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Abstract

This article analyzes the consequences of environmental tax policies when the government imposes a constraint on stabilizing public debt. A public sector of pollution abatement is financed by taxation and by issuing public debt. Considering a simple overlapping-generations model, the tax reform stimulates steady-state investment. Then, the environmental quality and the aggregate consumption increase if and only if (i) pollution abatement is large enough and (ii) there is under-accumulation of the per capita capital stock. This arises if environmental taxation allows a decrease of either income taxation or debt-output ratio.

JEL Classification: Q5, H23, H63.

Keywords: Environmental tax reform, Debt, Public emission abatement, Double dividend.

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1 Introduction

The growing environmental concerns motivate the developed countries to adapt their tax structure by introducing new taxes on pollutants. France, following the Scandinavian countries, has planned to adopt a carbon tax in the next few years. The French Environmental Protection Agency (ADEME) is entirely financed by revenues of taxes on pollutants, called General Tax on Polluting Activities. One of the advantages of the environmental tax is that it provides a public revenue which can be recycled. This is the reason why it is often preferred to subsidies or emission quotas. Several authors like Parry (1995) or Poterba (1993) argued that this revenue recycling could reduce or even annihilate the gross cost of the implementation of an environmental tax. The revenues of these taxes are used to limit the economic distortions of the reform by reducing other taxes, or alternatively, these revenues are allocated to pollution abatement programs. However, whatever the government's decision about the use of the tax revenues, public engagements in the environmental protection are often constrained by fiscal objectives which impose to control public deficits and public debt. Therefore, a pre-existing high level of public debt can be an obstacle for the launching of new environmental protection programs. This is basically the case in Europe during the global debt crisis of 2007-2010.

Accordingly, we study the impacts of environmental policies under a debt stabilization constraint, when public actions to protect the environment are at least partially financed by public funds. Can public debt be an obstacle for the financing of environmental policies? Reversely, could the environmental tax reduce efficiently the public debt burden, and protect the environment simultaneously?

To analyze the interactions between environmental policies and public debt, this article considers an overlapping generations model *à la* Diamond (1965) with an environmental intergenerational externality. Pollution emission occurs through production processes which deteriorate the environmental quality, harming the welfare of future generations. Public expenditure for pollution abatement are financed by taxation and debt. Moreover, a debt stabilizing constraint imposes a constant level of debt per output.

We take into account both the efficiency and the intergenerational distributional aspects of environmental taxes: like Bovenberg and Heijdra (1998), we examine whether a revenue-neutral increase in the pollution tax compensated by a change of the labor tax can yield a double dividend and whether a higher pollution tax can be Pareto welfare improving by benefiting all generations.

We show that the steady-state levels of the capital stock and of the public pollution abatement are the key factors that explain the consequences of the tax reform. Namely, if the capital stock is low and the public pollution abatement is large enough, an increase of the environmental tax, compensated by a decrease of the income tax, will increase both the environmental quality and the aggregate households' consumption. During the convergence to the steady-state, these benefits are no longer available for the first generation that has to implement the policy, but the policy may be welfare improving in the long run. Finally, an increase of the environmental taxation budget-balanced by a variation of the debt-output ratio may also increase the environmental quality and the aggregate consumption. Hence, the fiscal policy may improve the aggregate consumption and the environmental quality while reducing the debt-output ratio. Our findings confirm the empirical results of Raush (2013). Using an OLG dynamic general-equilibrium model of the U.S. economy, Raush (2013) shows that when a carbon tax is employed to consolidate public debt, the environmental policies allow the possibility of sustained welfare gains for future generations.

Whether an environmental tax reform can be designed without negatively affecting the economic welfare has given rise to a huge literature on the double dividend issue. Terkla (1984), Parry (1995), or Poterba (1993) first had the intuition that the recycling of the revenue of an environmental tax could reduce or even eliminate the gross cost of its implementation. As governments use the revenues from pollution taxes to decrease other distortionary taxes, environmental taxes may lead to a *double dividend*, according to Goulder's definition, by improving the environmental quality and achieving a less distortionary tax system (Goulder (1995)). Baumol and Oates (1988), Pearce (1991) and Oates (1991) suggested that these efficiency gains could be a powerful argument in favor of environmental taxation.

Beside these potential efficiency properties, environmental decisions have an impact on the welfare of both current and future generations, since environmental quality is a public good shared by different generations. These intergenerational issues on environmental externalities and taxation have been widely studied. John and Pecchenino (1994) and John *et alii* (1995) examine the effect of an environmental tax which revenues finance a public pollution abatement sector. Bovenberg and Heijdra (1998) examine the effects of a green tax on polluting capital when the tax revenue is redistributed by lump-sum intergenerational transfers. More generally, this literature concludes that a double dividend can be obtained at the expense of equity (Proost and van Regemorter (1995), Bovenberg and van der Ploeg (1996), Bosello *et alii* (2001), Chiroleu-Assouline and Fodha (2005), (2006) and (2014)).

Nevertheless, in these previous studies, government cannot fund pollution abatement programs by issuing public debt. In consequence, they only consider tax financing schemes. Debt financing has been introduced in dynamic models with environmental concerns (Bovenberg and Heijdra (1998); Heijdra *et alii* (2006), Fernandez *et alii* (2010)), but these contributions focus on a different issue than ours. Instead of using debt to finance a share of pollution abatement, debt policy makes possible to redistribute welfare gains from future to existing generations. In our model, the role of the public debt is twofold: as usual, it redistributes welfare among young and old generations, but first of all, it finances the public pollution abatement sector. Hence, the redistribution properties of the public debt are limited by the environmental actions of the government. Fodha and Seegmuller (2012) and (2014) analyze the consequences of some environmental tax reforms under a public debt stabilization constraint. In Fodha and Seegmuller (2012), the households can invest in private pollution abatement activities, in addition to public abatement. Fodha and Seegmuller (2014) consider the impacts of pollution on life expectancy. These articles point out the crucial role of the public debt on the dynamics of capital stock and environmental quality, and to reach the optimal allocation. In this paper, we rather analyze the impacts of the tax reforms on the capital stock and the environmental quality, while maintaining the debt to output ratio constant.

The rest of this paper is organized as follows. Section 2 presents an OLG model in which environmental externalities are provided as a by-product of production, and the government issues debts and imposes taxes on personal income and production for financing public emission abatement. Section 3 defines the intertemporal equilibrium and examines the multiplicity and the stability of steady states. Section 4 studies a possibility of the double dividend when the increase of the environmental tax is balanced through a decrease of the income tax. Section 5 considers the intergenerational distributive issues. Section 6 presents the consequences of an environmental tax reform balanced by the debt-output ratio. The final section provides the conclusions.

2 The model

We consider an overlapping generation model with discrete time ($t = 0, 1, ...,$ $+\infty$), capital accumulation, and environmental quality which degrades with production, but may be improved by public abatements. These government expenditures are financed by environmental taxation on production, labor income taxes, or public debt.

2.1 Household

At each period, a new generation is born. There is no population growth and population size of a generation is normalized to $N > 0$. Individuals live for two periods. They have preferences over their consumption bundle when young (c_t) and old (d_{t+1}), and environmental quality when young (E_t) and old (E_{t+1}). E_t is an externality for the household. The life-cycle utility is given by:

$$\ln c_t + \gamma v(E_t) + \beta (\ln d_{t+1} + \gamma v(E_{t+1}))$$

where $\beta \in (0, 1)$ is the discount factor, $\gamma > 0$ the relative weight of the environmental quality and $v(.)$ measures the welfare gains from the environmental quality. The young born in period t inelastically supplies one unit of labor and receives real wage (w_t). A personal income tax (τ_t^w) is imposed on the real wage and the after-tax income is shared between present consumption and savings (s_t). When old, the household is retired and entirely consumes the remunerated savings ($r_{t+1}s_t$) where r_{t+1} is the real interest rate.[1] Budget constraints of an individual born in period t are given by:

$$c_t + s_t = (1 - \tau_t^w)w_t, \quad d_{t+1} = r_{t+1}s_t$$

Then, the savings function is derived as:

$$s_t = \frac{\beta}{1+\beta}(1 - \tau_t^w)w_t$$

Because labor is inelastically supplied, the income tax does not distort labor market.

[1] We assume complete depreciation of capital. Since the period length is quite long in overlapping generations model with two-period lived households, this assumption is not restrictive.

2.2 Firms

A representative firm produces the unique final good using a Cobb-Douglas technology:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha},$$

where Y_t, L_t, and K_t are output, labor, and capital stock, respectively. The intensive production function is given by $y_t = k_t^{\alpha}$, where k_t and y_t are per worker capital stock and output. Production process emits pollutions as by-products and, therefore, the government imposes an environmental tax (τ^e) on its product sales. Profits write $(1 - \tau^e)Y_t - w_t L_t - r_t K_t$. The first order conditions for profit maximization are:

$$\begin{aligned} w_t &= (1 - \tau^e)(1 - \alpha)k_t^{\alpha}, \\ r_t &= (1 - \tau^e)\alpha k_t^{\alpha - 1}. \end{aligned}$$

2.3 Government

The government imposes taxes on income and sales. Moreover, debt (B_t) is issued in order to finance a share of government spending for emission abatement (G_t). The government budget constraint is:

$$B_t = r_t B_{t-1} - (\tau_t^w w_t N + \tau^e Y_t) + G_t.$$

with $B_{-1} \geqslant 0$ given.

To avoid explosive debt path, we assume that the government spending-output G_t/Y_t and debt-output B_t/Y_t ratios are constant over time, i.e. equal to $g \geq 0$ and $\delta \geq 0$, respectively (see also de la Croix and Michel (2002)). Therefore, the government budget constraint is:

$$\delta Y_t = r_t \delta Y_{t-1} - (\tau_t^w w_t N + \tau^e Y_t) + g Y_t.$$

2.4 Environmental quality

Pollution emission occurs through polluting production processes while the government spends on public emission abatement. Because environmental quality evolves in opposite direction than pollution, its law of motion is given as:

$$E_{t+1} = (1 - \eta)E_t + \theta G_t - \epsilon Y_t, \text{ with } E_0 \text{ given,}$$

where $\epsilon > 0$, $\theta > 0$, and $\eta \in (0, 1)$ are efficiency parameters measuring the pollution emission from production, the public emission abatement, and the capacity to converge to the natural environmental quality in the absence of any pollution flow.

3 Equilibrium, steady states and dynamics

The labor market equilibrium is given as $N = L_t$, for all t. Therefore, environmental quality per young household e_t satisfies:

$$e_{t+1} = (1 - \eta)e_t + (\theta g - \epsilon)k_t^\alpha$$

and the government budget constraint rewrites:

$$\delta y_t = r_t \delta y_{t-1} - (\tau_t^w w_t + \tau^e y_t) + g y_t.$$

Since g, δ and τ^e are kept constant over time, the government must adjust the income tax rate to balance the government budget:

$$\tau_t^w w_t = r_t \delta y_{t-1} - (\tau^e + \delta - g) y_t. \tag{1}$$

The market-clearing condition for capital market is described as:

$$k_{t+1} = s_t - \delta y_t \tag{2}$$

Defining $z_{t+1} = k_{t+1}/k_t^\alpha$ as an investment factor, equation (2) can be rewritten as:

$$z_{t+1} = \phi(z_t) = \frac{\beta(\mu + \alpha\tau^e) - \delta}{1 + \beta} - \frac{\alpha\beta\delta}{1 + \beta}\frac{1 - \tau^e}{z_t} \tag{3}$$

where $\mu \equiv 1 - \alpha - g$. By direct inspection of this equation, we immediately see that the following assumption is required to have positive values of z_t:

Assumption 1 (i) $\mu > 0$ and (ii) $\beta(\mu + \alpha\tau^e) > \delta$.

We are now able to define an intertemporal equilibrium:

Definition 1 *Under Assumption 1, an intertemporal equilibrium is characterized as a sequence of investment factors $(z_t)_{t=1}^{\infty}$, such that (3) is satisfied, given $z_0 > 0$.*

Dynamics are driven by a one-dimensional dynamic system, where z_t is a predetermined variable. Note that z_0 given implies two initial conditions k_0 and k_{-1}. In fact, the second initial condition comes from the initial condition on debt $B_{-1} \geqslant 0$ and the constant debt-output ratio $B_t = \delta Y_t$. Given the sequence (z_t), we are able to determine (k_t) defined by $k_{t+1} = z_{t+1} k_t^{\alpha}$. Finally, given (k_t), one deduces the dynamics of e_t.

The steady-state investment factors are solutions to:

$$P(z) = z^2 - \frac{\beta(\mu + \alpha\tau^e) - \delta}{1+\beta} z + \frac{\alpha\beta\delta(1-\tau^e)}{1+\beta} = 0 \tag{4}$$

Note that the corresponding stationary level of capital (k) and environmental quality (e) per capita are given by:

$$k = z^{\frac{1}{1-\alpha}} \tag{5}$$

$$e = \frac{\theta g - \epsilon}{\eta} z^{\frac{\alpha}{1-\alpha}} \tag{6}$$

Steady states and dynamics are determined as follows:

Proposition 1 *Let Assumption 1 be satisfied and $\bar{\delta}$ such that $\beta(\mu + \alpha\tau^e) = \bar{\delta} + 2\sqrt{\alpha\beta\bar{\delta}(1+\beta)(1-\tau^e)}$. When $\delta \in [0, \bar{\delta})$, there are two steady states, an unstable one $\underline{z}$ and a stable one $\overline{z}$, given as:*

$$\underline{z} = \frac{\beta(\mu + \alpha\tau^e) - \delta - \sqrt{\{\beta(\mu + \alpha\tau^e) - \delta\}^2 - 4(1+\beta)(1-\tau^e)\alpha\beta\delta}}{2(1+\beta)} \tag{7}$$

$$\overline{z} = \frac{\beta(\mu + \alpha\tau^e) - \delta + \sqrt{\{\beta(\mu + \alpha\tau^e) - \delta\}^2 - 4(1+\beta)(1-\tau^e)\alpha\beta\delta}}{2(1+\beta)}$$

$$\equiv \zeta(\tau^e, g, \delta) \tag{8}$$

When $\delta = \bar{\delta}$, a saddle-node bifurcation occurs and no steady state exists when $\delta > \bar{\delta}$.

Proof. The existence of two steady states requires that the discriminant of $P(z)$ must be positive, that is, $\{\beta(\mu + \alpha\tau^e) - \delta\}^2 > 4\alpha\beta\delta(1+\beta)(1-\tau^e)$. Under Assumption 1, this condition can be reduced to $\{\beta(\mu + \alpha\tau^e) - \delta\} > 2\sqrt{\alpha\beta\delta(1+\beta)(1-\tau^e)}$. This defines an upper bound $\bar{\delta}$ lower than $\beta(\mu+\alpha\tau^e)$ (Assumption 1). When $\delta < \bar{\delta}$ there exist two steady states, given by (7) and

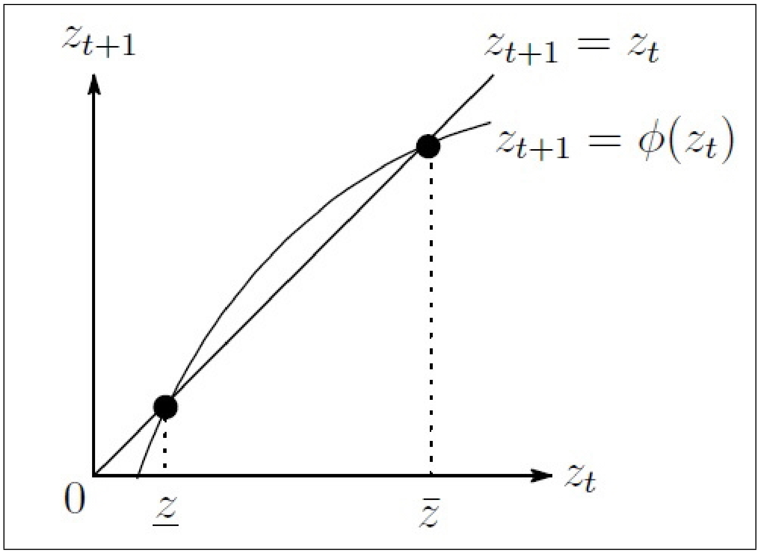

Figure 1: Dynamics with two steady states.

(8). When $\delta = \overline{\delta}$, the two steady states merge, and disappear for $\delta > \overline{\delta}$. We deduce the stability properties from the features of (3). Since

$$\lim_{z_t \to 0} \phi(z_t) = -\infty, \quad \lim_{z_t \to +\infty} \phi(z_t) = \frac{\beta(\mu + \alpha\tau^e) - \delta}{1 + \beta} > 0,\ \phi(z_t)' > 0,\ \phi(z_t)'' < 0$$

the lower steady state is unstable, whereas the larger one is stable (see Figure 1). ■

The configuration where there are two steady states ($\delta < \overline{\delta}$) is represented in Figure 1.[2] The lower steady state $\underline{z}$ is unstable, while the higher one $\overline{z}$ is stable. Therefore, for z_t lower than $\underline{z}$, the economy is relegated to a poverty trap, where z_t decreases to 0. Otherwise, the economy converges to the steady state $\overline{z}$. Note that since $k_{t+1} = z_{t+1}k_t^\alpha$, the convergence of the investment factor to a stationary value corresponds to the convergence of the capital stock k_t to its steady state level.

By direct inspection of (4), we see that without debt ($\delta = 0$), the trap disappears. The dynamics become $z_{t+1} = \frac{\beta(\mu+\alpha\tau^e)}{1+\beta}$ and may be explicitly

[2]Recall that, as it is clear from the proof of the proposition, δ lower than $\overline{\delta}$ satisfies Assumption 1.

given by a sequence of per capita capital (k_t) satisfying $k_{t+1} = \frac{\beta(\mu+\alpha\tau^e)}{1+\beta} k_t^\alpha$. Therefore, the higher steady state $k = \left(\frac{\beta(\mu+\alpha\tau^e)}{1+\beta}\right)^{1/(1-\alpha)}$ is globally stable, whereas the lower steady state $k = 0$ is unstable. There is no more any poverty trap. Indeed, the trap comes from a crowding-out effect due to the existence of public debt.

Using (3), we also deduce that a larger public spending-output ratio reinforces the level of the trap. Indeed, we can show that $\partial\overline{z}/\partial g < 0$, $\partial\underline{z}/\partial g > 0$ and $\partial\phi(z_t)/\partial g < 0$ for all $z_t > 0$. A larger income taxation is needed to balance the budget, implying a lower saving. For $g > \epsilon/\theta$, the associated level of environmental quality always raises at the low steady state, whereas it raises at the high steady state if the efficiency of public spending θ is large enough.

In the next sections, we will focus on the effect of fiscal policy on a steady state. Because z_t is a predetermined variable, we focus on the stable steady state $\overline{z}$.[3]

4 Environmental tax reform balanced by labor income taxation

We are interested in the effect of an increase of environmental taxation, given that public spending-output and debt-output ratios are constant. Therefore, income taxation will vary to balance the budget, modifying the level of the investment factor. We will focus on a possible improvement of both environmental quality and macroeconomic variables, i.e. capital accumulation, aggregate consumption.

As a preliminary step, we examine the effects of such an increase of environmental taxation on the investment factor and the labor income tax rate.

Proposition 2 *Let Assumption 1 be satisfied and assume $\delta \in [0, \bar{\delta})$. Following a raise of the environmental tax rate τ^e, the investment factor $\overline{z}$ increases, while the labor income tax rate τ^w evaluated at this steady state decreases.*

Proof. The effect on the steady state investment factor is derived by differ-

[3] To clarify notations, $\overline{x}$ will denote in the following the value of the variable x_t evaluated at the steady state $\overline{z}$.

entiating (8):

$$\left.\frac{d\overline{z}}{d\tau^e}\right|_{d\delta=dg=0} = \frac{\partial\zeta(\cdot)}{\partial\tau^e} = \frac{\alpha\beta(\overline{z}+\delta)}{\sqrt{\{\beta(\mu+\alpha\tau^e)-\delta\}^2-4\alpha\beta\delta(1-\tau^e)(1+\beta)}} > 0 \tag{9}$$

The steady state income tax rate is derived as a function of other policy instruments from the government budget constraint (1):

$$\tau^w = \frac{\alpha}{1-\alpha}\frac{\delta}{\overline{z}} + \frac{g-\tau^e-\delta}{(1-\tau^e)(1-\alpha)} \tag{10}$$

Using $\overline{z} = \zeta(\tau^e, g, \delta)$, it can be defined as $\tau^w \equiv \tau(\tau^e, g, \delta)$. By differentiating (10) with respect to τ^e, we obtain:

$$\left.\frac{d\tau^w}{d\tau^e}\right|_{dg=d\delta=0} = -\frac{1}{(1-\alpha)(1-\tau^e)}\left(\frac{1-g+\delta}{1-\tau^e} + \frac{\alpha(1-\tau^e)\delta}{\overline{z}^2}\frac{\partial\zeta(\cdot)}{\partial\tau^e}\right) < 0 \tag{11}$$

■

The environmental tax, in principle, imposes additional costs on polluting behavior, which reduces the steady state investment factor. However, recycling revenues provided from the increase of the environmental tax rate leads to lower income tax rates. Because the latter effect is greater than the former, this environmental tax reform will increase the steady state investment factor. Alternatively, income taxation is more harmful to investment or capital accumulation than environmental taxation. Considering the government budget, note that the decrease of τ^w comes from two direct effects and a general equilibrium effect. The first one is explained by the increase of government revenue coming from a larger environmental tax rate. The second direct effect goes through the fact that a higher environmental tax rate directly decreases the interest rate. Finally, the general equilibrium effect goes in the same direction: a higher level of capital induces a decrease in the interest rate. This leads to a smaller amount of debt reimbursement in the future and, thereby, lowering the income tax rate.

We focus now more specifically on the possible improvement of both environmental quality and macroeconomic variables of the model. Beside an increase in the amount of environmental quality per capita (i.e. $d\overline{e}/d\tau^e > 0$), we are interested in an increase in total amount of consumption per capita $\overline{C} \equiv \overline{c} + \overline{d}$ (i.e. $d\overline{C}/d\tau^e > 0$), called in the "macroeconomic effect". Using the resource constraint $\overline{y} = \overline{c} + \overline{d} + \overline{k} + g\overline{y}$, $\overline{C}$ is given by:

$$\overline{C} = (1-g)\overline{k}^{\alpha} - \overline{k} \tag{12}$$

Given the government policy, $k = \overline{k}_g$ maximizes $\overline{C}$, where:

$$\overline{k}_g = [\alpha(1-g)]^{\frac{1}{1-\alpha}} \tag{13}$$

From (5), the stationary investment factor corresponding to $\overline{k}_g$ is $\overline{z}_g = \alpha(1-g)$. This allows us to show:

Proposition 3 *Let Assumption 1 be satisfied and assume $\delta \in [0, \bar{\delta})$. The environmental tax reform produces positive environmental and macroeconomic effects if and only if (i) the public emission abatement is large enough ($g > \epsilon/\theta$), and (ii) there exists under-accumulation at the stable steady state ($\overline{k} < \overline{k}_g$).*

Proof. Differentiating (6) with $d\delta = dg = 0$, we get:

$$\left.\frac{d\overline{e}}{d\tau^e}\right|_{d\delta=dg=0} = \frac{\alpha(\theta g - \epsilon)}{\eta\overline{z}} \left.\frac{d\overline{k}}{d\tau^e}\right|_{d\delta=dg=0}$$

From (5) and (9), the positive environmental effect is obtained if and only if the public emission abatement is large enough, that is, $g > \epsilon/\theta$.
Differentiating now (12) with $d\delta = dg = 0$, and using (5) and (13), one obtains:

$$\left.\frac{d\overline{C}}{d\tau^e}\right|_{d\delta=dg=0} = \frac{(\overline{z})^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left(\frac{\overline{z}_g}{\overline{z}} - 1\right) \left.\frac{d\overline{z}}{d\tau^e}\right|_{d\delta=dg=0}$$

From (9), the macroeconomic effect is obtained if and only if $\overline{z} < \overline{z}_g$. ■

The environmental tax reform cuts the personal income tax, allowing a larger level of capital per capita. This raises aggregate consumption when there is under-accumulation, explaining the macroeconomic effect. Note that the requirement of under-accumulation of capital seems to be quite realistic, since this is equivalently ensured by a not too low real interest rate, which is experienced by most developed countries in the last decades. Recall that under-accumulation also means dynamic efficiency, which is a feature supported by the findings of Abel *et alii* (1989). Public emission abatements play an important role for the environmental effect. The public spending-output ratio g or efficiency of public emission abatements θ has to be large enough to ensure negative pollution flows. In this case, environmental quality is positive at the steady state and positively varies with the level of capital.

At this stage, public debt has no yet an explicit role on our results. However, since the macroeconomic effect occurs if and only if there is under-accumulation of capital, we now discuss its implication on the level of debt-ouput ratio δ.

Because $\partial\zeta(\tau^e, g, \delta)/\partial\delta < 0$ for all $\delta < \overline{\delta}$, there exists at most a unique debt-output ratio $\delta = \delta_g$ that corresponds to the maximized level of total consumption per capita:

$$\zeta(\tau^e, g, \delta_g) = \overline{z}_g = \alpha(1-g). \tag{14}$$

We deduce that there is under-accumulation if and only if $\delta > \delta_g$. Then, considering the three cases, (i) $\delta_g \leq 0$, (ii) $\delta_g \geq \overline{\delta}$, and (iii) $\delta_g \in (0, \overline{\delta})$, we clarify in which configuration both positive macroeconomic and environmental effects are fulfilled:

Proposition 4 *Let Assumption 1 be satisfied and assume $\delta \in [0, \overline{\delta})$. There is under-accumulation, and both positive macroeconomic and environmental effects may apply, if one of the following conditions is satisfied:*

(i) $\tau^e \leqslant 1 - \frac{1-g}{\alpha\beta}[\beta - \alpha(1+\beta)] \equiv \hat{\tau}^e$;

(ii) $\frac{\beta}{1+\beta} < 4\alpha(1-\alpha)$ *and* $\delta \in (\delta_g, \overline{\delta})$.

Proof. When $\delta_g \leq 0$, there is under-accumulation for all $\delta \in [0, \overline{\delta})$. However, because $\partial\zeta(\tau^e, g, \delta)/\partial\delta < 0$, $\delta_g \leq 0$ is equivalent to $\zeta(\tau^e, g, \delta_g) \geqslant \zeta(\tau^e, g, 0)$. Using (8) and (14), we deduce case (i) of the proposition.
To prove case (ii), we begin by determining $\overline{\delta}$. From Proposition 1, we recall that $\overline{\delta}$ is the lowest root of:

$$\begin{aligned} & [\beta(\mu + \alpha\tau^e) - \delta]^2 - 4\alpha\beta\delta(1+\beta)(1-\tau^e) = 0 \\ \Leftrightarrow \quad & \delta^2 - 2\beta[\mu + \alpha\tau^e + 2\alpha(1+\beta)(1-\tau^e)]\delta + [\beta(\mu + \alpha\tau^e)]^2 = 0 \end{aligned}$$

We deduce that:

$$\begin{aligned} \overline{\delta} \; = \; & \beta[\mu + \alpha\tau^e + 2\alpha(1+\beta)(1-\tau^e)] \\ & -2\beta\sqrt{\alpha(1+\beta)(1-\tau^e)[\mu + \alpha\tau^e + \alpha(1+\beta)(1-\tau^e)]} \end{aligned} \tag{15}$$

Note that $\delta_g < \overline{\delta}$ is equivalent to $\zeta(\tau^e, g, \delta_g) > \zeta(\tau^e, g, \overline{\delta})$. Using (8) and (14), this inequality rewrites $\overline{\delta} > \beta(\mu + \alpha\tau^e) - 2(1+\beta)\alpha(1-g)$. Substituting (15), one obtains $P(1-\tau^e) > 0$, with:

$$\begin{aligned} P(1-\tau^e) \;\; \equiv \;\; & \alpha\beta^2(1-\tau^e)^2 + \beta(1-g)[2\alpha(1+\beta) - \beta](1-\tau^e) \\ & + \alpha(1+\beta)(1-g)^2 \end{aligned}$$

The discriminant of this polynomial of degree 2 is given by $\beta^3(1-g)^2[\beta - 4\alpha(1-\alpha)(1+\beta)]$. When $\frac{\beta}{1+\beta} < 4\alpha(1-\alpha)$, it is negative, which shows that $P(1-\tau^e) > 0$ for all τ^e. This ensures that $\delta_g < \overline{\delta}$. Therefore, for $\delta \in (\delta_g, \overline{\delta})$, the steady state is characterized by under-accumulation.
Finally, note that of course, the configuration where $\delta_g \geq \bar{\delta}$ is not relevant to get under-accumulation. ■

In configuration *(i)* of Proposition 4, under-accumulation requires a sufficiently low environmental tax rate, *i.e.* $\tau^e \leqslant \hat{\tau}^e$. Following Proposition 2, this implies a sufficiently large labor income tax rate. As we have seen, this last one has a dominant effect on savings, and therefore capital accumulation. This explains that a low τ^e may ensure under-accumulation.

The second configuration of Proposition 4 is also of special interest. Our result requires a sufficiently large level of debt-output ratio. Therefore, debt plays a role. It is useful to notice that the saving rate $\beta/(1+\beta)$ is smaller than $1/2$, while under a standard parametrization, the capital share in income α belongs to $(1/4, 1/2)$. In this case, the inequality $\frac{\beta}{1+\beta} < 4\alpha(1-\alpha)$ is fulfilled and configuration (ii) of Proposition 4 may apply. Note that if $\tau^e > \hat{\tau}^e$ and sufficiently close to $\hat{\tau}^e$, the debt-output ratio does not need to be so large since δ_g tends to zero.

Because the environmental tax reform positively affects the steady-state investment factor $\overline{z}$, the conditions for the double dividend differ from those in the literature. Ono (2005) considers an environmental tax reform that cuts the social security tax in the absence of public emission abatement. Therefore, the environmental dividend is produced only when the capital per capita decreases. Moreover, the non-environmental dividend is obtained because there is over-accumulation at the steady state and capital per worker decreases.

5 Distributive issues

We now focus on the distributive issues of the tax reform balanced by labor income taxation. We investigate both the intergenerational (*i.e.* between generations) and the intragenerational (*i.e.* between life-cycle consumptions) impacts of the policies. First, we analyze if following the implementation of the fiscal reform at a given period, the total amount of consumption increases during all the dynamic path converging to the stable steady state. Second, we study if the tax reform implies an increase of consumption of both young and old at the steady state.

We start by extending our analysis considering an intertemporal equilibrium which is no more stationary, but converges to the stable steady state. The question we address is the following. Assuming that the steady state $\overline{z}$ satisfies the positive effects of the fiscal reform and we apply the fiscal reform at some given date, should the fiscal reform be detrimental for some generations and for the consumption at some periods? Can we determine which generations will benefit from the fiscal reform? These issues are solved in the following proposition:

Proposition 5 *Let Assumption 1 be satisfied and assume that $\delta \in [0, \bar{\delta})$. Furthermore, consider that Propositions 3 and 4 are satisfied, and $z_0 > \underline{z}$. Following an increase of τ^e at date t_0, the positive effects of the fiscal reform fails at $t = t_0$, but there exists $t_1 > t_0$ such that for all $t > t_1$, they occur.*

Proof. Assume that τ_0 increases permanently at $t = t_0$. By direct inspection of equation (3) and Figure 1, we deduce that z_t raises for all $t \geqslant t_0 + 1$. Since $k_{t+1} = z_{t+1} k_t^\alpha$, the same happens for k_t for all $t \geqslant t_0 + 1$.
At $t = t_0$, we have $c_t + d_t = (1 - g)k_t^\alpha - k_{t+1} = k_t^\alpha (1 - g - z_{t+1})$. Since k_t is predetermined and z_{t+1} increases, aggregate consumption falls, which means that the positive effects of the fiscal reform fail.
Considering now that $t \geqslant t_0 + 1$, $d(c_t + d_t) = (1 - g)\alpha k_t^{\alpha-1} dk_t - dk_{t+1}$. For t sufficiently large, namely $t > \widetilde{t}$, the capital stock is characterized by under-accumulation,[4] i.e. we have $(1 - g)\alpha k_t^{\alpha-1} > 1$, which implies that $d(c_t + d_t) > dk_t(1 - \frac{dk_{t+1}}{dk_t})$. Since at a stable equilibrium with $z_t = \overline{z}$ we have $k_{t+1} = \overline{z} k_t^\alpha$, there exists $t > \widehat{t}$ such that $dk_{t+1}/dk_t < 1$.
Therefore, when $t > t_1 = \max\{\widetilde{t}, \widehat{t}\}$, $d(c_t + d_t) > 0$ because $dk_t > 0$ for all

[4] This is always the case if the sequence of (k_t) is increasing though time.

$t \geqslant t_0 + 1$. Since $e_{t+1} = (1-\eta)e_t + (\theta g - \epsilon)k_t^\alpha$ also raises, the positive effects of the fiscal reform apply for all $t > t_1$. ■

One may note that aggregate consumption decreases at some periods, because consumption when old does. In fact, at $t = t_0$, consumption when young raises because, given the level of capital, the increase of the environmental tax rate reduces the labor income tax rate, which enhances the after-tax income to consume. In contrast, consumption when old goes down, because a higher environmental tax rate reduces the interest rate, i.e. the remunerated saving. At the following periods, the increase of c_t is reinforced by the raise of capital, which pushes up the wage and pushes down labor income taxation. The interest rate becomes even lower because of the larger capital accumulation. After some periods, this mechanism allows to get a positive effect on aggregate consumption.

This intergenerational issue on environmental externalities and taxation has already been widely studied (John and Pecchenino (1994), John *et alii* (1995), Howarth (1996), Fisher and van Marrewijk (1998)). The main result of all these studies is that environmental taxation implies such a welfare loss for the older generations experiencing the fiscal reform that its implementation can not be wished because the generation which would decide it would also bear the heaviest burden. This result of the literature originates in the fact that balanced environmental fiscal reforms have generally not been considered. We show here that this negative result for the political feasibility can be generalized to the balanced-budget reform case.

We now focus on the distributive effect of the policy between consumptions when young and old at the stable steady state. Should the fiscal reform not only improve aggregate consumption but also both $\overline{c}$ and $\overline{d}$? This is an important issue because if this occurs, utility for consumption of young and old consumers increase. Moreover, since under Proposition 4, the environmental tax reform increases utility for environmental quality, this leads to a double dividend.

Proposition 6 *Let Assumption 1 be satisfied and assume that $\delta \in [0, \overline{\delta})$. Furthermore, consider that Propositions 3 and 4 are satisfied. Following an increase of τ^e, the consumption of young $\overline{c}$ is increasing at the stable steady state. For $\mu < \frac{\alpha^2}{1-\alpha}$ and $\tau^e < \alpha - \frac{1-\alpha}{\alpha}\mu$, there exists $\widetilde{\delta} > 0$, such that for all $\delta < \widetilde{\delta}$, the consumption of old $\overline{d}$ is increasing in τ^e too.*

Proof. At the stable steady state, consumption when young $\overline{c}$ is given by:

$$\overline{c} = \frac{1}{1+\beta}\overline{k}^{\alpha}\left[\mu + \alpha\tau^e + \delta - \frac{\alpha\delta(1-\tau^e)}{\overline{z}}\right]$$

Because $\partial\overline{z}/\partial\tau^e > 0$ (see Proposition 2) and $\overline{k} = \overline{z}^{1/(1-\alpha)}$, we deduce that $\partial\overline{c}/\partial\tau^e > 0$.
Using the fact that saving is equal to $\overline{k}+\delta\overline{y}$, consumption when old $\overline{d}$ is equal to:

$$\overline{d} = (1-\tau^e)\alpha\left(\overline{z}^{\frac{\alpha}{1-\alpha}} + \delta\overline{z}^{\frac{2\alpha-1}{1-\alpha}}\right)$$

For $\delta = 0$, we have $\partial\overline{z}/\partial\tau^e = \alpha\overline{z}/(\mu+\alpha\tau^e)$. We deduce that:

$$\frac{\partial\overline{d}}{\partial\tau^e} = \frac{\alpha\overline{z}^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)(\mu+\alpha\tau^e)}\left[\alpha^2 - (1-\alpha)\mu - \alpha\tau^e\right]$$

Therefore, for $\delta = 0$, $\partial\overline{d}/\partial\tau^e > 0$ if and only if $\mu < \frac{\alpha^2}{1-\alpha}$, or equivalently $1-\alpha > g > 1-\alpha-\frac{\alpha^2}{1-\alpha}$, and $\tau^e < \alpha - \frac{1-\alpha}{\alpha}\mu$.
In this case, by continuity, there exists $\widetilde{\delta} > 0$, such that for all $\delta < \widetilde{\delta}$,[5] we have $\partial\overline{d}/\partial\tau^e > 0$. ■

If this proposition applies, welfare is increasing. Indeed, following an increase of the environmental tax rate, environmental quality, consumptions when young and old become larger. We obtain a double-dividend. Note that this requires a sufficiently low debt-output ratio. Otherwise, the increase of the environmental tax rate that decreases the return of assets, directly and through the raise of capital, implies a too large decrease of the remunerated debt. In this case, remunerated saving, i.e. consumption when old, decreases.

6 Environmental tax reform balanced by debt-output ratio

In our model, the government issue debt to finance current deficits. Instead of assuming that a larger environmental tax rate may be used to reduce

[5] Note that this condition is compatible with case (i) of Proposition 4 and case (ii) for τ^e sufficiently close to $\hat{\tau}^e$.

income taxation, the environmental policy can also be used to modify the debt-output ratio. We will address this issue now and investigate if such a policy may induce both the positive environmental and macroeconomic effects at the stable steady state.

To be more specific, we consider that following an increase of τ^e, the government budget is balanced by a modification of δ, taking the labor income tax rate as constant. Differentiating (10) with $d\tau^w = dg = 0$, the policy change is described as:

$$\left.\frac{d\delta}{d\tau^e}\right|_{d\tau^w=dg=0} = -\frac{\partial\tau(\cdot)/\partial\tau^e}{\partial\tau(\cdot)/\partial\delta} \tag{16}$$

Using (6) and (12), we note that debt-output ratio affects aggregate consumption and environmental quality only through the investment rate z. Therefore, using Proposition 3, the positive environmental and macroeconomic effects of the fiscal reform may be obtained if $\overline{z}$ is increasing following the new policy.

Proposition 7 *Let Assumption 1 be satisfied and assume $\delta \in [0, \overline{\delta})$. Consider an increase of environmental taxation budget-balanced by a variation of the debt-output ratio. If $\tau^e > 1 - (1-g)\beta/\alpha$, there is $\widehat{\delta} \in (0, \overline{\delta})$, such that $\overline{z}$ is increasing in τ^e if $\delta < \widehat{\delta}$. This goes through a decrease of δ. In this case, the environmental tax reform produces positive environmental and macroeconomic effects if and only if (i) the public emission abatement is large enough ($g > \epsilon/\theta$), and (ii) there exists under-accumulation at the stable steady state ($\overline{k} < \overline{k}_g$).*

Proof. To investigate the effect of the increase of τ^e on $\overline{z}$, we first note that:

$$\left.\frac{d\overline{z}}{d\tau^e}\right|_{d\tau^w=dg=0} = \frac{\partial\zeta(\cdot)}{\partial\tau^e} + \frac{\partial\zeta(\cdot)}{\partial\delta}\left.\frac{d\delta}{d\tau^e}\right|_{d\tau^w=dg=0}$$

where $\partial\zeta(\cdot)/\partial\tau^e > 0$ is given by (9) and

$$\frac{\partial\zeta(\cdot)}{\partial\delta} = -\frac{\overline{z} + \alpha\beta(1-\tau^e)}{\sqrt{\{\beta(\mu+\alpha\tau^e)-\delta\}^2 - 4\alpha\beta\delta(1-\tau^e)(1+\beta)}} < 0 \tag{17}$$

Therefore, a sufficient condition to have $d\overline{z}/d\tau^e > 0$ is $d\delta/d\tau^e < 0$. Using

(11) and (16), we deduce that $d\delta/d\tau^e$ and $\partial\tau(\cdot)/\partial\delta$ have the same sign. To find it, a derivative of (10) with respect to the debt-output ratio gives:

$$\frac{\partial\tau(\cdot)}{\partial\delta} = \frac{\alpha}{1-\alpha}\frac{1}{\overline{z}}\left(1 - \frac{\delta}{\overline{z}}\frac{\partial\zeta(\cdot)}{\partial\delta}\right) - \frac{1}{(1-\tau^e)(1-\alpha)} \tag{18}$$

Using (17) and (18), $\partial\tau(\cdot)/\partial\delta < 0$ is equivalent to $\psi(\delta) < 1$, with:

$$\psi(\delta) \equiv \frac{\alpha(1-\tau^e)}{\overline{z}}\left[1 + \delta\frac{1+\alpha(1-\tau^e)\beta/\overline{z}}{\sqrt{[\beta(\mu+\alpha\tau^e)-\delta]^2 - 4\alpha\beta\delta(1-\tau^e)(1+\beta)}}\right]$$

We can easily show that $\psi'(\delta) > 0$. Moreover, using (8), we have:

$$\begin{aligned} \psi(\overline{\delta}) &= +\infty \\ \psi(0) &= \frac{\alpha(1+\beta)(1-\tau^e)}{\beta(\mu+\alpha\tau^e)} \end{aligned}$$

Therefore, if $\psi(0) < 1$, there exists $\widehat{\delta} \in (0, \overline{\delta})$ such that $\psi(\delta) < 1$ for all $\delta < \widehat{\delta}$. Using Proposition 3, we deduce the proposition.[6] ■

Under Proposition 3, the positive environmental and macroeconomic effects occur if the fiscal reform raises z (or k). Here, a larger environmental tax rate implies a variation of the debt-output ratio, the income tax rate staying constant. We show that when a larger τ^e implies a lower debt-output ratio δ, capital accumulation raises. Since the income tax rate is constant, the debt-output ratio modifies k or z through the level of the crowding-out effect only. This configuration is especially interesting because the fiscal policy allows to improve aggregate consumption and environmental quality by reducing the debt-output ratio. Regarding the debt sustainability constraints faced by many countries today, this fiscal reform gives rise to a third dividend.

[6] Note that in order to ensure under-accumulation, the conditions obtained in this proposition should be in accordance with Proposition 4. Case (i) of Proposition 4 is satisfied for $1-(1-g)\beta/\alpha < \tau^e \leqslant \hat{\tau}^e$. This requires $\alpha > \beta(1-\beta)/(1+\beta)$, which is satisfied for β sufficiently close to 1. Case (ii) of Proposition 4 is fulfilled under the same condition on α, but τ^e larger and close to $\hat{\tau}^e$.

7 Conclusion

This paper examines the effects of environmental tax reform in an overlapping generations model by taking into account a debt stabilization constraint and public pollution abatement. We show that, when the budget-neutral reform allows a decrease in the income tax, the steady state investment factor increases. This result implies an increase in the pollution emission, the (first) environmental dividend cannot be obtained in the absence of public abatement. On the other hand, the second (*i.e.* economic) dividend is obtained when the economy is characterized by under-accumulation of the per worker capital stock. Finally, an increase of the environmental taxation budget-balanced by a variation of the debt-output ratio may also increase the environmental quality and the aggregate consumption. Hence, the fiscal policy allows to improve aggregate consumption and environmental quality by reducing the debt-output ratio.

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