Monetary Policy Switching in the Euro Area and Multiple Equilibria: An Empirical Investigation

Gilles Dufrénot, Anwar Khayat

Working Papers / Documents de travail

Monetary Policy Switching in the Euro Area and Multiple Equilibria: An Empirical Investigation

Gilles Dufrénot
Anwar Khayat

WP 2014 - Nr 08

Monetary Policy Switching in the Euro Area and Multiple Equilibria: an Empirical Investigation

Gilles Dufrénot*[1] and Anwar Khayat†[2]

[1]Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS, CEPII, Banque de France

[2]Aix-Marseille University (Aix-Marseille School of Economics), CNRS & EHESS

This version: March 2014
First draft: January 2014

Abstract

This paper provides evidence that the European Central Bank (ECB) has adjusted its interest rate since 1999 nonlinearly according to the macroeconomic and financial environment in the euro zone. Its policy function is described by a Taylor rule with regime shifts implying that the stance of reaction to the inflation-gap and output-gap has varied according to the credit risk in the private and sovereign bond markets, the monetary base and past levels of inflation, output and the shocks affecting the European economies. We provide evidence of regimes corresponding to low to high levels of inflation with the possibility of a situation near a zero low bound (ZLB) for the interest rate. We study the implications of such a rule for the economy in a simple new-Keynesian framework and show that it is consistent with several stable long-run steady states equilibria among which one that is consistent with the recent situation of a *near* liquidity trap in the euro area. We also find that around this liquidity trap steady state the equilibrium is locally determinate for most plausible parameter values. We discuss the issue of moving from a situation of low nominal interest rate to a policy that have been more typically implemented in the past by relying on an analysis of the impact of shocks (supply and demand) to the economy.

Keywords: Nonlinear Taylor rules; multiple steady state equilibria; Euro area.

JEL Classification Numbers: C54, E52, E58.

*Aix-Marseille School of Economics, Chateau La Farge, Route des Milles 13290 Aix-en-Provence, France. Email: gilles.dufrenot@univ-amu.fr; Tel.: 00.33.(0)4.42.93.59.60; Fax: 00.33.(0)4.42.38.95.85

†GREQAM, centre de la vieille charité, 2 rue de la charité 13002 Marseille, France. Email: anwar.khayat@gmail.com; Tel: +33.(0)4.91.14.07.27

1 Introduction

This paper shows that a regime-shift Taylor rule can be used to model the short-term interest rate in the euro zone from 1999 till 2012. This rule captures monetary regime changes, in the sense that the European Central Banks (ECB)'s targeted interest rate is characterized by important shifts when the monetary authorities worry about the macroeconomic and financial environment, for instance the occurrence of bubbles, situations of liquidity trap or increasing risks in some financial market segments. The observed changes in the short-term interest rate is not only the result of the ECB's response to deviations from inflation and output-targets, but also reflect the time variation in the stance of their response to macroeconomic and financial conditions.

Our paper relies on two important strands of the literature on monetary policy modelling: 1/ nonlinear interest rate rules and their implications for the macroeconomic equilibrium of monetary economies, 2/ time-varying Taylor rules.

On the one hand, a literature on Taylor rules find that the latter can display nonlinearities that create several long-run equilibria for monetary economies.
A key issue is whether several stable equilibria can coexist and whether it is possible for an economy to start in the neighborhood of an equilibrium and follow eventually a path yielding to another one. This can happen for instance if there are several long-run steady states that are locally stables. Such an issue is of particular interest in the current context of liquidity traps that characterizes the industrialized economies with a situation of zero low bound (ZLB) since the recent 2008 crisis. One has to worry about the possibility that the economy becomes permanently trapped in an unintended equilibria, as was the case for instance in Japan over the 2000s with a deflationary regime and low nominal interest rates. Whether or not interest rate rules can lead to several equilibria is, however, still highly controversial. In rational expectation models, ZLB solutions are sometimes considered as the results of self-fulfilling expectations and are ruled out. This is what is done when one

linearizes the macroeconomic model in the neighborhood of a desired equilibrium. However, there are many papers showing that once we consider the true nonlinear models, nonlinear Taylor rules yields to equilibria including endogenous cycles, bifurcation, chaotic dynamics[1], or simply the coexistence of several stable stationary steady states (see Section 2 below for a brief review of the literature on this issue).

The purpose of this paper is to examine this issue in the case of the euro area. We find that since 1999 two stable long-run steady states have co-existed, one of which is consistent with the current situation of a near liquidity trap. This result is obtained by considering a nonlinear Taylor rule that replicates the dynamics of the interest rate on the historical data and a rule that describes the targeted interest rate from a simple Keynesian model. We discuss the policy implications by examining the impulse response functions of supply and demand shocks. Since the Taylor rule is not linearized in the neighborhood of a specific steady state, the economy contains several long-run steady states and the economy can move away from a desired equilibrium of medium nominal interest rate to an undesired situation of liquidity trap. More interestingly, we wonder about the size of an inflation or demand shock that is needed to help the economy leaving a situation of *liquidity trap* or zero interest rate policy and return to a *normal* situation.

The second strand of the literature on which we rely concerns the modelling of time-varying interest rate rules. This motivates the type of nonlinearity considered for our Taylor rule. There are several reasons for considering regime-shift dynamics (also called regime-switching dynamics) in the Taylor rules. Firstly, the economy can be characterized by structural shifts, thereby implying a time-varying relationship with the interest rate. If these structural changes are neglected by the monetary authorities small changes in the interest rate could result in involuntary large effects on output and inflation (see Schorfheide (2005), Zampolli (2006)). Secondly, the effects of shifts in monetary policy rules can be a source of widening of the yield spreads (see Ang et al. (2011)). Thirdly, some previous

[1]See Benhabib et al. (2002b) and Benhabib and Eusepi (2005).

papers have pointed to sizable empirical differences between Taylor rules with time-varying coefficients over the conventional constant parameter Taylor rules to describe the ECB's monetary policy (see Assenmacher-Wesche (2006), Trecroci and Vassalli (2010)). Our paper propose a new contribution to this existing literature in several directions.

We explore the possibility of a regime-dependent Taylor rule for the Euro zone by considering a *true* ECB nonlinear Taylor rule. Indeed, due to the limitation of the data, many previous studies either consider a hypothetical ECB rule, by averaging national short-term interest rates, or by considering ECB linear Taylor rules with constant coefficients (see for instance, Belke and Polleit (2007), Gerdesmeier and Roffia (2004), de Haan et al. (2008), Sauer and Sturm (2007), Ullrich (2003)). Further, all the previous studies consider a period before the 2008 crisis. We broader the usual time span by including in the data the years following the recent Great recession. Another contribution concerns the modelling of the regimes. It is common wisdom in the papers that have considered regime-switching monetary policy rules to consider regimes on a pairwise basis. Authors usually distinguish between *normal* and *exceptional* regimes when important events happens, for instance high and low inflation regimes, regimes of bubbles in the returns and prices as opposed to no bubble regimes (see Alcidi et al. (2011), Davig and Leeper (2006)). Here, we find that the ECB monetary policy can sometimes be described by more than two regimes. This illustrates the fact that the ECB does not only shifts its behavior when *exceptional* events like a crisis occur. We let the data determines the number of regimes, because the time-varying coefficients in the Taylor rule can be explained by several factors: shift in the policymakers' decisions, structural instabilities in the macroeconomic fundamentals, model uncertainty, etc. In our case, it is assumed that he following factors cause a shift in the parameters: the ECB's concern about providing the economy with a sufficient amount of liquidity, the past level of the EONIA rate and credit risk in bond markets.

The remainder of the paper is organized as follows. In Section 2 we present a brief review of the literature. In Section 3 we estimate a standard linear Taylor rule for the ECB with constant coefficients, which serves as a benchmark model. Section 4 contains the description and estimations of different regime-shifts Taylor rules. In Section 5, we study the implications for the economy of such rule in presence of supply and demand shocks. Finally, section 6 concludes.

2 Literature review

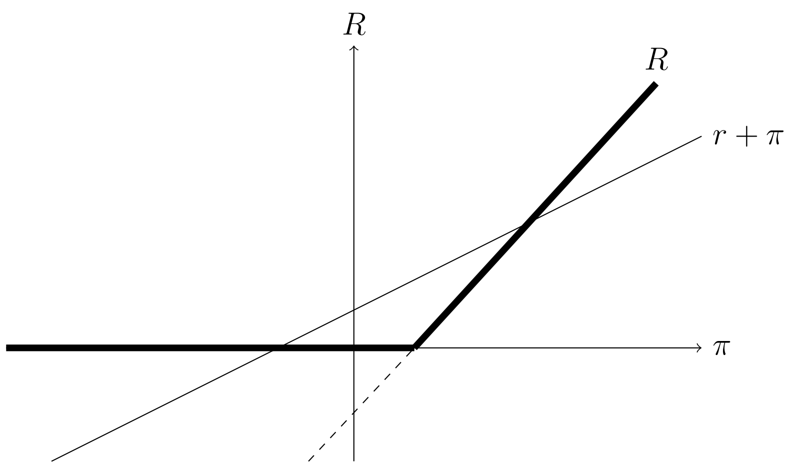
Since the seminal paper of Taylor (1993), feedback rules have become so common in monetary policy discussions and consensual. Central banks of largest developed economies used such rules to implement monetary policy (see Clarida et al. (1998)). Taylor stressed on the stabilizing effect of a feedback rule with an inflation coefficient higher than one, which implies that the monetary authority increases real interest rate whenever inflation increases, and an important part of the literature argued that such rule would contribute to macroeconomic stability since they guarantee the uniqueness of rational expectations equilibrium while feedback rules with a coefficient lower than one, passive rules, have destabilizing effect because the equilibrium become indeterminate[2].

The advocacy for active monetary policy has been challenged by another strand of the literature in two ways. On the one hand, it has been shown that the conclusion about local determinacy depends crucially on the specification of the model. Carlstrom and Fuerst (2001) shows that in a model in which consumption but not investment purchases are subject to a transactions constraint there is real indeterminacy if the monetary authority implements its policy using a forward-looking Taylor rule. Benhabib et al. (2001a) shows that assuming that consumption and real balances are Edgeworth complements ($U_{cm} > 0$) or substitues (($U_{cm} < 0$) is crutial to local determinacy under active policy rule, it also shows that the standard conclusion about local determincy under active policy rules does not hold when money enters the production function.

[2]See Woodford (2003) for more details.

In Benhabib and Eusepi (2005), the authors considered both local and global analysis. They introduced capital into a sticky prices monetary model and showed that an active policy rule may not be enough to guarantee global determinacy. Their main result is that even when an active rule allow the equilibrium to be locally determined (and the nominal interest rate affects marginal costs), global equilibria can be indeterminate because of a possible convergence to a cycle.

On the other hand, the analysis of local dynamics around the targeted steady state denies implicitly the zero lower bound on nominal interest (ZLB). The figure below plots the Fisher relation, $r + \pi$, the nominal interest rate, R, and its lower bound. If the ZLB does not exist, then monetary authority would react by adjusting the nominal interest rate following the Taylor rule for any economic condition (the dashed line). When the zero lower bound is taken into consideration (the thick line), a second steady state arises (the intersection point between the Taylor rule when hit by the ZLB and the Fisher relation) which necessitate the analysis of global and not only local dynamics.



Source: Benhabib et al. (2001b)

The seminal paper Benhabib et al. (2001b) is the seminal paper that analyzed the impact of the ZLB on macroeconomic stability, the important contribution of the paper was to show that it is possible for the economy to start near the targeted steady state and converge towards the liquidity trap steady state. After showing in models with both flexible and sticky prices that the ZLB implies the existence of a second steady state at which the monetary policy is passive, authors analyze both local and global equilibria. They start by proving that the equilibrium is locally determinate around the steady state at which monetary policy is active and indeterminate around the steady state at which the monetary policy is passive. Then they considered the global equilibria and showed that for plausible parametrization there exist an infinite number of equilibrium trajectories originating close to the steady state at which monetary policy is active that converges to that steady state.

The same authors also show in Benhabib et al. (2002b) that, under certain conditions, cycles and chaos (nonperiodic deterministic cycles) exist. They consider a flexible prices model in which the production function depends on real balances and which includes the ZLB on the nominal interest rate. They show that even when the targeted steady state is locally the unique equilibrium, cycles of any periodicity and chaos might exist.

One trivial answer to eliminate this multi equilibria problem, regardless of the possibility of implementing such solution, would be to allow the monetary authority to set negative nominal interest rates since it is due to the ZLB. Interestingly, Schmitt-Grohe and Uribe (2009) showed that even under global Taylor rules, equilibrium liquidity traps can still possible. Authors showed, in a flexible and sticky prices, discrete and continuous time frameworks, that under certain model specifications a continuum of rational expectations equilibria in each of which the nominal interest rate converges to zero and the inflation rate to a smaller rate than the targeted one.

In the spirit of results obtained in Carlstrom and Fuerst (2001) and Benhabib et al. (2001a) about the different impact of backward and forward-looking Taylor rules on macroeconomic stability, Eusepi (2007) used the flexible price of Benhabib et al. (2001a) but backed off the rational expectations assumptions and assumed that agents can learn over time by considering the date produced by the economy. The main result of this paper was the difference between backward and forward-looking Taylor rules on macroeconomic stability. Eusepi showed that under forward-looking Taylor rule, the economy can still converge to the liquidity trap steady state. But when a the monetary authority implements its policy through a backward-looking Taylor rule, then the unique learnable equilibrium is the steady state at which the monetary policy is active.

Benhabib et al. (2001b) did not include an effective policy to avoid such *undesired* outcomes, after its publication several authors tried to engineer policies to avoid liquidity traps. Several policies were suggested to both avoid the economy from falling into a liquidity trap and allow it to benefit from the positive characteristics of the Taylor rule by preserving it. The first attempt was in Benhabib et al. (2002a) where the authors built on their previous paper and suggested both monetary and fiscal policies that would avoid the economy falling into a liquidity trap. They showed that the economy can avoid such outcome if the government threatens to switch fiscal policy by implementing an aggressive enough fiscal stimulus if the inflation rate becomes lower than a threshold. They also showed that a monetary policy switch which follows the Taylor rule when inflation is close to the targeted rate and a money growth rule if it becomes lower can be effective, under the condition that fiscal policy would be non-Ricardian, in eliminating liquidity traps.

Chattopadhyay and Daniel (2013) stresses on the important relation between inflation expectations and central bank's inflation target. The authors suggest a monetary policy switch where the short run inflation target differs from the long run inflation target, they show that if the central bank switches from targeting the long run inflation target to a higher short run inflation target whenever the nominal interest rate becomes lower than a threshold than such policy would prevent the economy from falling into a liquidity trap under the condition that such switch would be highly persistent. Such policy would increases inflation expectations, reduces real interest rate and boost demand enough that the nominal interest rate would rises.

Policies that would avoid allow the economy to escape from a liquidity trap were also studied under learning. In Evans et al. (2008), the authors employ a new Keynesian model based on Benhabib et al. (2001b) but assumed that agents form their expectations using an adaptive forecasting rule. They started by showing that the steady state at which the monetary policy is active is locally stable under learning while the steady state at which the monetary policy is passive is locally unstable under learning. They also showed that if an exogenous shock leads to a strong downward revision of expectations relative to the targeted steady state then paths leading to a deflationary spiral are generated. Then the authors consider both monetary and fiscal policies that would avoid the economy from falling into a liquidity trap, but they show that an aggressive monetary policy is not enough to achieve this goal. They considered a combined aggressive monetary and fiscal policy and show that if the authorities commit to this policy then the targeted steady state is the unique outcome and liquidity traps are avoided. This policy relies on switching to an aggressive policy whenever inflation rate becomes lower than a defined threshold: first, implementing an aggressive monetary policy; second (if this is not sufficient) using fiscal policy to drive up inflation to meet the threshold in question. The value of this threshold is crucial. It should be between the inflation rates consistent with the steady state at which monetary policy is passive and the inflation rate at which monetary policy is active in order for the policy to be effective.

More recently, in Schmitt-Grohe and Uribe (2013), the authors write a model which displays the jobless recovery which characterizes liquidity traps. The main contribution of the paper is the combination between downward rigidities on wages combined to the ZLB on the nominal interest rate. In such framework, if inflation expectations are well anchored then negative shocks on the economy have only temporary impact and recovery is accompanied by job creation. But a negative confidence shock which affect inflation anchoring and lowers inflation expectations would result by a growth rate of wages above the full employment level (because wages are downwardly rigid) and unemployment emerges. This unemployment depresses demand and puts downward pressure on inflation. Nominal interest rate is then lowered which signals a decrease in inflation through the Fisher effect. The authors suggest a policy under which monetary authority pegs nominal interest rate to intended rate[3] (the

[3]Costas Azariasdis and Jess Benhabib argued that such policy would create a pseudo steady state (at

nominal interest rate consistent with the inflation target), they argue that such policy would avoid the economy to fall into liquidity traps. The beneficial impact of this policy comes from allowing real wages to fall as fast as the growth rate of the economy during the transition period when involuntary unemployment persists. Rigidities on nominal wages implies that the only way to decrease real wages is to increase inflation and when the central bank raises the nominal interest rate, it impacts positively inflation through the Fisher effect.

It is now consensual to say that Japanese economy has been in a liquidity trap regime since late 1990s. Since 2009 this experience was a real concern to policymakers in western countries. Bullard (2010) argues that assuming the Japanese experience is due to some cultural or institutional particularities, or that the economy would naturally return to its natural long-run outcome is denial. He stresses on another form of denial: accepting the existence of both steady states but pretending that the long-run outcome of the economy would only be the targeted steady state, because stable dynamics are present only around it. Even if the global analysis in the literature presented above shows a possible convergence towards the liquidity trap steady state, Bullard argues that US data present important facts which would lead to a conclusion that the liquidity trap steady state is locally stable, just like the targeted steady state (our results argues in favor of such conclusion in the case of the euro area). He finally criticizes the policies suggested in the literature as unrealistic and argues in favor of quantitative easing. A major part of the suggested policies is based on implementing an aggressive fiscal policy whenever inflation expectations are beneath a certain threshold, he points to the recent European experience as an example of how unpractical such suggestions are.

In Aruoba and Schorfheide (2013), the authors estimate a New Keynesian DSGE model using U.S. data and contribute to two strands of the literature. First, they consider the impact of the ZLB on the fiscal multiplier. They capture, as suggested by this strand of the

the point of discontinuity) and that the economy might oscillate around that point instead of converging to the intended steady state. See Bullard (2010) for more details.

literature, that the effect of an increase in government spending when the economy is at the ZLB can be substantially larger than one but they obtained smaller multipliers than those found in the literature. They also contribute to the strand of the literature presented in this section by answering if the U.S. economy fell into a deflation regime after the crisis or not. They concluded that a short lived switch, when compared to a more persistent one, to the deflation regime provide more plausible characterizations to the period from 2009 to 2010.

3 Estimating a Taylor rule for the ECB with constant parameters

We begin with a Taylor rule with constant parameters estimated on ex-post data. The rule is written as follows:

$$i_t = \alpha + \beta_\pi \pi_t + \beta_y y_t + \omega_z z_t + \sum_{j=1}^{2} \rho_j i_{t-j} + \epsilon_t \tag{1}$$

with the following definitions of the variables:

i_t : nominal interest rate at time t,

α : a constant,

π_t: inflation rate at time t,

y_t : output-gap at time t,

i_{t-1}, i_{t-2} : interest rate smoothing terms reflecting a gradual adjustment of the interest rate,

z_t: indicator of bond market risk in the money market at time t,

ϵ_t : error term which is assumed to be $iid(0, \sigma_\epsilon^2)$.

The rule links the policy rate to inflation and the output-gap as in many papers. We add the influence of a third factor reflecting the ECBs concern about financial stability. As was evidenced in previous empirical papers, central banks mitigate the fallout of financial busts if they think this is risky for the inflation and output objectives (see Gros and Grauwe

(2009)). The theoretical motivation for the introduction of a bond market risk variable in a Taylor rule is the existence of financial frictions as suggested for instance by Curdia and Woodford (2010).

Equation 1 is the best we succeeded to estimate among several linear specifications, including forward looking interest rate rules. This equation was selected on the basis of information criteria (AIC) and specification tests on the residuals. We ran OLS specifications using monthly data between February 1999 and July 2012. All the series come from the ECB statistical database.

The EONIA rate is used as a measure of the interest rate. For the inflation rate, we consider the average inflation over two consecutive months computed using the HICP monthly rate of change and the output gap is defined as the difference between the log of the volumes GDP and the log of the real potential GDP obtained by applying the Hodrick-Prescott filter to the real GDP series. We use the cubic spline interpolation to transform the quarterly GDP series into monthly ones. The credit risk indicator is the bond market sub-index of ECB's Composite Indicator of Systemic Stress; this sub-index is an aggregation of the realized volatility of the German 10-year benchmark government bond index, yield spread between A-rated non-financial corporations and government bonds (7-year maturity) and 10-year interest rate swap spread. This sub-indexis chosen because it is the best proxy that contains information on credit risk (through the yield spread between A-rated non-financial corporations and government bonds) that was publicly available and covering a reasonably large period of time. The bond market risk indicator is aggregated from daily data by taking the monthly averages.

The results of the OLS estimation are reported in Table 1, along with some specification tests on the estimated residuals. For purpose of comparison, we also report the values of the inflation and of the output-gap obtained in previous studies on Taylor rule estimates for the euro area, as well as the coefficients of the smoothing parameters (Table 2).

OLS estimates		
Variable	Estimation	Std-error†
Intercept	0.068226	0.036152*
$\bar{\pi}_t$	0.19628	0.088913**
y_t	3.0699	0.92173***
z_t	-1.4716	0.39007***
i_{t-1}	1.2651	0.077259***
i_{t-2}	-0.27988	0.076576***
Autocorrelation tests on the residuals (Ljung-Box)		
LB(1)	Statistic: 1.04262	p-value: 0.30721
LB(2)	Statistic: 7.12202	p-value: 0.02841
BDS	Statistic: 1.64	p-value: 0.1

Table 1: Results of the estimation of the linear Taylor rule
†: ***, ** and * denote significance at 1%, 5% and 10% levels, respectively.
When p-value < 0.10, the null *iid* is rejected. $R^2 = 0.99$, p-value vs constant model < 0.0001.

Authors	Inflation coefficient	Output-gap coefficient	Smoothing coefficient	Data and sample
Adema (2004)	1.8	1.72	0.75	Ex-post data 1994 Q1 - 2000 Q4
	1.89	0.46	0.64	Real time data 1994 Q1 - 2000 Q4
Cartensen and Calavecehio (2004)	1.01	1.36	0.95	Real time data 1999 M1 - 2004 M2
Gerdesmeier and Roffia (2005)	1.08	0.7	0.84	Ex-post data 1999 M1 - 2003 M6
	0.39	2.05	0.63	Real time data 1999 M1 - 2003 M6
	1.31	1.95	0.71	Survey data 1999 M1 - 2003 M6
Sauer and Sturm (2007)	-0.84	1.45	0.94	Ex-post data 1999 M1 - 2003 M10
	-0.27	3.01	0.98	Real time data 1999 M1 - 2003 M10
	6.62	9.24	0.98	Forecast data 1999 M1 - 2003 M10
Goster, Jacobs and De Haan (2008)	0.09	0.37	0.95	Ex-post data 1997 M1 - 2006 M12
	1.39	1.52	0.86	Real time data 1997 M1 - 2006 M12
Belke and Klose (2011)	[0.17; 2.98]	[-0.55; 1.42]	[0.53; 0.98]	Real time data 1999 Q1 - 2010 Q2

Table 2: Estimates of the coefficients of inflation rate, the output-gap and the smoothing parameters in some previous studies (Taylor rules in the euro zone)

Our estimates show that the coefficients are statistically significant and carry the expected signs, indicating a positive reaction to inflation and the output-gap and a negative reaction to the credit risk. The so-called Taylor principle is violated since $\beta_\pi < 1$ and the output-gap coefficient is high with a value of 3.07 (as in some previous studies of the literature, see Table 2). A coefficient of the inflation rate below 1 should not necessarily be interpreted as a source of macroeconomic instability and indeterminacy, as discussed in Section 2. And the high coefficient on the output-gap does not imply a high preference for the output-gap, since its captures both the ECBs preferences and the structural determinants of the economy[4]. The estimations also show a sluggish adjustment of the interest rate since the sum of the coefficients ρ_j,(j=1,2) is near 1. This supports previous findings in the literature (see Table 2). However, as discussed in Section 4, the interest rate smoothing is not a robust finding once the nonlinearity of the interest rate rule is accounted for. This means that the persistence of the interest rate hide a problem of specification.

Table 2 shows that, there is a high dispersion in the estimates of the inflation rate and output-gap, between ex-post and real time data, and also across real time and ex-post data themselves when one uses linear Taylor rule to account for the ECBs interest rate rule. Given the high dispersion of the estimates, it is difficult to form a judgment about the adequacy of our estimated rule in the sense that different estimates yield different conclusions.

At first sight, Figure 1 suggests that the interest rate estimated by the rule is consistent with the actual path of policy rates. Indeed, on average the estimated rule seems to overlap the historical data and the EONIA displays an adjustment towards the Taylor rule.

However, looking at Table 1, the specification tests on the residuals suggest that there are still some unexplained correlations in the residuals, as shown by the Ljung-Box and BDS tests. However, the motivations that lead us not to retain this rule for purpose of policymaking are not statistical. Our point is rather that, from a policy viewpoint, the linear rule is neither transparent to communicate, nor simple to verify by the private sector. Transparency and communication are the two pillars which make the Taylor rule a tool for

[4]The reader can refer to Hayo and Hofman (2006) for a discussion of the interpretation of the output-gap coefficient in Taylor rules.

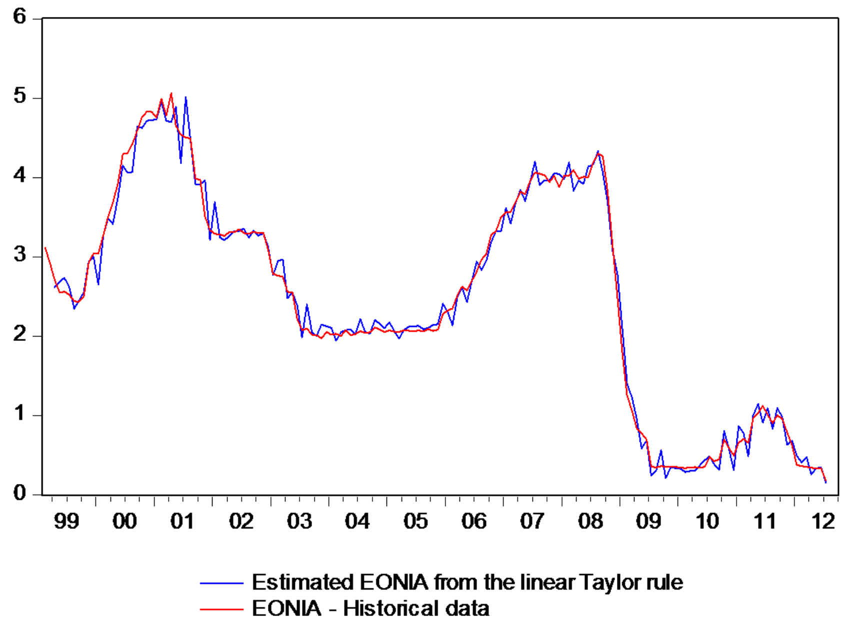


Figure 1: Historical and estimated interest rate (linear rule)

central bank, in the sense that they can anchor inflation expectations by telling the private agents what they are doing and by sticking to the announced rule. Figure 1 shows that there has been some changes in the level of EONIA several times and the data suggests regime switches. For instance, since 2008, the interest rate has decreased at low levels, which was not the case before the financial crisis. We also observe that the nominal rule has been kept stable during the years 2002, 2004-2005 and between mid-2007 and mid-2008. Conversely, significant cuts or increases have been observed at other periods. This suggests that the changes in the rates have been very incremental during some years, but have been characterized by rapid switches during other years. The linear specification does not tell us why. An equation involving multiple regimes for the interest rate would be more informative. Further, there are no reasons why, subject to the existence of several regimes in the setting of monetary policy, the reaction of the interest rate to inflation rate, output-gap and credit risk should remain the same.

The next section presents examples of such rules which adequately describe the dynamics of the EONIA between 1999 and 2012.

4 Taylor rules with multiple regimes: examples and modeling

4.1 Example 1

In the wake of the recent great recession, central banks in the industrialized countries have turned to the so-called unconventional monetary policies which involved an expansion of their balance sheet (quantitative easing). Their aim was to influence the market rates through channels that differ from the classic open-market channel. The expansion of the ECBs balance sheet was the result of provision of long-term loans in exchange of collateral (bank loans instead of the usual government bonds). The objective was to alleviate the raising funding risk in the banking sector. The ECBs policy was designed to set the interest rate with awareness of what increase in the quantum of monetary base was necessary to support the euro-area economy[5].

Consider the ECBs balance sheet as an advanced indicator of the monetary authorities preferences for inflation, the output-gap or credit risk. For instance, in a context of weakness of the economic recovery, an increase in the monetary base can be an indication that the ECB is willing to give a higher importance to the output-gap objective than in *normal* times. In this case, the optimal response of the central bank to changes in the explanatory variables can be studied using a nonlinear Taylor rule by making the parameter of this rule a function of the ECBs balance sheet:

$$i_t = \alpha(b_\tau) + \beta_\pi(b_\tau)\pi_t + \beta_y(b_\tau)y_t + \omega_z(b_\tau)z_t + \sum_{j=1}^{2} \rho_j(b_\tau)i_{t-j} + \epsilon_t \tag{2}$$

where b_τ is the ECBs balance sheet at time τ. If the central bank seeks to communicate

[5] Among the numerous papers that have studied the unconventional monetary policy followed by the ECB, we refer the reader to Giannone et al. (2012), Lenza et al. (2010), Peersman (2011).

about its policy action using this rule, the latter can be useful to inform the private sector and the public about two regimes: a *normal* regime and an *exceptional* regime. In normal times, the central bank recur to the interest rate as its policy instrument to stabilize the inflation rate and the output-gap. However, during some exceptional periods, for instance the periods following a great recession or a financial crisis, the ECB uses its balance-sheet as an additional instrument to implement its policy and this in turn affects the response of the interest rate to the macroeconomic variables. The delimitation between the two regimes can be captured by a threshold function:

$$\theta(b_\tau) = \begin{cases} \theta^1, \text{ if } b_\tau < c\text{: normal regime} \\ \\ \theta^2, \text{ if } b_\tau \geq c\text{: exceptional regime} \end{cases} \tag{3}$$

where $\theta = (\alpha, \beta_\pi, \beta_y, \omega_z, \rho_1, \rho_2)$. c is a threshold value of the ECBs balance sheet. The transition between the regimes in (3) is assumed to be sharp and to be described by an indicator function:

$$\theta_j(b_\tau) = I(b_\tau < c)\theta_j^1 + [1 - I(b_\tau < c)]\,\theta_j^2 \tag{4}$$

The index j denotes one element of the vector θ. I is the Heaviside function. Smoother responses can be allowed for by considering a continuous function f:

$$\theta_j(b_\tau) = f(b_\tau < c)\theta_j^1 + [1 - f(b_\tau < c)]\,\theta_j^2 \tag{5}$$

with this formulation, the Taylor rule indicates that the response of monetary policy is variant across time and regimes.

4.2 Example 2

In Equations 3, 4, 5, instead of the ECBs balance sheet, we can consider the credit risk in bond markets, z_τ. There are several reasons why this variable causes switches in the response of the interest rate to the macroeconomic variables.

Firstly, the risk-taking behavior of the banking sector and of investors in bonds markets drive the business cycle and can be considered as an advanced indicator of future economic activity. Increased leverages are usually associated with beliefs of forthcoming periods of prosperity. Conversely, shifts to less risky assets signals a forthcoming vulnerability of the real activity through a lending channel (credit conditions are characterized by tightening lending conditions that can be a factor of economic recessions[6]). This situation can lead the monetary authorities to react asymmetrically to output-changes depending upon whether credit risks signal loose or tight credit conditions (by cutting the interest rates in times of reduced risk, but not reacting in good times when increased risks reflect optimistic behaviors in the financial markets).

Secondly, risk-taking behaviors in financial markets induce changes in the policy interest rate through two channels. The direct channel is the liquidity channel. In times of turmoil, characterized by an increased risk, the ECB must ensure that the markets are not gripped by illiquidity problems and this leads to a decline of the short-term interest rate. This reaction is captured by the coefficient ω_z in Equation 1. There is also an indirect channel. Indeed, risk-taking behaviors are a source of bubbles and financial instability. The monetary authorities take into account these risks if they believe that the latter can have large negative effects on the economic activity and price stability. In this case, the coefficients β_π and β_y in equation (1) are conditioned by the level of credit risk in the economy. The values of these coefficients should differ when the observed changes in credit risks signal a forthcoming financial market slump and when the dynamics of inflation and output-gap are unrelated to the agents risk-taking behavior in equity and bond markets.

[6] See Bhattacharya et al. (2011), Lown and Morgan (2006).

4.3 Example 3

The third example refers to the case of endogenous monetary policy switching as defined by Davig and Leeper (2006). In this case, instead of b_τ in equations (4) and (5), the variable influencing the reactions of the ECB can be the interest rate itself. The interpretation of the Taylor rule is accordingly the following. Not only do changes in the economic variables (inflation, output, risk in the bond markets) drive monetary policy regimes. The latter also depends upon the ECB's past time preferences towards these variables. Such a rule is called a *self-exciting* Taylor rule and can be useful is the monetary authorities worry about the Lucas criticism: when the economic environment is modified, the private sector change the way in which it uses the Taylor rule to make decisions and the central bank in turn reacts to these changing behaviors by adjusting its own policy.

4.4 Regime-switching Taylor rules for the ECB : estimation and results

We estimate LSTR (logistic smoothed transition) equations on the EONIA to account for regime-switching dynamics. They encompass the linear rule as a particular case[7]. The estimated rule is as follows:

$$i_t = x_t^{'}\beta_1 + \sum_{m=2}^{M} x_t^{'}\beta_m f(l_t, \gamma_m, c_m) + \epsilon_t \tag{6}$$

where

$$f(l_t, \gamma_m, c_m) = \frac{1}{1 + exp(-\gamma_m(l_t - c_m))} \tag{7}$$

[7]This class of models was proposed in the early 1990s. For details, the reader can refer to v. Dijk et al. (2002), v. Dijk et al. (2002), Hillebrand et al. (2012). For application to monetary policy modeling, we refer the reader to Alcidi et al. (2011), Dufrenot et al. (2004), Jawadi and Sousa (2013).

l_t is the transition variable that determines which regime is activated, γ_m is the speed parameter of regime m, it allows comparison of how fast transition occurs when having multiple non-linear regimes, c_m is the threshold of the transition variable that identifies the transition for the non-linear regime m and $x_t^{'} = (1\ \bar{\pi}_t\ y_t\ z_t\ i_{t-1}\ i_{t-2})$ contains the variables defined above. It is clear that $f(l_t, \gamma_m, c_m) \to 1$ when $l_t \to \infty$ and $f(l_t, \gamma_m, c_m) \to 0$ when $l_t \to -\infty$ which is why l_t is called the transition variable and how a continuous transition between regimes is introduced by LSTR models.

The choice of the transition variables rely on the arguments provided before in our three examples. However, the assumptions according to which the lagged values of the interest rate, the ECBs balance sheet and credit risk contain information to be considered as advanced indicators of the changing reactions of the interest rate in the Taylor rule need to be tested formally. To begin with, F-test type are applied to test the null hypothesis of linear Taylor rule against alternative LSTR specifications with two regimes. Then, if the null is rejected, additional nonlinearity tests are done by testing n against alternative $n+1$ nonlinear regimes.

Once the assumption of linear Taylor rule is rejected against the alternatives of LSTR specifications, we then estimate nonlinear Taylor rule. For technical details about the testing and estimation methodology, the reader can refer to the references in footnote 7. To the extent that these methodologies are now widespread in the applied literature, we skip their presentation in this paper. We simply notice two points. Firstly, as is usually the case, the Fisher test is based on a third-order expansion of the logistic function to overcome the known problem of nuisance parameter. Secondly, the estimation of the model is based on the maximum likelihood estimator.

Table 3 contains the results of the linearity tests. In the first column, we report the number of regimes tested under the alternative hypothesis. Columns 2 till 4 contains the p-values (significance level) of the Fisher tests. The assumption of a linear Taylor rule should be rejected at the 5% or 10% level of significance if the reported values lie below 0.05 or 0.1.

When the transition variables are the ECBs balance-sheet and the credit risk variables, the tests concludes to a two-regime model, while a three-regimes model seems to be adequate when the lagged interest rate is considered as the transition variable.

Tables 4, 5 and 6 report the estimations and some tests on the estimated residuals. We first give general comments on the main implications from the estimations and then we discuss in deeper details the different regressions.

	Transition variables		
Number of regimes	i_{t-1}	u_t	z_t
2 regime	<0.001	<0.001	0.017
3 regimes	0.05	0.21	0.29
4 regimes	0.8		

Table 3: P-values of the non linearity tests of a linear Taylor rule against regime-shift Taylor rules

	Linear coefficients (L)		**Non linear coefficients (NL1)**		**Non linear coefficients (NL1)**	
	Estimate	Std-Error	Estimate	Std-Error	Estimate	Std-Error
Intercept	0.2271	0.0764***	-0.0859	0.1273	1.0867	0.6474**
$\bar{\pi}_t$	-0.0250	0.1473	0.3557	0.1845**	-0.5036	0.2586
y_t	3.6175	1.3264***	0.1700	1.8253	-1.7146	3.9949
z_t	-2.6715	0.7393	1.8470	1.0279**	-4.0803	1.7756
i_{t-1}	0.8871	0.1681***	0.5202	0.1945***	-0.6802	0.2305
i_{t-2}	0.0322	0.1539	-0.4901	0.1801	0.5362	0.1932***
γ			50	0***	30	0***
c			2.2048	0***	4.0644	0***
Specification tests on the residuals:						
LB(1) = 1.3918					p-value: 0.2381	
LB(2) = 4.4794					p-value: 0.1064	
BDS test = 2.921					p-value: 0.00348	

Table 4: Estimation of the nonlinear Taylor rule with the interest rate as the trasition variable

†: ***, ** and * denote significance at 1%, 5% and 10% levels, respectively. When p-value < 0.10, the null of iid is rejected.

	Linear coefficients (L)		Nonlinear coefficients (NL)	
	Estimate	Std-Error	Estimate	Std-Error
Intercept	0.0807	0.049*	0.082	0.0962
$\bar{\pi}_t$	-0.046	0.1054	0.244	0.1898
y_t	1.733	1.0507*	3.641	1.9716**
z_t	0.514	0.7933	-2.339	1.3511
i_{t-1}	1.207	0.0829***	-0.405	0.1911
i_{t-2}	-0.235	0.0830	0.305	0.1832**
γ			50	81.241
c			1.442	0.0372***
Specification tests on the residuals:				
LB(1) = 1.0204				p-value: 0.3124
LB(2) = 11.5673				p-value: 0.00308
BDS test = 0.3631				p-value: 0.7164

Table 5: Estimation of the nonlinear Taylor rule with ECB's balance sheet as the trasition variable

†: ***, ** and * denote significance at 1%, 5% and 10% levels, respectively. When p-value < 0.10, the null of iid is rejected.

	Linear coefficients (L)		Nonlinear coefficients (NL)	
	Estimate	Std-Error	Estimate	Std-Error
Intercept	0.0836	0.0443**	-0.0628	0.1644
$\bar{\pi}_t$	-0.1640	0.1216	0.6899	0.1904***
y_t	2.6352	1.0860***	-1.5276	2.2913
z_t	-0.0058	0.7147	-1.6887	1.6150
i_{t-1}	1.1559	0.0986***	0.1960	0.1950
i_{t-2}	-0.1709	0.0974	-0.1909	0.1908
γ			50	82.2702
c			0.6933	0.0407***
Specification tests on the residuals:				
LB(1) = 0.23696				p-value: 0.6264
LB(2) = 4.81006				p-value: 0.0902
BDS test = 1.5043				significance level: 0.1324

Table 6: Estimation of the nonlinear Taylor rule with the bond market risk as the trasition variable

†: ***, ** and * denote significance at 1%, 5% and 10% levels, respectively. When p-value < 0.10, the null of iid is rejected.

4.4.1 Switches in the interest rate and threshold values

Figure 2, which plots the EONIA rate and the estimated thresholds, shows three regimes in the ECB policy. The first regime begins in mid-2000 and ends by the end of the year 2001; it coincides with the Dot-Com bubble bust. Financial disruption that occurred in the euro area during this period explains the high rate regime. The low rate regime is triggered after the beginning of the financial crisis at the end of 2008 which confirms the concern of the ECB about reaching its policy rate effective lower bound. The third regime can be interpreted as a normal regime when the euro area is not facing major economic disruptions.

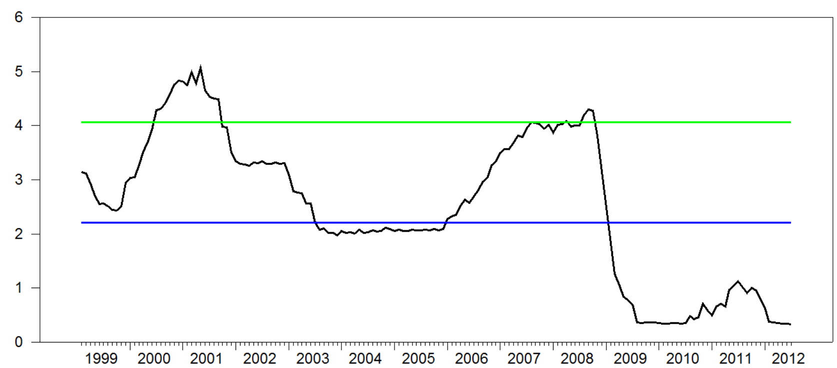


Figure 2: EONIA rate, in percent, and the estimated thresholds

As mentioned above, after the crisis policy rates reached and got stuck at their effective lower bounds in many economies. Like many central banks, the ECB reacted by using its balance sheet as a new instrument to implement its monetary policy. This use makes the ECB's balance sheet a suitable candidate to detect any regime switch motivated by the concern of interest rate hitting the effective lower bound. By looking at Figure 3 one can identify the timing of regime change which coincides with the beginning of the financial crisis. It is interesting to note here that using the lagged interest rate (Figure 2) or the ECB's balance sheet (Figure 3) predict both the same timing of switch in monetary regimes motivated by the ECB's concern about interest rate hitting the effective lower bound.

Figure 4 plots the variable z_t and the estimated threshold related to it. As mentioned

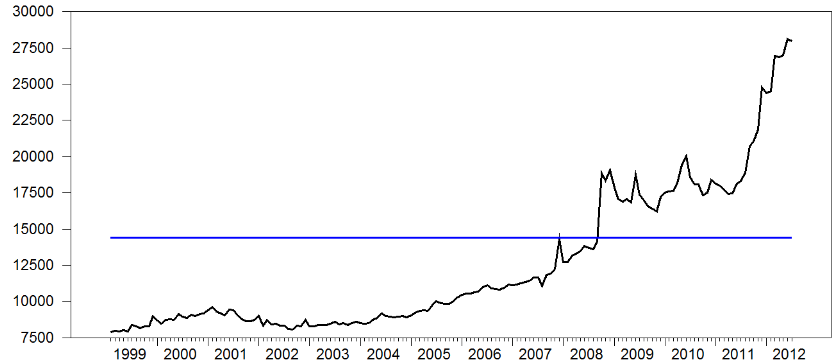


Figure 3: ECB's total assets and the estimated threshold

before, this indicator aggregates several economic features. One should be cautious when interpreting regime switches related to it because it contains information about financial health and lending in the economy but also information related to the European sovereign debt crisis. Only periods starting from mid-2007 to mid-2009 and from mid-2011 to the beginning of 2012 will be interpreted here since there is relatively no ambiguity about their inferred regime and they allow discussing both information stated above that are included in the indicator. On the one hand, in mid-2007 financial stress was a significant concern to the ECB and relatively stable situation on the sovereign debt market argue in favor of considering that the financial stress is the information contained in the bond market risk indicator that counts in this period. On the other hand, as pointed out in Eser et al. (2012), the negative development of the sovereign debt crisis in mid-2011 was ECB's major concern during the second period pointed out above. One can argue that information about sovereign debt has more importance during this period.

As a whole, it can be concluded from the different figures that the ECB has switched policy regimes after the beginning of the crisis. Indeed, the estimated thresholds for all transitions variables, despite differences between the economic information that they contain, show a switch in policy regimes after the beginning of the recent financial crisis. We would also like to stress that the results from using the bond risk indicator as the transition variable show that the ECB has changed its monetary regime to answer the concern of a rise in credit

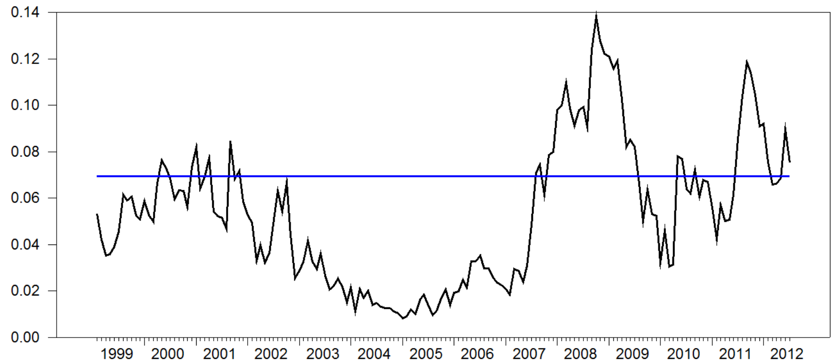


Figure 4: ECB's bond market risk indicator and the estimated threshold

spreads.

4.4.2 Regime-shifts in the EONIA when lagged interest rate is the transition variable

To write the nonlinear Taylor rule corresponding to the estimations in Table 4, we need to re-parametrize the model. This yields the following three-regime rule:

$$i_t = \begin{cases} 0.23 - 0.02\pi_t + 3.62y_t - 2.67z_t + 0.88i_{t-1} + 0.03i_{t-2}, \text{ if } i_{t-1} < 2.2 \\ 0.15 + 0.33\pi_t + 3.78y_t - 0.83z_t + 1.4i_{t-1} - 0.46i_{t-2}, \text{ if } 2.2 < i_{t-1} \leq 4.06 \\ 1.23 - 0.17\pi_t + 2.07y_t - 4.91z_t + 0.72i_{t-1} + 0.08i_{t-2}, \text{ if } i_{t-1} > 4.06 \end{cases} \tag{8}$$

The estimated equation manages to identify three regimes of respectively, low, medium and high interest rates. The first regime corresponds is identified when the interest rate crosses a threshold value of 2.2%. We have seen, in Figure 2, that this corresponds to a situation when the interest rate fell near a zero low bound (ZLB). In this regime, the ECB disregards the inflation objective (very low coefficient) but responds to output fluctuation and bond market risk. In this regime, the interest rate is very sluggish in comparison with the other two regimes. The third regime can be interpreted as one in which the ECB consider the economy in danger of facing a credit risk, thereby implying a strong cut in the interest rate (the coefficient is 4.91 for z_t, much higher than for the other two regimes). In the intermediate regime, the ECB seems to pay a lower attention to the bond market risk (the smallest coefficient for z_t is obtained for this regime) but focuses on its traditional goal, namely a trade-off between inflation and output stabilization.

4.4.3 Regime-shifts in the EONIA, ECB's balance sheet and bond market risk

Using the estimates in Tables 5 and 6, we obtain the following nonlinear Taylor rules, respectively when the ECBs balance sheet and the bond market risk are used as the transition variables:

$$i_t = \begin{cases} 0.08 - 0.05\pi_t + 1.73y_t + 0.51z_t + 1.22i_{t-1} - 0.24i_{t-2}, \text{ if } ECB_t < 14400 \\ 1.16 + 0.2\pi_t + 5.3y_t - 1.8z_t + 0.08i_{t-1} + 0.07i_{t-2}, \text{ if } ECB_t \geq 14400 \end{cases} \tag{9}$$

$$i_t = \begin{cases} 0.08 - 0.16\pi_t + 2.63y_t - 0.006z_t + 1.16i_{t-1} - 0.17i_{t-2}, \text{ if } z_t < 0.07 \\ 0.02 + 0.52\pi_t + 1.15y_t - 1.68z_t + 1.34i_{t-1} - 0.36i_{t-2}, \text{ if } z_t \geq 0.07 \end{cases} \tag{10}$$

The first rule tells us that the ECB can manage the size of its balance sheet and use the interest rate to implement its monetary policy. The threshold can be considered as a quantitative target which serves as a benchmark to determine the conditions in which the central bank reacts to changes in the macroeconomic and financial environment. As we noticed in Figure 3, the second regime ($ECB_t \geq 14400$) coincides with the timing of a low interest rate policy. In this regime, the interest rate adjusts freely to achieve monetary policy objectives (the sum of the autoregressive terms is only 0.15) and that is in sharp contrast with what happens in the other regime where the sum of the autoregressive terms is near 1. This calls for cautious about the interpretation of the equation in this regime. When monetary policy takes place in a context of large balance sheet ($ECB_t \geq 14400$), the observed movements of the interest rate in reaction to inflation, output or credit risk is not necessarily the result of a discretionary interest rate policy. Rather, the estimated equation distinguishes between a regime of active Taylor rule ($ECB_t < 14400$) and inactive Taylor rule ($ECB_t \geq 14400$). One reason is that, there are several interest rates: the lending rate, the deposit rate, the targeted rate which is here the EONIA. When the central bank has no quantitative objective (the balance sheet remains below a given level), the targeted rate is the main objective of the monetary policy and money supply is adjusted to achieve it, in regard to the macroeconomic determinants of the demand for reserves (inflation, output, liquidity). In the other regime, the central bank searches to activate other transmission channels than the standard interest rate channel to the activity. For this reason, it uses other tools (credit easing, quantitative easing or the interest rate paid on reserves). In the case of the ECB, the excess reserves held on banks accounts at the central bank do not earn any interest. For this reason in the second regime, the interest rate moves freely.

The second rule says that the ECB is concerned with the effects of a higher or lower risk on the economy (for instance a possible deflationary and depressing effects of a higher liquidity risk) and that it adapts its policy accordingly. Specifically a higher attention is paid to the risk when it increases (as suggested by the higher coefficient of -1.68 in the second regime which is much higher than 0.006 in the first regime) and the central bank would tend to intervene more strongly to smooth changes in inflation.

5 Macroeconomic implications of nonlinear Taylor rules

What are the implications of the nonlinear Taylor rules for macroeconomic policymaking? There is a rapidly growing literature on this topic. Studies suggest that nonlinear interest rate rules give rise to multiple macroeconomic equilibria in monetary economies (seminal papers are Benhabib et al. (2001b), Benhabib et al. (2002a)). A central point of the discussion is whether all the equilibria are desirable outcomes and whether they are meaningful.

Until now, the policy responses are diverse. A typical *undesirable* outcome is for instance a situation of low nominal interest steady state with a large *basin of attraction* yielding for instance to a long situation of deflationary spiral as was observed in Japan (see Bullard (2010) for a discussion). Some authors consider that in general the co-existence of several locally stable steady states is not a desirable result and propose to make some assumptions to obtain only one stable steady state. For instance the introduction of a learning process can yield the agents to choose the *good* equilibrium (see, Evans et al. (2008)). It is also possible to eliminate equilibria like liquidity trap, by introducing an expansive fiscal policy (Benhabib et al. (2002a), Eggertsson and Woodford (2004)). An important strand of the literature also points to the meaning of multiplicity, specifically when nonlinear Taylor rule explain the existence of endogenous cycles in monetary economies (see Airaudo et al. (2012), Benhabib et al. (2002b), Eusepi (2007)).

To study some of the implications of the nonlinear Taylor rules for the euro area monetary economy, we proceed by doing a simple exercise aiming at showing several features. We consider the rule estimated when the lagged interest rate is the transition variable. Firstly, there are several macroeconomic equilibria, among which two steady-states. One corresponds to a situation of a *near* liquidity trap and the other to a *normal* situation with an intermediate level of the nominal interest rate. This is shown by considering the framework of a simple Keynesian model. Secondly, we worry about the issue of moving from one equilibrium to the other. It is important to know under which circumstances the European economy is trapped in an undesired outcome (for us a situation of ZLB[8]) or whether shocks that are of an important size can help to avoid the equilibrium. We consider two types of shocks, supply and demand, and compare their respective ability in driving the economy towards an equilibrium or the other.

[8]In such situation, the conventional monetary policy loses its ability to react to negative shocks and the financial system's health might be endangered because many financial contracts are stated in nominal terms. See Bullard (2010) for a further discussion.

5.1 Nonlinear interest rate rule and multiple macroeconomic equilibria

As is seen from our estimation, the Taylor principle is always violated. This raises issues about the possibility of multiple equilibria if the Taylor rules are embedded into a macroeconomic model. The literature has extensively discussed one aspect of the multiplicity of equilibrium, namely the fact that sunspot equilibria exist and are not ruled out when there is no credibility that the central banks reaction to different shocks will be strong enough to make the agents coordinate their expectations to one fundamental equilibrium. We do not discuss this here. Another implication of the violation of the Taylor principle is the possibility of several fundamental equilibria when the economy can switch between several stable steady states defined in different regimes.

To illustrate this, we consider a simple model that consists of a Fisher equation and the nonlinear Taylor rule in which the lagged interest rate is the transition variable. This rule is taken for purpose of illustration and a similar analysis could be done by considering the other two rules in Section 4.

5.1.1 Derivation of the optimal interest rate

The Fisher equation is not considered in an ad-hoc manner, but derived from the interpretation of a new-Keynesian model[9]. Assume that the central bank minimizes the following intertemporal loss function:

$$
\begin{gathered}
E_t \sum_{\tau=t}^{+\infty} \beta^{\tau-t} V(\pi_t, y_t, z_t), \\
\text{where } V(\pi_t, y_t, z_t) = \frac{1}{2}\left[\lambda_1(\pi_t - \pi_t^*)^2 + \lambda_2(y_t - y_t^*)^2 + \lambda_3(z_t - z_t^*)^2\right]
\end{gathered}
\tag{11}
$$

where π_t, y_t, z_t are respectively the inflation rate, the output-gap and the ECBs bond risk

[9]For a similar interpretation, see Barnea and Liviatan (2011), Cochrane (2011), Walsh (2010).

indicator. A star indicate the targeted values of these variables. E_t denotes the expectation taken at time t and $0 < \beta < 1$ is the discount factor.

We also consider an augmented Phillips curve with forward looking expectations:

$$\pi_t = E_t \pi_{t+1} + \kappa_1 y_t + \kappa_2 z_t + u_t \tag{12}$$

u_t is a stochastic supply shock and κ_1 captures the degree of price stickiness.
We further introduce the following (IS) curve:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t) + v_t \tag{13}$$

i_t is the nominal interest rate and r_t the Wicksellian natural rate of interest defined as the real interest rate consistent with full employment in the absence of demand shocks. v_t is a stochastic demand shock and σ represents the intertemporal elasticity of substitution.

Equations (12) and (13) have been extensively discussed in the literature and are derived from micro-based macroeconomic models with representative agents adopting intertemporal decisions and setting prices la Calvo (see, for instance Woodford (2003)).

To obtain the first-order conditions (FOC) with respect to the inflation rate, the output-gap and the bond-risk, we write the Lagrangian:

$$L = \sum_{\tau=t}^{+\infty} \beta^{\tau-t} V(\pi_t, y_t, z_t) + \sum_{\tau=t}^{+\infty} \phi_\tau \left[\pi_\tau - E_\tau \pi_{\tau+1} - \kappa_1 y_\tau - \kappa_2 z_\tau - u_\tau\right] \tag{14}$$

ϕ_τ is a Lagrangian multiplier. We rewrite the Lagrangian as follows:

$$L = \sum_{\tau=t}^{+\infty} \beta^{\tau-t} V(\pi_t, y_t, z_t) + \sum_{\tau=t}^{s-2} \phi_\tau \left[\pi_\tau - E_\tau \pi_{\tau+1} - \kappa_1 y_\tau - \kappa_2 z_\tau - u_\tau\right]$$

$$+\phi_{s-1}\left[\pi_{s-1} - E_{s-1}\pi_s - \kappa_1 y_{s-1} - \kappa_2 z_{s-1} - u_{s-1}\right] + \phi_s\left[\pi_s - E_s \pi_{s+1} - \kappa_1 y_s - \kappa_2 z_s - u_s\right]$$

$$+ \sum_{\tau=s+1}^{+\infty} \phi_\tau \left[\pi_\tau - E_\tau \pi_{\tau+1} - \kappa_1 y_\tau - \kappa_2 z_\tau - u_\tau\right] \tag{15}$$

The FOC are the following:

$$\begin{aligned} &\text{w.r.t. } \pi_s : \beta^{s-t}\lambda 1(\pi_s - \pi^*) - \phi_s + \phi_{s-1} \\ &\text{w.r.t. } y_s : \beta^{s-t}\lambda_2 y_s - \phi_s \kappa_1 \\ &\text{w.r.t. } z_s : \beta^{s-t}\lambda_3 (z_s - z^*) - \phi_s \kappa_2 \end{aligned} \tag{16}$$

The combination of these three equations yields the following equality:

$$\pi_t - \pi^* = \frac{-1}{\lambda_1(\kappa_1 + \kappa_2)}\left[\frac{\lambda_2}{\beta}(y_{t-1} - y^*) - \lambda_2(y_t - y^*) + \frac{\lambda_3}{\beta}(z_{t-1} - z^*)\right] \frac{\lambda_3}{\lambda_1(\kappa_1 + \kappa_2)}(z_t - z^*) \tag{17}$$

Solving for i_t in the (IS) equation yields:

$$i_t = \sigma^{-1}\left[E_t y_{t+1} - y_t\right] + E_t \pi_{t+1} + r_t - \sigma^{-1} v_t \tag{18}$$

At steady state $v_t = 0$, $E_t y_{t+1} = y_t$ and we assume that the ECB reaches its targets. We thus have

$$i_t = E_t \pi_{t+1} + r_t \tag{19}$$

This equation is a modified Fisher equation stating that, at the steady state, the nominal interest rate equals the real interest rate (wickesellian natural rate) plus expected inflation. At the steady state, the optimal real interest rate should equal the Wicksellian real natural interest rate. This gives a path of the interest rate the monetary authorities should set if they search to minimize the intertemporal loss function (11) subject to the macroeconomic constraints represented by the IS and Phillips curves. The macroeconomic equilibrium is obtained by crossing this relationship with the Taylor rule followed by the central bank.

From (17) if $\beta \to 1$, (19) implies

$$i_t = r_t + \pi^* \tag{20}$$

Thus, when the discount rate tends to 1, the existence of several steady states due to the regime-switching dynamics of the interest rate can be interpreted as a consequence of switches in the long-run natural rate and or as a consequence of switches in the ECBs inflation target.

5.1.2 Macroeconomic equilibria

In Figure 6, we plot the regime-switching Taylor rule when the lagged interest rate is considered as the transition variable (red curve) and a Fisher equation estimated for each of the three regimes (blue curve). Both curves relates the inflation rate to the EONIA. In the Fisher equation, we have taken the real interest rate to be constant. A typical characteristic of our nonlinear Taylor rule is the discontinuity of the red curve due to the presence of switches between the different regimes. The two curves cross at three points representing a macroeconomic equilibrium in each regime. This is the interest rate corresponding to an optimum at the steady state: if there are no shocks, the monetary authorities do no longer wish to cut or raise the interest rate.

The first equilibrium corresponds to the point (1%;-0.2%) that is a situation of negative yearly inflation rate around 2.4% and a low nominal interest rate. This illustrates a context of *near* liquidity trap (the interest rate is small, but not zero) in an environment of deflation. At the other extreme we have a situation of high interest rate with a high interest rate, since the two lines cross at the point (4.8%; 0.5%): the interest rate is set at a high level to cope with an average yearly inflation of 6%. Finally, there is a third equilibrium corresponding to an intermediate level of the interest rate (3.1%) and where the inflation rate reaches a monthly level of 0.2% (that is on average 2.4% per year). As a conclusion, there are potentially three targeted interest rate corresponding to the different regimes. An important question concerns the stability of these equilibria: can one expects the economy to remain for a long time at these equilibria if small disturbances occur near the steady states.

The stability properties can be studied by comparing the slopes of the Taylor rule and of the fisher relation at the different steady states. Formally, we consider the following relationships within each regime j:

$$i_t = a_1^j E_t \pi_{t+1} + b_1^j + u_t^j \quad \text{and} \quad i_t = \theta_0^j + \theta_1^j \pi_t + \theta_2^j y_t + \theta_3^j z_t + v_t^j \tag{21}$$

This yields the following equation for the inflation rate:

$$\pi_t = \pi_0 \left(\frac{\theta_1^j}{a_1^j} \right)^t + \sum_{j=1}^{t} \left(\frac{\theta_1^j}{a_1^j} \right)^{j-1} x_{t-j} + \sum_{j=0}^{t-1} \left(\frac{\theta_1^j}{a_1^j} \right)^{j-1} \delta_{t-j} \tag{22}$$

where $x_t = \frac{\theta_0^j - b_1^j}{a_1^j} + \frac{\theta_2^j}{a_1^j} y_t + \frac{\theta_3^j}{a_1^j} z_t$. δ_t is a stochastic noise with a zero mean and a finite variance (which combines the supply and demand shocks and the error-term of the inflation rate rational expectation). The solutions of this equation are locally non-explosive if $\theta_1^j < a_1^j$. Using the different estimates, we obtain the following conclusions[10]:

The ECB regime-switching Taylor rule thus rules out hyperinflationary solutions, but any of the other two steady state equilibria are possible: low and intermediate interest rates. The first equilibrium corresponding to a situation of low interest is not necessarily undesirable at present time, because the steady state interest rate is still not at zero and could accordingly be changed to accommodate any further negative shocks in the euro area (cutting the interest

[10]We do not discuss here the issue of sunspots equilibria which may exist within each regime.

	Fisher equation a_1^j	Taylor rule θ_1^j	Conclusion
Regime 1 $i_{t-1} \leq 2.2$	0.49	0.22	Locally stable
Regime 2 $2.2 < i_{t-1} \leq 4.06$	1.63	0.84	Locally stable
Regime 3 $i_{t-1} > 4.06$	0.65	0.81	Explosive

rate can be proved effective until the zero low bound is reached). In the current context of a moderate deflation in the euro area, this equilibrium is at least as desirable as the other locally steady state.

However, the monetary authorities in Europe hold onto the belief that this situation should be transitory and their objective for the medium term is to reach the steady state that was prevailing before the financial crisis, which corresponds here to the steady state with moderate inflation. Moreover, like other central bankers, the ECB would like to avoid a situation in which the euro area would undergo a deflationary trap as was observed in Japan over the last decade. So, although, the equilibrium with a low interest rate may be desirable for the moment, the authorities are interested by shocks that may move the economy to the second stable steady state. Theoretically, this issue refers to global determinacy and has been discussed extensively in the literature (see the references at the beginning of this section). We focus the discussion here on the policy implications for our case. For this purpose, we conduct different experiments by examining the impulse response functions of the inflation rate, output-gap and inflation rate to supply and demand shocks.

5.2 Shocks and regime-switching equilibria

We now turn to a different exercise. To gain some insight into the dynamic properties of our nonlinear Taylor rules, we conduct impulse response analysis and analyze local equilibria in each regime. The model considered here consists of three simple equations: a demand curve, a Phillips curve and the nonlinear rule with the interest rate as the transition variable[11]:

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma^{-1}(i_t - E_t\pi_{t+1} - g_t) && (23)\\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t + u_t && (24)\\
i_t &= \theta_0^j + \theta_\pi^j \pi_t + \theta_y^j y_t + \rho_1^j i_{t-1} + \rho_1^j i_{t-2} && (25)
\end{aligned}
$$

Where j represents the regime at time t, σ the intertemporal elasticity of substitution, β the discount factor and κ the degree of price stickiness[12]. The output-gap and inflation shocks follow autoregressive processes:

$$
\begin{aligned}
g_t &= \rho_g g_{t-1} + \epsilon_t^g && (26)\\
u_t &= \rho_u u_{t-1} + \epsilon_t^u && (27)
\end{aligned}
$$

ϵ_t^g and ϵ_t^u are random noise variables drawn on normal distributions $N(0,1)$, σ_g and σ_u are the standard errors of the noises. The economy can receive adverse or positive shocks (in this case, the signs of the shocks are respectively negative and positive).

5.2.1 Local dynamics

The aim here is to verify local determinacy[13] when the monetary policy takes the form of the estimated Taylor rule in each regime presented above. Given the uncertainty about the parameters[14] β, σ and κ[15] we study the local determinacy for each combination on the following regions: $\beta \in (0.98, 0.9975)$, $\sigma \in (1, 3.5)$ and $\kappa \in (0.025, 0.525)$.

[11] For purpose of simplicity, we consider the standard IS curve without the Wicksellian interest rate.

[12] As explained in Woodford (2003), $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\omega+\sigma^{-1}}{1+\omega\theta}$ where α is the fraction of goods prices that remain unchanged each period, ω is the firms marginal cost with respect to its own output and θ is the price elasticity of demand of the goods produced by monopolistic firms.

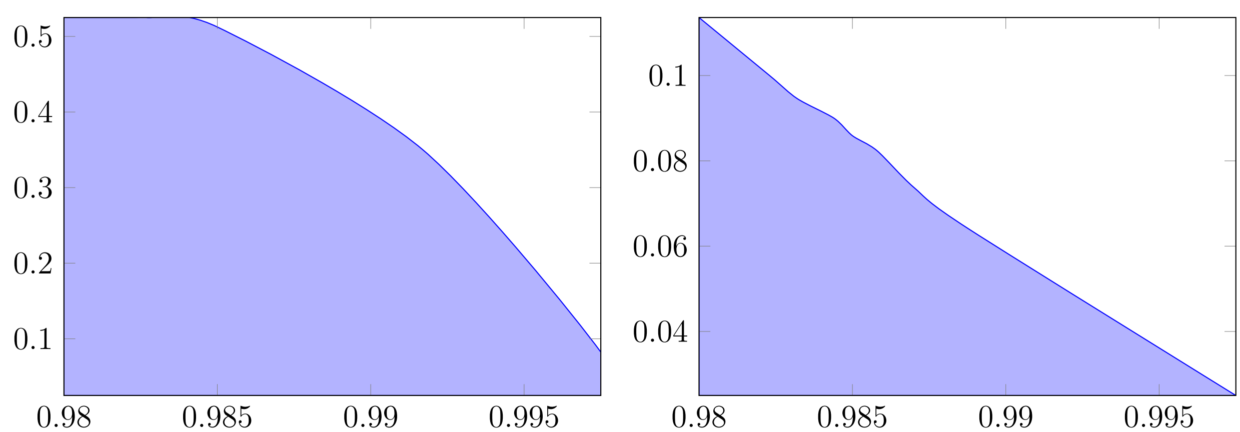
[13] For more details on local determinacy analysis, see appendix.

[14] See Ahmed et al. (2012), King et al. (1988) and Rotemberg and Woodford (1999) for more details.

[15] For simplification reasons we give a reasonable range of variation instead of varying the fundamental parameters which defines it.

Local determinacy is impacted by values of both β and κ but not those of σ. We find that the equilibrium is always determinate in the intermediate rates regime and figures 5 present regions where the equilibrium is determinate in the liquidity trap and high rates regimes, respectively.

Right figure in 5 shows that for most reliable parameters ($\kappa < 0.3$), the equilibrium in the liquidity trap regime is determinate, while the domain where the equilibrium in the high rates regime is much less important. Most of the theoretical literature (Benhabib et al. (2001b), Benhabib et al. (2002b), Benhabib et al. (2002a)...) present an unstable liquidity trap steady state and analyze global equilibria and how sunspot equilibria might lead the economy to a liquidity trap. Our findings (as argued in Bullard (2010)) suggest that liquidity trap equilibria can, as the targeted equilibria, be locally determinate for most of plausible parameters values.



(a) Liquidity trap regime

(b) High rates regime

Figure 5: Determinate equilibrium (blue area)
X-axis: β Y-Axis: κ

5.2.2 Demand and supply shocks

To conduct impulse response analysis, we use the standard calibration (see Davig and Leeper (2006)) as follows: $\beta = 0.99$, $\rho_g = \rho_u = 0.9$ and $\kappa = 0.18$. Our simulations are based on regime-dependent impulse response functions: generalized impulse response functions

(GIRF). We first solve the system of rational expectation equations and then we use the solution with backward dynamics to simulate the model. Local equilibria is determinate around steady states in both *liquidity trap* and *normal* but indeterminate around the steady state in the *high rates* regime.

GIRFs rather than standard IRF are considered because the moving average representation of the solution is not linear in the shocks, because the endogenous variables are conditional on the regimes visited during the entire past history as well as on the sign and direction and the shocks.

The standard impulse response function, for a variable y, which is mostly used in the literature is defined as the difference between two realizations identical up to $t-1$, the first realization y_{t+n} is hit by a shock $V_t = \delta$ at t while the second realization assumes that there is no shocks on y between t and $t+n$. We follow Koop et al. (1996) and define the standard impulse response function as:

$$I_Y(n,\delta,\varphi_{t-1}) = E\left[Y_{t+n}|V_t=\delta, V_{t+1}=0,...,V_{t+n}=0,\varphi_{t-1}\right] - E\left[Y_{t+n}|V_t=0, V_{t+1}=0,...,V_{t+n}=0,\varphi_{t-1}\right] \tag{28}$$

As noted in Koop et al. (1996), the standard impulse response functions depend on the history φ_{t-1} and the size of the shock δ for nonlinear models; asymmetries might also arise in these cases. Since we are considering here the nonlinear policy rule, the standard impulse response function is not adapted to our analysis. We compute the Generalized Impulse Response Functions (GIRF) defined as:

$$GIRF_Y(n,\delta',\varphi_{t-1}) = E\left[Y_{t+n}|V_t=\delta',\varphi_{t-1}\right] - E\left[Y_{t+n}|\varphi_{t-1}\right] \tag{29}$$

The GIRF is the difference between two conditional realizations, one that is shocked and the other without the shock. The purpose of our analysis is to analyze how the economy reacts to demand and supply shocks.

For each size δ', we generate 1000 shocks from $N(0, \sigma^2_{\delta'})$, simulate the model and compute the corresponding GIRF for each shock then we average out to obtain the final GIRF. We repeat this process 1000 times and obtain a distribution for each response at every timing $t + j$ where $j = 124$ and responses of each variable at $t + j$ are computed as the median of each distribution and presented in the following figures.

We assumed the economy to be initially in the low or intermediate rates regimes ($i_{t-1} \leq 2.20$ and $2.20 < i_{(t-1} \leq 4.06$) since the high rates regime presents indeterminacy (see appendix for more details). In both cases (low or intermediate rates regimes) we choose three different sizes of shocks, namely small, medium and large size. In the sequel, we employ the expression *liquidity trap* to characterize the low interest rate regime (though the latter is not strictly zero) and *normal* regime when the latter is in the intermediate regime.

Figures 7 and 8 display the response of the output-gap, inflation and interest rate to demand and supply shocks conditional on initially being in a regime of liquidity trap (low interest rate) or intermediate rate regime. The dashed lines represent the response corresponding to negative shocks, while the solid lines plot the responses of the positive shocks. The variables remains in the initial regime when there is a feedback dynamics, after the initial shock, to the zero line. They switch from one regime to the other when there is no such mean-reverting dynamics to the zero line and when we instead observe that the GIRFs converges to a value above or below the zero line. The shocks considered here are policy shocks. We assume that they are persistent in the sense that the public does not expect them to be reverted too quickly. Such an assumption is worth to avoid that the shocks become neutral on the economy (or equivalently that the policy become ineffective). Indeed, one needs to keep in mind that the effects of the shocks are channeled to the macroeconomic variables through the publics expectations.

Figure 7 reports the response of inflation, the interest rate and the output-gap to different standard deviation positive and negative supply shocks. Figure 8 displays similar responses to demand shocks.

With regards to the question on how to make an economy move from a situation of liquidity trap to a normal regime, the simulations yield to conclusions that are in line with those of the recent theoretical studies. Assume that the economy is initially in a liquidity trap regime. What is needed to leave this regime is a policy of reflation, which is obtained by positive supply and demand shocks (expansive monetary policy, deficit spending, tax increases). The figures show that large enough expansionary demand and supply shocks cause the interest rate and the inflation rate to switch to the intermediate regime. However, the impact on the output-gap is ambiguous. The dynamics results from two effects. On the one hand, large expansionary and inflationary shocks imply that the private sector anticipates higher future interest rates, when the economy will have left the liquidity trap regime. By forwarding the (IS) curve:

$$y_t = E_t y_{t+1} - \sigma^{-1} \sum_{s=t}^{T} E_t(i_s - \pi_{s+1} - g_s) \tag{30}$$

we see that the output-gap depends upon the expected path for future interest rate and a similar dependent relationship applies to the Phillips curve by substituting (30) for y_t in (24). People thus expects a lower interest rate which stimulates aggregate demand and raise the inflation rate.

On the other hand, owing to the switching Taylor rule, the ECB reacts to changes in the aggregate and inflation rate. In the rule estimated, the ECB seems is more reactive to changes in the output-gap than to inflation. As a consequence the positive effects induced by the expectation channeled can be countered by the increase in the interest rate by the ECB that resultsfrom the Taylor rule. Since the central bank is very reactive to changes in the output-gap, the total changes on this variable can eventually be null. In this case, the economy moves from a regime of liquidity trap with deflationary pressures to a regime of stagflation (with a higher inflation rate, but unchanged output-gap). Figure 8 suggests that, during the adjustment dynamics towards the long-run equilibrium, the output-gap can even become negative over some periods. Therefore, if the ECB follows the Taylor rule that we estimated, expansionary demand shocks and inflationary supply shocks can induce a switch between two undesired equilibria (liquidity trap and stagflation). The reason is the following. The public anticipates that, once the deflationary pressures usually observed in a situation of liquidity trap are subsided and higher levels of output-gap are obtained, the central bank will turn to a restrictive policy by raising the interest rate.

Conversely, the interest rate and inflation switch from a normal regime to a liquidity trap regime in case of large recessionary demand shocks or large deflationary shocks. Assume that the economy evolves initially in the intermediate regime and that large negative shocks hit the economy. As shown by the graph this causes a drop in both the inflation rate and the interest rate and a regime-shift for both variables. In addition, we see that this time the impact of the shocks on the output is non-neutral. The ECB would like to cut aggressively the interest rate to offset the negative shocks. However, once the economy reaches the liquidity trap regime, the nominal interest rate is bounded below by zero. In other terms, the interest rate that would be required to avoid the recessionary effects on the output-gap would be negative but that is not possible because of the zero low bound. As a consequence, the economy combines a situation of deflation and recession.

The size of the negative shocks causing a switch from a normal regime to a regime of low interest rate is lower than the size of the positive shocks implying a switch from a low interest rate regime to an intermediate regime. Therefore, it is more demanding from a policy viewpoint to leave a situation of liquidity trap than to make the economy stay in the intermediate regime after a severe recession.

Needless to say that a situation in which a central bank faces a trade-off between a situation of recession/deflation and a situation of stagflation is not desirable. This suggest that, specifically, in times of crises the ECB could change its policy instrument.

6 Conclusion

There are several proposals in the literature to avoid the economy a situation of liquidity trap. Benhabib et al. (2002a) and Evans et al. (2008) suggest a combination of both aggressive monetary and fiscal policies. Some economists suggest the use of regime-switching Taylor rules whereby the central banks changes the inflation target and allows it to temporary deviate from its fixed long-run target (see, for instance Chattopadhyay and Daniel (2013)). Others propose a permanent monetary expansion (Auerbach and Obstfeld (2005)), a depreciation of the exchange rate (Svensson (2004)) or pegging the nominal interest rate (Schmitt-Grohe and Uribe (2013)). This paper adopts the approach of a regime-switching monetary rule and examines the case of the ECB. We find that, undoubtedly, the stance of the reaction to the inflation rate and to output-gap has changed across time and this can be captured by a regime-switching dynamics. Moreover the ECB also seems to pay attention to financial risk variables.

One caveat of this rule is its macroeconomic implications. In an uncertain environment, whereby the European economies are hit by positive and negative demand shocks, it seems that the euro area might face a persistent liquidity trap regime as suggested by Bullard (2010). We suggest that the ECB authorities, owing the current situation of low interest rate, could refer to other monetary policies to exit the liquidity trap situation as suggested above.

7 Appendix

7.1 Local stability of the demand-supply model

The planner dynamic system[16] can be written in the form:

[16]The system without taking into account the equations describing the evolution of the exogenous stochastic processes.

$$E_t z_{t+1} = A z_t + a$$

where the vector of endogenous variables is:

$$\mathrm{z}_t \quad = \begin{pmatrix} y_t \\ \pi_t \\ i_{t-1} \\ i_{t-2} \end{pmatrix}$$

and the matrices of coefficients are:

$$\mathrm{A} \quad = \begin{pmatrix} 1+\sigma^{-1}(\theta_y+\kappa/\beta) & \sigma^{-1}(\theta_\pi-\beta^{-1}) & \sigma^{-1}\rho_1 & \sigma^{-1}\rho_2 \\ -\kappa/\beta & \beta^{-1} & 0 & 0 \\ \theta_y & \theta_\pi & \rho_1 & \rho_2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

and

$$\mathrm{a} \quad = \begin{pmatrix} \sigma^{-1}\theta_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The matrix A has the characteristic equation:

$$P(\lambda) = det(A - I\lambda) =$$

$$-[[(1+\sigma^{-1}(\theta_y+\kappa/)-\lambda)(^{-1}-\lambda)$$

$$+\frac{\sigma^{-1}\kappa}{\beta}(\theta_\pi-\beta^{-1})][((\rho_1-\lambda)\lambda+\rho_2)-(\frac{\theta_\pi\kappa}{\beta}+\theta_y(\beta^{-1}-\lambda))\sigma^{-1}(\rho_1\lambda+\rho_2)]$$

$$= 0$$

Our system contains two non-predetermined variables (y and π). Conditions in Blanchard and Kahn (1980) state that for our system to be have a unique solution, there should be two eigenvalues outside the unite circle. Analyzing in each regime how many eigenvalues are outside the unit circle allow us to conclude if the equilibrium is determinate or not. We now take calibrations used to compute the GIRFs as an examlple to better illustrate this. The following table presents the eigenvalues of A in each regime: We notice that there is exactly

Low rates	Intermediate rates	High rates
5.533	6.0433	3.8274
1.0043	1.022	0.9795
0.1841	0.153 + 0.2275 i	0.2582
-0.0296	0.153 - 0.2275 i	-0.0834

two eigenvalues outside the unit circle in both low and intermediate rates regimes while only one eigenvalue is outside the unit circle in the high rates regime. Then equilibriums in low and intermediate rates regimes are determinate, but this is not the case in the high rates regime where sunspot equilibria might arise.

7.2 Figures

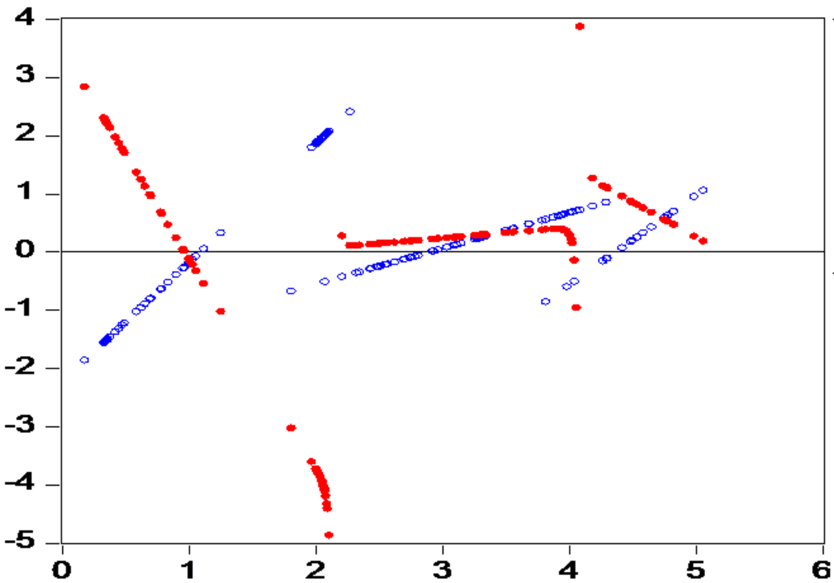
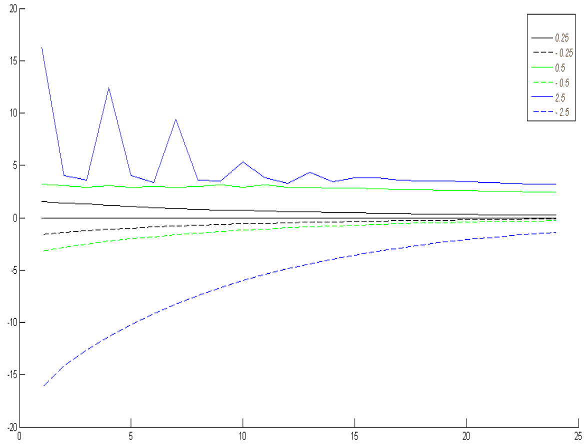
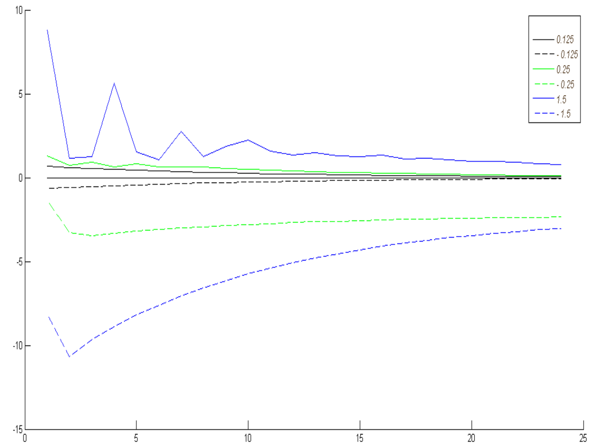


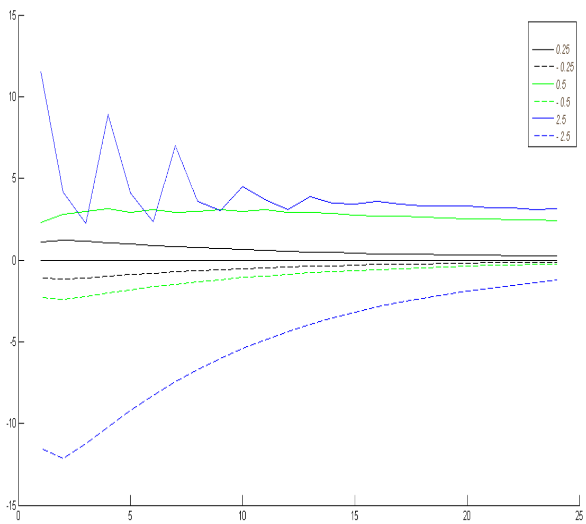
Figure 6: Nonlinear Taylor rule and Fisher equation
Blue curve: Fisher model - Red curve: nonlinear Taylor rule
X-axis: interest rate Y-Axis: inflation



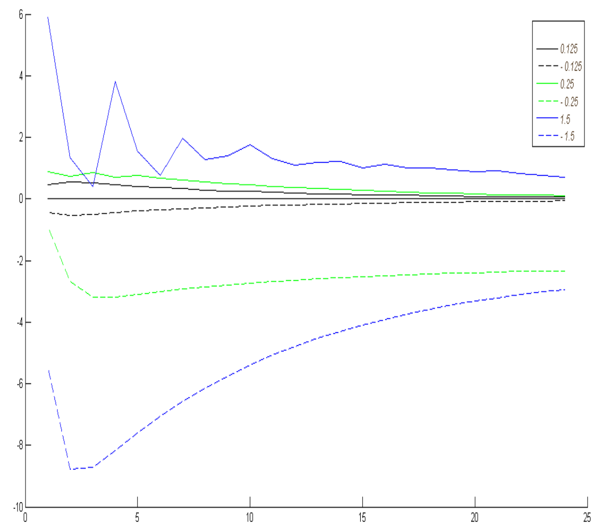
(a) Response of inflation (liquidity trap regime)



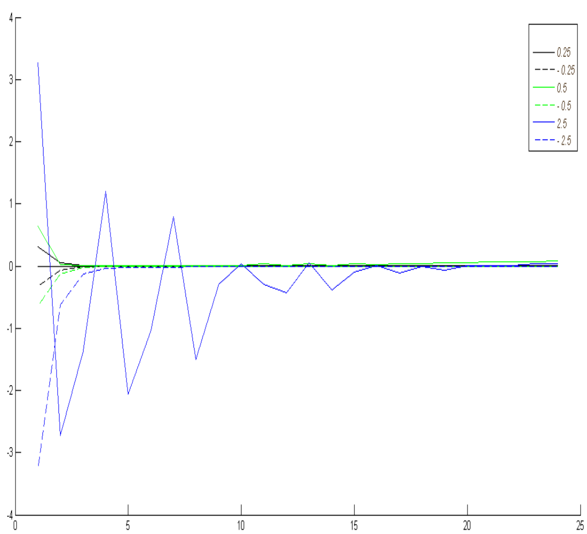
(b) Response of inflation (intermediate regime)



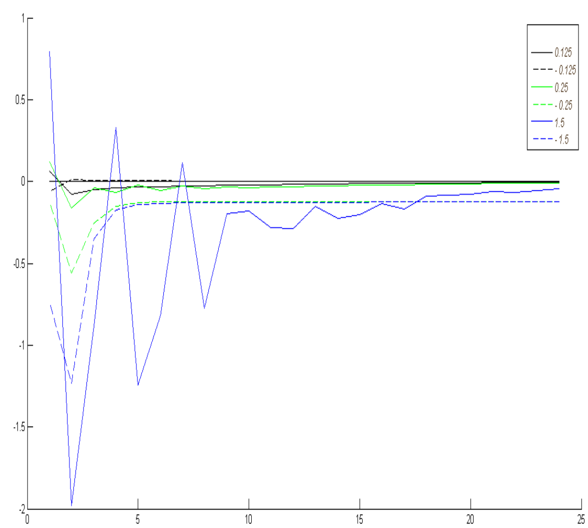
(c) Response of interest rate (liquidity trap regime)



(d) Response of interest rate (intermediate regime)

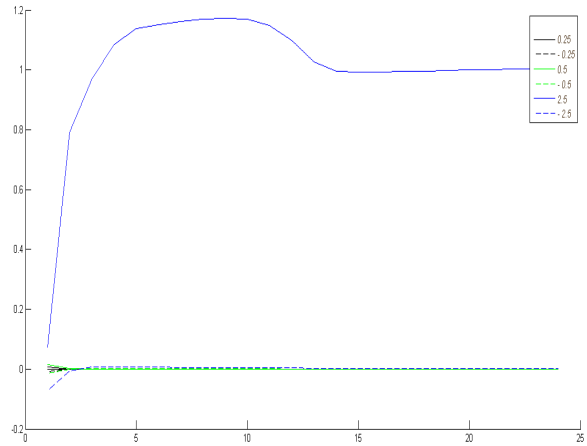


(e) Response of output-gap (liquidity trap regime)

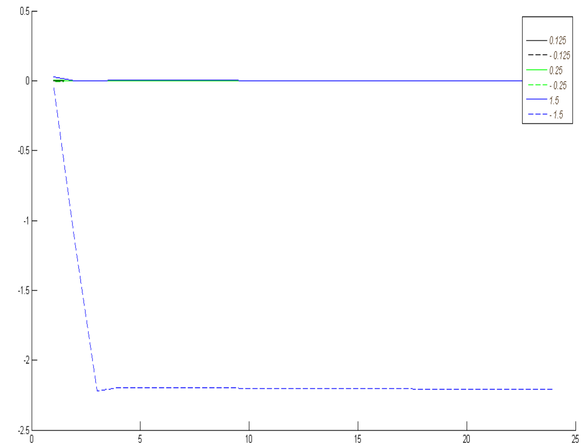


(f) Response of output-gap (intermediate regime)

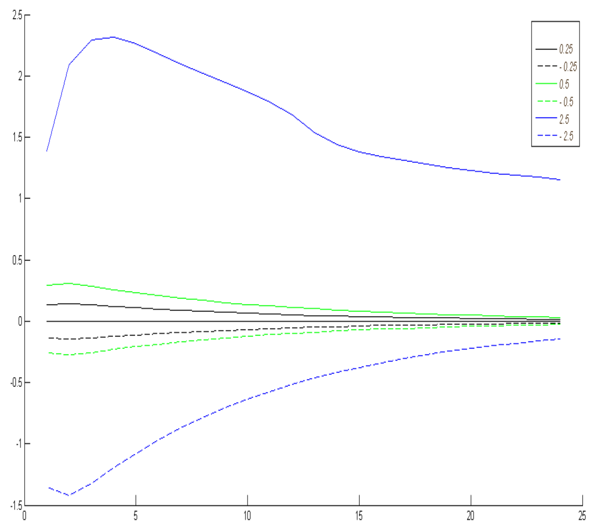
Figure 7: Responses to positive and negative supply shocks
Initial position (left: regime of liquidity trap; right: intermediate regime)



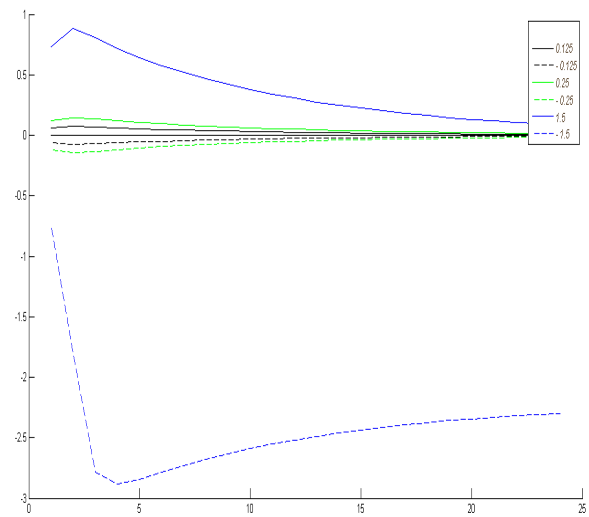
(a) Response of inflation (liquidity trap regime)



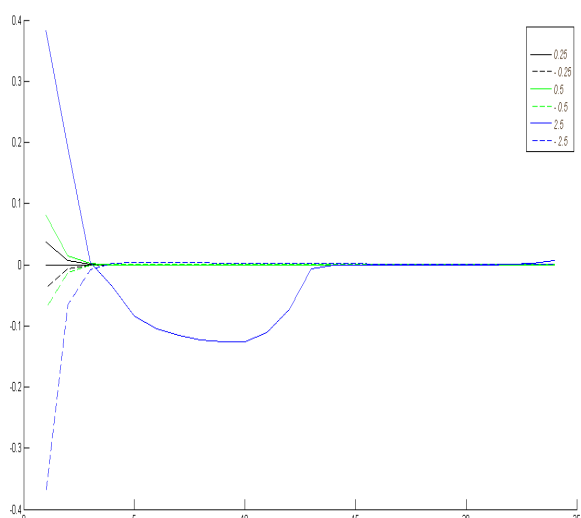
(b) Response of inflation (intermediate regime)



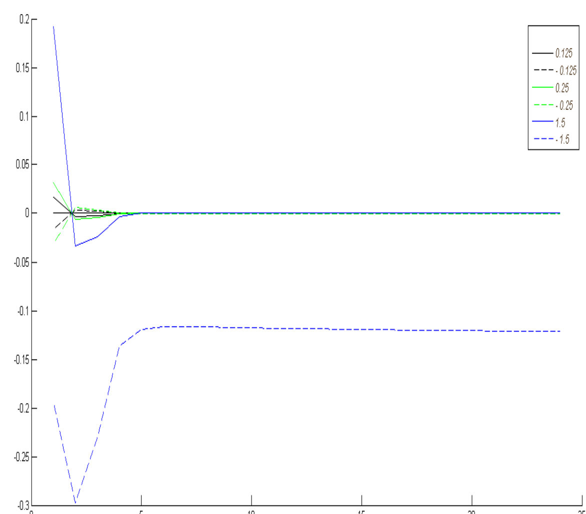
(c) Response of interest rate (liquidity trap regime)



(d) Response of interest rate (intermediate regime)



(e) Response of output-gap (liquidity trap regime)



(f) Response of output-gap (intermediate regime)

Figure 8: Responses to positive and negative demand shocks
Initial position (left: regime of liquidity trap; right: intermediate regime)

References

Ahmed, W., Haider, A., and Iqbal, J. (2012). Estimation of discount factor (beta) and coefficient of relative risk aversion (gamma) in selected countries. *MRPA Paper*, (39736).

Airaudo, M., Nistico, S., and Zanna, L.-F. (2012). Learning, monetary policy and asset prices. *LeBow College of Business, Drexel University: Department of Economics Working Paper Series*, (2012-12).

Alcidi, C., Flamini, A., and Fracasso, A. (2011). Policy regime changes, judgment and taylor rules in the greespan era. *Economica*, 78.

Ang, A., Boivin, J., Dong, S., and Loo-Kung, R. (2011). Monetary policy shifts and the term structure. *The Review of Economic Studies*, 78(2).

Aruoba, S. B. and Schorfheide, F. (2013). Macroeconomic dynamics near the zlb: A tale of two equilibria. *NBER Working Paper Series*, (19248).

Assenmacher-Wesche, K. (2006). Estimating central banks' preferences from a time-varying empirical reaction function. *European Economic Review*, 50(8).

Auerbach, A. J. and Obstfeld, M. (2005). The case for open-market purchases in a liquidity trap. *American Economic Review*, 91(1).

Barnea, E. and Liviatan, N. (2011). Reflections on the failure of the taylor principle under commitment. *Economics Letters*, 112(1).

Belke, A. and Polleit, T. (2007). How the ecb and the us fed set interest rates. *Applied Economics*, 39(17).

Benhabib, J. and Eusepi, S. (2005). The design of monetary and fiscal policy a global perspective. *Journal of Economic Theory*, 123(1).

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. (2001a). Monetary policy and multiple equilibria. *American Economic Review*, 91(1).

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. (2001b). The perils of taylor rules. *Journal of Economic Theory*, 96(1-2).

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. (2002a). Avoiding liquidity traps. *Journal of Political Economy*, 110(3).

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. (2002b). Chaotic interest-rate rules. *American Economic Review*, 92(2).

Bhattacharya, S., Goodhart, C., Tsomocos, D., and Vardoulakis, A. (2011). Minskys financial instability hypothesis and the leverage cycle. *FMG Special Papers*, (sp202).

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5).

Bullard, J. (2010). Seven faces of the peril. *Federal Reserve Bank of St. Louis: Review*.

Carlstrom, C. T. and Fuerst, T. S. (2001). Real indeterminacy in monetary models with nominal interest rate distortions. *Review of Economic Dynamics*, (4).

Chattopadhyay, S. and Daniel, B. C. (2013). Monetary policy switching to avoid a liquidity trap. *Working Paper*.

Clarida, R., Gali, J., and Gertler, M. (1998). Monetary policy rules in practice: Some international evidence. *European Economic Review*, 42.

Cochrane, J. H. (2011). Determinacy and identification with taylor rules. *Journal of Political Economy*, 119(3).

Curdia, V. and Woodford, M. (2010). Credit spreads and monetary policy. *Journal of Money, Credit and Banking*, 42(s1).

Davig, T. and Leeper, E. M. (2006). Endogenous monetary policy regime change. *NBER Working Papers*, (12405).

de Haan, J., Gorter, J., and Jacobs, J. (2008). Taylor rules for the ecb using expectations data. *Scandinavian Journal of Economics*, 110(3).

Dufrenot, G., Mignon, V., and Peguin-Feissolle, A. (2004). Business cycles asymmetry and monetary policy: a further investigation using mrstar models. *Economic Modelling*, 21(1).

Eggertsson, G. B. and Woodford, M. (2004). Optimal monetary and fiscal policy in a liquidity trap. *NBER Working Papers*, (10840).

Eser, F., Carmona, A. M., Iacobelli, S., and Rubens, M. (2012). The use of the eurosystem's monetary policy instruments and operational framework since 2009. *European Central Bank: Occasional Paper Series*, (135).

Eusepi, S. (2007). Learnability and monetary policy: A global perspective. *Journal of Monetary Economics*, 54(4).

Evans, G. W., Guse, E., and Honkapohja, S. (2008). Liquidity traps, learning and stagnation. *European Economic Review*, 52(8).

Gerdesmeier, D. and Roffia, B. (2004). Empirical estimates of reaction functions for the euro area. *Swiss Journal of Economics and Statistics*, 140(I).

Giannone, D., Lenza, M., and Reichlin, H. P. . L. (2012). The ecb and the interbank market. *Economic Journal*, 122(564).

Gros, D. and Grauwe, P. D. (2009). A new two-pillar strategy for the ecb. *CESifo Working Paper Series*, (2818).

Hayo, B. and Hofman, B. (2006). Comparing monetary policy reaction functions: Ecb versus bundesbank. *Empirical Economics*, 31(3).

Hillebrand, E., Medeiros, M. C., and Xu, J. (2012). Asymptotic theory for regressions with smoothly changing parameters. *CREATES Research Papers*.

Jawadi, F. and Sousa, R. M. (2013). Money demand in the euro area, the us and the uk: Assessing the role of nonlinearity. *Economic Modelling*, 32.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles. *Journal of Monetary Economics*, 21.

Koop, G., Pesaran, M. H., and Potter, S. M. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, 74(1).

Lenza, M., Pill, H., and Reichlin, L. (2010). Monetary policy in exceptional times. *Economic Policy*, 25(04).

Lown, C. and Morgan, D. P. (2006). The credit cycle and the business cycle: New findings using the loan officer opinion survey. *Journal of Money, Credit and Banking*, 38(6).

Peersman, G. (2011). Macroeconomic effects of unconventional monetary policy in the euro area. *European Central Bank: Working Paper Series*, (1397).

Rotemberg, J. J. and Woodford, M. (1999). The cyclical behavior of prices and costs. *NBER Working Papers*, (6909).

Sauer, S. and Sturm, J.-E. (2007). Using taylor rules to understand european central bank monetary policy. *German Economic Review*, 8(08).

Schmitt-Grohe, S. and Uribe, M. (2009). Liquidity traps with global taylo rules. *International Journal of Economic Theory*, 5.

Schmitt-Grohe, S. and Uribe, M. (2013). The making of a great contraction with a liquidity trap and a jobless recovery. *Working Paper*.

Schorfheide, F. (2005). Learning and monetary policy shifts. *Review of Economic Dynamics*, 8(2).

Svensson, L. (2004). The magic of the exchange rate: Optimal escape from a liquidity trap in small and large open economies. *Hong Kong Institute for Monetary Research Working Papers*, (072004).

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Cornergie-Rchester Conference Series on Public Policy*, 39.

Trecroci, C. and Vassalli, M. (2010). Monetary policy regime shifts: New evidence from time-varying interest rate rules. *Economic Inquiry*, 48(4).

Ullrich, K. (2003). A comparison between the fed and the ecb: Taylor rules. *ZEW Discussion Papers*, (03-19).

v. Dijk, D., Teräsvirta, T., and Franses, P. H. (2002). Smooth transition autoregressive models a survey of recent developments. *Econometric Reviews*, 21(1).

Walsh, C. E. (2010). *Monetary Theory and Policy, Third Edition*. MIT Press Books.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Economics*. Princeton University Press.

Zampolli, F. (2006). Optimal monetary policy in a regime-switching economy: the response to abrupt shifts in exchange rate dynamics. *Journal of Economic Dynamics and Control*, 30(9-10).