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► **To cite this version:**

François Fleury, Bernard Duprat. ANALYSIS OF AN UNPUBLISHED TREATISE OF AN 18TH CENTURY ENGINEER, ANTOINE D'ALLEMAN (1679-1760). 2015. halshs-01128919

HAL Id: halshs-01128919

<https://shs.hal.science/halshs-01128919>

Preprint submitted on 10 Mar 2015

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ANALYSIS OF AN UNPUBLISHED TREATISE OF AN 18TH CENTURY ENGINEER, ANTOINE D'ALLEMAN (1679-1760)

François Fleury¹, Bernard Duprat²

Keywords

Engineering trade, applied geometry, 18th century, unpublished treatise, Antoine d'Alleman, France.

Abstract

Military culture has occupied a central place in the constitution of constructive knowledge amongst the French intellectual elite since the beginning of the 17th century. The royal engineers, whose trade is rapidly institutionalized and developing in the 17th and 18th centuries, are important agents and vectors of this complex of practical knowledge backed by geometry, mathematics and the new physics.

Despite the recent scientific advances in structural mechanics and strength of materials, it is mainly the knowledge in geometry and mathematics that are put forth in the military architecture and engineering treatises as fundamental and strategic for the engineer, be they considered as prerequisites for access to the physical sciences, as necessary tools for good design and their in-field concretization, or as a reliable training method of rational thinking.

Antoine d'Alleman is “*Chevalier* and citizen” of the city of Carpentras, which at the time was the capital of the *Comtat Venaissin*. He is a specimen of this generation of architects and engineers, having studied a good number of military and scientific treatises, implementing his knowledge throughout a long career as topographer surveyor and hydraulics engineer in this papal territory, an independent state landlocked in the French kingdom. He designed and conducted in the *Comtat* important civil engineering works representative of those undertaken in the neighboring French provinces by the Corps of military engineers: roads, dikes, canals, aqueducts and water supply for cities, cartography. He also designed important buildings, hospitals, churches and chapels, at Carpentras and Orange. Besides his professional activity, he undertook the project of writing a treatise of architecture.

The review of the mathematical parts of these manuscripts informs us of the relationship that such an engineer could establish between theory and practice: a partial mathematization of the topographer surveyor's graphical and in-field operations, yielding a reliable foundation to his know-how. Within the limited framework of this paper, we will briefly illustrate how this process unfolds pragmatically, by analyzing a limited number of representative propositions of the Ms1127 manuscript, entitled pompously “The engineer, mathematical works of Mister d'Alleman or introduction to the science of engineering”.

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INTRODUCTION

Military culture has occupied a central place in the constitution of constructive knowledge amongst the French intellectual elite since the beginning of the 17th century. The royal engineers are important agents and vectors of this complex of practical knowledge backed by geometry, mathematics and the new physics. Military architecture and engineering treatises do yield information on the nature of these engineers' education, but they provide little information on how this formalized knowledge is appropriated, implemented, enhanced by experience and passed on by the great number of "ordinary" engineers, living substance of a renewed constructive culture.

The Inguimbertaine library of Carpentras (France), keeps three unpublished manuscripts written by Antoine d'Alleman (c. 1734). As "*Chevalier* and citizen" of that city, which at the time was the capital of the *Comtat Venaissin*, he had a long career as topographer surveyor and hydraulics engineer in this papal territory, an independent state landlocked in the French kingdom. He designed and conducted in the *Comtat* important civil engineering works representative of those undertaken in the neighboring French provinces by the Corps of Military Engineers: roads, dikes, canals, aqueducts and water supply for cities, cartography. He also designed important buildings, hospitals, churches and chapels, at Carpentras and Orange, or even a church for Mahon in the Minorque Island. Besides his professional activity, he undertook the project of writing a textbook of mathematics for engineers comprising a treatise of architecture³.

STANDARD THEMES OF THE MANUSCRIPT

The three volumes of the manuscript (d'Alleman, c. 1734) are an assembly of small notebooks, very uneven in number of pages, sewn together. Each volume focuses on a main theme, yet with some heterogeneity. The texts are often crossed out: d'Alleman corrects himself in the margins, scratches out certain parts, or relocates entire paragraphs further in another chapter. It is therefore clearly a draft, several times amended, revealing parts of its making process. We can witness how it is being structured as he goes along with the writing.

Manuscript Ms1129-1

The author dedicates the first ten pages of the Ms1129-1 to an "inventory of the most important works that I have done and conducted...". This chronological listing takes only three of the ten pages, and covers the period 1700 to 1760, mentioning the year, nature and location of the works. Other sources attest that a number of his projects are not included⁴. But even if the list does not give an exact image of this practitioner, it indicates what in his view is worth to be mentioned. d'Alleman appears first as a hydraulic engineer and a topographer surveyor, as he lists 18 times works of river bank planning, waterworks and levelling, undertaken in Provence or in the *Comtat*, for which he was also the "director of roads". As an architect, he puts forth 14 architectural projects all for ecclesiastical clients. The rest of his business covers occasional tasks of mapping, archeology, fortification⁵, or even a cannon prototype never tested. Even if his second and third manuscripts contain large parts dedicated to fortification, and although he does have

³ "treatise of architecture as part of the mathematics textbook for engineers I have just finished", so he tells Jesuit Estienne Souciet in a letter dated July 1734 (Ms1129-1, f° 117).

⁴ such as the retaining wall and cellars of the *Hotel de Murs* in Carpentras, attributed to the famous Franque architects, of which the negotiated price is known (Archives départementales de Vaucluse, 3 E 27 – 19 f° 224, 1752)

⁵ An archeological survey in Rome, one unique beginner's fortification project in Malta in 1702 and the "wall of plague" between Provence and the *Comtat Venaissin* in 1720.

the profile of the military engineer, he is not listed in the inventory of Anne Blanchard (1981), and has almost no experience in the big business of that time: defending and taking fortresses.

Manuscript Ms1129-2

In addition to various pages of drafts or interspersed calculations, Ms1129-2 presents three textual units: “demonstrated fortification” (97 pages), “introduction to engineering” (2 pages), and “treatise of arithmetic and rectilinear trigonometry” (65 pages). Elsewhere, that is on the cover page of Ms1127, d’Alleman gives a rather complete list of the scientific fields of his time, as can be found in the curricula of military engineers since the 17th century (Bousquet-Bressolier, 2008). Yet the short-list he has kept for his “textbook of mathematics” is very limited, and the structure of his treatise considerably deviates from that inventory. He exposes in some 60 pages only elementary arithmetics teaching how to count with the fractional units of the time, and calculations using the properties of the triangle in so-called trigonometric measurement operations.

On the one hand, we are dealing only with a collection of basics, probably close to mathematics taught in Jesuit colleges (Rousteau-Chambon 2013), and on the other hand, these documents are heterogeneous. But this heterogeneity does not seem to result from an accidental or awkward assembly of the pages which would have happened subsequently to their writing. Indeed some texts overlap several notebooks and one same notebook can contain two unrelated texts. As well as the numerous crossed out texts, the observed thematic heterogeneity betrays the author’s hesitations. They relate both to the structure of the text and to its writing. Conversely, when his rhetoric seems clear and strong, his writing well-formed and without cross-outs, then one is inclined to think that he is either writing out again a first draft, either borrowing sentences or expressions from other authors. The potential sources should therefore be sought for and matched, in order to understand the purpose of his writing project and his personal scientific culture within which it is undertaken.

Manuscript Ms1127

The Ms1127 manuscript provides also interesting data for the study of this very practical scientific culture. Following a number of definitions, d’Alleman writes a set of “propositions” that pertain to what he calls “geometric practice” of the engineer. These are more specialized than the simple arithmetics of the Ms1129, and they reveal clearly the partial mathematization process at hand. But before he starts on the mathematics, d’Alleman reproduces the definitions of categories that rule the Corps of the Fortifications Engineers since Vauban. Their hierarchy and functions are given, as well as the “qualities and knowledge necessary for the engineer”, in particularly messy lines. His third chapter “*Des mathématiques*”, clearly distinguishes “practical mathematics” from “speculative mathematics”. Clearly d’Alleman stands in the first category. This is confirmed by the following chapter on the practical measuring units, where he substitutes the concept of *measure* by that of *reference yardstick* serving as computational unit. His description of the instruments for measuring, drawing and topography goes in the same direction, and he finally gives to the sixth chapter, dedicated to “expressions for lines and surfaces”, the only objective of fixing a notation standard using letters for these entities, giving advice and graphical models, without any mathematical content, addressed more to a draftsman than to an engineer.

Most of the Ms1127 is dedicated to chapter seven: “*Des pratiques de géométrie*”, that comprises 76 neatly ordered “propositions” on 47 pages, followed by a trigonometric table⁶ on 15 pages. The propositions are presented according to main subtitles:

⁶ This table gives the length of the base of a 25 feet isosceles triangle depending on the angle of its summit.

Analysis of an unpublished treatise of an 18th century engineer, Antoine d'Alleman (1679-1760)

- 12 for the single straight line, to be drawn, continued, divided, subtracted;
- 8 teaching how to construct parallels or perpendiculars from different passing points;
- 14 to draw, measure and divide plane angles;
- 24 describe methods for constructing regular polygons, starting from the triangle up to the dodecagone, ending with a general method valid whatever the number of sides.
- 4 propositions teach how to draw scaled figures one from another;
- 3 then show how to draw ovals, and finally
- 2 for small scale plane cartography, respectively applied to an enclosure and a river.

D'ALLEMAN'S APPLIED GEOMETRY

Put aside l'"*essay d'architecture civile*", the chapter on applied geometry is the more developed and coherent textual unit of these manuscripts. It is analyzed here by comparing a chosen number of propositions with their equivalents in two reference French treatises of applied geometry, namely "*La géométrie pratique*" by Jacques Ozanam, published in 1684, and "*Pratique de la géométrie sur le papier et sur le terrain*" by Sébastien Leclerc (1682), first published in 1669.⁷

Definitions

Right before starting on the propositions, d'Alleman gives a definition for the straight line: "line *AB* is straight when looking from point *A* to point *B*, point *A* covers all the points that form line *AB*". Let us compare it to those of:

- Euclid in c. 300 BC : "A straight line lies evenly with the points on itself" (Joyce 1996)
- Leclerc in 1669: "Straight is a line that is evenly placed within its ends. In another way, it is that which goes from one point to another without any detour" (Leclerc 1682).
- Ozanam in 1684: "If the moving mathematical point goes not more to one side than to the other, then the line described by this movement is called straight" (Ozanam 1684)
- Roberval, before 1675: "When a solid revolves around two pivot points, a line can be stretched between these two points that will lie evenly within these same points, that will conserve the same position during the entire movement of the solid, and at the same time, each of all the points of the line will conserve its own position. (Roberval 1996)

Euclid's definition can be traced in those of Roberval and Leclerc, although quite differently interpreted. Even if Roberval's straight line is constituted of material points, the definition calls for a real mental effort, and is quite useless for practical operations such as drawing, surveying, or laying foundations. That of Ozanam explicitly uses the concept of *mathematical point*, but ends as a tautology. As Ozanam, d'Alleman disregards Euclid's heritage in his definition, which assumes de facto a rectilinear visual ray, stretched as a string line. It is strikingly effective: synthetic, univocal, and with an essential practical scope for the field surveyor aiming at staffs. The comparisons between the same authors of the definitions given for a perpendicular line confirm that d'Alleman stands out as the practical man. Instead of referring to the mathematical concept of plane angle, he resorts to the image of a perpendicular that should not "*lean* more towards *A* than towards *B*", as a surveyor's staff.

⁷ Ozanam is very popular and considered as a reference up to the end of the 18th century (Bousquet-Bressolier 2008). As for Leclerc, d'Alleman himself refers to this author ("that I have at home", he says) with whom he disagrees on the definition of the "ordre français" in architecture. The book of Leclerc is considered by Bousquet-Bressolier (2008) as a prototype of the new applied geometries that appear in the second half of the 17th century.

Geometric algorithms

Of the 76 propositions, we have chosen to compare herein the answers given by the three authors to the problem of drawing a perpendicular. When the crossing point is located within the given line, the three implement quite the same method (figure 1). The same letters are used for the same points⁸ and they correspond to those used by the translators of Euclid for the same problem⁹. d'Alleman's style is again effective, with no ambiguity: the text is a real algorithm that can be applied without the help of the figure. In comparison, that of Ozanam is somewhat tortuous, and Leclerc omits to specify that points D and E should lie on line AB .

Although Leclerc declares in the title of his book that he will treat of applied geometry *on the paper and on the terrain*, he never gives any variant for in-situ constructions. That given by d'Alleman is in its geometric principal similar to that for paper, yet he does not explain the analogy between the course of the end of a “*chaînette*” (small measuring chain) stretched from a fixed point and the trace given by a compass. He simply tells us to find point F by joining the ends of two chains of equal lengths, respectively attached to stakes in E and D .

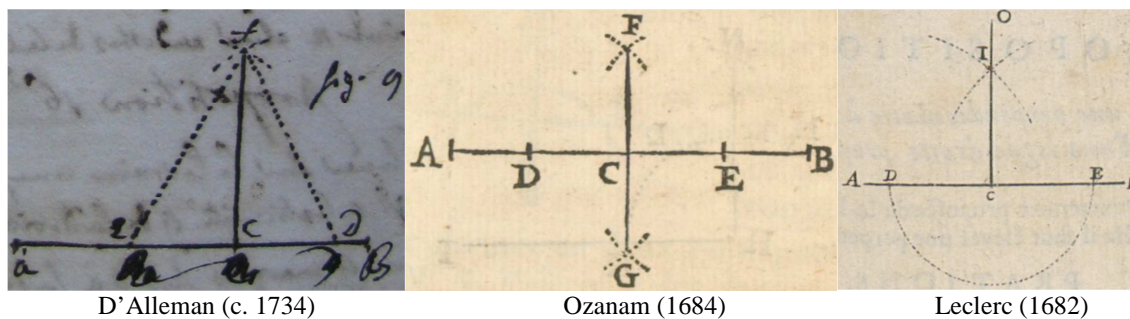


Figure 1: Drawing of the perpendicular to a given line AB , passing by a point C lying on AB

We get more variety amongst the authors when the perpendicular must be drawn from the end of a half-line (table 1), when drawing or working surface is constrained. That problem is not presented explicitly in Euclid, and as a matter of fact, the letters to name the points are chosen more freely. Each author presents a different combination of two techniques out of a total of three. One relies on Pythagore's theorem in a “3-4-5” triangle, the second is based on the inscribed triangle, one side being the diameter of the circle, and the last considers the perpendicular as the bisector of a side of a hexagone centered on the given half-line's end.

Table 2 shows which of these techniques are selected by the three authors, and their respective degrees of complexity, on paper and in-situ. The “3-4-5” technique is the only one implemented in-situ. It is indeed the easiest to teach and to remember, while emerging as the most efficient, minimizing the number of stakes to be driven in the ground, and the number of goings and comings. Ozanam does not recommend it on paper, where he prefers a rather complex method, difficult to implement in-situ. On paper, in addition to a transcription of that recommended for the field, d'Alleman favors the most efficient method for pencil and compass.

In many of the propositions, the techniques put forth by the three authors differ, and although d'Alleman may appear closer to Ozanam than to Leclerc, this is not systematic. The method proposed by d'Alleman for subdividing a segment in equal parts, for instance, is also found in the more mathematized 1690 geometry treatise of Leclerc, but not in his applied geometry of 1669,

⁸ Except for the intersection point of the arcs by Leclerc.

⁹ Such as D. Henrion in 1632.

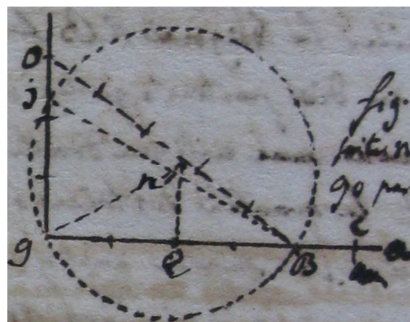
which proposes the same method as Ozanam's. Of both methods, d'Alleman chooses the one with the minimum operations, even if it uses more paper. It is thus very likely that d'Alleman borrows from a different number of sources, but clearly not in a systematic way. His manuscript gathers a personal selection of pragmatic and efficient solutions to classic useful situations. These examples are quite representative of most of the "propositions". No methodological errors are identified. Often, alternatives are included for the terrain, which take into account practical difficulties such as those resulting from accessibility or tools' availability.

"Proposition 15 : Raise a perpendicular gO at the end of line ga .

From point g with any compass opening that suits you, mark on line ga five equal parts gC and from the fourth division B as center, with opening gC having drawn an arc in O , from point g with opening of $3 gC$ parts, mark point O on arc O and from g draw line gO which will be perpendicular to line ga , which is obvious from 49 of 1st.¹⁰

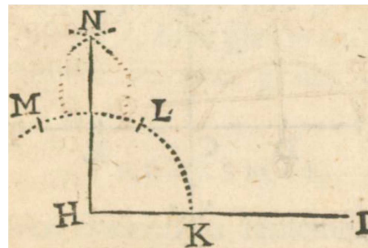
Else with a compass opening gn that suits you, from point n above line gC as center, draw circle $gjBg$. From point B where the circle crosses line ga , draw through the center n line Bj , and from point g to point j line gj which will be perpendicular to line ga .

Proposition 16 : Raise on the field a perpendicular gj at the end of line ga . From point g mark on ga distance gB of 4 toises and having attached to stake g and stake B a chainette at each, stretch them both towards j so that the one from g be three toises and that from B be five toises. Have a stake driven in point j . Pull a string line from g to j , it will be perpendicular to line ga ."

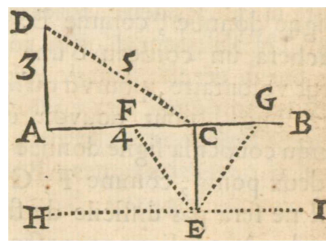


D'Alleman (c. 1734)

[On paper] "To pull from end H of a given line HI , a perpendicular, draw from the given end H with a compass opening freely chosen, but less than the given line HI , a portion of circle KLM , which crosses the given line HI at point K . Plot the same compass opening on arc KLM , from K to L , and from L to M . Draw from both points L, M , with the same compass opening, if you wish, two arcs, that intersect here in point N . Finally, draw line HN , which will be the sought perpendicular.



[In situ] Having measured on given line AB from A to C the length of 4 toises, attach to the given point A a string line 3 toises long, and to point C another string line 5 toises long. It is obvious from 48.I that if one stretches these two string lines, so that one joins together their two ends, one will have point D on the sought perpendicular."

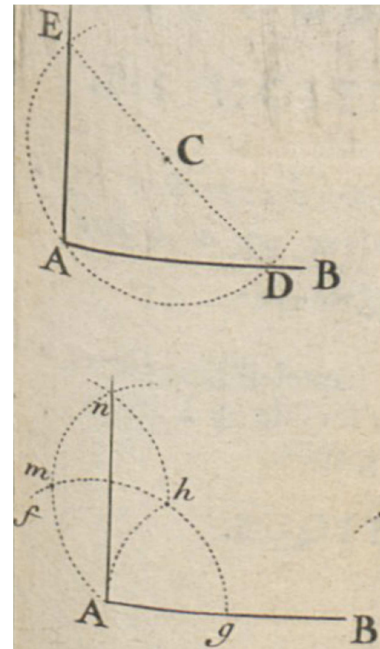


Ozanam (1684)

"Proposition 2: Raise a perpendicular from the end of a given straight line. A be the proposed extremity of line AB from which the perpendicular is to be raised.

Technique: Place as you wish point C above line AB . From that point C and interval CA draw the portion of circle EAD . Draw the straight line DCE through points D & C . Draw the wanted line AE . It will be perpendicular to AB & [passing through] the given extremity.

Else: From point A draw arc GHM . From point G draw arc AH . From point H draw arc AMN . From point M draw arc HN . Draw the wanted line AN ."



Leclerc (1682)

Table 1: Propositions for the drawing of a perpendicular from the end of a given line

¹⁰ Proposition 48 of the 1st book of Euclid's elements is the reciprocal of Pythagore's theorem. There is no proposition 49. Either d'Alleman is citing by heart, either number 9 is not well formed, and could be in fact a crooked 8.

	Pythagore “3-4-5”	Inscribed triangle	Hexagone
D’Alleman	Paper and in-situ	Paper	-
Ozanam	In-situ	-	paper
Leclerc	-	paper	paper
Complexity on paper			
Number of operations	7	3	6 or 5
Different compass openings	3	1	1
In-situ complexity			
Distance traveled (toises)	9	14	14
Number of stakes	3	3	5
Easy to teach	easy	easy	difficult

Table 2: Distribution of the different methods and their relative complexity

In rare occasions, the in-situ method departs completely from that given for a paper construction. This is the case for the division of a segment in equal parts. The classic *ruler-and-compass-only* discipline (or its in-situ equivalent) yields in front of the operational issue. Whilst the method for dividing a line on paper is purely geometric, the in-situ version is arithmetic, and consists simply in measuring the total length and dividing it numerically by the number of required parts.

Demonstrations

D’Alleman’s “demonstration” for proposition 13 (figure 1) proceeds as such: “line Cf [...] will be perpendicular to line aB , which is obvious because as points D and E are equally far from point C and as point f is also equally far from points D and E , the triangles ECf [and] fCD will be equal by the 4th axiom, and the line fC will not lean more to one side than to the other, and therefore will be perpendicular. Since point f is equally far from points D^* and E^* and [since] points D^* and E^* [are equally far] from point C^* , point f will not lean more towards D than towards E and angles fCD [and] fCE [will be] equal and consequently line fC^* perpendicular to line aB .”¹¹

A correct demonstration would be: $CE = ED$ and $FE = FD$ by construction, thus triangles FCE and FCD have their three sides equal two by two. By Euclid’s eight’s property, (and not by the “4th axiom”¹²) angles FED and FEC are equal. Since E , D and C are aligned with aB , then these two angles are right angles and FC is perpendicular to ED , and thus also to aB .

In the two other treatises compared, there are no demonstrations. D’Alleman very often gives a tautological explanation in the form of “which is obvious, since...”, but these are practically never mathematical demonstrations. Those which seem to claim that purpose are laborious, incomplete, and not founded on explicit properties, theorems or axioms. When these are referred to, indicating Euclid’s nomenclature, it is not always relevant, as we have seen.

CONCLUSION

The review of the mathematical parts of these manuscripts informs us of the relationship that such an “ordinary” engineer could establish between theory and practice: a partial mathematization of the topographer surveyor’s graphical and in-field operations, yielding a reliable foundation to his know-how. For d’Alleman, mathematics seems to be less an exercise for the elevation and refinement of the mind than a science that “teaches us how to measure and count [...] all that

¹¹ Here d’Alleman refers to points using crossed out letters of the corresponding figure. We have chosen to substitute them with the correct letters: B,C and D of the original are replaced here respectively by D^* , E^* and C^* .

¹² In Euclid, proposition 4 goes: “if two triangles have one angle equal and such that the corresponding sides are equal from one triangle to the other, then the third side will be the same for both triangles, as well as the two other angles, and the triangles will be equal.”

we consider under the name of magnitude or quantity”¹³. He considers that the engineer should know “The elements of Euclid because they give him assessed principles to guide him in his practice” and should “well possess applied geometry because it will teach him how to lie down on the field and on the paper the plans and profiles of the construction works, [which are] needed for taking or for defending a fortress as well as for its construction”.

Although d'Alleman recons the mathematician's effort to extract this science from its earthly gangue, he is more interested in the reliability of its application to real life that stems from the process, than he is in its formal rigor and beauty. The approximations in the demonstrations, the concern of putting side by side the paper and in-situ constructions, the form given to the concepts and his preference for efficient methods, all this confirms that d'Alleman is not a mathematician. He is merely an expert user, capable of choosing in the already published methods those relevant for his own applications, and competent for adapting them in a reliable way.

REFERENCES

- Alleman (d'), Antoine. c. 1734. *L'ingénieur, œuvres mathématiques de monsieur d'Alleman ou in-troduction à la science du génie*, in manuscrit Ms1127, Bibliothèque Inguimbertaine, Carpentras, France.
- Alleman (d'), Antoine. c. 1734 *Essay d'Architecture civile*, in manuscrit Ms1129-1, Bibliothèque Inguimbertaine, Carpentras, France.
- Alleman (d'), Antoine. c. 1734 *La fortification démontrée*, in manuscrit Ms1129-2, Bibliothèque Inguimbertaine, Carpentras, France.
- Blanchard, Anne. 1981. *Dictionnaire des ingénieurs militaires : 1691-1791*. Centre d'histoire militaire et d'études de défense nationale, Montpellier, France.
- Bousquet-Bressolier, Catherine. 2008. “Etudes et formation des ingénieurs sous Vauban”, *Comité français de cartographie*, n°195, march, (15-26).
- Joyce, David E. 1996. *Euclid's elements*. Internet web site of Clark University, Worcester, MA, USA <http://aleph0.clarku.edu/~djoyce/java/elements/elements.html>
- Le Clerc, Sébastien. 1682. *Pratique de la géométrie sur le papier et sur le terrain*. Paris, Jombert. Première édition 1669.
- Le Clerc, Sébastien. 1690. *Traité de géométrie*, Paris, Jombert.
- Orgeix, Emilie; Warmoes, Isabelle eds. 2013. *Les savoirs de l'ingénieur militaire (1751-1914) - Actes de la 5ème journée d'étude du Musée des Plans-Reliefs, 22 octobre 2010*.
- Ozanam, Jacques. 1684. *La géométrie pratique*. Paris, Michallet.
- Roberval, Gilles P. 1996. *Eléments de géométrie de G.P. Roberval*. Jullien V. ed. Librairie philosophique, J. Vrin, France.
- Rousteau-Chambon, Hélène. 2013. “Philippe de la Hire et le « Traité de mécanique »”, in *Les savoirs de l'ingénieur militaire (1751-1914) Actes de la 5ème journée d'étude du Musée des Plans-Reliefs, 22 octobre 2010*, Orgeix, E., Warmoes, I. eds. Ministère de la Culture et de la Communication, Direction des Patrimoines. Musée des Plans-Reliefs. (15-20).

¹³ “continuous quantity or magnitude [is] all that can be measured such as lines, planes, solids, time, speed or delay of movement, etc. [that is] what we call geometry.”