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Economics
The Open-Access, Open-Assessment E-Journal

Vol. 8, 2014-23 | June 16, 2014 | http://dx.doi.org/10.5018/economics-ejournal.ja.2014-23

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Abstract
This paper analyses the profitability of horizontal mergers in a Stackelberg model and their impact on welfare when there is uncertainty about the marginal costs of the newly merged firms. The authors consider that the merging firms decide their production strategy knowing the actual value of the production cost, while outsiders are a priori uncertain about the exact amount of cost efficiency/inefficiency that will result from the merger. Nevertheless, the key element of the model is that the merged entity can signal its own cost to some rivals (outsider-followers) when it behaves as a leader; while all outsiders remain uninformed when it behaves as a follower. They show that when there is role redistribution, the merging firms always have incentives to merge, irrespective of cost uncertainty, while a merger without role redistribution is ex ante profitable if and only if uncertainty is sufficiently great. As regards the social desirability of mergers, it is found that a merger between leaders always enhances welfare if participants have incentives to merge, such that private and collective interests coincide. Nevertheless, a merger with role redistribution leads to conflict between private and collective interests.

JEL D21 D80 L20 L40
Keywords Merger; competition authorities; uncertainty; asymmetric information

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Citation Nicolas Le Pape and Kai Zhao (2014). Horizontal Mergers and Uncertainty. *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 8, 2014-23. http://dx.doi.org/10.5018/economics-ejournal.ja.2014-23

Received November 27, 2013 **Published as Economics Discussion Paper** December 10, 2013
Revised April 25, 2014 **Accepted** June 10, 2014 **Published** June 16, 2014

1 Introduction

Following the seminal paper of Salant, Switzer and Reynolds (1983), many authors take for granted the efficiency gains associated with horizontal mergers, or consider that all competing firms have perfect knowledge about the future cost of the merged entity. In practice, outsiders and Competition Authorities[1] have a great deal of difficulty in knowing the exact future efficiency gains (or losses) when competing firms decide to merge. Merged firms are not just larger firms, but become more complex organizations. For instance, mergers create additional uncertainty for employees because of potential clashes of cultures and of management styles. This uncertainty can lead to dysfunctional outcomes: stress, job dissatisfaction, low trust in the organization, and a greater readiness to consider leaving the organization. These dysfunctionalities can, in turn, diminish productivity and increase production cost[2] (Morán and Panasian, 2005). On the other hand, improvement in productive efficiency could also result from a better allocation of resources within the merging firm. One can also imagine that the merger causes efficiency gains by increasing the incentives for the merging parties to invest in cost-reducing R&D.

We assume that the merger generates either potential efficiency gains or potential efficiency losses, and that there is uncertainty over the future production cost of the merged entity. We develop a Stackelberg model where, in the *pre-merger* game, firms are assumed to adopt asymmetric behaviors. Some firms play a leader strategy, since they are able to engage in irreversible action (R&D effort, advertising expenditure, financial decisions, managerial contracts...) and may benefit from a first-mover advantage. As regards the informational structure, we assume that the insider benefits from the *first-to-know* advantage of its own actual cost, since outsiders are unaware of this cost. Nevertheless, even if the cost information of the merged entity is private, the insider may choose to transmit its private information through its market conduct. Indeed when the insider behaves as a leader, followers observe perfectly the output level of the merged firm and infer the exact value of the merged firm's cost. Consequently, this information structure is quite different from the one proposed by Amir et al. (2009) and Hamada (2012), where after the merger all outsiders are uninformed about the merged firm's costs. We depart from their framework[3] by highlighting the role of sequentiality in output decision. The behavior of the merged entity defines the distribution of information amongst the non-merged firms: a leader strategy chosen by the insider generates asymmetric information amongst outsiders (outsider-followers are aware of insider's cost, while the outsider-leaders are not informed about it) and there is symmetric information amongst outsiders when the insider behaves as a follower. When the merged firm plays a follower role, the information gap between outsiders disappears, and all outsiders remain uninformed about the actual costs of the merged entity.

The framework of our model is related to two strands in the literature. The first typically focuses on the relationship between timing (simultaneous versus sequential decisions) and merger incentives in a context of deterministic markets. In the absence of uncertainty, and when a

[1]Merging firms in general have strong incentives to overestimate these gains in front of Competition Authorities.

[2]The failure of consolidation in the bio-technology and oil industries is a good illustration of this phenomenon and reflects the growing uncertainty caused by the rising R&D investments and raw-material costs.

[3]Hamada (2012) examines the effect of uncertainty on horizontal mergers, and focuses on the horizontal merger in a Cournot fashion. He uses a binomial distribution under which the merged firm will experience efficiency gains.

merged firm changes its behavior from a Cournot-Nash player to a Stackelberg leader, Levin (1990) shows that the private incentive to merge is higher, and the antagonism between the private and the collective advantage of the merger disappears. In a game where asymmetric roles among firms in the *pre-merger* situation (Stackelberg competition) are introduced, mergers can also improve welfare and boost profit: even without cost savings following the merger, if two followers decide to merge, and when the newly merged entity behaves as a leader, social welfare and the merging firms' profits increase (Daughety, 1990). In Stackelberg markets with linear costs, two leaders rarely have an incentive to merge, nor do two followers when the new entity stays in the same category (Huck, Konrad and Mueller, 2001). The second strand includes authors who introduce uncertainty into a merger game, but mainly consider that the output (or price) decision-making process is simultaneous (Cournot or Bertrand). It is generally admitted that the scope of profitable merger increases with uncertainty (Choné and Linnemer, 2008; Zhou, 2008a and 2008b, Amir et al., 2009). Banal-Estanol (2007) investigates merger incentives under cost uncertainty, and concludes that uncertainty always enhances merger incentives if the signals are privately observed. Hamada (2012) considers a Cournot oligopoly model with homogeneous products in a context of cost uncertainty, but he does not recognize the impact of the distribution of roles in the industry. He shows that increased uncertainty itself can prompt firms to merge. Our model can be viewed as a reassessment of the model developed by Hamada (2012) by investigating the extent to which uncertainty may alter incentives to merge. In a Stackelberg model Cunha, Sarmento and Vasconcelos (2014) consider the role of uncertainty in horizontal merger games where leaders compete with followers. But in contrast to our model, they consider that uncertainty over the production cost of the merged entity affects all players, including insiders who remain uninformed about the actual value of production costs in the *post merger* game. Insofar as we want to capture the mechanism of inferring the actual value of merged firm's costs when the insider behaves as a leader, we assume that the insider knows the actual value of its cost (when it decides its level of production).

In order to capture the impact of role distribution, we consider four scenarios of two-firm mergers: a merger between two leaders, a merger between two followers, a merger between one leader and one follower giving rise to a leader, and a merger between followers resulting in a leader.[4] The first two scenarios refers to the case where there is no role redistribution due to the merger, in the sense that both firms engaged in the merger keep the same role as in the *pre-merger* game. By contrast, the third scenario reflects the situation where only one firm changes the role (Huck, Konrad and Muller, 2001); the fourth one reflects the situation where the two followers merge and the result is a firm that behaviorally is a leader. For this particular class of mergers, in a deterministic environment it is found that if a merger is welfare increasing, it is profitable for the merging firms (Daughety, 1990). The scenario of a merger between a leader and a follower has been observed during the mid-1990s in the automobile industry, where Chrysler (a leader firm in the US market) decided to merge with Daimler-Benz (a follower with a very small market share) giving rise to the merged company Daimler-Chrysler (fifth largest automaker). The scenario of a merger between two followers resulting in a leader firm can also

[4]The reason that we focus only on bilateral merger is explained by some illustrations in automotive domain, e.g. Daimler-Chrysler in 1998, Porsche-VW during 2004-2008, Chrysler-Fiat in 2009, etc. From the theoretical viewpoint, Zhou (2008a) demonstrates that two-firm mergers are far more frequent than three- or four-firm mergers.

be illustrated by mergers and acquisitions observed in the American pharmaceutical industry. The merger in 2000 between Pharmacia AB and Upjohn Co, two competing companies with small market shares, has resulted in a new strong competitor with a high growth rate.

We show that the two-follower merger aiming to be a leader is more likely than one choosing a follower strategy. In the absence of role redistribution (all insider stay in the same category), we demonstrate that the merged firm has an interest in pooling private information about its production costs with outsiders; however, in the presence of role redistribution concealment is more profitable from the viewpoint of insider.

As regards "Merger Approval", Competition Authorities have to decide whether to approve or refuse a merger proposal without knowing[5] the actual costs of the merged entity. A merger between leaders always enhances welfare, as long as the participants have incentives to merge. This generates a unanimity of private and collective incentives, and it provides support for *laissez-faire* policy. Nevertheless, a merger with role redistribution may lead to conflict between the private and the collective, since some mergers that had been expected to be profitable may decrease expected welfare.

Ex post policy intervention is also used by Competition Authorities to judge the completed merger. According to Ottaviani and Wickelgren (2011), Competition Authorities can employ a "*wait and see*" approach, letting the merger go through in order to gain more accurate information about it. By studying two alternative criteria using two different timing[6] for policy intervention, we find that the timing of policy intervention has important implications for the choice between two possible welfare standards: the consumer welfare standard is more rigorous than the aggregate welfare standard in the case of *ex ante* intervention, while the consumer welfare standard becomes more lenient under *ex post* enforcement. Since prudent Competition Authorities (using *ex ante* intervention) should adopt a restrictive policy, our framework gives a supplementary reason to explain why US Horizontal Merger Guidelines and EC Merger Regulation are biased in favor of the consumers' interests.

The reminder of the paper is organized as follows. Section 2 presents the model and specifies the sub-game perfect equilibria for different scenarios of mergers. Section 3 analyzes the "Ex ante and Ex post profitability of merger". Section 4 investigates the welfare implications of mergers and also devotes some attention to Competition Authorities' distinct criteria (aggregate welfare standard, or consumer welfare standard), assuming that Competition Authorities may adopt an *ex post* enforcement. Finally, Section 5 discusses the main findings and concludes the paper. All proofs and some detailed expressions are in the Appendix.

2 Model

The timing of this game is summarized in the diagram (Figure 1) which shows both the decision structure and the information structure in a time dimension. *Pre-merger* competition is modelled as a standard Stackelberg game with complete information on marginal costs to all active firms.

[5] See US Merger Guidelines Section 4. Merging parties, arguably, know more about potential efficiency gains than Competition Authorities. Firms have strong incentives to dissemble about efficiency.

[6] See *ex ante* versus *ex post* merger control in Ottaviani and Wickelgren (2010).

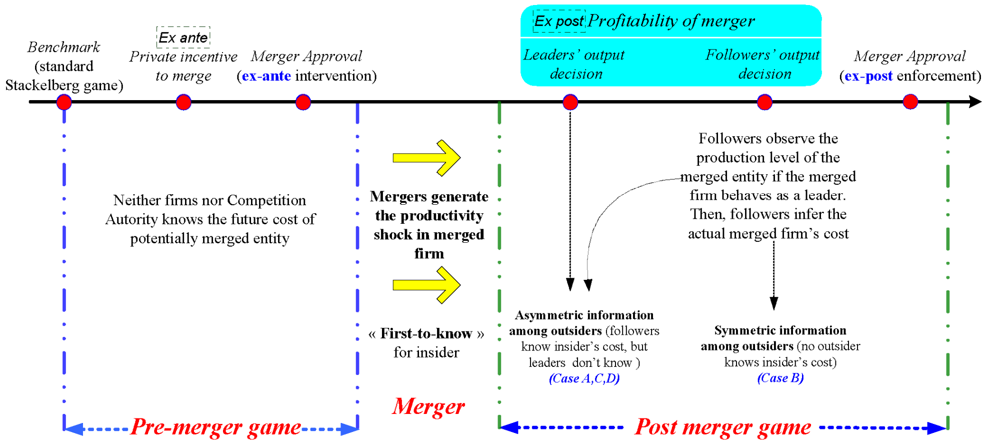

Figure 1: Game structure

By assumption, the merger may generate either efficiency gains or losses, and there is some uncertainty about what the actual value of the insider's marginal cost will be.

At the point of "*Ex ante* profitability of merger", all firms (including the merging firms) in an industry face uncertainty as to the efficiency gains that the merged firm could achieve. Thus merging firms evaluate the profitability of the merger without knowing their actual cost in the *post merger* game. We consider two alternative timings of antitrust intervention. *Ex ante* intervention refers to the situation where the Competition Authorities must decide the approval of the merger facing cost uncertainty. *Ex post* enforcement corresponds to the situation where the insider signals its private information through its market conduct and thus the Competition Authorities may infer[7] the production cost of the insider.

In the *post merger* game, we consider that the insider defines its production strategy knowing the value of its new marginal cost, while outsiders are *a priori* uninformed on the insider cost. Nevertheless, the strategic behavior of the merged entity can alter the outsider firms' information configuration: if the insider adopts a leader strategy then he creates asymmetric information amongst non-merged firms. Outsider-followers perfectly observe the output level of merged firm and infer the actual value of merged firm's costs. Indeed, the outsider-follower firms are aware of insider's costs, but outsider-leader firms are not informed about it. When the insider behaves as a follower, the information amongst outsiders is symmetric. We follow a body of literature which identifies different types of merger that may occur in Stackelberg competition. Following a standard approach in the literature (Daughety, 1990, Feltovich, 2001, Huck et al., 2001, Escrihuela-Villaraud and Fauli-Oller, 2008, Heywood and Mc Ginty, 2008, Brito and Cataleo-Lopes, 2011) we carefully examine four possible merger cases: a merger between two leaders (case A), a merger between two followers (case B), a merger between two followers

[7]We suppose that Competition Authorities incur no costs in acquiring information on the costs of the merged entity.

resulting in a newly-merged leader (case C) and a merger between one leader and one follower resulting in a newly-merged leader (case D).

2.1 The *pre-merger* game

The industry is composed of n initially active firms producing homogenous products who compete by setting quantity schedules. In the first stage, $m < n$ firms act as Stackelberg leaders and independently decide on their individual supply. In the second stage, $n-m$ Stackelberg followers make their quantity decisions after learning about the total quantity supplied by the leaders. Initially, we assume $m > 2$ and $n-m > 2$, the strict inequalities ensuring that in every case the outsiders include both leader and follower in the *post merger* situation.[8] All firms face the same constant average cost normalized to c. The market price is determined by the linear inverse demand curve $p = a - Q$ where $a > c$. The aggregate industry output is given by $Q = Q^l + Q^f$ with $Q^l = \sum_{i=1}^{m} q_i^l$ and $Q^f = \sum_{i=m+1}^{n} q_i^f$, q_i denotes the firm i's individual quantity. The superscript "l" stands for a leader and "f" represents a follower.

The equilibria are obtained by backward induction. At the second stage, each follower maximizes its profit (π_i^f) considering as given the production level of leader (Q^l). The best response function (q_i^f) of a follower firm results from:

$$\max_{q_i^f} \pi_i^f = (a - Q^l - Q^f - c)q_i^f$$

At the first stage, a leader selects its profit-maximizing output (q_i^l) anticipating the best response function of each follower:

$$\max_{q_i^l} \pi_i^l = \left[a - c - Q^l - Q^f(Q^l)\right] q_i^l$$

In the *pre-merger* game, the corresponding individual outputs and profits are:

$$q_i^l(m) = \frac{a-c}{m+1} \qquad \pi^l(n,m) = \frac{(a-c)^2}{(m+1)^2(n-m+1)}$$

$$q_i^f(n,m) = \frac{a-c}{(n-m+1)(m+1)} = \frac{1}{n-m+1} q_i^l$$
$$\pi^f(n,m) = \frac{(a-c)^2}{(m+1)^2(n-m+1)^2} = \frac{1}{n-m+1} \pi^l$$

[8] The particular cases: both $m = 0$ and $m = n$ correspond to a Cournot industry, the firms are in the simultaneous game. Stackelberg and Cournot models are similar, because in both competition is over quantity. However, as we have seen, the first move gives the leader in Stackelberg a crucial advantage. There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.

Obviously the distribution of roles among firms exhibits the first mover advantage:[9] each leader benefits from a higher market share and earns higher profits.

2.2 Merger scenarios

We focus on a bilateral (two-firm) merger. We suppose that the expected marginal cost of the merged firm is equal to the non-merged firm's cost "c" which is the same[10] as in the *pre-merger* game. The actual cost of the newly merged entity's "c_i" is random. Hence, we assume that $a > \max\{c, c_i\}$ and the variance of this random cost c_i is $Var(c_i) = \sigma^2$. The variance σ^2 represents the degree of uncertainty and captures the fluctuation of marginal costs. The merging firms generates efficiency gains if $c_i - c < 0$. This situation corresponds to the usual argument which emphasises the increase in productive efficiency generated by the merger itself. Conversely, when $c_i - c > 0$ the merger is assumed to cause efficiency losses (due to the clash of company culture, for instance).

Case A: Merger between two leaders

In this case, the industry is composed of $m-1$ leaders but still $n-m$ followers, since the newly merged entity behaves like a leader. Let $q_t^{j,i}$ represent the firm's output, the superscript $j = \{l, f\}$ stands for the firm's role (leader or follower), and the superscript $i = \{A, B, C, D\}$ corresponds to one of the four possible cases; the subscript $t = \{I, O\}$ signifies the firm's status (Insider or Outsider). For instance, consider $q_I^{l,A}$ as the merged firm's quantity, $q_O^{l,A}$ as outsider-leader firm's output and $q_O^{f,A}$ as outsider-follower's output. From the standpoint of information structure, since the insider is the first to know its production cost (or productivity), its output level will depend on the actual cost (c_i), namely $q_I^{l,A}(c_i)$; outsider-followers observe the output level of the insider and then perfectly infer the merged entity's cost, accordingly $q_O^{f,A}(c_i)$; since all leaders simultaneously decide the quantity level, outsider-leaders have no opportunity to observe the insider's production. Consequently, the outsider-leaders consider the expected value of the insider's production cost as c, we have $q_O^{l,A}(c)$.

By backward induction, we begin with the follower production stage. The optimizing question is

$$\max_{q_O^{f,A}} \pi_O^{f,A} = (p^A - c)q_O^{f,A} = [a - c - Q_O^{-f,A} - q_O^{f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_O^{f,A}$$

From the first-order-condition, we derive the best response function of followers (See detail in **Appendix A**):

$$(n-m+1)q_O^{f,A} = a - c - Q_O^{l,A}(c) - q_I^{l,A}(c_i) \quad (1)$$

In the first stage, outsider-leaders are not aware of the actual cost for the insider; they therefore consider the insider's cost as the expected value c and maximize the following profit func-

[9] The leader's profit under the sequential-game equilibrium will be higher than under Cournot equilibrium. Since the follower firm reacts in a "Nash fashion", the leader firm could just choose to produce the Cournot output level. In this case, the leader firm would earn exactly the Cournot profit. However, since in the sequential game the leader firm chooses to produce a different output level, it must be increasing its profit compared with the Cournot profit level. This kind of reasoning is called a *revealed profitability* argument.

[10] This assumption allows us to focus on the effect of uncertainty on mergers even without any uncertain efficiency gains.

tion:

$$\max_{q_O^{l,A}} \pi_O^{l,A} = (p^A - c)q_O^{l,A} = [a - c - Q_O^{f,A} - Q_O^{-l,A}(c) - q_O^{l,A} - q_I^{l,A}(c)]q_O^{l,A}(c)$$

For the insider, since it knows the real cost c_i

$$\max_{q_I^{l,A}} \pi_I^{l,A} = (p^A - c_i)q_I^{l,A} = [a - c_i - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c_i)]q_I^{l,A}(c_i)$$

We then obtain the following expressions for the equilibrium output (See detail in **Appendix B**):

$$\begin{aligned} q_I^{l,A}(c_i) &= \frac{2(a-c) - m(n-m+1)(c_i - c)}{2m} \\ q_I^{l,A}(c) &= \frac{(a-c)}{m} \\ q_O^{l,A}(c) &= \frac{(a-c)}{m} \\ q_O^{f,A}(c_i) &= \frac{2(a-c) + m(n-m+1)(c_i - c)}{2m(n-m+1)} \end{aligned} \tag{2}$$

The aggregate quantity is expressed as

$$Q^A = q_I^{l,A}(c_i) + (m-2)q_O^{l,A}(c) + (n-m)q_O^{f,A}(c_i)$$

Both the equilibrium profits and the expected equilibrium profits of firms are given as follows (See detail in **Appendix C**).

Insider:

$$\pi_I^{l,A} = \frac{[2(a-c) - m(n-m+1)(c_i - c)]^2}{4m^2(n-m+1)} \tag{3}$$

$$\mathbb{E}[\pi_I^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)} + \frac{n-m+1}{4}\sigma^2 \tag{4}$$

The merged entity knows its actual cost when it produces. $\pi_I^{l,A}$ represents the exact value of the merged firm's profit which will be used to analyze the *ex post* profitability of the merger. The expected profit of the merged firm is used to evaluate the *ex ante* profitability of the merger.

Outsider-leader:

$$\pi_O^{l,A} = \frac{(a-c)[2(a-c) + m(n-m+1)(c_i - c)]}{2m^2(n-m+1)} \tag{5}$$

$$\mathbb{E}[\pi_O^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)} \tag{6}$$

Outsider-leader firms commit to quantities before uncertainty is resolved; therefore only the expected value of the cost is relevant to them. An increased variance in the cost distribution with the same expected value will not change the profit of outsider-leader firms. Consequently, uncertainty has no effect on them, and each outsider-leader's expected profit is the same as that when the merged firm's cost is deterministic ($c_i = c$).

Outsider-follower:

$$\pi_O^{f,A} = \frac{[2(a-c)+m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)^2} \tag{7}$$

$$\mathbb{E}[\pi_O^{f,A}] = \frac{(a-c)^2}{m^2(n-m+1)^2} + \frac{1}{4}\sigma^2 \tag{8}$$

Insofar as the merged firm acts as a leader, outsider-follower firms know the exact marginal cost of the merged entity. Consequently, the merged firm's and outsider-follower firms can adjust their production accordingly. The sensitivity of firms' expected profits to uncertainty shows that the uncertainty effect affects the merged entity's profit more strongly than the outsider (followers) profit.

Consumer surplus (**CS**) and social welfare (**W**) are easily found to be:

$$CS^A = \frac{\{2[1-m(n-m+1)](a-c)+m(n-m+1)(c_i-c)\}^2}{8m^2(n-m+1)^2} \tag{9}$$

$$W^A = CS^A + \pi_I^{l,A}(c_i) + (m-2)\pi_O^{l,A}(c) + (n-m)\pi_O^{f,A}(c_i) \tag{10}$$

By simple calculation, we obtain the following expected values of CS and W.

$$\mathbb{E}[CS^A] = \frac{(a-c)^2[1-m(n-m+1)]^2}{2m^2(n-m+1)^2} + \frac{1}{8}\sigma^2 \tag{11}$$

$$\begin{aligned} \mathbb{E}[W^A] = & \quad \mathbb{E}[CS^A] + \mathbb{E}[\pi_I^{l,A}] + (m-2)\mathbb{E}[\pi_O^{l,A}] + (n-m)\mathbb{E}[\pi_O^{f,A}] \\ = & \quad \tfrac{(a-c)^2}{2}\left[\tfrac{m^2(n-m+1)^2-1}{m^2(n-m+1)^2}\right] + \left(\tfrac{n-m}{2}+\tfrac{3}{8}\right)\sigma^2 \end{aligned} \tag{12}$$

Note that both consumer surplus and social welfare are increasing functions with respect to the variance σ^2. Concretely, we have $\frac{\partial \mathbb{E}[CS^A]}{\partial \sigma^2} = \frac{1}{8}$ and $\frac{\partial \mathbb{E}[W^A]}{\partial \sigma^2} = \frac{n-m}{2} + \frac{3}{8}$. The extent of the uncertainty effect on expected social welfare evidently depends on the number of leaders and followers in the industry. In fact, the more leader firms, the lower the impact of uncertainty on welfare.

Case B: Merger between two followers

In this case, we assume that two followers take part in the merger. It is assumed that the distribution of roles in the industry is not altered by the merger decision, since the merged entity behaves like the two merging firms in the *pre-merger* game. The industry contains $n-1$ firms with m leaders, and neither outsider-leader firms nor outsider-follower firms can infer the actual marginal costs of the merged firm. Therefore, there is an informational symmetry between the outsider-leaders and the outsider-followers. The relevant equilibrium values are shown in Table 1. (See brief demonstration in **Appendix D**)

Table 1: Equilibrium values in case B

	Case B	
Equilibrium	Actual terms[a]	Expected terms[b]
Output	$q_I^{f,B}(c_i) = \frac{2(a-c)-(m+1)(n-m)(c_i-c)}{2(m+1)(n-m)}$ $q_O^{l,B}(c) = \frac{(a-c)}{(m+1)}$ $q_O^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$	$q_I^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$
Profit	$\pi_I^{f,B} = \frac{[2(a-c)-(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)^2}$ $\pi_O^{l,B} = \frac{(a-c)[2(a-c)-(m+1)(n-m)(2-c-c_i)]}{2(m+1)^2(n-m)}$ $\pi_O^{f,B} = \frac{(a-c)[2(a-c)-(m+1)(n-m)(2-c-c_i)]}{2(m+1)^2(n-m)^2}$	$\mathbb{E}[\pi_I^{f,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} + \frac{1}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)} - \frac{a-c}{m+1}$ $\mathbb{E}[\pi_O^{f,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} - \frac{a-c}{(m+1)(n-m)}$
Consumer surplus	$CS^B = \frac{\{2(a-c)[(m+1)(n-m)-1]-(m+1)(n-m)(c_i-c)\}^2}{8(m+1)^2(n-m)^2}$	$\mathbb{E}[CS^B] = \frac{(a-c)^2[(m+1)(n-m)-1]^2}{2(m+1)^2(n-m)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^B = CS^B + \pi_I^{f,B} + m\pi_O^{l,B} + (n-m-2)\pi_O^{f,B}$	$\mathbb{E}[W^B] = \mathbb{E}[CS^B] + \mathbb{E}[\pi_I^{f,B}] + m\mathbb{E}[\pi_O^{l,B}]$ $+(n-m-2)\mathbb{E}[\pi_O^{f,B}]$ $\frac{\partial \mathbb{E}[W^B]}{\partial \sigma^2} = \frac{3}{8}$

[a] **Actual terms** refer to the post merger game where the insider learns its own cost level. Merger profitability and *ex post* merger assessment are analyzed based on these values.

[b] **Expected terms** refer to the pre-merger game where the (merger) participants do not know the future productivity level. The private incentive to merge and *ex ante* enforcement merger control are studied by means of these expected values.

Case C: Merger between two followers resulting in a leader

Consider a special type of merger wherein two followers merge and form a firm behaving like a leader. As a result, there are $m+1$ leaders and $n-m-2$ followers. This case was examined by Daughety (1990) who found that horizontal merger was potentially profitable for the merged firm; and this merger might be advantageous from the viewpoint of social welfare when the insider marginal cost is not affected by the merger. We restudy this scenario by introducing two elements: cost uncertainty and information structure. Of course, the outcome found by Daughety (1990) corresponds to our result in the extreme situation where there is no uncertainty, and information is perfect and complete. The equilibrium values are displayed in Table 2.

Table 2: Equilibrium values in case C

Equilibrium	Case C: Actual terms	Case C: Expected terms
Output	$q_I^{l,C}(c_i) = \frac{2(a-c)-(m+2)(n-m-1)(c_i-c)}{2(m+2)}$ $q_O^{l,C}(c) = \frac{a-c}{m+2}$ $q_O^{f,C}(c_i) = \frac{2a-c[(m+2)(n-m)-m]+(m+2)(n-m-1)c_i}{2(m+2)(n-m-1)}$	$q_I^{l,C}(c) = \frac{(a-c)}{(m+2)}$
Profit	$\pi_I^{l,C} = \frac{[2(a-c)-(m+2)(n-m-1)(c_i-c)]^2}{4(m+2)^2(n-m-1)}$ $\pi_O^{l,C} = \frac{(a-c)[2(a-c)+(m+2)(n-m-1)(c_i-c)]}{2(m+2)^2(n-m-1)}$ $\pi_O^{f,C} = \frac{[2(a-c)+(m+2)(n-m-1)(c_i-c)]^2}{4(m+2)^2(n-m-1)^2}$	$\mathbb{E}[\pi_I^{l,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)} + \frac{(n-m-1)}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)}$ $\mathbb{E}[\pi_O^{f,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)^2} + \frac{1}{4}\sigma^2$
Consumer surplus	$CS^C = \frac{\{2(a-c)[(m+2)(n-m-1)-1]-(m+2)(n-m-1)(c_i-c)\}^2}{8(m+2)^2(n-m-1)^2}$	$\mathbb{E}[CS^C] = \frac{(a-c)^2[(m+2)(n-m-1)-1]^2}{2(m+2)^2(n-m-1)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^C = CS^C + \pi_I^{l,C} + m\pi_O^{l,C} + (n-m-2)\pi_O^{f,C}$	$\mathbb{E}[W^C] = \mathbb{E}[CS^C] + \mathbb{E}[\pi_I^{l,C}] + m\mathbb{E}[\pi_O^{l,C}]$ $+(n-m-2)\mathbb{E}[\pi_O^{f,C}]$ $\frac{\partial \mathbb{E}[W^C]}{\partial \sigma^2} = \frac{n-m}{2} - \frac{5}{8}$

Case D: Merger between one leader and one follower

Finally, we focus on the merger between one leader and one follower (the merged entity behaves like a leader). In the *post merger* market, the number of leaders is the same as that in case B, and the number of leaders equals $m-1$. This case, without taking into account the issue of information sharing and uncertainty, was studied by Huck, Konrad and Muller (2001), who observes that the merger between two firms from different categories increased the joint profits of firms. They compare the profitability of a two-follower merger with that of a leader-follower merger, and show that mergers between a leader and a follower are unambiguously profitable. In a model with random costs, the equilibrium values are shown in Table 3.

It is worth noting that the merged firm's profit, levels of consumer surplus and social welfare (prior to the completion of the merger) are increasing functions with respect to the variance, and thus increase, as uncertainty increases. By comparing the four aforementioned cases, we can make the following remarks:

Remark 1 *i). From the private perspective, cost uncertainty has the greatest impact on the merged firm's expected profit, when this entity is made up of two leaders. By contrast, it generates the weakest effect on expected profit when two followers merge without role redistribution. More precisely,* $\frac{\partial \mathbb{E}[\pi_I^A]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^D]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^C]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^B]}{\partial \sigma^2}$. *ii). In public view, the same ranking is found* $\frac{\partial \mathbb{E}[W^A]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^D]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^C]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^B]}{\partial \sigma^2}$. *iii). Furthermore, the intensity of the impact of uncertainty on the merged firm's profits and on social welfare depends upon the distribution of roles* (n,m)*, except for case B.*

Table 3: Equilibrium values in case D

	Case D	
Equilibrium	Actual terms	Expected terms
Output	$q_I^{l,D(c_i)} = \frac{2(a-c)-(m+1)(n-m)(c_i-c)}{2(m+1)}$ $q_O^{l,D}(c) = \frac{a-c}{m+1}$ $q_O^{f,D}(c_i) = \frac{2(a-c)+(m+1)(n-m)(c_i-c)}{2(m+1)(n-m)}$	$q_I^{l,D}(c) = \frac{(a-c)}{(m+1)}$
Profit	$\pi_I^{l,D} = \frac{[2(a-c)-(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)}$ $\pi_O^{l,D} = \frac{(a-c)[2(a-c)+(m+1)(n-m)(c_i-c)]}{2(m+1)^2(n-m)}$ $\pi_O^{f,D} = \frac{[2(a-c)+(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)^2}$	$\mathbb{E}[\pi_I^{l,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)} + \frac{n-m}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)}$ $\mathbb{E}[\pi_O^{f,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} + \frac{1}{4}\sigma^2$
Consumer surplus	$CS^D = \frac{\{2(a-c)[(m+1)(n-m)-1]-(m+1)(n-m)(c_i-c)\}^2}{8(m+1)^2(n-m)^2}$	$\mathbb{E}[CS^D] = \frac{(a-c)^2[(m+1)(n-m)-1]^2}{2(m+1)^2(n-m)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^D = CS^D + \pi_I^{l,D} + (m-1)\pi_O^{l,D}$ $+(n-m-1)\pi_O^{f,D}$	$\mathbb{E}[W^D] = \mathbb{E}[CS^D] + \mathbb{E}[\pi_I^{l,D}] + (m-1)\mathbb{E}[\pi_O^{l,D}]$ $+(n-m-1)\mathbb{E}[\pi_O^{f,D}]$ $\frac{\partial \mathbb{E}[W^D]}{\partial \sigma^2} = \frac{n-m}{2} - \frac{1}{8}$

In the cases A, C and D, the newly merged firm behaves as a leader, there is asymmetric information between outsider-leaders and outsider-followers. The greater the number of followers $(n-m)$ in a *pre-merger* market, the greater the intensity of uncertainty (regarding the merged firm's profit and welfare). By contrast, when there is symmetric information between outsiders, the impact of uncertainty on a merged firm's profit and on the welfare is independent of the number of followers.

If we now compare the impact of cost uncertainty on expected consumer surplus and on expected social welfare, we can observe that social welfare is more sensitive to uncertainty. Concretely, $\frac{\partial \mathbb{E}(W^i)}{\partial \sigma^2} > \frac{\partial \mathbb{E}(\pi_I^{j,i})}{\partial \sigma^2} > \frac{\partial \mathbb{E}(CS^i)}{\partial \sigma^2}$ $(i = \{A,B,C,D\}$ and $j = \{l,f\})$. This inequality may justify a Competition Policy based on a consumer welfare standard rather than an aggregate welfare standard when under *ex ante* intervention. More detailed analysis will be found in the welfare section. In the following section we provide a detailed account of the consequences of the merger on profits. We distinguish the *ex ante* from the *ex post* profitability of the merger.

3 Merger analysis

We assume that the "private incentive to merge" results from the comparison of the *ex ante* expected profits of the merged firm and the sum of merging parties' profits in the *pre-merger* game. The "*ex post* profitability of the merger" is determined by the difference between the actual profit earned by the newly merged entity and the sum of profits of merging firms in the benchmark case.

3.1 Private incentive to merge: *ex ante* profitability of merger

Let $\Delta^i_{\mathbb{E}[\pi]}$ $(i = \{A, B, C, D\})$ represent the private incentive to merge. The firms have an incentive to merge when $\Delta^i_{\mathbb{E}[\pi]} \geq 0$. The relationship between the merger incentive and cost uncertainty under different scenarios is shown in Table 4.

Table 4: Merger incentive and cost uncertainty

Scenarios	$n \geq 6$ and $3 \leq m \leq n-3$
Case A $\left(\Delta^A_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,A}] - 2\pi^l\right)$	$\Delta^A_{\mathbb{E}[\pi]} \geq 0$ when $\sigma^2 \geq \sigma^2_{\pi_A}$
Case B $\left(\Delta^B_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{f,B}] - 2\pi^f\right)$	$\Delta^B_{\mathbb{E}[\pi]} \geq 0$ when $\sigma^2 \geq \sigma^2_{\pi_B}$
Case C $\left(\Delta^C_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,C}] - 2\pi^f\right)$	$\Delta^C_{\mathbb{E}[\pi]} \geq 0$ always holds true
Case D $\left(\Delta^D_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,D}] - (\pi^l + \pi^f)\right)$	$\Delta^D_{\mathbb{E}[\pi]} \geq 0$ always holds true

With

$$\sigma^2_{\pi_A} = \frac{4(a-c)^2(m^2-2m-1)}{m^2(m+1)^2(n-m+1)^2} > 0$$
$$\sigma^2_{\pi_B} = \frac{4(a-c)^2[(n-m)^2-2(n-m)-1]}{(m+1)^2(n-m)^2(n-m+1)^2} > 0$$

This table shows that under scenarios C and D firms always have incentives to merge, irrespective of the extent of uncertainty. This result is in line with theoretical models of Stackelberg mergers in a deterministic environment. For instance, according to Daughety (1990), when two followers decide to merge and the newly merged entity behaves like a leader on the product market, firms have incentives to merge even without cost-saving (or efficiency gains). In addition, Huck, Konard and Muller (2001) show that the merger between one leader and one follower is profitable, in the absence of an information issue and cost fluctuation. Based on the Table 4, we derive the following proposition:

Proposition 1 *i). When there is role redistribution, the merging firms always have incentives to merge, irrespective of cost uncertainty. ii). A merger without role redistribution is ex ante profitable if the cost uncertainty is sufficiently large, i.e., if* $\sigma^2 \geq \min(\sigma^2_{\pi_A}, \sigma^2_{\pi_B})$.

Proof: See detail in **Appendix H.1**

Proposition 1 shows that the expected profit of the merged firm increases as variance increases. If the merger involves no role redistribution, and when uncertainty is sufficiently large (the extent of the variance exceeding a certain threshold, such as $\sigma^2_{\pi_A}$ and $\sigma^2_{\pi_B}$), the expected profit of the merged firm becomes larger than the sum of individual profits in the *pre-merger* game and firms choose to merge. In other words, in a deterministic environment, only mergers with a redistribution of roles are profitable. When uncertainty increases, we can expect the emergence of more diverse merger scenarios (with and without redistribution of roles). When the variance of a merged entity's cost is close to zero, the firms without role redistribution have no incentive to merge. As the variance grows, the expected profit for merged firms also increases, since the gains from the optimal quantity adjustment for the insider become greater. The redistribution of roles may be viewed as a mechanism that ensures the *ex ante* profitability of merger in markets where uncertainty is very low. These outcomes extend the contributions by Hamada (2012). Banal-Estanol (2007) finds that cost uncertainty always enhances the incentives to merge, and argues that an additional incentive to merge is driven by information sharing. Zhou (2008a) shows that under cost uncertainty merger incentives are reinforced by production rationalization. Moreover, our model emphasises that additional incentives are also created both by role redistribution and informational asymmetry.

3.2 Ex post profitability of merger

The *ex post* profitability of the merger is evaluated by considering the variation in actual profits (Δ^i_π). The difference between the merged firm's actual cost (c_i) for a merger of type i ($i = \{A,B,C,D\}$) and the *pre-merger* cost (c) is defined as "δ^i". For instance, $\Delta^A_\pi = \pi^{l,A}_I(\delta^A,n,m) - 2\pi^l(n,m)$ in the case A. We define δ^A_{sup} as the threshold value of δ^A that separates profitable from unprofitable mergers. When $\delta^A < \delta^A_{sup}$ (respectively $\delta^A > \delta^A_{sup}$) we have $\Delta^A_\pi > 0$ (respectively $\Delta^A_\pi < 0$). In addition, in order to avoid boundary problems in which some firms are inactive, we also define δ^A_{inf} as the value of δ^A below which outsiders are ruled out of the market. It is given by the conditions : $q^{l,A}_O = 0$ and $q^{f,A}_O = 0$. Note that when we have $\delta^A_{inf} < \delta^A < \delta^A_{sup}$, the merger is profitable and two categories of outsider remain in the market.

3.2.1 Incomplete information

With incomplete information, the merged firm knows its own marginal costs, whereas not all outsider firms are aware of the actual costs of the merged entity. In the cases A, C and D, outsider-leader firms are uninformed about the exact value[11] c_i, but outsider-follower firms are aware of c_i. In Table 5, we summarize the ranges of cost variation (δ^i) in different scenarios in which the merger is profitable.

To ensure that none of the outsider firms exit the market and that the merger is profitable, the potential cost change in different scenarios should satisfy the condition that δ^i lies in the interval

[11] In the case B where two followers take part in the merger, all outsider firms are uninformed about the exact value c_i.

Table 5: Merger profitability and potential efficiency gains (or losses)

Scenarios	$n \geq 6$ and $3 \leq m \leq n-3$
Case A $\left(\Delta_\pi^A = \pi_I^{l,A} - 2\pi^l\right)$	$\delta_{inf}^A < \delta \leq \delta_{sup}^A$
Case B $\left(\Delta_\pi^B = \pi_I^{f,B} - 2\pi^f\right)$	$\delta \leq \delta_{sup}^B$
Case C $\left(\Delta_\pi^C = \pi_I^{l,C} - 2\pi^f\right)$	$\delta_{inf}^C < \delta \leq \delta_{sup}^C$
Case D $\left(\Delta_\pi^D = \pi_I^{l,D} - (\pi^l + \pi^f)\right)$	$\delta_{inf}^D < \delta \leq \delta_{sup}^D$

With

$$\delta_{inf}^A = -\frac{2(a-c)}{m(n-m+1)} < 0 \qquad \delta_{sup}^A = \frac{2(a-c)}{m(n-m+1)} - 2\sqrt{2}\frac{a-c}{(n-m+1)(m+1)} < 0$$

$$\delta_{sup}^B = -2\sqrt{2}\frac{(a-c)}{(m+1)(n-m+1)} + \frac{2(a-c)}{(n-m)(m+1)} < 0$$

$$\delta_{inf}^C = -\frac{2(a-c)}{(m+2)(n-m-1)} < 0 \qquad \delta_{sup}^C = -2\sqrt{2}\frac{(a-c)}{(m+1)(n-m-1)\sqrt{n-m-1}} + \frac{2(a-c)}{(m+2)(n-m-1)} > 0$$

$$\delta_{inf}^D = -\frac{2(a-c)}{(m+1)(n-m)} < 0 \qquad \delta_{sup}^D = 2[\frac{a-c}{(m+1)(n-m)} - \frac{(a-c)}{(m+1)(n-m+1)}\sqrt{\frac{n-m+2}{n-m}}] > 0$$

$(\delta_{inf}^i, \delta_{sup}^i]$. Note that there is no constraint on the exit of outsider in case B.

By comparing δ_{sup}^i, we obtain:

$$\delta_{sup}^C > \delta_{sup}^D > 0 > \begin{cases} \delta_{sup}^A > \delta_{sup}^B & \text{if } m \text{ belongs to } [3, \frac{n}{2}) \\ \delta_{sup}^B > \delta_{sup}^A & \text{if } m \text{ belongs to } (\frac{n}{2}, n\text{-}3] \end{cases}$$

Since the values at the upper bound δ_{sup} in case C and in case D are greater than zero, a merger with anticompetitive effects can also lead to efficiency losses. The intriguing result is that the redistribution of roles may permit mergers with efficiency losses to become *ex post* profitable. By contrast, in the absence of the redistribution of roles, mergers that are *ex post* profitable necessarily imply efficiency gains. The redistribution of roles (the acquisition of a leader status following the merger) can be viewed as a strategy to eliminate efficiency losses. If there are more leaders in the *pre-merger* market (*i.e., $m \in (\frac{n}{2}, n-3]$*), a profitable merger between two leaders requires a greater degree of marginal cost reduction in comparison with a profitable merger between two followers. In other words, the conditions for efficiency gains, under which the two-follower merger is profitable, are less restrictive. By contrast, if there are more follower firms in the *pre-merger* market, a two-follower merger needs more efficiency gains to be profitable.

The higher δ_{sup}^i, the greater the permitted potential efficiency losses, the more likely mergers occur with an increase in the firm's marginal costs. Since the merger composed of two followers forming a leader (case C) generates potential efficiency losses higher than the merger between one leader and one follower (case D), to some extent the condition for a profitable merger in case C is less restrictive, and a merger of this type is more likely to take place. This result means that even if the merger leads to efficiency losses, the resulting leader can be profitable due to the effect of role redistribution. It is clear that the two-follower merger aiming at the leader strategy is more likely to take place than one choosing the follower strategy. Profitable mergers of type C are more likely to generate inefficiencies than in case D, since the former brings more advantages to the merged entity by changing the roles of both merging parties from follower to leader in the market. When the redistribution of roles relates to the two firms there is a greater chance that a merger with efficiency losses will occur.

3.2.2 Incomplete *vs.* complete information for all outsiders

Under complete information,[12] we assume that the insider communicates its private information concerning the value of its actual marginal costs, so that all players are aware of the value of c_i. This assumption allows us to consider whether the merged firm is interested in revealing its own costs to competing firms[13] or not.

Consider $\hat{\pi}_I^{j,i}$ ($i = \{A, B, C, D\}$ and $j = \{l, f\}$) the merged firm's profits under complete and perfect information (see expressions of $\hat{\pi}_I^{j,i}$ in **Appendix E**). It will be interesting to compare the profit of the insider in the incomplete information scenario with that of the complete information scenario (see Proposition 2).

Proposition 2 *Within the range of* $\delta^i \in (\delta^i_{inf}, \delta^i_{sup}]$*, i). the merged firm's profit will be greater with complete information than with incomplete information when there is no role redistribution. ii). the merged firm's profits will be greater with incomplete information, when roles are redistributed.*

Proof: See detail in **Appendix H.2**

The economic intuition behind proposition 2 is the following. In the absence of the redistribution of roles (cases A and B), the merged firm benefits from a higher equilibrium price under complete information than under incomplete information. Moreover, the merged entity produces more if the information is complete. Consequently, the merged firm benefits from higher equilibrium profits under complete information, and it has an interest in revealing information about its own cost to competing firms. This outcome is consistent with the well-known conclusion obtained in the information-sharing literature;[14] firms competing on quantity are willing to reveal their private information about production costs (but are not willing to reveal their private information about market demand).

[12] The framework under complete information is studied in the working paper Le Pape and Zhao (2010).

[13] Under some circumstances (case A, C and D), outsider-follower firms can observe the insider's output level, and then infer the exact value of its marginal cost.

[14] There are some important contributions to this information sharing literature without merger issue, such as, Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985), Li (1985), Shapiro (1986) and Raith (1996).

By contrast, in the presence of role redistribution (cases C and D) the result is reversed, and under incomplete information mergers are more profitable as compared with the scenario of complete information. A possible explanation of this result could be that under incomplete information, the advantage associated with the redistribution of roles offsets the disadvantage resulting from the transmission of this private information to outsider followers. On the one hand, the intrinsic value of the private information is mitigated by the leader role of the merged entity. On the other hand, the leader strategy acquired by the newly merged entity provides a strategic advantage over competitors, and this latter effect dominates. This result is also in line with Amir et al. (2009) who find that the merged firm always benefits from "*first-to-know*". Our finding is also in line with the conclusion of Zhou,[15] who states that "firms are less likely to merge when they possess more information" (Zhou, 2008a, p.68).

Overall, proposition 2 shows that a "*first-to-know disadvantage*" appears if the merged firm adopts the same strategic behavior as *ex ante* merging firms. In this case, the informational asymmetry created by the merger is detrimental to the merged entity.

Let $\hat{\delta}^i_{sup}$, $\hat{\delta}^i_{inf}$ denote respectively the upper bound and the lower bound under complete information (see **Appendix F**). By comparison with the boundary under incomplete information, we derive: i). Without role redistribution, profitable mergers necessarily generate efficiency gains. Moreover, the ceiling of these potential efficiency gains under incomplete information δ^i_{sup} (with $i = A, B$) is lower than that under complete information. ii). With role redistribution (cases C and D), profitable mergers can generate efficiency losses. A formal proof of these results is given in **Appendix H.3**

4 Welfare analysis and potential policy guidance

We investigate the welfare implications of mergers, and discuss possible antitrust policy toward horizontal mergers. Consumer welfare (CS) and social welfare (W) in the *pre-merger* game are given as follows:

$$CS = \frac{(a-c)^2(n+mn-m^2)^2}{2(m+1)^2(n-m+1)^2}$$

$$W = \frac{(a-c)^2[(m+1)(n-m+1)+1](n+mn-m^2)}{2(m+1)^2(n-m+1)^2}$$

4.1 Ex ante intervention of competition authorities

When Competition Authorities intervene *ex ante*, we assume that they are uninformed about the merged firm's costs, so that decisions are based on expected welfare.

First, we assume that the decision of the Competition Authorities is based on the following principle: a merger is approved whenever the expected change of social welfare is positive. Secondly, we will compare merger approvals based on social welfare with those based on consumer surplus.

[15] The reason for Zhou (2008a) is that mergers are driven by production rationalization under cost uncertainty. When firms have more information, they are able to rationalize their production even without a merger, thus having less incentive to merge.

Table 6: Private vs. social incentive to merge

	Welfare-enhancing	
Scenarios	Threshold $\sigma^2_{W_i}$	Comparison with $\sigma^2_{\pi_i}$
Case A	$\sigma^2_{W_A} = \frac{4(a-c)^2(2m+1)}{m^2(m+1)^2(n-m+1)^2[4(n-m)+3]}$	$\sigma^2_{\pi_A} > \sigma^2_{W_A} > 0$
Case B	$\sigma^2_{W_B} = \frac{4(a-c)\{(a-c)[2(n-m)+1]+2(m+1)(n-m)[(m+1)(n-m)-2](n-m+1)^2\}}{3(m+1)^2(n-m+1)^2(n-m)^2}$	1).$\sigma^2_{\pi_B} > \sigma^2_{W_B} > 0$ when $n > 6, m \in [3, n-3), a > \Phi + c$ 2).$\sigma^2_{W_B} > \sigma^2_{\pi_B} > 0$ when $n = 6, m = 3$ or $n > 6, m \in [3, n-3), a < \Phi + c$
Case C	$\sigma^2_{W_C} = \frac{4(a-c)^2[(2m+1)(n-m-1)+2n](n-3m-3)}{(m+1)^2(m+2)^2[(n-m)^2-1]^2[5-4(n-m)]}$	$\sigma^2_{W_C} > 0$ $(\nexists\, \sigma^2_{\pi_C})$
Case D	$\sigma^2_{W_D} = \frac{4(a-c)^2[2(n-m)+1]}{[4(n-m)-1](m+1)^2(n-m+1)^2(n-m)^2}$	$\sigma^2_{W_D} > 0$ $(\nexists\, \sigma^2_{\pi_D})$

$$\text{with} \quad \Phi = \frac{2(m+1)(n-m)(n-m+1)^2[(n-m)(m+1)-2]}{3(n-m)^2 - 4[2(n-m)+1]}$$

Consider $\Delta^i_{\mathbb{E}[W]} = \mathbb{E}[W^i] - W$ as the yardstick which judges whether the merger improves social welfare. In the case of $\Delta^i_{\mathbb{E}[W]} > 0$, the merger enhances welfare, and it will reduce welfare if $\Delta^i_{\mathbb{E}[W]} < 0$. Table 6 gives the thresholds $\sigma^2_{W_i}$, beyond which the merger always gives rise to welfare improvement.

Proposition 3 *i). Profitable mergers between leaders always constitute welfare-enhancing mergers. ii). In the presence of role redistribution, when uncertainty is sufficiently high, profitable mergers between two followers are also welfare-enhancing.*

Proof: See detail in **Appendix H.4**

This proposition illustrates the fact that mergers between leaders always harmonise private and social incentives. By contrast, when the merger implies two-follower without role redistribution, profitable mergers may decrease expected social welfare.

Antitrust criteria may refer to either social welfare or consumer welfare. We therefore propose an analysis of the effect of merger types on expected consumer surplus. The new criteria are given by the sign of the expression $\Delta^i_{\mathbb{E}[CS]} = \mathbb{E}[CS^i] - CS$, so that we evaluate the thresholds

$\sigma^2_{CS_i}$, beyond which the merger improves expected consumer surplus.

Corollary 1 *i). A Profitable merger between leaders always enhances the ex ante total welfare, but can reduce the ex ante consumer surplus. ii). Consider a profitable merger between followers leading to a follower, or a merger between a leader and a follower. If these two types of mergers improve the ex ante consumer surplus, they are also ex ante welfare-enhancing.*

Proof: See detail in **Appendix H.5**

This corollary shows that in general the *ex ante* consumer welfare standard is more rigorous than the *ex ante* total welfare standard. Consequently, an antitrust decision based on consumer surplus effectively guarantees the expected welfare enhancement. A possible explanation of this fact could be that consumer surplus is less sensitive to uncertainty than social welfare.

4.2 Ex post enforcement of competition authorities

We now consider the case where uncertainty is disclosed, and information on the magnitude of the cost variation becomes available to Competition Authorities for *ex post* enforcement. Since the intervention of antitrust agencies takes place *ex post*, Competition Authorities are aware of the actual value of a merged firm's costs. Assume Δ^i_W as the variation in social welfare (before and after the merger): $\Delta^i_W = W^i - W$. We find the ranges of δ^i_W, whereby the merger improves social welfare (see **Appendix G**). Furthermore, by comparing the upper bound of δ^i_W with the critical value δ^i_{sup}, demonstrated in the merger analysis section, we shed light on the following proposition.

Proposition 4 *i). If the merger is composed of two leaders, a profitable merger unambiguously improves ex post social welfare. ii). When two followers merge and become a leader, or when the merger stems from firms of different types, a profitable merger can reduce ex post social welfare.*

Proof: See detail in **Appendix H.6**

Proposition 4 shows that as long as the merger between leaders is profitable, it is always welfare-enhancing. This point is consistent with Zhou (2008a), who has found that if a merger (with efficiency gains) is profitable to the merging firms, it will also be welfare-improving. Moreover, this result provides support for a "*laissez-faire*" policy if the criteria depend upon social welfare. Nevertheless, in other cases, profitable mergers can reduce *ex post* social welfare. Therefore Competition Authorities must supervise bilateral mergers which consist of either one, or two followers more closely.

Suppose now that Competition Authorities adopt the *ex post* consumer surplus criteria, we find the ranges of δ^i_{CS} wherein the merger improves consumer surplus, and then we compare the upper bound of δ^i_{CS} (namely $\delta^i_{CS_{sup}}$), with both δ^i_{sup} and $\delta^i_{W_{sup}}$ to achieve the following corollary.

Corollary 2 *The ex post consumer welfare standard is less demanding than the ex post social welfare standard.*

Proof: See detail in **Appendix H.7**

Concerning the impact of the merger scenario, our model shows that in the case of a two-leader merger, when there are more than four leaders in the market, a profitable merger is unambiguously welfare-enhancing and consumer-surplus-improving. In the case of a two-follower merger with role redistribution, when there are sufficiently fewer leader firms in the market, the profitable merger generating efficiency losses can improve both consumer and aggregate welfare, and the welfare-enhancing merger ensures the improvement of consumer surplus. Finally, when the merger is composed of one leader and one follower, a merger that improves consumer surplus can damage social welfare.

The model provides some insight into the relationship between the criteria used by Competition Authorities and the timing of policy intervention. When Competition Authorities adopt *ex ante* intervention, the consumer welfare standard is more restrictive than the aggregate welfare standard. By contrast, when Competition Authorities choose *ex post* enforcement and are aware of the actual cost of a merged firm, the consumer welfare standard is less demanding than the aggregate welfare standard.

5 Concluding remarks

This paper extends the body of literature dealing with horizontal mergers under uncertainty in a homogeneous oligopoly where there are leaders and followers. We emphasize the role of cost uncertainty within sequential output decisions. We find that if there is role redistribution due to the merger, even in the absence of an uncertainty effect, firms have incentives to merge. Concerning the *ex post* profitability of a merger, when the merged firm cost is private information, we show that the two-follower merger aiming at a leader strategy is more likely to occur than the one satisfying the *status quo*. Furthermore, the merged firm has an interest in pooling the private signals to outsiders, in the absence of role redistribution while concealment is more profitable (for the insider) if we assume role redistribution. As for the social desirability of mergers, it is found that a merger between leaders always enhances welfare if participants have incentives to merge, harmonising private and collective intentions. Nevertheless, a merger with role redistribution leads to conflict between the private and collective dimensions.

This framework has a number of limitations, providing further developments for future research. 1). We could verify whether these findings hold if there was some noise in outsider-followers' knowledge about the cost impact of the merger after observing the merged entity's quantity choice. There are many reasons that might create such noise in outsider-followers' information (*e.g.*, any form of demand uncertainty will do that). 2). The possible issue with respect to policy implications is the following: In practice, how can a competition authority identify which firms were leaders or followers before the merger, and whether the merging parties have changed their roles from being a follower to a leader? This is not a concern specific to the current paper, but a general concern for the literature of horizontal mergers assuming sequential firm competition. 3). The endogenous Stackelberg issue in the context of cost uncertainty would also be taken into account.

Acknowledgements: We are grateful to Thierry Penard and Bernard Franck for thoughtful comments and suggestions. We also thank Said Souam and Jean-Pascal Gayant. This work is supported by the Fundamental Research Funds for the Central Universities and Philosophy and Social Science of Huaqiao University young scholars growth project (13SKGC-QG03). Last but not least, we are grateful for the very painstaking efforts made by the editor and the referees in providing us with very valuable suggestions and comments. All remaining errors are ours.

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Appendix:

A Best response function of followers

In the follower production stage. The optimizing question is:

$$\max_{q_O^{f,A}} \pi_O^{f,A} = (p^A - c)q_O^{f,A} = [a - c - Q_O^{-f,A} - q_O^{f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_O^{f,A}(c_i) \tag{13}$$

From the standpoint of information structure,

- $Q_O^l(c)$: outsider-leaders consider that the cost level of insider is equal to c
- $q_I^l(c_i)$: first-to-know
- $q_O^f(c_i)$: outsider-followers observe the production level and perfectly infer the cost level of merged entity c_i

the *FOC* (first-order-condition) is

$$2q_O^{f,A} = a - c - Q_O^{-f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)$$

perfect symmetry for outsider-followers:

$$Q_O^{-f,A} = (n - m - 1)q_O^{f,A}$$

reaction function of outsider-follower is

$$(n-m+1)q_O^{f,A} = a - c - Q_O^{l,A}(c) - q_I^{l,A}(c_i) \tag{14}$$

and note the sum

$$\begin{aligned} Q_O^{f,A} &= (n-m)q_O^{f,A} \\ &= (\frac{n-m}{n-m+1})(a-c) - (\frac{n-m}{n-m+1})\big(Q_O^{l,A}(c) + q_I^{l,A}(c_i)\big) \end{aligned} \tag{15}$$

B Best response function of leaders and equilibrium output

In the (first) leader production stage, outsider-leaders are not aware of the actual cost of the insider, thereby they take into account the expected value c

$$\max_{q_O^{l,A}} \pi_O^{l,A} = (p^A - c)q_O^{l,A} = [a - c - Q_O^{f,A} - Q_O^{-l,A}(c) - q_O^{l,A} - q_I^{l,A}(c)]q_O^{l,A}(c) \tag{16}$$

plug the sum of the follower quantity *Eq.* (15) into *Eq.* (16), and the maximization problem becomes

$$\max_{q_O^{l,A}} \pi_O^{l,A} = \frac{1}{n-m+1}[(a-c) - Q_O^{-l,A}(c) - q_O^{l,A}(c) - q_I^{l,A}(c)]q_O^{l,A}(c) \tag{17}$$

FOC:

$$2q_O^{l,A}(c) = (a-c) - Q_O^{-l,A}(c) - q_I^{l,A}(c)$$

perfect symmetry for outsider-leaders:

$$Q_O^{-l,A}(c) = (m-3)q_O^{l,A}(c)$$

The reaction function of outsider-leader is

$$(m-1)q_O^{l,A}(c) = a - c - q_I^{l,A}(c) \tag{18}$$

and note the sum

$$Q_O^{l,A}(c) = (m-2)q_O^{l,A}(c) = \frac{m-2}{m-1}(a - c - q_I^{l,A}(c))$$

For the insider (merged entity), when the insider knows the real cost c_i, the optimizing question is

$$\begin{aligned} \max_{q_I^{l,A}} \pi_I^{l,A} &= (p^A - c_i)q_I^{l,A} = [a - c_i - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c_i)]q_I^{l,A}(c_i) \\ &= \frac{1}{n-m+1}[(a-c) + (n-m+1)(c-c_i) - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_I^{l,A}(c_i) \end{aligned}$$

FOC:

$$2q_I^{l,A}(c_i) = (a-c) + (n-m+1)(c-c_i) - Q_O^{l,A}(c) \tag{19}$$

when the insider is not informed about the exact cost $E(c_i) = c$

$$\max_{q_I^{l,A}} \pi_I^{l,A} = (p^A - c)q_I^{l,A} = [a - c - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c)]q_I^{l,A}(c)$$
$$= \frac{1}{n-m+1}[(a-c) - Q_O^{l,A}(c) - q_I^{l,A}(c)]q_I^{l,A}(c)$$

FOC with respect to expected value c is

$$2q_I^{l,A}(c) = (a-c) - Q_O^{l,A}(c) \tag{20}$$

then yields

$$q_I^l(c) + \frac{1}{2}(n-m+1)(c-c_i) = q_I^l(c_i)$$

It is straightforward that in the case of $c_i < c$, we obtain $q_I^l(c_i) > q_I^l(c)$; otherwise, $q_I^l(c_i) < q_I^l(c)$.

Based on *Eqs.* (18), (19) and (20), it is possible to derive the leaders' equilibrium outputs:

$$q_I^{l,A}(c_i) = \frac{2(a-c) - m(n-m+1)(c_i - c)}{2m}$$
$$q_I^{l,A}(c) = \frac{(a-c)}{m}$$
$$q_O^{l,A}(c) = \frac{(a-c)}{m}$$

plugging them into the follower's reaction function *Eq.* (14) yields

$$q_O^{f,A}(c_i) = \frac{2(a-c) + m(n-m+1)(c_i - c)}{2m(n-m+1)}$$

and then we derive the aggregate output

$$Q = q_I^{l,A}(c_i) + (m-2)q_O^{l,A}(c) + (n-m)q_O^{f,A}(c_i)$$
$$= a - \frac{a}{m(n-m+1)} - [\frac{1}{2} - \frac{1}{m(n-m+1)}]c - \frac{c_i}{2}$$

C Real and expected profits

The profit of the *insider*:

$$\pi_I^{l,A} = (a - Q - c_i)q_I^{l,A}(c_i)$$
$$= \frac{a^2}{m^2(n-m+1)} + \frac{[m^2+2-m(n+1)]^2(c_i-c)^2}{4m^2(n-m+1)} - \frac{2ac_i}{m^2(n-m+1)}$$
$$+ \frac{c_i^2}{m^2(n-m+1)} + \frac{a(c_i-c)(\frac{2}{n-m+1} - m)}{m^2} + \frac{c_i(c_i-c)(m - \frac{2}{n-m+1})}{m^2}$$
$$= \frac{[2(a-c) - m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)}$$

Knowing that $\mathbb{E}[(c_i-c)^2]=\sigma^2$, $\mathbb{E}[c_i]=c$, $\mathbb{E}[c_i^2]=c^2+\sigma^2$, $\mathbb{E}[c_i-c]=0$, $\mathbb{E}[(c_i-c)c_i]=\sigma^2$, the expected profit of the *insider*:

$$\mathbb{E}[\pi_I^{l,A}] = \frac{(n-m+1)\sigma^2}{4} + \frac{c^2}{m^2(n-m+1)} - \frac{2ac}{m^2(n-m+1)} + \frac{a^2}{m^2(n-m+1)}$$
$$= \frac{(a-c)^2}{m^2(n-m+1)} + \frac{n-m+1}{4}\sigma^2$$

The profit of an *outsider-leader*:

$$\pi_O^{l,A} = (a-Q-c)q_O^{l,A}(c)$$
$$= \frac{(a-c)[2(a-c)+m(n-m+1)(c_i-c)]}{2m^2(n-m+1)}$$

and then the expected profit of an *outsider-leader* is

$$\mathbb{E}[\pi_O^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)}$$

The profit of an *outsider-follower*:

$$\pi_O^{f,A} = (a-Q-c_i)q_O^{f,A}(c_i)$$
$$= \frac{[2(a-c)+m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)^2}$$

the expected value is

$$\mathbb{E}[\pi_O^f] = \frac{(a-c)^2}{m^2(n-m+1)^2} + \frac{1}{4}\sigma^2$$

D Merger between two followers

Using a similar method (See **Appendix A and B**), the equilibrium outputs for followers are resolved on the basis of the following equations:

- $a-(n-m-2)q_O^{f,B}(c)-Q_O^{l,B}(c)-q_I^{f,B}(c)-c-q_O^{f,B}(c)=0$ (outsider-followers do not realize the insider's real costs)
- $a-(n-m-2)q_O^{f,B}(c)-Q_O^{l,B}(c)-q_I^{f,B}(c_i)-c_i-q_I^{f,B}(c_i)=0$ (insider knows his own cost level)
- $a-(n-m-2)q_O^{f,B}(c)-Q_O^{l,B}(c)-q_I^{f,B}(c)-c-q_I^{f,B}(c)=0$ (insider does not know his own cost level)

The expression of the followers' outputs can be found

$$q_O^{f,B}(c) = \frac{(a-c) - Q^{l,B}(c)}{(n-m)}$$
$$q_I^{f,B}(c_i) = \frac{2(a-c) - (n-m)(c_i - c) + 2Q^{l,B}(c)}{2(n-m)}$$
$$q_I^{f,B}(c) = \frac{(a-c) - Q^{l,B}(c)}{(n-m)}$$

and then, plugging these into the leader's profit function:

$$\max_{q_O^{l,B}} \pi_O^{l,B} = (p^B - c)q_O^{l,B} = [a - c - (n-m-2)q_O^{f,B}(c) - q_I^{f,B}(c) - Q_O^{l,B}(c)]q_O^{l,B}(c)$$

It is easy to calculate the leader output level:

$$q_O^{l,B}(c) = \frac{a-c}{m+1}$$

Putting the expression of q^l into the output for followers, we obtain

$$q_O^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$$
$$q_I^{f,B}(c_i) = \frac{2(a-c) - (m+1)(n-m)(c_i - c)}{2(m+1)(n-m)}$$
$$q_I^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$$

The equilibrium values in terms of price, profit, consumer surplus and social welfare are shown in Table 1. The other cases (case C and case D) can be resolved by a similar method.

E Merged firm's profit under complete and perfect information ($\hat{\pi}_I^{j,i}$)

$$\hat{\pi}_I^{l,A} = \frac{\left[(a-2c+c_i) + (c-c_i)\left[(m-1)n - (m-2)m\right]\right]^2}{m^2(n-m+1)}$$
$$\hat{\pi}_I^{f,B} = \frac{\left[a-2c+c_i+(c_i-c)(n-m)(m+1)\right]^2}{(n-m)^2(m+1)^2}$$
$$\hat{\pi}_I^{l,C} = \frac{\left[(a-2c+c_i) + (c-c_i)\left[m(n-m) + (n-2m)\right]\right]^2}{(m+2)^2(n-m-1)}$$
$$\hat{\pi}_I^{l,D} = \frac{\left[a-c+m(c-c_i)\right]\left[(a-2c+c_i)+(c-c_i)(n-m)(m+1)\right]}{(n-m)(m+1)^2}$$

See also in Le pape and Zhao (2010).

F $\hat{\delta}^i_{sup}$ and $\hat{\delta}^i_{inf}$

$$\hat{\delta}^A_{inf} = -\frac{a-c}{n-m+1} \qquad \hat{\delta}^A_{sup} = \frac{(a-c)[1-m(\sqrt{2}-1)]}{(m^2-1)(n-m+1)}$$

$$\hat{\delta}^B_{inf} = -(a-c) \qquad \hat{\delta}^B_{sup} = \frac{a-c}{(n-m)(m+1)-1} - \frac{\sqrt{2}(a-c)(n-m)}{m^3-m^2n+mn(n-1)+n^2-1}$$

$$\hat{\delta}^C_{inf} = -\frac{a-c}{n-m-1} \qquad \hat{\delta}^C_{sup} = \frac{a-c}{(m+1)(n-m-1)} - \frac{(a-c)(m+2)}{(m+1)^2(n-m+1)}\frac{1}{\sqrt{n-m-1}}$$

$$\hat{\delta}^D_{inf} = -\frac{a-c}{n-m} \qquad \hat{\delta}^D_{sup} = \frac{a-c}{m(n-m)} - \frac{(a-c)}{m(n-m+1)}\sqrt{\frac{n-m+2}{n-m}}$$

G δ^i_{Wsup}

$$\delta^A_{Wsup} = -\frac{2\left(a(-3+2m-2n)+c(3-2m+2n)+m\left(3+4m^2+7n+4n^2-m(7+8n)\right)\sqrt{\frac{(a-c)^2\left(4m^4-4m^3(1+2n)+8m\left(1+3n+n^2\right)+4\left(3+4n+n^2\right)+m^2\left(-19-4n+4n^2\right)\right)}{m^2(1+m)^2\left(3+4m^2+7n+4n^2-m(7+8n)\right)^2}}\right)}{m\left(3+4m^2+7n+4n^2-m(7+8n)\right)}$$

$$\delta^C_{Wsup} = 2\left(\frac{(a-c)(2n-2m-1)}{(2+m)\left(5+4m^2+m(9-8n)-9n+4n^2\right)}\right) - 2\sqrt{\frac{(a-c)^2\left(4m^5-12m^4n+m^3\left(17-8n+12n^2\right)+m\left(60-68n+4n^2-8n^3\right)+m^2\left(65-17n+16n^2-4n^3\right)+4\left(4-12n+n^2-n^3\right)\right)}{\left(2+3m+m^2\right)^2(1+m-n)\left(5-m-4m^2+n+8mn-4n^2\right)^2}}$$

$$\delta^D_{Wsup} = 2\left(\frac{c(-1+2m-2n)}{(1+m)\left(m+4m^2-8mn+n(-1+4n)\right)} + \frac{a(1-2m+2n)}{(1+m)\left(m+4m^2-8mn+n(-1+4n)\right)} - \sqrt{\frac{(a-c)^2\left(-8+4m^3-21n-12n^2-4n^3-12m^2(1+n)+3m\left(7+8n+4n^2\right)\right)}{(1+m)^2(m-n)\left(1-4m^2-3n-4n^2+m(3+8n)\right)^2}}\right)$$

H Proofs

H.1 The proof of Proposition 1

$$\begin{cases} \sigma^2_{\pi A} > \sigma^2_{\pi B} > 0, & \text{when } \frac{n}{2} < m \leq n-3; \\ \sigma^2_{\pi B} > \sigma^2_{\pi A} > 0, & \text{when } 3 \leq m < \frac{n}{2}. \end{cases} \qquad \blacksquare$$

H.2 The proof of Proposition 2

- $\pi^{l,A}_I < \hat{\pi}^{l,A}_I$ and $\pi^{f,B}_I < \hat{\pi}^{f,B}_I$
- $\pi^{l,C}_I > \hat{\pi}^{l,C}_I$ and $\pi^{l,D}_I > \hat{\pi}^{l,D}_I$ $\blacksquare$

H.3 The formal proof

Case A:	$\delta^A_{sup} < \hat{\delta}^A_{sup} < 0$	$0 > \delta^A_{inf} > \hat{\delta}^A_{inf}$
Case B:	$\delta^B_{sup} < \hat{\delta}^B_{sup} < 0$	$\nexists$
Case C:	$\delta^C_{sup} > \hat{\delta}^C_{sup} > 0$	$0 > \delta^C_{inf} > \hat{\delta}^C_{inf}$
Case D:	$\delta^D_{sup} > \hat{\delta}^D_{sup} > 0$	$0 > \delta^D_{inf} > \hat{\delta}^D_{inf}$ ■

H.4 The proof of Proposition 3

(a). In the case of a merger between leaders, the magnitude of variance guaranteeing the incentives to merge ensures the enhancement of the social welfare without ambiguity. $\sigma^2_{\pi_A} > \sigma^2_{W_A} > 0$.

(b). If the market size is sufficiently large ($a > c + \Phi$), the magnitude of variance guaranteeing a private incentive ensures welfare enhancement. $\sigma^2_{\pi_B} > \sigma^2_{W_B} > 0$ when $n > 6, m \in [3, n-3), a > \Phi + c$; otherwise, $\sigma^2_{W_B} > \sigma^2_{\pi_B} > 0$.

(c). When two followers come together in a newly-merged firm behaving as leader (case C), or when the merger is composed of one leader and one follower (case D), uncertainty should be greater than the critical value ($\sigma^2_{W_C}$ or $\sigma^2_{W_D}$) to guarantee the enhancement of welfare: $\sigma^2_{W_C} > 0$ $(\nexists\, \sigma^2_{\pi_C})$ or $\sigma^2_{W_D} > 0$ $(\nexists\, \sigma^2_{\pi_D})$.

See also Table 6. ■.

H.5 The proof of Corollary 1

(a). A profitable merger between leaders requires more uncertainty to guarantee the enhancement of consumer surplus compared to the welfare criteria, *i.e.*
$\sigma^2_{CS_A} > \sigma^2_{\pi_A} > \sigma^2_{W_A}$ with $\sigma^2_{CS_A} = \frac{4(a-c)^2(2mn+2m^2n-2m^3-1)}{m^2(m+1)^2(n-m+1)^2}$.

(b). In the case of the merger between followers without role redistribution, the variance guaranteeing the consumer surplus enhancement ensures welfare improvement and the private incentive to merge when the market size is sufficiently large, *i.e.*
$\sigma^2_{CS_B} > \max\{\sigma^2_{\pi_B}, \sigma^2_{W_B}\}$ if $a > \Phi + c$ *with* $\sigma^2_{CS_B} = \frac{4(a-c)^2\{2(n-m)[n(m+1)-m^2]-1\}}{(m+1)^2(n-m)^2(n-m+1)^2}$.

(c). In the case of a merged leader firm composed of two followers, when there are enough active firms in the market where the proportion of leaders is smaller than that of followers, the required uncertainty guaranteeing welfare enhancement coincides with that guaranteeing consumer surplus; otherwise, the reverse outcome appears, *i.e.*

$$\begin{cases} \sigma^2_{W_C} > \sigma^2_{CS_C} & \text{if } n > 12,\ 3 \leq m < \frac{n}{3} - 1 \\ \sigma^2_{CS_C} > \sigma^2_{W_C} & \text{otherwise} \end{cases}$$

$$with\ \sigma^2_{CS_C} = \frac{4(a-c)^2(3m-n+3)\{2(m+1)(m+2)n^2-2mn[2m(m+3)+5]+m[2m(m+1)(m+2)-3]-3(n+1)\}}{(m+1)^2(m+2)^2[(n-m)^2-1]^2}$$

(d). When the merger is composed of one leader and one follower, the uncertainty guaranteeing the improvement of consumer surplus ensures that welfare enhancement is guaranteed with no ambiguity, *i.e.*
$\sigma^2_{CS_D} > \sigma^2_{W_D}$ with $\sigma^2_{CS_D} = \frac{4(a-c)^2\{2(n-m)[n(m+1)-m^2]-1\}}{(m+1)^2(n-m)^2(n-m+1)^2}$. ■

H.6 The proof of Proposition 4

Case A: $\delta^A_{sup} < \delta^A_{W_{sup}} < 0$

Case B: Complicated (depending upon numerous parameters such as the market size "a", the marginal cost "c", the numbers of leaders and followers "n" and "m", *etc.*)

Case C: $0 < \delta^C_{W_{sup}} < \delta^C_{sup}$, if $n > 12$ and $m \in [3, \frac{n}{3} - 1)$
$\delta^C_{W_{sup}} < 0 < \delta^C_{sup}$, otherwise

Case D: $\delta^D_{W_{sup}} < 0 < \delta^D_{sup}$ ■

H.7 The proof of Corollary 2

(a). In the case of a two-leader merger, (1). when there are three or four leaders in the premerger market, the profitable merger always improves the consumer surplus, but possibly damages social welfare; (2). when there are more than four leaders in the market, the profitable merger is unambiguously welfare-enhancing and consumer-surplus-improving.

$$\begin{cases} \delta^A_{CS_{sup}} < \delta^A_{sup} < \delta^A_{W_{sup}} < 0 & \text{if } m = 3 \text{ or } 4 \\ \delta^A_{sup} < \delta^A_{CS_{sup}} < \delta^A_{W_{sup}} < 0 & \text{if } m \geq 5 \end{cases}$$

$$with\ \delta^A_{CS_{sup}} = \frac{-2(a-c)}{m(m+1)(n-m+1)}.$$

(b). In the case of a two-follower merger with role redistribution, when there are sufficiently fewer leader firms in the market, the profitable merger generating efficiency losses can improve both consumer and aggregate welfare, and the welfare-enhancing merger ensures the improvement of consumer surplus.

$$\begin{cases} 0 < \delta^C_{W_{sup}} < \delta^C_{CS_{sup}} < \delta^C_{sup} & \text{if } n > 12,\ m \in [3, \frac{n}{3} - 1) \\ \delta^C_{CS_{sup}} < \delta^C_{W_{sup}} < 0 < \delta^C_{sup} & \text{otherwise} \end{cases}$$

$$with\ \ \delta^{C}_{CS_{sup}} = \frac{2(a-c)(n-3m-3)}{(m+1)(m+2)[(n-m)^2-1]}.$$

(c). When the merger is composed of one leader and one follower, the merger which improves consumer surplus can damage social welfare. $\delta^{D}_{CS_{sup}} < \delta^{D}_{W_{sup}} < 0 < \delta^{D}_{sup}$ *with* $\delta^{D}_{CS_{sup}} = \frac{-2(a-c)}{(m+1)(n-m)(n-m+1)}$. ■

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The Editor