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Longevity, age-structure, and optimal schooling

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Abstract

The mechanism stating that longer life implies larger investment in human capital, is premised on the view that individual decision-making

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governs the relationship between longevity and education. This relationship is revisited here from the perspective of optimal period school life expectancy, obtained from the utility maximization of the whole population characterized by its age structure and its age-specific fertility and mortality. Realistic life tables such as model life tables are mandatory, because the age distribution of mortality matters, notably at infant and juvenile ages. Optimal period school life expectancy varies with life expectancy and mortality. The application to French historical data from 1806 to nowadays shows that the population age structure has indeed modified the relationship between longevity and optimal schooling.

keywords: longevity, schooling, school life expectancy, age structure.

JEL C02, C65, C80, D90, H50, I26, I28, J10, J24

1 Introduction

Ben-Porath (1967) suggested a “mechanism” according to which longer life spans imply larger investment in human capital. This mechanism is central to several new growth theories in the field of unified growth theory. For example, Boucekkine et al. (2002, 2004) argued that early increases in life expectancy, mediated by the Ben-Porath mechanism, are at the origin of modern growth: longer lives would induce longer schooling and higher rates of human capital accumulation, giving rise to a new growth regime which allows the escape from the Malthusian trap.

French historical data are accurate enough to give us an insight into the relationship between longevity and education, at the moment of both the schooling and the mortality transitions. From the accurate estimates of female life expectancy at birth by French *départements* from 1806-10 to 1901-05 of Bonneuil (1997a) and from French schooling rates in 1837, 1850, 1867, and 1876 (Bonneuil, 2014), we regressed the time series of the growth rate of female life expectancy (tested to be stationary) on the growth rate of the female schooling rate (also tested to be stationary) in 1837-50, 1850-67, and 1867-76. On 82 French *départements*, these regressions yield 11 positive correlations (for *départements* located erratically on the territory), 6 negative, and 65 non significant. Then, in spite of the scarcity in time, the absence of clear correlation raises doubts that the relationship between longevity and schooling would be unequivocal.

In a quantity-quality trade-off model *à la* Becker (1991), Hazan and Zoabi (2006) show that the Ben-Porath mechanism may not always work because an increase in longevity affects not only the return to schooling (quality of children), but also the return to quantity or the optimal total number of children. The latter effect mitigates the Ben Porath mechanism and can in principle negate it. Under homothetic preferences, Hazan and Zoabi (2006) find that when fertility is endogenous, an exogenous increase in children's longevity has no effect on schooling. Hazan (2010) questions the Ben-Porath mechanism explicitly. He starts from the cohort model of Boucekkine et al. (2002): all individuals of all cohorts are identical and make decisions about their lifetime consumption, schooling, and work time. Attending school for a longer time has a cost in terms of foregone labor income but this schooling time also induces a gain because longer schooling means higher wages in the labor market. In the case of a perfectly rectangular survival function, increased longer longevity leads to longer schooling only if the total expected number of hours spent at work during one's lifetime also rises. Hazan (2010) tested this property on US data for consecutive 10-year cohorts born between 1840 and 1970, to find that the total number of hours worked did not increase, and to conclude that the Ben-Porath mechanism was not relevant for the US during this period. Cervellati and Sunde (2009), however, argued that the connection between the total number of hours spent at work during an individual's lifetime and the Ben-Porath mechanism does not hold for non-perfectly rectangular survival functions.

All the studies referred to so far are based on individual decision-making where agents decide about their optimal consumption stream and their lifetime accumulation of human capital over lifetime for a given ad-hoc survival function. Hazan (2010) and Cervellati and Sunde (2009) used a continuous time (homogeneous) cohort model, incorporating a schooling time decision model like in Boucekkine et al. (2002). Cervellati and Sunde (2013) illustrated their argument on a discrete-time version of their cohort model. Boucekkine et al. (2007) introduced within-cohort heterogeneity into this model, which adds formidable complications. However, decisions relative to education are not individual; on the contrary, at least in continental Europe, education is run mainly by the State. For example, control over education in France dates back at least to 1837, when the government started to invest substantial human and material resources in schooling. It withdrew university degrees from private education in 1880, made primary schooling free in 1881, and education became non-clerical and mandatory in 1882. The State finances schools and teachers, with individuals who send their children to state-run schools, which constitute the large majority of teaching establishments, having practically nothing to pay. State schools have been organized in this way until now in most European countries. Nineteenth century France offers an exemplary case, and Ben Porath's hypothesis that governments would respond to increasing longevity by lengthening schooling time can be tested in the context of a rapidly changing demography and deliberate State policies to increase schooling time for boys and girls.

We innovate by relying on the alternative criterion of optimal period schooling, which is the only one so far to involve the whole age structure, in contrast to individual follow-up. The State's expenditures depend on the ratio of schooled children to tax-payers, and this ratio is a function of the age structure. For an equivalent mortality level and population size, at each date, a population in a low fertility regime has comparatively more contributors and fewer schooled children than a population in a higher fertility regime.

A basic formulation of the optimal period schooling equivalent could be the following: given the age structure of the population, its current fertility and mortality levels, would a planner seeking to maximize social welfare (say with respect to the Benthamite criterion) lengthen schooling in response to rising life expectancy? In contrast to the individual-cohort perspective adopted in the related literature, the key component of the period schooling optimum problem is that the decisions have to be taken on the basis of the overall demographic structure. Therefore, the current age structure of the population determines schooling decisions, whereas it is ignored in the literature on individual schooling decisions. Put simply, the main reason supporting the argument that longer life implies longer education is that the proportion of people above the maximum school completion age increases as mortality decreases. But this holds true only if the proportion of people under that age does not increase faster. Following a cohort, as did Hazan (2010) or Cervellati and Sunde (2009, 2013), is equivalent to fixing fertility at a constant value over time. The conditions of the moment

however vary with fertility, so that if the proportion of people in school increases, compared to the proportion out of school, greater longevity may not be enough to offset higher fertility, leading to *a decrease* in schooling, at the optimum. The relationship between length of life and schooling will thus depend on the balance between the additional young from higher fertility and the additional old from improved survival.

We shall then work with the concept of the *fictitious cohort*,¹ where the forces of change affecting people of successive ages living at the same time are applied to a fictitious group of people born at a same time and who would experience these age-specific forces successively as they age. A fictitious cohort built *for each date* and its associated stable population for this date will yield the *conditions of the moment* at that date. The tool of fictitious cohort is common in mathematical demography (Bonneuil, 1997b). The associated concept to the fictitious cohort is “school life expectancy”, which is defined as “the total number of years of schooling (primary to tertiary) that a child can expect to receive, assuming that the probability of his or her being enrolled in school at any particular future

¹A fictitious cohort over the period, say, $[t, t + 1)$ consists of, say, 100 people, born during this period and experiencing over their lifetimes the mortality schedule prevailing in this period. Its attrition over age characterizes the mortality conditions of the moment $[t, t + 1)$. This is the basis for computing the widely used period life expectancy, which is the expected age at death of the fictitious cohort, and the total fertility rate, which is the no less widely used expected total number of children born to a woman of the fictitious cohort during her entire life if she were exposed to the fertility conditions of the moment $[t, t + 1)$.

age is equal to the current enrollment ratio at that age.”² The *period* school life expectancy is then the sum over all ages of the probabilities at a given date to remain in school.

The conditions of the moment consist in the expected age of the oldest schooled individual at time t , the consumption schedule by age at t , the leisure schedule by age at t , the fertility schedule by age at t , and the life table at t . To make decisions, social planners use their knowledge of these conditions of the moment, not their knowledge of the stocks of population or of the school experience of each cohort present at t . In demography, family policies are based on period fertility rates, whose sum is the total fertility rate. This rate is used to define population policies, no matter the total number of children born to each cohort of women at time t . Similarly, period life expectancy summarizes the mortality rates of the moment in a fictitious cohort, and insurance policies are decided on the basis of period life expectancy, not of cohort life expectancies. Social planners cannot modify past variables and are aware that, if the conditions at date t were maintained for long enough, the population would converge to the stable population associated with the fertility schedule and the life table at t (“ergodicity” theorem (Cohen, 1979)). The decisions of social planners are based not on the mortality, fertility, educational, consumption, and income histories of each cohort, but on current conditions, which are measured through age-specific mortality, fertility, and schooling rates and through consumption and age-specific

²<https://www.cia.gov/library/publications/the-world-factbook/fields/2205.html>

income prevailing at the current period.

We shall study the optimal schooling rule adopted by a social planner and see how this rule responds to increased longer longevity. For the sake of clarity, we shall assume that the planner fixes the income transfer rates from working to non-working ages (people at school and pensioners) in such a way that the budget is never in deficit. Allowing the planner to borrow (subject to a no-Ponzi game condition) complicates the technicalities, but handling the age structure is already onerous (leading to nonlinear functional integral equations). So, we impose zero deficit at any date. By doing so, we offer a symmetrical setting to the one adopted in the cohort-individual literature. In the latter, the individual (representing the fictitious cohort) is followed-up over his/her lifetime (inter-temporal optimization). We focus on the demand side, with emphasis on the population age structure, to identify the cases where this mechanism fails to be optimal, even when education supply is not at stake.

In Section 2, we derive optimal period schooling from conditions of the moment through a maximization program to be solved by the social planner. In Section 3, we show how the period optimal schooling is modified when life expectancy or fertility are varied. We shall specify the model with realistic model life tables, and, in Appendix, present analytical yet unrealistic cases (constant, linear, and Gompertz mortality). We shall situate empirical cases of French *départements* in the space defined by fertility, life expectancy, and schooling. We shall draw the trajectory of France as a whole in this space and discuss optimal

schooling with demographically realistic assumptions. Our main result is that the relationship between longevity and schooling is mediated by infant mortality and by fertility, so that Ben Porath's claim that longer life is positively associated with longer schooling holds true only in countries with low mortality and low fertility. The contrary holds true in countries before or at the onset of the demographic transition.

2 Period optimal schooling, through the fictitious cohort

Instead of tracking individuals' life-cycle decisions as in Ben Porath's framework, we consider an alternative setting where schooling decisions are taken by a government which optimizes social welfare at each date. Its program is:

$$\max_{c(t,\cdot), \ell(t,\cdot), A_S(t)} V(t) = \int_0^{\bar{A}} p(t, a) u(c(t, a), \ell(t, a)) da, \quad \forall t, \quad (1)$$

where $A_S(t)$ is the age of the oldest schooled individual, \bar{A} is the maximal life span, and where, at age a and time t , $p(t, a)$ represents the total number of people and $c(t, a)$ consumption per head; each individual of age a has one unit of time available, shared between $\ell(t, a)$ in leisure and $1 - \ell(t, a)$ in labor supply for active people or in time spent in school for schooled children, and u is the continuously differentiable strictly concave utility function, assumed additively separable in consumption and leisure.

The budget constraint of the government at date t is:

$$\int_0^{\bar{A}} p(t, a)c(t, a) da = \int_{A_S(t)}^{A_R} (1 - \ell(t, a))p(t, a)w(t, a) da, \quad (2)$$

where $w(t, a)$ denotes wages at age a and time t . Equation (2) equalizes total consumption of individuals at every age at date t with labor income earned by all active individuals at t . The age at retirement A_R is not a control variable, but taken here as constant for the sake of simplicity, because we focus on schooling.

Problem $\{1, 2\}$ is age-structured, in contrast to models of Ben Porath's mechanism based on individual follow-up. One may question the fact that the government's objective function is not inter-temporal. First, governments have short-term horizons often limited by the electoral cycle. Second, by maximizing at each date, we isolate the pure effect of the age structure, whereas individual follow-up isolates pure life cycle effects and misses collective disparity. Third, planners wish to know what to decide from the conditions of the moment: they are free to repeat this exercise with projected demographic forces into the future, and then, at each future date, to adapt policies to the projected conditions of the moment (this is exactly what is done with fertility and mortality). And the age structure associated with the demographic forces of the moment are obtained by the ergodicity theorem (Cohen, 1979).

Using the concept of fictitious cohort, we define the school life expectancy as:

$$S(t) := \int_0^{A_S(t)} (1 - \ell(t, b)) db. \quad (3)$$

It corresponds to no actual cohort, but to a fictitious one, which at age b would

have spent $(1-\ell(t, b))$ time units in school. It measures the conditions of schooling of the moment t . In contrast to the literature focusing on individual schooling decisions, the schooling variable $S(t)$ measures aggregate time at school spent by all individuals aged between 0 and $A_S(t)$ at time t . In practice, it is the total time spent in primary, secondary, and tertiary education at a given period (e.g. Done, 2012). $A_S(t)$ is a control variable, whereby the planner decides at which age individuals can no longer remain in school. Schooling starts at $t = 0$ by construction. In practice, governments may set the age for entry into schooling and the age for exit. We consider only the latter, following the literature developing Ben Porath's hypothesis.

We specify wages to depend on the school period life expectancy, as:

$$w(t, a) = \iota(a) \exp(\theta(S(t))), \quad (4)$$

where ι describes a schedule of wages across age, after controlling for date. We assume that for given total schooling S , productivity at work is an increasing function of S , say $\exp(\theta(S))$, where $\theta(\cdot)$ is an increasing and concave production function. This follows the specification in the literature around Ben Porath with individual maximization. Wages at age a and time t depend on the age profile ι and on school life expectancy $S(t)$ in a multiplicative and separable way, for the sake of tractability. Equation (4) implies that a higher school life expectancy $S(t)$ increases wages at date t for all ages. There is no empirical evidence to support this claim, but it may hold true anyhow: for example, Done (2012) reports that,

in sub-Saharan Africa, school life expectancy is 9 years of age with low wages across all ages while in the Western world, schooling life expectancy is 16 years of age with much higher wages at each age.

We now solve the program. State variables are the age-specific population $p(t, a)$ and the total schooling time $S(t)$; control variables are consumption $c(t, a)$, leisure $\ell(t, a)$, and the age $A_S(t)$ of the oldest schooled individual.

The Lagrangian is built from (1) and (2), after replacing with the right-hand side of (4). The first-order condition obtained by differentiating with respect to consumption c is:

$$u'_c(c(t, a), \ell(t, a)) = \lambda, \quad \forall a. \quad (5)$$

The first-order conditions obtained by differentiating with respect to $\ell(t, a)$:

$$\text{for } a > A_S(t) \quad u'_\ell(c(t, a), \ell(t, a)) = \lambda e^{\theta(S(t))} \iota(a)$$

$$\text{for } a \leq A_S(t) \quad p(t, a) u'_\ell(c(t, a), \ell(t, a)) = \lambda \int_{A_S(t)}^{A_R} p(t, b) (1 - \ell(t, b)) e^{\theta(S(t))} \theta'(S(t)) \iota(b) db. \quad (6)$$

The first-order conditions obtained by differentiating with respect to $A_S(t)$ yields the relationship between schooling time and population:

$$\frac{1}{\theta'(S(t))} = \int_{A_S(t)}^{A_R} \frac{p(t, a)}{p(t, A_S(t))} (1 - \ell(t, a)) \frac{\iota(a)}{\iota(A_S(t))} da. \quad (7)$$

Equation (7) is also:

$$\int_{A_S(t)}^{A_R} p(t, a) (1 - \ell(t, a)) \iota(a) \theta'(S(t)) da = \iota(A_S(t)) p(t, A_S(t)), \quad (8)$$

or, for the sake of economic interpretation:

$$\int_{A_S(t)}^{A_R} p(t, a)(1 - \ell(t, a))\iota(a)(1 - \ell(t, A_S(t)))e^{\theta(S(t))}\theta'(S(t)) da = \quad (9)$$

$$(1 - \ell(t, A_S(t)))e^{\theta(S(t))}\iota(A_S(t)) p(t, A_S(t)),$$

where the left-hand side is the marginal benefit from increasing $A_S(t)$, which depends on the curvature of the return function through $S(t)$ and on the age structure of the active population. The right-hand side is the marginal social loss, which consists of the income forgone by postponing entry into the labor market until age $a = A_S(t)$. In contrast to the formulas derived in Boucekine et al. (2002), here the schooling decision depends on the age-structure.

Equation (7) is the counterpart of Equation (9) of Cervellati and Sunde (2009: 5) associated with the individual-based life-cycle model of the Ben Porath mechanism. Notably, the age class size $p(t, a)$ at time t and age a has replaced the unconditional probability for an individual of surviving to age a . These authors followed an individual, assimilated to a cohort, and used *cohort* school life expectancy (but there is a single cohort) as control variable. By construction, they did not deal with population, but only with survival of the cohort along with age. The decisive difference with us is that (7) involves not only survival, but age pyramids, which mix survival *and* fertility.

We now combine fertility and mortality of the moment. This operation results in an age-structured population reflecting the combined conditions of the moment. For this population associated with the fictitious cohort, fertility and mortality are constant over time; their values are those taken at moment t . By

definition, this population is stable.

3 Stable population associated with the conditions of the year t

A stable population is a population closed to migration and characterized by constant fertility and mortality. These forces determine the age structure. Conversely, a stable age structure characterizes given fertility and mortality flows.

The stable population associated with the conditions of the moment t , namely mortality $\mu(t, a)$ and fertility $\phi(t, a)$, has its population growth rate $\rho(t)$ given by the Lotka equation:

$$1 = \int_0^{\bar{A}} \sigma(t, a) e^{-\rho(t)a} \phi(t, a) da, \quad (10)$$

where $\sigma(t, a) = 1 - \exp(-\int_0^a \mu(t, u) du)$ is the period survival function at t . An empirical population has no reason to be stable, but, following Lotka, at the moment t , its forces of death ($\mu(t, a)$) and its forces of life ($\phi(t, a)$) are synthesized in the intrinsic growth rate $\rho(t)$ solution of (10). At each time t fixed, consider a population of size $P_0(0) := P(t)$ endowed with a fertility schedule $\phi_0(a) := \phi(t, a)$ for all ages a and a mortality distribution $\mu_0(a) := \mu(t, a)$. According to Lotka's equation, its growth rate is equal to the intrinsic growth rate $\rho_0 = \rho(t)$. This population with constant fertility $\phi_0(\cdot)$ and mortality $\mu_0(\cdot)$ has an age pyramid

$$p_0(\tau, a) := P_0(0) e^{\rho_0 \tau} \frac{\sigma_0(a) e^{-\rho_0 a}}{\int_0^{\bar{A}} \sigma_0(b) e^{-\rho_0 b} db} \quad (11)$$

at time τ , where $\sigma_0(a) = \exp(-\int_0^a \mu_0(u) du)$, and a population size $P_0(\tau) = P_0(0) \exp(\rho_0 \tau)$. The time τ runs from 0 to ∞ for each date t , so that the stable population characterized by $p_0(\tau, a)$ corresponds at time τ to the conditions of the fixed moment t . The age structure

$$\frac{\sigma_0(a) e^{-\rho_0 a}}{\int_0^{\bar{A}} \sigma_0(b) e^{-\rho_0 b} db} \quad (12)$$

is independent of the time τ .

We now replace the previous notation indexed by 0, which we introduced for pedagogical reasons, by the notations $\phi(t, a)$, $\mu(t, a)$, $\sigma(t, a)$, $\rho(t)$, and $P(t)$. At time τ , the total number of people is

$$p_t(\tau, a) := P(t) e^{\rho(t)\tau} \frac{\sigma(t, a) e^{-\rho(t)a}}{\int_0^{\bar{A}} \sigma(t, b) e^{-\rho(t)b} db} \quad (13)$$

aged a in this stable population (e.g. Bonneuil, 1997) depends on the mortality $\mu(t, a)$ and the population growth rate $\rho(t)$, which themselves do not depend on the kinematics inherent in the stable population and indexed by the time τ . We leave the subscript t to $p_t(\tau, a)$ to remind that the forces of death and fertility are those at t , but that the associated stable population involves another time, denoted by τ . The population size $P(t)$ at time t is the initial population for the kinematics of the associated stable population $p_t(\tau, a)$, $\tau = t, \dots$.

By replacing $p_t(t, a)$ by its expression in Equation (13) for $\tau = t$, Equation (7) becomes:

$$\frac{1}{\theta'(S(t))} = \int_{A_S(t)}^{A_R} \frac{\sigma(t, b)}{\sigma(t, A_S(t))} (1 - \ell(t, b)) \frac{\iota(b)}{\iota(A_S(t))} e^{-\rho(t)(b - A_S(t))} db. \quad (14)$$

At constant fertility, if the life expectancy increases, $\mu(t, a)$ decreases for every age a and $e^{-\int_a^b \mu(t, u) du}$ increases, but $\rho(t)$ also increases and $e^{-\rho(t)(b-a)}$ decreases. So the two effects are in opposite directions.

In Equation (14), the term depending on period life expectancy $e_0(t)$ defined as $e_0(t) := \int_0^\infty a\mu(t, a)\sigma(t, a) da = \int_0^\infty \sigma(t, a) da$ is

$$\frac{\sigma(t, b)}{\sigma(t, A_S(t))} e^{-\rho(t)(b-A_S(t))} = e^{-\int_{A_S(t)}^b \mu(t, a) da} e^{-\rho(t)(b-A_S(t))}, \quad (15)$$

whose derivative is:

$$e^{-\int_{A_S(t)}^b \mu(t, u) du} e^{-\rho(t)(b-A_S(t))} \left(-\int_{A_S(t)}^b \frac{\partial \mu(t, u)}{\partial e_0(t)} du - (b - A_S(t)) \frac{\partial \rho(t)}{\partial e_0(t)} \right). \quad (16)$$

The direction of the co-variation of longevity and schooling is given by the sign of $\partial S(t)/\partial e_0(t)$. Because

$$\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} = -\frac{1}{\theta^2(S(t))} \frac{\partial \theta'(S(t))}{\partial e_0(t)} \frac{\partial S(t)}{\partial e_0(t)}, \quad (17)$$

and thanks to the assumption that θ is increasing and concave ($\frac{\partial \theta'(S(t))}{\partial e_0(t)} < 0$), $\partial S(t)/\partial e_0(t) = \partial(1/\theta'(S(t)))/\partial e_0(t)$.

The effect on $\frac{1}{\theta'(S(t))}$ of an increase in period life expectancy $e_0(t)$ on $\frac{1}{\theta'}$ depends then on the sign of:

$$\begin{aligned} \frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} &= \int_{A_S(t)}^{A_R} (1 - \ell(t, b)) \frac{\iota(b)}{\iota(A_S(t))} e^{-\int_{A_S(t)}^b \mu(t, u) du} e^{-\rho(t)(b-A_S(t))} \\ &\quad \left(-\int_{A_S(t)}^b \frac{\partial \mu(t, u)}{\partial e_0(t)} du - (b - A_S(t)) \frac{\partial \rho(t)}{\partial e_0(t)} \right) db. \end{aligned} \quad (18)$$

or

$$\begin{aligned} \frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} &= \int_{A_S(t)}^{A_R} (1 - \ell(t, b)) \frac{\iota(b)}{\iota(A_S(t))} e^{-\int_{A_S(t)}^b \mu(t, u) du} e^{-\rho(t)(b-A_S(t))} \\ &\quad \left(-\int_{A_S(t)}^b \frac{\partial(\rho + \mu(t, u))}{\partial e_0(t)} du \right) db. \end{aligned} \quad (19)$$

From (10), we obtain $\frac{\partial \rho(t)}{\partial e_0(t)}$ from the Lotka equation:

$$\frac{\partial \rho(t)}{\partial e_0(t)} = \frac{\int_0^{\bar{A}} (-\int_0^a \frac{\partial \mu(t,v)}{\partial e_0(t)} dv) e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da}{\int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da}, \quad (20)$$

which is positive, because $\frac{\partial \mu(e_0(t),u)}{\partial e_0(t)} < 0$: empirical observation as well as mortality models indicate that mortality decreases at all ages when life expectancy increases.

Substituting $\frac{\partial \rho(t)}{\partial e_0(t)}$ into Equation (19) yields:

$$\begin{aligned} \frac{\partial(\frac{1}{\theta'(S(\bar{t}))})}{\partial e_0(t)} &= \int_{A_S(t)}^{A_R} (1 - \ell(t,b)) \frac{\iota(b)}{\iota(A_S(t))} e^{-\int_{A_S(t)}^b \mu(t,u) du} e^{-\rho(t)(b-A_S(t))} \\ &\quad \left(- \int_{A_S(t)}^b \frac{\int_0^{\bar{A}} (-\int_0^a \frac{\partial \mu(t,v)}{\partial e_0(t)} dv) e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da + \frac{\partial \mu(t,u)}{\partial e_0(t)} \int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da}{\int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da} \right) du db \\ &= \int_{A_S(t)}^{A_R} (1 - \ell(t,b)) \frac{\iota(b)}{\iota(A_S(t))} e^{-\int_{A_S(t)}^b \mu(t,u) du} e^{-\rho(t)(b-A_S(t))} \\ &\quad \left(- \int_{A_S(t)}^b \frac{\int_0^{\bar{A}} (\int_0^a (\frac{\partial \mu(t,u)}{\partial e_0(t)} - \frac{\partial \mu(t,v)}{\partial e_0(t)}) dv) e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da}{\int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t,a) \phi(t,a) da} \right) du db. \end{aligned} \quad (21)$$

Marchand and Thélot (1991) explain that, until the nineteenth century, usual daily work coincided with daylight, and that work hours had been remaining the same, be it in town or in countryside, varying only with seasons. They do not mention any age component, but their description is consistent with the fact that age had no effect on the duration at work. OECD publishes incidences of employment by usual weekly hours worked and by quinquennial age class only from 2000 onwards.³ From these data, we computed the mean total number of worked hours by age class: for France, this number was 36.5 (sd=0.3) for men and women aged 20-24, 37.9 (sd=0.3) for the 25-29, 38.3 (sd=0.2) for the 30-34,

³http://stats.oecd.org/Index.aspx?DataSetCode=AVE_HRS&Lang=fr

38.3 (sd=0.3) for the 35-39, the 40-44, and the 45-49 age classes. The proximity of these numbers allows us to assume that the age profile of labor supply in one individual's day is constant with age, then:

$$\forall b \in [A_S(t), A_R], \quad 1 - \ell(t, b) = 1 - \ell(t, A_S(t)). \quad (22)$$

Then $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ has the sign of:

$$\begin{aligned} f(e_0(t), \phi(t, \cdot)) &:= \int_{A_S(t)}^{A_R} \frac{\iota(b)}{\iota(A_S(t))} e^{-\int_{A_S(t)}^b \mu(t, u) du} e^{-\rho(t)(b-A_S(t))} \\ &\quad \left(- \int_{A_S(t)}^b \frac{\int_0^{\bar{A}} \left(\int_0^a \left(\frac{\partial \mu(t, u)}{\partial e_0(t)} - \frac{\partial \mu(t, v)}{\partial e_0(t)} \right) dv \right) e^{-\rho(t)a} \sigma(t, a) \phi(t, a) da}{\int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t, a) \phi(t, a) da} \right) du db, \end{aligned} \quad (23)$$

where $\phi(t, \cdot)$ is the fertility pattern underlying the value of the population growth rate $\rho(t)$ with respect to Equation (10).

Figure 1 shows the value of $\frac{\partial \mu(t, a)}{\partial e_0(t)}$ from Ledermann model life tables, which are very realistic,⁴ contrary to analytical formulas, which at best capture only certain age intervals (after 40 years for a Gompertz curve, for example). Figure 1 shows that infant ages, up to 4 years, are determinant in the computation of $\frac{\partial \mu(t, u)}{\partial e_0(t)} - \frac{\partial \mu(t, v)}{\partial e_0(t)}$: with increasing life expectancy, this importance of infant ages gradually vanishes, and the sign of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ becomes more dependent on what occurs at old ages. In contrast to what happens at low life expectancy, $\frac{\partial \mu(t, u)}{\partial e_0(t)} - \frac{\partial \mu(t, v)}{\partial e_0(t)}$ for old ages dominates for high life expectancy. It is lower than values of this expression for younger ages, so that the sign of $\frac{\partial \mu(t, u)}{\partial e_0(t)} - \frac{\partial \mu(t, v)}{\partial e_0(t)}$ changes.

⁴Model life tables are of common use in demography. Ledermann model life tables are semi-parametric, resulting from regressions performed at each age on some 300 empirical life tables.

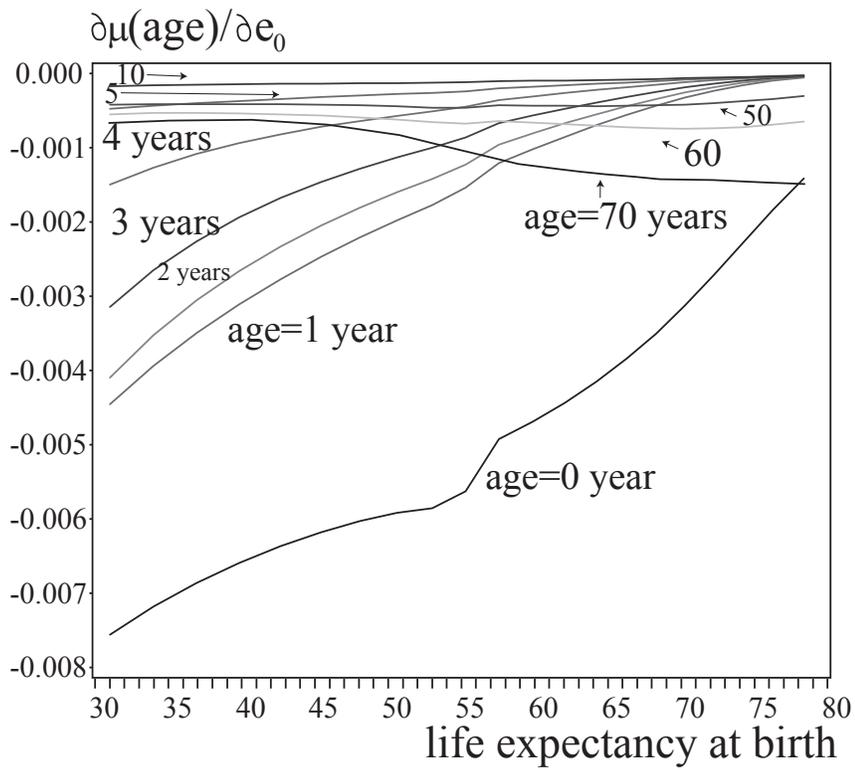


Figure 1: Value of $\frac{\partial \mu(t,a)}{\partial e_0(t)}$ with respect to life expectancy. Ledermann model life tables.

Subsequently, the sign of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ changes, too. *The shape of the life table is then determinant in the relationship between longevity and optimal schooling time.*

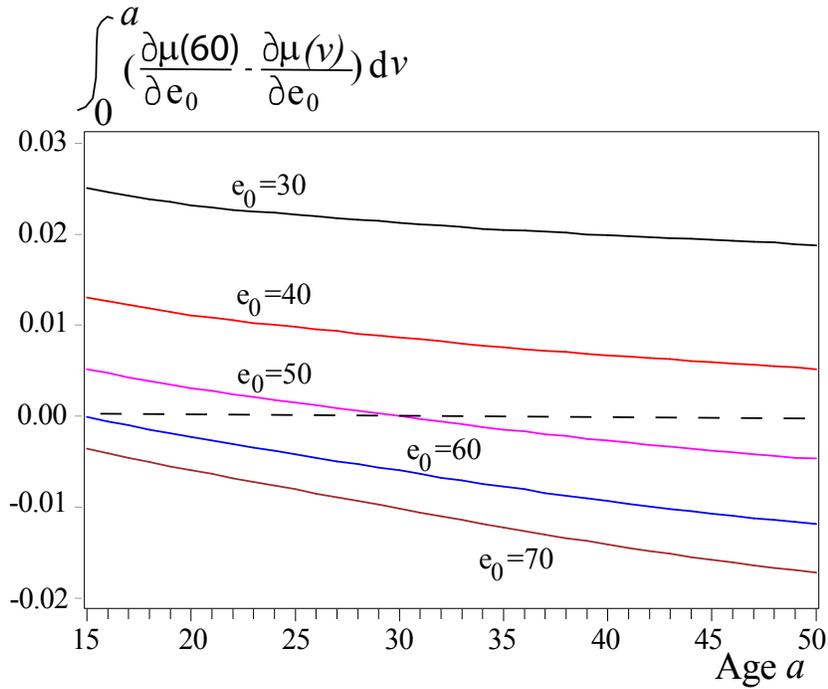


Figure 2: Example of values taken by $\int_0^a (\frac{\partial \mu(u)}{\partial e_0} - \frac{\partial \mu(v)}{\partial e_0}) dv$ as a function of age a and with respect to life expectancy e_0 . Ledermann model life tables.

The sign of $\frac{\partial \mu(t,u)}{\partial e_0(t)} - \frac{\partial \mu(t,v)}{\partial e_0(t)}$ is modulated by $e^{-\rho(t)(t)a} \sigma(t,a) \phi(t,a)$ and by $e^{-\int_{A_S(t)}^b \mu(t,u) du} e^{-\rho(t)(b-A_S(t))}$: we propose to specify the fertility schedule and examine the relationship between longevity and schooling through their determi-

nants which are fertility and mortality.

4 Empirical analysis

Our theoretical analysis has shown that the relationship between longevity and schooling is mediated by the age structure, if the social planner maximizes collective well-being from the conditions prevailing at the period, which is realistic. The case of nineteenth century France is exemplary because France experienced a century of literacy transition, with the literacy rate rising from 23% on average in 1806 to 96% in 1906, and a century of demographic transition, with an average life expectancy rising from 36 years in 1806-10 to 47 years in 1906-10 and a Coale overall fertility index that fell from 0.39 in 1806 to 0.22 in 1906. France is also a well-documented case (Bonneuil, 1997; Bonneuil and Rosental, 1999, 2008). It allows us to examine the diversity of demographic and literacy conditions in a historical follow-up, marked by State schooling policies.

4.1 The relationship between schooling and longevity varies with the demographic transition

Equation (23) shows that $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ depends on the improvement in mortality at young ages. At the onset of the demographic transition, mortality decreases faster at young ages than at any other age. This is due, among other reasons, to the fact that a large proportion of infants and children in the old demographic

regime died from water-borne diseases, often through dehydration resulting from diarrhea caused by poor water quality. The mortality decline was triggered by improvements in hygiene, the cleaning of contaminated reservoirs, the filtering of water supplies, especially in cities, and the eradication of unsafe health practices (Bonneuil and Fursa, forthcoming). Perrenoud and Bourdelais (1998) also point out the diminishing virulence of smallpox in Europe as contributing to the mortality decline among children, especially in the 2-4 age group. So, when life expectancy at birth e_0 remains under approximately 50 years, the term $\frac{\partial \mu(t,v)}{\partial e_0(t)}$ at low age v is relatively high, and the term $\int_0^a (\frac{\partial \mu(t,u)}{\partial e_0(t)} - \frac{\partial \mu(t,v)}{\partial e_0(t)}) dv$ is positive for most values of u , as Figure 2 shows on Ledermann's model life tables (which are built on historical data). Then from Equation (23), $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} < 0$ (expected schooling decreasing with longevity). The major decline in infant mortality relative to mortality at other ages implies a larger proportion of children surviving and going to school. Juvenile mortality did not decrease so quickly, however. The population includes more young children, but not many teenagers, whose mortality decreased less rapidly. As a consequence, the average total number of expected schooling years *decreases* while longevity increases.

Conversely, in the post-transition era, life expectancy at birth e_0 is higher, infant mortality is already low, and mortality at older ages has declined substantially, so that the term $\int_0^a (\frac{\partial \mu(t,u)}{\partial e_0(t)} - \frac{\partial \mu(t,v)}{\partial e_0(t)}) dv$ is negative for many values of u , as Figure 2 shows. Then $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} > 0$. As a consequence, the average total number of expected schooling years *increases* with longevity.

4.2 Mortality

Mortality studies have shown that mortality is regular with age only over short periods, say a year, subsequently across different cohorts living at the same time. Cohort mortality patterns on the contrary look irregular, because a cohort successively experiences uneven conditions. This is the reason why models of mortality are meaningful only for fictitious cohorts at a given period, and always unrealistic for actual cohorts followed-up over time.

In the Appendix, we present two toy cases: constant and linear mortality, to show that the relationship between longevity and schooling rapidly becomes complicated. We also present the more realistic case of Gompertz mortality, because it is a well-known mortality model, realistic only after 40 years of age. However, as infant mortality plays a major role, only the case of model life tables is worth considering.

The model life tables we use (Ledermann, 1969) are semi-parametric, indexed by a single parameter $e(t)$, close to the life expectancy at birth:

$$\log_{10}(1 - e^{-\int_0^5 \mu(t, a+u) du}) = \gamma_a^1 + \gamma_a^2 \log_{10}(100 - e(t)) \quad (24)$$

where γ_a^1 and γ_a^2 , $a = 0, 5, \dots, 85$ were estimated by Ledermann (1969) from some three hundred empirical life tables. With this semi-parametric formula, we compute $\frac{\partial \mu(t, u)}{\partial e_0(t)}$ and $\frac{\partial \rho(t)}{\partial e_0(t)}$ numerically.

4.3 Fertility

We introduced the age structure through the simple stable population model. The population growth rate $\rho(t)$ then appears in Equations (19) and (20). Rather than treating this population growth rate as exogenous, we gain better insight into the demographic processes at work by revealing its dependence on fertility with respect to Equation (10).

Coale and Trussel (1974) calibrated the semi-parametric model of realistic five-year fertility schedules:

$$\phi(t, a) = M(t)n(a)e^{m(t)\nu(a)} \quad (25)$$

where $n(a)$ and $\nu(a)$ are given distributions by age. The parameter $M(t) \geq 0$ represents a fertility level, $m(t) \geq 0$ conditions the shape of the distribution: $m > 0$ reflects family limitation, $m = 0$ the absence of family limitation. We obtain the yearly fertility schedule by cubic spline on the cumulated function $\int_{15}^a \phi(t, u) du$.

4.4 Age profile of wages

Statistics of wages by age are scarce. None are mentioned in the important book by Chevallier (1887) on wages in nineteenth century France. Only recent statistics are available, such as those published by Aeberhardt, Pouget, and Skalitz (2007) for the period 1978-2005: wages earned by men over 45 increase slightly compared to those of younger age classes. This is due mainly to

the increase over this period in the proportion of highly skilled wage-earners, for whom wages increase the most with age, whereas the age profile is almost flat for unskilled and low-skilled workers (Aubert and Crépon, 2003). For the year 1978, for men under 30, the ratio of wages to those of men aged over 45 is 0.37 and for men aged 30-45 it is 0.88. The unknown age profiles before the year 1978 should be closer to the uniform distribution, especially before WWII when the French population was mostly rural. Because $\iota(b)/\iota(A_S(t)) \geq 1$ for $b > A_S(t)$ and increases with age b , the age profile $\iota(\cdot)$ reduces the weight given by high mortality to young age classes. Consequently, using the 1978 French age profile for $\iota(\cdot)$ *lower bounds* the probability that $\partial(1/\theta'(S(t)))/\partial e_0(t)$ is negative, while using the uniform distribution for $\iota(\cdot)$ *upper bounds* it. For each year between 1978 and 1988, we fit an age profile by yearly age to the three values presented by Aeberhardt, Pouget, and Skalitz (2007) through a continuous function consisting of one constant before age 21 (to avoid that the interpolated wage before 21 takes too low a value) and three linear interpolations from ages 21 to 65 by one-year interval, with continuity constraints at ages 21, 30, and 45. We then solve a system of linear equations equating the mean wages obtained by interpolation to the recorded values and minimizing the variation of the slope over the age interval 22-65. Table 4.4 presents the resulting 1978 yearly age profile grouped into five-year classes to save space.

The computation of $\partial(1/\theta'(S(t)))/\partial e_0(t)$ is robust to the age profile $\iota(\cdot)$: as expected, with the uniform distribution, this quantity remains in the negative

Table 1: Age profile of wages regrouped in five-year age classes

age	percent								
15-19	0.56	20-24	0.71	25-29	1.43	30-34	2.02	35-39	2.33
40-44	2.65	45-49	2.76	50-54	2.65	55-59	2.55	60-64	2.45

half plane for longer than with the 1978 age profile, but the difference is tiny: at fixed total fertility rate and shape parameter m , $\partial(1/\theta'(S(t)))/\partial e_0(t)$ under the uniform distribution for $\iota(\cdot)$ crosses the zero line at a life expectancy at birth which is less than 0.5% on average higher than with the 1978 age profile.

4.5 Results

Figure 3 shows the computation of $f(e_0(t), \phi(t, \cdot))$ for Lederman life tables (with $e_0(t)$ varying) and for Coale-Trussel fertility schedules $\phi(t, \cdot)$ (with m and M varying). As we mentioned, both fertility and mortality patterns are both realistic, and the result no longer comes from a simplifying assumption on these schedules. The value of the total fertility rate ($\text{TFR}(t) := \int_0^{\bar{A}} \phi(t, a) da$) is given on each (m, M) to help situate the level of fertility.

Figure 3 shows that $f(e_0(t), \phi(t, \cdot))$ and subsequently $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ change with respect to life expectancy and with fertility,

For example, with $m \approx 0$ for nineteenth century France (Bonneuil, 1997a), we situate French *départements* over the course of the demographic transition. The population of the Calvados *département* was one of the earliest to embark on

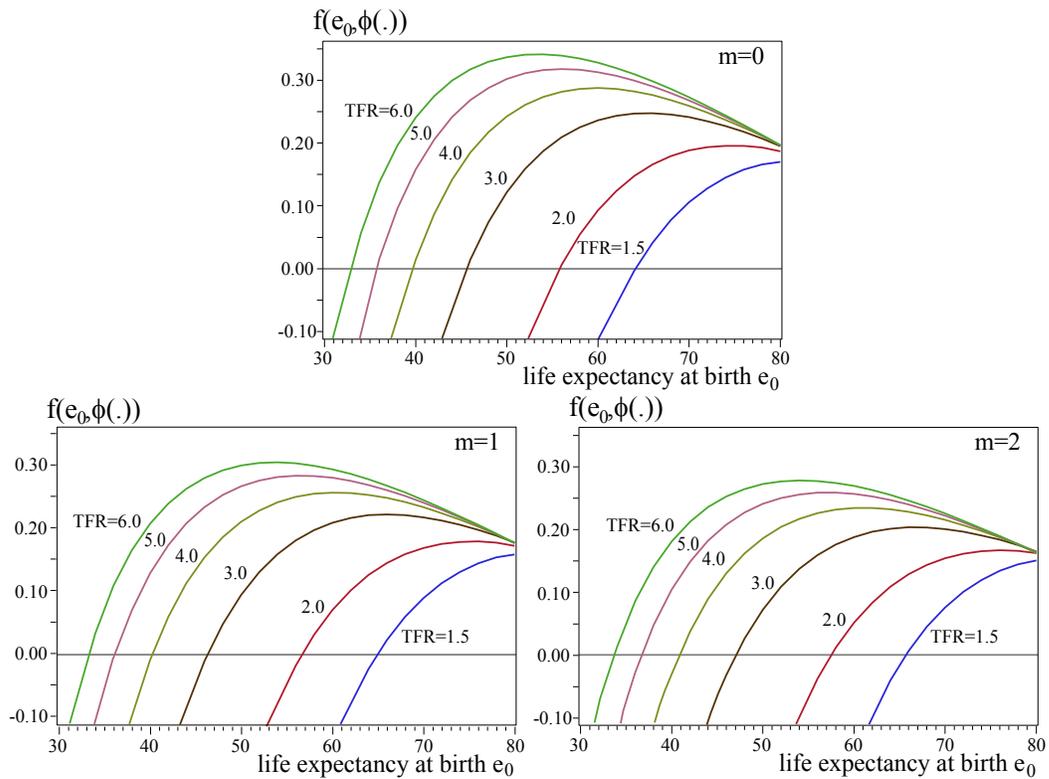


Figure 3: Value of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$, varying with life expectancy (Ledermann model life tables) and Coale-Trussel fertility, m the parameter of family limitation, and TFR the total fertility rate. The wage age profile $\iota(\cdot)$ is the French one in 1978 presented in Table 1. In this Figure, $\rho(t)$ is always positive.

fertility decline and experienced an almost stationary fertility during the nineteenth century: TFR=3.11 and $e_0 = 47.5$ in 1806; TFR= 2.74, and $e_0 = 45.8$ in 1906 (Bonneuil, 1997a). Calvados then began in the positive half plane of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} = 0$ in Figure 3a to move to negative values. This comes from the fact that this *département* started its demographic transition early, with relatively low mortality and fertility levels, but mortality decreased slightly while fertility did not decline fast enough, leading Calvados to increase the proportion of young age classes and to cross the zero line of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)} = 0$ in the direction of negative values. At the other end of the spectrum of the transition, the French *département* of Finistère, whose population was one of the last to enter the fertility decline (TFR= 8.09, $e_0 = 28.7$ years in 1806; TFR= 4.11, $e_0 = 39.5$ years in 1906), $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0}$ remains negative during this period (Figure 3a), although traveling toward positive values, which are attained at the beginning of the twentieth century. This comes from the fact that these demographic conditions correspond to a “young” age pyramid, which puts $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0}$ in the negative half plane of Figure 3a). The move toward positive values corresponds to the aging process caused mostly by declining fertility. So, for the same country, at the same epoch, namely nineteenth-century France, we find locations with positive signs, others with negative signs.

Figure 4 shows what the direction of the relationship between longevity and period school life expectancy would have been if a social planner had solved Equation (1), taking into account the demographic conditions of the moment.

In nineteenth-century France, even if the relationship between the increase in schooling rate and in life expectancy is ambivalent at the *département* level, as we mentioned in the introduction, the demographic transition at the national level was accompanied by a series of State policies most often directed toward increasing schooling for both sexes, finally consistent with the maximization program (1).

Between 1830 and 1895, the still young age structure of the French population should have prompted governments to reduce the length of schooling in order to maximize well-being. Schooling was still not mandatory and was dominated by clerics. However, French governments, aware that the country's future (at the possible price of optimal well-being) depended upon the people's literacy, began to implement policies in favor of schooling: the Guizot law of 1833 required that villages with at least 500 inhabitants have a boy's school (*école primaire élémentaire*), that higher-level primary schools (*école primaire supérieure*) be set up for vocational training of poor pupils, and that each *département* had a teaching training college for primary school teachers (*école normale primaire*). The Falloux law of 1850 required each municipality to open a school for girls, and the Duruy law of 10 April 1867 provided for the development of girls schools and for free school access for boys and girls in municipalities (*communes*) that so wished.

High schools for girls were created in 1880, but only the wealthiest could access them at that time. Jules Ferry, minister of Public Education and Arts, addressing

the President of the Republic on the 25 of January 1880, described the immense improvement in primary schooling: from 1837 to 1877, he claimed that the total number of pupils had increased by 82 percent, the total number of schools by 36 percent, (75 percent for secular schools), girls schools had multiplied four-fold and the total number of teachers had risen by 85 percent, with a pupil-teacher ratio that had fallen from 53 to 1 to 48 to 1.

From 1896 onward, Figure 4 shows that, except during the two World Wars, $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ becomes definitely positive, while both life expectancy at birth and expected schooling were increasing. This is corroborated by the Astier law of 1919, which fostered the development of technical schools. The return of $f(e_0, \phi(.))$ during the two World Wars to negative values is attributable to the fall in both fertility and mortality. In terms of the relationship between longevity and schooling, it is also coincided —although certainly by chance— with the introduction of fees to attend high school by the Vichy regime (1940-44). The high values of $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ after the war are consistent with the increase in schooling, notably through the Berthoin reform of 1959, which made schooling mandatory until 16 years of age (although it was not enforced until 1971).

In the problem of the relationship between longevity and schooling, one could ignore the role played by fertility, because children attending school are of a certain age. This would be a mistake, as it would mean using a mortality schedule which ignores the specific pattern of infant mortality. The relationship in fact depends on fertility as well as on mortality, because fertility determines the

population growth rate and the age structure, which appears as $\frac{p_t(t,a)}{p_t(t,A_S(t))}$ in Equation (14) for the general case or through $\rho(t)$ in Equation (23) in the case of the stable population of the moment.

A very good way to capture the conditions of the moment in a real population is to fit the stable population associated with the fertility and the mortality schedules of this moment.⁵

Figure 4 shows the trajectory of $f(e_0, \phi(\cdot))$ for these stable populations associated with the conditions of each year between 1806 and 1988 in the whole of France. The age-specific fertility distribution is known only from 1892 onward (for the whole of the country). From 1806 to 1892, the age-specific schedule of 1892 is used with the TFR deduced from the Coale fertility index reconstructed by Bonneuil (1997a). Over the course of France's demographic transition, which took place during the nineteenth century, when life expectancy was still low, $f(e_0, \phi(\cdot))$ is negative. With life expectancy increasing, this function becomes positive, with two abrupt returns to negative values during the World Wars, on account of the higher wartime mortality.

⁵*Reminder: we never assume that the empirical population is stable; we compute the conditions of the moment t given by Lotka's intrinsic growth rate $\rho(t)$ (equation (10)) and by the associated stable population having the mortality and fertility schedules of the moment t . We do this at each year t between 1806 and 1988. The time series of the conditions of the moment are reflected by the time series of the associated stable populations $((p_t(\tau, a))_{\tau, a})_{t=t_0, \dots, t_f}$ and the associated intrinsic growth rates $(\rho(t))_{t=t_0, \dots, t_f}$ (where the initial date is t_0 and the last date t_f).*

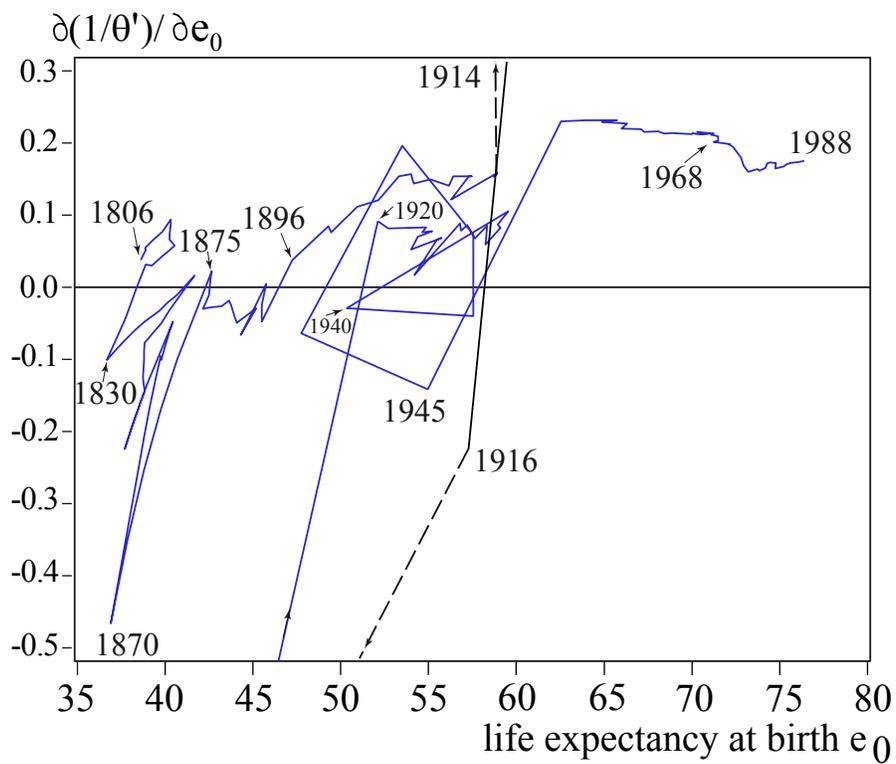


Figure 4: Value of $\frac{\partial(\frac{1}{\theta'(s(t))})}{\partial e_0(t)}$ (same sign as $f(e_0(t), \phi(t, \cdot))$), for the whole of France
 (Data: Bonneuil (1997a) for data from 1896 to 1892 and Ined for data after 1892.)

The wage age profile $\iota(\cdot)$ is the French one presented in 1978 in Table 1.

After the effect of declining fertility, we again find the key role played by infant mortality, which declined continuously (except during crises such as the 1870-71 Franco-Prussian war or the World Wars) between 1806 and 1988. The zero line was crossed by $\frac{\partial(\frac{1}{\theta'(S(t))})}{\partial e_0(t)}$ when the variations of the mortality rate μ became dominated by the changes at old ages, and specifically in Figure 4, for period life expectancies over 46 years.

So, from Figures 3 and 4, in the case of θ strictly concave, the Ben-Porath proposition that schooling grows with longevity appears to be optimal only in post-transitional countries, and in conditions of sufficiently low mortality and low fertility. In countries currently experiencing the transition, especially in those where mortality is still high and fertility not high enough to attain the complete replacement of the population, the age structure is relevant, and longer life is no longer associated with schooling in a Ben Porath-like manner.

5 Conclusion

Schooling has been a political process, whereby planners adapted (or not) to changing conditions. Policies involving demographic forces and favoring collective well-being over the interests of individual cohorts and the situations inherited from the past should adapt to conditions of the moment. By maximizing the collective utility associated with the conditions of the moment, we escaped the premise that individual decision-making governs the relationship between

longevity and education.

By doing so, we revealed the role played by the age structure in the relationship between longevity and schooling. We showed that optimal period school life expectancy varies with life expectancy and mortality, and that the direction of this relationship is negative for young age pyramids, associated with high fertility and high mortality. The effect of an increase in life expectancy on expected schooling is thus dependent on the age structure. The realistic case of model life tables associated with model fertility schedules shows that the relationship between longevity and schooling depends on both life expectancy and fertility, in a non linear manner. With varying fertility and mortality, as was the case during the demographic transition, the direction of this relationship changes. The positive relationship between longevity and schooling claimed by Ben Porath (1967) holds true only for ‘old’ enough age pyramids, corresponding to post-transitional populations. It does not hold true for “young” populations.

Our analysis of French historical data from 1806 to the present has shown that the population age structure has indeed modified the relationship between longevity and optimal schooling. The French case shows that governments in the nineteenth century may not have complied with the objective of maximal well-being, and may have had higher views of what was good for the country, if not for the people. The maximization program, however, indicates what should have been done in order to maximize well-being: the deviation from this program tells us about the gap between economics and politics.

The method of the fictitious cohort, fundamental in demography and essential to understanding how a situation is changing at a given moment, may have been mistakenly ignored in economics. Our study of the relationship between longevity and schooling shows that the certainties derived from individual-based frameworks are overturned when age structure and conditions of the moment are fully addressed.

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Appendix: constant, linear, and Gompertz mortality

Constant mortality: $\forall u, \frac{\partial \mu(t,u)}{\partial e_0(t)} = k$ **constant** From Equation (20),

$$\frac{\partial \rho(t)}{\partial e_0(t)} = -k \quad (26)$$

and from Equation (10):

$$\frac{\partial \left(\frac{1}{\theta'(S(t))} \right)}{\partial e_0(t)} = 0, \quad (27)$$

the increase in life expectancy has no effect on schooling.

In particular, if $\mu(t, u) = \mu$, $e_0(t) = \frac{1}{\mu}$, $\frac{\partial \mu(t,u)}{\partial e_0(t)} = -\frac{1}{e_0(t)^2}$ is constant with age and Equation (27) holds true.

Linear mortality: $\mu(t, x) = Bx$ The linear case is not realistic, but is of interest here because it is the simplest departure from the case of mortality constant with age. Then

$$\begin{aligned} \sigma(t, a) &= e^{-\frac{B}{2}a^2} \\ e_0(t) &= \int_0^{\bar{A}} \sigma(t, a) da = \left(\frac{\pi}{2B} \right)^{\frac{1}{2}} \end{aligned} \quad (28)$$

which yields:

$$\begin{aligned}\frac{\partial \mu(t, u)}{\partial e_0(t)} &= -\frac{\pi}{e_0(t)^3} u \\ \frac{\partial \rho(t)}{\partial e_0(t)} &= \frac{\pi}{2e_0(t)^3} \frac{V_m + A_m^2}{A_m}\end{aligned}\tag{29}$$

where A_m and V_m are the period mean age at procreation and the variance of the age at procreation:

$$\begin{aligned}A_m &= \int_0^{\bar{A}} a e^{-\rho(t)(t)a} \sigma(t, a) \phi(t, a) da \\ V_m &= -A_m^2 + \int_0^{\bar{A}} a^2 e^{-\rho(t)(t)a} \sigma(t, a) \phi(t, a) da\end{aligned}\tag{30}$$

Finally:

$$\begin{aligned}\frac{\partial(\frac{1}{\theta'(s(t))})}{\partial e_0(t)} &= (1 - \ell(t, A_S(t))) \frac{\pi}{2e_0(t)^3} \\ &\int_{A_S(t)}^{A_R} (b - A_S(t))(b + A_S(t) - A_m - \frac{V_m}{A_m}) \sigma(t, b) \frac{\iota(b)}{\iota(A_S(t))} e^{-\rho(t)(b - A_S(t))} db\end{aligned}\tag{31}$$

Equation (31) shows that for $A_S(t)$ high enough, $\frac{\partial(\frac{1}{\theta'(s(t))})}{\partial e_0(t)} > 0$, which would validate the Ben-Porath mechanism. The problem is that the decrease of $\frac{\partial \mu(t, u)}{\partial e_0(t)}$ with age described in Equation (29) is contrary to real-world experience.

Gompertz mortality The Gompertz law of mortality is a close approximation of the observed force of mortality after 40 years of age. It relies on two parameters A for the level and γ for the increase with age:

$$\mu(t, a) = A\gamma^a \quad \text{with } \gamma > 1 \text{ and } A < 1.\tag{32}$$

The survival function is then an exponential of an exponential: in no circumstances can a simple exponential portray an empirical human survival function.

Differentiating with respect to life expectancy $e_0(t)$ yields:

$$\frac{\partial \mu(t, u)}{\partial e_0(t)} = \left(\frac{\partial \ln(A)}{\partial e_0(t)} + u \frac{\partial \ln(\gamma)}{\partial e_0(t)} \right) \mu(t, u) \quad (33)$$

which is negative because $\frac{\partial \ln(A)}{\partial e_0(t)} < 0$ and $\frac{\partial \ln(\gamma)}{\partial e_0(t)} < 0$. After integration by parts:

$$\int_0^b \frac{\partial \mu(t, u)}{\partial e_0(t)} du = \frac{A}{\ln \gamma} \left(\frac{\partial \ln A}{\partial e_0(t)} (\gamma^b - 1) + \frac{\partial \ln \gamma}{\partial e_0(t)} \left(b\gamma^b - \frac{1}{\ln \gamma} (\gamma^b - 1) \right) \right) \quad (34)$$

Hence:

$$\int_{A_S(t)}^b \frac{\partial \mu(t, u)}{\partial e_0(t)} du = \frac{A}{\ln \gamma} \left(\frac{\partial \ln A}{\partial e_0(t)} (\gamma^b - \gamma^{A_S(t)}) + \frac{\partial \ln \gamma}{\partial e_0(t)} \left(b\gamma^b - A_S(t)\gamma^{A_S(t)} - \frac{1}{\ln \gamma} (\gamma^b - \gamma^{A_S(t)}) \right) \right) \quad (35)$$

and

$$\frac{\partial \rho(t)}{\partial e_0(t)} = \frac{1}{A_m} \frac{A}{\ln \gamma} \int_0^{\bar{A}} \left(\frac{\partial \ln A}{\partial e_0(t)} (1 - \gamma^a) - \frac{\partial \ln \gamma}{\partial e_0(t)} \left(a\gamma^a - \frac{1}{\ln \gamma} (\gamma^a - 1) \right) \right) \sigma(t, a) \phi(t, a) e^{-\rho(t)a} da \quad (36)$$

with again $A_m = \int_0^{\bar{A}} a e^{-\rho(t)a} \sigma(t, a) \phi(t, a) da$. Equations (35) and (36) introduced into Equation (19) yield a computable formula. It remains however hard to interpret because the relationship of A and γ as functions of e_0 are not straightforward. This is better to work with model life tables, which are obtained from empirical data, and for which there is no need to compute which values of the parameters correspond to a given life expectancy. The difficulty arises because Gompertz, although good at describing mortality after 40 years of age, gives a poor fit before that age, especially at infant and juvenile ages.