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Holdout threats of the union during wage bargaining

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Abstract. We investigate a wage bargaining between the union and the firm where the parties’ preferences are expressed by varying discount rates and the threat of the union is to be on go-slow instead of striking. First, we describe the attitude of the union as hostile or altruistic where a hostile union is on go-slow in every disagreement period and an altruistic union never threatens the firm and holds out in every disagreement period. Then we derive the subgame perfect equilibrium of the bargaining when the union’s attitude is determined exogenously. Furthermore, we determine necessary conditions for the extreme equilibrium payoffs of both parties independently of the union’s attitude and calculate the extreme payoffs for a particular case of discount rates.

JEL Classification: J52, C78

Keywords: union - firm bargaining, varying discount rates, holdout threats, go-slow threats, subgame perfect equilibrium

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1 Introduction

It is well known that strikes and other wasteful conflicts can be observed between rational agents during wage negotiations. Kennan and Wilson (1989, 1993) emphasize that strikes are the signaling devices of the firm’s willingness to pay to the workers. Therefore, if the firm is more profitable, workers have high wage expectations. Ingram et al. (1993) find empirical evidences both for and against this explanation of the occurrence of strikes. By using noncooperative bargaining theories one may analyze wage expectations of unions and outcomes of union-firm negotiations in an appropriate way (see e.g. Kennan and Wilson (1989, 1993), Osborne and Rubinstein (1990) and Binmore et al. (1990)). Especially, the private information of the firm’s willingness to pay can stimulate the strikes. Other inefficiencies in the wage bargaining are shown, for instance, in Crawford (1982) who analyzes uncertain commitments and in Haller and Holden (1990) and Fernandez and Glazer (1991) who point out multiple equilibria in bargaining game.

Depending on labor laws, strike actions may not be protected legally in some countries. Although necessary federal legislations were accepted in 1930’s workers’ rights to strike, people who work for the federal government are not allowed to strike in the US. In particular, all public officers, including teachers, are forbidden to strike in New York
While several works on wage bargaining with strike strategies of the union have been presented, other threats of the union did not receive enough attention so far. Although holdout threats are frequently ignored in the literature (see e.g. Fudenberg et al. (1985), Hart (1989) and Kennan and Wilson (1989)), Cramton and Tracy (1992, 1994) prove that, as well as the strikes, the holdout threats after the expiration of the contract can also provide a significant wage increase. By investigating the labor negotiations in the US, they analyze the problem of the firm’s willingness to pay caused by the private information. They conclude that most of the conflicts during collective bargaining are ended off by the holdout threats of the union such as work-to-rule or go-slow actions instead of strike. After the expiration of the actual contract, workers continue to work with the existing wage level until a new contract is signed. For instance, between 1970 and 1989 the holdout threats appeared four times more frequently then the strikes during the wage negotiations in the US labor market.

In order to analyze the effects of the union’s threats on wage levels, Moene (1988) indicates four different threats: work-to-rule, go-slow, wild cat strikes and official strikes or lockouts. Work-to-rule is a non-official industrial action in which the workers severely slow down their working efforts to the minimum required level by the rules of their contract. Differently from work-to-rules, go-slow is an official threat of the union where the workers announce officially how much they reduce their work efforts. Moene (1988) argues that holdout threats of the union give a higher wage increase than strikes. The analysis of the holdout threats of the union may help to study real world collective wage bargaining where the strikes are prohibited. For instance, Moene and Wallerstein (1997) examine the go-slow threats of the union in Scandinavian countries.

Several works on noncooperative wage bargaining refer to Rubinstein’s alternating offer model. One can assume that the players are risk neutral during the bargaining process. Therefore, constant discount factors are used although players’ incentives and patience levels show variability in real life. For instance, while the demand of a firm shifts daily or weekly, protracted strikes or slowdown strikes affect the cost of the conflict. In the same vein, the union’s preferences may vary during the uncertainty of the collective bargaining. This clearly motivates the interest of assuming discount rates varying in time. In particular, Cramton and Tracy (1994) emphasize that it is more accurate to apply non-stationary bargaining models for real life situations. Binmore (1987) analyses preferences that do not necessarily satisfy the stationarity assumption, see also, e.g., Coles and Muthoo (2003). Rusinowska (2001, 2002b, 2004) generalizes the original model of Rubinstein to bargaining models with preferences described by sequences of discount rates or/and bargaining costs varying in time. Ozkardas and Rusinowska (2014, Forthcoming) generalize the wage bargaining model of Fernandez and Glazer (1991) by assuming varying discount rates.

The aim of this paper is to examine the effects of the union’s holdout threats such as go-slow on the wage determination when the parties’ preferences vary in time. In order to apply the go-slow strategies of the union, we modify the wage bargaining model of Ozkardas and Rusinowska (2014). First, we restrict our analysis to history independent
strategies with no delay. We specify two different attitudes of the union, either *hostile* or *altruistic*, and determine the subgame perfect equilibria in the wage bargaining for each of the attitudes. More precisely, we say that the union is hostile if it is on go-slow in every period when there is no agreement. An altruistic union always holds out and continues to work with the same effort and wage during disagreement periods. Then we generalize and apply the method used in Houba and Wen (2008) to the situation when the strikes are not allowed and the union can threaten the firm with being on go-slow.

The rest of the paper is organized as follows. In Section 2 the generalized wage bargaining model where the union can threaten the firm with go-slow action is described in details. Section 3 concerns the subgame perfect equilibrium of the wage bargaining depending on the union’s attitude, i.e., hostile or altruistic. In Section 4 we derive the necessary conditions for the supremum of the union’s subgame perfect equilibrium payoffs and the infimum of the firm’s subgame perfect equilibrium payoffs, and then calculate the extreme payoffs for a particular case of the discount rates. Our conclusions are presented in Section 5.

### 2 Description of the model

Since the wage bargaining models that include the strike option cannot explain properly the wage negotiation processes if the legal interdiction on making strikes exists, we investigate the holdout threats of the union. More precisely, we introduce a modification of the bargaining model of Fernandez and Glazer (1991) and Ozkardas and Rusinowska (2014). We assume that the union cannot strike for threatening the firm, but it can decide to go-slow in a disagreement period.

As in the original model of Fernandez and Glazer (1991) and the generalized wage bargaining model investigated in Ozkardas and Rusinowska (2014), the union and the firm make alternating offers during the negotiations. There is an existing wage contract which has come up for renegotiation. We suppose that all workers are unionized and they have equal skills. We assume that the risk neutrality of both the firm and the union is relinquished, and hence the varying discount rates are introduced.

Inspired by the works of Rusinowska (2002a) and De Marco and Morgan (2008, 2011), we introduce in the model different attitudes of the union. Rusinowska (2002a) analyzes the bargaining model under an assumption of players’ attitudes towards their opponents’ payments. She determines the type of a player as jealous or friendly to examine the effects over his/her opponent’s payoff while his/her own payoff is constant. De Marco and Morgan (2008, 2011) introduce and study the concepts of the (strong) friendliness equilibrium and the slightly altruistic (correlated) equilibrium.

In our wage bargaining model we assume that the union and the firm divide the added value normalized to 1. Under the existing wage contract the firm makes a wage payment of $w_0$ on a daily basis where $w_0 \in [0, 1]$. By the new contract $W \in [0, 1]$ the union and the firm will get $W$ and $1 - W$, respectively. We assume that the attitude of the union towards the firm can be either *hostile* or *altruistic*. The type of the union is a common knowledge. If the union is hostile, then it makes go-slow threats in every disagreement period. Under the go-slow decision, the payoff of the union is the existing wage $w_0$ and the payoff of the firm is the discounted added value according to the rate of go-slow minus wage spending, i.e., $\lambda - w_0$, where $\lambda \in [w_0, 1]$ is the given rate of go-slow. On the other hand, if the union
is altruistic, then it does not make any threat to the firm in disagreement periods, i.e., the payoffs of the union and the firm are \( w_0 \) and \( 1 - w_0 \), respectively\(^1\). In other words, if an agreement is not reached, regardless of the union’s attitude, the union gets \( w_0 \) (i.e., the existing wage), but the firms bear the go-slow decision of the union with a decrease of its payoff from \( 1 - w_0 \) to \( (\lambda - w_0) \) where \( \lambda \in [w_0, 1] \). If the go-slow rate \( \lambda \) of the union is close to the minimum level \( w_0 \), then the union’s go-slow threat has the maximum effect on the firm’s payoff. Inversely, if \( \lambda = 1 \), then there is no threat of the union over the firm.

Players bargain sequentially over discrete time and a potentially infinite horizon. They make new wage offers alternately in which the other party is free to accept or to reject. After a rejection of an offer, the union decides whether to go-slow or not according to its attitude. More precisely, the bargaining procedure is as follows. In period 0, the union makes the first offer of \( W \) and for every \( t \) the union makes an offer \( W \), then the agreement is not reached, regardless of the union’s attitude, the union gets \( w_0 \) and for every \( t \) the union makes an offer \( W \), then the union’s go-slow threat has the maximum effect on the firm’s payoff. Inversely, if \( \lambda = 1 \), then there is no threat of the union over the firm.

We assume that the preferences of the union and the firm are described by sequences of discount factors varying in time. \((\delta_{u,t})_{t \in \mathbb{N}}\) is the discount factor of the union in period \( t \in \mathbb{N} \) and \((\delta_{f,t})_{t \in \mathbb{N}}\) is the discount factor of the firm in period \( t \in \mathbb{N} \) where \( \delta_{i,0} = 1 \), \( 0 < \delta_{i,t} < 1 \) for \( t \geq 1 \) and \( i = u, f \).

The result of the wage bargaining is either a pair \((W, T)\) where \( W \) is the wage contract agreed upon and \( T \in \mathbb{N} \) is the number of proposals rejected in the bargaining, or a disagreement denoted by \((0, \infty)\) where the parties never reach an agreement.

We use the following notations for each \( t \in \mathbb{N} \):

\[
\delta_u(t) := \prod_{k=0}^{t} \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^{t} \delta_{f,k} \quad \text{and} \quad (1)
\]

for \( 0 < t' \leq t \), \( \delta_u(t', t) := \frac{\delta_u(t)}{\delta_u(t' - 1)} = \prod_{k=t'}^{t} \delta_{u,k}, \quad \delta_f(t', t) := \frac{\delta_f(t)}{\delta_f(t' - 1)} = \prod_{k=t'}^{t} \delta_{f,k} \quad \text{(2)}\)

and for every \( t \in \mathbb{N}_+ \)

\[
\Delta_u(t) := \frac{\sum_{k=t}^{\infty} \delta_u(t, k)}{1 + \sum_{k=t}^{\infty} \delta_u(t, k)}, \quad \Delta_f(t) := \frac{\sum_{k=t}^{\infty} \delta_f(t, k)}{1 + \sum_{k=t}^{\infty} \delta_f(t, k)} \quad \text{(3)}
\]

\[
\tilde{\Delta}(t) := 1 - \Delta_f(2t + 1) + \sum_{m=t}^{\infty} (1 - \Delta_f(2m + 3)) \prod_{j=t}^{m} \Delta_u(2j + 2) \Delta_f(2j + 1) \quad \text{(4)}
\]

Moreover, we introduce the following definition:

**Definition 1** Let \((s_u, s_f)\) be the following family of strategies:

\(^1\) Note that for \( \lambda = 1 \) we recover the case of the altruistic union.
- **Strategy of the union** $s_u$: in period $2t$ ($t \in \mathbb{N}$) propose $W_{2t}^u$; in period $2t + 1$ accept an offer $y$ if and only if $y \geq Z_{2t+1}^u$;

- **Strategy of the firm** $s_f$: in period $2t$ propose $Z_{2t+1}^u$; in period $2t + 1$ accept an offer $x$ if and only if $x \leq W_{2t}^u$.

The union’s attitude specifies additionally its go-slow decision.

The utility of the result $(W, T)$ for the union is equal to

$$U(W, T) = \sum_{t=0}^{\infty} \delta_u(t) u_t$$

where $u_t = W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$u_t = w_0$ if there is no agreement in period $t \in \mathbb{N}$ regardless of the union’s attitude.

The utility of the result $(W, T)$ for the firm is equal to

$$V(W, T) = \sum_{t=0}^{\infty} \delta_f(t) v_t$$

where $v_t = 1 - W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$v_t = \lambda - w_0$ if the union is hostile,

$v_t = 1 - w_0$ if the union is altruistic.

The utility of the disagreement is equal to

$$U(0, \infty) = V(0, \infty) = 0$$

We assume that the series that define $U(W, T)$ and $V(W, T)$ are convergent.

### 3 Subgame perfect equilibria under different attitudes of the union

In this subsection, we analyze the SPE of the wage bargaining depending on the attitude of the union. First, consider the case of the hostile union. Let $W_{2t}^H$ and $Z_{2t+1}^H$ denote the SPE offers when the union is hostile.

**Theorem 1** Consider the generalized alternating offer model of wage bargaining with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t \in \mathbb{N}},$ where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u,f$. Assume that the union is hostile. Then there is the unique SPE of the form $(s_u, s_f)$ introduced in Definition 1, in which the offers of the parties are given by

$$W_{2t}^H = w_0 + (1 - \lambda) \tilde{\Delta}(t)$$

and for each $t \in \mathbb{N}$

$$Z_{2t+1}^H = w_0 + (1 - \lambda) \Delta_u (2t + 2) \tilde{\Delta}(t + 1)$$
Proof: One can show that \((s_u, s_f)\) is a SPE of this game if and only if the offers satisfy the following infinite system of equations, for each \(t \in \mathbb{N}\)

\[
(1 - W^{2t}) + (1 - W^{2t}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k) = (\lambda - w_0) + (1 - Z^{2t+1}) \sum_{k=2t+1}^{\infty} \delta_f(2t+1, k)
\]

and

\[
Z^{2t+1} + Z^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k) = w_0 + W^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u(2t+2, k)
\]

which can be equivalently written by

\[
W^{2t} - Z^{2t+1} \Delta_f (2t+1) = (1 - \lambda + w_0) (1 - \Delta_f (2t+1))
\]

\[
Z^{2t+1} - W^{2t+2} \Delta_u (2t+2) = w_0 (1 - \Delta_u (2t+2))
\]

The infinite system of (12) and (13) is a regular triangular system \(AX = Y\) with \(A = [a_{ij}]_{i,j \in \mathbb{N}^+}, X = [(x_i)_{i \in \mathbb{N}^+}]^T, Y = [(y_i)_{i \in \mathbb{N}^+}]^T\), where for each \(t, j \geq 1, a_{t,t} = 1, a_{t,j} = 0\), for \(j < t\) or \(j > t+1\) and for each \(t \in \mathbb{N}\)

\[
a_{2t+1,2t+2} = -\Delta_f (2t+1), \quad a_{2t+2,2t+3} = -\Delta_u (2t+2)
\]

Moreover, we have

\[
x_{2t+1} = W^{2t}, \quad x_{2t+2} = Z^{2t+1}
\]

\[
y_{2t+1} = (1 - \lambda + w_0) (1 - \Delta_f (2t+1)), \quad y_{2t+2} = w_0 (1 - \Delta_u (2t+2))
\]

We know that any regular triangular matrix \(A\) possesses the (unique) inverse matrix \(B\), i.e., there exists \(B\) such that \(BA = I\), where \(I\) is the infinite identity matrix. The matrix \(B = [b_{ij}]_{i,j \in \mathbb{N}^+}\) is also regular triangular, and its elements are the following:

\[
b_{t,t} = 1, \quad b_{t,j} = 0 \text{ for each } t, j \geq 1 \text{ such that } j < t \quad (14)
\]

for each \(t \in \mathbb{N}\)

\[
b_{2t+1,2t+2} = \Delta_f (2t+1), \quad b_{2t+2,2t+3} = \Delta_u (2t+2) \quad (15)
\]

and for each \(t, m \in \mathbb{N}\) and \(m > t\)

\[
b_{2t+2,2m+2} = \prod_{j=t}^{m-1} \Delta_u (2j + 2) \Delta_f (2j + 3) \quad (16)
\]

\[
b_{2t+2,2m+3} = \prod_{j=t}^{m-1} \Delta_u (2j + 2) \Delta_f (2j + 3) \Delta_u (2m + 2) \quad (17)
\]

\[
b_{2t+1,2m+1} = \prod_{j=t}^{m-1} \Delta_u (2j + 2) \Delta_f (2j + 1) \quad (18)
\]
\[ b_{2t+1,2m+2} = \prod_{j=t}^{m-1} \Delta_u (2j + 2) \Delta_f (2j + 1) \Delta_f (2m + 1) \]  

(19)

Hence, \( AX = Y \) is equal to

\[
\begin{bmatrix}
1 - \Delta_f (1) & 0 & 0 & \cdots & \cdots \\
0 & 1 & -\Delta_u (2) & 0 & \cdots \\
0 & 0 & 1 & -\Delta_f (3) & \cdots \\
\vdots & \vdots & & \ddots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
W^0 \\
Z^1 \\
W^2 \\
Z^3 \\
\vdots
\end{bmatrix} =
\begin{bmatrix}
(1 - \lambda + w_0) (1 - \Delta_f (1)) \\
w_0 (1 - \Delta_u (2)) \\
(1 - \lambda + w_0) (1 - \Delta_f (3)) \\
w_0 (1 - \Delta_u (4)) \\
\vdots
\end{bmatrix}
\]

By applying \( X = BY \), where

\[
B =
\begin{bmatrix}
1 & \Delta_f (1) & \Delta_f (1) & \Delta_u (2) & \cdots & \cdots \\
0 & 1 & \Delta_u (2) & \Delta_f (3) & \cdots & \cdots \\
0 & 0 & 1 & \Delta_f (3) & \cdots & \cdots \\
\vdots & \vdots & & \ddots & \ddots & \ddots \\
\end{bmatrix}
\]

we have

\[
\overline{W}^t_H = (1 - \lambda + w_0) (1 - \Delta_f (2t + 1)) + w_0 \Delta_f (2t + 1) (1 - \Delta_u (2t + 2)) + (1 - \lambda + w_0) \Delta_f (2t + 1) \Delta_u (2t + 2) (1 - \Delta_f (2t + 3)) + \cdots
\]

and therefore \( \overline{W}^t_H \) and \( \overline{Z}^{2t+1}_H \) are given by (8) and (9), respectively.

\[\Box\]

**Example 1** Let us apply this result to the wage bargaining with constant discount rates, i.e., we have \( \delta_{f,t} = \delta_f \) and \( \delta_{u,t} = \delta_u \) for each \( t \in \mathbb{N} \), and therefore for each \( j \in \mathbb{N} \), \( \Delta_f (2t + 1) = \delta_f \) and \( \Delta_u (2t + 2) = \delta_u \). By inserting this into (8), we get

\[
\overline{W}^t_H = w_0 + \frac{(1 - \delta_f) (1 - \lambda)}{1 - \delta_f \delta_u}
\]

If additionally we assume that \( \delta_f = \delta_u = \delta \), then \( \overline{W}^t_H = w_0 + \frac{1 - \lambda}{1 + \delta} \).

**Example 2** Consider the model in which the union and the firm have the following sequences of discount factors varying in time: for each \( t \in \mathbb{N} \)

\[
\delta_{f,t} = \delta_{f,t+1} = \frac{1}{2}, \quad \delta_{f,t+2} = \delta_{u,t+2} = \frac{1}{3}
\]

By virtue of (8) the offer of the union in period \( 2t \) in the SPE is equal to

\[
\overline{W}^t_H = w_0 + \frac{2 (1 - \lambda)}{3}.
\]

If the union is supposed to be altruistic, i.e., it is never on go slow in disagreement periods, then we obtain the unique SPE that leads to the minimum wage contract \( w_0 \). Let us denote the SPE offers when the union is altruistic as \( \overline{W}^t_A \) and \( \overline{Z}^{2t+1}_A \). We have the following fact:
Fact 1 Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors \((\delta_{i,t})_{t\in\mathbb{N}}\), where \(\delta_{i,0} = 1, 0 < \delta_{i,t} < 1\) for \(t \geq 1, i = u, f\). Assume that the attitude of the union is altruistic. Then there is the unique SPE of the form \((s_u, s_f)\), where
\[
W^2_{A} = Z^2_{A} = w_0
\]
for each \(t \in \mathbb{N}\).

Proof: Suppose that the union is altruistic. One can show that if \((s_u, s_f)\) is a SPE, then it must hold for each \(t \in \mathbb{N}\)
\[
(1 - W^{2t}) + \left(1 - W^{2t}\right) \sum_{k=2t+1}^{\infty} \delta_f (2t + 1, k) = (1 - w_0) + \left(1 - Z^{2t+1}\right) \sum_{k=2t+1}^{\infty} \delta_f (2t + 1, k)
\]
and
\[
Z^{2t+1} + Z^{2t+1} \sum_{k=2t+2}^{\infty} \delta_u (2t + 2, k) = w_0 + W^{2t+2} \sum_{k=2t+2}^{\infty} \delta_u (2t + 2, k)
\]
and hence we get
\[
W^{2t} - Z^{2t+1} \Delta_f (2t + 1) = w_0 (1 - \Delta_f (2t + 1))
\]
\[
Z^{2t+1} - W^{2t+2} \Delta_u (2t + 2) = w_0 (1 - \Delta_u (2t + 2))
\]
Obviously, \(W^{2t} = Z^{2t+1} = w_0\) for each \(t \in \mathbb{N}\) is a solution of this system of equations, and we know from the infinite matrices theory that this system has only one solution. One can also show that \((s_u, s_f)\) with \(W^2_{A} = Z^2_{A} = w_0\) for \(t \in \mathbb{N}\) is a SPE. ■

Note that we have the following:
\[
W^{2t}_H = W^{2t}_{A} + (1 - \lambda) \hat{\Delta} (t)
\]
where \((1 - \lambda) \hat{\Delta} (t) \geq 0\), and therefore \(W^{2t}_H \geq W^{2t}_{A}\).

4 On the subgame perfect equilibrium payoffs

By applying the Shaked and Sutton (1984) method to the wage bargaining model of Fernandez and Glazer (1991), Houba and Wen (2008) derive the extreme equilibrium payoffs. We generalize their method and apply it to the model with the sequences of discount rates varying in time, where the strikes are not allowed and the sole threat of the union is to be on go-slow during disagreement periods.

Let \(M^{2t}_u\) be the supremum of the union’s SPE payoffs in any \(2t\) period and \(m^{2t+1}_f\) be the infimum of the firm’s SPE payoffs in any \(2t + 1\) periods, \(t \in \mathbb{N}\). The following propositions present the necessary conditions on \(m^{2t+1}_f\) and \(M^{2t}_u\), for \(t \in \mathbb{N}\), respectively:
Proposition 1 We have for all \((\delta_{u,t})_{t \in \mathbb{N}}, (\delta_{f,t})_{t \in \mathbb{N}}\), \(0 \leq w_0 \leq \lambda \leq 1\) and \(t \in \mathbb{N}\)

\[
m_f^{2t+1} \geq \begin{cases} 
1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2) & \text{if } (23) \\
(\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) & \text{if } (24) 
\end{cases}
\]

\[
\Delta_u (2t + 2) \leq \Delta_f (2t + 2) \text{ or }
\Delta_u (2t + 2) > \Delta_f (2t + 2) \text{ and }
(1 - \Delta_f (2t + 2)) (1 - \lambda) > (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2)) \quad (23)
\]

\[
\Delta_u (2t + 2) > \Delta_f (2t + 2) \text{ and }
(1 - \Delta_f (2t + 2)) (1 - \lambda) \leq (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2)) \quad (24)
\]

Proof: We consider an arbitrary odd period \(2t + 1, t \in \mathbb{N}\). If the union holds out after rejecting the firm’s offer, the union will get at most \(w_0 (1 - \Delta_u (2t + 2)) + M_u^{2t+2} \Delta_u (2t + 2)\). Hence the firm could get at least \(1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2)\) from making an irresistible offer and at least \((1 - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) = 1 - w_0 (1 - \Delta_f (2t + 2)) - M_u^{2t+2} \Delta_f (2t + 2)\) from making an unacceptable offer. The firm will make either the least irresistible offer or an unacceptable offer, depending on these two payoffs.

If the union is on go slow after rejecting the firm’s offer, the union will get at most \(w_0 (1 - \Delta_u (2t + 2)) + M_u^{2t+2} \Delta_u (2t + 2)\). Hence the firm will get at least \(1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2)\) from making an irresistible offer or \((\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2)\) from making an unacceptable offer.

Consequently, we get the following: for all \((\delta_{u,t})_{t \in \mathbb{N}}, (\delta_{f,t})_{t \in \mathbb{N}}\), \(0 \leq w_0 \leq \lambda \leq 1\) and \(t \in \mathbb{N}\)

\[
m_f^{2t+1} \geq \min \begin{cases} 
\max \left\{ 1 - w_0 (1 - \Delta_f (2t + 2)) - M_u^{2t+2} \Delta_f (2t + 2) \right. & (a) \\
1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2) \right. & (b) \\
\left. (\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) \right. & (b) \\
\left. (\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) \right. & (c) \\
\end{cases}
\]

Consider now an arbitrary \(t \in \mathbb{N}\). If \(\lambda < 1\), then we have \(1 - w_0 (1 - \Delta_f (2t + 2)) > (\lambda - w_0) (1 - \Delta_f (2t + 2)) + M_u^{2t+2} \Delta_f (2t + 2)\). Hence we get \((25a) > (25c)\).

Assume that \(\Delta_u (2t + 2) \leq \Delta_f (2t + 2)\). Then we have

\[
1 - w_0 (1 - \Delta_f (2t + 2)) - M_u^{2t+2} \Delta_f (2t + 2) \leq 1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2),
\]

therefore we get \((25a) \leq (25b)\). Moreover, we have

\[
(\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) \leq 1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2),
\]

and hence \((25c) \leq (25b)\).

Assume that \(\Delta_f (2t + 2) < \Delta_u (2t + 2)\). Then we have the following:

\[
1 - w_0 (1 - \Delta_f (2t + 2)) - M_u^{2t+2} \Delta_f (2t + 2) > 1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2),
\]

we get \((25a) \geq (25b)\) and \((\lambda - w_0) (1 - \Delta_f (2t + 2)) + (1 - M_u^{2t+2}) \Delta_f (2t + 2) < 1 - w_0 (1 - \Delta_u (2t + 2)) - M_u^{2t+2} \Delta_u (2t + 2)\) if and only if \((1 - \Delta_f (2t + 2)) (1 - \lambda) > (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2))\).

Hence, we get \((25b) > (25c)\), otherwise we have \((25c) \geq (25b)\).
Proposition 2 We have for all \((\delta_{u,t})_{t \in \mathbb{N}}, (\delta_{f,t})_{t \in \mathbb{N}}\), 0 \leq w_0 \leq \lambda \leq 1 and \(t \in \mathbb{N}\)

\[
M_{u}^{2t} \leq \begin{cases} 
   w_0 (1 - \Delta_u (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_u (2t + 1) & \text{if } (27) \\
   1 - (\lambda - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1) & \text{if } (28) 
\end{cases} \tag{26}
\]

\[
\Delta_f (2t + 1) < \Delta_u (2t + 1) \quad \text{and} \quad \left( w_0 + m_{f}^{2t+1} \right) \left( \Delta_f (2t + 1) - \Delta_u (2t + 1) \right) > 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1) \tag{27}
\]

\[
\Delta_f (2t + 1) \geq \Delta_u (2t + 1) \quad \text{or} \quad \Delta_f (2t + 1) < \Delta_u (2t + 1) \quad \text{and} \quad \left( w_0 + m_{f}^{2t+1} \right) \left( \Delta_f (2t + 1) - \Delta_u (2t + 1) \right) \leq 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1) \tag{28}
\]

**Proof:** We consider an arbitrary even period \(2t, t \in \mathbb{N}\). If the union holds out after its offer is rejected, the firm will get at least \((1 - w_0) (1 - \Delta_f (2t + 1)) + m_{f}^{2t+1} \Delta_f (2t + 1)\). Hence, the union’s SPE payoffs must be smaller or equal to \(w_0 (1 - \Delta_f (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_f (2t + 1)\) from making the least acceptable offer or \(w_0 (1 - \Delta_u (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_u (2t + 1)\) from making an unacceptable offer.

If the union is on go slow after its offer is rejected, the firm will get at least \((\lambda - w_0) (1 - \Delta_f (2t + 1)) + m_{f}^{2t+1} \Delta_f (2t + 1)\) by rejecting the union’s offer. Hence, the union’s SPE payoffs must be smaller or equal to \(1 - \lambda (1 - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1)\) from making the least acceptable offer, or \(w_0 (1 - \Delta_u (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_u (2t + 1)\) from making an unacceptable offer.

Consequently, we have for all \((\delta_{u,t})_{t \in \mathbb{N}}, (\delta_{f,t})_{t \in \mathbb{N}}\), 0 \leq w_0 \leq \lambda \leq 1 and \(t \in \mathbb{N}\)

\[
M_{u}^{2t} \leq \max \begin{cases} 
   \max \left\{ w_0 (1 - \Delta_f (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_f (2t + 1) \quad (a) \right. \\
   w_0 (1 - \Delta_u (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_u (2t + 1) \quad (b) \\
   \left. 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1) \quad (c) \right\} 
\end{cases} \tag{29}
\]

For every \(t \in \mathbb{N}\) and \(\lambda < 1, 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1)\) > \(w_0 (1 - \Delta_f (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_f (2t + 1)\), and hence we get \((29c) > (29a)\).

Assume that \(\Delta_f (2t + 1) \geq \Delta_u (2t + 1)\). Then \((29a) \geq (29b)\), and since \((29c) > (29a)\), we have \(M_{u}^{2t} \leq 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1)\).

If \(\Delta_f (2t + 1) < \Delta_u (2t + 1)\), then \((29a) < (29b)\) and \(w_0 (1 - \Delta_u (2t + 1)) + (1 - m_{f}^{2t+1}) \Delta_u (2t + 1) > 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1)) - m_{f}^{2t+1} \Delta_f (2t + 1)\) if and only if \((w_0 + m_{f}^{2t+1}) \left( \Delta_f (2t + 1) - \Delta_u (2t + 1) \right) > 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1)\). Hence, \((29b) > (29c)\), otherwise we have \((29c) > (29b)\). 

We can use Propositions 1 and 2 to determine the extreme equilibrium payoffs for particular cases of the discount rates varying in time. Fact 2 shows one of the cases, when in every period the generalized discount factor of the firm is not smaller than the generalized discount factor of the union.
Fact 2 Let $0 \leq w_0 \leq \lambda \leq 1$, and let $(\delta_{u,t})_{t \in \mathbb{N}}$ and $(\delta_{f,t})_{t \in \mathbb{N}}$ be the sequences of discount rates such that $\Delta_f(t) \geq \Delta_u(t)$ for every $t \in \mathbb{N}$. Then we have for every $t \in \mathbb{N}$

\[ M_u^{2t} = w_0 + (1 - \lambda) \tilde{\Delta}(t) \]  
\[ m_f^{2t+1} = (1 - w_0) - (1 - \lambda) \Delta_u(2t + 2) \tilde{\Delta}(t + 1) \]

where $\tilde{\Delta}(t)$ is given in (4).

Proof: Let $\Delta_f(2t + 2) \geq \Delta_u(2t + 2)$ and $\Delta_f(2t + 1) \geq \Delta_u(2t + 1)$ for every $t \in \mathbb{N}$. From Propositions 1 and 2 we have for every $t \in \mathbb{N}$:

\[ m_f^{2t+1} + M_u^{2t+2} \Delta_u(2t + 2) = 1 - w_0 (1 - \Delta_u(2t + 2)) \]

and

\[ M_u^{2t} + m_f^{2t+1} \Delta_f(2t + 1) = 1 - (\lambda - w_0) (1 - \Delta_f(2t + 1)) \]

which is a regular triangular system and possesses a unique solution. This solution is given by (30) and (31).

\[ \blacksquare \]

Remark 1 Note that $M_u^{2t}$ and $m_f^{2t+1}$ defined in (30) and (31) are equal to the SPE payoffs obtained by the union and the firm under the “always going slow” case. More precisely, this SPE strategy profile is given by the following strategies:

- In period 2t the union proposes $w_0 + (1 - \lambda) \tilde{\Delta}(t)$, in period 2t + 1 it accepts an offer if and only if $y \geq w_0 + (1 - \lambda) \Delta_u(2t + 2) \tilde{\Delta}(t + 1)$, it is always on go-slow if there is a disagreement.

- In period 2t + 1 the firm proposes $w_0 + (1 - \lambda) \Delta_u(2t + 2) \tilde{\Delta}(t + 1)$, in period 2t it accepts $x$ if and only if $x \leq w_0 + (1 - \lambda) \tilde{\Delta}(t)$.

This $M_u^{2t} = \tilde{W}_H^{2t} = w_0 + (1 - \lambda) \tilde{\Delta}(t)$ can be interpreted as follows: the union gets the existing wage plus the gain from being on go-slow which depends on the go-slow rate $\lambda$ and $\tilde{\Delta}(t)$ determined by the discount factors of both parties.

Remark 2 When the go-slow rate $\lambda = 1$, then $M_u^{2t} = w_0$ which gives the minimum wage contract. This SPE is acquired by the never-go-slow strategies of the union. On the other hand, when the go-slow rate $\lambda = w_0$, then we have $M_u^{2t} = w_0 + (1 - w_0) \tilde{\Delta}(t)$ which is equal to the SPE payoff obtained by the generalized alternating strike strategies shown in Ozkardas and Rusinowska (2014, Forthcoming).

Remark 3 Note that for some cases of the discount rates the solutions on $M_u^{2t}$ and $m_f^{2t+1}$ do not satisfy the necessary conditions. We give some examples below:

- Let $\Delta_f(2t + 2) \geq \Delta_u(2t + 2)$, $\Delta_f(2t + 1) < \Delta_u(2t + 1)$ and

\[ (w_0 + m_f^{2t+1}) (\Delta_f(2t + 1) - \Delta_u(2t + 1)) > 1 - \lambda (1 - \Delta_f(2t + 1)) - \Delta_u(2t + 1) \]

for every $t \in \mathbb{N}$. We have the infinite system for $t \in \mathbb{N}$:

\[ m_f^{2t+1} + M_u^{2t+2} \Delta_u(2t + 2) = 1 - w_0 (1 - \Delta_u(2t + 2)) \]

and

\[ M_u^{2t} + m_f^{2t+1} \Delta_u(2t + 1) = w_0 (1 - \Delta_u(2t + 1)) + \Delta_u(2t + 1) \]

which is a regular triangular system and has a unique solution of $M_u^{2t} = w_0$. But this unique solution does not satisfy the necessary condition.
- Consider the case where $\Delta_f (2t + 2) < \Delta_u (2t + 2)\), $\Delta_f (2t + 1) < \Delta_u (2t + 1)$, $(w_0 + m_f^{2t+1}) (\Delta_f (2t + 1) - \Delta_u (2t + 1)) > 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1)$ and $(1 - \Delta_f (2t + 2)) (1 - \lambda) > (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2))$ for every $t \in \mathbb{N}$.

We have the infinite system for $t \in \mathbb{N}$:

$$m_f^{2t+1} + M_u^{2t+2} \Delta_u (2t + 2) = 1 - w_0 (1 - \Delta_u (2t + 2))$$

and $M_u^{2t} + m_f^{2t+1} \Delta_u (2t + 1) = w_0 (1 - \Delta_u (2t + 1)) + \Delta_u (2t + 1)$ which has a unique solution $M_u^{2t} = w_0$ and $m_f^{2t+1} = 1 - w_0$, but this solution does not satisfy one of the necessary conditions.

- Consider the case where $\Delta_f (2t + 2) < \Delta_u (2t + 2)\), $\Delta_f (2t + 1) < \Delta_u (2t + 1)$, $(w_0 + m_f^{2t+1}) (\Delta_f (2t + 1) - \Delta_u (2t + 1)) > 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1)$ and $(1 - \Delta_f (2t + 2)) (1 - \lambda) > (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2))$ for every $t \in \mathbb{N}$.

We obtain the following infinite system of equations for $t \in \mathbb{N}$:

$$m_f^{2t+1} + M_u^{2t+2} \Delta_f (2t + 2) = (\lambda - w_0) (1 - \Delta_f (2t + 2)) + \Delta_f (2t + 2)$$

and $M_u^{2t} + m_f^{2t+1} \Delta_f (2t + 1) = w_0 (1 - \Delta_u (2t + 1)) + \Delta_u (2t + 1)$, and hence

$$M_u^{2t} = w_0 + (1 - \lambda) \left( \Delta_u (2t + 1) - \sum_{m=t}^{\infty} (1 - \Delta_u (2m + 3)) \prod_{j=t}^{m} \Delta_u (2j + 1) \Delta_f (2j + 2) \right)$$

but it does not satisfy one of the necessary conditions.

- Let $\Delta_f (2t + 2) < \Delta_u (2t + 2)\), $\Delta_f (2t + 1) \geq \Delta_u (2t + 1)$ and $(1 - \lambda) (1 - \Delta_f (2t + 2)) \leq (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2))$ for every $t \in \mathbb{N}$.

We have the infinite system for $t \in \mathbb{N}$:

$$m_f^{2t+1} + M_u^{2t+2} \Delta_f (2t + 2) = (\lambda - w_0) (1 - \Delta_f (2t + 2)) + \Delta_f (2t + 2)$$

and $M_u^{2t} + m_f^{2t+1} \Delta_f (2t + 1) = 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1))$ and therefore $M_u^{2t} = 1 - \lambda + w_0$, but it does not satisfy the necessary condition.

- Let $\Delta_f (2t + 2) < \Delta_u (2t + 2)\), $\Delta_f (2t + 1) < \Delta_u (2t + 1)$, $(w_0 + m_f^{2t+1}) (\Delta_f (2t + 1) - \Delta_u (2t + 1)) \leq 1 - \lambda (1 - \Delta_f (2t + 1)) - \Delta_u (2t + 1)$ and $(1 - \Delta_f (2t + 2)) (1 - \lambda) \leq (M_u^{2t+2} - w_0) (\Delta_u (2t + 2) - \Delta_f (2t + 2))$ for every $t \in \mathbb{N}$.

We have the infinite system for $t \in \mathbb{N}$:

$$m_f^{2t+1} + M_u^{2t+2} \Delta_f (2t + 2) = (\lambda - w_0) (1 - \Delta_f (2t + 2)) + \Delta_f (2t + 2)$$

and $M_u^{2t} + m_f^{2t+1} \Delta_f (2t + 1) = 1 - (\lambda - w_0) (1 - \Delta_f (2t + 1))$ and hence $M_u^{2t} = 1 - \lambda + w_0$, but it does not satisfy one of the necessary conditions.

5 Conclusion

We investigated the SPE for the union-firm wage bargaining model with discount rates varying in time when the strikes are not allowed and the sole threat of the union is to decrease the output level by using the go-slow option. First, we modified the generalized bargaining model presented in Ozkardas and Rusinowska (2014) by introducing the go-slow action of the union and studied the SPE under different attitudes of the union. Then we used an extended version of the analysis presented in Houba and Wen (2008) to deliver the necessary conditions for the extreme payoffs and we calculated the extreme payoffs of the parties for a particular case of the discount rates when strikes are prohibited.

In the wage bargaining literature, the union’s threats different from strikes are usually not taken into consideration. An important feature of our model lies on introducing such threats in the union-firm bargaining. In order to model real life situations in a more accurate way, we also consider varying discount rates.
It is worthy of note that although strikes are not allowed, the union can achieve a wage increase during the wage bargaining. We show that threatening the firm with the go-slow decision in every disagreement periods gives a significant wage increase to the union. This result is also supported by the supremum of the union’s subgame perfect equilibrium payoff for some particular cases of the sequences of discount rates. More precisely, the “always going slow strategy” leads in some cases to the maximum wage that the union can achieve. In other words, while the union always gets the existing wage, it prefers to threat and punish the firm by being on go slow in every period when there is no agreement. In this case, the firm’s added value decreases with the go-slow rate. The firm’s loss during the go-slow is equal to the actualized value of the union’s wage increase. Furthermore, the subgame perfect equilibrium payoffs for some cases are the same as our results on the wage bargaining with strike decisions of the union (see e.g. Ozkardas and Rusinowska (2014)). Depending on the go-slow rate $\lambda$, the supremum of the union’s subgame perfect equilibrium payoffs can be supported by the generalized alternating strike strategy or the never strike strategy of the union defined in Ozkardas and Rusinowska (Forthcoming).
Bibliography


