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Abstract This paper studies the effect of investor's bounded rationality on market dynamics. In an order driven market, we consider a few-types model where two risky assets are exchanged. Agents differ by their behavior, knowledge, risk aversion and investment horizon. The investor's demand is defined by a utility maximization under constant absolute risk aversion. Relaxing the assumption of perfect knowledge of the fundamentals enables to identify two components in a bubble. The first one comes from the unperceived fundamental changes due to trader's belief perseverance. The second one is generated by chartist behavior. In all simulations, speculators make the market less efficient and more volatile. They also increase the maximum amount of assets exchanged in the most liquid time step. However, our model is not showing raising average volatility on long term. Concerning the fundamentalists, the unknown fundamental has a stabilization impact on the trading price. The closer the anchor is to the true fundamental value, the more efficient the market is, because the prices change smoothly.

Keywords Agent-based modeling · market microstructure · fundamental value · trading volume · efficient market

JEL classification C63 · D44 · G12 · G14

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1 Introduction

The main goal of financial markets is to guarantee an optimal transfer of resources from supply to demand. This aim can be attained only if exchanges do actually occur in a considered period, i.e. the market is liquid. Amihud et al. (2005) write that "liquidity is a complex concept. Stated simply, liquidity is the ease of trading a security ". Hence liquidity is a property of the system, which cannot be attained by just one agent with not enough influence on the market to create a context of easy trade. It is, however, an important feature which assures the functioning of the financial market through the behavior of the agents. It impacts the price, the volatility, and the amount of quoted orders.

During the last crisis, there was no way analysts could anticipate the liquidity and price falls that took place. And, to the best of our knowledge, there is no known mean to impact on the market liquidity and we are just faced with ex post observations and attempts to understand the data. As an example: Air France -KLM was valued under 5€ on the CAC40, even if a consensus of analysts estimates that the book value was at least 6 times higher. However, nobody wanted to hold this asset so it was undervalued from the start! This is neither predictable nor rational. Actually, it has been shown that agent's behavior is drastically driven by asset exposure to liquidity risk (Amihud, 2002). For example, a fly to quality is observed, traders prefer to hold less risky and more liquid assets even if their returns is lower. Liquidity is studied by micro structure theory, but it is usually taken as an exogenous parameter which influences the agents' behaviors. However, it has been little studied in agent-based computational economics, where it could be endogeneized as the emerging result of actual transactions, then taken into account by the agents.

The contribution of agent based economics is to produce models that integrate agents' bounded rationality, as well as their heterogeneity in terms of information and cognition. Several authors have already proven that this assumption of heterogeneity is necessary to reproduce, with models, dynamics of actual behaviors (like experimental data) (Bao et al., 2012).

The main goal of this paper is to focus on heterogeneous knowledge about fundamentals and it impact on liquidity dynamics in a financial market. We build an agent-based model, for which we make choices to produce the modeling structure and the rationality of agents. The comparison among different simulations show that the information that is available to different agents has an impact on price dynamics and market liquidity. Different stylized facts are thus produced. The introduction of an idiosyncratic perceived fundamental value enables to identify different bubble types: some that can be attributed to anchoring (Lord et al., 1979; Westerhoff, 2004), and some that are generated by chartists behavior, based on trend extrapolation (Hommes, 2006). As seen in other models (Giardina and Bouchaud, 2003; Hommes et al., 2005; Lux and Marchesi, 1998), we observe a destabilization power of chartists. We also witness the stabilization impact of the anchor on the price variance, since the trading price evolves more slowly than in the case of perfect knowledge of fundamentals. Finally, we test the agent's aggressiveness (Parlour, 1998) on the price efficiency. As expected, if agents take into account the market liquidity as a parameter of price valuation, we observe a rise in liquidity and a fall in efficiency. It would be possible to evaluate, with this mean, the price of liquidity in the system, but at this stage, we do not perform econometric tests, just observe stylized facts.

This paper is organized as follows : Section 2 reports our model assumptions and their literature, describes the functioning of the model in English. Section 3 discusses results and Section 4 concludes.

2 Model and results

2.1 Choosing a suitable model to study liquidity

The field of microstructure focuses on the concept of liquidity, impact of traders type or information revealed. Without doubt, one of the most studied is the Kyle model (Kyle, 1985) where the author focuses on the optimal behavior of discretionary traders and their effect on patterns of trade. An other example is Grossman and Miller (1988), who point out the importance of liquidity for behavior, prices and the viability of the market. In their paper, they study how agents submit orders in more or less liquid market so as to avoid disclosing their private information. Chu et al. (2009) highlight a strong preference of investors for liquid assets amid heightened price volatility during the last financial crisis. In their study, extended asset guarantee is a way to improve asset liquidity. The liquidity theory predicts that the level of liquidity and liquidity risk are priced (Amihud et al., 2005; Karolyi et al., 2012). Empirical studies find the effect of liquidity on asset prices to be statistically significant and economically important, controlling for traditional risk measures and asset characteristics. This result could be generated by high transaction cost, demand pressure and inventory risk or private information.

The branch of finance in ABM aims at replicating some market stylized facts. The most famous example is the Santa Fe Institute (SFI) market (LeBaron, 2002), the first agent-based financial market platform, which was used to study the impact of agent interactions and group learning dynamics in a financial setting. It is made of learning agents who trade two assets, a risky and a non risky one. The asset price is defined under a simple market clearing mechanism. Indeed, starting from the evidence that agents are boundly rational, Chiarella et al. (2013) point out the importance of learning (which they model using genetic algorithms) to capture many realistic features of limit order markets. In an other context, Lux and Marchesi (1998) model the agent's mood (optimist or pessimist feeling) to reproduce bubbles. An interesting stylized fact of this paper is that bubbles grow and burst exogeneously and cyclically. Hommes et al. (2005) highlight that with adaptive agents, the fundamental steady state becomes unstable and multiple steady states may arise. Pouget (2000), in her paper on market efficiency, highlights that agents type, investor's market power and motivation drive the main market price oscillations. For her, the time horizon of each investor is the main explanation to inefficiency in markets.

These already quite complex results rely on market dynamics where agents are assumed to have perfect knowledge of the fundamental and a special agent, the market maker, deals with market liquidity; most of the time there are only two assets, one risky and one non risky. We build our model relaxing these three assumptions, which are usually not abandoned all at once.

First, we relax the strong assumption of perfect knowledge of the fundamentals. According to the works of Tversky and Kahneman (1974) on agent's decision in unexpected context, traders are reluctant to change their beliefs and keep an influence of original belief for long. They process information with misunderstanding and update their expectation very slowly, while using the market as a source of new information. Fundamental value, hence, is neither unique nor exogenous to the market (Orléan, 2011). We use this idea to build a learning model that is adapted to heterogeneous initial beliefs of agents.

For practical advantage and mathematical simplicity, the market maker is usually used as market structure. Lux (1995), Iori (2002), Hommes et al. (2005) or Harras and Sornette (2011) have developed the same kind of market maker to provide liquidity. Hommes et al. (2005) argue "an advantage of the simple price adjustment rule is that the model remains analytically tractable". Beja and Goldman (1980) invoke a market maker mechanism in order to justify sluggish Walrassian price adjustment. Howewer, Foucault et al. (2005) has written that "a trader who monitors the market and occasionally competes with the patient traders by submitting limit orders, can significantly alter the equilibrium. His intervention forces patient traders to submit aggressive limit orders and hence narrows the spreads. This feature may provide important guidance for market design". LeBaron (2006) writes in his survey that the most realistic mechanism to replicate a financial market in ABM would be to use an order book. In this market structure, there is no counter part as market maker who impacts on the market price or provides liquidity. The liquidity is endogenous to the model, generated only by the execution of quoted orders. While working on liquidity, we decided to produce a more demanding model structure, and hence run an order book.

The model we base our market upon is Yamamoto's model (2011), which we extend by adding a second risky asset. The reason to use two risky and one non-risky assets to deal with liquidity can be explained : if one risky asset only is modeled, in case of a shock on this asset, all risk averse traders leave the market to invest in the risk free one. In a multi-risky assets, traders reallocate their portfolio without leaving the market, which as a result does not become illiquid. An other advantage of multi-assets markets is the possibility to manage the risk. According to the portfolio theory, the mean-variance criteria highlights that it is possible to minimize the portfolio risk in case of multi-risky assets (if $cov < 1$, the correlation coefficient between the assets). As Chowdry and Nanda (1991) state, multi assets enable to study liquidity that is essential for both viability and dynamics of market. Finally, a two risky assets model seems more realistic than a unique risky asset and can be extended to a n-risky one. To the best of our knowledge, the main Agent-based Computational Economics (ACE) in finance are build with a unique risky asset and a risk free. We can mentioned a paper from Westerhoff (2004) and one from Chiarella et al. (2007) in which two risky assets are traded but with the intervention of a specialist (the market maker).

As a summary, the novelty of our model stays in the aggregation of an order book structure where two risky assets are traded and where fundamentalists don't have access to the true fundamental value. In the following subsections we describe the models we use to build upon as well as the choices we specifically added to the structure and rationality of our agents.

2.2 Market structure

Gode and Sunder (1993) has proven with their Zero-Intelligence traders that computational market structure plays an important role in the market efficiency and price convergence. They argue that it is very important to fix a structure once, to be able to compare while testing different elements of rationality.

Domowitz (1993) has shown that in the early 90's over thirty important financial markets in the world had some of order-driven market features in their design. Whereas some markets were driven by prices in the recent past (the NASDAQ until 2002), today, most stock exchanges operate on an order book. The more realistic modeling choice is thus an order driven market. As said before, this market structure is the most difficult to compute, but necessary when focusing on liquidity (LeBaron, 2006).

Our model is based on an order driven market where two stocks are traded. At any time t, an agent is randomly chosen to enter the market or not. She can invest her wealth in the two risky assets (*Stocks*) and in a risk free one (*Bond*). An order driven market is characterized by an order book which contains the list of interested buyers and sellers. For each entry it keeps the number of shares and the price that the buyers (sellers) are bidding (asking) for each asset and its limit execution date. The submitted prices are not continuous, they are defined as a multiple of a "tick" size. Orders are executed according to time submission and quoted price.

The market price is defined by a matching process between buy and sell orders quoted in the book. When the best quoted buy order meets a counter part, an exchange occurs. The market price of the asset is defined by the price at which this exchange is realized. If the best bid (or ask) meets no counter part, no trade occurs. A mid point $(b_t^{best} + a_t^{best})/2$ is defined as market price. If (at least) a part of the order book is empty, we assume that the new market price is equal to the previous one ($p_t = p_{t-1}$). For order book examples, see the papers of Chiarella et al. (2009), Foucault et al. (2005), Tedeschi et al. (2012) or Yamamoto (2011).

Note that bid, ask and market price need to be positive. Agents are heterogeneous in their initial endowment. Traders' portfolios differ by their weight affected to each component, the assets and bond. In this model, agents are not allowed to engage in short selling and are not monetary constrained. For simplicity no quoted orders can be modify or cancelled. Indeed, the trader who has submitted orders has to wait for its execution or for its limit execution date before submit a new one. Finally, the risky assets are assumed to be independent ($cov = 0$), and this fact is common knowledge.

In this context, the order type is decided according to the submission price and the bid-ask spread. Agents face two types of orders : the market order (MO) and the limit order (LO). The first one enables to exchange a defined quantity of assets very quickly, since agents agree to deal at any price. On the opposite, the second one is characterized by an amount of assets and a limit execution price. The limit order insures the trading price but not its execution. Here, a market order is chosen when the agent's bid (ask) is higher (lower) than the best quoted ask (bid). Otherwise, the agent submits at a limit price. The type of order is a parameter that defines a simulation: it is set at the beginning and is used as a functioning rule at each time-step.

When the length of the market order is larger than the best counter part, the remaining volume is executed against other quoted limit orders. If there is not enough quoted counter part, the remaining volume is executed as new orders are submitted. Hence, the order type influences greatly the market liquidity. As an example, in the case of two limit orders, if the best bid is lower than the best ask, no change occurs. The market stays illiquid until either a higher bid (lower ask) than the quoted ask (bid) or a market order is submitted.

2.3 Trader's model

We make several assumptions about our agents in the system, some that are very usual in ACE financial models, and others that are related to our present issue regarding the impact of information on liquidity and prices. Within a typology of agents that is rather usual, fundamentalists and chartists who have different risk aversion, we add several features of bounded rationality that are relevant to this type of modeling.

2.3.1 Agents with types and risk-aversion

In 1980, Beja and Goldman highlight that agents types affect the quality of the price signal. In general, modelers distinguish two (Chiarella et al., 2007; Jacob-Leal, 2012) or three (Chiarella et al., 2009; Hommes, 2006) types of agents that are fundamentalists, chartists and noise traders.

Following the financial theory, the first ones trade in order to make the market converge to its fundamental value and are considered as informed traders. They trade in order to minimize the gap between the fundamental value and the trading price. Their goal is to bring money to the liquidity demander and guarantee an optimal allocation of resources. They are assumed to be the most risk averse agents and have long term investment horizons. According to this features, a pure fundamentalist market should be efficient in the sense of Fama (trading price close to fundamental price - Fama, 1970). Graphically, the trading price should oscillate strongly in a small path around its fundamental value. It should look like a with noise. However, this market should be relatively less liquid than an heterogeneous one (fundamentalists and chartists). Indeed, an efficient market populated by pure fundamentalists is illiquid until a new information arise. In this paper the liquidity is measured by the volume of exchange.

At the opposite, chartists are describe as speculators and have a destabilizing impact. They try to predict price evolution so as to surf on the bubbles and hence exploit the market trends to make profit. As a consequence they revise their expectation frequently and prefer short time investments. Chartists don't care about fundamental changes. They increase the market depth thanks to their short term horizon. A pure chartists market or an heterogeneous one should be more liquid but less efficient. Graphically, the market price should oscillate around short trends that are independent of the fundamental value. We also expect that the spread between fundamental price and trading price is wider in the case of chartists (or heterogeneous) market than in the case of fundamentalists one.

The more a trader is risk averse, the less she trades. The longer the investment horizon, the longer the agent holds her assets and hence the less she trades as well. Amihud and Mendelson (1986) distinguish different types of traders for each liquidity degree. At the equilibrium, in a market populated by risk neutral agents, the long term investors – fundamentalists – buy assets relatively illiquid and with a high trading cost because they expect to hold it for a long time. Whereas short term investors – chartists – prefer liquid asset with less trading cost in order to surf on the trend.

We choose here to model each individual agent as a mix of the two most usual components: fundamentalists and chartists, as can be found in Yamamoto (2011). This formulation is motivated by Harras and Sornette (2011) who mention that "agent forms her opinion based on a combination of different sources". The fundamentalist's source of an agent expects that the forward price converges to its fundamental, while the chartist's one assumes that the future price follows the past trend (Hommes, 2006). The key parameters of this agent model are g_1 and g_2, which are generated at initialization, for each trader, following an exponential law of variance σ_1^2 and σ_2^2, respectively. A pure fundamentalist strategy has $g_2^i = 0$, whereas a pure chartist strategy has $g_1^i = 0$. When both values are higher than 0, the agent is a mixed of both, which implies that she takes into account the chartist and fundamentalist expectations and make an average according to the g_1 and g_2 weight (following Eq. (5) in *appendix 5.1*). From the two parameters values, the time horizon of investment and the risk aversion of each agent is also calculated (Eq. (11) and (12)). The more the agent tends to be fundamentalist, the more risk averse and long term investor she is. The converse is true : the higher the tendency to be chartist, the lower risk aversion and the longer the investment horizon.

2.3.2 Bounded rationality and market depth as information

The choice we make is to define our agents as boundedly rational. We try to replicate some real-life choices rather than optimal decision. Lord and al. (1979) focus on the belief perseverance of traders: when an agent has formed an opinion, it is hard for her to change her mind. People are reluctant to search for evidences contradicting their beliefs. Even if they find such an evidence, they treat it with excessive skepticism. Sometimes, people miss-interpret evidence that goes against their hypothesis as actually being in their favor. As example, belief perseverance predicts that when people formulate expectation on fundamental value, they may continue to believe in it long time after the proof of fundamentals misunderstanding has emerged. In our paper, we distinguish two cases : a perfect knowledge of the true fundamental value and a belief perseverance one. We treat in more details this point in *section 2.4*.

According to their preferences, traders try to maximize their utility function under Constant Absolute Risk Aversion (CARA). The parameter of aversion toward risk is function of the trader components

(Eq. (12)) and has a direct impact on the optimal demand of assets (Eq. (8) and (9)). The order size ($s_t^{i,j}$) an agent is willing to trade is assumed to be equal to the absolute difference in the optimal demand for asset j at time t and $t-1$. The sign of this difference gives us the agent's position (Eq. (13)). When the difference is positive, the trader buys, if it's null, she doesn't enter the market, otherwise she sells. Remark that this type of utility is independent of wealth, as Chiarella et al. (2009) mentioned. This is not consistent with the intuition and some empirical results about aversion toward risk. Usually wealthier people bear more easily risk than poorer ones (Prospect theory - Kahneman and Tversky, 1979). The risk premium is decreasing with wealth. A possible evolution is to rewrite the model with an Harmonic Absolute Risk Aversion (HARA) function or at least a Decreasing Absolute Risk Aversion (DARA).

The submitted bid (or ask) is different of the expected forward price ($\hat{p}$) according to the agent's mood. In an optimistic mood ($M_t > 0$), the agent accepts to buy ($b_t > \hat{p}_t$) or sell ($a_t > \hat{p}_t$) at higher price. She believes that the market will continue to raise. The buyer expects to make profit on the future sell and the seller expects to perceive an additional premium ($a_t - \hat{p}_t$). On the contrary, if $M_t < 0$, the trader is in a pessimist way, she expects a fall. Sellers and buyers accept to exchange but at a lower price than they predicted. They try to diminish their loss in case of a fall.

After defining the price (a_t or b_t) and the quantity (s_t) at which the agent is willing to trade, she adapts her order according to the market depth. With an order book – where at least a part of the book is observable – Parlour (1998) shows how the order placement decision is influenced by the state of the book. Particularly the depth available at the inside quotes. Empirical researches point out that investors place more aggressive orders when the same side of the order book is thicker, and less aggressive orders when it is thinner (Handa et al., 2003). In concrete terms, with a probability $Prob_t^i$, the trader adapt her behavior according to the order imbalance (x_t^{ob}) which is defined by the log difference between the depth of the five best bids and asks (as Yamamoto, 2011).

$$Prob_t^i = \tanh(\beta_i * abs(x_t^{ob})) \quad (1)$$

$$x_t^{bid} = \log(\frac{depth\ of\ the\ 5\ best\ bids}{depth\ of\ the\ 5\ best\ asks}) \quad or \quad x_t^{ask} = \log(\frac{depth\ of\ the\ 5\ best\ asks}{depth\ of\ the\ 5\ best\ bids}) \quad (2)$$

where $ob = bid$ or ask respectively for a buy order or a sell order. β_i reflects the sensitivity of the probability of switching in response to the depth of the order book. The β parameter follows a uniform law on the interval $[0; \beta_{max}]$. It is constant over time but differs for each asset and agent. According to the order imbalance, the agent submits more or less aggressive orders. She faces five specific cases :

1. Submit a bid knowing that the depth of the buy side is thicker than the other ($x_t^{bid} > 0$). In this context, the risk of non execution of the order is high. There is a bigger demand of asset than supply, therefore, only the more expensive orders have a chance to be executed. With a probability $Prob_t^i$ – which is function of market depth and the sensitivity of agent to adapt, Eq. (1) – the agent submits at a more aggressive price. If she had decided to submit a limit order out of the bid-ask spread, then she would have changed it for an order at the best bid price plus a tick size. The order would have taken the first place in the order book. If she had decided to submit a limit order in the bid-ask spread, then she would have preferred to submit a market order. She takes more risk on price fluctuation but she is confidant in her probability of execution.
2. Submit an ask knowing that the depth of the buy side is thicker than the other ($x_t^{ask} < 0$). The demand is strong and the consider agent is a supplier. The probability of execution is high. With a probability $Prob_t^i$, she submits a less aggressive order. She expects to earn more. If she had chosen to submit a market order, then she would have preferred to submit a limit order at the best bid plus a tick size. If she had decided to submit a limit order in the bid-ask spread, then she would have decreases her price submission to the best ask.
3. Submit a bid knowing that the depth of the buy side is thinner than the other ($x_t^{bid} < 0$). The buyer faces the same advantage as the seller in the previous case. With a probability $Prob_t^i$, the agent submits a less aggressive order, to buy at a lower price. If she had previously decided to submit a market order, then she would have preferred to trade at the ask minus a tick size. If her limit order had been in the bid-ask spread, then she would have submitted at the best bid.
4. Submit a sell knowing that the depth of the buy side is thinner than the other ($x_t^{ask} > 0$). With a probability $Prob_t^i$, the agent trades more aggressively. She prefers to loose a bit (the bid-ask spread) than to take a non-execution risk! If her original choice had been to submit a limit order in the spread, then she would have preferred to submit a market order. No control on the order price, but

she knows that she would have been the first in the order priority. If she had decided to submit out of the bid-ask spread, then she would have submitted at the best ask minus a tick size to be sure to be the first in the ask book.

5. In any other cases, the agent doesn't update her order, she submits at the normal price (a_t or b_t) defined by the Eq. (14) for a buy order and (15) for a sell order.

To sum up, the traders are heterogeneous in their fundamentalist and chartist components, their investment horizon and their risk aversion. Moreover, they are bounded rational. Their mood and their aggressiveness influence the price at which they are willing to trade. In the case of belief perseverance, we also assume that traders are reluctant to change their minds about fundamental value estimation, even if they have evidence of misunderstanding (Fischoff, Slovic and Lichtenstein, 1977 and Lord, Ross and Lepper, 1979 mentioned by Barberis and Thaler, 2003). Concerning the submission process, at each period one agent is randomly chosen and follows the four next steps:

1. for each asset available on the market, she formulates expectations on the forward price according to her features (g_1^i and g_2^i).
2. she defines the amount of assets she wants to trade according to a CARA utility function.
3. she adapts her expectation according to her personal mood (simple rule of thumb). It is assumed to be time and asset dependent.
4. after having formulated an order, she corrects it according to the market depth and submits.

Her order is quoted in the order book and can't be modified or removed. The complete mathematical model is developed in the appendix (see *appendix 5.1*).

2.4 Fundamental value

The fundamental value is usually defined as: the expected dividend of the firm ($\mathrm{E}_t[y_{t+k}]$) corrected by the risk premium required for risky asset ($\alpha\sigma^2 Z$) divided by the actualization rate (R). This value is assumed to be unique and equal to:

$$f = \sum_{k=1}^{\infty} \frac{\mathrm{E}_t(y_{t+k}) - \alpha\sigma^2 Z}{R^k}$$

There are lots of criticism about the definition of the fundamental value. The first problem comes from the heterogeneity of actualization rates, which one to select? A constant or a time evolutive? How to fixe it? The second problem is in the forward dividend. On which economic variable do we base our dividend expectation? Moreover the dividend expectations come from the market, and so, the fundamental value becomes intrinsic to the market whereas it must be intrinsic to the firm but exogenous to the market! With heterogeneous bounded rational agents and unpredictable future, the fundamental value becomes idiosyncratic.

In this paper, we distinguish two cases: a perfect knowledge one and an imperfect knowledge one, also namely "belief perseverance". In both settings, we assume that the true fundamental value (f) follows a random walk and agents have access to the entire history of asset prices.

In the case of perfect knowledge of the fundamental value, all fundamentalists have access to the good information. Because everybody has the same information and processes it correctly, the estimation of the fundamental value ($\hat{f}$) is assumed to be unique and right. For each fundamentalist i, it is mathematically expressed as:

$$\hat{f}_t^i = \hat{f}_t = f_t \quad ,\forall i \tag{3}$$

In the case of belief perseverance (imperfect knowledge of the fundamental value with adaptive learning), the forward fundamental value becomes idiosyncratic. It is a well known fact that traders make errors in their expectations and are overconfident (Barberis and Thaler, 2003). Tversky and Kahneman (1974) highlight that people make estimation of prices by starting from an initial value that is insufficiently adjusted across time. This is why we produce heterogeneity in our agents by giving them different initial believes: even if they get the same piece of information in time, they do not necessarily deduce the same fundamental value for the asset. Our formulation of the estimated fundamental value is inspired by Westerhoff (2004), so as to fit the anchoring assumption that is one aspect of bounded rationality. The fundamental value ($\hat{f}_t^i$) of agent (i) in the case of belief perseverance is designed as:

$$\begin{aligned} \hat{f}_t^i &= \gamma_1 \; p_{t-1} + \gamma_2 \; \hat{f}_{t-1}^i + \gamma_3 \; \hat{f}_{origin}^i \\ &+ N_t + a(\hat{f}_{t-1}^i - \hat{f}_{t-2}^i - N_t) \\ &+ b \; (f_{t-1} - \hat{f}_{t-1}^i) \\ \hat{f}_t^i &\neq \hat{f}_t^j \quad ,\forall i \neq j \end{aligned} \tag{4}$$

The first line represents the anchor. It is defined by the last observed price (p_{t-1}), her previous ($\hat{f}_{t-1}^i$) and her original ($\hat{f}_0^i$) estimation of the fundamental value. γ_1, γ_2 and γ_3 represent the weight given to each component and add up to 1. The second line describes the anchor correction. The first component reflects the arrival of new information (N_t), common knowledge. The second component highlights the faith related to it. As an example, if the recent update of the estimated fundamental value has been above the news impact ($\hat{f}_{t-1}^i - \hat{f}_{t-2}^i > N_t$), the fundamentalist tends to overreact to news. The a parameter is the degree of misperception, which represents the time needed to process information. The third line ($f_{t-1} - \hat{f}_{t-1}^i$) – which is the spread between the last true fundamental value and the estimated one – is the one that represents learning. The b parameter affected to this learning is assumed to be close to zero, agents are reluctant to change their mind. All parameters (a, b, γ_1, γ_2, γ_3) are fixed and equal among traders.

3 Simulations analysis

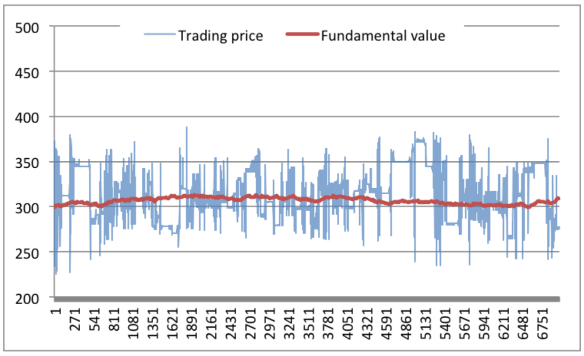
This section describes the dynamics of two types of market that can be produced within our frame: in the first sub-section the market is made of traders that are pure fundamentalists ("one-type model") than in the second one is made of traders that are a mix of fundamentalist and chartist components ("two-type model"). It has to be remembered that even agents who are all 100% fundamentalists are not necessarily homogenous, since they differ in time horizon and aversion to risk. Within these two sub-frames, we study specifically the influence of the type of information agents get on the market dynamics – more precisely we study volatility and liquidity when agents have perfect knowledge of the fundamental value of the assets, and when the agents have limited information. We hence face four different cases: OTP (One-Type model – Perfect knowledge), OTB (One-Type model – Belief perseverance), TTP (Two-Types model – Perfect knowledge) TTB (Two-Types model – Belief perseverance). We change diverse parameters within each set of simulations to be able to establish 1/ the relevance of our model which produces certain stylized facts which are usually recognized as relevant 2/ the influence of our main assumption: the disparition of perfect knowledge and the idiosyncratic learning of agents.

We ran 200 simulations for each trading round, each simulation being a succession of 8,000 time-steps, which we observe after 1000 steps have already been run (so that agents have time to learn and in order to exclude impacts of computer initialization). The model setup is summarized in the table 3 of *appendix 5.2.*

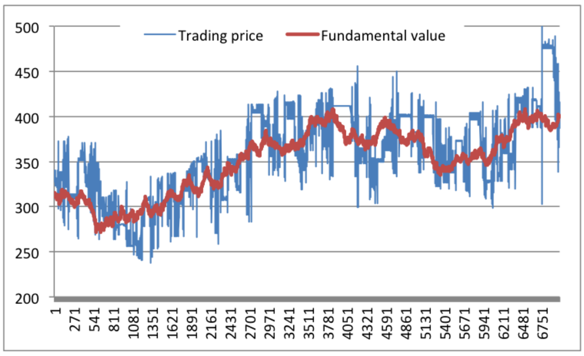
3.1 Dynamics in one-type markets

3.1.1 Benchmark: perfect information

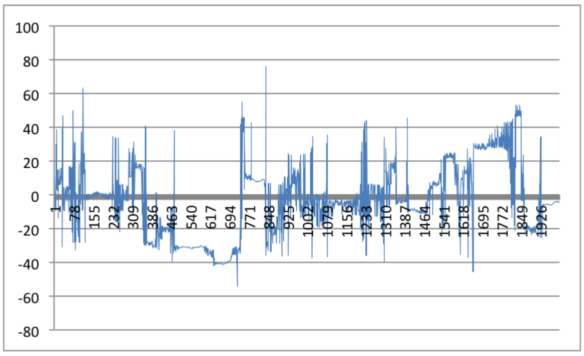
As said before, fundamentalists trade according the fundamental value of assets, of which they are here perfectly aware: they submit orders to make the market efficient – current price equals the fundamental price. Even when all agents follow this rule, strong market efficiency is never verified, and we observe an oscillation of the trading price around its fundamental. This inefficiency is linked to the heterogeneity of our population, which directly impacts the market dynamics. Thus, to model agents heterogeneity, we focus on the the variance distribution of the fundamentalist component (σ_{g1}^2), which is the key parameter of our one-type markets. Increasing $\sigma_{g_1}^2$ makes the traders "more fundamentalists" in the sense that on average they become more risk averse and more long time investors (see Eq. (11) and Eq. (12)). We simulated trading rounds for increasing value of $\sigma_{g_1}^2$, and identified a decreasing volume of exchange (table 1). This is consistent with what can be expected from the variation of



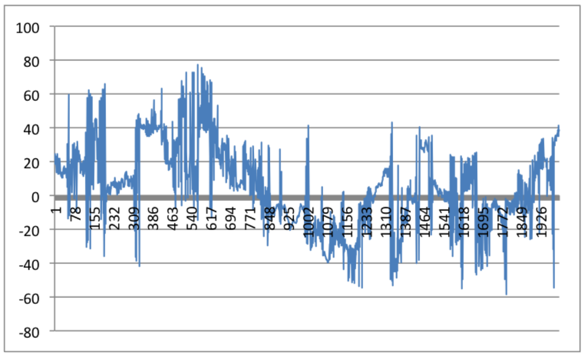
(a) Asset 1: Trading price



(b) Asset 2: Trading price



(c) Asset 1 : Spread between trading price and fundamental price on a shorter period, 0-2000



(d) Asset 2 : Spread between trading price and fundamental price on a shorter period, 0-2000

Fig. 1: Price dynamics in OTP case ($\sigma^2_{g_1} = 0.6$, $\sigma^2_{f_1} = 0.2$, $\sigma^2_{f_2} = 1$)

Asset 1	$\sigma^2_{g_1} = 0.1$		$\sigma^2_{g_1} = 0.6$		$\sigma^2_{g_1} = 1$		$\sigma^2_{g_1} = 10$	
	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev
vol. per simu.	710	56	592	92	554	70	359	46
max. vol./time step	11.3604	6.23	11.02806	9.17	10.94757	6.25	9.628196	6.98
price variance	669	205	695	227	710	245	754	245

Table 1: Market liquidity in OTP case, $\sigma^2_f = 0.2$

risk aversion: the more risk averse the agents, the less they trade. The average trading volume per simulation falls from 710 to 359. This values are similar for each traded asset. Surprisingly, on average the most liquid time-step is not strongly affected by the variance distribution of the fundamentalist component. For $\sigma^2_{g_1} = 0.1$, the most liquid time-step is 11.36. It decreases slowly until 9.63 for $\sigma^2_{g_1} = 10$. When considering Eq. (8) and (9), one could assume that the volume of exchange is correlated to price oscillations rather than agents expectations. Indeed the amount an agent agrees to deal is defined by the absolute difference of assets demand between time t and $t-1$.

According to empirical data (Elyasiani et al. 2000), a decreasing trading volume implies an increase in the volatility of the market price: less assets are quoted and traded on the market, so the price movements are larger. In our model, a 50% liquidity fall implies an increasing price variance of 15%. The probability not to find any counter part (no quoted bid or ask) increases also from 2.6% to 4%. When fundamentalists' investment horizon is larger and their risk-aversion increases, the market liquidity falls. So, this model is able to replicate the finding of Beja and Goldman (1980): increasing the fundamentalist power in a previous stable system makes it less stable.

The variance of the trading price is high and linked to the variance of the fundamental value. When the true fundamental value changes, the fundamentalists perceive this movement and update their expectations according to news. In our simulations, whatever the original fundamental value is, the trading price oscillates of ±26%. This value falls to ±20% and to ±13% if we exclude respectively the 10% and the 20% extreme data. So, usually the price oscillates strongly in a small path and sometimes, peaks appear and destabilize the market for a short period. As an example, for an initial true fundamental value of 300 ECU and $\sigma^2_{g_1} = 0.6$, all prices are contained in the interval $[-80; +80]$ around its fundamental, which is a wide trading price fluctuation around the fundamental, already

$\sigma^2_{f_1} = 0.2$ & $\sigma^2_{f_2} = 0.2$	Benchmak		Belief perseverance	
Asset 1	Mean	Std. Dev	Mean	Std. Dev
price	308.1574	12.32511	306.6011	3.695452
mean spread	7.789	4.584102	6.931	11.74591
variance	698	216	552	113
Asset 2	Mean	Std. Dev	Mean	Std. Dev
price	310.4384	14.41455	308.0654	4.270272
mean spread	9.645	6.012068	7.979	11.80833
variance	723	240	546	142

Table 2: Perfect vs. imperfect knowledge of the fundamentals in one-type market

pointed out by Iori and Porter (2012). In a 95% confidence interval, the trading prices are contained in $[-38; +58]$. The spread asymmetry seems to mean that the market is overvalued, which is the case 63% of the time. The mean spread – difference between the trading price and the true fundamental value – is such that asset 1 is overvalued by 8 ECU, and asset 2 by 10 ECU. The average trading prices are 308.157 and 310.438 ECU.

In addition, fundamentalists submit on average 20% of market orders so as to buy and 26% so as to sell: they are more impatient to sell than to buy, which is consistent with the literature.

Fig. 1 is a typical example of the OTP case. The trading price oscillates in a large spread around its fundamental value, and never diverges. The executed orders are above as much as below the fundamental price. If we focus on a smallest time interval, we distinguish two main dynamics that seem cyclical. Fig. 1c and Fig. 1d reflect them. We distinguish a short period of high price variation and a longer period of low variation. The periods of low variation appear mainly in the interval [-30;+30] around the fundamental. This spread is equivalent to the one in which the 20% extreme data are excluded. In this interval, fundamentalist seems to estimate that the market is relatively efficient – which explain the low variance of the trading price – until a bullish or bearish shock appears and destabilizes the market – which becomes very volatile. After a short time, the price variance goes back to a low level, and the trading price converges to its true fundamental value. This is consistent with Eq. (8). The less the price varies, the more fundamentalists exchange and the faster the price converges.

Even in OTP case, we highlights that trading prices may differ from their fundamental value.

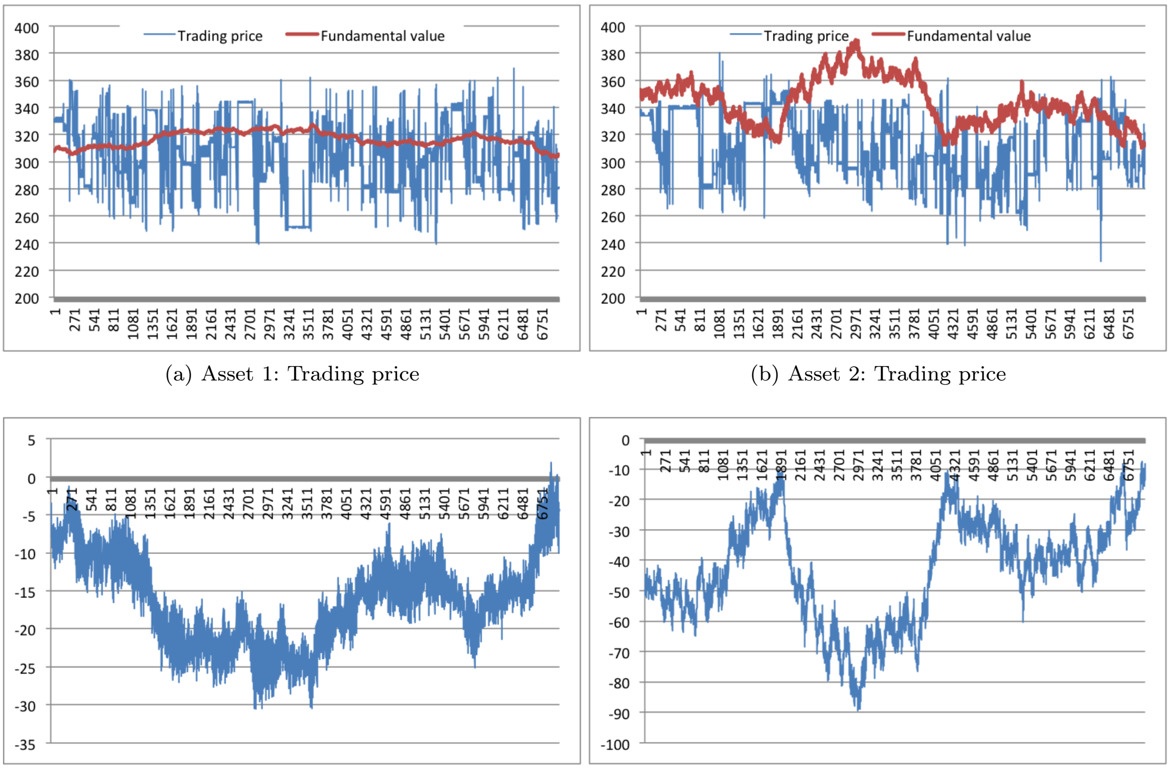
3.1.2 Belief perseverance

In this sub-section, we focus on changes in the dynamics of a one-type market where the assumption of perfect knowledge of the fundamental value is relaxed : OTB case. Traders base their own predictions on their previous expectations (anchor) and a learning element (see Eq. (5)), and each agent is given a personal initial anchor.

If the original anchor is chosen close to the initial true fundamental value, the introduction of personal believes about fundamental value will not change the market dynamics significantly. All values being the same, we found trading prices around 306 ECU for asset 1 and 308 ECU for asset 2, which are respectively 0.858 ECU and 1.666 ECU less overvalued than in the OTP case (see Table 2). The trading price oscillates about ±26%. The 95% confidence interval makes the path thinner [-14%;+18%]. And the market is overvalued 60% of the time, which means that the price seems to have a smoother oscillation and a lower variance. Moreover, when the variance of the true fundamental increases, the variance of the market price is not really affected. The true fundamental value is perceived by the fundamentalists as being more stable than it really is.

The trading price is directly impacted, becoming more stable and the market liquidity is weakly decreasing. All these results are what was expected. Fundamentalists thus integrate the news they perceive in the trading price. If the fundamental value is perceived as stable, there will be no "non-integrated" news, and so no more trade. Finally, the belief perseverance makes runs more similar, the standard deviation of the average price is at least twice lower compare to the OTP case.

If the anchor is not consistent with the fundamental value, the market price will not reflect the fundamental at all. Indeed, agents have a tendency to give less importance to news than to the original fundamental expectation ($\hat{f}^i_0$) because of the anchor. In this case, the trading price is stable and evolves around the original fundamental expectation. This is due to the fact that the anchoring is strong and learning is slow, hence re-ajustment is slow. The result is an inefficient market: the market price never reflects its fundamental.



(a) Asset 1: Trading price

(b) Asset 2: Trading price

(c) Asset 1: Spread between the perceived fundamental value and the true one

(d) Asset 2: Spread between the perceived fundamental value and the true one

Fig. 2: Price dynamics in OTB case ($\sigma^2_{g_1} = 0.6$, $\sigma^2_{f_1} = 0.2$, $\sigma^2_{f_2} = 1$)

Fig.2 illustrates the OTB case in which the original anchor is generated close to the initial true fundamental value. When the true fundamental value varies slightly ($\sigma^2_{f_1} = 0.2$), the trading price seems to follow it (Fig. 2a). The market is relatively efficient, but Fig. 2c highlights some misunderstanding in the fundamental changes. When the true fundamental value has a high variance ($\sigma^2_{f_2} = 1$), the market is inefficient (Fig. 2b). It doesn't integrate the fundamental changes. Indeed, fundamentalists submit orders following their estimation of the fundamental value, but they misestimate it which is due to belief perseverance (Fig. 2d). The anchor makes agents minimize or misunderstand the changes in the fundamental price. The variance of the traded asset is relatively low but the spread between the true fundamental value and the trading price is high. The market efficiency is so dependent of agents beliefs and the variance of the true fundamental value (σ^2_f).

When the agents have their own believes about fundamental value (their own anchor), the trading price doesn't reflect the evolution of the true fundamental value (Fig. 3a). A way to correct this is to give more importance to the learning process, defined in Eq. (5), by increasing parameter b. This enables traders to update their expectations quickly, and makes the price more informative in the case of high fundamental variance ($\sigma^2_{f_2} = 1$). The effect of the anchor is weaker, but still exists (Fig. 3b). When the true fundamental value falls, the trading price also decreases, but with a lag.

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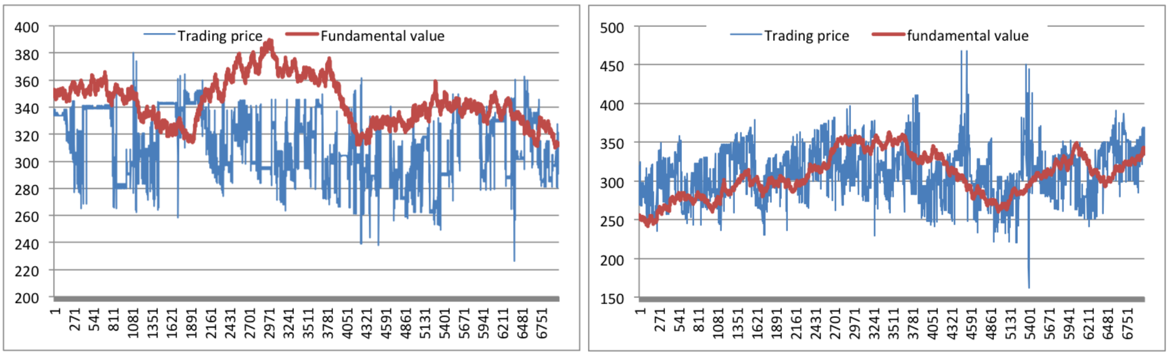
* *

Fundamentalists are by definition risk averse. Increasing their risk aversion and their investment horizon (with $\sigma^2_{g_1}$ parameter) make the market less liquid and more volatile. Moreover, the exchanging volume is strongly affected by the price oscillation. The more the price fluctuates, the less agents exchange, the more the bid-ask spread rises.

Our model is able to generate endogenous bubbles, even with a pure fundamentalists-market. In the OTP case, bubbles burst quite fast and their occurrence probabilities are low. The price is overestimated by 3% on average. We can consider the price as informative at each time step – even if the market price is not equal to the fundamental price, it replicates its trend– whatever the degree of heterogeneity of fundamentalists.



(a) Asset 1: Low learning process (b=0.005) (b) Asset 1: Fast learning process (b=1)

Fig. 3: Impact of learning speed in OTB case ($\sigma^2_{g_1} = 0.6$, $\sigma^2_{f_1} = 1$, $\sigma^2_{f_2} = 1$)

In the OTB case, when the fundamental price varies slightly ($\sigma^2_{f_1} = 0.2$), the dynamics of the trading price is close to the OTP case. However, we observed a more stable price just as a more stable volume of exchange across the simulations. The standard deviation of the market price falls from 12 to 4. The price variance also decreases from 700 to 550. Relaxing the perfect knowledge assumption enables to model some perseverance beliefs and persistent bubbles in a pure fundamentalist market. We assume that the market is on average informative, the mean trading price is close to the mean fundamental value. With different values of the b parameter, the market is able to follow the fundamental changes. We don't adapt this learning parameter in order to respect the works of Tversky and Kahneman: people begin with a starting value, one supplied to them or generated by them, and insufficiently adjust their estimates around this anchor. In the following sub-section, we choose to give individual anchors in which the original estimation of the fundamental value ($\hat{f}^i_0$) is close to the original true fundamental value (f_0). Two motivations for this choice: 1/ we assumed that traders are fundamentalist, they so have a correct information about past fundamental 2/ if $\hat{f}^i_0$ is generated far from the true fundamental, then the market will be never efficient.

As expected, the amount of market order submission is the same in the OTP case as in the OTB one – hence this number of submission is similar whatever the access to the real information about the fundamental value. This result is consistent with the fact that the ratio of submitted market orders is dependent of the informational advantage, not the information *per se*. There is no informational advantage neither in belief perseverance nor in perfect knowledge. In the first case, nobody knows the true fundamental value and in the second case, everybody knows it.

We do not develop the description of a pure chartists market here, since it generates very usual results (a few data can be found in *appendix 5.4*): chartists have a pure destabilizing impact, they make the price increase to infinity or drop to zero. Chartists are indeed known to produce quick reinforcement dynamics: when they observe a small rise, they trade at a higher price, which makes the price rise again. They are assumed to improve market liquidity, but this is not necessarily true for each time-step: the maximum volume is at least twice higher than in a pure fundamentalists market (26 versus 11) but not on average.

If their risk aversion and their investment horizon decrease ($\sigma^2_{g_2}$ increases), the market liquidity increases from 278 ($\sigma^2_{g_2} = 0.1$) to 908 ($\sigma^2_{g_2} = 10$). A market populated by fundamentalists who are strongly risk averse ($\sigma^2_{g_1} = 10$) is as liquid as a market populated by chartists who are weakly risk lover ($\sigma^2_{g_2} = 0.1$). However, the standard deviation of exchanging volume is, at least, 3 times higher for the chartists-market (due to the destabilizing effect of chartists).

3.2 Dynamics in two-types markets

In this sub-section, each agent is a mixed of fundamentalist ($\sigma^2_{g_1}$) and chartist ($\sigma^2_{g_2}$) components. It has to be remembered that the weight of each component is generated according to an exponential law, and the mean and the variance of this law are correlated. For easy understanding, we can write that a market with $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 10$ corresponds to a market where the "mean agent" is at 19% fundamentalist and 81% chartist in her expectations. For easy reading, we call this market as 19%/81%. In the same spirit, a 43%/57% market corresponds to $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$.

In addition of the belief perseverance, because traders are not myopic, they are able to adapt their orders before submitting (see Eq. (1)). More specifically, according to the market depth, the participants resolve the trade-off between accepting the non-execution risk and paying the bid-ask spread. This choice, which is made according to the individual ability to adapt to the visible part of the order book (β_t^i), has a direct impact on the market efficiency.

Different parameters that drive the market dynamics are treated in the following sub-frames as the ratios of agents types, the agents' aggressiveness and the knowledge of fundamentals.

3.2.1 Benchmark: perfect information

As we can expect, even if chartists have a destabilizing impact on the market, when a fundamentalists trend exists, the market never diverges. In our simulations, the trading price never exceeds 100 times the fundamental value even if fundamentalists are in minority. Indeed, in a 19%/81% market, the trading price evolves in a large spread around its fundamental value [−150; +250] but stays in this path. Minimizing the chartist component (43%/57% market) doesn't impact drastically the average trading price but makes its volatility significantly lower (849 vs. 200). In the same way, the limits and the variance of the spread between the trading price and the true fundamental value are lower. This is coherent with the chartist destabilizing impact.

To compare this data to the previous case: in the OTP case, the market price is above the fundamental price 63% of time. The trend induces by the fundamentalists is a kind of white noise around the fundamental. So the trading price tends to be as above as below the fundamental price, the positive and negative spread counter-balance each other and the market is relatively efficient. Hence when chartists trade, because of their backward looking, they just amplify fundamentalists' trends: in the TTP case, the trading price is above the fundamental value 62% of time in the 43%/57% market, and 57% of time in the 19%/81%market. Increasing the weight of chartist's component makes the trading price above as much as below the fundamental value. However, it also makes the price more volatile, and more overvalued on average. So, the market price is less frequently over the fundamental value, but it is on average more overvalued! This result is justified by the positive spreads which are much bigger than the negative one and the extreme cases which do not counter-balance each other. It is also worth noticing that simulations display different qualitative emerging patterns, with much less predictability than in case of one-type market. This is in accordance with the reinforcement dynamics of chartists.

Concerning the traders' choice between market and limit order, in the 43%/57% market, traders use MO to buy in 20.5% of cases and to sell in 24%. In the 19%/81% market – where agents are essentially chartist in their behavior –, the ratios of submitted MO decrease and converge to 20% to buy and 22% to sell. Thus "impatience" to sell is still present in our two-types markets. The decreasing amount of submitted MO is consistent with O'Hara and Easley (1995) who points out that the chartists are the liquidity suppliers, they submit LO and the fundamentalists because of their informational advantage capture this liquidity by submitting MO. Our model is able to reproduce this stylized fact.

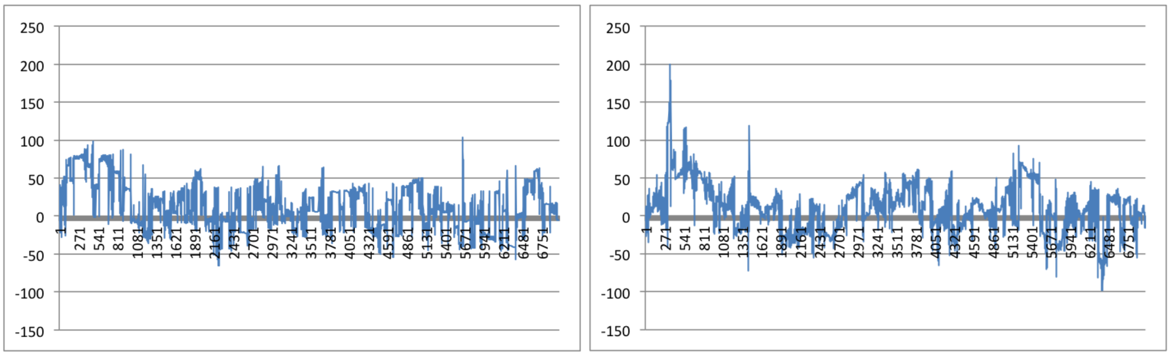
Unfortunately, the trading volumes of our two-types markets are unexpectedly low. In the chartists-market (see *appendix* 5.4), the liquidity is around 300 for $\sigma_{g_2}^2 = 0.6$ and increases to 900 for $\sigma_{g_2}^2 = 10$. In the OTP case ($\sigma_{g_1}^2 = 0.6$), it is around 600. In the TTP case ($\sigma_{g_1}^2 = 0.6$ and $\sigma_{g_2}^2 = 1$), it is as low as 391. Chartists should at least couner-balance the decreasing part of fundamentalists submissions, since we know they are liquidity suppliers! As an example, an increase in the ratio of chartist (from 57% to 81%) makes the price variance twice higher and the trading volume 10% lower. This result is in opposition with empirical study and logically incoherent: adding chartists or noise traders permits to increase the market liquidity usually.

However, the result can be explained by our structure: the asset demand defined by Eq. (14) and Eq. (15). The first and second differentials in the case of two independent assets are :

$$\frac{\partial \pi_t}{\partial \hat{p}_{t+\tau}} = \frac{1}{\alpha\ p_t\ \hat{p}_{t+\tau} Var} > 0 \quad and \quad \frac{\partial \pi_t}{\partial Var} = -\frac{\ln\left(\frac{\hat{p}_{t+\tau}}{p_t}\right)}{\alpha\ p_t\ Var^2} < 0$$

$$\frac{\partial \pi_t}{\partial^2 \hat{p}_{t+\tau}} = \frac{-1}{\alpha\ p_t^2\ \hat{p}_{t+\tau} Var} < 0 \quad and \quad \frac{\partial \pi_t}{\partial^2 Var} = \frac{2\ln\left(\frac{\hat{p}_{t+\tau}}{p_t}\right)}{\alpha\ p_t Var^3} > 0$$

The positive impact of the expected price and the negative impact of the variance on the asset demand are verified. The second differential enables to highlight a stronger effect of the price variance. So, due to the increasing price variance, the fundamentalists trade less. However, the decrease of fundamentalist



(a) Asset 1: $\sigma^2_{f_1} = 0.2$ (b) Asset 2: $\sigma^2_{f_2} = 1$

Fig. 4: Spread between the market price and its fundamental with myopic agents in TTP case ($\beta = 0$, $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$)

trading is not totally counter-balanced by the chartists' trades. Indeed, the volume of exchange is correlated to the price variance rather than to the price increase.

An other explantation is that indeed agents want to trade more and they submit higher volume, but they do not meet counterpart. The trading price is defined by a mid point between bid and ask, and no trade occurs. So, the price variance increases, but the liquidity volume stays small.

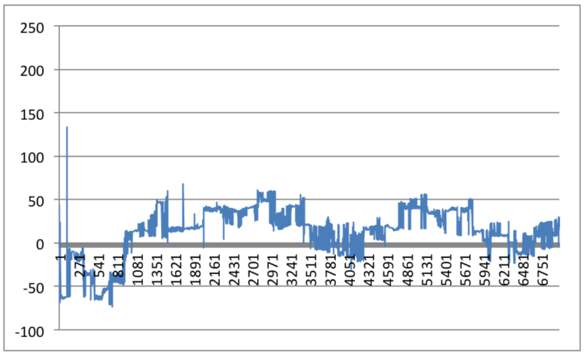
In Fig. 4, we plot the spread between the trading price and the true fundamental value. We distinguish two graphics in which the fundamental value has a low and high variance. In both graphics, the average price of the trading round is relatively close to the fundamental. The price never diverges, but its variance is huge. Thus, the price is less informative than in the OTP case. Indeed, the chartist component makes the price oscillates strongly in a wider path around its fundamental value: destabilization effect. The market prices follow cyclical process as in the OTP case, but bubbles have larger amplitude and need more time to burst. Moreover, as expected, the bubbles are easily detected when the variance of the fundamental value is high. Indeed, so that to have efficient market, fundamentalist make numerous price adjustments and chartists easily observe more persistent trend, that they amplify.

To go further, agents are non-myopic in real world ($\beta \neq 0$). They adapt their orders according to market observations. Traders change their behaviors and become more or less aggressive according to the market depth. With $\beta = 2$ (Fig .5), agents are strongly influenced by the order book depth. The result is a deeper market. As an example, the liquidity is multiplied by 1.5 in the 43%/57% market. The amount of assets exchanges in a time-step is also hugely impacted, because of traders adapting their submissions in order to increase the probability of order execution. However, the market efficiency is negatively impacted: the market price is on average overvalued by 14.761 ECU. The mean spread increases by 5.544 ECU compared to the same market with a null β. The price stays far from the fundamental value for a relative long period. The bubbles need twice more time to burst (around 3000 time-steps). This market price does not reflect the fundamentals, it reflects the "fear" of non execution[1]. The β parameter has an impact on the "impatience" and the "fear" of traders which impact the market efficiency. It also impacts negatively the ratio of submitted MO. Indeed, traders submit on average 16% of it so as to buy and 19% so as to sell – no matter what the weight of fundamentalist vs chartist components is. This fall is explained by the agents who take into account the market depth, and adapt their orders: the more liquid the market, the less risk they take and the less market orders are submitted. Nonetheless, the "impatience" at the sell is still verify.

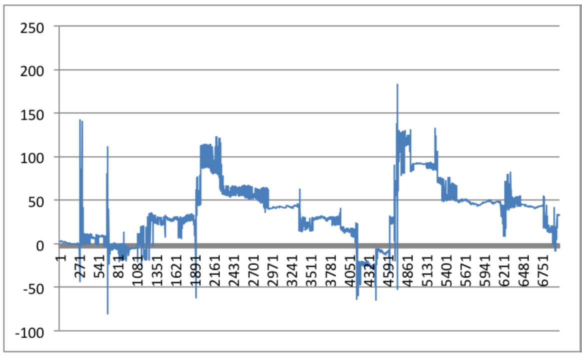
3.2.2 Belief perseverance

The switch from the TTP to the TTB case should give us the same main dynamics change as from the OTP to the OTB case. With the anchoring phenomena, the news about fundamentals are misunderstood or underestimated: the true fundamental value is perceived by the fundamentalists as being more stable than it really is. The expected results are: a more stable price just as a more stable

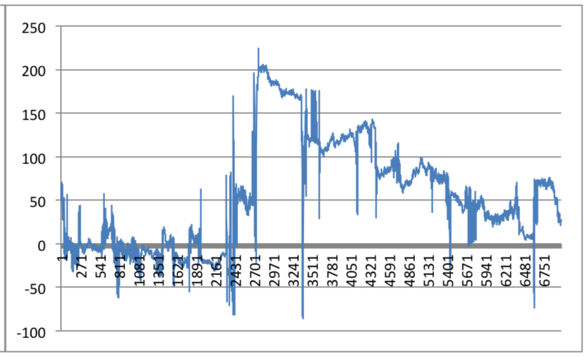
[1] In a market with myopic agents, traders don't revise their expectations according to the market depth, the price oscillates around its fundamental value (Fig. 4).



(a) Asset 1: $\sigma_{f_1}^2 = 0.2$



(b) Asset 2: $\sigma_{f_2}^2 = 0.2$



(c) Asset 2: $\sigma_{f_2}^2 = 1$

Fig. 5: Spread between the market price and its fundamental with non-myopic agents in TTP case ($\beta = 2$, $\sigma_{g_1}^2 = 0.6$ and $\sigma_{g_2}^2 = 1$)

volume of exchange across the simulations, a lower price variance and more persistent bubbles than in TTP case. Compared to the OTB case, an increase in the chartists component should make the market price less informative (increasing average market price and mean spread) and more volatile (increasing variance).

With belief perseverance, the trading price evolves in a path around the anchor. When the fundamental value has a low variance ($\sigma_f^2 = 0.2$), the market price integrates the largest part of fundamental news and it is less volatile than in the TTP case. Nonetheless, the destabilization power of chartist is still present. Indeed, in a TTB case with 43%/57%, the spread between the trading price and the true fundamental value is equal to 8.156 ECU. In the case of 19%/81%, the market price is less informative of 1.597 ECU. The spread increases to 9.753 ECU and the market volatility is timed by 2.5.

When the variance of the fundamental price is high ($\sigma_f^2 = 1$), the trading price evolves far away from its true fundamental value (as in OTB case), and "double bubbles" can appear (Fig. 6b and Fig. 6d - for example between time-steps 250 and 1000). The first bubble – due to chartist component – corresponds to the difference between the estimated fundamental value and the trading price. Because our fundamentalists are belief perseverant, and according to our setup, they estimate the fundamental value around 300 ECU (on average). The trading price is frequently over 325 ECU during this period, so the market can be considered overvalued of 25 ECU by the fundamentalist. At the same time, we observe that the estimated fundamental value is wrong (the true fundamental value being 80 ECU lower than the estimated one) (Fig. 6d). A second bubble appears. The belief perseverance is responsible for the error in estimation of fundamental - it takes for information to be integrated on the market , and sometimes the process just fails.

As can be expected, this general result is dependent on the variance of the fundamental value. Indeed, the "double bubbles" appear also when the fundamental value has a low variance (Fig. 6a and Fig. 6c), but they are not as easy to observe (lower amplitude, faster bursting). The belief perseverance of fundamentalists is minimized by the low variance of the true fundamental value. A quick look to Fig. 6a and Fig. 6b confirms our finding: when $\sigma_{f_1}^2 = 0.2$, the spread between the estimated fundamental value and the true one evolve in the interval $[-5; +30]$, while it is $[+5; +145]$ when $\sigma_{f_2} = 1$.

When agents are non-myopic ($\beta \neq 0$), and hence take into account the depth of the order book, the price rises. The destabilization power of chartist in the case of belief perseverance and non-myopia is less obvious. Indeed, the average market price are around 314.6 ECU for asset 1 and 325 for asset 2 – whatever the weight of fundamentalist vs chartist. The destabilization power of chartist stays in the price volatility: an increase of chartist ratio from 57% to 81% makes the variances 3 times higher[2]. Moreover, the standard error is also positively impacted (times by 3), noticing that simulations display different emerging patterns. This is surely du to the impact of the market depth in the price valuation, which evolve at each time-step, and which is increased by chartist behavior. So, the market price reflects the "fear" of no order execution and the chartist's herding. This is why the runs are less similar among trading round. Finally and non surprisingly, the volume of exchange is hugely increase by non-myopic agent. However, as mentioned previously, we observe a fall in liquidity when the weight of chartists component is heavy. The amount of exchange decreases form 632 to 486 for the first asset and from 760 to 618 for the second one[3]. Our assumption – the price variance is mainly responsible of the liquidity fall – seems to be confirmed.

In Fig. 7b and Fig.7d, we focus on the high variance case of the true fundamental value, that is the most caricatural one. Between time-steps 800 and 2500, the market is clearly inefficient. The market price seems to be driven by an excess of optimism, due to chartist behavior (Fig. 7b). But, Fig. 7d highlights a fundamental value overvalued by 100 ECU. So the spread between the true fundamental value and the trading price is driven by non-myopia, chartist behavior and belief perseverance of fundamentalist. After that, between the time-steps 2500 and 4000, the trading price stays relatively constant and the true fundamental value converges to the estimated one. The spread difference tends to zero (Fig. 7d). The bubble bursts partially, even if fundamentalists don't revise their expectations. The true fundamental value rises and goes back to a value close to the anchor. Thus, this price convergence is independent of agents expectations. After the time-step 4000, the trading price falls to the true fundamental value. This part of the price convergence is due to agents' trade and to their slow learning process. The anchor has a strong impact on the market dynamics, when the variance of the fundamental price increases, the trading price does not follow it, or with a delay.

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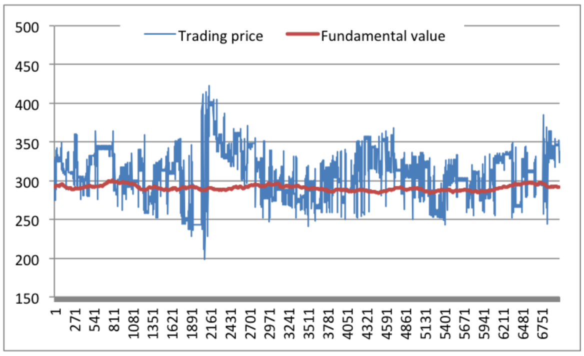
Fundamentalists and chartists are heterogeneous in their investment horizons, risk aversion, aggressiveness, moods and knowledge of the fundamentals. They also differ by their roles, liquidity suppliers or demanders. A pur chartist strategy ($\sigma^2_{g_1} = 0$) makes the market inefficient until it disappears. Chartists base their forward expectations on the past trend. They submit essentially limit orders. A fundamentalist strategy ($\sigma^2_{g_2} = 0$) makes the trading price evolves around its fundamental value with more or less fluctuations. So, in the OTP case, the marker is considered as relatively efficient. In our two-types market, agents are an aggregation of fundamentalist and chartist components. The more chartists are, the less market is efficient. Indeed, increasing chartist component makes the market price overvalued on average. The market price oscillates more frequently and in a wide spread. In the TTP case (43%/57%), we have observed at least an increase of the average price by 2 ECU and a variance increase of 22% compared to the OTP case. The chartist component makes also decreasing the amount of submitted market orders (relative low impact) and increasing the maximum amount of asset exchange in a time-step. This is coherent with the chartist's destabilization power. However, an unexpected results, already mentioned is the market liquidity fall. The volume of exchange is more correlated to the price variance than to the expectation of rising price, but the amount of exchange becomes more stable across simulations (decreasing standard error).

In the spirit of the bounded rationality, we have tried to relax the strong assumption of perfect knowledge of the fundamentals. To do that, we propose an adaptive learning of the fundamental value with belief perseverance. The main results of this essay are a decreasing price variance, smallest extreme values and the market efficiency function of the fundamental variance and the anchor. Indeed, if the fundamental value has a high variance, the market will be inefficient because of the belief perseverance. But, if the anchor is closed to the true fundamental value and that fundamental value has a low variance, the TTB case is more efficient than the TTP one – in the sense that the market price is on average closed to the true fundamental value. However, the spread between the trading price and the

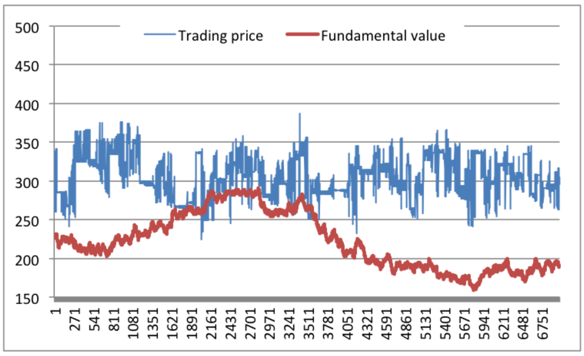
[2] It has to be remembered that the market volatility was multiplied by 2.5 with myopic agents.
[3] As point of comparaison, the trading volume in a 43%/57% market with myopic agents is 392 for the first asset and 420 for the other one.

true fundamental value has a higher mean and variance. The market price does not integrate all the fundamental changes in the case of belief perseverance.

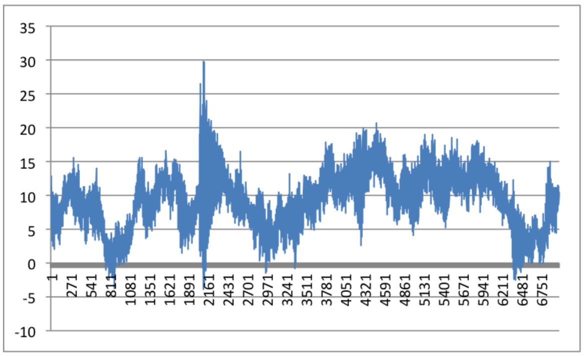
Finally, this model deals with agents' behavior according to market depth. The β parameter, because of arbitrage between higher price and no execution risk, makes long term trend easily identified by chartists, and more persistent bubbles (higher amplitude and duration). The result is a non efficient market in any way. However, the β parameter permits also to improve the market liquidity. A quick comparison between Fig. 6 and Fig. 7 permits to understand the main dynamics imply by the β parameter. When agents are myopic ($\beta = 0$), the trading price oscillates frequently around its true fundamental value. Some trends appear but they have low amplitude and the duration is short (between time-steps 250 and 1000 as an example). As previously mentioned, the price evolves in an horizontal wide path. In the TTB case with myopic agent, the main reason of bubbles is a non perceived change in the fundamental (belief perseverance). When agents are receptive to the order book statement ($\beta \neq 0$), the market price can stay over- or under-valued for a longer period. Long periods of rise (or fall) are observable and the market liquidity increases by 35% at least. In addition of unperceived change in the fundamental, the bubbles are self-sustained by a "fear" of order non execution. Traders care less about fundamentals, the market efficiency is negatively impact. The amount of submitted market orders decreases. It falls from 20% to 16% for the buy and from 23% to 19% for the sell. The market is deeper, traders can be more patient. They don't need to take too much risk and so prefer to submit limit orders. From a certain point of view the difference between $\beta = 0$ and $\beta \neq 0$ is the liquidity pricing. This idea will be quantitatively investigating in a future work.



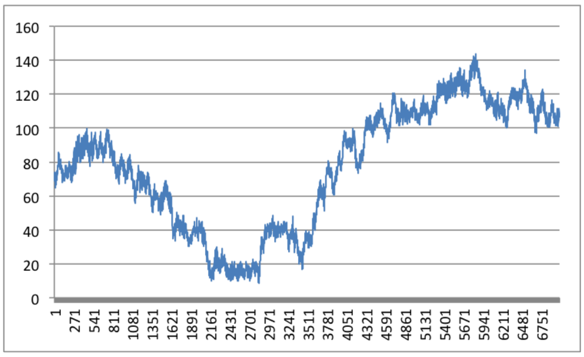
(a) Asset 1: Trading price ($\sigma^2_{f_1} = 0.2$)



(b) Asset 2: Trading price ($\sigma^2_{f_1} = 1$)

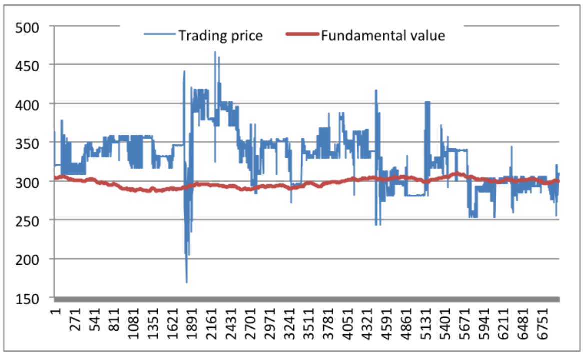


(c) Asset 1: Spread between the perceived fundamental value and the true one

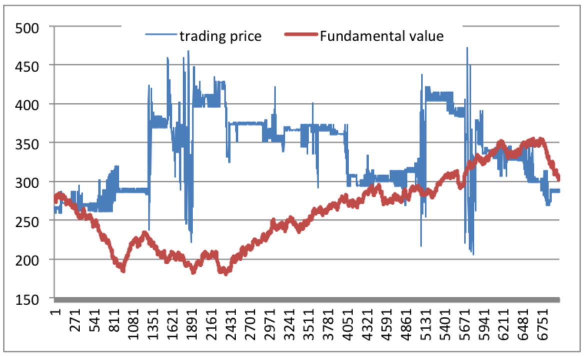


(d) Asset 2: Spread between the perceived fundamental value and the true one

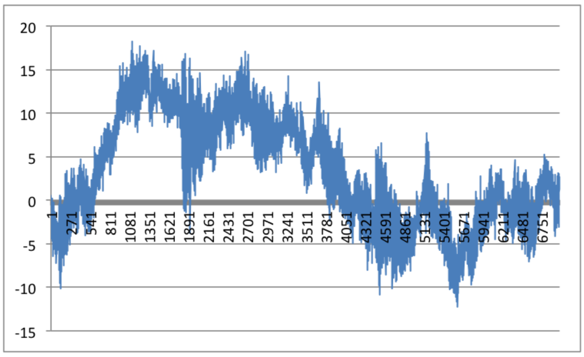
Fig. 6: Price dynamics with myopic agent in TTB case ($\beta = 0$, $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$)



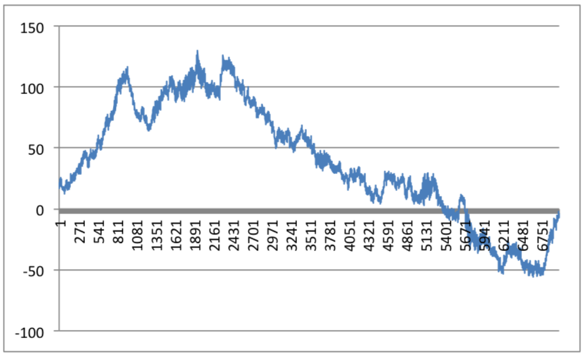
(a) Asset 1: Trading price ($\sigma^2_{f_1} = 0.2$)



(b) Asset 2: Trading price ($\sigma^2_{f_1} = 1$)



(c) Asset 1: Spread between the perceived fundamental value and the true one



(d) Asset 2: Spread between the perceived fundamental value and the true one

Fig. 7: Price dynamics with non-myopic agent in TTB case ($\beta = 2$, $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$)

4 Conclusion

In this paper we present an order driven market, where two assets are traded, in order to study the liquidity dynamics. We show that agent types influence the market dynamics. Indeed, fundamentalists make the market oscillates around its fundamental price, while chartists make it diverge. Moreover, according to Beja et Goldman (1980), increasing the fundamentalist power in a previous stable system or adding chartists make the price oscillates in a higher spread and varies more but never diverges. In fact, chartists – due to their reinforcement dynamics – amplify fundamentalists trend and market inefficiency. They also impact negatively the amount of submitted market order per run, but traders are still more impatient at the sell than at the buy.

In any cases, this model is able to generate endogenous bubbles bloom and burst, which is the basic qualitative feature that is interesting in an artificial market. As usual in financial ABM, our model is also able to reproduce long term memory.

Concerning the liquidity, as we can expect, chartists increase the maximum amount of assets exchange in a time-step. Unexpectedly, they make the market less liquid on average than fundamentalists. This is counter-intuitive when comparing to Shiller (2000) and Odeon et Barber (2000), but has a strong explanation within the logic of our model. Nonetheless, with this liquidity fall due to chartist component, we observe an increasing price variance as an increasing bid-ask spread. That is consistent with the literature.

Relaxing the assumption of perfect knowledge of the fundamentals permit to identify interesting change in price dynamics. Our model is able to generate bubbles in a pur fundamentalist market. Indeed, when agents do not have perfect knowledge of fundamentals (belief perseverance), the trading prices may evolve independently of the true fundamental values. This is due to the fact that the anchoring is strong and learning is slow, hence re-adjustment is slow. A possible way to correct this misunderstanding is to give more importance to the learning process. Notice that our model is not calibrated on real world, thus we have to improve this parameter. However, we know that a high value is not consistent with the behavioral research.

The belief perseverance permits to identify to main components in a bubble. The bubbles can be due to chartist following trend and (/or) to the misunderstanding of the fundamental's change.

Finally, it also may have positive impacts. Indeed, when the fundamental value has a low variance, the fact that agents do not know it perfectly has a stabilizing impact on the market dynamics. The small shocks in the fundamental value are unperceived, so the trading price oscillates less and in a smaller spread. In fact, the trading price reflects the main fundamental trends.

About the myopia, the fact that traders' choices are influenced by the quoted order book makes the market inefficient on long periods. The arbitrage between bid-ask spread and risk of no-execution makes bubbles take more time to burst. Trends are identified easily by chartists, and fundamentalists care less about fundamentals. In this case, the market price is driven by the "fear" of non execution. Because of this strategy of order submission, we observe, as with increasing chartists' power, a decreasing amount of submitting market orders. The result is a huge improve of the market liquidity and a loss in market efficiency. From a certain point of view, this inefficiency corresponds to the liquidity cost that is not taking into account in our definition of the fundamental value.

Hence, our model realizes many interesting features that allow us to explore liquidity in an artificial market, although we have added several complex elements to the usually used framework. The influence of a bad perception of the fundamental value, in particular, seems to produce rather expected dynamics, which means that we can carry on using this framework.

We still have one problem that is not consistent with the literature : there is a large liquidity fall when chartists are more present, the liquidity can fall on average of one third, because of the rise in the price variance. This is due to the asset demand equation, which is based on price and asset variation during the two last time-steps, see Eq. (13). We will have to explore a way to minimize this fall by adding noise traders agents (NTA), think about the asset demand equation and permits to more than one agent to submit at each time step.

From now, we wish to explore the importance of belief and its impact on self-fulfilling prophecy. As example, how two independent assets in their fundamentals may have a correlation in their price fluctuation. We also wish to explore new ways of getting information for the agents, and in particular to see the impact of a spread of information through diverse shapes of social network.

An other project, more econometric, is to focus on the co-movement in moments of assets returns. When fundamentals are correlated, we search the long run common dynamics between the asset volatility

5 Appendix

5.1 Mathematical Model

In this section, we focus on the mathematical equations of our order-driven market, where two risky assets are traded. It is based on a modified version of Chiarella et al. (2009) by Yamamoto (2011).

In our model, each trader is characterized by a fundamentalist component (g_1^i) and a chartist one (g_2^i). At period t, one randomly chosen trader formulates her expectation about the future return that will prevail in the interval $(t + \tau^i)$.

$$\hat{r}^i_{t,t+\tau^i} = \frac{1}{g_1^i + g_2^i}\left[g_1^i . \ln\left(\frac{\hat{f}_t}{p_t}\right) + g_2^i . \bar{r}_t^i\right] \tag{5}$$

where τ^i is the investment horizon of agent i, and p_t denotes the spot price of the considering asset. The weights g_1^i and g_2^i are generated following an exponential law of variance $\sigma^2_{g_1}$ and $\sigma^2_{g_2}$, respectively. Note that a pure fundamentalist strategy has $g_2^i = 0$, whereas a pure chartist strategy has $g_1^i = 0$. The choice of a positive distribution is justified by the works of Hommes and Wagener, 2009 and Hommes et al., 2007. They state that positive feedback for uninformed traders prevail in financial markets. By positive feedback traders, the literature means traders who buy and sell on momentum. Bao et al. (2012) state that with positive feedback traders, there is a self fulfilling oscillation around the fundamental value. Whereas with negative feedback traders, agents learn and make the price converge to its fundamental value.
The average stock return ($\bar{r}_t^i$) computed by chartists is defined by the expected trend based on the observations of the spot returns over the last τ^i time steps.

$$\bar{r}_t^i = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} r_{t-k} = \frac{1}{\tau^i}\sum_{k=1}^{\tau^i} \ln\frac{p_{t-k}}{p_{t-k-1}} \tag{6}$$

The forecasted return of the agent ($\hat{r}^i_{t,t+\tau^i}$) allows her to formulate the future expected price.

$$\hat{p}_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i}) \tag{7}$$

It has to be remembered that two risky assets are traded, therefore Eq. (5) to (7) are applied to each one.

Once the expected prices are defined ($\hat{p}^1_{t+\tau^i}, \hat{p}^2_{t+\tau^i}$), the agent tries to maximize her utility function according to a budget constraint. We assume that the optimal demand of assets is defined by the maximization of a constant absolute risk aversion utility function (CARA) under a gaussian return of assets as :

$$\max_{W^i_{t+\tau^i}} \ \mathbb{E}^i_t[U(W^i_{t+\tau^i}, \alpha^i)] = \max_{W^i_{t+\tau^i}} \ \mathbb{E}^i_t[-exp(-\alpha^i . W^i_{t+\tau^i})]$$

$$W_t^i = z_t^{i,1}.p_t^1 + z_t^{i,2}.p_t^2 + C_t^i$$

where W_t^i reflects the agent's wealth and α^i her risk aversion. The wealth is composed by $z_t^{i,j}$ that denotes the amount of asset j owned by agent i at time t, p_t^j that is the spot price and C_t^i that is the cash invest in a risk free asset, like saving account or bond.
Regarding to the maximization, the optimal demand at the expected prices ($\hat{p}^{i,1}_{t+\tau^i}$, $\hat{p}^{i,2}_{t+\tau^i}$) may be expressed for asset 1 as :

$$\pi_t^{i,1}(\hat{p}^1_{t+\tau^i} \ , \hat{p}^2_{t+\tau^i}) = \frac{\ln\left(\frac{\hat{p}^1_{t+\tau^i}}{p_t^1}\right)}{\alpha^i \ p_t^1 \ Var_1^i} \tag{8}$$

and for asset 2 :

$$\pi_t^{i,2}(\hat{p}^1_{t+\tau^i} \ , \hat{p}^1_{t+\tau^i}) = \frac{\ln\left(\frac{\hat{p}^2_{t+\tau^i}}{p_t^2}\right)}{\alpha^i \ p_t^2 \ Var_2^i} \tag{9}$$

The two assets are assumed to be independent ($Cov = 0$). In this case, $\pi_t^{i,1}$ is equal to the optimal demand of Yamamoto's paper (2011). The variance (Var_j^i) reflects the risk investment of asset j evaluated by agent i. It is assumed to be equal to the variance of the logarithmic of the return rate.

$$Var_j^i = \frac{1}{\tau^i} \sum_{k=1}^{\tau^i} [r_{t-k}^j - \bar{r}_t^{i,j}]^2 \tag{10}$$

where $\bar{r}_t^{i,j}$ is the average spot return of asset j. A general writing is given by Eq. (6). The investment horizon (τ^i) and the risk aversion (α^i) are dependent of agent's features. We define them as Yamamoto (2011):

$$\tau^i = \tau \; \frac{1 + g_1^i}{1 + g_2^i} \tag{11}$$

$$\alpha^i = \alpha \; \frac{1 + g_1^i}{1 + g_2^i} \tag{12}$$

where τ and α are respectively a reference time horizon and a reference degree of aversion toward risk.

Finally, the agent has to define and submit her order. The amont of assets ($s_t^{j,i}$) the agent is willing to trade is determined by the absolute difference in the optimal demand for asset j at time t and $t-1$ as:

$$s_t^{i,j} = abs(\pi_t^{i,j} - \pi_{t-1}^{i,j}) \tag{13}$$

The sign of this difference ($\pi_t^{i,j} - \pi_{t-1}^{i,j}$), gives the agent's position. Notice that the submission price may differ of the expected one ($\hat{p}_{t+\tau^i}^{i,j}, Eq.$ (7)) according to the agent's mood ($M_t^{i,j}$). The buy price is defined as :

$$b_t^{i,j} = \hat{p}_{t+\tau^i}^{i,j}(1 + M_t^{i,j}) \tag{14}$$

The sell price is:

$$a_t^{i,j} = \hat{p}_{t+\tau^i}^{i,j}(1 + M_t^{i,j}) \tag{15}$$

where $M_t^{i,j}$ is randomly assigned from a uniform distribution of mean zero.

5.2 Parameters setup

For the perceived fundamental value, the parameters are arbitrary fixed and are justified by the interpretation of the Kannehman's evidences. We tried to reflect the difficulty of agents to estimate the probability of occurrence, their excess of optimism, their wishful thinking, and their believes perseverances.

Description	Parameter	Value
Market variables		
Traders amount	Nb_agents	1000
Share amount	-	∞
Tick size	-	10^{-3}
Original fundamental value	asset 1 : f_0^1 asset 2 : f_0^2	$f_0^1 = 300$ $f_0^2 = 300$
Original market price	p_0^1 p_0^2	$p_0^1 = f_0^1 = 300$ $p_0^2 = f_0^2 = 300$
Fundamental value (Random Walk)	f_t^1 f_t^2	$f_{t-1}^1 + \mathcal{N}(0, \sigma_{f_1}^2)$ $f_{t-1}^2 + \mathcal{N}(0, \sigma_{f_2}^2)$
Assets covariance	$Cov_{1,2}$	0
Agent components		
Weight	fundamentalist : g_1 chartist : g_2	exponential law of variance $\sigma_{g_1}^2$ exponential law of variance $\sigma_{g_2}^2$
Original endowment	wealth : W_0^i asset 1 : $z_0^{i,1}$ asset 2 : $z_0^{i,2}$ bond : C_0^i	12.000 ECU $\mathcal{U}(0; 20)$ $\mathcal{U}(0; 20)$ $W_0^i - z_0^{i,1}.p_0^1 - z_t^{i,2}.p_0^2$
Time horizon reference	τ	300
Risk aversion reference	α	0.1
Agent mood	$M_t^{i,j}$	$\mathcal{U}(-0.25; +0.25)$
Propensity of adaptation	β_i β_{max}	$\mathcal{U}(0; \beta_{max})$ 2
Perfect knowledge case		
Fundamental value of agent i	$\hat{f}_t^i$	f_t
Belief perseverance case		
Fundamental value of agent i	$\hat{f}_t^i$	$\gamma_1\ p_{t-1} + \gamma_2\ \hat{f}_{t-1}^i + \gamma_3\ \hat{f}_0^i$ $+N_t + a(\hat{f}_{t-1}^i - \hat{f}_{t-2}^i - N_t)$ $+b\ (f_{t-1} - \hat{f}_{t-1}^i)$
Anchor component	γ_1 γ_2 γ_3	0.02 0.4 0.58
Information confidence	a	0.98
Learning	b	0.005
Original expectation (idiosyncratic)	$\hat{f}_0^i$	$f_0 + \mathcal{N}(0, \sigma_f^2)$
New information	N_t	$\mathcal{N}(0, 1)$

Table 3: Parameters setup

5.3 Demand function

Based on the Markowitz criteria, each agent try to maximize her utility :

$$U(W,\alpha) = -\exp(-\alpha W) \tag{16}$$

The utility function is defined as a constant absolute risk aversion class, where α is the risk aversion coefficient of the trader. W_t reflects the trader wealth at time t which is given by a cash amount C_t and the quantity of the risky assets z_t^j holding times their spot prices p_t^j.

$$W_t = z_t^1.p_t^1 + z_t^2.p_t^2 + C_t \tag{17}$$

The wealth at time $t+\tau$ (assuming the order is executing at a price p before $t+\tau$) is given by :

$$W_{t+\tau} = z_{t+\tau}^1.p^1 + z_{t+\tau}^2.p^2 + C_{t+\tau} \tag{18}$$

$$= W_t + s_t^1.p^1.\rho_{t+\tau}^1 + s_t^2.p^2.\rho_{t+\tau}^2 \tag{19}$$

where $\rho_{t+\tau} = p_{t+\tau}/p - 1$ is the return from t to $t+\tau$. As a zero order approximation, the agent's expectation for future returns are taken to be Gaussian where future return is assumed to be $\rho_{t+\tau} = p_{t+\tau}/p - 1 \simeq ln(p_{t+\tau}/p)$. Based on her knowledge, the trader tries to maximize at time t her forward utility $(U_{t+\tau})$:

$$\max_{W_{t+\tau}} \mathbb{E}_t[U(W_{t+\tau},\alpha)] = \max_{W_{t+\tau}} \mathbb{E}_t[-exp(-\alpha.W_{t+\tau})] \tag{20}$$

Because the utility is exponential and the return are assumed to be Gaussian, it may be expressed as :

$$\hat{U}(W_{t+\tau},\alpha) = -\exp\left(-\alpha.\mathbb{E}_t[W_{t+\tau}] + \alpha^2.\sigma_t^2[W_{t+\tau}]/2\right) \tag{21}$$

where σ^2 reflects the risk of investment. The variance and the expected wealth are:

$$\sigma_t^2[W_{t+\tau}] = s_t^1.{p^1}^2\sigma_t^2(\rho_1) + s_t^2.{p^2}^2\sigma_t^2(\rho_2) + 2.s_t^1.s_t^2.corr_t(\rho_1,\rho_2) \tag{22}$$

$$\mathbb{E}_t[W_{t+\tau}] = W_t + s_t^1.p^1\mathbb{E}_t(\rho_1) + s_t^2.p^2\mathbb{E}_t(\rho_2) \tag{23}$$

The Eq. (21) could be rewritten as:

$$\hat{U}_{t+\tau} = \hat{U}_t \times e^{\left(-\alpha[s_1p^1\mathbb{E}_t(\rho_1)+s_2p^2\mathbb{E}_t(\rho_2)]+\frac{\alpha^2}{2}[s_1^2{p^1}^2\sigma_t^2(\rho_1)+s_2^2{p^2}^2\sigma_t^2(\rho_2)+2.s_t^1.s_t^2.cov_{1,2}]\right)} \tag{24}$$

Differentiating the expected utility function (24) with respect to s_t^j gives:

$$\begin{cases} \frac{d}{ds_t^1}\hat{U}_{t+\tau} = -\hat{U}_{t+\tau}\left[\alpha p_t^1 \ln\left(\frac{p_{t+\tau}^1}{p^1}\right) - \alpha^2 s_t^1 {p^1}^2\sigma_t^2(\rho_1) - 2s_t^2 Cov_{1,2}\right] \\ \frac{d}{ds_t^2}\hat{U}_{t+\tau} = -\hat{U}_{t+\tau}\left[\alpha p_t^2 \ln\left(\frac{p_{t+\tau}^2}{p^2}\right) - \alpha^2 s_t^2 {p^2}^2\sigma_t^2(\rho_2) - 2s_t^1 Cov_{1,2}\right] \end{cases} \tag{25}$$

Setting the expression to zero we determine the optimal amount of stocks $(S_t^* = \pi(p))$ that the agent wishes to hold in her portfolio for a given price level p.

$$\begin{cases} \pi_t^{i,1}(\hat{p}_{t+\tau^i}^1\ ,\hat{p}_{t+\tau^i}^2) = \dfrac{\alpha^i p_t^2\left[\ln\left(\dfrac{\hat{p}_{t+\tau^i}^1}{p_t^1}\right)\alpha^{i^2}p_t^1p_t^2Var_2^i - 2\ln\left(\dfrac{\hat{p}_{t+\tau^i}^2}{p_t^2}\right)Cov_{1,2}\right]}{\alpha^{i^4}\ p_t^{1^2}\ Var_1^i\ p_t^{2^2}\ Var_2^i - 4\ Cov_{1,2}^2} \\ \pi_t^{i,2}(\hat{p}_{t+\tau^i}^1\ ,\hat{p}_{t+\tau^i}^2) = \dfrac{\alpha^i p_t^1\left[\ln\left(\dfrac{\hat{p}_{t+\tau^i}^2}{p_t^2}\right)\alpha^{i^2}p_t^1p_t^2Var_1^i - 2\ln\left(\dfrac{\hat{p}_{t+\tau^i}^1}{p_t^1}\right)Cov_{1,2}\right]}{\alpha^{i^4}\ p_t^{1^2}\ Var_1^i\ p_t^{2^2}\ Var_2^i - 4\ Cov_{1,2}^2} \end{cases} \tag{26}$$

If we assumed the assets independent (Cov=0), it correspond to Eq. (8) and (9) :

$$\begin{cases} \pi_t^{i,1}(\hat{p}_{t+\tau^i}^1\ ,\hat{p}_{t+\tau^i}^2) = \dfrac{\ln\left(\dfrac{\hat{p}_{t+\tau^i}^1}{p_t^1}\right)}{\alpha^i\ p_t^1\ Var_1^i} \\ \pi_t^{i,2}(\hat{p}_{t+\tau^i}^1\ ,\hat{p}_{t+\tau^i}^1) = \dfrac{\ln\left(\dfrac{\hat{p}_{t+\tau^i}^2}{p_t^2}\right)}{\alpha^i\ p_t^2\ Var_2^i} \end{cases} \tag{27}$$

5.4 Chartists

Chartists are created so as to represent agents who would try to make profit by surfing on the bubbles. Without fundamentalists to impose a market trend, they have a pure destabilizing impact. They make the price increases strongly or falls to 0. In a chartists-market, the price is purely speculative and never reflects its fundamental.

The key parameter for chartists is $\sigma^2_{g_2}$, the variance distribution. It impacts the investment horizon and the risk aversion in the opposite direction of fundamentalists. An higher variance implies a bigger weight affected to chartist component, and so a decreasing aversion toward risk and a shorter investment horizon. In the following table, we collect the percentage of market divergence in trading rounds. By market divergence we mean a price exceeded 10 times, 100 times and 10^4 times the fundamental value. The fall is bounded to a tick size beyond zero, the asset never disappears.

Divergence	$\sigma^2_{g2} = 0.1$		$\sigma^2_{g2} = 0.6$		$\sigma^2_{g2} = 1$		$\sigma^2_{g2} = 10$	
FV	%	t	%	t	%	t	%	t
x10	98.5%	1656	100 %	1042	100%	835	100%	630
x100	90.5%	2532	97.0%	1781	98.5%	1912	100%	1186
x10000	60.5%	4020	84.5%	3303	85.5%	3394	95.5%	2522

Table 4: Frequency of asset 1 divergence and time needed

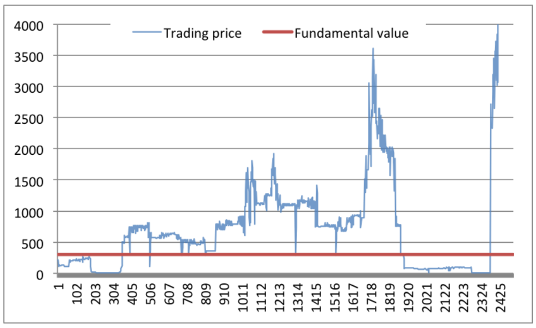
All in all, the price of asset 1 diverges 92.5% of the time. We get similar results with the dynamics of asset 2. The market diverges faster and more frequently when $\sigma^2_{g_2}$ increases (see Table 5). Hence, a higher chartists weight reinforces the effect of self-fulfilling prophecy. When chartists expect a rise, they submit orders at a higher price, thus trading price increases and the chartists expectations come true. The price enhancement is based on agents speculations and not on fundamental components.

In a market where only chartists interact the price could increase infinitely until one of them decides to sell at a relative low price. This decision could be motivated by the explosive variance of the trading price or a liquidity needed. If low price sellers are in minority, the market dynamics are, at best, impacted for a short time. If a panicked rush appears, the price falls down for a longer time, until a new bubble borns. The bubbles burst and bloom cyclically until the asset price exceeds the market limits. In this kind of market, driven by self-fulfilling prophecies, the price is not informative (Fig. 8).

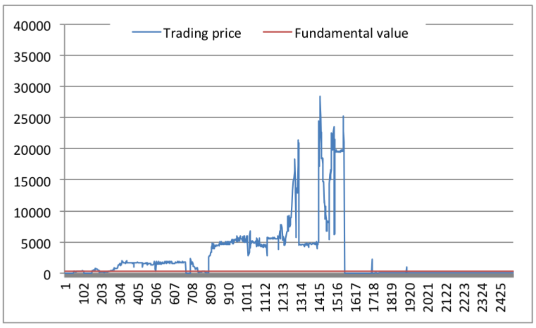
As an example for $\sigma^2_{g_2} = 0.1$ and $Divergence = 10^4$, the average price is higher than $17000ECU$, and a variance exceeded 10^9. The trading volume of asset 1 is around 300, which is approximately similar to a market populated by fundamentalists with $\sigma^2_{g_1} = 10$ (359) and half trading volume than a market with $\sigma^2_{g_1} = 0.1$ (710). The standard deviation of exchanging volume is, at least, 3 times higher for the chartists-market (due to the destabilizing effect of chartists). The maximum volume of exchange in a time-step is twice bigger than the fundamentalists-market: 26 versus 11.

The low aggregate trading volume of chartists may come from the computational method. Indeed, the volume of exchange is only estimated with non-diverging markets – in a market where the price doesn't rise too much, it can be due to an illiquid market or a volatile one. For the few cases where the market is not considered as diverging, the trading price is above the fundamental value 49% of time. It verifies the explanation of cycles with bubbles burst and bloom.

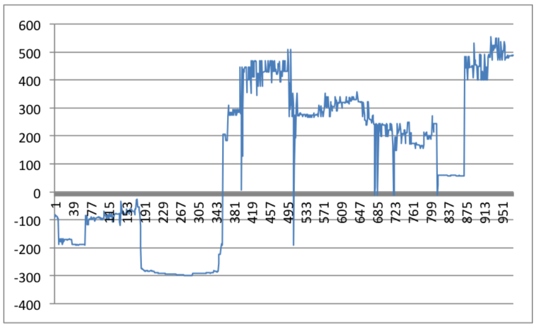
Concerning the traders' choice between market (MO) and limit order (LO), chartists submit less MO than fundamentalists. This could be justify by a wide bid-ask spread and a relative short term horizon. In any case, this result is in accordance with the microstructure theory. O'Hara and Easley (1995) has pointed out that " the discretionary uninformed traders can increase liquidity (in the period in which they trade) of the market". The submitters of limit orders are considered as provider of liquidity. The informed traders capture this liquidity by submitting market orders. So, it is predictable that fundamentalists submit more market orders than chartists. Moreover, we can expect that in an heterogeneous market, the ratio of submitted market orders decrease, and the market liquidity increases compare to a fundamentalists-market.



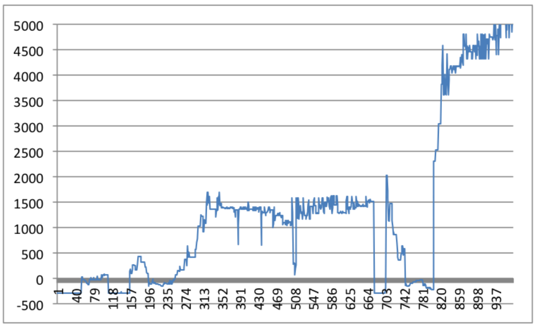
(a) Asset 1: Trading price ($\sigma^2_{f_1} = 0.2$)



(b) Asset 2: Trading price ($\sigma^2_{f_1} = 0.2$)



(c) Asset 1: Spread between the trading price and the fundamental value on a shorter period 0-1000



(d) Asset 2: Spread between the trading price and the fundamental value on a shorter period 0-1000

Fig. 8: Price dynamics in a market populated by chartists ($\sigma^2_{g_2} = 0.6$)

5.5 Tables and graphics

Table 5: Frequency of asset 1 divergence and time needed

Divergence	$\sigma^2_{g2}=0.1$		$\sigma^2_{g2}=0.6$		$\sigma^2_{g2}=1$		$\sigma^2_{g2}=10$	
FV	%	t	%	t	%	t	%	t
x10	98.5%	1656	100 %	1042	100%	835	100%	630
x100	90.5%	2532	97.0%	1781	98.5%	1912	100%	1186
x10000	60.5%	4020	84.5%	3303	85.5%	3394	95.5%	2522

Divergence=10 means that the trading price exceed 10 times the fundamental value at least once in the simulations
t = the average time needed for divergence

Table 6: Divergence table, when asset-1 diverges

$\sigma^2_{f_1}=0.01$

Divergence	$\sigma^2_{g2}=0.6$	$\sigma^2_{g2}=1$	$\sigma^2_{g2}=10$
10	99.5%	99.5%	100%
100	96%	97.5%	99%
10000	73.5%	83%	94%

$\sigma^2_{f_1}=0.2$

Divergence	$\sigma^2_{g2}=0.6$	$\sigma^2_{g2}=1$	$\sigma^2_{g2}=10$
10	99.5%	100%	100%
100	96%	98.5%	99.5%
10000	77%	83%	94.5%

$\sigma^2_{f_1}=1$

Divergence	$\sigma^2_{g2}=0.6$	$\sigma^2_{g2}=1$	$\sigma^2_{g2}=10$
10	100%	100%	100%
100	97%	98.5%	99%
10000	76%	81%	94%

For each σ^2_{g2}, 9 pools of trading rounds are simulated with different variance for the white noise of the fundamental value ($f_t = f_{t-1} + \mathcal{N}(0, \sigma^2_f)$ with $\sigma^2_f = 0.01,\ 0.2,\ 1$)

Table 7: Impact of components ($\sigma^2_f = 0.2$)

Parameter	$\sigma^2_{g_1}=0.6$	$+\hat{f}_i$	$\sigma^2_{g_1}=0.6$, $\sigma^2_{g_2}=1$	$+\hat{f}_i$	$+\beta=2$
Average price	308.157	306.601	309.623	307.580	314.148
(Std. Err)	(12.33)	(3.70)	(13.34)	(6.04)	(9.11)
Price variance	695.665	552.525	849.258	726.803	801.683
	(227.26)	(112.85)	(223.35)	(155.92)	(334.26)
Mean spread	7.79	6.93	9.21	8.16	14.70
	(4.58)	(11.74)	(6.09)	(12.30)	(15.80)
Spread interval	[-72;+77]	[-68; +77]	[-98; +132]	[-90; +115]	[-93; +141]
Overvalued	61.44%	59.84%	62.15%	60.39%	66.83%
Liquidity	592.8899	573.1003	391.1416	392.1275	632.5406
	(92.22)	(33.32)	(26.55)	(21.38)	(76.86)
MO (buy)	20.17%	20.64%	20.55%	20.77%	16.62%
MO (sell)	26.13%	25.06%	23.73%	23.22%	19.32%

Table 8: OTP with $\sigma_f^2 = 0.2$

	$\sigma_{g_1}^2 = .6$		$\sigma_{g_1}^2 = 1$		$\sigma_{g_1}^2 = 10$	
Asset 1	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
price	308.1574	12.32511	308.5255	12.43877	308.7847	12.14842
variance	695.6651	227.2629	710.3875	245.5468	754.0125	245.0574
mean fv	300.3684	11.89047	300.6404	11.02654	300.6333	10.57084
volume	592.8899	92.22462	554.1616	70.9133	359.2986	46.13003
mean volume	0.0846986	0.0131749	0.0791659	0.0101305	0.0513284	0.00659
max volume	11.02806	9.170719	10.94757	6.253219	9.628196	6.97539
empty	0.0307221	0.0439115	0.0348336	0.0460844	0.0348007	0.0443637
overvalued	61.44%	0.0753254	61.44%	0.0738832	61.85%	0.0855326
mean spread	7.788996	4.584102	7.885097	5.147205	8.151459	6.281391
min spread	-72.33428	8.516987	-72.69171	8.482047	-72.95131	8.329592
max spread	77.45177	6.372596	78.35316	6.433966	78.23292	8.157135
variance spread	648.9922	207.5843	666.5788	230.6979	702.8659	223.1803
means95	7.869221	4.670239	7.956016	5.264398	8.326653	6.382589
mins95	-40.52318	9.37619	-40.9088	10.46586	-44.06135	11.27737
maxs951	56.98935	12.31665	58.10474	12.68831	57.43411	13.31851
variances95	518.0513	176.0479	531.5191	192.232	564.9372	187.6149
Asset 2	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
price	310.4384	14.41455	309.6829	12.31885	309.1788	13.56409
variance	695.5531	267.0946	714.1698	237.3936	776.5419	271.0681
mean fv	300.9104	13.22514	299.6201	10.7646	299.1682	12.00967
volume	613.6832	156.6822	579.0842	143.0713	370.6732	105.7914
mean volume	0.087669	0.0223832	0.0827263	0.0204388	0.0529533	0.0151131
max volume	14.4399	10.52302	14.77144	9.396637	12.61206	6.8184
empty	0.0300579	0.0442258	0.0370929	0.0456139	0.0406014	0.047353
overvalued	64.04%	0.0873622	64.94%	0.0843774	64.08%	0.0894455
mean spread	9.528054	5.892816	10.06278	5.886229	10.01056	6.95305
min spread	-73.41645	8.496879	-74.2861	7.131068	-74.22578	8.279589
max spread	78.76266	5.920565	78.93036	5.644677	80.62684	6.275525
variance spread	663.3532	241.2208	672.9731	212.7183	742.3849	260.2845
means95	9.644843	6.012068	10.231	6.012701	10.2157	7.098899
mins95	-39.22203	10.75663	-39.87547	10.63538	-42.48869	12.32808
maxs951	57.90253	12.26719	58.19551	13.06891	59.31074	12.56349
variances95	533.1659	208.8357	536.8655	180.1373	601.8252	228.8337
MO buy-1	20.17%	0.0188271	20.12%	0.0214991	20.19%	0.0242499
MO sell-1	26.13%	0.0193556	26.20%	0.0212283	26.21%	0.0248144
MO buy-2	19.70%	0.0230664	19.56%	0.0244686	19.60%	0.0267621
MO sell-2	26.81%	0.0231043	27.00%	0.0237113	27.11%	0.0272122

Table 9: OTB with $\sigma^2_{f_1} = 0.2$ and $\sigma^2_{f_2} = 1$

	$\sigma^2_{g_1} = .6$		$\sigma^2_{g_1} = 1$		$\sigma^2_{g_1} = 10$	
Asset 1	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
price	306.1534	3.463512	306.3909	4.428084	306.6645	4.943354
variance	557.3468	131.1669	567.7487	128.1391	618.8802	186.5933
volume	578.2245	31.8367	536.9683	33.07209	350.9792	25.6745
max volume	7.205507	4.983599	7.271991	4.155743	6.43139	3.839755
empty	0.0097464	0.0213598	0.0100921	0.0211485	0.0134243	0.0280378
overvalued	59.74%	0.1677892	60.10%	0.1824225	57.49%	0.1557758
mean spread	6.911655	12.32845	7.322211	13.05146	5.173439	11.53773
min spread	-69.69829	13.87851	-68.97779	15.19092	-71.71536	13.62496
max spread	77.33325	14.94849	77.48692	14.6541	75.0051	13.00802
variance spread	604.4926	142.0178	615.7138	139.9558	663.056	191.6678
mean spreadEFV	0.9585063	11.68905	1.137591	12.01733	-1.263458	10.33704
min spreadEFV	-16.12562	11.66179	-16.06191	12.56467	-18.0219	11.39867
max spreadEFV	18.04531	12.37507	18.28009	12.12234	15.47089	10.54068
variance spreadEFV	48.20557	33.40747	51.33188	38.64856	47.13037	42.22906
Asset 2	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
price	306.9574	4.857592	307.3198	4.740281	307.6364	6.105452
variance	554.5131	124.6286	546.4546	148.3676	610.1107	165.1104
volume	591.5599	104.284	545.064	98.79757	362.3642	78.3615
max volume	9.448241	6.288714	9.1585	5.0728	9.462259	7.067709
empty	0.0084321	0.0185964	0.0099371	0.0189447	0.0154214	0.0247669
overvalued	57.24%	0.337121	57.42%	0.334296	53.53%	0.3484412
mean spread	10.51849	58.13192	11.59553	60.38223	5.454497	56.81206
min spread	-104.4057	60.1539	-103.0915	62.62739	-107.7331	58.155
max spread	117.198	61.72648	118.8464	61.65015	111.0119	59.30671
variance spread	1760.692	1047.526	1823.554	1307.714	1642.148	920.5805
mean spreadEFV	3.766442	58.18193	4.482041	60.67168	-1.915285	57.38136
min spreadEFV	-65.32578	60.14335	-64.62811	62.65806	-66.80576	57.027
max spreadEFV	72.74141	59.81421	72.84327	60.14862	63.62937	58.75076
variance spreadEFV	1211.545	996.9575	1279.267	1242.215	1023.606	855.6559
MO buy-1	20.72%	0.0127661	20.73%	0.0156313	20.77%	0.0182882
MO sell-1	24.94%	0.0130618	25.05%	0.0165479	25.17%	0.0184851
MO buy-2	20.66%	0.0171273	20.54%	0.0162071	20.46%	0.0211096
MO sell-2	25.33%	0.0169755	25.44%	0.0169223	25.63%	0.0213242

Table 10: Two-types market with myopic agent. $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$

$\beta = 0$, $\sigma^2_{f_1} = \sigma^2_{f_2} = 0.2$

	TTP		TTB	
Asset 1	Mean	Std. Er.	Mean	Std. Er.
price	309.623	13.33735	307.580	6.040267
variance	849.258	223.3471	726.8023	155.9246
volume	391.1416	26.55105	392.1275	21.38328
max volume	8.123096	5.535868	6.525318	4.079126
empty	0.0118771	0.0111059	0.0068514	0.0087016
overvalued	62.15%	0.0817064	60.39%	0.152901
mean spread	9.217	6.093379	8.156	12.2993
min spread	-98.574	25.37524	-90.252	27.31835
max spread	132.057	44.44724	115.275	41.7988
variance spread	804.3789	190.9701	767.7095	172.077
mean spreadEFV	XXX	XXX	0.825	10.62857
min spreadEFV	XXX	XXX	-16.574	10.69423
max spreadEFV	XXX	XXX	18.169	12.00475
variance spreadEFV	XXX	XXX	50.32136	42.07813
Asset 2	Mean	Std. Er.	Mean	Std. Er.
price	310.860	12.16608	309.691	7.112836
variance	947.3698	285.6258	778.3532	201.7131
volume	420.4131	75.1874	415.8279	65.98666
max volume	12.38223	6.11285	9.802436	5.959227
empty	0.0187443	0.0155055	0.0098757	0.0093289
overvalued	63.67%	0.0956571	61.59%	0.1764587
mean spread	10.548	7.088149	9.332	14.26773
min spread	-110.418	24.05953	-95.646	25.73981
max spread	145.036	44.72508	128.776	41.6798
variance spread	913.5817	269.3361	815.5288	216.4183
mean spreadEFV	XXX	XXX	-0.034	12.61678
min spreadEFV	XXX	XXX	-17.304	13.32003
max spreadEFV	XXX	XXX	17.020	12.26027
variance spreadEFV	XXX	XXX	44.1114	34.73074
MO buy-1	20.55%	0.0106554	20.77%	0.0093301
MO sell-1	23.73%	0.0110661	23.22%	0.0095739
MO buy-2	20.51%	0.012634	20.56%	0.0118371
MO sell-2	24.27%	0.0125587	23.72%	0.0122481

Table 11: Two-types market with myopic agent. $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 10$

$\beta = 0$, $\sigma^2_{f_1} = \sigma^2_{f_2} = 0.2$

	TTP		TTB	
Asset 1	Mean	Std. Er.	Mean	Std. Er.
price	309.645	15.22974	309.288	10.85575
variance	1956.68	591.2399	1771.537	505.3346
volume	353.1909	31.70824	359.5172	25.59655
max volume	7.523469	3.864274	7.465332	4.121011
empty	0.0356364	0.0350324	0.0249586	0.0138739
overvalued	57.86%	0.0987944	58.02%	0.1550106
mean spread	9.580	10.39823	9.753	16.83569
min spread	-142.921	25.00724	-138.842	29.81492
max spread	243.640	78.684	230.3917	74.51019
variance spread	1910.569	582.1038	1813.58	515.8413
mean spreadEFV	XXX	XXX	0.768	12.62883
min spreadEFV	XXX	XXX	-19.384	13.306
max spreadEFV	XXX	XXX	21.280	12.97876
variance spreadEFV	XXX	XXX	56.97127	45.44262
Asset 2	Mean	Std. Er.	Mean	Std. Er.
price	309.436	16.16384	308.833	10.64477
variance	2311.15	877.6242	1808.425	585.7817
volume	378.5888	62.57369	424.1773	51.19907
max volume	11.76656	5.955194	9.868778	4.701071
empty	0.0381564	0.0161272	0.0229529	0.0119421
overvalued	57.00%	0.1161541	56.94%	0.1359406
mean spread	8.390	13.24927	9.082	14.8614
min spread	-149.588	22.47362	-139.976	25.73726
max spread	251.085	89.33567	223.124	71.53082
variance spread	2272.728	835.8982	1867.683	612.2452
mean spreadEFV	XXX	XXX	0.539	11.09187
min spreadEFV	XXX	XXX	-19.180	11.22918
max spreadEFV	XXX	XXX	21.270	12.4479
variance spreadEFV	XXX	XXX	57.38593	42.02272
MO buy-1	19.91%	0.01346	20.10%	0.0104076
MO sell-1	22.39%	0.011808	22.13%	0.0103705
MO buy-2	20.51%	0.0136254	20.17%	0.0106023
MO sell-2	22.53%	0.0138985	22.17%	0.0103072

Table 12: Two-types market with non-myopic agent. $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 1$

$\beta = 2$, $\sigma^2_{f_1} = \sigma^2_{f_2} = 0.2$

	TTP		TTB	
Asset 1	Mean	Std. Er.	Mean	Std. Er.
price	314.795	15.44511	314.148	9.111996
variance	902.4987	396.3353	801.6831	334.2617
volume	617.0394	81.45461	632.5406	76.85992
max volume	20.66251	17.8293	20.13479	14.53847
empty	0.0153543	0.0153994	0.0132714	0.0133887
overvalued	68.77%	0.1205703	66.83%	0.1936148
mean spread	14.761	9.566618	14.704	15.80046
min spread	-96.166	28.07585	-93.905	33.52497
max spread	142.326	46.79219	141.445	51.15623
variance spread	857.4721	374.2211	857.0026	370.3938
mean spreadEFV	XXX	XXX	1.018	11.98978
min spreadEFV	XXX	XXX	-16.786	12.65067
max spreadEFV	XXX	XXX	18.544	12.47587
variance spreadEFV	XXX	XXX	53.58562	44.75238
Asset 2	Mean	Std. Er.	Mean	Std. Er.
price	326.547	16.78076	325.308	13.47082
variance	1432.033	673.134	1334.09	759.1288
volume	710.5836	213.8777	760.9063	234.3343
max volume	31.0924	16.87446	31.98838	19.91671
empty	0.0345514	0.0286827	0.0266264	0.0184325
overvalued	76.55%	0.133096	73.90%	0.1690035
mean spread	26.751	12.05893	25.109	18.35389
min spread	-114.198	23.42406	-109.162	26.22176
max spread	190.632	51.53113	177.906	54.00222
variance spread	1380.472	651.9696	1378.23	792.632
mean spreadEFV	XXX	XXX	0.644	12.16455
min spreadEFV	XXX	XXX	-18.09	12.84863
max spreadEFV	XXX	XXX	19.532	13.09841
variance spreadEFV	XXX	XXX	55.80524	47.23891
MO buy-1	16.48%	0.011571	16.62%	0.0101137
MO sell-1	19.35%	0.0109981	19.24%	0.0103971
MO buy-2	15.35%	0.0132538	15.62%	0.0131221
MO sell-2	19.98%	0.0138066	19.92%	0.0121331

Table 13: Two-types market with non-myopic agent. $\sigma^2_{g_1} = 0.6$ and $\sigma^2_{g_2} = 10$

$\beta = 2$, $\sigma^2_{f_1} = \sigma^2_{f_2} = 0.2$

	TTP		TTB	
Asset 1	Mean	Std. Er.	Mean	Std. Er.
price	315.087	16.98607	315.167	14.94488
variance	2609.926	1105.48	2287.299	873.6935
volume	465.7326	60.29947	486.1503	56.78002
max volume	16.48685	10.91818	17.94335	15.1951
empty	0.0342214	0.0179805	0.030465	0.0149577
overvalued	61.47%	0.1295715	61.90%	0.1602467
mean spread	15.581	15.18423	15.843	19.14285
min spread	-143.546	27.16042	-140.049	29.04924
max spread	265.219	93.9506	247.705	76.04802
variance spread	2588.241	1084.544	2327.795	889.5185
mean spreadEFV	XXX	XXX	1.172	11.5666
min spreadEFV	XXX	XXX	-18.966	12.2095
max spreadEFV	XXX	XXX	22.232	11.48083
variance spreadEFV	XXX	XXX	53.58916	38.04949
Asset 2	Mean	Std. Er.	Mean	Std. Er.
price	328.902	21.07345	325.296	16.9841
variance	4071.31	3295.502	3125.436	1374.545
volume	557.2725	148.1856	618.7041	143.2585
max volume	25.26889	11.02454	22.013	10.33209
empty	0.0536886	0.0235475	0.0401043	0.0157431
overvalued	67.48%	0.1281061	66.86%	0.1482752
mean spread	28.35	19.27285	26.347	20.72252
min spread	-148.941	23.38636	-142.411	25.78911
max spread	329.651	147.5279	294.439	85.96492
variance spread	4032.141	3308.326	3170.04	1413.612
mean spreadEFV	XXX	XXX	1.88	11.3333
min spreadEFV	XXX	XXX	-19.091	11.54524
max spreadEFV	XXX	XXX	24.555	13.2024
variance spreadEFV	XXX	XXX	56.59346	47.42207
MO buy-1	16.36%	0.0112969	16.63%	0.0101674
MO sell-1	18.42%	0.0109538	18.36%	0.0102335
MO buy-2	15.71%	0.0113833	15.94%	0.011504
MO sell-2	19.02%	0.0123566	18.96%	0.0121034

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