Technological Changes and Global Value Chains

Ramezan Ali Marvi

Working Papers / Documents de travail

Technological Changes and Global Value Chains

Ramezan Ali Marvi

WP 2014 - Nr 22

Technological Changes and Global Value Chains

Ramezan Ali Marvi
Bocconi University and AMSE

May 21, 2014

Abstract

This paper focuses on the pattern of task and income distribution within a Global Value Chain. Using the recently developed WIOD database, collecting data on the trade in value added within a world Input/Output matrix, we reveal a high heterogeneity of countries in terms of their trends of skill premia. The latter is a stylized fact at odds with the assumption of a recent theoretical model of Global Value Chains (Costinot et al. [2013]), which we extend by allowing for different types of labor and different types of production stages. The model generates a pattern of vertical specialization in which the position of each country in the chain is a function of two factors: its productivity and skill intensity of its labor endowments. Moreover, the wage of each labor type depends on the position of the country, its skill intensity and productivity of skilled workers. As a result, depending on the model parameters and labor endowments, technological innovations will induce various trends in the relative position of countries, prices, wages and exports, in line with the stylized fact. The model thus represents a suitable candidate for addressing the heterogeneity of countries in terms of skill premia.

JEL Codes: F10, F16, F19
Keywords: Global Value Chains, technological changes, wage premium, vertical specialization

1 Introduction

There has been a growing trend in the ratio of trade happening within Global Value Chains (hereafter, GVCs) out of the total world trade. Johnson and Noguera [2012] argue that production fragmentation across countries has been rising during last four decades and in particular after 1990. Hummels et al. [2001] focuse on 10 OECD and four emerging market countries and show that vertical specialization accounts for 21% of these countries' exports, and grew almost 30% between 1970 and 1990. Miroudot et al. [2009] analyze trade flows in intermediate goods and services among OECD countries and their partners for the period 1995-2005. They discuss that trade in intermediate inputs takes place mostly among developed countries and represents 56% and 73% of overall trade flows in goods and services, respectively. In spite of these facts, there is a sparse theoretical literature on GVCs and specially on the pattern of task and income distribution within a GVC. An important piece of this literature is a study by Costinot, Vogel and Wang (2013)(hereafter, CVW) that rationalizes the pattern of vertical specialization within a GVC. Focusing on the productivity of countries, they provide a theoretical framework to explain the interdependence of countries and income distribution within a production chain. They show that a unique outcome in the free trade equilibrium emerges in which countries specialize vertically and position of each country in the chain is defined based on its productivity level. In their setting, the labor input is homogenous, so the model is silent about the observations related to the heterogeneity of the labor market. In particular, their model tells nothing about the observations on the trend of skill premia inside countries. As we will see in the next section, regardless of the level of development, the skill premia has been rising in some countries, falling in some others and almost constant in the rest.

There is a huge literature including traditional (HO) theories and the new research to address the relationship between trade and inequality trend within countries. Traditional theories predict that trade will lead to an increasing skill premia in North and a decreasing one in South. So, these theories cannot justify this observation. Some parts of the new research (e.g. skill-biased technological change (Acemoglu [2003]) and Trade in tasks (Feenstra and Hanson) predicts an increasing skill premia both in North and south and some others (e.g. Search frictions and unemployment (Davidson and Matusz [2009] and Trade and innovation (Dinopoulos and Segerstrom [1999])) can generate a wide range of effects on labor-market outcomes. Although, some theories can potentially explain the different trends of skill premia across (both south and north) countries, there is not a study that rationalizes the pattern of vertical specialization within GVCs and at the same time justifies these kind of observations on skill premia. So, we need a new framework in which a sequential production is done by multiple countries and labor factors are heterogeneous.

This paper develops the CVW's model by allowing for two types of labor and two types of production stages. CVW's framework consists of a world with an arbitrary number of countries, one type of labor, a continuum of intermediate goods and one final good produced sequentially in a multistage production. Each stage of production is subject to a mistake rate which is different across countries. This mistake rate has actually an inverse relation with total productivity of the country. They found that such a framework yields a unique free trade equilibrium in which there is full employment across countries and countries with lower mistake rates at all stages specialize in later stages. In other words, there is a vertical specialization based on the mistake rates of countries in the unique equilibrium.

To extend their model, we consider two different types of labor and two different types of stages. Each labor force is either skilled or unskilled and each stage is either high-tech or low-tech. High-tech stages need a production technology that is a Leontief function of the intermediate input from previous stage and a Cobb-Douglas combination of both labor types. Whereas, low-tech stages use a production technology that is a Leontief function of the intermediate good from previous stage and unskilled labor. These modifications enable us to represent technological changes in a wider domain. In this setting, the vertical specialization is preserved, but these modifications result in some new results about the position of countries in the chain and the effect of technological changes on the skill premia within countries. Now, the position of countries depends not only on mistake rates, but also on the endowment of skilled and unskilled labor. Moreover, the contribution level of each country to both type of stages is a function of these two factors. The lower is the mistake rate of a country, the more downstream is the position of that country and the higher is the skill ratio of a country, the bigger is the share of high-tech stages that it produces. In turn, the wage of labor factors and skill premium are defined by the position of the country in the chain, the share of high-tech stages produced by the country and the productivity ratio of skilled labor over unskilled labor in high-tech stages.

This paper contributes to a growing theoretical literature on GVCs. A number of papers try to analyze theoretically different aspects of GVCs. Fujita and Thisse [2006] focus on the trade costs of goods, communication costs between headquarters and production facilities and wage differentials across regions to analyze the location decision of plants. Baldwin and Venables [2010] model the interaction between international cost differences and benefits of co-location of related stages. This interaction determines the fragmentation level of production stages. Antràs and Chor [2013], take as given a GVC, try to rationalize the optimal way of organizing production (FDI vs outsourcing) along each linkage of a GVC.

Our paper is closely related to several studies on trade and skill premia. Traditional HO models predict a rising effect of trade on the relative demand for low-skilled workers in south and a rising effect on the relative demand for high-skilled workers in north. Consequently, we have a decreasing effect on the skill premium of south and an increasing effect on the skill premium of north. However, some studies (e.g. Hanson and Harrison [1995] and Robbins [1996]) show that this effect on developing countries is not symmetric. In fact, Some developing countries experienced a decline in the skill premium. Some papers (e.g. Feenstra and Hanson, Zhu [2004], and Zhu and Trefler [2005]) address this puzzle by resorting to outsourcing and technology transfer. These two phenomena shift a portion of input production from the North to the South, which is the most skilled-intensive in the South, and the most unskilled-intensive in the North. Some other studies (e.g. Acemoglu [2002, 2003], Thoenig and Verdier [2003], Dinopoulos and Segerstrom [1999]) focus on the role of skill biased technological changes. They argue that trade creates a tendency for the relative price of skilled-intensive goods to increase. So, the development of technologies used in the production of these goods is more profitable. This induces further skill biased technological change, which contributes to the increase in wage inequality.

The rest of the paper is organized as follows. Section II explains stylized facts. Section III describe the model setup and derives the main results relevant to any free trade equilibrium. Section IV analyzes the effects of technological changes on the patterns of vertical specialization and inequality across and within countries including simulation results. Finally section V states some concluding remarks.

2 Stylized Facts

We use WIOD[1] database to derive the trend of skill premia for different countries. WIOD database covers 40 countries[2] and contains annual data on wages and employment by skill type (low-, medium- and high-skilled) of 35 industries for the period 1995-2009. This includes data on hours worked and compensation for three labor types. Skills in this database are defined based on educational attainment levels. We calculate hourly wage for each skill category and then calculate skill premia of medium-skilled and low-skilled (wage ratio of medium-skilled (M) over low-skilled (L)) and skill premia of high-skilled and low-skilled. We find that both types of skill premia have similar trend for almost all 40 countries. Regardless of short time cyclical variation, we can categorize countries based on their trend in skill premia to 5 groups: 37.5 percent of them experienced a rising trend, 30 percent witnessed a falling trend, 22.5 percent have a constant trend, 7.5 percent experienced a rising and then a falling trend and finally 2.5 percent witnessed a falling and then a rising trend. Putting aside the last two groups (due to their small sizes), each group contains both developed and developing countries.

Figures (1)-(3) depict three examples related to the first three groups. For better capturing of trends, we have normalized the skill premium of the starting year (1995) to 1. The solid line is the trend of normalized value of skill premium between high skilled and low skilled and the dashed line is the trend of normalized value of skill premium between medium skilled and low skilled.

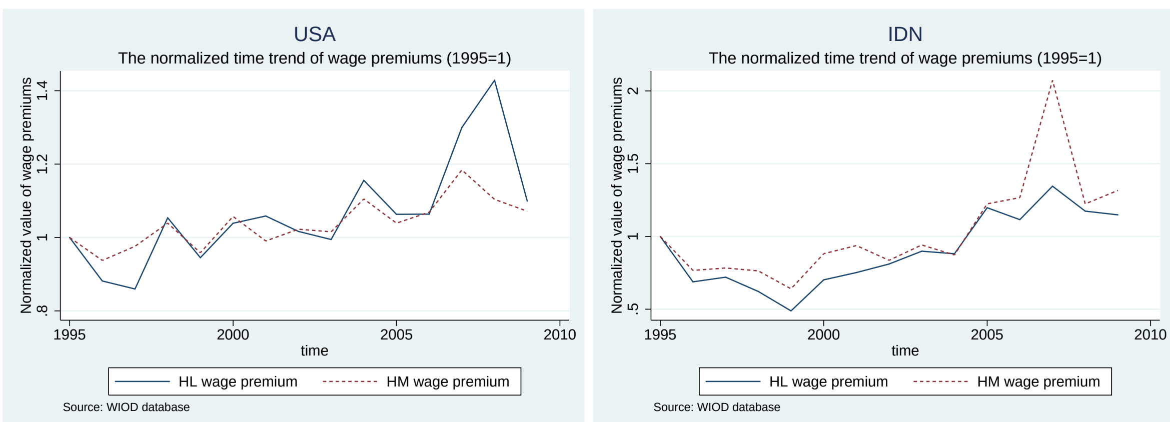

Figure 1: Example of countries with rising trend of wage premium

It can be seen from these figures that countries are heterogenous in terms of the time trend of skill premia within them. In addition, the time trend of the skill premium between high skilled and low skilled is very similar to the time trend of skill premium between medium skilled and low skilled. So, without loss of generality, we can theoretically focus on the case with only two types of labor. It is also evident for the figures that the time behavior of skill premia does not depend on the level of development. All kinds of trends could be found in both developed and developing countries.

[1] World Input-Output Database

[2] These countries are:
Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, South Korea, Spain, Sweden, Taiwan, Turkey, United Kingdom and United States

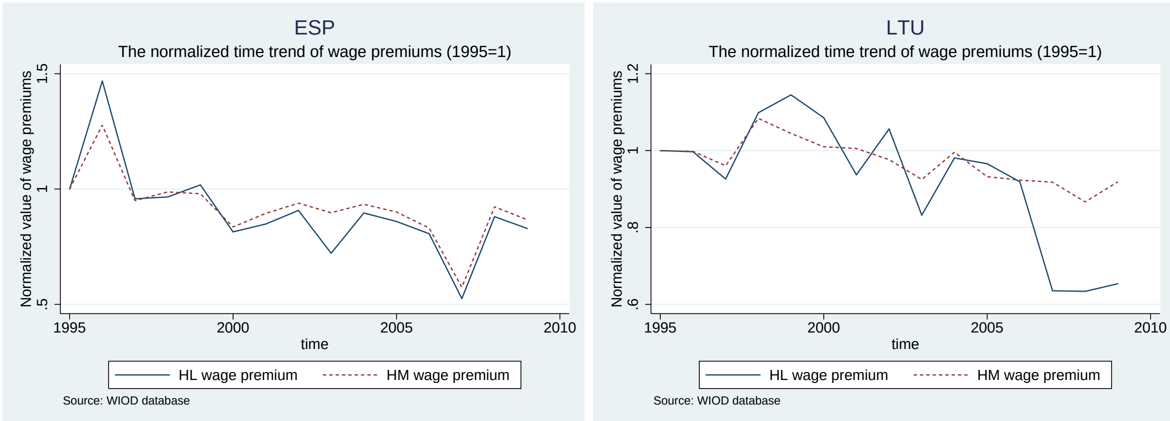

Figure 2: Example of countries with declining trend of wage premium

Finally, we can see that there is a sharp change in the trend of either type of skill premia during 2007-2009. It might be due to the recent crisis. In the rest of the paper, we will try to rationalize these heterogeneities in skill premia trends both for developed and developing countries.

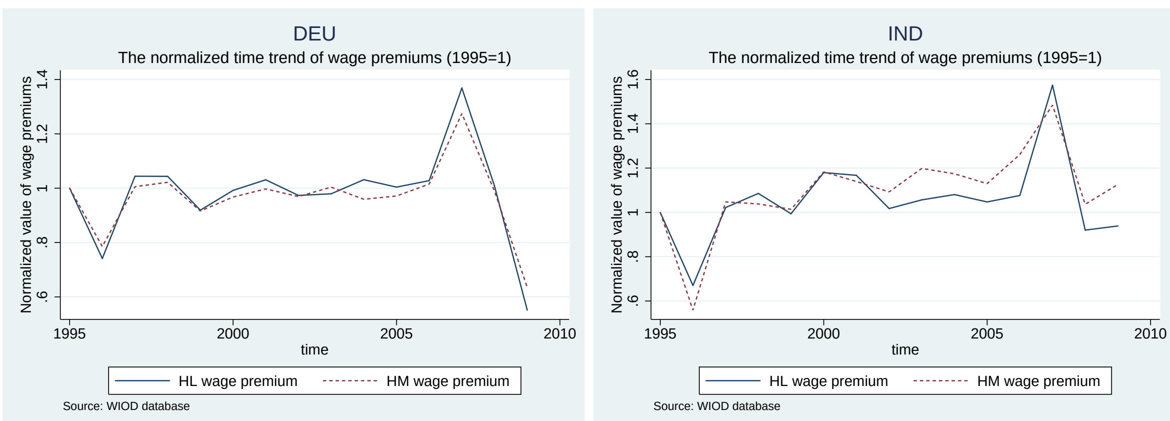

Figure 3: Example of countries with constant trend of skill premia

3 The model

3.1 Setup

Consider a world with Multiple countries $c \in \mathcal{C} \equiv \{1, ..., C\}$ that are producing a final good q. To produce the final good, a continuum of stages $s \in \mathcal{S} \equiv (0, S]$ must be performed in a sequential way. Production at any stage is subject to a country-specific mistake rate. Mistakes occur at a exogenous constant Poisson rate, $\lambda_c > 0$. When a unit of intermediate good meets a mistake, at any stage, it is entirely lost. Following CVW, we order countries in the way that λ_c is strictly decreasing in c. In other words, more productive countries are bigger in label. Unlike to CVW, the stages of production are not homogenous here. There are two types of production stages: low-tech stages and high-tech stages. Low-tech stages need two factors of production: unskilled labor L and intermediate good from previous stage. High-tech stages use three factors: skilled labor H, unskilled labor L and the intermediate good. Each type of labor is inelastically supplied and immobile across countries. Let L_c and H_c denote the endowment of unskilled and skilled labor in country c, respectively. Similarly, w_c^L and w_c^H represent the wage of unskilled and skilled labor in country c, respectively.

For the sake of simplicity, and without loss of generality, we assume that $\mathcal{S}$ is composed of finite subintervals of each stage type, as shown in figure (4). Here σ_i^z ($i \in \{1, 2, ..., n\}$ and $z \in \{l, h\}$) is the i^{th} subinterval of type z, $\mathcal{S}_h$ is the set of all high-tech subintervals and $\mathcal{S}_l$ is the set of all low-tech subintervals:

$$\mathcal{S}_l = \sigma_1^l \cup \sigma_2^l \cup ... \cup \sigma_n^l \quad , \quad \mathcal{S}_h = \sigma_1^h \cup \sigma_2^h \cup ... \cup \sigma_n^h$$

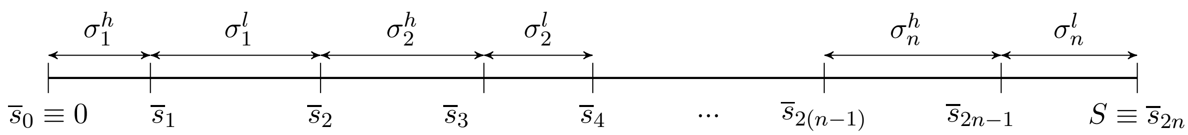

Figure 4: The continuum of total stages is partitioned by subintervals of high-tech and low-tech subintervals

Consider two consecutive stages, s and $s+ds$, with ds infinitesimal. The production function for a firm from country c that is undertaking a high-tech stage $s+ds$ is:

$$q(s+ds) = (1 - \lambda_c ds) Min\{q(s), H^\alpha L^{1-\alpha}\} \quad (0 < \alpha < 1) \tag{1}$$

where q_s is the intermediate input from stage s and α is an exogenous coefficient that represents the productivity of high-skilled labor in the labor content of the production technology. Similarly, the production function in a low-tech stage is:

$$q(s+ds) = (1 - \lambda_c ds) Min\{q(s), L\} \tag{2}$$

There is no trade cost and all markets are perfectly competitive. The world price of intermediate good $q(s)$ is $p(s)$. As CVW, we assume that $q(0)$ is in infinite supply and thus $p(0) = 0$. Moreover, $q(S)$ corresponds to q and is considered as numeraire, $p(S) = 1$. Furthermore, we also assume that: if a firm produces intermediate good $s+ds$, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Formally, for any intermediate good $s+ds$, we assume the existence of $s_\Delta < s + ds \leq s_\Delta + \Delta$ such that if $q(s+ds) > 0$, then $q(s') > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. As CVW point out, this assumption implies that each unit of q is produced by a finite number of firms.

3.2 Free Trade Equilibrium

Free trade equilibrium requires all markets to be cleared and all firms to maximize their profits. Profit maximizing behavior of firms leads to:

$$\begin{cases} p(s+ds) \leq (1 + \lambda_c ds) p(s) + w'_{hc} ds & \text{if } s' \in int(\mathcal{S}_h) \\ \\ p(s+ds) \leq (1 + \lambda_c ds) p(s) + w_{lc} ds & \text{if } s' \in int(\mathcal{S}_l) \end{cases} \tag{3}$$
$$(\forall s' \in (s, s+ds], \text{with equality if } Q_c(s') > 0)$$

where

$$w'_{hc} = \frac{(w_c^L)^{1-\alpha} (w_c^H)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \quad \text{and} \quad w_{lc} = w_c^L$$

and $Q_c(s')$ denotes the total output of country c at stage s'. For boundary points, we have these relationships:

$$p(0) = 0 \quad , \quad p(S) = 1 \quad and \quad p(\overline{s}_i) = \overline{p}_i = \lim_{s \to (\overline{s}_i)^-} p(s) \quad \forall i \in \{1, 2, ..., 2n\} \tag{4}$$

The last expression guarantees that the price of intermediate goods is continuous in the borders of subintervals. Let call w'_{Hc} the wage profile of country c at high-tech stages. Equation (3) says that, regardless of the stage type, the price of intermediate component $s + ds$ could not be larger than its unit cost of production. If some firms of country c are actually producing $s + ds$, then this price is equal to the unit cost. This result arises from the competitiveness of markets. The reasoning is similar to that of CVW with the exception of replacing wage profile by wage in the high-tech stages.

Market clearing for goods implies that:

$$\sum_{c=1}^{C} Q_c(s_2) - \sum_{c=1}^{C} Q_c(s_1) = -\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c Q_c(s) ds \quad \forall s_1 < s_2 \tag{5}$$

This equation states that the change in the total supply of intermediate goods between stages s_1 and s_2 is equal to the amount of goods that are lost (because of mistakes) in all countries between these stages.

Labor market clearing for the low-skilled requires that:

$$L_c = (\frac{1-\alpha}{\alpha} \frac{w_c^H}{w_c^L})^{\alpha} \int_{s \in \mathcal{S}_h} Q_c(s) ds + \int_{s \in \mathcal{S}_l} Q_c(s) ds \tag{6}$$

and for the high-skilled, it implies that:

$$H_c = (\frac{\alpha}{1-\alpha} \frac{w_c^L}{w_c^H})^{1-\alpha} \int_{s \in \mathcal{S}_h} Q_c(s) ds \tag{7}$$

Equation (6) states that the total demand for unskilled labor in country c equals its total endowment of that kind of labor. This demand is equal to the sum of what is consumed in high-tech stages and low-tech stages. Similarly, equation (7) says that the total demand for skilled workers in country c clears its total endowment of skilled workers. However, the demand for skilled labor originates only from high-tech stages. Now, we can define a free trade equilibrium in the same way as CVW:

Definition 1: A free trade equilibrium corresponds to output levels $Q_c(.) : \mathcal{S} \to \mathcal{R}^+$ for all $c \in \mathcal{C}$, wages $(w_c^L, w_c^H) \in \mathcal{R}_+^2$ for all $c \in \mathcal{C}$, and intermediate good prices $p(.) : S \to \mathcal{R}^+$, such that (3)-(7) hold.

Before characterizing the pattern of international specialization in a free trade equilibrium, we label the countries undertaking σ_i^z by $c_{i,1}^z, c_{i,2}^z, ...$ and $c_{i,n_{iz}}^{iz} (n_{iz} \geq 1)$ as shown in figure (5):

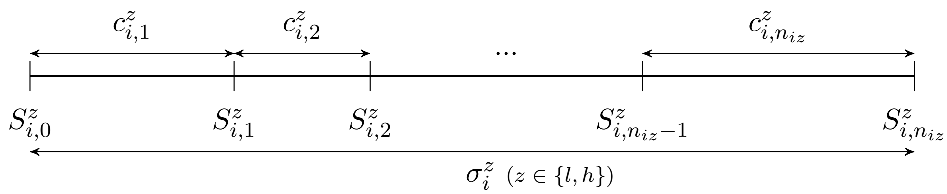

Figure 5: Countries that are doing σ_i^z

and label unskilled and skilled labor use of country $c_{i,j}^z$ during the interval σ_i^z by $L_{i,j}^z$ and $H_{i,j}^z$, $(z \in \{l,h\}\,,\, j \in \{1,2,...,n_{iz}\})$, respectively. Since the skilled labor use of low-tech stages is zero, we can set $H_{i,j}^h \equiv H_{i,j}$. These notations help us to explain the following propositions in an easier way.

Proposition 1: In any free trade equilibrium, for every $z \in \{l,h\}$,there exists a sequence of stages $0 < S_1^z \leq S_2^z ... \leq S_C^z$ such that for all $s \in \mathcal{S}_z$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if and only if $s \in (S_{c-1}^z, S_c^z]$.

This proposition states that in any free trade equilibrium, there is vertical specialization within the set of all high-tech subintervals ($\mathcal{S}_h$) and within the set of all low-tech subintervals ($\mathcal{S}_l$). According to each of these vertical specializations, the countries that are active in the later stages are more productive that the active countries in the early stages. This proposition is similar to the first proposition in CVW, except the fact that here we have two parallel vertical specializations. Actually, the case in which less productive country is active at later stages (but with different stage type) than more productive firm is not impossible here.

Proposition 2: In any free trade equilibrium, for every $\sigma_i^z \in \mathcal{S}$, there exists a sequence of stages $S_{i,0}^z < S_{i,1}^z < S_{i,2}^z ... < S_{i,n_{iz}}^z$ such that for all $s \in \sigma_i^z$ and $c_{i,j}^z \in \{c_{i,1}^z, c_{i,2}^z, ..., c_{i,n_{iz}}^z\}$, $Q_{c_{i,j}^z}(s) > 0$ if and only if $s \in (S_{i,j-1}^z, S_{i,j}^z]$. The first and last terms of this sequence are equal to:

$$(S_{i,0}^z, S_{i,n_{iz}}^z) = \begin{cases} (\overline{s}_{2(i-1)}, \overline{s}_{2i-1}) & if \quad z = h \\ \\ (\overline{s}_{2i-1}, \overline{s}_{2i}) & if \quad z = l \end{cases}$$

Furthermore, $\forall i < j$ we have $c_{i,n_{iz}}^z \leq c_{j,1}^z$.

Proposition 2 shows that in any free trade equilibrium there is also vertical specialization within each subinterval of sages in which more productive countries produce and export at later stages of production. The intuition behind proposition 1 and proposition 2 is similar to CVW. One possibility is to look at them from the standpoint of the hierarchy literature (e.g. Robert E. Lucas [1978], Rosen [1982] and Garicano [2000]): the efficiency of final good production requires that countries with higher productivity work on larger amount of inputs. Another explanation is that since new intermediate goods require intermediate goods produced in previous stages, skilled labor and unskilled labor, prices must be increasing along the supply chain. So, the non-labor cost share is relatively higher at later stages. Or, equivalently, labor-cost share is relatively lower in countries with higher wages. These countries are those with higher productivity at all stages. However, There is a difference between this proposition and that of CVW. In the CVW framework, productivity differences across countries define their location in the GVC, whereas, there are two defining factors here: productivity differences across countries and the share of skilled labor in the endowments of countries. In fact, the second factor defines the degree of participation in each type of stages.

We call the vector $(S^z_{i,1}, S^z_{i,2}, ..., S^z_{i,n_{iz}})$ the pattern of vertical specialization in interval σ^z_i and the set of all vectors $\{(S^z_{i,1}, S^z_{i,2}, ..., S^z_{i,n_{iz}})\, \forall i \in \{1,2,...,n\}$ and $z \in \{l,h\}\}$ the pattern of vertical specialization. We denote by $Q^z_{i,j} \equiv Q_{c^z_{i,j}}(S^z_{i,j})$ the total amount of intermediate good $S^z_{i,j}$ produced by country $c^z_{i,j}$. The total amount of intermediate goods produced and exported by country c is equal to:

$$Q_c = \sum_z \sum_i \sum_{j=1}^{n_{iz}} 1_{(c=c^z_{i,j})} Q^z_{i,j}$$

where $1_{(c=c^z_{i,j})}$ is an indicator function. Finally, let $\lambda^z_{i,j}$ represents the mistake rate of j^{th} country which is active in subinterval σ^z_i (i.e. $\lambda_{c^z_{i,j}}$). Using these notations, we can state lemma 1.

Lemma 1: The pattern of vertical specialization within i^{th} high-tech subinterval (σ^h_i) satisfies [3]:

$$S^h_{i,j} = S^h_{i,j-1} - (\frac{1}{\lambda^h_{i,j}})Ln(1 - \frac{\lambda^h_{i,j} H_{i,j}}{\widetilde{Q}^h_{i,j-1}}) \quad \forall j \in \{1,2,...,n_{ih}\} \tag{8}$$

and for the low-tech subinterval σ^l_i, we have:

$$S^l_{i,j} = S^l_{i,j-1} - (\frac{1}{\lambda^l_{i,j}})Ln(1 - \frac{\lambda^l_{i,j} L^l_{i,j}}{Q^l_{i,j-1}}) \quad \forall j \in \{1,2,...,n_{il}\} \tag{9}$$

Moreover, the pattern of export levels satisfies:

$$Q^z_{i,j} = e^{-\lambda^z_{i,j}(S^z_{i,j} - S^z_{i,j-1})} Q^z_{i,j-1} \quad \forall z \in \{l,h\}\,,\, j \in \{1,2,...,n_{iz}\} \tag{10}$$

Lemma 1 is derived from market clearing Conditions for goods and labors. According to this lemma, labor use of countries producing in σ^z_i depends on their mistake rates and their level of production. Equation (10) is a result of the fact that intermediate goods would be destroyed at a constant rate at each stage when produced in country $c^z_{i,j}$.

Let $w'_{i,j}$ denote the wage profile of j^{th} country producing in subinterval σ^h_i (i.e. $w'_{Hc^h_{i,j}}$), $w^l_{i,j}$ proxy the wage of j^{th} country producing in subinterval σ^l_i (i.e. $w_{Lc^l_{i,j}}$), $p^z_{i,j}$ represents $p_{c^z_{i,j}}$ and $N^z_{i,j} = S^z_{i,j} - S^z_{i,j-1}$ denote the measure of stages performed by country $c^z_{i,j}$ within the interval σ^z_i. Lemma 2 explains the relationships between world income distribution and export prices.

Lemma 2: The world income distribution and export prices satisfy these conditions:

a. For the high-tech subintervals, the wage profiles follow the equation below:

$$w'_{i,j+1} = w'_{i,j} + (\lambda^h_{i,j} - \lambda^h_{i,j+1})p^h_{i,j}, \quad \forall c^h_{i,j} < c^h_{i,n_{ih}} \quad (\forall i \text{ and } \forall \sigma^h_i) \tag{11}$$

[3]Here, we use below notation to avoid unnecessary complications:

$$\widetilde{Q}^h_{i,j-1} \equiv (\frac{\alpha}{1-\alpha}\frac{w^L_c}{w^H_c})^{1-\alpha} Q^h_{i,j-1}$$

and prices of intermediate goods are:

$$p^h_{i,j} = e^{\lambda^h_{i,j} N^h_{i,j}} p^h_{i,j-1} + (e^{\lambda^h_{i,j} N^h_{i,j}} - 1)(\frac{w'_{i,j}}{\lambda^h_{i,j}}), \quad \forall c^h_{i,j} \in \{c^h_{i,1}, ..., c^h_{i,n_{ih}}\} \quad (\forall i \text{ and } \forall \sigma^h_i) \tag{12}$$

b. For the low-tech subintervals, the wages of unskilled labor satisfy:

$$w^l_{i,j+1} = w^l_{i,j} + (\lambda^l_{i,j} - \lambda^l_{i,j+1}) p^l_{i,j}, \quad \forall c^l_{i,j} < c^l_{i,n_{il}} \quad (\forall i \text{ and } \forall \sigma^l_i) \tag{13}$$

and the export prices will behave as follow:

$$p^l_{i,j} = e^{\lambda^l_{i,j} N^l_{i,j}} p^l_{i,j-1} + (e^{\lambda^l_{i,j} N^l_{i,j}} - 1)(\frac{w^l_{i,j}}{\lambda^l_{i,j}}), \quad \forall c^l_{i,j} \in \{c^l_{i,1}, ..., c^l_{i,n_{il}}\} \quad (\forall i \text{ and } \forall \sigma^l_i) \tag{14}$$

c. With the boundary conditions:

$$\begin{cases} p^z_{i,0} = \overline{p}_i \, , \, p^z_{i,n_{iz}} = \overline{p}_{i+1} \quad if \quad z = l \\ \\ p^z_{i,0} = \overline{p}_{i-1} \, , \, p^z_{i,n_{iz}} = \overline{p}_i \quad if \quad z = h \\ \\ p^l_{n,n_{nl}} = p_C = 1 \, , \, p^h_{1,0} = 0 \end{cases}$$

Equation (11) states that the wage profile $w'_{i,j+1}$ in country $c^h_{i,j+1}$ is greater than the wage profile $w'_{i,j}$ in country $c^h_{i,j}$ because the mistake rate of country $c^h_{i,j+1}$ is smaller. Indeed this equation is a result of the fact that the unit cost of producing the "cutoff good" $S^h_{i,j}$ in country $c^h_{i,j}$ $((1+\lambda^h_{i,j})ds + w'_{i,j}ds)$ is equal to that of country $c^h_{i,j+1}$ $((1+\lambda^h_{i,j+1})ds + w'_{i,j+1}ds)$. Regarding that the wage profile is a combination of skilled and unskilled wages, we can use this equation to justify the heterogeneous behavior of skill premia across countries. Equation (13) has the same intuition as equation (11) except the fact that in the low-tech subintervals the unit cost of producing the intermediate goods is only a function of unskilled wage and not wage profile.

Equation (12) shows that the price of the high-tech cutoff good produced by country $c^h_{i,j}$ depends on the price of the intermediate good imported from country $c^h_{i,j-1}$ plus the total labor cost in country $c^h_{i,j}$. Total labor cost consists of both skilled and unskilled labor cost. So, we have the wage profile in the right hand side of this equation. The same intuition applies for equation (14).

Using proposition 1, proposition 2, lemma 1 and lemma 2, proposition 3 shows existence and uniqueness of the free trade equilibrium.

Proposition 3: There exists a unique free trade equilibrium. In this equilibrium, the pattern of vertical specialization and export levels are defined by proposition 1, proposition 2 and lemma 1 and the pattern of world income distribution and export prices are illustrated by lemma 2.

4 Technological changes

Before starting to analyze the effect of technological changes on GVCs, we define an important index for value chains based on the share of high-tech stages in the interval of all stages.

Definition 2: The degree of high-tech intensity of a chain could be defined as:

$$\gamma = \frac{|\ \mathcal{S}_h\ |}{|\ \mathcal{S}\ |} = \frac{S - \sum_{i=1}^{n}(\overline{s}_{2i} - \overline{s}_{2i-1})}{S}$$

Introduction of this index yields more possibilities to represent technological changes in our framework. CVW has two proxies for modeling global technological changes: increasing in complexity (increase in S) and standardization (decreasing in the mistake rate of all countries). However, we can consider other possibilities as well. For instance, increase in complexity, depending on the amount and direction of γ change, could be analyzed in many different cases. Or, technological innovation may change γ without any variation in complexity and mistake rates. To be more precise, we represent global technological innovations by below options:

1. Increasing in complexity that not only induces an increase to S, but also causes γ to change across spectrum
2. Increasing in the level of skilled labor productivity (increase in α)
3. Decreasing in the degree of high-tech intensity of a chain without any change in the complexity and mistake rates

For the sake of simplicity, we focus on the case in which there are just one subinterval of each type (figure (6)). Moreover, we skip innovations that lead to a chain with more than two subintervals. As we will see, this simple case also provides main results including rationale for our observation about the heterogeneity of skill premia trends across countries.

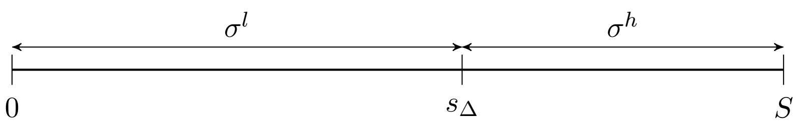

Figure 6: An example with $C = 5$ and $n = 2$

Before analyzing the effect of technological changes on the patterns of vertical specialization and wages, it is useful to introduce some other formal definitions.

Definition 3: Let $(S^{'}_{h1}, S^{'}_{h2}, ..., S^{'}_{hC})$ denote the initial pattern of vertical specialization in the set of all high-tech subintervals ($\mathcal{S}_h$) and $(S_{h1}, S_{h2}, ..., S_{hC})$ represent the new pattern. A country c is moving up (down) the high-tech part of the value chain if $S_{hc} \geq S^{'}_{hc}$ and $S_{hc-1} \geq S^{'}_{hc-1}$ ($S_{hc} \leq S^{'}_{hc}$ and $S_{hc-1} \leq S^{'}_{hc-1}$). The same definition applies to the low-tech part of the chain.

Definition 4: A country is a high-tech (low-tech) producer, if it produces only high-tech (low-tech) intermediate goods. It is bi-tech producer, if it is active in both types of stages.

Definition 5: Inequality in the wage of type z labor between countries c_1 and c_2 ($c_1 > c_2$) is increasing (decreasing) if $(w^z_{c_1})'/(w^z_{c_2})' > w^z_{c_1}/w^z_{c_2}$ ($(w^z_{c_1})'/(w^z_{c_2})' < w^z_{c_1}/w^z_{c_2}$).

4.1 Increase in complexity

As CVW mentioned, in some cases, technological innovations lead to an increase in the number of operations required for the production of the final good. In this part, we analyze the consequences of an increase in the measure of production stages. It is assumed that the utility gain from this increase in complexity is zero. This assumption does not affect on our results about the pattern of task allocation, inequality changes across countries and skill premia within countries. However, it's easy to see that allowing for utility gains will affect real wages. Since it is rarely the case that technological changes lead to the transformation of low-tech stages to high-tech stages, we confine our attention to the cases where $\Delta s_{\Delta} \geq 0$.

Proposition 4: Provided that the number of subintervals remains fixed and the new measure of high-tech stages is high enough, any increase in complexity will cause all countries to move up the high-tech part of the chain. Reaction of countries in the low-tech part depends on the direction and size of change in γ.

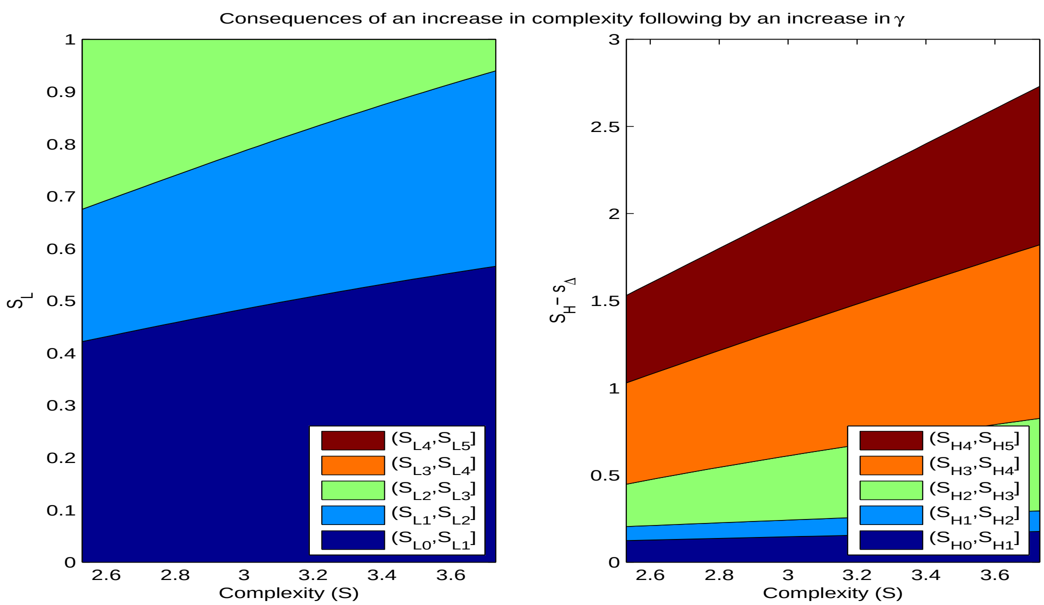

Figure 7: Changes in the pattern of vertical specialization

Proposition 5: Provided that the number of subintervals remains fixed, any increase in complexity may lead to a rise or fall in inequality of type z labor income between each pair of countries. Moreover, the skill premia within a country depends on the direction and size of γ change, as well.

To be more intuitive, Consider a simple example with 5 countries and two subintervals. Figure (7) depicts the effects of increasing in complexity following by an increase in γ. Here, we assume that $\alpha = 0.7$. It's also assumed below values for mistake rates and labor endowments:

$$\begin{cases} (\lambda_1, L_1, H_1) = (0.7802, 0.9500, 0.2000) \\ (\lambda_2, L_2, H_2) = (0.6256, 0.4563, 0.1200) \\ (\lambda_3, L_3, H_3) = (0.3685, 0.6447, 0.3000) \\ (\lambda_4, L_4, H_4) = (0.1835, 0.9890, 0.4500) \\ (\lambda_5, L_5, H_5) = (0.0811, 0.5268, 0.4300) \end{cases}$$

As shown in the figure (7), an increase in S from 2.5 to 3.8 when s_{Δ} remains constant would cause all countries to move up in the high-tech part of the chain. In the low-tech part, countries 4 and 5 do not produce and countries 1-3 move up the chain. The overall intuition behind these changes is simple. An increase in complexity will decrease the total output at all high-tech stages of production. Since the production technology is Leontief, the labor component of the production in each high-tech stage also decreases. Moreover, labor market clearing needs skilled labor supply to be equal to skilled labor demand. So, the measure of stages performed by each country in the high-tech subinterval increases. Thus, all countries in the high-tech part will move p the chain. On the other hand, the unskilled labor demand of bi-producers in the high-tech part decreases due to the decrease in the total output of each high-tech stage. So, they will use more unskilled labor in the low-tech part and move up the chain.

Figure (8) depicts the relevant changes in wages and skill premia. An obvious observation is that countries react heterogeneously to any increase in complexity. Moreover, the wage of both types of labor in all countries decreases. As noted before, this is a direct result of the assumption that increasing the complexity does not affect the utility. Finally, the skill premia of country 4 increases as a result of a decrease in its contribution to the low-tech part. Indeed, the unskilled labor use of country 4 in the high-tech part increases and since the supply of skilled labor is fixed, the wage ratio of skilled labor over unskilled labor increases.

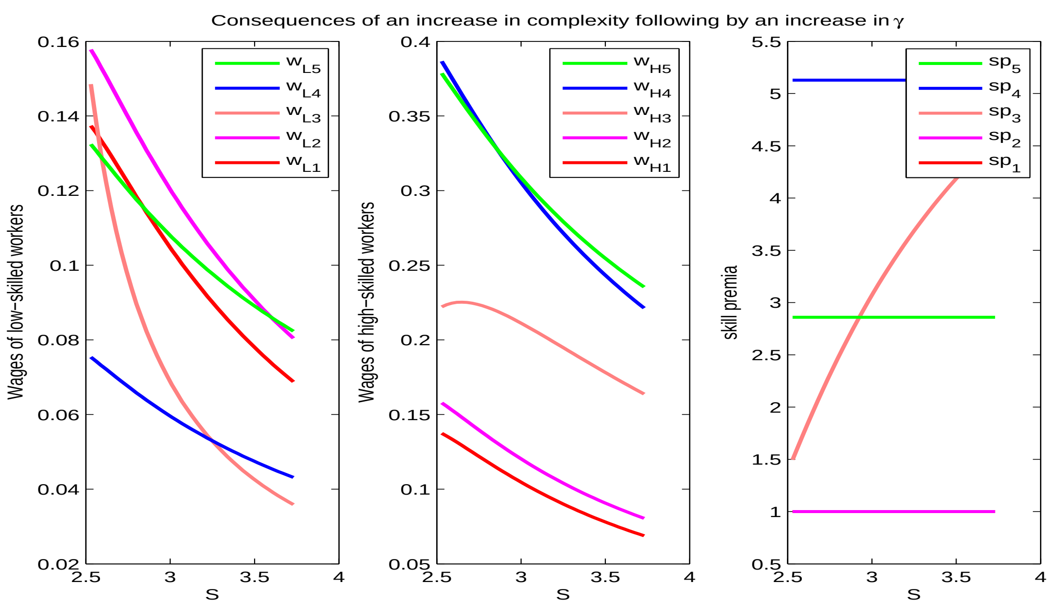

Figure 8: Changes in wages and skill premia

The other cases related to increasing in complexity and decreasing in the high-tech intensity of the chain are explained in the appendix.

4.2 Increase in α

Another possibility for technological change is increasing in the productivity level of skilled labor in the Cob-Douglas combination of high-tech stages' production function. This type of innovation affects the chain in the following way:

Proposition 6: Depending on the relationship between mistake rates and the skill ratio of the endowments, increasing in α would have different effects on the pattern of task allocation within the high-tech part. In particular, if the mistake rate of every country is a negative function of the skill ratio of its endowments, then any increase in the level of α leads all high-tech producers with the skill ratio above 0.5 to move down in the high-tech part of the chain. In the low-tech part, if the unskilled labor use of the most productive bi-producer increases high enough, then all bi-producers move down in the low-tech part. If it decreases or the increase is smaller than the increase in other bi-producers, then all bi-producers move up in the low-tech part.

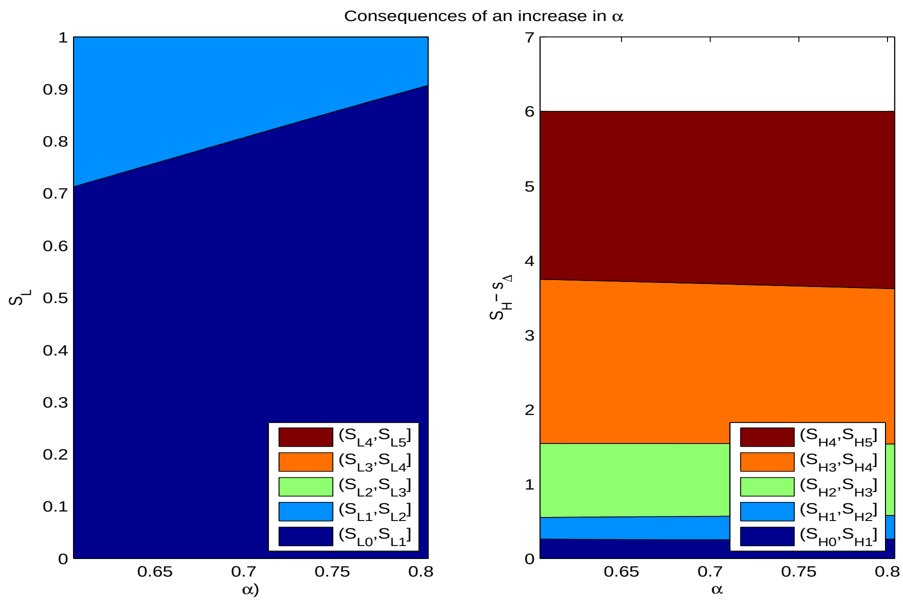

Figure 9: Changes in the pattern of vertical specialization

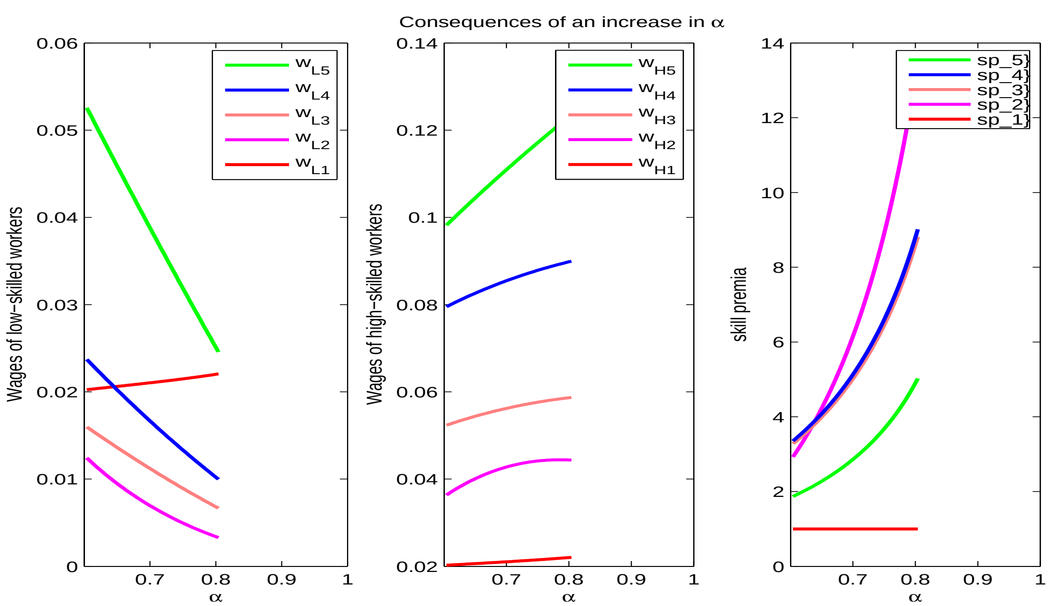

Figure 10: Changes in wages and skill premia

Proposition 7: Any increase in the level of α has an heterogenous effect on the wage inequality of type z between pairs of countries. The effect on skill premia is to raise in some countries and zero effect in the others.

Figures (9) and (10) show an example on the effect of this kind of technological change on the pattern of task allocation and wages within a GVC.

4.3 Increase in α accompanied by decrease in γ

As we explained in the introduction section, the observation about the trend of skill premia for some countries reveals that it is rising for some countries, falling for some others and constant for the remaind ones. However, we saw in previous parts that there is not any single technological change that generates a rising trend of skill premia in some countries and a falling trend in some others. To rationalize this fact, we can consider cases where more than one type of technological innovations happen at the same time. For instance, figures (11) and (12) show the change in the pattern of vertical specialization, wages and skill premia produced by an increase in the productivity level of the skilled labor in the Cobb-Douglas combination and a decrease in the high-tech intensity of the chain. We can see that these two types of innovations can generate all types of trends in skill premia.

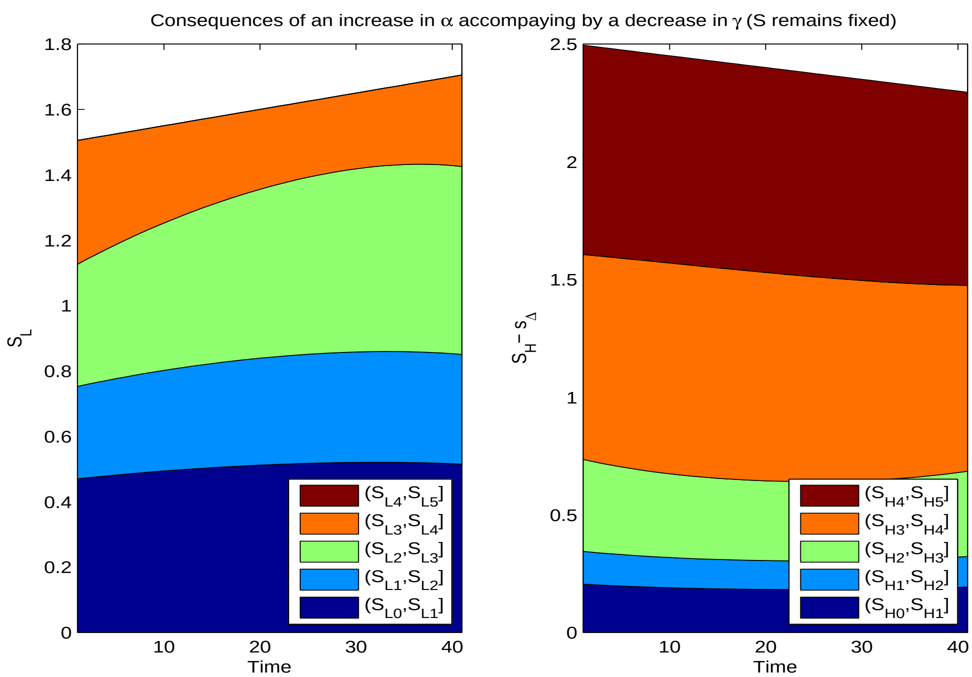

Figure 11: Changes in the pattern of vertical specialization

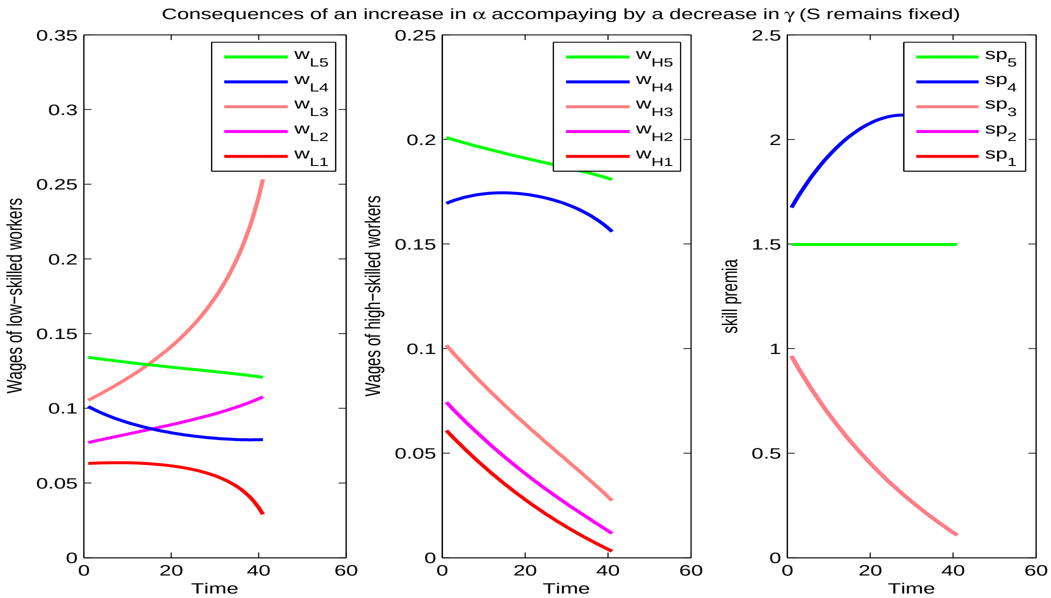

Figure 12: Changes in wages and skill premia

5 Conclusions

In this paper, we have developed a rudimentary framework to rationalize the pattern of task and income distribution within a global value chain and at the same time address the heterogeneity of countries in their trends of skill premia. The model is based on the Costinot et al. [2013]'s setting and focuses on an international sequential production that is subject to mistake. However, we adopt some heterogeneity assumptions on labor and production stages.

In the new setting, the unique vertical specialization of countries is preserved but with some different results from Costinot et al. [2013] findings. First, the vertical specialization happens in both types of stages. Second, beside the absolute productivity differences, the skill ratios of endowments are also a source of comparative advantage among countries. In fact, these ratios are defining in the level of contribution of each country to the each type of stages. Finally, the assumptions on the heterogeneity of labor and stages enable us to address the stylized facts on the trends of skill premia across countries derived from the WIOD database. Indeed, the effects of technological innovations on different types of labor and different types of stages are not symmetric. This is one source of different behavior of both developed and developing countries on some variables including skill premium.

This paper assumes that trade costs are zero and all firms inside a country are similar. Another limiting aspect of the paper is the strong assumption that the mistake rate of countries is constant over all stages. There is space for future research to relax each of these assumption and study more features of global value chains.

References

Daron Acemoglu. Directed technical change. *The Review of Economic Studies*, 69(4):781–809, 2002.

Daron Acemoglu. Pattern of skill premia. *The Review of Economic Studies*, 70(2):199–230, 2003.

Pol Antràs and Davin Chor. Organizing the global value chain. *Econometrica*, 2013.

Richard Baldwin and Anthony Venables. Relocating the value chain: Offshoring and agglomeration in the global economy. *NBER working paper*, (16611), 2010.

Arnaud Costinot, J. Vogel, and S. Wang. An elementary theory of global supply chains. *Review of Economic Studies*, 2013.

Carl Davidson and Steven J. Matusz. *International Trade with Equilibrium Unemployment*. Princeton, NJ: Princeton Univ. Press, 2009.

Elias Dinopoulos and Paul Segerstrom. A schumpeterian model of protection and relative wages. *The American Economic Review*, 89(3):450–472, 1999.

Robert Feenstra and Gordon Hanson.

Masahisa Fujita and Jacques-Francois Thisse. Globalization and the evolution of the supply chain: Who gains and who loses? *International Economic Review*, 47:811–836, 2006.

Luis Garicano. Hierarchies and the organization of knowledge in production. *Journal of Political Economy*, 108(5):874904, October 2000.

Gordon H. Hanson and Ann Harrison. Trade, technology, and wage inequality. *NBER Working Papers*, (5110), 1995.

David Hummels, Jun Ishii, and Kei-Mu Yi. The nature and growth of vertical specialization in world trade. *Journal of International Economics*, 54:75–96, 2001.

Robert C. Johnson and Guillermo Noguera. Fragmentation and trade in value added over four decades. *NBER working paper*, (18186 version), June 2012.

Sébastien Miroudot, Rainer Lanz, and Alexandros Ragoussis. Trade in intermediate goods and services. *ECD Trade Policy Papers*, (93):OECD Publishing, 2009.

Donald J. Robbins. Hos hits facts: Facts win; evidence on trade and wages in the developing world. *Development Discussion Paper*, (557):Harvard Institute for International Development, 1996.

Jr. Robert E. Lucas. On the size distribution of business firms. *Bell Journal of Economics*, 9 (2):508–523, Autumn 1978.

Sherwin Rosen. Authority, control, and the distribution of earnings. *Bell Journal of Economics*, 13(2):311–323, 1982.

Mathias Thoenig and Thierry Verdier. A theory of defensive skill-biased innovation and globalization. *The American Economic Review*, 93(3):709–728, 2003.

Susan Zhu. Trade, product cycles and inequality within and between countries. *Canadian Journal of Economics*, 37(4), 2004.

Susan Zhu and Daniel Trefler. Trade and inequality in developing countries: a general equilibrium analysis. *Journal of International Economics*, 65(1), 2005.

6 appendix

6.1 Simulations

6.1.1 Increasing in complexity

Figure (13) shows the changes in the pattern of vertical specialization for the situation in which Complexity increases but γ decreases. Here, the same values for mistake rates and labor endowments are assumed and the productivity coefficient of the skilled labor in the Cob-Douglas combination is set to $\alpha = 0.8$. The complexity increases from 4 to 5.8 and s_{Δ} grows from 1 to 2.86 in a rate slightly greater than the growth rate of complexity. It can be seen that all countries move up the high-tech part of the chain and the number of countries active in the low-tech part increases when S increases and γ decreases.

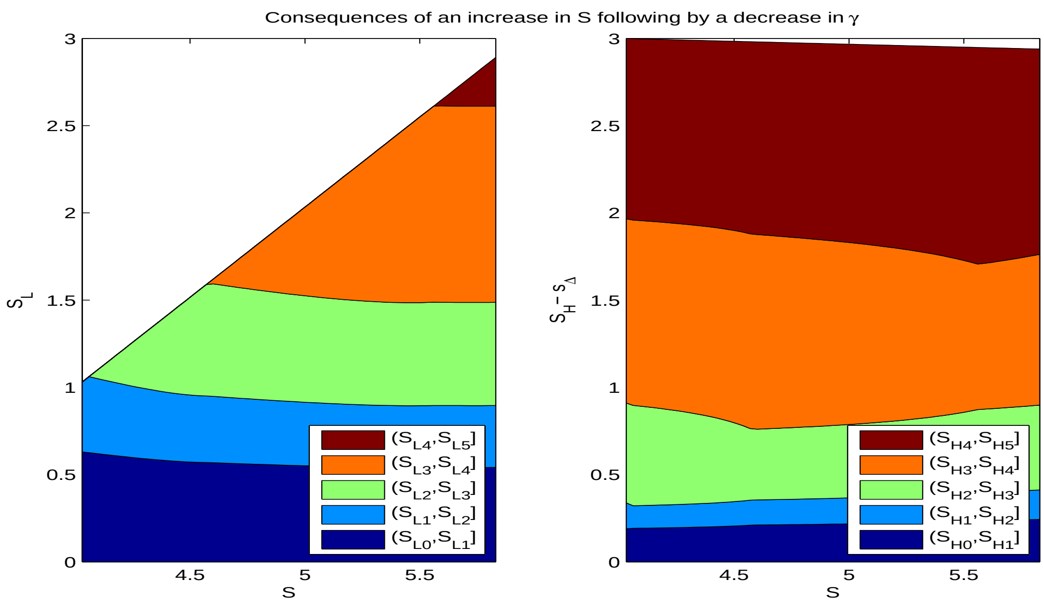

Figure 13: Changes in the pattern of vertical specialization

The reaction of skilled and unskilled labor wages and the skill premia are depicted in figure (14). We can see that the wage of unskilled labor in countries (3)-(5) increases and in countries (1) and (2) decreases. The reason is that the demand for unskilled labor in countries (3)-(5) increases as a result of producing in the low-tech part of the chain. Moreover, as noted before, countries (1) and (2) move down the low-tech part of the chain and so the demand for unskilled labor decreases. The wage of skilled labor decreases in all countries. This is due to the fact that the high-tech intensity of the chain decreases and there is not any utility gain of increasing in complexity, as well. The right column in the figure shows changes in skill-premia that is compatible with the changes in the wage of skilled and unskilled labor.

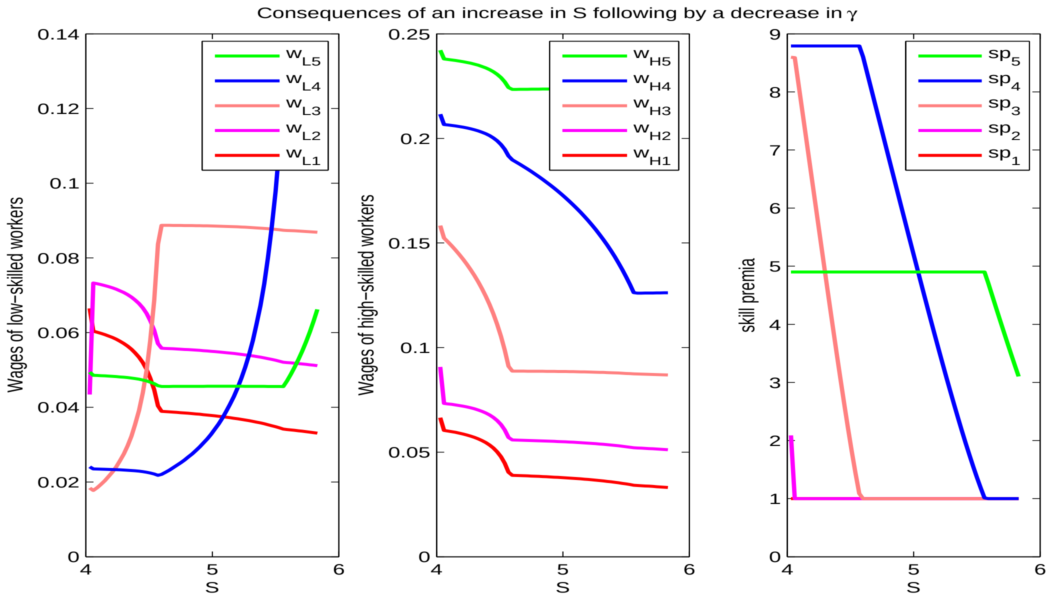

Figure 14: Changes in wages and skill premia

6.1.2 Decrease in the high-tech intensity of the chain

Technological innovation could be the transforation of some high-tech stages to low-tech stages. We can represent this case by a decrease in the level of the high-tech intensity of the chain. As figure (15) shows any decrease in the level of high-tech intensity leads all countries to move up in the high-tech part. In the low-tech part, the impact is heterogenous across countries. Some countries may move down the chain and some of them transfer from a high-tech producer to a bi-producer country.

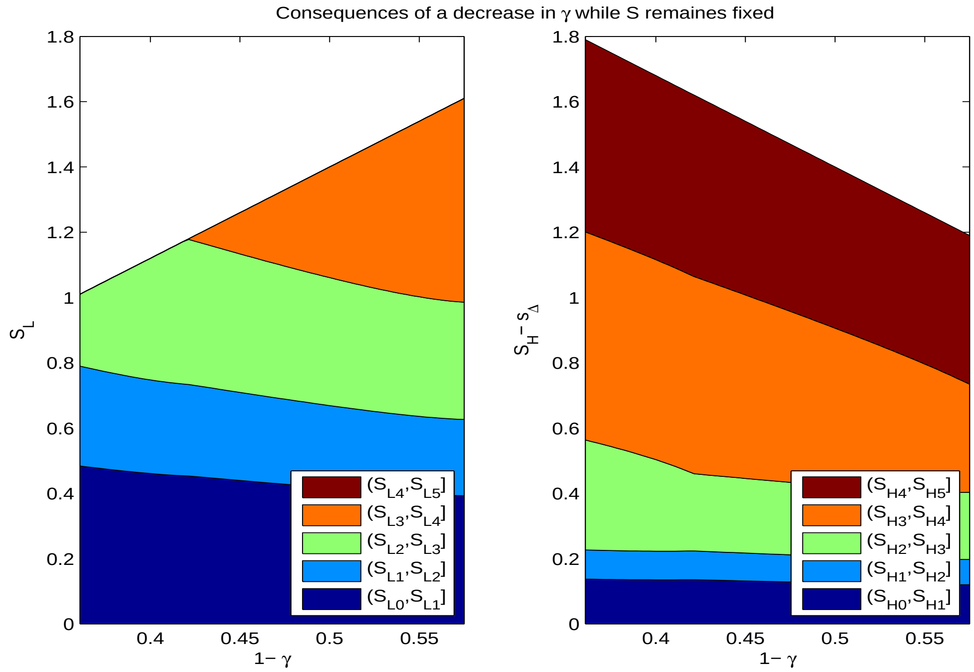

Figure 15: Changes in the pattern of vertical specialization

Figure (16) depicts the example on the effects of this technological innovation on pattern of wages and skill premia. Any decrease in the level of high-tech intensity results in a decrease in the skill premia of some countries and a constant skill premia for the others. The impact on the wage inequality of type z between any pair of countries varies based on the chain and technology parameters.

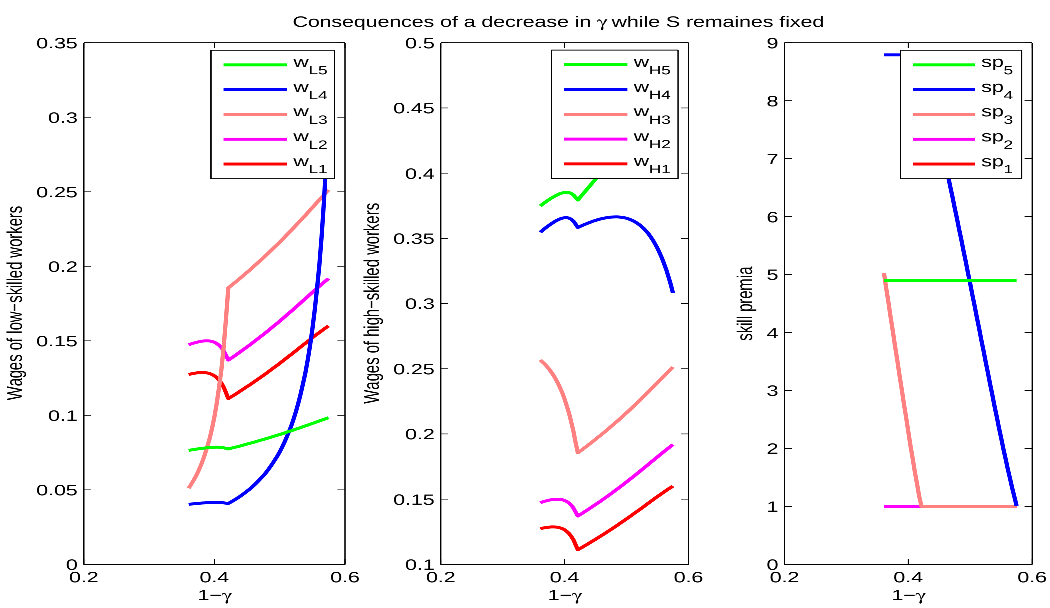

Figure 16: Changes in wages and skill premia

6.2 Proofs

Proof of Proposition 1

As we note before, for technical reason, we assume that if a firm produces intermediate good $s + ds$, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Formally, for any intermediate good $s + ds$, we assume the existence of $s_\Delta < s + ds \leq s_\Delta + \Delta$ such that if $q(s + ds) > 0$, then $q(s') > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. The proof of this proposition is similar to the proof of the proposition 1 in CVW. We define $\Delta(s) \equiv (s_\Delta, s_\Delta + \Delta)$ for some s_Δ satisfying previous conditions. The proof, however, does not depend on which exact s_Δ we pick up. Like CVW, we provide the proof in four steps:

Step 1: $p(.)$ is continuous.

For each subinterval σ_i^z, we can easily use the result of the first step in the proof of proposition 1 of CVW and show that $p(.)$ is continuous within each subinterval. For the boundary points, we assumed:

$$p(\overline{s}_i) = \overline{p}_i = \lim_{s \to (\overline{s}_i)^-} p(s) \quad \forall i \in \{1, 2, ..., 2n\}$$

so, there is not a jump in price when moving from a subinterval to its neighbors.

Step 2: If $s_1 > s_2$ then $p(s_2) > p(s_1)$.

By step 1, we know that $p(.)$ is continuous and for each σ_i^z, $p'(s) > 0 \, \forall s \in \sigma_i^z$. These two facts indicate that $p(.)$ is strictly increasing.

Step 3: If $c_2 > c_1$ then $w^{'}_{hc_2} > w^{'}_{hc_1}$ and $w^l_{c_2} > w^l_{c_1}$.

Factor market clearing implies that country c_1 produces at least one intermediate good in a high-tech stage (s_1) and one in a low-tech stage (s_2). By assumption, this requires $Q_{c_1}(s) > 0$ for all $s \in \Delta(s_1)$ and $s \in \Delta(s_2)$. Equation (3) indicates that:

$$\begin{cases} p(s_1) = (1 + \lambda_{c_1} ds) p(s_1 - ds) + w^{'}_{hc_1} ds \\ \\ p(s_1) \leq (1 + \lambda_{c_2} ds) p(s_1 - ds) + w^{'}_{hc_2} ds \end{cases} \qquad (1)'$$

and

$$\begin{cases} p(s_2) = (1 + \lambda_{c_1} ds) p(s_2 - ds) + w^l_{c_1} ds \\ \\ p(s_2) \leq (1 + \lambda_{c_2} ds) p(s_2 - ds) + w^l_{c_2} ds \end{cases} \qquad (2)'$$

Equations (1)' and (2)' plus $\lambda_{c_2} < \lambda_{c_1}$ yield $w^{'}_{hc_2} > w^{'}_{hc_1}$ and $w^l_{c_2} > w^l_{c_1}$.

Step 4: If $c_2 > c_1$ and $Q_{c_1}(s_1) > 0 (s_1 \in \mathcal{S}_z)$, then $Q_{c_2}(s) = 0 \, \forall s \in \mathcal{S}_z, s < s_2$.

[The same proof as CVW]

To finish the proof of proposition 1, define $S_c^Z \equiv Sup\{s \in \mathcal{S}_z | Q_c(s) > 0\} \, \forall c \in \mathcal{C}$. Regarding step 4, we must have $S_0^h \equiv 0 < S_1^h < ... < S_C^h = \overline{s}_{2n-1}$ and $S_0^l \equiv \overline{s}_1 < S_1^l < ... < S_C^l = S$, such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c^z(s) > 0$ if $S_{c-1}^z < s < S_c^z$ and $Q_c^z(s) = 0$ if $s < S_{c-1}^z$ or $s > S_c^z$. Moreover, since $Q_c^z(s) > 0$ requires $Q_c^z(s') > 0$ for all $s' \in (s - ds, s]$, we must also have $Q_c^z(S_c) > 0$ and $Q_c^z(S_{c-1}) = 0$ for all $c \in \mathcal{C}$. So, $Q_c^z(s) > 0$ if and only if $s \in (S_{c-1}^z, S_c^z]$. Lastly, the good market clearing condition implies that $S_C^h = \overline{s}_{2n-1}$ and $S_C^l = S$.

Proof of Proposition 2

Consider an arbitrary σ_i^z that is undertook by countries $c_{i,1}^z, c_{i,2}^z, ...$ and $c_{i,n_{iz}}^z$. Applying proposition 1 of CVW, it is straightforward to obtain this proposition.

Proof of Lemma 1

The proof of this lemma is very similar to the proof of lemma 1 in CVW. It's easier to first prove the equation (10) and then equations (8) and (9). Proposition 2 and equation (5) yield:

$$Q_{i,j}^z(s_2) - Q_{i,j}^z(s_1) = -\lambda_{i,j}^z \int_{s_1}^{s_2} Q_{i,j}^z(s)ds, \text{ for all } s_2 \in (S_{i,j-1}^z, S_{i,j}^z] \tag{15}$$

Taking the derivative of this expression with respect to s_2 gives us:

$$\frac{dQ_{i,j}^z(s)}{ds} = -\lambda_{i,j}^z Q_{i,j}^z(s), \text{ for all } s \in (S_{i,j-1}^z, S_{i,j}^z]$$

The solution of this equation would satisfy:

$$Q_{i,j}^z(S_{i,j}^z) = e^{-\lambda_{i,j}^z(S_{i,j}^z - S_{i,j-1}^z)} \lim_{s \to (S_{i,j-1}^z)^+} Q_{i,j}^z(s) \tag{16}$$

Proposition 2 and equation (5) would also imply:

$$Q_{i,j}^z(S_{i,j-1}^z + ds) - Q_{i,j-1}^z(S_{i,j-1}^z - ds) = -[\lambda_{i,j}^z \lim_{s \to (S_{i,j-1}^z)^+} Q_{i,j}^z(s) + \lambda_{i,j-1}^z Q_{i,j-1}^z(S_{i,j-1}^z - ds)]ds$$

ds is infinitesimal, so this imply:

$$\lim_{s \to (S_{i,j-1}^z)^+} Q_{i,j}^z(s) = \lim_{s \to (S_{i,j-1}^z)^-} Q_{i,j-1}^z(s) = Q_{i,j-1}^z(S_{i,j-1}^z) \tag{17}$$

Regarding this equation, equation (16) and the definition of $Q_{i,j}^z \equiv Q_{i,j}^z(S_{i,j}^z)$, we can obtain equation (10).

To derive equations (8) and (9), we can use proposition 2 and equations (7) and (6). We only derive equation (8). Equation (9) could be obtained in the same way. By proposition 1 and equation (7), we can write:

$$\left(\frac{1-\alpha}{\alpha}\frac{w_c^H}{w_c^L}\right)^{1-\alpha} \int_{S_{i,j-1}^z}^{S_{i,j}^z} Q_{i,j}^z(s)ds = H_{i,j}^z \tag{18}$$

Regarding equations (16) and (17) we can also get:

$$\int_{S_{i,j-1}^z}^{S_{i,j}^z} Q_{i,j}^z(s)ds = \frac{1}{\lambda_{i,j}^z}[Q_{i,j-1}^z(S_{i,j-1}^z) - Q_{i,j}^z(S_{i,j}^z)] \tag{19}$$

Equations (18) and (19) imply:

$$H_{i,j}^z = \left(\frac{1-\alpha}{\alpha}\frac{w_c^H}{w_c^L}\right)^{1-\alpha} \frac{1}{\lambda_{i,j}^z}[Q_{i,j-1}^z(S_{i,j-1}^z) - Q_{i,j}^z(S_{i,j}^z)] \tag{20}$$

Equation (8) is obtained from equations (10) and (20) and the definition of $Q_{i,j}^z \equiv Q_{i,j}^z(S_{i,j}^z)$.

Proof of Lemma 2
We only prove part (a) of this lemma. Proof of part (b) is similar. We first consider equation (11). Proposition 2 and condition (3) yield:

$$p(S^z_{i,j}+ds)-(1+\lambda^z_{i,j+1}ds)p(S^z_{i,j})-w^z_{i,j+1}ds \geq p(S^z_{i,j}+ds)-(1+\lambda^z_{i,j}ds)p(S^z_{i,j})-w^z_{i,j}ds$$

$$p(S^z_{i,j})-(1+\lambda^z_{i,j}ds)p(S^z_{i,j}-ds)-w^z_{i,j}ds \geq p(S^z_{i,j})-(1+\lambda^z_{i,j+1}ds)p(S^z_{i,j}-ds)-w^z_{i,j+1}ds$$

for any σ^z_i and any $c^z_{i,j} \in \{c^z_{i,1}, c^z_{i,2}, ..., c^z_{i,n_{iz}}\}$. After some algebra, we can get:

$$(\lambda^z_{i,j}-\lambda^z_{i,j+1})p(S^z_{i,j}) \geq w^z_{i,j+1}-w^z_{i,j} \geq (\lambda^z_{i,j}-\lambda^z_{i,j+1})p(S^z_{i,j}-ds)$$

Considering that $p(.)$ is continuous and ds is infinitesimal, we can write:

$$w^z_{i,j+1}-w^z_{i,j}=(\lambda^z_{i,j}-\lambda^z_{i,j+1})p(S^z_{i,j}), \text{ for all } c^z_{i,j} \in \{c^z_{i,1}, c^z_{i,2}, ..., c^z_{i,n_{iz}}\}$$

This equation is equivalent to equation (11) in which $p^z_{i,j} \equiv p(S^z_{i,j})$.

To show equation (12), consider proposition 2 and condition (3). We can derive:

$$p(s+ds)=(1+\lambda^z_{i,j}ds)p(s)+w^z_{i,j}ds, \text{ for all } s \in (S^z_{i,j-1}, S^z_{i,j}]$$

Dividing by ds, we gain:

$$\frac{dp(s)}{ds}=\lambda^z_{i,j}p(s)+w^z_{i,j}$$

The solution of this differential equation must satisfy:

$$p(S^z_{i,j})=e^{\lambda^z_{i,j}(S^z_{i,j}-S^z_{i,j-1})}\lim_{s\to(S^z_{i,j-1})^+}p(s)+[e^{\lambda^z_{i,j}(S^z_{i,j}-S^z_{i,j-1})}-1](\frac{w^z_{i,j}}{\lambda^z_{i,j}})$$

Since $p(.)$ is continuous and regarding $N^z_{i,j} \equiv S^z_{i,j}-S^z_{i,j-1}$ and $p^z_{i,j} \equiv p(S^z_{i,j})$, this equation is equivalent to equation (12).

Proof of Proposition 3
We provide the proof in four steps. Before going through these steps, we use a new labeling for the cutoff stages and border points of subintervals. Let m, m_h and m_l denote the total number of subintervals, the total number of high-tech subintervals and total number of low-tech subintervals produced by border points and cutoff stages, respectively. $\mathcal{I}=\{1,2,...,m\}$ is the set of all these m subintervals as shown in figure (17).

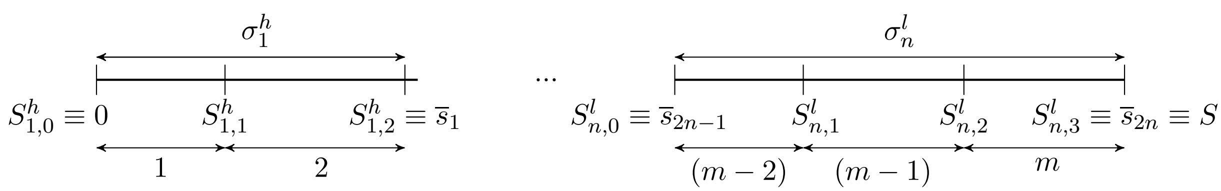

Figure 17: An example of new labeling

In addition, we define $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ as:

$$(S'_0, S'_1, ..., S'_m)=(S^h_{1,0}, S^h_{1,1}, ..., S^l_{n,n_{nl}})$$

$$(Q'_0, Q'_1, ..., Q'_m) = (Q(S'_0), Q(S'_1), ..., Q(S'_m))$$

Step 1: $(S^z_{i,0}, ..., S^z_{i,n_{iz}})$ and $(Q^z_{i,0}, ..., Q^z_{i,n_{iz}})$ satisfy equations (8)-(10) $(\forall \sigma^z_i)$ if and only if $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ satisfy

$$S'_i = S'_0 + \sum_{j=1}^{i} (\frac{1}{\lambda_j}) ln[\frac{Q'_0 - \sum_{k=1}^{j-1} \lambda_k \widetilde{L}_k}{Q'_0 - \sum_{k=1}^{j} \lambda_k \widetilde{L}_k}], \text{ for all } i \in \mathcal{I} \tag{21}$$

$$Q'_i = Q'_0 - \sum_{j=1}^{i} \lambda_j \widetilde{L}_j, \text{ for all } i \in \mathcal{I} \tag{22}$$

To prove this step, notice that $(S^z_{i,0}, ..., S^z_{i,n_{iz}})$ and $(Q^z_{i,0}, ..., Q^z_{i,n_{iz}})$ satisfy equations (8)-(10) if and only if $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ satisfy following equations:

$$S'_i = S'_{i-1} - (\frac{1}{\lambda_i}) Ln(1 - \frac{\lambda_i \widetilde{L}_i}{Q'_{i-1}}) \, , \, for\, all\, i \in \mathcal{I} \tag{23}$$

$$Q'_i = e^{-\lambda_i (S'_i - S'_{i-1})} Q'_{i-1} \, , \, for\, all\, i \in \mathcal{I} \tag{24}$$

Regarding our labeling, this one-to-one correspondence is straightforward. So, we must show that $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ satisfy equations (23) and (24) if and only if they satisfy equations (21) and (22). By equations (23) and (24), we have:

$$Q'_i = Q'_{i-1} - \lambda_i \widetilde{L}_i$$

and by iteration, we get:

$$Q'_i = Q'_0 - \sum_{j=1}^{i} \lambda_j \widetilde{L}_j, \text{ for all } i \in \mathcal{I}$$

which is equation (22). Now, regarding equations (23), again we can iterate and obtain:

$$S'_i = S'_0 - \sum_{j=1}^{i} (\frac{1}{\lambda_j}) ln[1 - \frac{\lambda_j \widetilde{L}_j}{Q'_{j-1}}], \text{ for all } i \in \mathcal{I}$$

Substituting equation (22) in this expression, we can derive equation (21). It's a simple algebra to show that if $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ satisfy equations (21) and (22), then they will satisfy equations (23) and (24).

Step 2: There exists a unique set of vectors $(S^z_{i,0}, ..., S^z_{i,n_{iz}})$ and $(Q^z_{i,0}, ..., Q^z_{i,n_{iz}})$ $(\forall \sigma^z_i)$ that satisfy equations (8)-(10) and boundary conditions

$$\begin{cases} (S^z_{i,0}, S^z_{i,n_{iz}}) = (\overline{s}_{2(i-1)}, \overline{s}_{2i-1}) & \text{if } z = h \\ \\ (S^z_{i,0}, S^z_{i,n_{iz}}) = (\overline{s}_{2i-1}, \overline{s}_{2i}) & \text{if } z = l \end{cases}$$

Again, it is straightforward from our labeling that this statement is equivalent to show that there exists a unique pair of vectors $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ that satisfy equations (23) and (24) and the boundary conditions: $S'_0 = 0$ and $S'_I = S$. To prove the latter, define $\widetilde{Q'}_0 = \sum_{i=1}^{I} \lambda_i L_i$. Considering step 1, it's obvious that there does not exist a pair of vectors $(S'_0, S'_1, ..., S'_m)$ and $(\widetilde{Q'}_0, Q'_1, ..., Q'_m)$ that satisfy equations (21) and (22). Thus, there does not exist such a pair of vectors that satisfy (23) and (24). So, we focus on $Q'_0 > \widetilde{Q'}_0$. Considering equation (21), we can see that $\partial S'_I/\partial Q'_0 < 0$ for all $Q'_0 > \widetilde{Q'}_0$. In addition, it is easy to check that:

$$\lim_{Q'_0 \to \widetilde{Q'}_0^+} S'_I = +\infty \quad \text{and} \quad \lim_{Q'_0 \to +\infty} S'_I = S'_0$$

So, conditional on setting $S'_0 = 0$, there exists a unique $Q'_0 > \widetilde{Q'}_0$ such that vectors $(S'_0, S'_1, ..., S'_m)$ and $(Q'_0, Q'_1, ..., Q'_m)$ satisfy equation (21) and (22) and $S'_I = S$.

Step 3: Within each high-tech subinterval, for any vector $(N^h_{i,1}, ..., N^h_{i,n_{ih}})$ there exists a unique pair of vectors $(w'_{i,1}, ..., w'_{i,n_{ih}})$ and $(p^h_{i,0}, ..., p^h_{i,n_{ih}})$ satisfying equations (11) and (12) and the boundary conditions $p^h_{i,0} = \overline{p}_{i-1}$ and $p^h_{i,n_{ih}} = \overline{p}_i$.

The proof of this step is exactly the same as the proof of proposition 2 of CVW (step 3).

Step 4: Within each low-tech subinterval, for any vector $(N^l_{i,1}, ..., N^l_{i,n_{il}})$ there exists a unique pair of vectors $(w^l_{i,1}, ..., w^l_{i,n_{il}})$ and $(p^l_{i,0}, ..., p^l_{i,n_{il}})$ satisfying equations (13) and (14) and the boundary conditions $p^l_{i,0} = \overline{p}_i$ and $p^l_{i,n_{il}} = \overline{p}_{i+1}$.

The proof of this step is also exactly the same as the proof of proposition 2 of CVW (step 3).

Steps (1)-(4) indicate the existence and uniqueness of $(\{(S^z_{i,0}, ..., S^z_{i,n_{iz}}), (Q^z_{i,0}, ..., Q^z_{i,n_{iz}})\}\, \forall \sigma^z_i)$, $(\{(w'_{i,1}, ..., w'_{i,n_{ih}}), (p^h_{i,0}, ..., p^h_{i,n_{ih}})\} \forall \sigma^h_i)$ and $(\{(w^l_{i,1}, ..., w^l_{i,n_{il}}), (p^l_{i,0}, ..., p^l_{i,n_{il}})\} \forall \sigma^l_i)$ that satisfy equations (8)-(14) with boundary conditions:

$$\begin{cases} (S^z_{i,0}, S^z_{i,n_{iz}}) = (\overline{s}_{2(i-1)}, \overline{s}_{2i-1}) & \text{if } z = h \\ \\ (S^z_{i,0}, S^z_{i,n_{iz}}) = (\overline{s}_{2i-1}, \overline{s}_{2i}) & \text{if } z = l \end{cases}$$

$$\begin{cases} p^z_{i,0} = \overline{p}_i \,,\, p^z_{i,n_{iz}} = \overline{p}_{i+1} & \text{if } z = l \\ \\ p^z_{i,0} = \overline{p}_{i-1} \,,\, p^z_{i,n_{iz}} = \overline{p}_i & \text{if } z = h \\ \\ p^l_{n,n_{nl}} = p_C = 1 \,,\, p^h_{1,0} = 0 \end{cases}$$

If we consider these output levels ($\forall z \in \{l, h\}\,,\, j \in \{1, 2, ..., n_{iz}\}$):

$$Q^z_{i,j}(s) = e^{-\lambda^z_{i,j}(s - S^z_{i,j-1})} Q^z_{i,j-1}, \quad \forall s \in (S^z_{i,j-1}, S^z_{i,j}]$$

and these intermediate good prices:

$$p^h_{i,j}(s) = e^{\lambda^h_{i,j}(s - S^h_{i,j-1})} p^h_{i,j-1} + (e^{\lambda^h_{i,j}(s - S^z_{i,j-1})} - 1)(\frac{w'_{i,j}}{\lambda^h_{i,j}}), \quad \forall s \in (S^h_{i,j-1}, S^h_{i,j}] \text{ and } \forall \sigma^h_i$$

$$p^l_{i,j}(s) = e^{\lambda^l_{i,j}(s - S^l_{i,j-1})} p^l_{i,j-1} + (e^{\lambda^l_{i,j}(s - S^l_{i,j-1})} - 1)(\frac{w^l_{i,j}}{\lambda^l_{i,j}}), \quad \forall s \in (S^l_{i,j-1}, S^l_{i,j}] \text{ and } \forall \sigma^l_i$$

By construction, these output levels, input prices and the relevant wages satisfy conditions (3)-(6). So, a free trade equilibrium exists. Moreover, $(\{(S^z_{i,0}, ..., S^z_{i,n_{iz}}), (Q^z_{i,0}, ..., Q^z_{i,n_{iz}})\}\, \forall \sigma^z_i)$,

$(\{(w'_{i,1},...,w'_{i,n_{ih}}),(p^h_{i,0},...,p^h_{i,n_{ih}})\}\forall\sigma^h_i)$ and $(\{(w^l_{i,1},...,w^l_{i,n_{il}}),(p^l_{i,0},...,p^l_{i,n_{il}})\}\forall\sigma^l_i)$ are unique and regarding propositions 1 and 2 and Lemmas 1 and 2, the free trade equilibrium is unique as well.

Proof of Proposition 4

Before going through the proof, it is useful to note that since $w^H_c \geq w^L_c$, the Cobb-Douglas combination of labors in the high-tech stages and the overall production structure result in the partitioning of countries to three categories: a group with only one country (c_0) that is a bi-producer and $w^H_{c_0} > w^L_{c_0}$, the group of bi-producers that have mistake rates bigger than λ_{c_0} in which the skill premium is equal to one and the group of countries more productive than c_0 and are only active in high-tech stages. Let call c_0, the "border country". Furthermore, let Q_0 and $\widehat{Q}_0$ represent the input at stages 0 and s_Δ respectively.

Denote the number of countries producing at low-tech stages by n_1. We know that skilled labor market clearing entails the number of countries that are active in high-tech part to be equal to n. Let $(l_1, l_2, ..., l_{n_1})$ and $(\widetilde{L}_1, ..., \widetilde{L}_n)$ represent respectively the unskilled labor use of countries $\{1, ..., n_1\}$ in the low-tech part and the labor content that countries use in the high-tech part, where:

$$\widetilde{L}_c = H^\alpha_c (L_c - l_c)^{1-\alpha}$$

It is evident from this equation that $\widetilde{L}_c$ has a reverse relationship with l_c. By lemma 1, we know that the amount of H_c and l_c depend on α and cutoff stages (S^h_j). For the proof of the first part, we consider the high-tech part as a CVW type supply chain with $(L_1, ...L_2)$ replaced by $(\widetilde{L}_1, ..., \widetilde{L}_n)$ and for the proof of the second part, we consider the low-tech part as a CVW type supply chain with $(L_1, ...L_2)$ replaced by $(l_1, ..., l_n)$. Regarding the border country (c_{n_1}), increasing in complexity leads to one of these cases: increasing n_1, decreasing n_1 and changing the contribution of n_1 in each type of stages while n_1 remains fixed. First, we analyze the third case. Two options for the change in the contribution of n_1 in each type of stages could be imagined:

a. If the share of unskilled labor that n_1 devotes to low-tech production decreases ($l_{c_{n_1}} \downarrow$), then $\widetilde{L}_{c_{n_1}} \uparrow$. Moreover, since $\Delta s_\Delta \geq 0$ then $Q_0 \downarrow$ and $\widehat{Q}_0 \downarrow$. Thus, considering propositions 3 and 5 of CVW and the fact that $\widetilde{L}_c$ does not change for other countries, we can conclude that all countries in the high-tech part will move up the chain. In this case, it is easy to check that all countries in the low-tech also move up the chain.

b. If the share of unskilled labor that n_1 devotes to low-tech production decreases ($l_{c_{n_1}} \uparrow$), then $\widetilde{L}_{c_{n_1}} \downarrow$. Moreover, since $\Delta s_\Delta \geq 0$ then either $Q_0 \downarrow$ or $Q_0 \uparrow$. In the former case, the same reasoning as part (a) applies. The latter case could be followed by two different situations: either $\widehat{Q}_0 \downarrow$ or $\widehat{Q}_0 \uparrow$. Again, in the first situation, all countries move up the high-tech part and non border countries in the low-tech part also move up the chain. In the second situation, depending on the size of the drop in $\widetilde{L}_{c_{n_1}}$, which is a negative function of the measure of high-tech stages, we will have different results. If the drop is low enough (or the measure of high-tech stage is high enough), then all countries in the high-tech part will move up the chain.Here, non border countries in the low-tech part move down the chain.

If n_1 decreases, we can follow the same reasoning as part (a) for all countries that would be a border country at some point. If n_1 increases, the same reasoning as part (b) applies.

Proof of Proposition 5

Following the proof of proposition 4 and regarding the proposition 5 of CVW, it is straightforward to get this proposition.

Proof of Proposition 6
Define "border country", $\widetilde{L}_c$ and l_c as proof of proposition 4. Consider the high-tech part as a CVW type supply chain with $(L_1, ...L_2)$ replaced by $(\widetilde{L}_1, ..., \widetilde{L}_n)$ and the low-tech part as a CVW type supply chain with $(L_1, ...L_2)$ replaced by $(l_1, ..., l_n)$.

For the proof of the first part, consider country c with the skill ration of greater than 0.5 that only produce in high-tech part. Since $\frac{H_c}{L_c} > 1$, $\widetilde{L}_c$ is an increasing function of α. On the other hand, if λ_c is a decreasing function of $\frac{H_c}{L_c}$, the most productive countries experience an increase in $\widetilde{L}_c$ when α increases. Moreover, we can get below relationship for each non border bi-producer:

$$\widetilde{L}_c = (\frac{1-\alpha}{\alpha})^{1-\alpha} H_c$$

which is a declining function of α. Thus, any increase in α causes the labor content of high-tech stages to decrease for all non border bi-producers. Consequently, since the measure of high-tech stages remains fixed, the high-tech producers with $\frac{H_c}{L_c} > 1$ must move down the high-tech part to satisfy the conditions for skilled and unskilled labor market clearing.

In order to prove the second part, note that for every non border bi-producer we have:

$$\frac{w_c^H}{w_c^L} = \frac{\alpha}{1-\alpha}.\frac{L_c^H}{H_c} = 1$$

So, any increase in α will decrease $\frac{L_c^H}{H_c}$ and since H_c is fixed, this is equivalent to decline in $L_c^H \equiv (L_c - l_c)$ and a rise in l_c. Thus, if the increase in the unskilled labor use of the border bi-producer in the low-tech part is too high, the unskilled labor market clearing induces bi-producers to move down the low-tech part. On the other hand, if the increase is below the increase in other bi-producers or it decreases, then unskilled labor market clearing condition requires all bi-producers to move up the low-tech part.

Proof of Proposition 7
The first part is derived directly from the previous proposition, lemma 1 and lemma 2. The proof of the second part is straightforward. Note that if a country is non border bi-producer (refer to the proof of proposition 4 for the definition of this) then the skill premia always remains constant because the optimality condition requires it to put the most possible labor on the low-tech part. Otherwise, from the Cobb-Douglas combination of the labor content of high-tech stages, we can derive:

$$\frac{w_c^H}{w_c^L} = \frac{\alpha}{1-\alpha}.\frac{L_c^H}{H_c}$$

that is an increasing function of α.