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Building Blocks for a Disequilibrium Model of a European Team Sports League

Wladimir Andreff

Professor Emeritus at the University Paris 1 Panthéon Sorbonne,
Honorary President of ESEA and IASE,
former President of AFSE (French Economic Association, 2007-08)

61, chemin du Pouget – 30350 Lédignan – France
tel.: +33 4 66 85 28 04
andreff@club-internet.fr
Abstract: The paper presents two building blocks for elaborating on a disequilibrium economics model that fits with empirical evidence of a European team sport (soccer) league where teams are win-maximising and operate under a soft budget constraint. Going beyond the standard equilibrium model justifies the introduction of price rigidity, heterogeneous talent units, and a differentiated economic behaviour of clubs’ fans as compared to TV viewers. A European soccer league appears to be an economic regime with repressed inflation, characterised by a double excess demand on both labour and product markets. Step by step, the disequilibrium model is enriched with labour market segmentation between superstars and journey men players, then a market for televised sport differentiating between free-to-air and pay-per-view TV channels.

Key words: disequilibrium economics, sports economics, team sport league, win-maximising, European soccer, price rigidity, soft budget constraint, excess demand.

JEL: L83, J42, D59.
A standard equilibrium model of a team sports league was first elaborated on in the context of North American professional team sports leagues assuming that teams are profit-maximising (El Hodiri and Quirk, 1971; Fort and Quirk, 1995). When economists adapted this model to European team sports leagues they dropped the assumption of profit-maximising teams for a team’s win-maximising objective function. They did not care enough of how much irrelevant would become some crucial hypotheses such as price flexibility or a balanced budget constraint when a team is no longer a profit-maximising price-taking entity. They did not mind that managing a club without a profit-maximising objective boils down to accepting that its budget constraint might not be strictly enforced with the subsequent survival of sports teams in the red. Price rigidity and soft budget constraint do not fit with an equilibrium model and pave the way for either excess demand or excess supply in the markets for teams’ inputs (labour, talents) and outputs (gate receipts, TV rights revenues). It is this article’s contention that a disequilibrium model should fit well with a European team sport (soccer) league where a number of teams actually run financial deficits.

It is not intended to build up a sophisticated and comprehensive disequilibrium model; this is beyond the scope of this article. The purpose of this article is to convince - or appeal - to model builders that they would be better off considering a switch from equilibrium to disequilibrium models when a team sports league encompasses teams that do not maximize profits. For this purpose it suffices to establish a few mathematical inequalities that make up the first building blocks of a required disequilibrium team sports league model.

The article reads as follows. The outcome of a literature review in disequilibrium sports economics is an empty shell (1). Beyond the standard equilibrium model of a European team sports league, other hypotheses and contrasting empirical evidence are pointed out while two varieties of disequilibrium models are distinguished (2). Then a first building block for a disequilibrium model of a European team sports league comprises a labour market for talent
in excess demand (3). Dropping the hypothesis of a homogenous talent unit leads to integrating labour market segmentation for differently talented players (4). A second building block deals with a market for sport matches in short supply (5). Differentiating the demand side between fans and TV viewers and, for the latter’s demand, introducing a trade-off between watching a free-to-air and a pay-per-view TV channel is a last step so far (6). Avenues for further research are sketched (7) before concluding (8).

1. Literature review in disequilibrium sports economics: an empty shell

Searching the sports economics literature shows no article devoted to a disequilibrium team sport league model. A few sports markets in disequilibrium are surveyed in Andreff (2012) such as an excess demand for some sports events, possible excess demand or excess supply in the different markets for sporting goods, and an excess supply of some sport arenas and televised sports. As regard team sports leagues, the only departure from a standard equilibrium model is due to two Australian mathematicians, Tuck and Whitten (2012). Their dynamic simulation model of a win-maximising sports league allows for non-equilibrium solutions when the complex stochastic dynamic characteristics of a league provide teams with incentives to under-perform and tank. However, this mathematical model is not an application of disequilibrium economics properly speaking.

None of the famous disequilibrium economists, for instance Clower, Leijonhufvud, Barro, Grossman, Malinvaud and others, was appealed by sports economics so that there is no application of disequilibrium economics to sports so far. Disequilibrium economics modelling started up with a re-assessment of how much the standard general equilibrium model is relevant for a market economy with frictions and imperfections and grew up to its standardised form in Benassy (1982, 1983). It eventually became the most relevant theory to
understand those disequilibria that plagued centrally planned economies with Kornaï (1980).

More recently Kornaï et al. (2003) provided a generalisation to all the situations where
disequilibrium is a self-reproducing and self-sustained economic regime, in particular in those
industries with non profit organisations. European soccer teams are a case in point as long as
(it is assumed that) they behave as win-maximising and not profit-maximising entities.

2. Beyond the standard equilibrium model of a European team sports league

A standard equilibrium model of a European team sports league was designed by Késenne
(1996, 2000) with win-maximising teams, which is usually assumed a relevant hypothesis in
the context of European soccer. This model was elaborated further on and compared with the
standard model of North American leagues (Szymanski, 2003). In order to build up the
European league variant of the standard model, Késenne altered two assumptions:
ap/ Teams are win-maximising but still wage-takers on the labour market for talent.
b/ Therefore they recruit as much talent as possible within their budget constraint, a behaviour
that leads to an arm’s race for talent (Rosen and Sanderson, 2001).

The standard model of a European team sports league thus writes as:

\[ \text{Max } t_i \]  \hspace{1cm} (1)  
\[ R_i (m_i, t_i) - s. t_i - c_i^0 = 0 \]  \hspace{1cm} (2)

with \( t_i \) the quantity of talent in a team \( i \), \( R_i \) a team \( i \)'s revenue function, \( m_i \) the size of its local
market, \( s \) the market unit cost of talent (or wage) and \( c_i^0 \) a fixed cost (stadium, management).

Using the Lagrangian objective function, first order conditions are:

\[ 1 + \lambda_i \left( \frac{\partial R_i}{\partial t_i} - s \right) = 0 \]  \hspace{1cm} (3),  hence \( R M_i = s - 1 / \lambda_i < s \)  \hspace{1cm} (4)

\[ R_i - s. t_i = 0 \]  \hspace{1cm} (5).
From (4) marginal revenue of talent is lower than marginal cost. For a given unit talent cost, a team’s demand for talent that maximises its wins is bigger than if the team were profit-maximising; in a European league, a team spends more on recruiting talent than in a North American league. From (5) a team’s demand for talent is not given by its marginal revenue curve but by its average revenue curve (revenue per unit of talent): \( \bar{R} = \frac{R_i}{t_i} = s \).

Szymanski (2004) stressed that two other assumptions are at odds with European leagues:

c/ In a post-Bosman global labour market for soccer players, free entry of players makes irrelevant the assumption of a fixed supply of talents adopted with North American leagues.

d/ Assuming that teams are wage-takers is controversial given the heavy impact of some large clubs - Manchester United, Chelsea, FC Barcelona, Real Madrid, Milan AC, Juventus, PSG, and a few others – on wage and transfer fee determination.

2.1. Some alternative assumptions

Though Szymanski (2004) and Szymanski and Késenne (2004) underlined various limitations introduced by the aforementioned assumptions in the standard equilibrium model of a team sports league, attention must be drawn to some alternative hypotheses. The first three of them are crucially required to allow writing a balanced team’s budget constraint such as (2):

H1: Infinite flexibility must be assumed for the variations of either the quantity of talent \( t_i \) or market wage \( s \) or both on the market for team \( i \)’s input; the same assumption pertains to the team \( i \)’s product market for it to reach equilibrium.

H2: Identical units of talent can always be marginally added or substituted for, which requires the assumption of an infinitely divisible and homogenous unit of talent. No differentiation between any two talent units or two players is taken into account even though, in the real world, journey men players are not as much talented as superstars.
H3: All clubs are run in such a way as to exactly *break even* in accounting terms, *i.e.* they strictly *stick to their budget constraint* having simultaneously a zero profit and a zero deficit. None of these hypotheses is required for a disequilibrium model. They are dropped in the following argument.

Beyond these three assumptions, a last means for team $i$ to adjust to its budget constraint consists in augmenting its revenues $R_i$ by increasing its market size $m_i$, that is attracting more gate receipts and/or TV rights revenues on its output market. This may happen through either a flexible ticket price or TV subscription fee on the one hand and, on the other hand, an unrestricted variation in the number of ticket holders and pay-per-view TV viewers. Analysing a team’s trade-off between output price and quantity flexible variations implies a need to integrate the detailed functioning of a second (product) market in the model which is rare in the literature about team sports leagues. A major exception is found in Késenne (2007) who demonstrates that equilibrium ticket price with price being assumed to be infinitely flexible, is higher in a league with win-maximising teams than in a league with profit-maximising teams. However, in this model, neither stadium attendance and gate receipts nor TV rights revenues depend on any consumer demand function that would differentiate fans, season ticket holders, occasional spectators, TV viewers, and couch potatoes who may have different behaviours in the face of a league supply of sport fixtures.

H4: A realistic hypothesis must be examined that fans and TV viewers have not the same consumer demand function. This means that a sport show is a differentiated (and not homogenous) product whereas the product market of a team sport league is segmented according to different varieties of a same sport show consumed either in attending a stadium or in switching on a TV set, a lap top or a mobile phone.

### 2.2. Two varieties of disequilibrium models
In disequilibrium models, there are always two markets, one for input (labour) and one for output (product). Imagine that H1 above is dropped for the two markets, and then the whole economy – the league - is in a double disequilibrium regime with either excess demand or excess supply on both labour and product markets. Recall that price rigidity, whether in the form of a fixed price, a (monopoly/oligopoly) sticky price, or price inelasticity generates market disequilibrium and quantitative adjustment, *i.e.* rationing (Benassy, 1982). A disequilibrium model of a team sports league must theoretically be able to combine excess supply on one market with excess demand on the other market, or excess supply on both, or excess demand on both, into one of the three possible theoretical disequilibrium economic regimes: repressed inflation, classical unemployment or Keynesian unemployment (Table 1). In this variety of disequilibrium models *à la* Benassy the focus is on *price rigidity or inelasticity.*

**Insert Table 1 about here**

In a deregulated labour market, win or utility-maximising teams in European soccer leagues overbid and overspend for talents, run deficits season after season, and sink into a debt crisis (2.3 below). Lasting deficit and debt suggest that teams enjoy a *soft budget constraint* in their maximising calculation which points out a second variety of disequilibrium models in which firms (here teams) can either endeavour a *hard* (*≥ 0*) or enjoy a *soft* (*≤ 0*) budget constraint, and not only a balanced budget (*= 0*) as in (2) above. Kornaï (1980) has demonstrated that firms which enjoy a soft budget constraint take stock of it to form an excess demand for inputs, typical of a shortage economy. If all or most firms (teams) are run with a soft budget constraint, then the whole economy (league) functions all the time in disequilibrium. It never returns to equilibrium: there are always a number of teams in the red.
Following Kornai’s model, there are five circumstances (hypothesis H5) that ex ante secure a firm (a team) to enjoy a soft budget constraint:

H5a: The team is price-maker on its input and/or output markets. If the team is wage-taker as in the standard equilibrium model, then its budget constraint is hard or balanced (never a deficit).

H5b: The team can influence taxation rules, avoid or postpone tax payment (tax arrears are tolerated by tax authorities). If the team cannot, then its budget constraint is hard.

H5c: The team can receive state or municipality grants or subsidies to cover current expenses or finance investment. If not, the team’s budget constraint is hard.

H5d: The team can be granted credit by banks whatever its deficit, debt and insolvency. If it does not find any credit due to its deficit, debt or insolvency, its budget constraint is hard.

H5e: The team’s investment is not dependent on retained profits because it can find external finance for investing, including from fans or ‘sugar daddy’ investors. When investment is strictly constrained by the amount of profits, a team’s budget constraint is hard.

To conclude on these circumstances, when its budget constraint is soft, spending more than its revenues is not a matter of life and death for the firm (team), as Kornai states it, and it will survive its deficit. A high survival rate of soccer teams, even when they are in the red, would confirm that it is so.

Eventually, soft budget constraint and price inelasticity must be at the core of a disequilibrium team sports league model.

2.3. European soccer empirical evidence

A lot of facts and empirical evidence witnessed in European soccer leagues provide some grounds to go beyond the standard equilibrium model since the aforementioned H1, H2 and
H3 hypotheses do not seem to be actually fulfilled in the real word. More recently, differentiation in the consumer demand for soccer games successfully passed through econometric testing (H4) whereas H5 is more than likely to be relevant.

Hypothesis H1, as regard the team sports league’s product market, is at odds with a demand for games inelastic to price or the fact that teams fix their price in the inelastic portion of their demand curve, as various studies have empirically tested it. References are many in the sports economics literature - Noll (1974), Fort and Quirk (1995), Coates and Harrison (2005), Coates and Humphreys (2007) -, and for European soccer leagues (Andreff, 1981; Bird, 1982; Dobson and Goddard, 1995; Falter and Pérignon, 2000). In the European context, both league monopsony on the labour market and player unionisation, as well as collective bargaining, though less widespread than in North American leagues, fuel some stickiness in wage determination.

With regard to H2, assuming a homogenous talent unit is rather abstract concept. How many homogenous talent units are embedded in Lionel Messi as compared for instance with Jean-Marc Bosman? The metrics to make it sure are missing. The managers of a real soccer team actually do not know what a talent unit is; they can only assess how much talented one player is – as a bundle of several heterogeneous talents. Thus a team maximises the number of heterogeneously talented units enshrined in live and entire players it recruits. Moreover, the terms of trade and wage determination are dramatically different for players who face a high demand for their talents – the superstars – from those for journey men players. The arm’s race observed in European soccer targets superstars in excess demand, not journey men players. Each superstar holds an embedded specificity of his talents and a monopoly position on the supply side of the labour market, and thus benefits from a rent included in his wage (Borghans and Groot, 1998) and transfer fee, a quite different market situation from excess
supply of sometime unemployed journey men players. Segmentation of the soccer labour market has been observed by several authors (e.g. Bourg, 1983)\textsuperscript{7}.

A balanced budget constraint (H3) introduces a strong assumption which is not that realistic compared with the current weak governance and bad financial performance of many European soccer teams and leagues (Andreff, 2007). According to UEFA (2011) data, 56\% of all European top division soccer teams were making losses in 2010. Being in the red is more the rule than exception, which translates into leagues operating off their economic equilibrium. Storm and Nielsen (2012) underlined that a number of European professional soccer teams chronically operate on the brink of insolvency for over a decade or so, without going out of business. Teams are promoted or relegated according to their sporting outcomes but they almost never disappear from soccer business due to financial deficits. The teams’ survival rate is very high\textsuperscript{8} despite soccer business generating losses. Spanish soccer teams are used to spend larger amounts on player wages and transfers than their revenues, and exhibit a rising indebtedness (Barajas and Rodriguez, 2010). The French soccer league experimented quite more years in the red than in the black over a decade (Aglietta \textit{et al.}, 2008) even though deficits and debts were smaller than in English, Italian and Spanish leagues. Even in the German league, professional soccer teams are not run as if they were compelled to break even since they are non-profit associations (\textit{Vereins}) committed to win as many games as possible for the fans, not to make profit.

A distinction between two varieties of sport show consumers (H4) has been successfully tested, with fans and season ticket holders on the one hand and, on the other hand, TV viewers, casual spectators and couch potatoes (Simmons, 1996; Buraimo and Simmons, 2008). There is a fans’ demand for attending games in stadiums which is distinguished from a demand for televised games (Buraimo, 2008). Thus there are two products, \textit{i.e.} two sport shows derived from the same game. The demand for seating close to the pitch is registered at
stadium gates while the demand for televised games is registered through audience records on public and free-to-air TV channels, and the number of subscribers to pay-per-view and commercial TV companies. Fans demand a maximum number of their favourite team games with an individual utility growing with as many attended games as possible, and with a wishful expectation of home team wins. TV viewers and couch potatoes rather demand uncertain and high quality games; thus they are attracted to more balanced games, \textit{i.e.} showing a sufficiently high uncertainty of outcome (Forrest \textit{et al.}, 2005).

At first glance it is not a same disequilibrium that prevails in the fan and TV viewer markets. The first one may be either in excess demand – when fans are queuing at the gate and then scalping – or excess supply – when a game is played in a nearly empty stadium. Both happen in European soccer. It is more difficult to assess which disequilibrium affects the TV viewer market. A preliminary quantitative signal leans toward the assumption of an excess supply of non-free televised sports overall though it is not an academic proof.

Turning now to hypothesis \textit{H5}, big soccer teams definitely influence wage determination in the market for talent (Szymanski, 2004) – they are wage-makers - and, though less so, a league and its teams are price-makers on their product market in particular when the league is pooling TV rights for sale (H5a). In European soccer, many teams fail to pay taxes without being liquidated (H5b), and when they enter administration the reconstructed tax arrears are almost always among the debt obligations that are not being met (Storm & Nielsen, 2012). A number of soccer teams rent sports facilities to a municipality at a subsidised rate (H5c). The enforcement of EU competition policy has restricted the possibility of municipal subsidies to professional soccer teams. But subsidisation still occasionally happens, like in the Italian \textit{Calcio}, with the government having stepped in league financing (the \textit{salve calcio} state plan in 2002, Baroncelli and Lago, 2006). Catalan banks grant credits (H5d) to FC Barcelona, and
Castillon banks to Real Madrid whatever their balance sheets (Ascari and Gagnepain, 2007). Teams often succeed in renegotiating and rescheduling repayments to the banks. Plenty of facts validate H5e. Many European soccer teams are seen as too big to fail by their stakeholders, and they always find some institution (bank, TV channel, etc.) to bail them out or grant them a loan even though irrecoverable; soft subsidies are provided by sugar daddies or sponsors in the form of additional cash and capital in cases of looming insolvency (Storm & Nielsen, 2012). This is widespread enough to be under the scrutiny of the coming UEFA Financial Fair Play initiative (Franck, 2013). Some TV companies either take over some teams or finance a league again and again through increased TV rights from one broadcasting contract to the other (Andreff, 2011). Such money inflows allow overinvestment in talent at team level. German teams are used to gambling on success from which ensues a constant overinvestment in talents and players (Dietl and Frank, 2007), a characteristics of organisations with a soft budget constraint. Many European soccer teams purchase more inputs than they can afford with their revenues; they attempt to endlessly recruit more expensive players and the richest clubs overbid each other to attract the best superstars.

3. First building block: a labour market for player talents in excess demand

The simplest disequilibrium model of a European team sports league comprising of win-maximising teams that operate in a labour market with some wage rigidity, is:

\[
\begin{align*}
Max & \quad t_i \\
R_i (m_i, t_i) - s. t_i - c^0_i & \leq or \geq 0
\end{align*}
\]  

Assuming that the stadium and club management are not significant inputs of a team’s revenue function, the fixed cost can be left out and then it follows:

\[
R_i (m_i, t_i) \leq or \geq s. t_i
\]
Due to wage rigidity, limited talent mobility across the teams will not be enough to trigger equality between marginal cost and average talent revenue - as in Equation (5) - while the market wage will not adjust to its equilibrium level. Thus, compared to equilibrium, team $i$ will form either excess demand for talents – when the budget constraint (7’) is $\leq 0$ – or a too short demand when it is $\geq 0$. In the first event, team $i$ spends more on acquiring talents than the revenues it makes, which fits with the empirical evidence of the above-mentioned soccer leagues empirical evidence. In the second case, team $i$ does not spend all its revenues on talent purchase, it is left with surplus (above normal)-profit, and it demands a shorter quantity of inputs than in equilibrium.

Now assume that team $i$ meets a soft budget constraint, (6’) to (8’) transform into:

$$\begin{align*}
\text{Max} & \quad t_i \\
R_i(m_i, t_i) - s_i t_i - c_i^0 & \leq 0 \\
\text{and without fixed cost} & : \quad R_i(m_i, t_i) \leq s_i t_i
\end{align*}$$

and if most teams behave this way, two logical consequences are: a/ the league itself repeatedly is in the red when aggregating the net financial results of all its teams (as soon as the profits of some teams are more than compensated by losses of most teams); b/ there is necessarily a permanent excess demand on the labour market for talents triggering an endless arm’s race.

Without a hard or balanced budget constraint there is no brake on a growing demand for talents while the number of players talented enough to play in professional soccer leagues is limited, say to $T_0$. The labour market for talents is in disequilibrium due to a teams’ aggregated excess demand in the face of a limited supply of player talents to the league $T_0$:

$$\sum_{i=1}^{n} t_i = T \geq T_0 \quad (9),$$

where the number of teams in the league is $n (i = 1, \ldots, n)$. 

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With an excess demand for talents, marginal revenue productivity\textsuperscript{11} of labour
\[ RM_i = \frac{\partial R_i(m_i, t_i)}{\partial t_i} \]
cannot equalise marginal unit cost of labour \( s \) when the last unit of the \( T_0 \) talents is recruited and the disequilibrium in the labour market for talents implies:
\[ RM_i = \frac{\partial R_i(m_i, t_i)}{\partial t_i} \leq s \quad (10) \]

All the labour units, up to the last recruited one, are overpaid when excess demand prevails in the labour market. Due to their aggregate overall excess demand, teams are rationed by a short supply of talents and are eager to pay a salary much higher than marginal labour productivity of talent, \textit{i.e.} to overpay players in order to attract them in a context of harsh competition across the teams on the demand side of the labour market\textsuperscript{12}. Then, recruited players provide a lower labour productivity than the salary they are paid for, which sounds like the exact opposite of Scully’s sense of player exploitation (Scully, 1974). In European leagues with win-maximising teams operating under a soft budget constraint players are paid more than they would have been at equilibrium wage. Since all teams are embarked on an arm’s race to recruit players, namely few available superstars, they accept to pay a wage higher than marginal revenue productivity of labour in order to outbid competing teams - in all European leagues after Bosman case. Paying more than equilibrium wage and recruiting less than the quantity of talents they demand, rationed teams are involved in an endless skyrocketing race of payroll increases, which is actually observed in European soccer as regards superstars.

Another implication of excess demand in the labour market for player talents is that soccer teams with a soft budget constraint attempt to recruit too many players, although they cannot afford as many as they would have wished (demanded) due to the shortage of superstars. Teams spend their money without counting losses - and sometimes they cook the books to hide this reality - due to soft budget constraint. Moreover, operating on the demand side of an excess demand input market teams are always scared of being short of inputs without being
able to find one more superstar in the market - due to inequality (9). In such event, like enterprises in former centrally planned economies, teams “hoard” labour as a reaction to circumvent the consequences of operating on a shortage input market. In European soccer, the very existence of a reservation and transfer system until the Bosman ruling of 1995 enabled teams to keep their players. After Bosman case, teams recruited on their rosters more players than they really needed\textsuperscript{13}. Thus, there is some slack in each team; teams are overmanned - look at the rosters of various European soccer teams and the number of substitutes never used over a season. This slack is beneficial to players in terms of the relationship between wage and both working time (very few players play all the season games during 90 minutes) – \textit{i.e.} work intensity – and labour productivity.

\section{4. Heterogeneous units of talent: labour market segmentation}

Now let us introduce a qualitative differentiation of talents between the most talented superstars and less talented journey men players. It was assumed above that the supply of player talents is limited. Is it as much likely to be true for journey men players as for superstars? Obviously not since one can witness in all European top soccer leagues a number of journey men players who are unemployed at the start of every season\textsuperscript{14}. Unemployment is a crucial index of excess supply. Such observation apparently contradicts inequality (9) which meant excess demand on the labour market for player talents. Therefore, for a journey man player segment of the labour market excess supply is to be modelled instead of excess demand on a superstar segment. Beforehand, the next point is to be considered: is a teams’ excess demand for superstars on one market segment compensated or not by excess supply of journey men players on the other segment of the market? It is assumed here that there is not
full compensation: excess demand for superstars exceeds excess supply of journey men players and then inequality (9) still remains relevant for the labour market overall.

Let $T_s$ stand for the overall number of available superstars and $T_a$ the overall number of available journey men players, then it follows that:

$$T_s + T_a = T_0 \quad (15).$$

Now a team has to maximise an assortment of superstar and journey men player talents in order to maximise its wins (16), and its soft budget constraint is to be rewritten in such a way as to take this assortment into account (17), below $t_{si}$ is defined as the demand for superstars by team $i$, $t_{ai}$ the demand for journey men players by team $i$, $s_s$ the market wage for superstars, and $s_a$ the market wage for journey men players. Thus for team $i$:

$$Max (t_{si} + t_{ai}) \quad (16)$$

under a soft budget constraint:

$$R_i(m_i, t_{si} + t_{ai}) - s_s \cdot t_{si} - s_a \cdot t_{ai} \leq 0 \quad (17)$$

and

$$\sum_{i=1}^{n} (t_{si} + t_{ai}) \geq T_0 \quad (18).$$

If, as assumed, excess demand for superstars more than compensates excess supply for day men players, the labour market disequilibrium in the superstar segment becomes:

$$\sum_{i=1}^{n} t_{ai} \geq T_s \quad (19)$$

In the superstar segment of the market, excess demand coincides with a wage higher than marginal revenue productivity of labour (20):

$$RM_{si} = \frac{\partial R_i(m_i, t_{ai})}{\partial t_{si}} \leq s_s \quad (20)$$

Superstars are not directly competing against each other, with any superstar being in a monopoly position over his/her practically non substitutable specific talent. In the journey men player segment of the market, excess supply of talents (21) drives market wage down to
be lower than marginal revenue productivity of labour (22) for these lower-quality and more competing talents:

$$\sum_{i=1}^{n} t_{ai} \leq T_a \quad (21)$$

$$RM_{ai} = \frac{\partial R_{i}(m_{ai}, t_{ai})}{\partial t_{ai}} \geq s_{ai} \quad (22)$$

Inequality (22) shows that day men players are subject to exploitation in a Scully’s sense; they are paid less than their marginal productivity. They suffer from being in excess supply as well as from the monopsonistic situation of the league\(^{15}\) (coordinated team owners) in the labour market; thus they bear a rent levied by owners on their salaries. An opposite asymmetry prevails on the superstar segment where the league’s monopsony is countervailed by a strong monopoly situation of each superstar due to the uniqueness of his talents, skills, reputation, performances, record of achievements and so on, and his/her absolute exclusivity over them.

Now a disequilibrium model of a league with win-maximising teams operating under a soft budget constraint in a segmented labour market describes by the same token the arm’s race for superstar talents fuelled by excess demand, superstars’ skyrocketing wages (higher than their marginal labour productivity) that trigger teams’ payroll overruns, the unemployment of journey men players in excess supply used as a safety valve or an adjustment variable by team owners, and their lower wages paid at a rate below their marginal labour productivity.

5. Second building block: a market for sport shows in short supply

Let us assume for a while that there is no difference between team fans attending a stadium and TV viewers. Thus, on the demand side are found only homogenous fans who basically demand attending as many games of their favourite team \(i\) as they can financially afford\(^{16}\); \(d_{hi}\) stands for a fan \(h\) demand of team \(i\) games. A fan’s utility is assumed to increase in strict
proportion to the number of games attended. Since a European top soccer league is a cartel of teams supplying professional soccer games in a country, it is in a monopoly position to fix its output supply, i.e. the number of fixtures once given the number of teams in the league; \( n_i \) stands for all the team \( i \) games supplied by the league over a season. This monopoly supply creates a games shortage on the supply side of the market for soccer matches and enables teams to maintain a price high enough to make a profit that includes a monopoly rent.

Consider one fan \( h \): as any consumer, her demand function of team \( i \) games depends on his/her initial money endowment \( m_{0h} \) (savings, assets), her income share available for paying tickets at the gate \( Inc_h \), and a fixed ticket price of a game \( p \). First, imagine that there was just one fan per team \( i \), her behaviour could be described as:

\[
\text{Max } d_i \quad (23)
\]

\[
d_i ( m_{0h}, Inc, p ) \geq n_i \quad (24)
\]

\[
p > 0 \quad (25)
\]

In European soccer, each team plays twice against all other teams, one home and one away game, so that the supply of team \( i \) games is \( n_i = 2 (n - 1) \) while the overall league supply of games over a season is \( 2 n (n - 1) \). This is total supply of professional soccer games in a country since creating another top soccer league in the same country is prohibited. Stadium capacity is assumed to be constant over a season and then the supply side overall is fixed by multiplying all stadiums’ capacities by the overall number of games \( 2 n (n - 1) \).

Now assume that the overall number of a team sport league fans is \( g \) (\( h = 1, \ldots, g \)) and the number of team \( i \)’s fans is \( g_i \) (\( h_i = 1, \ldots, g_i \)), with \( g_i \leq g \). All team \( i \)’s fans are subject to the same constraint, a shortage in the supply by the league of their favourite team games which number is restricted to \( n_i = 2 (n - 1) \). Thus, for any team \( i \)’s fan \( h \), maximising utility boils down to:

\[
\text{Max } d_{hi} \quad (26)
\]
\[ d_{hi} (m_{0h}, \text{Inc}_h, p) \geq 2 (n - 1) \] (27)

\[ \overline{p} > 0 \] (28).

Whatever fans’ expenditures over a season, the number of games they actually attend cannot be bigger than the number of their favourite team \( i \) games supplied by the league: \( 2 \ (n - 1) \). They are rationed by the league to this maximum number of team \( i \)’s games over the whole season\(^{19} \). Inequality (27) expresses the potential excess fan demand facing the league’s short supply. With a fixed price the market will adjust in quantity which is known as market rationing; fans adjust to available supply of games at a given price \( \overline{p} \). The short side of the market, \( i.e. \) the league’s supply of games, rations the longer side, \( i.e. \) fans demand. Moreover, since fans’ demand is trivially inelastic to a game fixed price, the price variable can be dropped, as in most disequilibrium models with price rigidity. Then the constraint (27) simplifies to:

\[ d_{hi} (m_{0h}, \text{Inc}_h) \geq 2 (n - 1) \] (29).

At the level of a whole league, that is for all games and all fans, market disequilibrium clearly is short supply (shortage) of games compared to fans’ demand - from their standpoint, they are in excess demand written as:

\[ \sum_{i=1}^{n} \sum_{h=1}^{g_i} d_{hi} (m_{0ih}, \text{Inc}_h) \geq 2n(n - 1) \] (30).

At first sight, it could be objected that fans’ demand does not take into account some variable reflecting game quality. There are various candidates for a game quality variable such as the team \( i \)’s standing in the championship, its win percentage, its quantity of talents \( t_i \) or its quotation by bookmakers. With regards to overall market disequilibrium (30), one could think of introducing some index of competitive balance on the left-hand side demand function \( d_{hi} \). However, this variable is not considered here under the assumption that a genuine team \( i \)’s fan is not attracted to the stadium by game quality, the opponent team quality or his/her favourite
team quality. The typical fan attends simply because this is the emotional and usual fan behaviour whatever their favourite team standing, win percentage, competitive balance and so on. This assumption will be dropped further when other game spectators such as TV viewers or couch potatoes are differentiated from fans.

Finally, relations (6) to (22) and (26) to (30) describe two building blocks of a team sport league’s economy that exhibits a double excess demand in labour and product markets which is known to reproduce a repressed inflation regime (Benassy 1983), also coined a shortage economy by Kornaï (1980), to which labour market segmentation has been added. As long as the budget constraint on soccer teams is soft – i.e. the break even point is not actually reached or enforced as a permanent governance rule -, European soccer leagues will remain in such a regime. The arm’s race among the clubs to acquire the most talented players in the market will go on. Fans will go on being frustrated of waiting (queuing) for more games of their favourite team since their willingness to attend and pay is bigger than the number of their favourite team games fixed by the league.

6. Differentiating between fans and TV viewers then between free-to-air and pay-per-view TV channels: a product market differentiation

Following now the distinction between two sorts of sport fans that emerged in the sports economics literature (2.3 above), it is assumed that fans and TV viewers do not behave the same way in the market for sports matches. The latter is to be split into two segments for two differentiated products. As regard fans’ demand and the corresponding segment of the market for sport shows nothing is changed compared to above inequalities (26) to (30). However, a second segment of the market for televised sport shows must be introduced into the model together with a consumer demand function for televised games by TV viewers.
Assuming that TV viewers audience is attracted by outcome uncertainty and not by a specific team \( i \) games, their variable of interest is no longer the number of their favourite team \( i \)'s games attended. One variable that encapsulates a game uncertainty of outcome lies in respective win percentages of two opponent teams \( i \) and \( j \), that is \( w_i \) and \( w_j \). Thus TV viewers’ utility is maximised when the outcome uncertainty of a televised game is the highest that is both win percentages of the two teams are the highest; their utility function is assumed to increase in win percentages. It is only beyond a given threshold of the couple \( (w_i, w_j) \) that a game is considered as attractive enough to represent a high quality product worth to be watched on. Beyond this threshold, that is for \( \bar{w} = \min (\bar{w}_i, \bar{w}_j) \), viewers switch on their TV, otherwise they do not. It follows that TV viewers’ demand is high for high quality games while only a proportion of overall games supplied by the league (assumed all to be televised) over a season will pass the quality threshold and will be effectively viewed on. Let \( d_{kij} \) stand for the number of high quality games between a team \( i \) and an opponent team \( j \) that a TV viewer \( k \) \((k = 1, \ldots, r)\) demands. A TV viewer utility is assumed to increase in strict proportion with the number of high quality games watched. Assume also that TV viewers of the \( k \) vintage do like only watching televised games for free and address their demand exclusively to free-to-air TV channels. Since all the games televised by these channels are not high quality, \textit{i.e.} passing the \( \bar{w} \) threshold, TV viewers \( k \) are rationed. If, say, only a proportion \( \lambda \) \((0 \leq \lambda \leq 1)\) of all televised games on free-to-air channels are high quality, then it follows for any TV viewer \( k \) :

\[
\begin{align*}
\text{Max } d_{kij} & \quad (31) \\
d_{kij} (m_{ok}, Inc_k, \bar{w}) & \geq 2 \lambda n (n - 1) \quad (32) \\
p = 0 & \quad (33).
\end{align*}
\]

Overall market excess demand derives from aggregating all the individual TV viewers’ demands:
This was the observed real situation when there was just one public free-to-air monopoly TV channel in operation per country. It may still be so now, when there are a few coordinated or collusive oligopolistic public and private TV companies. In practice, this translates into high audience ratios (or high TV market shares) when the $2 \times n \times (n-1)$ high quality games, with high outcome uncertainty, are broadcast on the one hand. On the other hand, only low audience is reached for a number $n (1 - \lambda)$ of boring games without enough outcome uncertainty. Relation (34) paves the way for empirical studies about audiences of televised sport on free-to-air channels in order to check whether they are low or not.

Now consider that televised games are no longer broadcast for free by public and free-to-air but by pay-per-view and commercial TV channels at a subscription fee $p^*$. The market situation (equilibrium or disequilibrium) will crucially depend on the level of the subscription fee - fixed ahead for all over the season - by TV companies in oligopoly situation. Of course, TV viewers $v (v = 1, \ldots, z)$ – those willing to pay for televised sport - will take $p^*$ into account in their demand function $d_{vij}$ when maximising their utility so that:

\[
\text{Max } d_{vij} \\
\quad d_{vij} (m_{0v}, \text{Inc}_v, \bar{w}, p^*) \geq \text{or } \leq 2 \alpha n (n-1) \\
p^* > 0
\]

with $\alpha$ the proportion ($0 \leq \alpha \leq 1$) of all games televised on pay-per-view channels which are high quality and that TV viewers are willing to pay for.

If by chance a TV channel exactly fixed ex ante the single value of the subscription fee $p^*$ that transforms inequality (36) into an equality, then this TV company would have found the price that TV viewers are exactly willing to pay for a proportion $\alpha$ of all the broadcast games without spending more than they can afford given their savings $m_{0v}$ and incomes $\text{Inc}_v$. In such
circumstances, $p^*$ is the market equilibrium price. In the current functioning of TV channels that overbid to obtain the broadcasting rights of European major soccer leagues, a TV channel which besides must cover the cost of such rights can find the equilibrium subscription fee only by chance. If a TV channel ex ante fixes a subscription fee lower than $p^*$, inequality (36) will be ≥ and TV viewers will be rationed in high quality games but will pay a low subscription fee for this ration, lower than their willingness to pay. If, as it is more likely to happen with oligopolistic commercial TV companies, a TV channel ex ante fixes a subscription fee higher than $p^*$, a number of potential subscribers will not subscribe, inequality (36) will be ≤ and the sport (soccer) broadcast market will be in excess supply.

7. Avenues for further research

Beyond the two building blocks shown here a lot of work remains to be done for achieving a complete disequilibrium model of a European team sports league. The first task is to check for, and analyse, plausible interactions between the labour and product markets as the one suggested in footnote 16. Before doing so, a third building block should be elaborated on. Indeed, TV viewers are not supplied with televised sport fixtures directly by the league but through an intermediary market for sport TV broadcasts on the supply side of which stand either a monopolistic league that pools TV rights or teams that negotiate with TV channels on their own right. On the demand side, oligopolistic competition between TV channels does not usually translate into flexible prices (TV rights), all the more so when channels are facing a monopoly league and the bid-winning TV channel is cursed. Thus this intermediary market is likely to operate off equilibrium. Once completed with a third market, the disequilibrium model should be resolved, and its mathematical properties exhibited, a task which calls for help from skilled model builders. Then, before data collection, some indices should be
designed in order to assess the existence and handle metrics of excess demand (e.g. outlying inflation for superstar wages, length of a fans waiting list, scalping) and excess supply (e.g. journey men player unemployment, rates of stadium utilisation, TV audience ratios). A further step of course should be robust econometric testing of the various building blocks after including appropriate disequilibrium indices.\textsuperscript{22}

In order to join a classical sports economics approach, at least to some extent, competitive balance should be introduced in the model, not only as a threshold to be reached like in (32), but also in the labour market block. Then competitive balance will be one of the interactive variables between two markets to be examined. An interesting question will emerge: is a sport contest more – or less – balanced with a team sports league in economic equilibrium or disequilibrium? There from a more extensive comparison between the standard equilibrium and newly-suggested disequilibrium models of a team sports league could be conducted with regard to mathematical properties, fitness to the empirical evidence of open leagues like in European soccer, and policy recommendations. A preliminary intuition derived from the first building block is that a salary cap would not moderate payroll growth as long as the labour market for superstars were to be in excess demand, that is as long as the teams’ budget constraint were not to be hardened up to not tolerating any financial deficit. From the disequilibrium model standpoint, the assessment of the UEFA Financial Fair Play, for example, should be different compared to the one derived from the standard equilibrium model. Once actually enforced, Financial Fair Play rules are likely to alleviate repressed inflation and somewhat curb superstar wage inflation in hardening the teams’ budget constraint.

A last avenue for further research could consist in disentangling the three theoretical situations of excess demand ($< 0$), excess supply ($> 0$) and equilibrium ($= 0$) in each building block. Then, fragmenting each side of a market into sub-groups of teams with different
(budget) constraints, it would be possible to compare, within a same model, the effect of, for instance, a soft, a hard and a balanced budget constraint. It would be of particular interest to allow deficit for a given sub-group of teams and impose the precondition of breaking even on other teams in the league, a differentiation which seems relevant to European soccer leagues. Then the model resolution will exhibit how the excess demand for superstars and competitive balance would evolve under such an assumption. Disentangling the three theoretical solutions fits here with the real world of some teams sticking to their budget constraint while others are overinvesting in talents, a major concern for policy makers in charge of European soccer today.

8. Conclusion

This article has explored two building blocks for a disequilibrium model of European team sports leagues in excess demand on both markets for talents and sport shows with win-maximising teams. The suggested model presents two breakthroughs. For one, it fits well with a number of actual facts characterising European soccer leagues such as price rigidity, teams’ soft budget constraint, arm’s race for superstars, labour market segmentation, and a product market differentiation between fans and TV viewers. Second, with an infinite number of solutions, not only a unique equilibrium, it can adapt to different market situations and disequilibria in European soccer.

This first attempt opens several avenues for further research regarding how the building blocks presented here interact - and then a better model specification, how design excess demand and excess supply empirical indices. We anticipate econometric testing that includes such indices, which can deepen a comparison between the respective strengths and
weaknesses of the standard equilibrium and disequilibrium models of a European team sports league and, finally, can deliver appropriate policy recommendations.

References:


**Table 1: Economic disequilibrium regimes**

<table>
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<tr>
<td>Labour market</td>
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<td>excess supply</td>
<td>excess demand</td>
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* This regime is mathematically unstable and eventually degenerates into repressed inflation (Benassy, 1982).
End notes:

1 I thank the three reviewers appointed by the IJSF for their comments; any remaining mistake is of my own.

2 The article basically focuses on European professional soccer teams and leagues.

3 With the anecdotal exception of Barro (2000).

4 See the Journal of Sports Economics 7 (1) 2006 special issue on the financial crisis of European football and a part of the JSE 8 (6) 2007 issue.

5 A disequilibrium model is more general than an equilibrium model which the solution is reached when the budget constraint exactly equals zero. In all the many (infinite number of) events when the budget constraint is either > 0 or < 0, disequilibrium self-sustains and lasts. A unique solution (= 0) is not crucially looked for in disequilibrium modelling where supply-demand equilibrium is considered as pure hazard or strictly theoretical. Kornaï (1980) compares Walras economic equilibrium with the point of absolute congealing in physics. Both have no empirical existence while being at the roots of theoretical reasoning and practical metrics.

6 A superstar effect on wage determination has been clearly exhibited in Italian soccer by Lucifora and Simmons (2003).

7 That a qualitative differentiation between player talents triggers labour market segmentation, though in American baseball, is also referred to in Hill and Spellman (1983) and Vrooman (1996) since there are two different wage rates for differently talented players in their models.

8 A high survival rate is clearly verified in English soccer by Kuper and Szymanski (2009).

9 Though tested for NBA basketball, fans wish at least two-thirds of their favourite team wins (Rascher and Solmes, 2007).

10 In France in 2010, 98,000 hours of sports were televised by pay-per-view and commercial TV companies against 2,000 hours by public and free-to-air channels (CSA, 2011). A TV viewer willing to subscribe to all the supply of sports broadcasts would have been offered to pay over €2,000 per year. If addicted TV viewers can envisage spending such an amount for watching sports all day and all year long, nobody can spend more than 24 hours (98,000 / 365 = 268 hours) per day watching his/her TV screen.

11 As one is used to define marginal productivity – since the entire model is not resolved so far (see 7 below).

12 When a market is in disequilibrium, one side (for instance supply) of the market is shorter than the other one (demand); therefore the excess demand. In an excess supply situation, the demand side is shorter than the supply side. This means that, in an excess demand market, aggregating all the microeconomic demands (of all teams) comes out with a bigger quantity of talents than the (aggregated) quantity supplied by suppliers (all players). In
excess supply, the aggregate quantity of supplied talents is bigger than the (aggregated) demand of all teams.

Usually those economic agents on the short side of a market have a stronger bargaining power than those on the longer side; they successfully negotiate and bargain on their own terms - prices (thus they are price-makers) and transaction conditions -, and obtain a better pay off for what they deliver to the market.

13 Since a soft budget constraint leads to “hoard” labour within the enterprise (team) – thus hedging against future labour market shortage – all European soccer teams are eager to recruit as many players as possible, including disposing of a great number of potential substitutes to seat on the touch-bench.

14 Some of them, often not accounted for as unemployed, simply revise downwards the terms of their supply of talent and switch to a lower division team or a weaker foreign league. Supply (and demand) revision and switching from one market to another are at the heart of rationing schemes and adjustment processes in disequilibrium models.

15 Though weaker in European soccer leagues than in North American major leagues.

16 In addition, they wish, expect or even bet on their favourite team winning as many games as possible (see footnote 9). However, they realistically cannot have a straightforward demand for home and away wins, due to outcome uncertainty. Their demand for wins is indirect and takes the form of their support (or request) to their team overinvesting in talents, which exactly fits with team behaviour as described in inequalities 6 to 8. This is an interaction between the product and labour markets.

17 Inc_h is the share of his/her overall income that a fan h can afford to pay for game attendance.

18 This assumption materialises price rigidity in tune with the literature mentioned in 2.3 above.

19 Here it is assumed that a fan wants to attend all his/her favourite team’s games, home and away, and that he/she can financially afford it. Of course, it could easily be assumed that he/she wants attending only home games, then the supply he/she faces would simplify to (n – 1) without any major change in the model.

20 Some insights are found in Andreff and Bourg (2006) and in Késenne (2007, p. 22-25) for a tentative modelling in an equilibrium framework.

21 Empirical evidence of a league’s monopoly pooling TV rights facing oligopolistic TV companies in European soccer (Buraimo, 2006; Cave and Crandall, 2001) suggest that prices are not flexible; transactions are often plagued with overbidding, a winner’s curse (Andreff, 2012), and a higher than equilibrium price.

22 Just to give an example, derived from (21), \( T_a - \sum_{i=1}^{n} t_{i} \) is an (unemployment) index signalling an excess supply in the labour market for day men players.