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On the Equilibrium and Welfare Consequences of *Keeping Up With The Joneses*

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Abstract

This paper provides an analysis of the social consequences of people seeking to *keep up with the Joneses*. All individuals attempt to reach a higher rank than the Joneses, including the Joneses themselves. This attitude gives rise to an equilibrium in which all individuals have equal utilities but unequal (gross) incomes. Due to a rat-race effect, individuals devote too much energy to climbing the social scale in this equilibrium. However, laissez-faire equilibrium is an equal-utility constrained social optimum. Unexpectedly, numerical simulations show that this theory could account for the observed distribution of intermediate wages.

Key words: Keeping up with the Joneses, Social interactions, Well-being, Inequalities, Efficiency.

JEL Classification numbers: D3, D6, D8, I3, Z1.

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1 Introduction

"So far as concerns the present question, the end sought by accumulation is to rank high in comparison with the rest of the community in point of pecuniary strength." - Veblen (1899)

In this paper, inspired by Veblen (1899), we provide an analysis of the social consequences of people seeking to Keep Up with the Joneses, KUJ henceforth. In accordance with Veblen (1899), individuals’ utilities not only depend on their incomes but also on their social “status”. All people attempt to reach a higher social status than the Joneses, including the Joneses themselves. Here KUJ means doing as well and even better than one’s social ”neighbors”.

The idea that the individual well-being is to some extent relative to that of others and that we all try to keep up with wealthier than us dates back to Smith (1776) who noticed that happiness is not linked to the stock of acquisitions but to the progressive state of acquiring (see Book I, chapter VIII: of the wages of Labour, paragraph 42). Smith also pointed out that prosperity makes the poorest incapable of being content with the consumption which had formerly satisfied them (see Book I, chapter VIII: of the wages of Labour, paragraph 34). Duesenberry (1949) was the first to empirically observe this relative income phenomenon. The well-know Easterlin paradox (Easterlin 1974) tells us that while the progressive state of income acquisition is correlated with happiness, increased income does not lead itself to increased happiness. As a consequence of this transitory effect of income on life satisfaction, which acts like a focusing illusion (Kahneman et al. 2006), the Keeping-Up-with-the-Joneses behavior leads to a rat-race in which the individual’s goal is to place her or himself higher in the (gross) income hierarchy. Another aspect of KUJ behavior is conformism. As noted by Leibenstein (1976) and Granovetter and Soong (1986), status-seeking people are incited to acquire trendy goods in order to conform with those people they compare themselves with. Reversely, the status rat-race also leads to avoiding the consumption

\footnote{See Arrow (1975) for a view of Veblen as an economic theorist.}
of overly popular products. In words, individuals express consumption (and wealth) concerns relative to that of their peers (see for instance Ghiglino and Goyal 2010 and Xiaodong et al. 2014).

Our model can be compared with James Mirrlees’ path-breaking paper. Contrary to Mirrlees (1971), individuals are identical ex ante. But, since they wish to become VIP’s, they turnout to be heterogenous ex post with VIP’s consuming a great deal of energy to keep their envied social position, common people having a low status but not consuming as much energy in pursuing that end. This gives rise to an equilibrium in which all individuals have equal utilities but unequal (gross) incomes. We derive other interesting insights in this essay. First, we show that there is a single social optimum, relative to which all effort is lower than in the case of laissez-faire. This is the consequence of individuals devoting their energy to climbing the social scale. Next, this social optimum generates true inequalities, that is, inequalities in terms of utilities. The reason for this is that a social planner takes advantage of the dispersion of individuals on the ranking scale. Finally, we find that KUJ equilibrium is a true-equality constrained social optimum. It follows that if “you” dislike true inequalities, that is inequalities in term of utility, then “you” should favor observed (gross) income inequalities, and “you” should firmly reject the idea of any redistributive tax policy. We also give an assessment of the empirical potential of KUJ theory. Unexpectedly, the simulated income distribution rather correctly replicates the observed distribution of wages in France, within an extensive ”middle class”.

The jealousy hypothesis is indirectly confirmed in the empirical investigations by Easterlin, who finds that “growth does not buy happiness”. This unfortunate outcome can be interpreted as a consequence of KUJ attitude. If all incomes increase by the same rate, income hierarchy is not affected. Consequently, individuals do not perceive any improvement (in their social status). In that regard, we would like to stress that the analytical tool that we construct in the following assumes rational expectations in the sense that individuals’ decisions generate the income distribution on which they base their calculations. Although rational expectations became a nec plus ultra in economic theory about forty years ago, it seems that myopic or adaptative
expectations à la Milton Friedman might better account for observed facts in the so-called empirical economics of Happiness. Indeed, myopic individuals would realize ex post that, despite their efforts, they did not succeed in reaching a higher status since everyone else did too. In other words, while the Smithes were attempting to keep up with the Joneses, the Joneses themselves moved towards the Harpers who, in turn, moved towards the Rebieres... As a result, the Smithes, Joneses, Harpers and Rebieres, and everybody else report that their happiness is either equal or even falling when asked by Happiness economists.

Section 2 builds our modeling of KUJ equilibrium. The welfare implications of keeping-up-with-the-Joneses are studied in section 3 while section 4 examines the empirical potential of KUJ hypothesis. The conclusion summarizes our findings and sets a non-exhaustive agenda for further investigations.

2 Environment and KUJ equilibrium

Let us consider an environment à la James Mirrlees. Following this author, individuals’ income, $y$, is an increasing function of their effort, $k$. But, contrary to Mirrlees (1971), all individuals have the same efficiency at work, implying that they are perfectly identical ex ante. In other words, there is a single technology $y = H(k)$, with $H(k)$ being an increasing (strictly) concave function which satisfies $H(0) > 0$. The set of individuals is a continuum whose measure is normalized to 1. Individuals’ investments generate an endogenous c.d.f., $\Pi(k)$, which represents the proportion of individuals whose investments are strictly lower than $k$, i.e. $\Pi(k) = \text{Prob}[\text{Effort} < k]$. This definition of $\Pi(.)$ makes the exposition easier. A priori this function is assumed to be piecewise continuous. Its mass points are denoted by $(K_1 < ... < K_i < ... < K_n)$ where the positive integer $n$ can be unbounded. The corresponding frequencies are $(\phi(K_1), ..., \phi(K_n))$. If $n$ is zero, $\Pi(.)$ is continuous on $[0, \infty)$.  

KUJ has been broadly empirically tested through well-being sample surveys and experiments since the 90s. See for instance Clark and Oswald (1996), or more recently Card et al. (2012).

3 This (reasonable) assumption makes the analysis simpler.
These people are jealous. They are all willing to keep up with the Joneses, implying that they are affected by their location on Lorenz curve. In this context, the utility of an individual, $U(k,.)$, is decreasing in her effort, while increasing in her rank in income hierarchy as well as in her output. We retain the following form

$$U(k,.) = -k + J(\Pi(k))H(k)$$

(1)

In the previous expression, the term $J(.)$, referred to as the “KUJ multiplier”, is a strictly increasing (continuous) function of the share, $\Pi(k)$, of strictly lower ranked participants. It is worth noting that, according to our definition, $\Pi(k) = \Pi(k^-)$. Thus, if $K_i$ is a mass point, $\Pi(K_i^+) = \Pi(K_i) + \Phi(K_i)$. With no loss in generality, the top, $J(1)$, can be set to 1 while the bottom, $J(0)$, assumed to be strictly positive, is denoted by $\mu$ ($0 < \mu < 1$). This specification of utility $U(.)$ nicely captures the ambition of keeping up with the Joneses. It is not purely additive, implying that the downgrading (or upgrading) effect is all the higher as individuals are rich. This is in conformity with Thorstein Veblen’s message (1899).

Each individual maximizes $U(k)$ with respect to her effort $k$ for a given c.d.f. $\Pi(k)$. It is also assumed that function $V(k) = -k + H(k)$ goes to $-\infty$ as $k$ goes to $+\infty$ and that the derivative of $V(k)$ is strictly positive for $k = 0$. This implies that $V(.)$ has a single maximum $\tilde{k} > 0$ with $V(\tilde{k}) = \tilde{V}$. On the other hand, $W(k)$ denotes $-k + \mu H(k)$.

In this environment an equilibrium can be defined as below.

**Definition 1** A KUJ equilibrium is a distribution function $\Pi^*(.)$ such that, for all $k$ in its support, $U(k)$ is maximized with respect to $k \geq 0$.

It can be noted that this definition implies that $U(k)$ is a constant $U^* \geq U(0) > 0$, 

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4 Results extend to the case in which the multiplier associated with a mass point $K_i$ is higher than $J(\Pi(K_i))$ but lower than $J(\Pi(K_i) + \Phi(K_i))$. 

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for all \( k \) in the support of an equilibrium distribution. Let us study this equilibrium. In Appendix A, it is proved that the equilibrium distribution is continuous and that its support, denoted by \( S^* \), is a bounded interval \([A, B]\) with \( 0 \leq A < B < \infty \). For simplicity, the analysis focus on the case in which the lower bound, \( A \), is strictly positive. In \( S^* \), \( U(k) \) is a constant \( U^* > 0 \). Consider the behavior of the KUJ multiplier \( J(k) = J(\Pi(k)) \) in the interval \([A, B]\). We have \( J(A) = \mu \). In the interval \([A, B]\), \( J(k) \) is strictly increasing and equal to one for \( k = B \). In this interval, \( J(k) \) satisfies (see equation (1))

\[
J(k) = \frac{k + U^*}{H(k)}
\]

(2)

So \( J(k) \) has a derivative \( J'(k) \) such that

\[
J'(k) = \frac{H(k) - (k + U^*)H'(k)}{H(k)^2}
\]

One can check that if \( J'(A) \geq 0 \), then \( J'(k) > 0 \) in \([A, B]\). As \( U(A) \) should be equal to \( U^* \), it follows that \( J'(A)H(A) = 1 - \mu H'(A) \geq 0 \). Consider the behavior of \( U(k) \) in \([0, A]\). In this interval, \( U(k) = W(k) \) should be lower than \( U(A) \). The concavity of \( H(.) \) then imposes that \(-1 + \mu H'(A) \geq 0 \). See Figure 1. It results that \(-1 + \mu H'(A) = 0 \). Consequently, \( S^* \) is an interval \([A, B]\) whose lower bound is determined by \(-1 + \mu H'(A) = 0 \). The latter equation has a solution \( A^*(>0) \) if and only if \(-1 + \mu H'(0) > 0 \). From \( A^* \), we deduce \( U^* = U(A^*) \).

Let us now turn to the upper bound. Since \( \Pi^*(B) = 1 \), \( B^* \) satisfies \(-B + H(B) = U(A^*) \). For \( k > B^* \), \( U(k) = V(k) = -k + H(k) \). This implies that \( B^* \geq \bar{k} \). If not, \( U(k) = V(k) \) would be strictly increasing for \( B^* < k < \bar{k} \). Consequently, \( B^* \) is the highest solution to \(-B + H(B) = U(A^*) \). See Figure 1.

Results are summarized in the proposition below.

**Proposition 1** If and only if \(-1 + \mu H'(0) > 0 \), the support of \( \Pi^*(.) \) is an interval \([A^*, B^*] \) with \( 0 < A^* < B^* \). \( A^* \) is the solution to \(-1 + \mu H'(A) = 0 \) while \( B^* \) is the
highest solution to $-B + H(B) = U^* = U(A^*)$. In this interval, KUJ equilibrium satisfies $\Pi^*(k) = J^{-1}((k + U^*)/H(k))$.\footnote{One can see that if $-1 + \mu H'(0) \leq 0$, then $S^*$ is an interval $[0, B^*]$, with $B^*$ being the highest solution to $-B + H(B) = \mu H(0)$.}

In fact, the derivation of a KUJ equilibrium is simple. The investment of individuals at the bottom of hierarchy does not depend on investment distribution. And the same holds for individuals at the top. Between the two extremes, $\Pi^*(k)$ is set so as to ensure that utilities are constant and equal to $U(A^*)$. From $\Pi^*(k) = 1$, hence, $J(k) = 1$, one deduces the upper bound of the support (which should be greater than the lower bound).

Figure 1 illustrates the determination of an equilibrium. As $W(k)$ is maximized in $A^*$, $U(k)$ coincides with $W(k)$ for $0 \leq k \leq A^*$, is equal to $U^* (= U(A^*))$, for $A^* \leq k \leq B^*$, and coincides with $V(k)$, beyond $B^*$.

Figure 1: KUJ Equilibrium, $W'(0) > 0$. 
Individuals are identical \textit{ex ante}. But, since they wish to become VIP’s, they turn out to be heterogenous \textit{ex post}, with VIP’s consuming a great deal of energy to keep their envied social position, common people having a low status but not consuming as much energy to pursuing that end. This gives rise to an equilibrium in which all individuals have equal utilities, but unequal (gross) incomes. Income inequality is not very surprising. Indeed, starting from a situation of income equality, a (very) small increase in effort is sufficient to reach the top of the social hierarchy, implying that such a symmetric outcome cannot be an equilibrium. On the other hand, one could predict that all individuals will be prompted to “do the maximum”. This is what can occur when beyond some ceiling, the marginal return to effort is reduced to zero. It can be noted that equilibrium existence is ensured for all strictly increasing KUJ multiplier, $J(\Pi(k))$. Numerical simulations given in section 4 throw some light on the comparative statics.

3 Welfare analysis

The welfare criterion is the weighted average of utilities. Let $S(\Pi(.)$ denote the support of $\Pi(.)$. Aggregate welfare is

$$\Sigma(\Pi(.)) = -\int_{S(\Pi(.))} kd\Pi(k) + \int_{S(\Pi(.))} J(\Pi(k))H(k)d\Pi(k)$$

We can state that an equilibrium is inefficient: all participants invest too much. In what sense? To answer this question, we rank the individuals uniformly (and continuously) on the segment $[0, 1]$. As the ranking is uniform, there is a share $r$ of individuals whose ranks are lower than $r$ (i.e. whose ranks lie in the interval $[0, r]$). In market equilibrium $\Pi^*(.)$, the effort of $r$-workers satisfies $r = \Pi^*(k)$ or $k = k^*(r)$, with $k^*(.)$ being the reciprocal of $\Pi^*(.)$. As KUJ equilibrium is continuous, we have

$$\Sigma(k^*(.)) = \int_{0}^{1} [-k^*(r) + J(r)H(k^*(r))]dr = \int_{A^*}^{B^*} [-k + J(\Pi^*(k))H(k)]d\Pi^*(k)$$
Making use of this ranking of individuals, let us study the effect of a small change
\(dK\) in all efforts in the neighborhood of equilibrium. Since all investments vary
by the same amount, the rank \((r)\) associated with \((k^*(r) + dK)\) is left unchanged.
Consequently, the change in aggregate welfare is

\[
d\Sigma = \int_0^1 \left[ -1 + J(r) \frac{dH(k^*(r))}{dk(r)} \right] dK dr
\]

On the other hand, in KUJ equilibrium, all individuals have the same utility \(U^*\),
implying that

\[
-1 + J(r) \frac{dH(k^*(r))}{dk(r)} = - \frac{dJ(\Pi^*(k^*(r)))}{dk(r)} H(k^*(r)) < 0
\]

We then obtain

\[
\frac{d\Sigma}{dK} = - \int_0^1 \frac{dJ(\Pi^*(k^*(r)))}{dk(r)} H(k^*(r)) dr < 0
\]

This proves that

**Proposition 2** *In the neighborhood of KUJ equilibrium, lowering individuals’ efforts
improves welfare.*

This result clearly comes from KUJ attitude. From Proposition 2, one can easily
deduce that introducing a small income tax would improve welfare. Proposition 2
fails to locate KUJ equilibrium relative to social optimum. Making use of the ranking
model of effort dispersion, \([k(r)]_0^1\), we can prove the following

**Proposition 3** (i) *Assuming that \(-1 + \mu H'(0) > 0\), KUJ model has a single social
optimum, \([k^S(r)]_0^1\), such that \(-1 + J(r)(dH(k(r))/dk(r)) = 0\) for all \(r\) in \([0, 1]\). (ii)
\(k^*(r) > k^S(r)\) for all \(r\) in \([0, 1]\).

From the ranking optimum \(k^S(r)\), we deduce the optimum distribution \(\Pi^S(k)\) which
is the reciprocal of \(k^S(r)\).
Statement (i) results from the fact that the two models of effort dispersion, either the distribution $\Pi(k)$, or the ranking function $k(r)$, are isomorphic. See Appendix B. In words, deciding on a non-decreasing function $k(r)$ is not different from deciding on a distribution function $\Pi(k)$. So the proof of statement (i) amounts to observing that, due to the concavity of $H(.)$, $k^S(r)$ is strictly increasing in $r$. At first glance this result might look counter-intuitive, however. Indeed, one could object that, if defined with a c.d.f. $\Pi(k)$, a social optimum might have mass points. This can be excluded. The reason is that any mass point $K_i$ of $\Pi(k)$ corresponds to a subinterval of $[0,1]$ in which $k(r)$ is constant and equal to $K_i$. Consequently, any mass point generates a constraint on $[k(r)]_0^1$. Since $J(r)$ is strictly increasing, such constraints would be binding for the social problem. Symmetrically, one can easily exclude the case in which the social optimum defined as a distribution $\Pi^S(k)$ would be constant in a subinterval of $[A^S = k^S(0), B^S = k^S(1)]$. This is because the ranking of individuals on $[0,1]$ would create a discontinuity of $k^S(r)$ in such a case. Hence, the social optimum, $\Pi^S(k)$, which is unique, is also continuous and strictly increasing in the interval $[A^S, B^S]$. See Appendix B and C.

Statement (ii) results from the strict concavity of $k(r) + J(r)H(k(r))$ (in $k(r)$). In words, this result means that, relative to the social optimum, laissez-faire generates too much investment. In a precise sense, that is, for all $r$ in $[0,1]$. As statement (i) makes clear, this comes from the ranking effect, measured by the term $dJ(\Pi(k)/dk)H(k)$. In KUJ language, this result is the consequence of individuals devoting their energy $k$ to climb up the social scale, aiming at having more people below them, or, equivalently, fewer people above them. This generates a rat-race effect which makes individuals’ efforts excessive. On the contrary, a social planner would decide on investments $[k(r)]_0^1$ for a given ranking of individuals.

One implication of statement (i) is interesting per se. It turns out that, in the so-

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6If $X$ is a random variable whose c.d.f. $F(.)$ is continuous and strictly increasing, then the random variable $Y = F(X)$ is uniformly distributed on $(0,1)$. The proof of Proposition 3 just generalizes this property to a piecewise continuous non-decreasing function $F(.)$, then allowing for mass points.
cial optimum, the utility \( U(r) = U(k^S(r)) \) (whose derivative reduces to \( U'(r) = J'(r)H(k^S(r)) > 0 \)) is strictly increasing. By this path, we have thus reached the result that, in this context, although individuals are homogenous ex ante, utility inequality is better for welfare. This is an unexpected outcome, since \textit{a priori}, one could surmise that, due to the concavity of \( H(.) \), the social optimum should be symmetric.

As the ranking model makes clear, a social planner takes advantage of the dispersion of individuals across the ranking scale. The reason is that the marginal product of effort grows with the individuals’ position in the hierarchy. In other words, rich people profit more from an increase in their (gross of effort) revenue. Or, equivalently, people are increasingly sensitive to social degrees when their income is higher. Consequently, aggregate welfare is higher with a hierarchy of individuals.

It is worth noticing that Proposition 3 dramatically depends on the definition of aggregate welfare. A (virtual) situation in which true (i.e. utility) inequality would be favored because “rich people should be very rich” is a bit shocking. Extending the analysis to other welfare criteria is an interesting line for further investigations. At this stage, we can ask the following question. Which income distribution would an egalitarian planner select? We can prove that

**Proposition 4.** \( KUJ \) equilibrium is an equal-utility constrained social optimum

The analysis can be restricted to a continuous distribution whose support is connected.\(^7\) In this case, the proof of Proposition 4 consists in observing that the equal-utility constraint \( U'(r) = 0 \) implies that, for all \( r \) in \([0, 1]\),

\[-k'(r) + J(r)H'(k(r))k'(r) + J'(r)H(k(r)) = 0\]

or,

\[-1 + J(k(r))H'(k(r)) + \frac{dJ(\Pi(k(r)))}{dk(r)}H(k(r)) = 0\]

\(^7\)Mass points of probability (strictly) lower than one are clearly incompatible with the equal-utility constraint. The same holds for holes in the support which are mapped into a discontinuity of \( k(r) \) (see Appendix B). Conversely, one can see that the equal-income social optimum is another equal-utility constrained social optimum.
The latter condition is satisfied by KUJ equilibrium $\Pi^*(k)$. It follows that this (strictly) egalitarian social planner only has a single degree of freedom which is used to make $U(0) = -k(0) + \mu H(k(0))$ as high as possible. This maximum is reached for $k(0) = A^*$. Since $k(r)$ cannot be decreasing, it results that $k(1) = B^*$\(^5\). This shows that KUJ equilibrium is an equal-utility constrained social optimum.

The implications of Proposition 4 can be expressed in the following maxima: Whoever dislikes true inequalities, that is inequalities in terms of utility, should like observed (gross) income inequalities, and should then fight against any redistributive tax policy.

4 Empirical potential and predictions of KUJ theory

"Ah! Now my love we will show that Jones woman that her husband is not the only Adonis that can wear pink socks and a fuzzy hat!" - Pop (1913)\(^9\)

In this section, we carry out numerical simulations aimed at throwing some light on different issues. Can KUJ equilibrium replicate observed wage dispersion? How does a change in KUJ attitude affect income dispersion?

The KUJ multiplier is specified as follows

$$J(\Pi(k)) = \exp(-\lambda(1 - \Pi(k)))$$

In the previous expression for the KUJ multiplier, the positive parameter $\lambda$ can be regarded as a measure of individuals’ KUJ propensity, also referred to as their “social ambition”. The higher this parameter, the higher the effect of social rank on individuals’ utilities. From section 2 (Proposition 1), we know that the bottom

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\(^5\)Notice that the derivative $dJ(\Pi(k(r)))/dk(r)$ is equal to zero for $r = 0$ but $J'(r) > 0$. This is because, for $r = 0$, $dJ(\Pi(k(r)))/dk(r)$ is zero but, since $r = \Pi(k(r))$, $(d\Pi(k(r))/dk(r))k'(r) = 1$ for all $r$ in $[0,1]$.

and the top of (gross) income distribution are respectively determined by $H(A^*)$ with 
$-1 + \exp(-\lambda)H'(A^*) = 0$, and by $H(B^*)$ with $B^*$ being the highest solution to 
$U(B) = U^*$, or 
$-B + H(B) = -A^* + \exp(-\lambda)H(A^*)$.

We also know that the equilibrium value of the KUJ multiplier $J^*(k)$ is deduced from

$$J^*(k) = \frac{k - A^* + \exp(-\lambda)H(A^*)}{H(k)}$$

The specification of the multiplier is equivalent to

$$\Pi(k) = 1 + \ln \frac{J(k)}{\lambda}$$

It follows that

$$\Pi^*(k) = 1 + \frac{1}{\lambda} \ln \left[ \frac{k - A^* + \exp(-\lambda)H(A^*)}{H(k)} \right]$$

Finally, the cumulative distribution of (gross) income $y$, denoted by $G^*(y)$, derives from

$$G^*(y) = 1 + \frac{1}{\lambda} \ln \left[ \frac{H^{-1}(y) - A^* + \exp(-\lambda)H(A^*)}{y} \right]$$

with $y$ lying on the range $[H(A^*), H(B^*)]$.

In the following calculations, the production function, $H(.)$ is specified as 
$H(k) = a + b k^\alpha$ with $a, b$, and $\alpha$ being three strictly positive parameters such that $\alpha < 1$.

Notice that $H'(0) = \infty$, implying that $A^* > 0$, as in KUJ equilibrium ($W'(0) > 0$).

KUJ propensity, $\lambda$, is strictly positive.

Figure 2 reports the actual distribution of wages in France in 2010. It can be noted that the lower tail of the distribution is quite flat. In addition, individuals showing similar behavior should belong to the same "community". For this reason, we limit our study on the one hand to the poverty threshold, that is to say to 60% of the
median income\textsuperscript{10} and on the other hand to the 95th percentile so that monthly gross incomes vary between 1,317 and 5,586 euros. Thus, in accordance with The Theory of Leisure Class, which here extends to a large “middle class”, the actual truncated wage distribution is rather better replicated by the simulated distribution. The two curves are presented on Figure 3.

The following numerical exercises give an assessment of the predictions of KUJ theory. Notice that in next figures, all distribution functions (including the observed one) have the same normalized support $[0, 1]$. Figure 4 shows the simulated distribution for different values of KUJ propensity, $\lambda$. KUJ attitude gives rise to inequalities. The intuition behind this is simple. Individuals are homogenous, implying that, in the absence of “social ambition” ($\lambda = 0$), they all earn the same income. In the presence of social ambition, equality becomes impossible, as explained in the comment on Proposition 1. Notice that beyond some threshold, an increase in KUJ parameter would tend to lower income inequalities. This is because all efforts go to zero as parameter $\lambda$ goes to infinity. Besides, one can show that an increase in KUJ propensity lowers individuals’ utilities\textsuperscript{12} but, in terms of (gross) incomes, the effect on “perceived” wealth (or poverty) is ambiguous.

Another interesting issue is the way in which economic development affects income inequalities and wealth. This can be appraised through an increase of parameter $b$ which can be seen as reflecting technological progress. Figure 5 depicts the effect of such improvement of labor efficiency. Similar to KUJ ambition, above some limit, an increase of labor efficiency would tend to reduce income dispersion. The reason for this non-monotonicity is that individuals’ incomes are close to parameter $a(= H(0))$ as parameter $b$ goes to zero. In other words, individuals cannot differentiate when labor efficiency is very low. On the other hand, in conformity with intuition, it can shown that economic development increases perceived wealth. An increase of labor efficiency,

\textsuperscript{10}This amounts to dropping about 10\% of the bottom of the distribution.

\textsuperscript{11}In other words, a c.d.f. $G(y)$ of support $[z, Z]$ is mapped into a c.d.f. $\Gamma(x) = G(z + (Z - z)x)$ of support $[0, 1]$.

\textsuperscript{12}Knowing that all individuals have the same utility in KUJ equilibrium, the proof amounts to see that an increase in social ambition (parameter $\lambda$) reduces the utility $U^\ast = U(A^\ast) = -A^\ast + J(0)H(A^\ast)$ of individuals at the lower end of the social scale. KUJ multiplier $J(0) = \exp (-\lambda)$ falls.
leads to a cumulative distribution which is stochastically dominant. Expressed in words, the gross income associated with any social rank increases, implying that the social rank, \( G(y) \), associated with any income level, \( y \), decreases. One can see that utilities also benefit from this technological change. It can be noted that these comparative statics exercises have very different meanings. In the second case, wealth is affected by a “material” phenomenon whereas, in the first case, the change in utilities and incomes results from a sociological phenomenon. This highlights the role of social values (like social ambition) in the economic sphere as well as the practical importance that value-oriented policies may have in the real world. Contrary to K. Marx but in accordance with J.M. Keynes, “ideas” may have economic consequences. They may exert an influence on the so-called economic “infra-structures”. This is a domain where “free lunches” could be found, although a marxian economist would probably object that KUJ behavior, like individualism, is determined by the economic environment.

To sum up, these first simulations, based on arbitrary parameters values, permit us predict that the KUJ model does have empirical potential. We acknowledge that, to some extent, KUJ theory is difficult to refute, since the plausibility of parameters values cannot be easily assessed. A natural way of circumventing this problem would be to assume that, excepting KUJ intensity \( \lambda \), other parameters are common to different countries. In that regard, notice also that the inference method to apply in the estimation of inequalities is not obvious. One could argue that the distance between observations and estimations should be linked to a measure of inequalities.

\[13\] See Appendix D. Notice that in this analysis, the support of the distribution is not normalized to (0, 1).
Figure 2: Income distribution for France, full distribution

Figure 3: Actual and simulated income distribution for France

Source: Eurostat Anonymized SES Microdata, 2010. Note: top and bottom 1% have been dropped.

alpha=0.5, a=500, b=500, lambda=2.4
Figure 4: Simulated income distribution sensitivity to KUJ

![Graph showing income distribution sensitivity to KUJ with different values of \( \lambda \).]

Note: \( \alpha = 0.5, a = 500, b = 500 \)

Figure 5: Simulated income distribution sensitivity to parameter \( b \)

![Graph showing income distribution sensitivity to \( b \) with different values of \( b \).]

Note: \( \alpha = 0.5, a = 500, \lambda = 2.4 \)
5 Concluding remarks

Perhaps, the main insight of this essay may be that income inequality can result from the desire for (advantageous) inequality. This insight call into question the precise meaning of studies showing that income inequalities “explain” life expectancy, mental health, toxicomania, obesity.\footnote{See Wilkinson and Pickett (2010). Whatever their interpretation may be, the empirical results given by the authors are puzzling.} It cannot be ruled out that all these social issues, including income inequalities, are driven by one common cause: the KUJ attitude. We also showed that the KUJ hypothesis would have an empirical potential in replicating the observed wage dispersion.

In this conclusion we would like to note that our KUJ model could be helpful in many other contexts where, whatever its form may be, a rat-race prevail, as, for instance, informative advertising\footnote{Butters (1977) and Grossman and Shapiro (1984) are seminal papers.} with endogenously differentiated goods, or in the population density of cities where workers’ on-the-job performance increases with their proximity to firms. Indeed, the proximity to a place of work can be regarded as a means of dealing with shocks, as with human capital in the Schultz/Nelson-Phelps view.\footnote{See Acemoglu and Autor (2012).} Our analysis of KUJ raises different issues. As noted in the introduction, building a dynamic model with adaptative expectations would make the analysis more expressive. Besides, what is going on when the Lorenz curve is not known for anyone? Why are tax returns public in some countries - like Norway where daily local newspapers report your neighbors’ incomes - whereas they are secret in many others? And, still assuming imperfect information, what are the social consequences of the poor mimicking rich, or hiding their poverty, and of the rich seeking to demonstrate to the poor that they are rich. Which goods are produced to help rich in their ostentation strategy? In France, Hermes offers a handbag which it advertises as reserved to women \textit{with the necessary age and elegance}. Introducing imperfect information would allow for \textit{conspicuous consumption} in the sense of Thorstein Veblen.\footnote{Granovetter and Soong (1986) study market equilibrium in the presence of “reverse bandwagons” effects.} Symmetrically,
which goods do poor people use in their (paltry) attempt at cheating. We leave these different questions for further investigation. We also assign to a companion paper the task of appraising the extent to which international disparities in income inequalities can be imputed to cultural specificities of KUJ social rules.

References


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Appendix

A. Equilibrium distribution $\Pi^*(.)$

This appendix shows that $\Pi^*(.)$ is continuous and that its support is connected and bounded above.  
In a preliminary step, we show that $\Pi^*(.)$ is continuous for $k \geq 0$. Suppose it is not.  
This means that $\Pi^*(.)$ has at least one mass point $K_i \geq 0$ of probability $\phi(K_i) > 0$.  
Suppose that a participant, $D$, who invests $K_i$ initially, deviates and decide on a higher investment $K_i + \epsilon$. Since $\Pi(K_i^+) = \Pi(K_i) + \phi(K_i)$, we can deduce that, due
to the continuity of $J(.)$ and $H(.)$, her utility will jump upwards, implying that $K_i$ is not an optimum. This proves by contradiction that $\Pi^*(.)$ is continuous.

Next step shows that if the support of $\Pi^*(.)$, denoted by $S^*$, contain two intervals $[A, B]$ and $[C, D]$ with $D > C > B > A \geq 0$, then it also contains the interval $[B, C]$. Suppose $S^*$ does not contain $[B, C]$. We then have $\Pi^*(k) < \Pi^*(B)$ for $A < k < B$, $\Pi^*(k) = \Pi^*(B)$ for $B \leq k \leq C$ and, $\Pi^*(k) > \Pi^*(C) = \Pi^*(B)$ for $C < k < D$. For all $k$ in the support, $U(k) = U^*$. This implies that $U(B) = U(C) = U^*$. As $\Pi^*(k) = \Pi^*(C) = \Pi^*(B)$ for all $k$ in $[B, C]$, we also have $J(B) = J(C) = J(k)$ in the interval $[B, C]$. Since $H(.)$ is strictly concave, $U(k) = -k + J(B)H(k)$ is strictly concave. Consequently, $U(k) > U(B) = U^*$ for all $k$ in $]B, C[$. This contradiction proves that the support of $\Pi^*(.)$ is an interval $[A, B]$ such that $0 \leq A < B \leq \infty$.

We can now show that the support of $\Pi^*(.)$ is bounded above. Suppose it is not. Consequently, this support is an interval $[A, \infty]$ with $A \geq 0$. For all $k > 0$, $U(k) \leq V(k)$. It results that $U(k)$ tends to $-\infty$ when $k$ tends to $+\infty$. As $U(0) > 0$, this is a contradiction which sets that $S^*$ is bounded above. In other words, $S^*$ is an interval $[A, B]$ with $0 \leq A < B < \infty$.

**B. Isomorphism between the distribution model $\Pi(k)$ and the ranking model $k(r)$**

If $\Pi(.)$ is continuous and strictly increasing - like KUJ equilibrium $\Pi^*(.)$ - the mapping between the ranking model, $k(r)$, and the distribution model, $\Pi(k)$, is a simple change in variables. This appendix shows how this mapping extends to piecewise continuous non-decreasing functions. Namely, we show that any piecewise continuous c.d.f., $\Pi(k)$ translates into a piecewise continuous non-decreasing “ranking function” $k(r)$.

Let us first consider an interval $[k_1, k_2]$ on which $\Pi(k)$ is strictly increasing. On this interval $\Pi(.)$ has a reciprocal $k(r) = \Pi^{-1}(r)$. For all $r$ in $[r_1 = \Pi(k_1), r_2 = \Pi(k_2)]$, the
share of individuals whose ranks are lower than \( r \) is \( \text{Prob}[\text{Rank} < r] = \text{Prob}[\text{Effort} < \Pi^{-1}(r)] = \Pi(\Pi^{-1}(r)) = r \).

Let us now consider the case of a mass point, \( K \), of \( \Pi(\cdot) \) such that \( \Pi(K^+) = \Pi(K) + \Phi(K) \), with \( 0 < \Phi(K) \leq 1 \). Remember that, according to our definition, \( \Pi(K) = \text{Prob}[\text{Effort} < K] \).

One can see that in this case, \( k(r) \) is a constant equal to \( K \) for all \( r \) in the interval \([r_3 = \Pi(K), r_4 = \Pi(K) + \Phi(K)]\).

In this interval, the KUJ multiplier, \( J(\Pi(k(r))) \), is constant and equal to \( J(r_3) \). This implies that

\[
\Sigma([k(.)]_{r_3}^{r_4}) = \int_{r_3}^{r_4} [-k(r) + J(r_3)H(k(r))]dr = \Phi(K)[-K + J(\Pi(K))H(K)]
\]

Finally, we have to deal with holes in the support of \( \Pi(k) \).

Suppose \( \Pi(\cdot) \) is constant in an interval \([k_5, k_6]\) while increasing in the left hand neighborhood of \( k_5 \) as well as in the right hand neighborhood of \( k_6 \). One can see that the rank associated with this interval is \( R = \Pi(k_5) = \Pi(k_6) \). In addition, \( k(R) = k_5 \) and \( k(R^+) = k_6 \). In words, the ranking function \( k(\cdot) \) is discontinuous at \( R \) and jumps upwards.

**C. Social optimum**

Due to the isomorphism between \( \Pi(\cdot) \) and \( k(\cdot) \), the social optimum can be defined as a ranking function \( k^S(r) \) on \([0, 1]\) which maximizes

\[
\Sigma(k(.)) = \int_0^1 [-k(r) + J(r)H(k(r))]dr
\]

It follows that, assuming that \(-1 + \mu H'(0) > 0\), as in Proposition 1, \( k^S(r) \) is determined by \(-1 + J(r)[dH(k(r))/dk(r)] = 0 \) for all \( r \) in \([0, 1]\).
Since $H(.)$ is strictly concave and $J(.)$ is strictly increasing, $k^S(r)$ is strictly increasing. Consequently, the case in which the social optimum $k^S(r)$ would be constant on a subinterval of $[0,1]$ is clearly excluded. Indeed, since $J(r)$ is strictly increasing, the maximum of $\int_{r_3}^{r_4}[-k(r) + J(r)H(k(r))]dr$ with respect to $[k(r)]_{r_3}^{r_4}$ is higher than the maximum of $(r_4 - r_3)[-K + J(r_3)H(K)]$ with respect to $K$. In words, assuming that $k(r)$ is constant on an interval $[r_3,r_4]$ with $(0 \leq r_3 < r_4 \leq 1)$ would generate a constraint on the planner’s problem. The same holds for a discontinuity of $k(r)$.

Since $k^S(r)$ is strictly increasing, the optimal distribution, $\Pi^S(k)$ is the reciprocal of $k^S(r)$. This implies that $\Pi^S(k)$ is strictly increasing in its (connected) support $[A^S = k^S(0), B^S = k^S(1)]$. Mass points and holes in the support of $\Pi^S(k)$ are not compatible with optimality.

**D. Impact of labor efficiency on perceived wealth**

Let us prove that an increase in labor efficiency, $b$, improves “perceived” wealth, measured by gross incomes. For any production function such that $H(k,b) = H(bk)$, we obtain $dH^{-1}(y)/db = -k/b$. On the other hand, due to the envelope theorem, $\partial U^*/\partial b = \exp(-\lambda)H'[(A^*b)]A^*$. This means that true wealth (utilities) is enhanced. Since $\exp(-\lambda)H'[(A^*b)] = 1/b$, it results that the derivative of $G(y)$ with respect to $b$ has the same sign as $A^* - k$, which difference is (strictly) negative for all $y$ in the interval $[H(A^*), H(B^*)]$. In words, for almost all $y$ in the support of $G(.)$, the share of individuals whose incomes are lower than $y$ decreases. According to the criterion of stochastic dominance, parameter $b$ increases (perceived) wealth, i.e., reduces (perceived) poverty.