a job next period for a worker currently unemployed at time t is $D_{x,t+1}$, which is the average value of working next period. Then as pointed out by GT, unemployed workers do not have a prior knowledge of which firm might be paying higher wages next period.

With these two notions, one can define the worker surplus at firm i, denoted $H_t(i)$, and the average worker surplus conditional on being a new hire, denoted $H_{x,t}$ by:

$$H_t(i) := D_t(i) - U_t, \quad H_{x,t} := D_{x,t} - U_t$$

Combining these equations one has the following relationship for the worker surplus at firm i:

$$H_t(i) = W_{r,t}(i) - b - \frac{V(T_t(i))}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\chi H_{t+1}(i) - z_t H_{x,t+1} \right) \tag{21}$$

where $\chi = 1 - \rho$.

Then the worker's contribution to the welfare of the family depends on the real salary earned if she/he works in firm i, minus what she/he looses if she/he works, plus her/his future contribution if she/he is not separated next period, minus the value incurred if she/he is searching for a job.

In the second step, as remarked by Thomas (2008), hours per worker are determined by firms and workers by maximizing the joint surplus of their employment relationship, i.e they maximize over T_t the sum the household surplus $H_t(i)$, and the firm surplus, $J_t(i)$, which leads to the following first order condition:

$$P^r_{q,t}\Gamma_t = \frac{V'(T_t(i))}{U'(C_t)} = T_t(i)^{\widetilde{w}} C_t \tag{22}$$

i.e, hours adjust to the point where the marginal value product of labor $P^r_{q,t}\Gamma_t$ equals the marginal rate of substitution between consumption and leisure for the worker. Because the marginal product of labor is the same for all producers, then hours across producers has to be also the same, i.e $T_t(i) = T_t$ for all i. Note that hours are independent of the monthly wage. The wage however will depend on the hours worked.

2.4 Wage Setting

Staggered Nash wage bargaining is introduced as in GT. First, as in Calvo (1983), let us suppose that each period, a fraction of firms $(1 - \theta_w)$ renegotiates its wage contracts, and its adjustment is independent of its history, meaning that firms do not need to know their history on being capable or not to renegotiate wages.

Assume also that while the duration of the contract of each firm does remains uncertain, the aggregate duration is $\frac{1}{1-\theta_w}$. This way θ_w could be interpreted as being a wage stickiness measure parameter.

For simplicity, I assume that firms which cannot renegotiate their wage contract at time t, take the nominal wage as it was in the last period, i.e $W_t(i) = W_{t-1}(i)$. Additionally, assume that those firms who are able to renegotiate wage contracts at time t, bargain with their respective existing workforce, which includes the recent new hires. Thus, workers hired in between contract periods receive the same wage as existing workers.[12]

Let $W^o_t(i)$ be the wage of the firm i that renegotiates its contract at date t. When renegotiating wage contracts, firms face the same problem. Hence they all set the same wage. Thus the choice of W^o_t does not depend on i. Assuming Nash bargaining, the contract wage W^o_t is chosen, so as to solve the following problem:

$$\begin{aligned} &\max_{W_t(i)} \quad H_t(i)^{\eta} J_t(i)^{1-\eta}, \\ &\text{subject to :} \quad W_t(i) = \begin{cases} W_{t-1}(i), & \textit{with} \quad \textit{proba} \quad \theta_w \\ W^o_t, & \textit{with} \quad \textit{proba} \quad (1-\theta_w) \end{cases} \end{aligned}$$

where $\eta \in [0,1]$ is the worker's relative bargaining power, assumed to be constant across time, and where $J_t(i)$ and $H_t(i)$ are defined by (16) and (21) respectively.

[12] As pointed out by GT, constant returns to scale guarantees that bargaining with individual marginal workers is equivalent to bargaining with a union maximizing average worker surplus. The wage is chosen in such a way that the firm and the marginal worker share the surplus from the marginal match. For further discussion on the validation of this assumption, please refer to Gertler et al. (2008), Thomas (2008) and references therein.

The first-order necessary condition for the Nash bargaining solution is given by:

$$\eta\xi_t J_t(r) = (1-\eta)\Sigma_t(r)H(r) \tag{23}$$

where $J_t(r)$ and $H_t(r)$ represent the firm's and worker's surplus for a firm renegotiating its wage at time t respectively, which exact expressions can be found in the Appendix (equations (33) and (34)), and:

$$\xi_t := \frac{\partial H_t(r)}{\partial W_t^o} P_{q,t}, \quad \Sigma_t(r) := -\frac{\partial J_t(r)}{\partial W_t^o} P_{q,t}$$

Remark that ξ_t is the effect of a rise in the contract wage on the worker surplus and $\Sigma_t(r)$ is the opposite of the effect of a rise in the contract wage on the firm's surplus. Using the exact values of ξ_t and $\Sigma_t(r)$ one can note that $\Sigma_t(r) > \xi$ on average. As remarked by GT, this implies that shifts in the contract wage have a larger impact in absolute value on firm's surplus than on worker's surplus.

For a further analysis, let us rewrite the Nash condition by:

$$\nu_t(r)J_t(r) = (1-\nu_t(r))H_t(r) \tag{24}$$

where:

$$\nu_t(r) := \frac{\eta}{\eta + (1-\eta)\frac{\Sigma_t(r)}{\xi}} \tag{25}$$

Using the expressions of $J_t(r)$ and $H_t(r)$, one can derive the following relationship for the wage dynamics:

$$\mathbb{E}_t\Delta_t W_{r,t}^o = \mathbb{E}_t W_t^{tar}(r) + \beta\chi\theta_w \mathbb{E}_t \Delta_{t+1}\Lambda_{t,t+1}W_{r,t+1}^o \tag{26}$$

where:

$$W_t^{tar}(r) := \mathbb{E}_t\left[\nu_t(r)\left(P_{q,t}^r\Gamma_t T_t + \frac{\kappa}{2}X_t^2(r)\right) + (1-\nu_t(r))\left(b + \frac{V(T_t)}{U'(C_t)} + \beta\Lambda_{t,t+1}z_t H_{x,t+1}\right)\right] \tag{27}$$

Note that in the case of full flexibility, $\theta_w = 0$, the ratio $\frac{\Sigma_(r)}{\xi}$ equals 1 and so $\nu_t(r) = \eta$. Then one recovers the conventional case of period-by period wage bargaining in the case without staggered wages.

2.5 The Final Good Firm (Retailers)

As in Gertler et al. (2008) and Blanchard & Galí (2010), let me introduce staggered prices, via the final good producers. Suppose that there exists a continuum of monopolistically final good producers indexed by $j \in [0,1]$. Each final good firm uses the intermediate goods produced by the representative intermediate firm described in Section 2.3, and transforms one unit of intermediate good in a differentiated final good to be resold to the households or exported in exchange for oil. Let us denote $\widetilde{Q}_t(j)$ the quantity of output sold by the final good firm j and $P_{q,t}$ its nominal price. Each final good firm has the following CES production function:

$$\widetilde{Q}_t := \left(\int_0^1 \widetilde{Q}_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj\right)^{\frac{\varepsilon_p}{\varepsilon_p-1}}$$

Each final good producer maximizes its profit in two steps. First, given the aggregate domestic price $P_{q,t}$ and the aggregate demand $\widetilde{Q}_t$, it chooses a quantity of output $\widetilde{Q}_t(j)$. The first order condition of this problem gives the demand addressed to each retailer j:

$$\widetilde{Q}_t(j) = \left(\frac{P_{q,t}(j)}{P_{q,t}}\right)^{-\varepsilon_p} \widetilde{Q}_t \tag{28}$$

In the second step, the final good firm, being a monopolistically one, chooses prices $P_{q,t}(j)$ in order to maximize its profits. As in Calvo (1983), let us assume that there is a fraction $(1-\theta_p)$ of final good firms that can re-optimize their prices at time t. Denote $P^o_{q,t}(j)$ this re-optimized price. The rest of the firms that cannot reset their prices, leave them as the period before (i.e $P_{q,t} = P_{q,t-1}$). As in the case of wages, $\frac{1}{1-\theta_p}$ represents the average duration of the price contract and so θ_p could be interpreted as a price stickiness measure.

Using equation (28) one can derive an expression for the final good price aggregator:

$$P_{q,t} = \left(\int_0^1 (P_{q,t}(j))^{1-\varepsilon_p} dj\right)^{\frac{1}{1-\varepsilon_p}}$$

Firms that are able to re-optimize prices will choose the same price, because all of them face the same problem. Then one has the following Calvo aggregate equation:

$$P^{1-\varepsilon_p}_{q,t} = (1-\theta_p)(P^o_{q,t})^{1-\varepsilon_p} + \theta_p P^{1-\varepsilon_p}_{q,t-1}$$

The maximization of the intertemporal profit leads to the following equation:

$$\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\theta_p)^k \Lambda_{t,t+k}\widetilde{Q}^o_{t+k}\left(\frac{P^o_{q,t}}{P_{q,t+k}} - \frac{\varepsilon_p}{\varepsilon_p - 1}P^r_{q,t+k}\right)\right] = 0$$

In the case of flexible prices $(\theta_p = 0)$, one has:

$$P^r_{q,t} = \frac{\varepsilon - 1}{\varepsilon_p} = \frac{1}{\mathcal{M}_p}$$

where $\mathcal{M}_p$ is the price markup.

Note that producing $\widetilde{Q}_t(j)$ units of final good j requires $Q_t(i)$ units of intermediate good i which is purchased from the intermediate firm at real price $P^r_{q,t}$. Therefore $P^r_{q,t}$ represents the real marginal cost for the production of final goods.

2.6 GDP, Monetary Policy and Government

I define nominal GDP at time t by:

$$S_{c,t}Y_t := P^r_{q,t}Q_t - S_{e,t}E_t - \frac{\kappa}{2}\int_0^1 X^2_t(i)L_t(i)di$$

Note that with this assumption GDP is the value added of production.[13]

[13] See Blanchard & Galí (2007) and Acurio-Vásconez et al. (2015) for further discussion on this assumption.

Let $\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$ be the domestic inflation. Suppose that the Central Bank sets the nominal short-term interest rate, i_t, by the following monetary policy:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} (\Pi_{q,t})^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \varepsilon_{i,t},$$

where $ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$ with $e_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$.

Finally, the Government budget constraint is given by:

$$(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t + b,$$

where G_t stands for the nominal government spending. I assume that the real government spending $G_{r,t} = \frac{G_t}{Pq,t}$ is an exogenous process given by:

$$\ln(G_{r,t}) = (1 - \rho_g)(\ln(\omega Q)) + \rho_g \ln(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}$$

where ω represents the share that the government takes from the total output (Q_t) for its own spending, Q represent the steady state of Q_t, and $e_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$ is a Gaussian white noise.

3 Model Evaluation

3.1 Calibration

In what follows, I assume a monthly frequency for the model. As pointed out by GT, a month calibration properly capture the high rate of job finding in U.S.data[14]. Aggregation and the steady state calculation are shown in the Appendix.

There exists 19 structural parameters in the model, in addition to those related to shocks. Four of these parameters are conventional in the Business Cycle literature for U.S: the discount factor, β, is set at $0.99^{\frac{1}{3}}$, the depreciation rate, δ, is set at $\frac{0.025}{3}$ the government spending output share, ω, is fixed at 0.18 and the price markup at steady state, $\mathcal{M}_p$, is set at $\frac{8}{7}$. There are also four parameters that are specific to the Nash bargaining process and the search and matching framework. I calibrate them as in GT. The monthly separation rate, ρ, is fixed at 0.035. This choice implies that jobs last about $2\frac{1}{2}$ years on average. The elasticity of matches to unemployment, γ, is set at 0.5[15]. The scale parameter that captures the efficiency of the search and matching process, ψ, is normalized at 1. The bargaining power parameter, η, is set at 0.5. The average probability of finding a job, z, is set at 0.46 following the results obtained by Shimer (2005) for the U.S.average monthly job-finding rate. As shown in the Appendix in equation (44) the unemployment benefits, b, can be calibrated if one calibrates $\widetilde{b}$, which is defined as the ratio of the unemployment flow value, $b + \frac{V(T_t)}{U'(C_t)}$, to the steady state flow contribution of the worker to the match $(P_q^r \Gamma T_t + \frac{\kappa}{2} X^2)$. The calibration of $\widetilde{b}$ is controversial. Shimer (2005) calibrated it at 0.4, Hall (2009) proposes a value of 0.7 and the Bayesian estimation performed in Gertler et al.

[14]This time path fits with the assumption that workers that have lost their jobs in time $t-1$ starts searching immediately and can still be hired at time t, but start working in the next period. Also, as emphasized by Thomas (2008), monetary policy decisions in the U.S.Federal Reserve are at least eight per year, then a monthly calibration is better to treat this problem.

[15]As pointed out by GT, this value is in the range of plausible ones, reported in Petrongolo & Pissarides (2001) survey

(2008) yields 0.73 for this parameter, close to the one in Hall (2009). However, their estimation recovers also a value of 0.9 for η, which is out of the range of this parameter, usually between 0.4 and 0.7. In order to be as close as possible to the literature, I calibrate this parameter at 0.5, which is the prior mean chosen by Gertler et al. (2008) for their Bayesian estimation. Also following GT, I calibrate the probability that a firm may not renegotiate the wage contract at 8/9, which assume that wages are on average renegotiated each 9 months. Finally, following GT and Thomas (2008), I set the hiring cost as a fraction of domestic output in steady state, $\frac{\kappa X^2 L}{2Q}$, at 1 percent.

For the Taylor rule coefficients, I use the calibration of Sveen & Weinke (2008), then the response to inflation, ϕ_π, the response to the output, ϕ_y, and the persistence of the Taylor rule, ϕ_i, are set at 1.5, $\frac{0.5}{12}$, and $0.7^{\frac{1}{3}}$ respectively.

Remaining values are calibrated using the estimated values of Acurio-Vásconez (2015). Then the share of oil consumption for households, x_c, oil's output elasticity at steady state, α_e, oil's elasticities of substitution, η_c and η_p, are calibrated at 0.0286, 0.0597, 0.5056 and 0.1387 respectively. The capital "share," α, is fixed at 0.3607 and the price rigidity, θ_p, is calibrated at $0.5164^{\frac{1}{3}}$. Finally, the $AR(1)$ autoregressive parameters are at the power $\frac{1}{3}$.

Parameters' calibration is summarized in Table 1 for the structural parameters, and in Table 2 for the autoregressive parameters.

Table 1: Structural Parameters Calibration

Parameter		Calibration	Parameter		Calibration
Discount factor	β	$0.99^{\frac{1}{3}}$	Calvo wage parameter	θ_w	$\frac{8}{9}$
Depreciation rate	δ	$\frac{0.025}{3}$	Taylor rule response to inflation	ϕ_π	1.5
Markup	M_p	$\frac{8}{7}$	Taylor rule response to output	ϕ_y	$\frac{0.5}{12}$
Gov. spending output share	ω	0.18	Taylor rule persistence	ϕ_i	$0.7^{1/3}$
Separation rate	ρ	0.035	Dist. parameter in consump.	x_c	0.0286
Elast. of matches to unemployment	γ	0.5	Oil's output elasticity	α_e	0.0597
Scale efficiency matching	ψ	1	Elast. substitution in production	η_p	0.1387
Bargaining power	η	0.5	Elast. substitution in consump.	η_p	0.5056
Steady state prob. of finding a job	z	0.46	"Share" parameter on capital	α	0.3607
$\widetilde{b}$		0.5	Calvo price parameter	θ_p	$0.5164^{1/3}$

Table 2: $AR(1)$ Coefficients Calibration

Parameter		Calibration	Parameter		Calibration
Real oil price	ρ_{se}	$0.9565^{\frac{1}{3}}$	Oil productivity	ρ_{ae}	$0.6576^{\frac{1}{3}}$
Real capital price	ρ_{sk}	$0.9221^{\frac{1}{3}}$	TFP	ρ_{alk}	$0.7334^{\frac{1}{3}}$
Government	ρ_g	$0.8910^{\frac{1}{3}}$	Oil Prod. in Gov.	$\rho_{ae,g}$	$0.1741^{\frac{1}{3}}$
Monetary	ρ_i	$0.6576^{\frac{1}{3}}$	TFP in Gov.	$\rho_{alk,g}$	$0.5944^{\frac{1}{3}}$

3.2 Simulations and Results

Figure 1 presents the impulse response functions (thereafter IRFS) of the economy to a two-percent increase in the real price of oil[16]. The model recovers most of the responses obtained with the model developed in Acurio-Vásconez (2015): the small but persistent increase in domestic inflation, the increase in hours worked, the muted response of oil in production and the short lived increase in investment and domestic output. The transmission channel for these responses are explained in Acurio-Vásconez (2015). With respect to the responses of the new variables introduced in this paper, one can observe that an increase in the real price of oil provokes a rise in unemployment that stagnates one year after the shock. This effect is explained as follows. Because of the low substitutability of oil, an increase in its price obligates firms to reduce cost somewhere else, hence one observes a decrease in hiring cost, which translates into a contemporaneous decrease in vacant posts. This then provokes the rise in unemployment. Note that firms could try to reduce costs by reducing wages instead of hiring cost. However after an oil shock, workers need as well more income to maintain their consumption level, so they bargain for higher wages. Then firms prefer to demand more hours of work, which will increase wages, but they reduce vacant posts. It is worth noticing that one observes an increase in real wages, rather than a decrease, in contrast with what was found in Acurio-Vásconez (2015). However, the labor market modelization is different in these two models. As explained in Acurio-Vásconez (2015), if oil substitutability is low, after the increase in oil prices, producers increase their labor demand, in order to produce a larger quantity of domestic output and pay for oil; and consumers increase their labor supply in order to pay for bills. In that particular model, labor supply is larger than labor demand and so the interaction provokes the decrease in real wages observed. However while in Acurio-Vásconez (2015) labor is just hours worked, in this model, labor is composed by hours and number of workers, then it is possible to increase labor demand by increasing hours demand and decreasing vacancies. The increase in hours demand and the households' need of larger wages provoke, after the bargaining, the increase in real wages in this model. Another important point to raise is that when one allows for unemployment, the response of domestic output, while still increasing at the moment of the shock, decreases by 0.14 percent one month after and continues to decrease, in contrast with what we found in Acurio-Vásconez (2015), where GDP came back to its steady state three quarters after the shock.

It is worth stressing that the response of unemployment is particularly sensitive to the calibration of two parameters: the frequency of wage renegotiation, θ_w, and the wage bargaining power of households, η. The frequency of renegotiation is observable from data[17] so its calibration is not problematic. The calibration of parameter η on the other hand, is a subject of controversy. As noted by numerous authors, there is little direct evidence on what an "appropriate" value of this parameter should be. As a sensitive exercise, let me reduce the bargaining power from 0.5 to 0.1 and do the same exercise as before. The blue solid line in Figure 2 represents the IRFS with the baseline calibration, while the dashed green line represents the IRFS of the model where the bargaining power of households has been diminished, ceteris paribus. As we can observe, with this change in calibration the increase in unemployment is being reduced by 1.3 percent on average, in comparison with the baseline calibration, while the responses of the rest of the variables do not have obvious

[16] I use the estimated standard deviation in Acurio-Vásconez (2015) to facilitate comparison.

[17] The latest studies on the subject found that most of wages in U.S.are renegotiated once a year. See Gottschalk (2005), Gertler et al. (2008) among others.

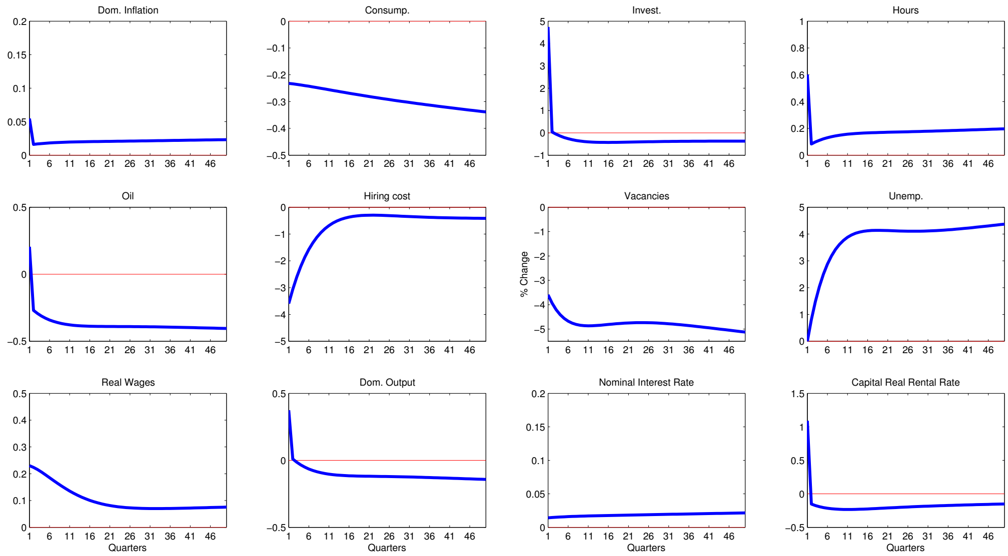

Figure 1: Response to 2 Percent Increase on Real Price of Oil

changes, besides from those directly affected as wages, hiring cost and, vacancies. These responses are explained as follows. On one hand, if households have less bargaining power when negotiating wage contracts, even if they ask for higher wages in order to offset the increase in oil prices, they will not be able to obtain the same wage contract as before, so that real wages do not increase as much as before right after the shock. On the other hand, firms still lower their hiring cost by reducing vacancies to cope with the increase in oil prices, but because they can negotiate lower wage contracts, the reduction in hiring costs are significantly smaller. Then unemployment increases around 2.5 percent, one year after the shock. It appears then that the reduction of the wage bargaining power of households could reduce the impact of an oil shock in unemployment.

This last result then raises another question: Could flexible wages[18] and low bargaining power for the households prevent the stagflatonary effects of oil shocks? In other to shed some light to this question, let me realize one more experiment. Figure 3 represents the IRFS of the economy in two different scenarios: the baseline calibration, and one where wages have been flexibilized (changing θ_w from 8/9 to 0.1), and the bargaining power of household reduced (changing η from 0.5 to 0.1). It seems that wage flexibility has an important effect in unemployment, preventing it from suddenly increasing after the oil shock. This is because when wages are renegotiated frequently and workers have almost no bargaining power, firms will be able to negotiate a lower wage contract and so reduce production cost, without strongly reducing hiring cost, i.e without limiting vacancies. This income reduction however reduces the rise in investment and output. Then while it seems that the two policies together, wage flexibility and the reduction in the bargaining power of households, could play a role on the reduction of unemployment reaction to oil shocks, but it can also cause a decrease in household income, which will have an effect in investment and then in GDP.

[18]Note that wage flexibility refers in this model to an increase in the frequency of wage negotiation.

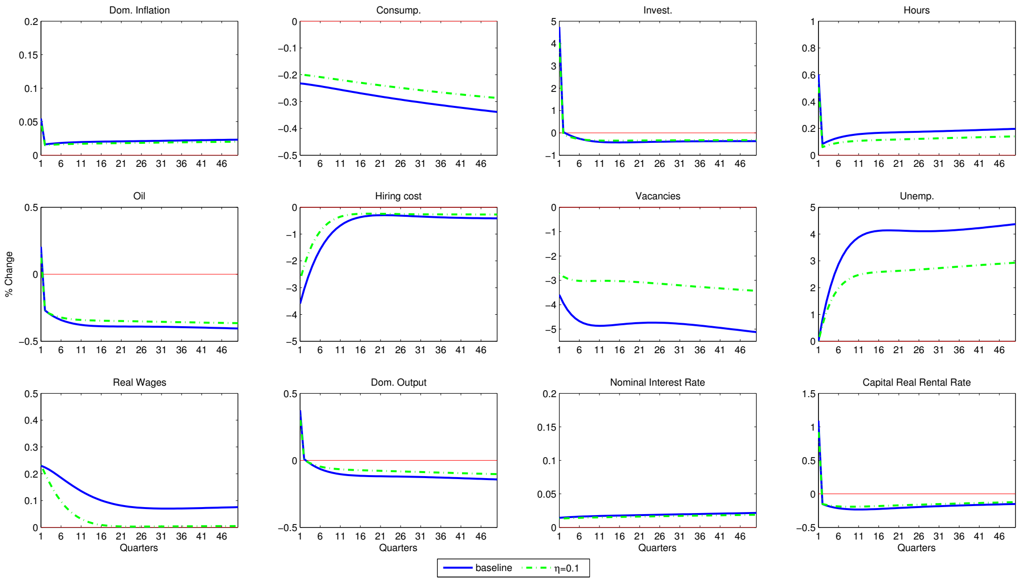

Figure 2: Response to 2 Percent Increase on Real Price of Oil–Bargaining Power Comparison

Comparison of two models, baseline (solid blue line) and its counterpart with reduced bargaining power (dashed green line)

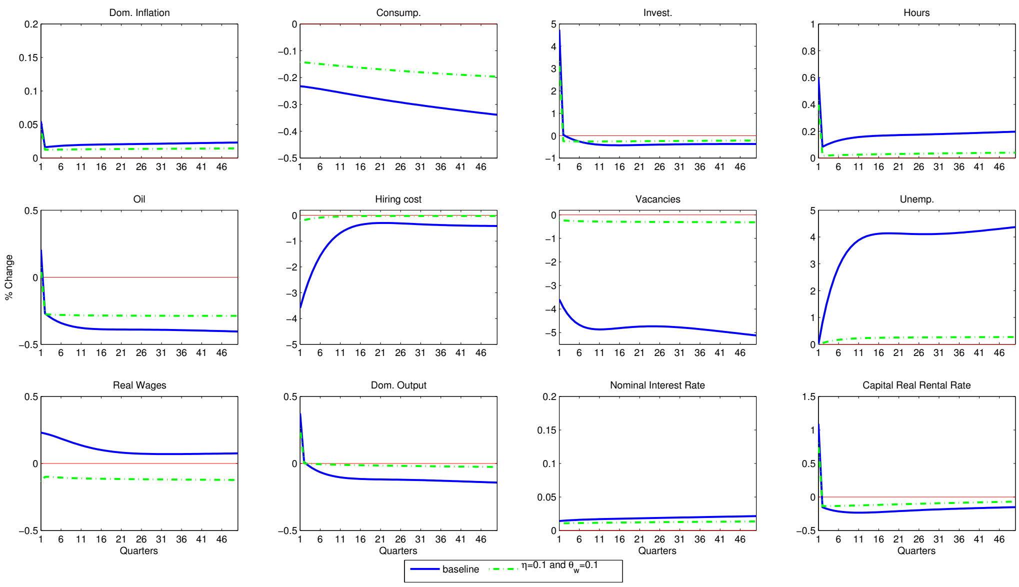

Figure 3: Response to 2 Percent Increase on Real Price of Oil—Wage Flexibility Comparison

Comparison of two models, baseline (solid blue line) and its counterpart with reduced bargaining power and flexible wages (dashed green line)

4 Conclusion

This papers addresses the question of the effects of oil price increases into unemployment and wage dynamics. It constructs a medium scale DSGE model that mixed two fundamental ideas: the introduction of oil in consumption and production and the recent literature on unemployment modelization. The model is able to recover most of the well-known stylized facts observed after the oil shock in the 2000s'. A sensitivity analysis shows that the reduction in the bargaining power of households for wage contracts negotiation reduces the impact of an oil shock in unemployment without negatively affecting domestic output. However, it also shows that even though the two policies together, wage flexibility and the reduction in the bargaining power of households, could substantially reduce the reaction of unemployment to oil shocks, they can also cause a negative reaction in wages, affecting negatively the income of households and consequently investment and GDP.

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A Appendix A

A.1 Firms and Workers

The problem of each firm i is:

$$\begin{aligned}
\max_{E_t(i),X_t(i),K_t(i),W_t(i)} \quad & \mathbb{E}_t\left[\sum_{k=0}^{\infty} d_{t,t+k}\left(P^r_{q,t+k}Q_{t+k}(i) - realcost(Q_{t+k}(i))\right)\right], \\
\text{subject to :} \quad & L_{t+k}(i) = (1-\rho+X_{t+k}(i))L_{t+k}(i) \\
& Q_{t+k}(i) = \left(x_p(A_{E,t+k}E_{t+k}(i))^{\sigma_p} + (1-x_p)(A_{LK,t+k}(K_{t+k}(i)^{\alpha}L_{t+k}(i)^{1-\alpha}))^{\sigma_p}\right)^{1/\sigma_p}
\end{aligned}$$

where $d_{t,t+k}$ is the stochastic disc out factor from date t to $t+k$, defined as:

1. Stochastic discount factor from date t to date $t+1$

$$d_{t,t+1} := \frac{\beta\lambda_t}{\lambda_{t+1}} = \beta\Lambda_{t,t+1}, \quad \Lambda_{t,t+1} = \frac{\lambda_t}{\lambda_{t+1}}$$

2. Stochastic discount factor from date t to date $t+k$

$$d_{t,t+k} := \frac{\beta^k\lambda_t}{\lambda_{t+k}} = \beta^k\Lambda_{t+k,t}$$

Let us denote $F_t(i)$ the present real value of the firm i. Then we have:

$$\begin{aligned}
F_t(i) =& \mathbb{E}_t d_{t,t}\left[P^r_{q,t}Q_t - W_{r,t}(i)L_t(i) - r^k_t S_{k,t}K_t(i) - S_{e,t}E_t(i) - \frac{\kappa}{2}X^2_t(i)L_t(i)\right] + \\
&+\mathbb{E}_t d_{t,t+1}\left[P^r_{q,t+1}Q_{t+1} - W_{r,t+1}(i)L_{t+1}(i) - r^k_{t+1}S_{k,t+1}K_{t+1}(i) - S_{e,t+1}E_{t+1}(i) - \frac{\kappa}{2}X^2_{t+1}(i)L_{t+1}(i)\right] \\
&+... \\
&= P^r_{q,t}Q_t - W_{r,t}(i)L_t(i) - r^k_t S_{k,t}K_t(i) - S_{e,t}E_t(i) + \\
&+\beta\Lambda_{t,t+1}\left[P^r_{q,t+1}Q_{t+1} - W_{r,t+1}(i)L_{t+1}(i) - r^k_{t+1}S_{k,t+1}K_{t+1}(i) - S_{e,t+1}E_{t+1}(i) - \frac{\kappa}{2}X^2_{t+1}(i)L_{t+1}(i)\right] \\
&+... \\
&= P^r_{q,t}Q_t - W_{r,t}(i)L_t(i) - r^k_t S_{k,t}K_t(i) - S_{e,t}E_t(i) - \frac{\kappa}{2}X^2_t(i)L_t(i) + \beta\mathbb{E}_t\Lambda_{t,t+1}F_{t+1}(i)
\end{aligned}$$

The first order condition with respect to $E_t(i)$ and $K_t(i)$ are given by:

$$\begin{aligned}
E_t(i): \quad & \frac{\partial}{\partial E_t(i)}F_t(i) = 0 \Leftrightarrow S_{e,t} = P^r_{q,t}\frac{\partial}{\partial E_t(i)}Q_t(i) \\
K_t(i): \quad & \frac{\partial}{\partial K_t(i)}F_t(i) = 0 \Leftrightarrow r^k_t S_{k,t} = P^r_{q,t}\frac{\partial}{\partial K_t(i)}Q_t(i)
\end{aligned}$$

Using these equations one can write:

$$\begin{aligned}
r^k_t S_{k,t} =& \frac{P^r_{q,t}}{K_t(i)}\alpha Q_t(i)^{1-\sigma_q}\left[Q^{\sigma_q}_t(i) - E^{\sigma_q}_t(i)\right] \\
=& \frac{P^r_{q,t}}{K_t(i)}\alpha Q_t(i) - \frac{P^r_{q,t}}{K_t(i)}\alpha Q^{1-\sigma_q}_t(i)E^{\sigma_p}_t(i) \\
=& \alpha P^r_{q,t}\frac{Q_t(i)}{K_t(i)} - \alpha\frac{E_t(i)}{K_t(i)}S_{e,t}
\end{aligned}$$

Defining $J_t(i)$ as in the paper one has:

$$\begin{aligned}
J_t(i) :=&\frac{\partial F_t(i)}{\partial L_t(i)}\\
=&P^r_{q,t}\frac{\partial Q_t(i)}{\partial L_t(i)} - W_{r,t}(i) - \frac{\partial}{\partial L_t(i)}\left[\frac{\kappa}{2}\left(\frac{\widetilde{q}_t V_t(i)}{L_t(i)}\right)^2 L_t(i)\right] + \beta\frac{\partial}{\partial L_t(i)}\mathbb{E}_t\Lambda_{t,t+1}F_{t+1}(i)\\
=&P^r_{q,t}\frac{\partial Q_t(i)}{\partial L_t(i)} - W_{r,t}(i) + \frac{\kappa}{2}\left(\widetilde{q}_t V_t(i)\right)^2 L_t(i)^{-2}+\\
+\beta\frac{\partial}{\partial L_t(i)}&\mathbb{E}_t\Lambda_{t,t+1}\left[P^r_{q,t+1}Q_{t+1}(i) - W_{r,t+1}(i)L_{t+1}(i) - r^k_t S_{k,t+1}K_{t+1} - S_{e,t+1}E_{t+1} - \frac{\kappa}{2}X^2_{t+1}(i)L_{t+1}(i)\right]+\\
&+\beta^2\frac{\partial}{\partial L_t(i)}\mathbb{E}_t\Lambda_{t,t+2}F_{t+2}(i)
\end{aligned}$$

Having X_t fixed, one has:

$$J_t(i) =P^r_{q,t}\Gamma_t T_t - W_{r,t}(i) - \frac{\kappa}{2}X^2_t(i) + \beta(1-\rho+X_t(i))\mathbb{E}_t\Lambda_{t,t+1}J_{t+1}(i)$$

The first order condition with respect to $V_t(i)$ gives

$$-\frac{\partial}{\partial V_t(i)}\left(\frac{\kappa}{2}X^2_t(i)L_t(i)\right) + P^r_{q,t}\frac{\partial Q}{\partial V_t(i)} - W_{r,t}(i)\frac{\partial L_t(i)}{\partial V_t(i)} + \beta\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial F_{t+1}(i)}{\partial V_t(i)} = 0$$

Remark that:

$$\begin{aligned}
\frac{\partial}{\partial V_t(i)}\left(\frac{\kappa}{2}\frac{\widetilde{q}_t^2 V_t^2(i)}{L_t^2(i)}L_t(i)\right) =&\frac{\kappa}{L_t(i)}\widetilde{q}_t V_t(i) = \kappa X_t(i)\widetilde{q}_t, \quad \textit{and}\\
\frac{\partial F_{t+1}(i)}{\partial V_t(i)} =&\widetilde{q}_t\frac{\partial F_{t+1}(i)}{\partial L_{t+1}(i)}
\end{aligned}$$

Then one has:

$$\begin{aligned}
\kappa X_t(i)\widetilde{q}_t =&\beta\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial F_{t+1}(i)}{\partial L_{t+1}(i)}\widetilde{q}_t\\
\kappa X_t(i) =&\beta\mathbb{E}_t\Lambda_{t,t+1}J_{t+1}(i)
\end{aligned}$$

Using these last equations one has:

$$\begin{aligned}
\kappa X_t(i) =&\beta\mathbb{E}_t\Lambda_{t,t+1}\left(P^r_{q,t}\Gamma_{t+1}T_t - W_{r,t+1}(i) - \frac{\kappa}{2}X^2_{t+1}(i) + \beta(1-\rho+X_{t+1}(i))\Lambda_{t+1,t+2}J_{t+2}(i)\right)\\
=&\beta\mathbb{E}_t\Lambda_{t,t+1}\left(P^r_{q,t+1}\Gamma_{t+1}T_{t+1} - W_{r,t+1}(i) - \frac{\kappa}{2}X^2_{t+1}(i)+\right.\\
&\left.\beta(1-\rho)\Lambda_{t+1,t+2}J_{t+2}(i) + \beta X_{t+1}(i)\Lambda_{t+1,t+2}J_{t+2}(i)\right)\\
=&\beta\mathbb{E}_t\Lambda_{t,t+1}\left(P^r_{q,t}\Gamma_{t+1}T_{t+1} - W_{r,t+1}(i) - \frac{\kappa}{2}X^2_{t+1}(i) + (1-\rho)\kappa X_{t+1}(i) + \kappa X^2_{t+1}(i)\right)
\end{aligned}$$

Then we recover equation (18).

Denote $\chi = 1-\rho$. Using the definition of worker surplus one can write:

$$\begin{aligned}
H_t(i) =& W_{r,t}(i) - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\chi D_{t+1}(i) + (1-\chi) U_{t+1} \right) \\
& - \beta \mathbb{E}_t \Lambda_{t,t+1} \left(z_t D_{x,t+1}(i) + (1-z_t) U_{t+1} \right) \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \chi \beta \mathbb{E}_t \Lambda_{t,t+1} D_{t+1}(i) + \beta \mathbb{E}_t \Lambda_{t,t+1} U_{t+1} - \chi \beta \mathbb{E}_t \Lambda_{t,t+1} U_{t+1} + \\
& - \beta z_t \mathbb{E}_t \Lambda_{t,t+1} D_{x,t+1} - \beta \mathbb{E}_t \Lambda_{t,t+1} U_{t+1} - \beta z_t \mathbb{E}_t \Lambda_{t,t+1} U_{t+1} \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \chi \mathbb{E}_t \Lambda_{t,t+1} \left(D_{t+1}(i) - U_{t+1} \right) - \beta z_t \mathbb{E}_t \Lambda_{t,t+1} \left(D_{x,t+1} - U_{t+1} \right) \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\chi H_{t+1}(i) - z_t H_{x,t+1} \right)
\end{aligned}$$

A.2 Firm/worker surplus for/and in a renegotiating firm

One can rewrite equation (16) as:

$$J_t(i) = \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi)^k \Lambda_{t,t+k} \left(P^r_{q,t+k} \Gamma_{t+k} T_{t+k} - \frac{W_{t+k}}{P_{q,t+k}} + \frac{\kappa}{2} X^2_{t+k}(i) \right)$$

because

$$\begin{aligned}
J_t(i) =& \mathbb{E}_t \Bigg[P^r_{q,t} \Gamma_t T_t - \frac{W_t}{P_{q,t}} + \frac{\kappa}{2} X^2_t(i) + \\
& + (\beta\chi) \Lambda_{t,t+1} \left(P^r_{q,t+1} \Gamma_{t+1} T_t - \frac{W_{t+1}}{P_{q,t+1}} + \frac{\kappa}{2} X^2_{t+1}(i) \right) + \\
& + (\beta\chi)^2 \Lambda_{t,t+2} \left(P^r_{q,t+2} \Gamma_{t+2} T_{t+2} - \frac{W_{t+2}}{P_{q,t+2}} + \frac{\kappa}{2} X^2_{t+2}(i) \right) + ... \Bigg] \\
=& P^r_{q,t} \Gamma_t T_t - \frac{W_t}{P_{q,t}} + \frac{\kappa}{2} X^2_t(i) + \\
& + (\beta\chi) \mathbb{E}_t \Lambda_{t,t+1} \Bigg(P^r_{q,t+1} \Gamma_{t+1} T_{t+1} - \frac{W_{t+1}}{P_{q,t+1}} + \frac{\kappa}{2} X^2_{t+1}(i) + \\
& \qquad (\chi\beta) \Lambda_{t+1,t+2} \left(P^r_{q,t+2} \Gamma_{t+2} T_{t+2} - \frac{W_{t+2}}{P_{q,t+2}} + \frac{\kappa}{2} X^2_{t+2}(i) \right) + ... \Bigg) \\
=& P^r_{q,t} \Gamma_t T_t - \frac{W_t}{P_{q,t}} + \frac{\kappa}{2} X^2_t(i) + \beta\chi \mathbb{E}_t \Lambda_{t,t+1} J_{t+1}(i)
\end{aligned}$$

Denote:

$$\overline{W}_t(i) := \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi)^k \Lambda_{t,t+k} \frac{W_{t+k}(i)}{P_{q,t+K}} \tag{29}$$

then one can write:

$$J_t(i) = \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi)^k \Lambda_{t,t+k} \left(P^r_{q,t+k} \Gamma_{t+k} T_{t+k} + \frac{\kappa}{2} X^2_{t+k}(i) \right) - \overline{W}_t(i) \tag{30}$$

In the same way one can write the worker surplus defined in equation (21) as

$$H_t(i) = \overline{W}_t(i) - \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi)^k \Lambda_{t,t+k} \left(b + \beta \Lambda_{t+k,t+k+1} z_{t+k} H_{x,t+k+1}\right) \tag{31}$$

On the other hand, let us denote $W_t(r)$ the re-optimize wage for a firm that can renegotiate its wage contract at time t, One has:[19]

$$W_t(r) = W_t^o$$

Then in $t+1$ one have:

$$\begin{aligned} \mathbb{E}_t W_{t+1}(r) =& \theta_w W_t(r) + (1-\theta_w)\mathbb{E}_t W_{t+1}^o \\ =& \theta_w W_t^o + (1-\theta_w)\mathbb{E}_t W_{t+1}^o \end{aligned}$$

By induction one has:

$$\begin{aligned} \mathbb{E}_t W_{t+2}(r) =& \mathbb{E}_t \left[\theta_w W_{t+1}(r) + (1-\theta_w) W_{t+2}^o\right] \\ =& \mathbb{E}_t \left[\theta_w \left(\theta_w W_t^o + (1-\theta_w)\mathbb{E}_t W_{t+1}^o\right) + (1-\theta_w) W_{t+2}^o\right] \\ =& \mathbb{E}_t \left[\theta_w^2 W_t^o + \theta_w(1-\theta_w) W_{t+1}^o + (1-\theta_w) W_{t+2}^o\right] \\ & ... \\ \mathbb{E}_t W_{t+k}(r) =& \mathbb{E}_t \left[\theta_w W_{t+k}(r) + (1-\theta_w) W_{t+k+1}^o\right] \\ =& \mathbb{E}_t \left[\theta_w^k W_t^o + \theta_w^{k-1}(1-\theta_w) W_{t+1}^o + ... + (1-\theta_w) W_{t+k}^o\right] \end{aligned} \tag{32}$$

Using this expression one can rewrite equation (29) as:

$$\begin{aligned} \overline{W}_t(r) =& \mathbb{E}_t \left[\frac{W_t(r)}{P_{q,t}} + (\beta\chi)\Lambda_{t,t+1}\frac{W_{t+1}(i)}{P_{q,t+1}} + (\beta\chi)^2 \Lambda_{t,t+2}\frac{W_{t+2}(r)}{P_{q,t+2}} + ...\right] \\ =& \mathbb{E}_t \Bigg[\frac{W_t^o}{P_{q,t}} + (\beta\chi)\Lambda_{t,t+1}\frac{1}{P_{q,t+1}}\left(\theta_w W_t^o + (1-\theta_w) W_{t+1}^o\right) + \\ & + (\beta\chi)^2 \Lambda_{t,t+2}\frac{1}{P_{q,t+2}}\left(\theta_w^2 W_t^o + \theta_w(1-\theta_w) W_{t+1}^o + (1-\theta_w) W_{t+2}^o\right) + ... \\ & ... + (\beta\chi)^k \Lambda_{t,t+k}\frac{1}{P_{q,t+k}}\left(\theta_w^k W_t^o + \theta_w^{k-1}(1-\theta_w) W_{t+1}^o + ... + (1-\theta_w) W_{t+k}^o\right) + ...\Bigg] \\ =& \mathbb{E}_t \Bigg[\frac{W_t^o}{P_{q,t}} + (\beta\chi)\frac{\Lambda_{t,t+1}}{P_{q,t+1}}\theta_w^k W_t^o + ... + (\beta\chi)^k \frac{\Lambda_{t,t+k}}{P_{q,t+k}}\theta_w^k W_t^o + ... \\ & + (\beta\chi)\frac{\Lambda_{t,t+1}}{P_{q,t+1}}(1-\theta_w) W_{t+1}^o + (\beta\chi)^2 \frac{\Lambda_{t,t+2}}{P_{q,t+2}}\theta_w(1-\theta_w) W_{t+1}^o + ... + (\beta\chi)^k \frac{\Lambda_{t,t+k}}{P_{q,t+k}}\theta_w^{k-1}(1-\theta_w) W_{t+1}^o + ...\Bigg] \\ =& \mathbb{E}_t \left(1 + (\beta\chi\theta_w)\frac{P_{q,t}}{P_{q,t+1}}\Lambda_{t,t+1} + ... + (\beta\chi\theta_w)^k \frac{P_{q,t}}{P_{q,t+k}}\Lambda_{t,t+k} + ...\right)\frac{W_t^o}{P_{q,t}} + \\ & + \mathbb{E}_t \left(1 + (\beta\chi\theta_w)\frac{P_{q,t}}{P_{q,t+1}}\Lambda_{t,t+1} + ... + (\beta\chi\theta_w)^k \frac{P_{q,t}}{P_{q,t+k}}\Lambda_{t,t+k} + ...\right)(\beta\chi)(1-\theta_w)\frac{\Lambda_{t,t+1}}{P_{q,t+1}} W_{t+1}^o + ... \end{aligned}$$

Let us denote:

$$\Delta_t := \sum_{s=0}^{+\infty} (\beta\chi\theta_w)^s \frac{P_{q,t}}{P_{q,t+s}}\Lambda_{t,t+s}$$

[19] As explained in the body of the paper, all the firms face the same problem. Then they choose the same wage, so the optimal wage for a renegotiating firm does not depend on i

then one has:

$$\begin{aligned}
\overline{W}_t(r) =& \mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} + \mathbb{E}_t(\beta\chi)(1-\theta_w)\Delta_{t+1}\frac{\Lambda_{t,t+1}}{P_{q,t+1}}W_{t+1}^o + \mathbb{E}_t(\beta\chi)^2(1-\theta_w)\Delta_{t+2}\frac{\Lambda_{t,t+2}}{P_{q,t+2}}W_{t+2}^o + ...\\
=& \mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} + \mathbb{E}_t\sum_{k=1}^{+\infty}(1-\theta_w)(\beta\chi)^k\frac{\Lambda_{t,t+k}}{P_{q,t+k}}\Delta_{t+k}W_{t+k}^o\\
=& \mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} + \mathbb{E}_t\sum_{k=0}^{+\infty}(1-\theta_w)(\beta\chi)^{k+1}\frac{\Lambda_{t,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o
\end{aligned}$$

Let us denote $J_t(r)$ the firm surplus of a firm that can renegotiate its wage contract at time t and $H_t(r)$ the worker surplus for a worker working in a renegotiating firm. Using this expression and equations (30) and (31), one can write:

$$\begin{aligned}
J_t(r) =& \mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(P_{q,t+k}^r\Gamma_{t+k}T_{t+k} + \frac{\kappa}{2}X_{t+k}^2(r) - (1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)\\
&- \mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} \qquad (33)\\
H_t(r) =& \mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} \qquad (34)\\
&-\mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(b + \frac{V(T_t)}{U'(C_t)} + \beta\Lambda_{t+k,t+k+1}z_{t+k}H_{x,t+k+1}.\right.\\
&\left.-(1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)
\end{aligned}$$

A.3 Nash Bargaining

Using the definition of ξ_t one has:

$$\begin{aligned}
\xi_t =& P_{q,t}\frac{\partial}{\partial W_t^o}\mathbb{E}_t\left[\Delta_t\frac{W_t^o}{P_{q,t}}\right.\\
&- \mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(b + \frac{V(T_t)}{U'(C_t)}\right.\\
&\left.\left.+\beta\Lambda_{t+k,t+k+1}z_{t+k}H_{x,t+k+1} - (1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)\right]\\
\xi_t =& \mathbb{E}_t\Delta_t
\end{aligned}$$

Then one has:

$$\xi_t = 1 + (\beta\chi\theta_w)\mathbb{E}_t\frac{P_{q,t}}{P_{q,t+1}}\Lambda_{t,t+1}\xi_{t+1} \qquad (35)$$

Using equation (16) one can write the surplus for a renegotiating firm as:

$$\begin{aligned}
J_t(r) =& P_{q,t}^r\Gamma_tT_t - \frac{W_t^o}{P_{q,t}} - \frac{\kappa}{2}\left(\theta_wX_t^2(W_t^o) + (1-\theta_w)X_t^2(W_{t+1}^o)\right) \qquad (36)\\
&+ \beta\theta_w\mathbb{E}_t(\chi + X_t(W_t^o))\Lambda_{t,t+1}\widetilde{J}_{t+1}(W_t^o) + \beta(1-\theta_w)\mathbb{E}_t(\chi + X_t(W_{t+1}^o))\Lambda_{t,t+1}\widetilde{J}_{t+1}(W_{t+1}^o) \qquad (37)
\end{aligned}$$

where

$$\begin{aligned}\kappa X_t(W_t^o) =&\beta \mathbb{E}_t \Lambda_{t,t+1} J_{t+1}(W_t^o)\\ \widetilde{J}_{t+1}(W_t^o) :=& P_{q,t+1}\Gamma_{t+1}T_{t+1} - \frac{W_t^o}{P_{q,t+1}} - \frac{\kappa}{2}X_{t+1}^2(i) + \beta(\chi - X_{t+1}^2(i))\Lambda_{t+1,t+2}J_{t+2}(i)\end{aligned}$$

Then using the definition of $\Sigma_t(r)$ one has:

$$\begin{aligned}\Sigma_t(r) =& - P_{q,t}\mathbb{E}_t\left(-\frac{1}{P_{q,t}} - \frac{\kappa}{2}\theta_w\frac{\beta^2}{\kappa^2}\Lambda_{t,t+1}^2 2\widetilde{J}_{t+1}(W_t^o)\frac{\partial}{\partial W_t^o}\widetilde{J}_{t+1}(W_t^o) + \beta\theta_w\chi\Lambda_{t,t+1}\frac{\partial}{\partial W_t^o}\widetilde{J}_{t+1}(W_t^o)\right.\\ &\left. + \theta_w\beta X_t(W_t^o)\Lambda_{t,t+1}\frac{\partial}{\partial W_t^o}\widetilde{J}_{t+1}(W_t^o) + \beta\theta_w\widetilde{J}_{t+1}(W_t^o)\frac{\beta}{\kappa}\Lambda_{t,t+1}^2\frac{\partial}{\partial W_t^o}\widetilde{J}_{t+1}(W_t^o)\right)\\ =&1 - \beta\theta_w(\chi + X_t(W_t^o))P_{q,t}\mathbb{E}_t\Lambda_{t,t+1}\frac{\partial}{\partial W_t^o}J_{t+1}(W_t^o)\end{aligned}$$

Note that $\widetilde{J}_{t+1}(W_t^o) = J_{t+1}(r)$ and $X_t(W_t^o) = X_t(r)$. Then

$$\Sigma_t(r) =1 + \beta\theta_w(\chi + X_t(r))\mathbb{E}_t\Lambda_{t,t+1}\frac{P_{q,t}}{P_{q,t+1}}\Sigma_{t+1}(r) \tag{38}$$

To derive the optimal wage equation (eq. 26) let us replace the values of equations (33) and (34) in (24):

$$\begin{aligned}&\nu_t(r)\left(\mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(P_{q,t}^r\Gamma_{t+k}T_{t+k} + \frac{\kappa}{2}X_{t+k}^2(r) - (1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)\right.\\ &\qquad\left.-\Delta_t\frac{W_t^o}{P_{q,t}}\right) =\\ &= (1-\nu_t(r))E_t\left(\Delta_t\frac{W_t^o}{P_{q,t}} + \right.\\ &\left.-\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(b + \frac{V(T_t)}{U'(C_t)} + \beta\Lambda_{t+k,t+k+1}z_{t+k}H_{x,t+k+1} - (1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)\right)\end{aligned}$$

$$\begin{aligned}\mathbb{E}_t\Delta_t\frac{W_t^o}{P_{q,t}} =&\mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi)^k\Lambda_{t,t+k}\left(\nu_t(r)\left(P_{q,t+k}^r\Gamma_{t+k}T_{t+k} + \frac{\kappa}{2}X_{t+k}^2(r)\right) + \right.\\ &(1-\nu_t(r))\left(b + \frac{V(T_t)}{U'(C_t)} + \beta\Lambda_{t+k,t+k+1}z_{t,k}H_{x,t+k+1}\right) + \\ &\left.-(1-\theta_w)(\beta\chi)\frac{\Lambda_{t+k,t+k+1}}{P_{q,t+k+1}}\Delta_{t+k+1}W_{t+k+1}^o\right)\\ =&\mathbb{E}_t\left[\nu_t(r)\left(P_{q,t}\Gamma_tT_t + \frac{\kappa}{2}X_{t+k}^2(r)\right) + (1-\nu_t(r))(b + \frac{V(T_t)}{U'(C_t)} + \beta\Lambda_{t,t+1}z_tH_{x,t+1})\right.\\ &\left.-(1-\theta_w)(\beta\chi)\frac{\Lambda_{t,t+1}}{P_{q,t+1}}\Delta_{t+1}W_{t+1}^o + (\beta\chi)\Lambda_{t,t+1}\Delta_{t+1}\frac{W_{t+1}^o}{P_{q,t+1}}\right]\\ =&\mathbb{E}_t\left[\nu_t(r)\left(P_{q,t}\Gamma_tT_t + \frac{\kappa}{2}X_{t+k}^2(r)\right) + (1-\nu_t(r))\left(b + \frac{V(T_t)}{U'(C_t)}\right.\right.\\ &\left.+\beta\Lambda_{t,t+1}z_tH_{x,t+1}) + (\beta\chi\theta_w)\Lambda_{t,t+1}\Delta_{t+1}\frac{W_{t+1}^o}{P_{q,t+1}}\right]\end{aligned}$$

Then one obtains equation (26). Remark that iterating this equation one has:

$$\begin{aligned}
\mathbb{E}_t\Delta_t W_t^o =& \mathbb{E}_t W_t^{tar}(r) + \mathbb{E}_t(\beta\chi\theta_w)\Delta_{t+1}W_{r,t+1}^o\Lambda_{t,t+1}\\
\mathbb{E}_t\Delta_t W_{t+1}^o =& \mathbb{E}_t W_{t+1}^{tar}(r) + \mathbb{E}_t(\beta\chi\theta_w)\Delta_{t+2}W_{r,t+2}^o\Lambda_{t+1,t+2}\\
&...
\end{aligned}$$

Then one can rewrite (26) as:

$$\begin{aligned}
\mathbb{E}_t\Delta_t W_t^o =& \mathbb{E}_t W_t^{tar}(r) + \mathbb{E}_t(\beta\chi\theta_w)\Lambda_{t,t+1}\left(W_{t+1}^{tar}(r) + \beta\chi\theta_w\Delta_{t+2}W_{r,t+2}^o\Lambda_{t+1,t+2}\right)\\
=& \mathbb{E}_t\left(W_t^{tar}(r) + (\beta\chi\theta_w)\Lambda_{t,t+1}W_{t+1}^{tar}(r) + (\beta\chi\theta_w)^2\Delta_{t+2}\Lambda_{t+1,t+2}W_{r,t+2}^o\right)\\
=& \mathbb{E}_t\sum_{k=0}^{+\infty}(\beta\chi\theta_w)^k\Lambda_{t+k,t+k+1}W_{t+k}^{tar}(r)\Delta_{t+k+1}
\end{aligned}$$

Then one has

$$\xi_t W_t^o = E_t\sum_{k=0}^{+\infty}(\beta\chi\theta_w)^k\Lambda_{t+k,t+k+1}W_{t+k}^{tar}(r)\xi_{t+k} \tag{39}$$

A.4 Aggregation and Equilibrium

Let me define,

$$K_t = \int_0^1 K_t(i)di, \quad E_t = \int_0^1 E_t(i)di, \quad L_t = \int_0^1 L_t(i)di, \quad V_t = \int_0^1 V_t(i)di, \quad M_t = \int_0^1 M_t(i)di$$

Define also the average wage across workers by:

$$W_t = \int_0^1 W_t(i)\frac{L_t(i)}{L_t}di$$

By Calvo assumption, one has the following wage aggregate equation:

$$\begin{aligned}
W_t =& \int_{set-wages} W_t(i)\frac{L_t(i)}{L_t}di + \int_{not-set-wages} W_t(i)\frac{L_t(i)}{L_t}di\\
=& (1-\theta_w)W_t^o\int_0^1\frac{L_t(i)}{L_t}di + \theta_w\int_0^1 W_{t-1}(i)\frac{L_t(i)}{L_t}di\\
=& (1-\theta_w)W_t^o + \theta_w\int_0^1 W_{t-1}(i)\frac{L_t(i)}{L_t}di
\end{aligned}$$

The intermediate firms are homogeneous, then:

$$Q_t = \int_0^1 Q_t(i)di$$

At the equilibrium, total domestic supply of the intermediate good Q_t must be equal to the total demand by the domestic final good firm $\int_0^1 \widetilde{Q}_t(j)dj$, then using equation (28) one has:

$$Q_t = v_{p,t}\widetilde{Q}_t$$

where

$$v_{p,t} := \int_0^1 \left(\frac{P_{q,t}(j)}{P_{q,t}}\right)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} dj$$

is a price dispersion measure. Remark also that one has:

$$\begin{aligned}
v_{p,t} =& P_{q,t}^{\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} \int_0^1 P_{q,t}(j)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} dj \\
=& P_{q,t}^{\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} \left(\theta_p P_{q,t-1}^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} \int_0^1 \left(\frac{P_{q,t-1}(j)}{P_{q,t-1}}\right)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} dj + (1-\theta_p)(P^o_{q,t})^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}}\right) \\
=& \theta_p \Pi_{q,t}^{\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} v_{t-1} + (1-\theta_p)\left(\frac{P^o_{q,t}}{P_{q,t}}\right)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}}
\end{aligned}$$

Aggregating the production function one has (Cf. Appendix Acurio-Vásconez (2015)):

$$v_{p,t}\widetilde{Q}_t = \left(x_p A^\rho_{E,t} E^\rho_t + (1-x_p) A^\rho_{LK,t} (K^\alpha_t (L^d_t)^{1-\alpha})^\rho\right)^{1/\rho}$$

Given constants returns to scale in intermediate production and perfect capital an oil mobility, the ratios $\frac{E_t(i)}{K_t(i)}$ and $\frac{Q_t(i)}{K_t(i)}$ are the same across firms, then the marginal product of labor Γ is also the same across firms. Equation 22 implies that hours are also equalized across producers. Then using equations (13) and (15) one has

$$\begin{aligned}
S_{e,t} =& P^r_{q,t} x_p A^{\sigma_p}_{E,t} \left(\frac{E_t}{Q_t}\right)^{\sigma_p - 1} \\
r^k_t S_{k,t} =& P^r_{q,t} \alpha (1-x_p) A^{\sigma_p}_{t,LK} \left(\frac{K_t}{Q_t}\right)^{\alpha\sigma_p - 1} \left(\frac{\bar{L}_t}{Q_t}\right)^{(1-\alpha)\sigma_p} \\
\Gamma_t =& (1-x_p)(1-\alpha) A^{\sigma_p}_{t,LK} \left(\frac{K_t}{Q_t}\right)^{\alpha\sigma_p} \left(\frac{\bar{L}_t}{Q_t}\right)^{(1-\alpha)\sigma_p}
\end{aligned}$$

Define X_t the unconditional average value of the hiring rate:

$$X_t := \int_0^1 X_t(i) \frac{L_t(i)}{L_t} di$$

From equation (18) one has:

$$\kappa X_t(i) \frac{L_t(i)}{L_t} = \beta \mathbb{E}_t \Lambda_{t,t+1} \left[P^r_{q,t} \Gamma_{t+1} T_{t+1} \frac{L_t(i)}{L_t} - W_{r,t+1}(i) \frac{L_t(i)}{L_t} + \frac{\kappa}{2} X^2_{t+1}(i) \frac{L_t(i)}{L_t} + \chi\kappa X_{t+1}(i) \frac{L_t(i)}{L_t}\right]$$

Taking the integral on both sides one has:

$$\begin{aligned}
\kappa X_t =& \beta \mathbb{E}_t \Lambda_{t,t+1} \left[P^r_{q,t} \Gamma_{t+1} T_{t+1} - \frac{1}{P_{q,t+1}} \int_0^1 W_{t+1}(i) \frac{L_t(i)}{L_t} di + \frac{\kappa}{2} \int_0^1 X^2_{t+1}(i) \frac{L_t(i)}{L_t} di + \right. \\
& \left. + \chi\kappa \int_0^1 X_{t+1}(i) \frac{L_t(i)}{L_t} di\right] \\
=& \beta \mathbb{E}_t \Lambda_{t,t+1} \left[P^r_{q,t+1} \Gamma_{t+1} T_{t+1} - W_{r,t+1} + \frac{\kappa}{2} X^2_{t+1} + \chi\kappa X_{t+1}\right] \\
& + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \left(\frac{\kappa}{2} X^2_{t+1}(i) - W_{r,t+1}(i) + \chi\kappa X_{t+1}(i)\right) \frac{L_t(i)}{L_t} di - \left(\frac{\kappa}{2} X^2_{t+1} - W_{r,t+1} + \chi\kappa X_{t+1}\right)\right]
\end{aligned}$$

Then one has:

$$\kappa X_t = \beta \mathbb{E}_t \Lambda_{t,t+1} \left[P^r_{q,t+1} \Gamma_{t+1} T_{t+1} - W_{r,t+1} + \frac{\kappa}{2} X^2_{t+1} + \chi \kappa X_{t+1} \right] + \Omega_{x,t} \tag{40}$$

where

$$\Omega_{x,t} := \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \left(\frac{\kappa}{2} X^2_{t+1}(i) - W_{r,t+1}(i) + \chi \kappa X_{t+1}(i) \right) \frac{L_t(i)}{L_t} di - \left(\frac{\kappa}{2} X^2_{t+1} - W_{r,t+1} + \chi \kappa X_{t+1} \right) \right]$$

Define the average value of workers, the average worker surplus and the average value of the firm marginal surplus respectively by:

$$D_t := \int_0^1 D_t(i) \frac{L_t(i)}{L_t} di, \quad H_t := \int_0^1 H_t(i) \frac{L_t(i)}{L_t} di, \quad J_t := \int_0^1 J_t(i) \frac{L_t(i)}{L_t} di$$

Multiplying equation (21) by $\frac{L_(i)}{L_t}$ and taking integral in both sides one has:

$$\begin{aligned}
H_t =& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \chi \left(D_{t+1}(i) - U_{t+1} \right) \frac{L_t(i)}{L_t} di - z_t H_{x,t+1} \right] \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \chi D_{t+1}(i) \frac{L_t(i)}{L_t} di - \chi U_{t+1} - z_t H_{x,t+1} \right] \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \chi D_{t+1}(i) \frac{L_t(i)}{L_t} di - \chi U_{t+1} - z_t (D_{x,t+1} - U_{t+1}) \right] \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \chi D_{t+1}(i) \frac{L_t(i)}{L_t} di + \chi U_{t+1} - z_t (D_{x,t+1} - U_{t+1}) \right] \\
=& W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\int_0^1 \chi D_{t+1}(i) \frac{L_t(i)}{L_t} di - (\chi - z_t) U_{t+1} - z_t D_{x,t+1} \right]
\end{aligned}$$

Remark that one also has $U_{t+1} = D_{t+1} - H_{t+1}$, then:

$$H_t = W_{r,t} - b - \frac{V(T_t)}{U'(C_t)} + (\chi - z_t) \beta \mathbb{E}_t \Lambda_{t,t+1} H_{t+1} + \Omega_{H,t} \tag{41}$$

where:

$$\Omega_{H,t} := \beta \mathbb{E}_t \Lambda_{t,t+1} \left[\chi \left(\int_0^1 D_{t+1}(i) \frac{L_t(i)}{L_t} di - D_{t+1} \right) - z_t \left(D_{x,t+1} - D_{t+1} \right) \right]$$

In the same way, using equation (16) one has:

$$\begin{aligned}
J_t =& P^r_{q,t} \Gamma_t T_t - W_{r,t} + \int_0^1 \frac{\kappa}{2} X^2_t(i) \frac{L_t(i)}{L_t} + \beta \chi \mathbb{E}_t \Lambda_{t,t+1} \int_0^1 J_{t+1}(i) \frac{L_t(i)}{L_t} di \\
=& P^r_{q,t} \Gamma_t T_t - W_{r,t} + \frac{\kappa}{2} X^2_t + \beta \chi \mathbb{E}_t \Lambda_{t,t+1} J_{t+1} + \Omega_{J,t} \qquad (42)
\end{aligned}$$

where

$$\Omega_{J,t} := \frac{\kappa}{2} \left(\int_0^1 X^2_t(i) \frac{L_t(i)}{L_t} di - X^2_t \right) + \beta \chi \mathbb{E}_t \Lambda_{t,t+1} \left(\int_0^1 J_{t+1}(i) \frac{L_t(i)}{L_t} di - J_{t+1} \right)$$

Aggregating the government constraint one has:

$$G_t = B_t - (1 - i_{t-1} B_{t-1}) - (1 - L_t) b P_{q,t} - Tax_t$$

Finally the resource constraint is given by:

$$S_{c,t} C_t + S_{k,t} I_t + G_{r,t} = P^r_{q,t} Q_t - S_{e,t} E_t - \frac{\kappa}{2} \int_0^1 X^2_t(i) L_t(i) di$$

A.5 Steady State

I denote Z the steady state for the variable Z_t. The following equations define the steady state value of the model.

Households

$$\begin{aligned}
S_c =& \left((1-x_c) + x_c S_e^{\frac{\sigma}{\sigma_c-1}}\right)^{\frac{\sigma_c-1}{\sigma}} \\
C_e =& x_c S_e^{\frac{1}{\sigma_c-1}} S_c^{\frac{-1}{\sigma_c-1}} C \\
C_q =& (1-x_c) S_c \frac{-1}{\sigma_c-1} C \\
i =& \frac{1}{\beta} - 1 \\
r^k =& \frac{1}{\beta} - 1 + \delta \\
I =& \delta K \\
\Lambda =& 1 \\
S_c C =& P_q^r Q - S_e E - \frac{\kappa}{2} X^2 L - \delta S_k K - \omega Q
\end{aligned}$$

Employment Dynamics

$$\begin{aligned}
X =& \rho \\
L =& \frac{z}{\rho + z} \\
\rho L =& M \\
S =& 1 - L
\end{aligned}$$

Firms, Government and GDP

$$\begin{aligned}
P_q^r =& \frac{1}{\mathcal{M}_p} \\
\left(\frac{Q}{E}\right)^{1-\sigma_p} =& \frac{S_e}{P_q^r x_p A_E^{\sigma_p}} \\
\frac{K}{Q} =& \frac{\alpha}{r^k S_k} \left(P_q^r - \frac{E}{Q}\right) \\
\left(\frac{\bar{L}}{Q}\right)^{(1-\alpha)\sigma_p} =& \frac{1 - x_p A_E^{\sigma_p} \left(\frac{E}{Q}\right)^{\sigma_p}}{(1-x_p) A_{LK}^{\sigma_p} \left(\frac{K}{Q}\right)^{\alpha\sigma_p}} \\
\Gamma =& (1-x_p)(1-\alpha) A_{LK}^{\sigma_p} \left(\frac{K}{Q}\right)^{\alpha\sigma_p} \left(\frac{\bar{L}}{Q}\right)^{(1-\alpha)\sigma_p - 1}
\end{aligned}$$

Remark that:

$$\begin{aligned}\int_0^1 \frac{\kappa}{2} X_{t+1}^2(i)\frac{L_t(i)}{L_t}di =& \frac{\kappa}{2}\int_0^1 \left(\frac{L_{t+1}(i)}{L_{t+1}} - \chi\right)^2 \frac{L_t(i)}{L_t}di \\ =& \frac{\kappa}{2}\left[\int_0^1 \frac{L_{t+1}^2(i)}{L_{t+1}^2}\frac{L_t(i)}{L_t}di - 2\chi\int_0^1 \frac{L_{t+1}(i)}{L_{t+1}}\frac{L_t(i)}{L_t}di - \chi^2 L_t\right]\end{aligned}$$

at steady state employment share are constants, i.e. $L(i) = L$, then:

$$\begin{aligned}\int_0^1 \frac{\kappa}{2} X_{t+1}^2(i)\frac{L_t(i)}{L_t}di =& \frac{\kappa}{2}(1 - 2\chi + \chi^2) \\ =& \frac{\kappa}{2}X^2\end{aligned}$$

Then one has

$$\kappa X = \beta(P_q^r \Gamma T - W_r + \frac{\kappa}{2}X^2 + \chi\kappa X) \tag{43}$$

Finally one has also

$$\begin{aligned}x_p =& \alpha_e^{\frac{1}{\eta_p}} \mathcal{M}_p^{\sigma_p} \\ Q =& \left(x_p A_E^{\sigma_p} E^{\sigma_p} + (1 - x_p) A_{LK}^{\sigma_p}(K^\alpha L^{1-\alpha})\right)^{\frac{1}{\sigma_p}} \\ v_p =& 1 \\ Q =& \widetilde{Q} \\ G_r =& \omega Q \\ S_c \frac{Y}{Q} =& P_q^r - S_e\frac{E}{Q} - \frac{\kappa}{2}X^2\frac{L}{Q} \\ \bar{L} =& LT \\ P_q^r \Gamma =& T^{\widetilde{w}} C\end{aligned}$$

Nash Bargaining, Wages and Surplus

At steady state, $\nu(r) = \nu$ and $\Sigma(r) = \Sigma$, then:

$$\begin{aligned}\nu =& \frac{\eta}{\eta + (1-\eta)\frac{\Sigma}{\xi}} \\ \xi =& \frac{1}{1 - \chi\beta\theta_w} \\ \Sigma =& \frac{1}{1 - \beta\theta_w}\end{aligned}$$

At steady state one has $W^{tar}(r) = W^{tar}$ and $W = W^o$ then $W_r = W_r^o$. Using equation (26) one can write:

$$\begin{aligned}\xi W_r^o =& W^{tar} + (\beta\chi\theta_w)W_r^o \\ \xi(1 - \beta\chi\theta_w)W_r^o =& W^{tar} \\ W_r^o =& W^{tar}\end{aligned}$$

At steady state, hiring rates are identical across firms, then $X_t(i) = X_t$, then:

$$D_x = \int_0^1 D(i)\frac{X(i)L(i)}{XL}di = D \Rightarrow H = H_x$$

Using equations (21) and (23) one then has:

$$\begin{aligned}
H =& \frac{1}{1-(x-z)\beta}\left(W_r - b - \frac{V(T)}{U'(C)}\right) = \frac{1}{1-(x-z)\beta}\left(W_r - b - \frac{CT^{1+\widetilde{\omega}}}{1+\widetilde{\omega}}\right) \\
J =& \frac{1-\eta}{\eta}\frac{\Sigma}{\xi}H
\end{aligned}$$

Using equation (27) one also has:

$$\begin{aligned}
W_r = W^{tar} =& \nu(P_q^r\Gamma T + \frac{\kappa}{2}X^2) + (1-\nu)(b + \frac{V(T)}{U'(C)} + \beta z H) \\
=& \nu(P_q^r\Gamma T + \frac{\kappa}{2}X^2) + (1-\nu)\left(b + \frac{V(T)}{U'(C)} + \frac{\beta z}{1-(x-z)\beta}W_r - \frac{\beta z}{1-(x-z)\beta}\left(b + \frac{V(T)}{U'(C)}\right)\right)
\end{aligned}$$

Using equation (43) one can replace the value of W_r in this last equation, having:

$$\begin{aligned}
W_r =& P_q^r\Gamma T\left(\nu + (1-\nu)\frac{\beta z}{1-(x-z)\beta}\right) + \frac{\kappa}{2}X^2\left(\nu + (1-\nu)\frac{\beta z}{1-(x-z)\beta}\right) \\
&- (1-\nu)\kappa X\frac{z(1-\chi\beta)}{1-(x-z)\beta} + (1-\nu)\frac{(1-\chi\beta)}{1-(x-z)\beta}\left(b + \frac{V(T)}{U'(C)}\right) \\
=& \frac{1-\chi\beta}{1-(x-z)\beta}\left(\nu\ \left(P_q^r\Gamma T + \frac{\kappa}{2}X^2 + z\kappa X\right) + (1-\nu)\left(b + \frac{V(T)}{U'(C)}\right)\right) \\
&+ \frac{\beta z}{1-(x-z)\beta}P_q^r\Gamma T + \frac{\kappa}{2}X^2\frac{\beta z}{1-(x-z)\beta} - \kappa X\frac{z(1-\chi\beta)}{1-(x-z)\beta}
\end{aligned}$$

then:

$$\begin{aligned}
W_r\left(1 - \frac{\beta z}{1-(\chi-z)\beta}\right) =& \frac{1-\beta\chi}{1-(x-z)\beta}\left(\nu\ \left(P_q^r\Gamma T + \frac{\kappa}{2}X^2 + z\kappa X\right) + (1-\nu)\left(b + \frac{V(T)}{U'(C)}\right)\right) \\
W_r =& \left(\nu\ \left(P_q^r\Gamma T + \frac{\kappa}{2}X^2 + z\kappa X\right) + (1-\nu)\left(b + \frac{V(T)}{U'(C)}\right)\right)
\end{aligned}$$

Remark that one can rewrite the steady state of equation (27) as:

$$W_r = \nu\left(P_q^r\Gamma T + \frac{\kappa}{2}X^2 + z\kappa X\right) + (1-\nu)\widetilde{b}(P_q^r\Gamma T + \frac{\kappa}{2}X^2)$$

where:

$$\widetilde{b} := \frac{b + \frac{V(T)}{U'(C)}}{P_q^r\Gamma T + \frac{\kappa}{2}X^2} \tag{44}$$

and

$$W_r = P_q^r\Gamma T + \frac{\kappa}{2}X^2 - \frac{\kappa X}{\beta}(1-\chi\beta)$$

I assume that the steady state of oil and capital real prices are equal to 1 as well as the steady states of both technologies processes. Then:

$$S_e := S_k := A_E := A_{LK} := 1$$

B Appendix B: Log-linearized Model

Small letters represent the log-deviation of each variable with respect its steady state, $z_t := log(Z_t) - log(Z) \approx \frac{Z_t - Z}{Z}$. For the rental rate of capital (r_t^k), the investment (I_t), both transition probabilities $\widetilde{q}_t$ and z_t, the stochastic discount factor $\Lambda_{t,t+1}$, and the Nash bargaining variables, ν, ξ and Σ, the log-deviation will be represented with a hat over the variable. One has: Small case letters represent the log-deviation of each variable with respect its steady state, $z_t := log(Z_t) - log(Z) \approx \frac{Z_t - Z}{Z}$. For the rental rate of capital (r_t^k), the investment (I_t), the transition probabilities $\widetilde{q}_t$ and z_t, the stochastic discount factor $\Lambda_{t,t+1}$, and the Nash bargaining variables, ν, ξ and Σ, the log-deviation will be represented with a hat over the variable. One has the following equations:

$$s_{c,t} = \left(\left(\frac{S_e}{S_c} \right)^{\frac{\sigma}{\sigma-1}} \right) x_c s_{e,t} \tag{B.1}$$

$$c_{q,t} = c_t - \frac{1}{\sigma_c - 1} s_{c,t} \tag{B.2}$$

$$c_{e,t} = c_t + \frac{1}{\sigma_c - 1}(s_{e,t} - s_{c,t}) \tag{B.3}$$

$$c_t = \mathbb{E}_t[c_{t+1}] - (i_t - \mathbb{E}_t[\pi_{c,t+1}]) \tag{B.4}$$

$$i_t = (1 - \beta(1-\delta))\mathbb{E}_t[\widehat{r}_{t+1}] + \mathbb{E}_t[\pi_{k,t+1}] \tag{B.5}$$

$$i_t = \phi_i i_{t-1} + \phi_i(\phi_\pi \pi_{q,t} + \phi_y y_t) + \varepsilon_i \tag{B.6}$$

$$\delta \widehat{I}_t = k_{t+1} - (1-\delta) k_t \tag{B.7}$$

$$Q^{\sigma_p} q_t = x_p (A_E E)^{\sigma_p} (a_{e,t} + e_t) + (1 - x_p) A_{LK}^{\sigma_p} (K^\alpha L^{1-\alpha})^{\sigma_p} (a_{lk,t} + \alpha k_t + (1-\alpha)(l_t + t_t)) \tag{B.8}$$

$$\gamma_t = (1 - \sigma_p) q_t + \sigma_p a_{lk,t} + \alpha \sigma_p k_t + ((1-\alpha)\sigma_p - 1)(l_t + t_t) \tag{B.9}$$

$$s_{e,t} = (\sigma_p - 1)(e_t - q_t) + p_{q,t}^r + \sigma_p a_{e,t} \tag{B.10}$$

$$\hat{r}_t + s_{k,t} = p_{q,t}^r + (1 - \sigma_p) q_t + (\alpha \sigma_p - 1) k_t + (1-\alpha)\sigma_p (l_t + t_t) + \sigma_p a_{lk,t} \tag{B.11}$$

$$\hat{\Lambda}_{t,t+1} = -\pi_{c,t+1} - c_{t+1} + c_t \tag{B.12}$$

$$\pi_{q,t} = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} P_{q,t}^r + \frac{(1-\beta\theta_p)(1-\theta_p)\varepsilon_p}{\theta_p(1+\varepsilon_p)} \varepsilon_{p,t} + \beta \mathbb{E}_t \pi_{q,t+1} \tag{B.13}$$

$$m_t = (1-\gamma) v_t + \gamma s_t \tag{B.14}$$

$$l_{t+1} = l_t + \rho x_t \tag{B.15}$$

$$\hat{\widetilde{q}}_t = m_t - v_t \tag{B.16}$$

$$\hat{z}_t = m_t - s_t \tag{B.17}$$

$$s_t = -\frac{L}{S} l_t \tag{B.18}$$

$$
\begin{aligned}
\hat{\tilde{q}}_t + v_t =& x_t + l_t && \text{(B.19)}\\
\pi_{k,t} =& \pi_{q,t} + s_{k,t} - s_{k,t-1} && \text{(B.20)}\\
\pi_{c,t} =& \pi_{q,t} + s_{c,t} - s_{c,t-1} && \text{(B.21)}\\
s_{e,t} =& \rho_{se} s_{e,t-1} + e_{se,t} && \text{(B.22)}\\
s_{k,t} =& \rho_{sk} s_{k,t-1} + e_{sk,t} && \text{(B.23)}\\
g_{r,t} =& \rho_g g_{r,t-1} + \rho_{gae} e_{ae,t} + \rho_{galk} e_{alk,t} + e_{g,t} && \text{(B.24)}\\
a_{e,t} =& \rho_{ae} a_{e,t-1} + e_{ae,t} && \text{(B.25)}\\
a_{lk,t} =& \rho_{alk} a_{lk,t-1} + e_{alk,t} && \text{(B.26)}\\
\varepsilon_{i,t} =& \rho_i e_{i,t-1} + e_{ei,t} && \text{(B.27)}\\
\varepsilon_{p,t} =& \rho_p \varepsilon_{p,t-1} - \nu_p e_{p,t-1} + e_{p,t} && \text{(B.28)}
\end{aligned}
$$

B.1 Budget constraint

Let us log-linearize the budget-constraint equation:

$$
\begin{aligned}
\frac{S_{c,t} - S_c}{S_c} + \frac{C_t - C}{C} =& \frac{Q}{S_c C}(P^r_{q,t} - P^r_q) + \frac{P^r_q}{S_c C}(Q_t - Q) - \frac{E}{S_c C}(S_{e,t} - S_e) - \frac{S_e}{S_c C}(E_t - E)\\
&- \frac{I}{S_c C}(S_{k,t} - S_k) - \frac{S_k}{S_c C}(I_t - I) - \frac{\kappa}{2 S_c C}\int_0^1 2X(i)L(i)di(X_t - X)\\
&- \frac{\kappa}{2 S_c C}\int_0^1 X^2(i)di(L_t - L)\frac{1}{S_c C}(G_{r,t} - G_r)
\end{aligned}
$$

from where one obtains:

$$
\frac{C}{Q}(s_{c,t} + c_t) = \frac{P^r_q}{S_c}(p^r_{q,t} + q_t) - \frac{S_e E}{S_c Q}(s_{e,t} + e_t) - \frac{S_k I}{S_c Q}(s_{k,t} + \hat{I}_t) - \frac{\kappa X^2 L}{2 S_c Q}(2x_t + l_t) - \frac{G_r}{S_c Q} g_{r,t} \tag{B.29}
$$

B.2 Hiring rate

Remark that $d_{x,t} = d_t$ and then $h_{x,t} = h_t$. Let us log-linearize equation (40):[20]

$$\begin{aligned}
\kappa XL(x_t + l_t) =& \beta\mathbb{E}_t \left[P_q^r \Gamma LT(\Lambda_{t,t+1} - \Lambda) + \Gamma TL(P_{q,t}^r - P_q^r) + TP_q^r L(\Gamma_t - \Gamma)\right. \\
& + P_q^r \Gamma T(L_t - L) + P_q^r \Gamma L(T_t - T) \\
& - W_r L(\Lambda_{t,t+1} - \Lambda) - L(W_{r,t} - W_r) - W(L_t - L) \\
& + \frac{\kappa}{2} X^2 L(\Lambda_{t,t+1} - \Lambda) + \kappa XL(X_{t+1} - X) + \frac{\kappa}{2} X^2 (L_t - L) \\
& + \chi\kappa XL(\Lambda_{t,t+1} - \Lambda) + \chi\kappa L(X_{t+1} - X) + \chi\kappa X(L_t - L) \\
+ \int_0^1 \frac{\kappa}{2} X^2(i) L(i) di(\Lambda_{t,t+1} - \Lambda) &+ \int_0^1 \kappa X(i) L(i) di(X_{t+1} - X) + \int_0^1 \frac{\kappa}{2} X^2(i) di(L_t - L) \\
& - \frac{\kappa}{2} X^2 L(\Lambda_{t,t+1} - \Lambda) - \kappa XL(X_t - X) - \frac{\kappa}{2} X^2 (L_t - L) \\
- \int_0^1 W_r(i) L_(i) di(\Lambda_{t,t+1} - \Lambda) &- \int_0^1 \chi\kappa L(i) di(X_t - X) + \int_0^1 \chi\kappa X(i) di(L_t - L) \\
+ \int_0^1 \chi\kappa X(i) L(i) di(\Lambda_{t,t+1} - \Lambda) &+ \int_0^1 \chi\kappa L(i) di(X_{t+1} - X) + \int_0^1 \chi\kappa X(i) di(L_t - L) \\
& + W_r L(\Lambda_{t,t+1} - \Lambda) + L(W_{r,t} - W_r) + W_r(L_t - L) \\
& \left. - \chi\kappa XL(\Lambda_{t,t+1} - \Lambda) - \chi\kappa L(X_t - X) - \chi\kappa X(L_t - L)\right] \\
=& \beta\mathbb{E}_t \left[P_q^r \Gamma L(\hat{\Lambda}_{t,t+1} + p_{q,t}^r + \gamma_{t+1}) - W_r L(\hat{\Lambda}_{t,t+1} + w_{r,t+1})\right. \\
& \left. + \frac{\kappa}{2} X^2 L(\hat{\Lambda}_{t,t+1} + 2x_{t+1}) \chi\kappa XL(\hat{\Lambda}_{t,t+1} + x_{t+1})\right] \\
& + \beta\mathbb{E}_t \left[P_q^r \Gamma L - W_r L + \frac{\kappa}{2} XL + \chi\kappa XL\right] l_t
\end{aligned}$$

From where one then has:

$$x_t = \mathbb{E}_t \hat{\Lambda}_{t,t+1} + \frac{\beta}{\kappa X} \mathbb{E}_t \left[P_q^r \Gamma T(p_{q,t+1}^r + \gamma_{t+1} + t_{t+1}) - W_r w_{r,t+1}\right] + \beta\mathbb{E}_t x_{t+1} \tag{B.30}$$

B.3 Nash Bargaining

Log-linearizing equation (35) one has:

$$\hat{\xi}_t = (\chi\beta\theta_w)\mathbb{E}_t \left[\hat{\Lambda}_{t,t+1} - \pi_{q,t+1} + \hat{\xi}_{t+1}\right] \tag{B.31}$$

On the other hand, log-linearizing equations (25) and (38) one obtains:

$$\begin{aligned}
\hat{\nu}_t(r) =& (1 - \nu)(\hat{\xi}_t - \hat{\Sigma}_t(r)) \\
\hat{\Sigma}_t(r) =& \beta\chi\theta_w x_t(r) + \beta\theta_w \mathbb{E}_t \left[\hat{\Lambda}_{t,t+1} - \pi_{q,t+1} + \hat{\Sigma}_{t+1}(r)\right]
\end{aligned}$$

Remark that averaging across firms one has:

$$\hat{\Sigma}_t = \beta\chi\theta_w x_t + \beta\theta_w \mathbb{E}_t \left[\hat{\Lambda}_{t,t+1} - \pi_{q,t+1} + \hat{\Sigma}_{t+1}\right] \tag{B.32}$$

Then one has:

$$\hat{\nu}_t = (1 - \nu)(\hat{\xi}_t - \hat{\Sigma}_t) \tag{B.33}$$

[20] Because $\Omega_x = 0$, one can not log-linearize directly $\Omega_{x,t}$.

B.4 Wage Dynamics

Log-linearizing equation (26) one obtains:

$$w^o_{r,t} + \hat{\xi}_t = \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi\theta_w)^k (\hat{\Lambda}_{t+k,t+k+1} + \hat{\xi}_{t+k+1} + w^{tar}_{t+k}(r))$$

Using the log-linear equation for $\hat{\xi}_t$ then:

$$\begin{aligned} w^o_{r,t} + \hat{\xi}_t =& \mathbb{E}_t \sum_{k=0}^{+\infty} (\beta\chi\theta_w)^k (\hat{\xi}_{t+k} + (\beta\chi\theta_w)\pi_{q,t+k} + w^{tar}_{t+k}(r)) \\ w^o_{r,t} =& \beta\chi\theta_w \pi_{q,t} + w^{tar}_t(r) + \beta\chi\theta_w w^o_{r,t+1} \end{aligned} \tag{B.34}$$

From equation (B.30) one can write:

$$\begin{aligned} x_t - x_t(r) =& -\frac{\beta W_r}{\kappa X}\mathbb{E}_t(w_{r,t+1} - w_{r,t+1}(r)) + \beta\mathbb{E}_t\left[x_{t+1} - x_{t+1}(r)\right] \\ =& -\frac{\beta W_r}{\kappa X}\mathbb{E}_t(w_{r,t+1} - w_{r,t+1}(r)) \\ & - \beta\frac{\beta W_r}{\kappa X}\mathbb{E}_t(w_{r,t+2} - w_{r,t+2}(r)) \\ & - \beta^2\frac{\beta W_r}{\kappa X}\mathbb{E}_t(w_{r,t+3} - w_{r,t+3}(r)) \\ & \ldots \end{aligned}$$

On the other hand, let us log-linearize the aggregate wage equation:

$$\begin{aligned} WL(w_t + l_t) =& (1-\theta_w)L(W^o - W_t) + (1-\theta_w)W(L_t - L) + \theta_w \int_0^1 L(i)di(W_{t-1} - W) + \\ & + \theta_w \int_0^1 W(i)di(L_t - L) \\ w_t =& (1-\theta_w)w^o_t + \theta_w w_{t-1} \end{aligned}$$

Then in real terms one has:

$$w_{r,t} = (1-\theta_w)w^o_{r,t} + \theta_w w_{r,t-1} - \theta_w \pi_{q,t} \tag{B.35}$$

Then log-linearizing the expressions on equation (32) one can write by recurrence:

$$\mathbb{E}_t\left[w_{t+k} - w_{t+k}(r)\right] = \theta^k_w(w_{r,t} - w^o_{r,t})$$

and then:

$$x_t(r) = x_t + \frac{\beta\theta_w W_r}{\kappa X(1-\beta\theta_w)}(w_{r,t} - w^o_{r,t}) \tag{B.36}$$

Log-linearizing equation (27), one has:

$$\begin{aligned} W_r w^{tar}_t(r) =& \mathbb{E}_t\Big[\nu P^r_{q,t}\Gamma T(p^r_{q,t} + \gamma_t + t_t) + \nu\kappa X^2 x_t(r) + & \text{(B.37)} \\ & +\nu\left(\frac{\kappa}{2}X^2 - \beta z H - b - \frac{V(T_t)}{U'(C)} + P^r_{q,t}\Gamma T\right)\hat{\nu}_t(r) + \beta z H(1-\nu)(\hat{z}_t + \hat{\Lambda}_{t,t+1} + h_{t+1}) + \\ & -\frac{\nu V(T)}{U'(C)}(c_t + (1+\widetilde{\omega})t_t)\Big] & \text{(B.38)} \end{aligned}$$

and log-linearizing equations (41) and (42) one also has:

$$\begin{aligned}
h_t =& \frac{W_r}{H} w_{r,t} + \beta\chi \mathbb{E}_t \left[h_{t+1} + \hat{\Lambda}_{t,t+1}\right] - \beta z \mathbb{E}_t \left[\hat{z}_t + h_{t+1} + \hat{\Lambda}_{t,t+1} - \frac{V(T)}{U'(C)}(c_t + (1+\widetilde{\omega})t_t)\right] \\
j_t =& P_q^r \Gamma T (p_{q,t}^r + \gamma_t + t_t) - W_r w_{r,t} + \kappa X^2 x_t - \beta\chi(\hat{\Lambda}_{t,t+1} + j_{t+1})
\end{aligned}$$

Then using these equations and those obtained when log-linearizing the bargaining equations, one has:

$$\begin{aligned}
h_t - h_t(r) =& \frac{W_r}{H}(w_{r,t} - w_{r,t}(r)) + \beta(x - z)\mathbb{E}_t \left[h_{t+1} - h_{t+1}(r)\right] \\
j_t - j_(r) =& - \frac{W_r}{J}(w_{r,t} - w_{r,t}(r)) + \frac{\kappa X^2}{J}(x_t - x_t(r)) - \beta\chi \mathbb{E}_t \left[j_{t+1} - j_{t+1}(r)\right] \\
\hat{\Sigma}_t - \hat{\Sigma}_t(r) =& - \frac{W_r}{\kappa} \frac{\beta^2\theta_w^2}{1-\beta\theta_w^2} \frac{1}{1-\beta\theta_w}(w_{r,t} - w_{r,t}^o) \\
\hat{\nu}_t - \hat{\nu}_t(r) =& - (1-\nu)(\hat{\Sigma}_t - \hat{\Sigma}_t(r))
\end{aligned}$$

Consider now a firm renegotiating at time $t+1$, denoted r'. The following relationships are verified:

$$\begin{aligned}
\mathbb{E}_t \left[w_{r,t+k} - w_{r,t+k}(r')\right] =& \theta_w^{k-1} \mathbb{E}_t \left[w_{r,t+1} - w_{r,t+1}^o\right] \\
\mathbb{E}_t \left[(x_{t+k} - x_{t+k}(r')\right] =& - \frac{\beta\theta_w^k W_r}{\kappa X(1-\beta\theta_w)} \mathbb{E}_t \left[w_{r,t+1} - w_{r,t+1}^o\right] \\
\mathbb{E}_t \left[h_{t+1} - h_{t+1}(r')\right] =& \frac{W_r}{H(1-\beta\theta_w(\chi - z))} \mathbb{E}_t \left[w_{r,t+1} - w_{r,t+1}^o\right] \\
\mathbb{E}_t \left[j_{t+1} - j_{t+1}(r')\right] =& - \frac{W_r}{J(1-\beta\theta_w)} \mathbb{E}_t \left[w_{r,t+1} - w_{r,t+1}^o\right] \\
\mathbb{E}_t \left[\hat{\nu}_{t+1} - \hat{\nu}_{t+1}(r')\right] =& \frac{W_r}{\kappa} \frac{\beta^2\theta_w^2}{1-\beta\theta_w^2} \frac{1-\nu}{1-\beta\theta_w}(w_{r,t+1} - w_{r,t+1}^o)
\end{aligned}$$

Log-linearizing the Nash condition for a firm renegotiating wages at time $t+1$, one has:

$$\mathbb{E}_t \left[j_{t+1}(r') + \frac{1}{1-\nu}\hat{\nu}_{t+1}(r')\right] = \mathbb{E}_t h_{t+1}(r')$$

Then:

$$\mathbb{E}_t h_{t+1} = \mathbb{E}_t \left[j_{t+1} + \frac{1}{1-\nu}\hat{\nu}_{t+1} + \zeta(w_{r,t+1} - w_{r,t+1}^o)\right]$$

where:

$$\zeta := -W_r \left(\frac{1}{\kappa(1-\beta\theta_w)} \frac{\beta^2\theta_w^2}{1-\beta\theta_w^2} - \frac{1}{J(1-\beta\theta_w)} - \frac{1}{H(1-\beta\theta_w(\chi - z))}\right)$$

Log-linearizing equation (17) one has

$$x_t = \mathbb{E}_t \left[\hat{\Lambda}_{t,t+1} + j_{t+1}\right]$$

Then replacing for j_{t+1} one can re-write

$$\mathbb{E}_t \left[h_{t+1} + \hat{\Lambda}_{t,t+1}\right] = x_t + \mathbb{E}_t \left[\frac{1}{1-\nu}\hat{\nu}_{t+1} + \zeta(w_{r,t+1} - w_{r,t+1}^o)\right]$$

Finally, one can re-write equation (B.37) as

$$w_t^{tar}(r) = w_t^{tar} + \phi_w(w_{r,t} - w_{r,t}^o) + \psi_w \mathbb{E}_t \left[w_{r,t+1} - w_{r,t+1}^o \right] \tag{B.39}$$

where

$$w_t^{tar} := \phi(p_{q,t}^r + \gamma_t + t_t) + \phi_x x_t + \phi_z \hat{z}_t + \phi_\nu \hat{\nu}_t + \psi_\nu \mathbb{E}_t \hat{\nu}_{t+1} + \phi_c(c_t + (1+\widetilde{\omega})t_t) \tag{B.40}$$

and

$$\begin{aligned}
\phi_w :=& \frac{1}{(1-\beta\theta_w)} \left(\beta\theta_w \nu X - \frac{\nu(1-\nu)}{\kappa} \frac{\beta^2\theta_w^2}{1-\beta\theta_w^2} \left(P_q^r \Gamma T + \frac{\kappa}{2} X^2 - \beta z H - b - \frac{\nu C T^{1+\widetilde{\omega}}}{1+\widetilde{\omega}} \right) \right) \\
\psi_w :=& \frac{\beta z H(1-\nu)}{W_r} \zeta \\
\phi :=& \frac{\nu P_q^r \Gamma T}{W_r} \\
\phi_x :=& \frac{1}{W_r} (\nu \kappa X^2 + \beta z H(1-\nu)) \\
\phi_z :=& \frac{\beta z H(1-\nu)}{W_r} \\
\phi_\nu :=& \frac{\nu}{W_r} \left(P_q^r \Gamma T + \frac{\kappa}{2} X^2 - \beta z H - b - \frac{C T^{1+\widetilde{\omega}}}{1+\widetilde{\omega}} \right) \\
\psi_\nu :=& \frac{\beta z H}{W_r} \\
\phi_c :=& -\frac{\nu C T^{1+\widetilde{\omega}}}{1+\widetilde{\omega}}
\end{aligned}$$

Replacing equation (B.39) in equation (B.34) and using (B.35) one can derive a simplified expression for the real wage

$$\begin{aligned}
w_{r,t} =& \frac{1-\theta_w}{\theta_w \widetilde{\phi}} w_t^{tar} + \frac{1+\phi_w}{\widetilde{\phi}} w_{r,t-1} + \frac{\beta\chi - \psi_w}{\widetilde{\phi}} w_{r,t-1} + \frac{1}{\widetilde{\phi}} \left(\frac{\beta\chi}{1-\theta_w} - 1 - \phi_w \right) \pi_{q,t} \\
&+ \frac{\beta\chi\theta_w - \psi_w}{\widetilde{\phi}} \mathbb{E}_t \pi_{q,t+1}
\end{aligned} \tag{B.41}$$

where

$$\widetilde{\phi} := \frac{1}{\theta_w} + \beta\chi\theta_w + \phi_w - \psi_w$$

Equations B.1 to B.28, B.29, B.30, B.31, B.32, B.33, B.40 and B.41 complete the log-linear model.