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JEL Codes: C78, D85 , G21, G23

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Cash Providers: Asset Dissemination over Intermediation Chains*

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Abstract

Many financial assets are disseminated to final investors via chains of over-the-counter transactions between intermediaries (investors or dealers). We build a model where an agent buying some units of the asset can offer to sell part of them to an OTC partner. Intermediation chains are endogenously formed and impact the asset's market liquidity, its issuance, and who ultimately holds the asset. An increase in the intermediaries' funding liquidity (e.g. a lower haircut on the asset) makes intermediation less necessary but also makes it cheaper to issue the asset, increasing the total volume to be distributed and the number of intermediaries and agents holding the asset. We derive implications on liquidity in OTC markets, the dissemination of "toxic" assets and the collateral policy of central banks and CCPs.

Journal of Economic Literature Classification Number: C78, D85, G21, G23.

Keywords: OTC markets, collateral, intermediation chains, shadow banking, systemic risk.

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1 Introduction

Many financial assets are disseminated to final investors through a number of intermediaries trading over-the-counter. These intermediaries buy assets and sell some of the units to their customers, who typically hold them to maturity, and to other dealers. The benefit for the issuer of a new asset depends on the price at which the asset is traded, and thus on the availability and resources of the intermediaries and their access to final investors, either directly through a single intermediary, or indirectly through intermediation chains.

Our objective in this paper is to analyze the formation of intermediation chains and their impact on investment. We provide a strategic model of the building up of chains based on the search for agents with a surplus of liquidity that needs to be invested. We characterize equilibrium behavior, for both the origination of an asset and the terms of trade (prices and units sold), in a game with possibly several successive intermediaries. We can thus analyze the asset's market liquidity, the length of intermediation chains and how the asset disseminates across these chains. We show how these elements depend on various market conditions such as the links between intermediaries in the OTC market, their funding liquidity, and the asset's collateral value.

The game first involves an origination stage in which an "originator" has an investment opportunity and can issue an asset backed on its return. A typical example would be a bank with securitizable mortgages. Three external sources of financing can be combined: unsecured borrowing at the market rate, secured borrowing using the asset as collateral, and the sale of some units of the asset. The originator conducts his sales on an OTC market and makes offers to his 'partners', financial institutions with which he usually trades. A partner who accepts the offer uses his cash and the three external financing sources just described to finance his purchase. In particular, he can sell some of the units bought (possibly at a different price) to an OTC partner, in which case the process goes on until an offer is turned down by everyone, or until no new offer is made.

The offers made by an intermediary depend on two ingredients: how much financing he needs to find, and which behavior he expects from his partners when he makes an offer. With high financing needs, he will be eager to ensure that his offer is accepted, which can be

achieved by conceding a rebate. A partner who does not have enough cash to purchase the asset may still find it profitable to buy at a low price and try to sell to his own partners, thus intermediating between two agents. This is the source of intermediation chains. Now the rebate level depends on his expectations on his partners, which themselves depend on their expectations on their own partners, and so on. We show that a single ‘benefit’ function embeds all these successive expectations, which includes both the direct benefit an intermediary draws from having access to his partners, and the indirect benefit of these partners having access to another layer of intermediaries, and so on. All the funding that an intermediary can expect to draw from his network of partners is subsumed in this function, which allows us to study how such a network reduces the funding costs of the originator and to derive the impact of the OTC market on optimal investment.

We derive a number of empirical implications, which can be compared to the empirical evidence on OTC markets, as we discuss in the text. First, we consider the price at which the asset is traded between intermediaries. We show that they tend to concede larger rebates when their financing needs are higher, which implies that the spread between the asset’s fundamental value and its market price goes down over time, as the asset is passed along an intermediation chain. Moreover, rebates are proportional to the haircut imposed on the asset when it is used as collateral: the intermediaries can more easily finance the purchase of the asset when it has a lower haircut, which makes its price closer to its fundamental value.

Second, our framework is particularly useful to study the level of intermediation, as measured by the length of intermediation chains. Changing parameters affecting the liquidity of the financial system leads to two conflicting effects. First, lower haircuts on the asset, the access to cheaper unsecured funding or the availability of more intermediaries allow each intermediary to accommodate a higher volume, which increases market liquidity and makes long chains less necessary. But due to the increased liquidity of the asset, the originator issues a higher quantity and the total volume to be distributed increases, which gives rise to longer intermediation chains. Different markets will thus react differently to changes in the funding conditions of intermediaries, depending on how sensitive origination is to funding costs. We show in particular that if origination is sufficiently elastic to costs, a higher collateral value for the asset leads to longer chains and more dissemination of the asset.

The model delivers implications for several markets and contexts. First, we derive several comparative statics results on market liquidity that can be compared to the empirical literature on OTC markets. Our results on the length of intermediation chains in particular are related to the empirical literature on dealer networks.

Second, the model can be used to study the dissemination of securitized loans through the shadow banking sector. Several studies show that the riskiness of U.S. subprime loans was underestimated and led to artificially high collateral values (see e.g. [Rajan, Seru, and Vig \(2013\)](#)). In that case, an implication of our model is that a high volume of origination and dissemination follow simply due to financial intermediaries' search for cheap financing.

Third, the impact of the asset's collateral value on its trading and origination has important implications for agents setting reference haircuts such as central banks and CCPs. We show in particular that by accepting assets at a lower haircut a central bank can actually reduce the demand for central bank funding by facilitating private transactions. Moreover, we show that all intermediaries along a chain provide their cash entirely to avoid costly unsecured borrowing. Thus, after the transactions take place all intermediaries along a chain are 'tight' in cash and have engaged all their assets as collateral. If the asset's value turns out to be lower than its collateral value, a systemic event occurs in which all intermediaries simultaneously lack the cash to repay their secured loans, and the collateral kept by the CCP does not cover the losses. This 'systemic' event is due to a common exposure and the fact that strategic trades generate an extreme use of their cash, and not to contagion, since our intermediaries have no engagement between themselves.

The end of this section relates our work to the theoretical literature. [Section 2](#) describes the model, [Sections 3](#) and [4](#) solve for the equilibrium, [Section 5](#) derives comparative static results and [Section 6](#) details applications of our model and how it relates to empirical studies.

Related literature. Our approach to OTC markets is related to the literature that analyzes the terms of trades and the role of intermediaries in a decentralized market. The limited access to potential trading partners and asymmetric information constitute the main sources of departure from the perfect market paradigm.

Two main approaches have been followed to model limited market access in a tractable

way, namely bilateral matching and networks. Bilateral random matching with search, initiated by [Rubinstein and Wolinsky \(1987\)](#), is the basis of recent models of OTC markets starting with [Duffie, Garleanu, and Pedersen \(2005\)](#). Traders meet randomly in pairs, bargain and either reach an agreement or fail to do so and search for another partner. Applied to OTC markets, such an analysis sheds light on how the bargaining protocol and the option values as represented by the availability of future partners affect the pricing and the liquidity of assets ([Duffie, Garleanu, and Pedersen \(2007\)](#)). The most related papers to ours are [Atkeson, Eisfeldt, and Weill \(2012\)](#), who introduce profits from intermediation, and [Neklyudov \(2012\)](#), who matches the core-periphery structure of dealer networks. These models however are not adequate to study intermediation chains as they focus on a stationary Markovian environment. In our analysis instead intermediation chains start with an originator and have an end. By assuming a particular random network structure (not bilateral) we are able to solve the technical difficulties through an equilibrium benefit function, which plays the role of a value function in dynamic programming.

The network approach provides another way to model restrictions to trade, as transactions must occur between pairs of linked agents. Such a modeling is convenient to understand the impact of “local” monopoly power, held by an intermediary who is the only one to connect some agents (see for example [Blume *et al.* \(2009\)](#) in a two-sided context). Most of these papers concentrate on the trading of a single indivisible object. The main questions are whether the owner of this object can sell it to an interested buyer, whether this buyer is the one with the highest valuation, and how much the intermediaries will extract from the sale. This is the approach followed by [Condorelli and Galeotti \(2012\)](#), [Manea \(2013\)](#) or [Gofman \(2011\)](#) on an OTC market. We instead assume a perfectly divisible asset in order to study dissemination. A financial intermediary who is involved in a transaction chain keeps part of the units purchased, possibly on behalf of his customers, and resells another part to other intermediaries when he is liquidity constrained. Ultimately, the originated units are disseminated across various intermediaries and their customers. Finally, [Malamud and Rostek \(2012\)](#) develop a very general model of decentralized trading and mainly study the implications on an asset’s liquidity, but not the build-up of intermediation chains per se. Another important departure from the above referred papers is that the volume of assets to

be sold is endogenously chosen by an originator, which enables us to study the impact of the OTC market on investment.

Asymmetric information is also an important concern in a decentralized trading environment and has been recently introduced in network models. In a common-value model à la Akerlof in which traders are heterogeneously informed about the value of the asset, [Glode and Opp \(2014\)](#) argue that intermediation chains mitigate the adverse selection problem. Indeed, in their model, the overall informational gap between the seller and the buyer is ‘spread’ along the chain: as intermediaries’ information is assumed to be increasing along the chain, each pair of consecutive intermediaries faces a ‘weak’ adverse selection problem. In a private-value model, in which traders’ valuations differ but are independent across traders, [Condorelli and Galeotti \(2012\)](#) analyze a dynamic bargaining process over a single object. Our model can also be seen as a private-value model as intermediaries are privately informed on their cash holdings and this drives the terms of trade. As intermediaries face different offers along a transaction chain and have different cash endowments, the transaction prices and volumes differ across potential pairs, thereby providing a rich framework for deriving empirical implications.

2 The model

This section first presents the model, discusses its potential applications and then studies the financing decision of an intermediary.

2.1 The game

The originator and the asset: An originator chooses to originate k assets at a cost $C(k)$. The asset delivers a random payoff $\tilde{\rho} \in \mathbb{R}^+$ per unit according to a continuous distribution $F(\cdot)$. We denote ρ the expected value of $\tilde{\rho}$.

To simplify, the originator has no cash and needs to finance $C(k)$, by using unsecured and secured borrowing and by selling some units of the asset. There is no liquid market for the asset, which can only be sold over-the counter to an intermediary¹. Intermediaries typically

¹For simplicity, we call all agents except the originator “intermediaries”, even though some of them may

have some cash but may also rely on the same three financing sources that we describe now in detail.

Secured and unsecured borrowing: Against one unit pledged as collateral, external financiers are ready to lend some amount ℓ at the secured interest rate. $\rho - \ell$ is the *haircut* per unit of collateral. To complement secured borrowing, each intermediary can also borrow on the unsecured market at an exogenous interest rate r , so that he has to repay $1 + r$ after $\tilde{\rho}$ is realized. No collateral is then required.

The OTC network: Each intermediary, including the originator, knows the same number d of other intermediaries, who each have a probability q to be available for trade when they receive an offer (independent draws). No intermediary can thus be contacted more than once and the set of all intermediaries available for trade is the outcome of a Galton-Watson process, frequently analyzed in the computer science literature (see e.g. [Kleinberg and Raghavan \(2005\)](#)). The probability that at least one intermediary is available is

$$h = 1 - (1 - q)^d. \tag{1}$$

Intermediaries have unlimited liability and are risk neutral. Each intermediary starts with some amount of cash ω independently drawn from a distribution $G(\cdot)$. Cash is invested at the secured interest rate, normalized to zero.

The trading process: Once the originator has decided on the investment level k , he can decide on a take-it or leave-it offer (p, v) made to all his partners in the following layer. Each intermediary who receives the offer decides whether to accept it and buy v units of the asset at price p , or refuse. If $d > 1$ and several intermediaries accept the offer, then the originator randomly chooses one of them, I , to trade with.

Intermediary I then has the same financing options as the originator, and can decide in turn on a take-it or leave-it offer made to all his partners in the following layer. The game continues until an offer is turned down by everyone, or until no new offer is made. The initial

not buy and sell in equilibrium.

volume k can thus be distributed on a network of intermediaries selling sequentially on an over-the-counter market.

Figure 1 gives an example with two layers. One intermediary is inactive in the first layer, one rejects the offer, and the third one accepts it. This intermediary makes a new offer, which is rejected by two intermediaries and accepted by one.² Figure 2 shows the flow of cash and assets in a realized intermediation chain obtained with the simulations we present in section 4.2. 0.43 units are originated, 0.11 of them are pledged as collateral in exchange for 6.60 units of cash, the 0.32 others are sold to I_1 for 31.25 units of cash, the total covering the origination costs. I_1 collects the 31.25 units of cash by selling part of the assets for 25.70, using his own cash equal to 1.95, and borrowing 3.60 against the remaining assets. The chain ends with I_3 , who has a higher amount of cash and does not need to sell the assets he buys.

[Insert Fig. 1 and 2 here.]

Information: The return ρ , the network characteristics d and q , the distribution G of cash are common knowledge. When making an offer, an intermediary I does not know how many partners are available nor their amount of cash. Whether an intermediary knows what happened before he receives an offer has no influence on his behavior.

Technical assumptions:

(A1): The originator's cost function C is differentiable, strictly increasing and convex: $C' > 0, C'' \geq 0$. Furthermore, $C(0) = 0$ and $C'(0) \leq \frac{\rho+r\ell}{1+r}$, which ensures a positive level of origination.

(A2): The distribution of cash $G(\cdot)$ admits a continuous and log-concave density G' , positive on \mathbb{R}^+ . $G'(0)$ is finite and $\lim_{\omega \rightarrow \infty} \frac{G'(\omega)}{1-G(\omega)}\omega < 1$.

The monotony and convexity assumptions on C are standard. The last assumption simplifies the presentation by avoiding to consider the case where the originator does not invest: he chooses to issue a positive volume, even without access to the OTC market (see section 4.1).

Most common distributions (Gaussian, uniform, beta, or exponential) are log-concave. The conditions at the boundary ensure that the distribution of cash is not too concentrated

²All figures and tables are in the Appendix A.

on very small or very large values.

Discussion: the model is kept quite general so as to accommodate a number of applications, which will be detailed in section 6. The “originator” can be a bank extending and securitizing loans, but also a company or a municipality issuing bonds and selling them to primary dealers. The model can also be applied to the secondary market, in which case the originator is simply an agent holding the asset and considering selling some units via partners on the OTC market. As will be discussed later, these different examples may imply different shapes, and in particular elasticities, for the function C . If intermediaries are interpreted as dealers then ω is a demand coming from customers, so that a dealer can buy assets for the amount ω without incurring funding costs.

It is convenient to assume that agents cannot default (unlimited liability) so as to avoid risk-shifting effects. This is not an unrealistic assumption if the model is applied to a situation where institutions need cash in the short term (e.g. overnight). There may still be a risk for the lender, such as a delayed reimbursement: if an intermediary borrows to cover his financing needs and the realized $\tilde{\rho}$ is low he may not have enough cash to reimburse immediately. As a guarantee against this risk, the lender asks for collateral and applies a haircut $\rho - \ell$. ℓ is typically set equal to a value-at-risk at a conservative level, and thus depends on the riskiness of the asset. Without collateral the borrower has to pay an extra premium r .

The particular trading protocol we consider entails two frictions. First, the originator can only contact a limited (and random) number of agents, who have a limited amount of cash, making access to the network of intermediaries beneficial. Second, there is two-sided asymmetric information between any buyer and seller on their respective cash holdings. The first friction is key as it gives rise to the intermediation chains we are interested in. The second friction enriches the trade-off faced by the intermediaries and is a source of potential inefficiencies, as some opportunities may not be exploited. The intermediaries could use more sophisticated trading mechanisms to address these informational inefficiencies, especially when $d > 1$,³ but it is well known that no mechanism can fully suppress this friction (Myerson and Satterthwaite (1983)).

³See for instance Zhu (2012) for a model of OTC trading with two-sided asymmetric information and repeated contacts between trading partners.

In our model the only gains from trade come from differences in funding costs and in available cash across different agents, other elements like diversification or differences in information are on purpose kept out of the model. This explains why the knowledge of the originator and the issued amount has no influence on an intermediary behavior.

2.2 The financing decision

We describe here the problem faced by I , who has himself received an offer, denoted by (p_0, v_0) ,⁴ and can make a new offer, thus intermediating between two agents. To decide whether or not to accept the initial offer, I evaluates how much profit can be achieved by accepting it, and accepts if profit is non-negative. This profit depends on how the purchase is financed. In particular, I 's optimal behavior depends on his expectation that an offer to sell some units of the asset is accepted.

Financing needs: assume I has ω in cash and accepts the offer (p_0, v_0) . Since the v_0 units can be used as collateral, the amount $\ell v_0 + \omega$ is available at zero interest to finance the purchase $p_0 v_0$. I 's *financing needs* are defined as the residual amount:

$$y = \max[(p_0 - \ell)v_0 - \omega, 0]. \quad (2)$$

I needs to borrow this quantity if he makes no offer, in which case his profit is

$$(\rho - p_0)v_0 - ry. \quad (3)$$

Offers: let I make an offer (p, v) to his usual partners. I cannot sell more units than he buys, so v has to be lower than v_0 .

If (p, v) is not accepted, I is in the same situation as if he had made no offer: y must be borrowed on the unsecured market at the cost ry , and the expected profit is as in (3).

If (p, v) is accepted, I keeps $v_0 - v$ units, receives pv from the sales, and can borrow $\ell(v_0 - v)$ on the unsecured market. The amount $pv + \ell(v_0 - v) + \omega$ is thus available at zero interest cost. If this amount is lower than $p_0 v_0$, the remaining financing needs are equal to

⁴This includes the case of the originator, with $v_0 = k, p_0 = C(k)/k$.

$p_0v_0 - (pv + \ell(v_0 - v) + \omega)$, which can be rewritten as $y - (p - \ell)v$: the financing needs are reduced by the value of the sale minus the loss in collateral. In the sequel we call $(p - \ell)v$ the *net cash value* of the transaction. There are three cases to consider: (i) the offer's net cash value *under finances* the needs: $y - (p - \ell)v$ is positive and has to be borrowed on the unsecured market; (ii) the offer's net cash value *over finances* the needs, $y - (p - \ell)v$ is negative, and the intermediary keeps some cash which earns zero interest (this is surely the case when y is null); (iii) the net cash value *exactly covers* the needs: no unsecured borrowing is needed and no cash is retained. In all these three cases, the interests to be paid due to unsecured borrowing or those received due to extra cash can be written as $r \max[y - (p - \ell)v, 0]$. The profit expected from an accepted offer (p, v) is thus equal to:

$$\rho \underbrace{(v_0 - v)}_{\text{Assets kept}} + \underbrace{pv - p_0v_0}_{\text{Intermediation profit}} - \underbrace{r \max[y - (p - \ell)v, 0]}_{\text{Funding costs}}. \quad (4)$$

Beliefs on acceptance: an offer may be turned down because no partner is available or no available partner finds it attractive. An intermediary forms some belief $\Phi(p, v)$ that an offer (p, v) is accepted. A natural assumption is that offers with a price larger than ρ are not accepted: $\Phi(p, v) = 0$ for $p > \rho$. Furthermore the probability of acceptance is bounded by the probability that at least one intermediary is available: $\Phi(p, v) \leq h$.

Profit and optimal behavior: given the belief described by Φ , using (4) and rearranging, the intermediary I 's expected profit from offer (p, v) is

$$\pi_I(p_0, v_0, y; p, v) = (\rho - p_0)v_0 - ry + \Phi(p, v)Q_I(y; p, v) \text{ where} \quad (5)$$

$$Q_I(y; p, v) = \begin{cases} r(p - \ell)v - (\rho - p)v & \text{if } (p - \ell)v \leq y \\ ry - (\rho - p)v & \text{if } (p - \ell)v \geq y \end{cases} \quad (6)$$

We call $Q_I(y; p, v)$ the *transaction value* for I of the offer. Comparing with (3), the additional gain obtained by the offer (p, v) is equal to the expected transaction value, that is the expected probability of acceptance $\Phi(p, v)$ multiplied by the transaction value.

Upon the receipt of offer (p_0, v_0) , intermediary I determines his optimal behavior in two steps. First, I chooses an *optimal* offer maximizing the expected transaction value over the

feasible offers. Second, I accepts (p_0, v_0) if this optimal offer yields a non-negative profit. Formally, an *optimal offer* solves

$$\Pi_I(p_0, v_0, y) = \max_{p \leq \rho, v \leq v_0} \pi_I(p_0, v_0, y; p, v). \quad (7)$$

and (p_0, v_0) is thus accepted if the value $\Pi_I(p_0, v_0, y)$ is non-negative.

The transaction value at an optimal offer is surely non negative (as I can make no offer) and is always less than ry as can be checked from (6). So we have

$$(\rho - p_0) - r(p_0 - \ell)v_0 \leq (\rho - p_0)v_0 - ry \leq \Pi_I(p_0, v_0, y) \leq (\rho - p_0)v_0 - r(1 - h)y. \quad (8)$$

Acceptance threshold: We now show that the intermediaries who accept an offer are those with enough cash. First, an intermediary with no financing needs surely makes a non-negative profit by accepting an offer as can be seen from (8). Under assumption (A2), there are such intermediaries. Second, profit is decreasing in y and thus increasing in the intermediary's cash ω . This implies that there is a threshold level of ω above which an offer (p_0, v_0) is accepted and this threshold is surely lower than $(p_0 - \ell)v_0$. This defines the *threshold function* W_Φ as:

$$W_\Phi(p_0, v_0) = \inf\{\omega \text{ such that } \Pi(p_0, v_0, \max[(p_0 - \ell)v_0 - \omega, 0]) \geq 0\}. \quad (9)$$

Offered prices are within two natural bounds. First a proposer never chooses a price larger than $p_0 > \rho$. Second he never chooses a price less than $\underline{p} = \frac{\rho + r\ell}{1+r}$: from (6), the transaction value is smaller than $-(\rho - p)v + r(p - \ell)v$ which is negative for $p < \underline{p}$.

Thus, we only need to consider prices between \underline{p} and ρ . For prices in this interval, the trade-off is the following. An offer at price \underline{p} (or less) is accepted by any intermediary I : at this price, I does not make a loss by using collateral and unsecured borrowing only, even if I has no cash as can be seen from the left hand side of equation (8). Hence $W_\Phi(\underline{p}, v_0) = 0$. An offer at price ρ is accepted only by those who have no financing needs (from (8) I 's profit is bounded above by $-r(1 - h)y$ which is negative for any positive y): $W_\Phi(\rho, v_0) = (p_0 - \ell)v_0$. Thus, considering offers for a fixed v_0 , making offers more attractive by decreasing the asked

price below ρ lowers the transaction value but lowers the risk of refusal and costly unsecured borrowing. How this trade-off is solved depends on Φ , which will be endogenized in the next section.

3 Equilibrium of the financing game

The game has two components: the investment decision of the originator, and the financing decisions of all intermediaries in the ensuing network. We first need to solve the second component, before analyzing the originator's choice.

3.1 The financing game

The situation faced by an intermediary is entirely characterized by the offer (p_0, v_0) he receives and his liquidity ω . The number of intermediaries between the initial proposer and the intermediary under consideration does not matter. The setting is thus Markovian with states described by (p_0, v_0, ω) . We look for a *stationary equilibrium* of this financing game, such that two intermediaries in the same state behave identically. Equilibrium entails a condition of correct expectations on the behavior of the receivers: assuming that the intermediaries' probability of acceptance follows a certain function, the optimal response induces the same function. To make this precise, let us introduce the probability that at least one intermediary at a given layer of the network is active and has more cash than ω , denoted by $H(\omega)$:

$$H(\omega) = 1 - [1 - q + qG(\omega)]^d. \quad (10)$$

We have $H(0) = h$, the probability that at least one intermediary is available. An intermediary facing offer (p_0, v_0) expects receivers to accept an offer (p, v) with probability $\Phi(p, v)$. He accepts (p_0, v_0) if his cash is above the threshold $W_\Phi(p_0, v_0)$ defined by (9). Hence the probability that (p_0, v_0) is accepted is $H(W_\Phi(p_0, v_0))$. Equilibrium requires that this is equal to $\Phi(p_0, v_0)$.

Definition 1. *An equilibrium is characterized by a threshold W and an acceptance probability Φ such that $W(p_0, v_0) = W_\Phi(p_0, v_0)$ and $\Phi(p_0, v_0) = H(W(p_0, v_0))$ for any (p_0, v_0) .*

At an equilibrium, an intermediary I who decides to make an offer to his partners only needs to know how they react. He thus behaves as if he faced only one layer of partners whose financing costs depend on their own cash ω and an extra “benefit”. We first solve a game with one layer only where this benefit is exogenously given, which then allows us to construct the equilibrium recursively by endogenizing this benefit when additional layers of partners are introduced.

3.2 A game with one layer only

We consider a game with two steps. In the first step, there is an intermediary I who faces offer (p_0, v_0) (with $p_0 \leq \rho$) and has financing needs y . I has to decide to either refuse the offer or accept it and possibly make an offer to his partners. In the second step, I 's active partners accept or reject the offer. If the transaction takes place between I and partner R , R receives in addition to the transaction value a benefit that depends on his financing needs only:⁵ R 's profit is augmented by a benefit $r\beta(z)$ if his financing needs are equal to z . The benefit is null for null financing needs, $\beta(0) = 0$, and is non-decreasing. We denote this game Γ_β .⁶ We make the following assumptions on β :

$$\beta(0) = 0 \text{ and there is } \bar{z} \geq 0 \text{ such that } \beta \text{ increases for } z \leq \bar{z} \text{ and } \beta(z) = \bar{\beta} \text{ for } z > \bar{z}. \quad (11)$$

$$\beta(z) - \beta(z') \leq h(z - z') \text{ for } z' < z. \quad (12)$$

These assumptions mean that the receiver's benefit is weakly increasing in his financing needs but not too much (as $h < 1$). They will be satisfied by the equilibrium benefit function we derive in the next section.

The single-layer game is easily solved by backward induction. An intermediary R who receives an offer (p, v) and has cash ω must finance $z = \max[(p - \ell)v - \omega, 0]$ on the unsecured

⁵This assumption as well as the technical assumptions (11) and (12) used in Proposition 1 below are automatically satisfied in our recursive construction of the equilibrium. In our general game, the benefit function β is determined endogenously at equilibrium, and must be equal to β^* . Thus, the properties stated in Proposition 1 apply as well.

⁶Strictly speaking, there is a game for each value of (p_0, v_0, y) .

market. Thus R 's expected profit if he accepts (p, v) is

$$\pi_R(p, v, \omega) = (\rho - p)v - rz + r\beta(z) \text{ with } z = [(p - \ell)v - \omega]^+. \quad (13)$$

R optimally accepts the offer if this profit is non-negative. π_R is increasing in ω up to $\omega = (p - \ell)v$ and then constant equal to the non-negative value $(\rho - p)v$. This defines the threshold value $w(p, v)$ as the minimum cash for R that yields him a non-negative profit. Thus, I 's profit is given by (5) where the acceptance probability Φ is defined by $\Phi(p, v) = H(w(p, v))$. I 's optimal behavior is defined accordingly. The next proposition derives I 's optimal behavior:

Proposition 1. *Let intermediary I face offer (p_0, v_0) and have positive financing needs y in Γ_β , with β satisfying (11) and (12). Define:*

$$\beta^*(y) = \max_{0 \leq \omega \leq y} H(\omega)(\omega + \beta(y - \omega)) \quad (14)$$

$$\tau(y) = \arg \max_{0 \leq \omega \leq y} H(\omega)(\omega + \beta(y - \omega)) \quad (15)$$

$$\text{and } \pi_I^*(p_0, v_0, y) = (\rho - p_0)v_0 - ry + r\beta^*(y). \quad (16)$$

At equilibrium, I accepts the offer if and only if $\pi_I^(p_0, v_0, y) \geq 0$. In that case, I makes a new offer (p, v) with $w(p, v) \in \tau(y)$, I 's expected transaction value is positive equal to $r\beta^*(y)$ and I 's expected profit is $\pi_I^*(p_0, v_0, y)$. p and v satisfy:*

$$(p - \ell)v = y \text{ and } (\rho - p)v = r(y - \omega - \beta(y - \omega)), \omega \in \tau(y). \quad (17)$$

The proposition states that I 's optimal behavior is characterized by the functions β^* and τ as defined by (14) and (15), which we call the *benefit* and the *target*. Both are functions of I 's financing needs but do not depend on ρ , ℓ and r . Knowing I 's financing needs y is thus enough to determine I 's equilibrium expected transaction value, which is equal to r times $\beta^*(y)$; then, provided that together with the purchase of (p_0, v_0) this results in a positive expected profit for I , $\pi_I^*(p_0, v_0, y) \geq 0$, I makes an offer. According to (17), I 's optimal offer (p, v) reduces to choosing a cash level in the target⁷, that is who will be indifferent between

⁷As β is not assumed to be concave, several values may be in the target.

accepting or rejecting the offer. p and v are then determined by this indifference condition and the fact that I 's offer exactly covers his financing needs.

To understand this result, we first interpret $r\beta^*(y)$ as an upper bound on I 's expected transaction value. Once the transaction takes place between I and a receiver R with cash ω , the sum of I 's transaction value and R 's profit can be shown to satisfy

$$\pi_R(p, v, \omega) + Q_I(y; p, v) \leq r[\omega + \beta(z)]. \quad (18)$$

This inequality simply says that the overall gain for I and R can only come from R 's cash and extra benefit because the transfers of the payment and the asset (accounting for the collateral value) creates no value. This implies in turn that I 's transaction value is not larger than the interest on the cash and benefit of a receiver who accepts the offer because their profit is non-negative, that is for those with cash at least equal to the threshold $w(p, v)$.

Furthermore, thanks to (12), I 's optimal offers surely do not over-finance his needs.⁸ The financing needs of R at the threshold are thus lower than $y - \omega$ so that, assuming here that they are positive, we obtain

$$Q_I(y; p, v) \leq r[\omega + \beta(y - \omega)] \text{ at } \omega = w(p, v). \quad (19)$$

Multiplying by the probability of acceptance, it follows that I 's expected transaction value at (p, v) is bounded above by $rH(\omega)(\omega + \beta(y - \omega))$ at $\omega = w(p, v)$, which is itself bounded by $r\beta^*(y)$.

Thus $r\beta^*(y)$ provides an upper bound to the expected transaction value. According to Proposition 1, this upper bound is reached. From the preceding argument, this can be true only if I makes an offer whose threshold is in $\tau(y)$ and transaction value is exactly equal to the cash and benefit of this threshold. This is performed by the offer characterized by the equations (17). The first equation requires that I exactly covers his needs and the second one that the profit of R with cash equal to ω is null, i.e. that ω is the threshold associated

⁸ The receiver's profit is decreasing in his financing needs thanks to ((12)) as can be seen from (13) so that neither I nor R gain in an offer over-financing y . If $(p - \ell)v > y$, decreasing slightly the price leaves unchanged I 's transaction value and increases the receivers' profit for those who have positive financing needs, hence increases the acceptance probability. The offer cannot be optimal.

to $w(p, v)$ (the main point is to show that the offer is feasible, which is true if the profit is positive).

To illustrate, let us consider the game with a null benefit function ($\beta = 0$), denoted \mathcal{G}_1 , which will also be used in the next section.

Null benefit function: The null function satisfies (11) and ((12)). R 's profit with positive financing needs can be written simply as $(1 + r)(\underline{p} - p)v + r\omega$. The equilibrium benefit for I in \mathcal{G}_1 is defined as:

$$B_1^*(y) = \max_{0 \leq \omega \leq y} H(\omega)\omega \quad (20)$$

Assumption A2 on the distribution G ensures that H is log-concave and that $H(\omega)\omega$ reaches its maximum at a (unique) positive finite ω^* (see the Online Appendix E.1).⁹ It follows that the target is unique given by $\tau(y) = y$ for $y \leq \omega^*$, and by $\tau(y) = \omega^*$ for $y \geq \omega^*$.

For $y \leq \omega^*$, an optimal offer must have a price equal to ρ and the optimal offer is unique. For $y > \omega^*$, the offer characterized by (13) is not the unique optimal one: the price can be increased up to ρ while keeping the value $(p - \underline{p})v$ constant (v decreases so surely $v < v_0$ and $(p - \ell)v$ decreases so is lower than y). Such an offer does not over-finance y and targets ω^* so that I 's nor R 's profit are unchanged: this is an optimal offer.

Inspection of the definition (15) of τ yields the following corollary.

Corollary 1. *The target's financing needs $y - \tau$ are weakly increasing in y , and there is $\underline{y} > 0$ for which $\tau(y) = y$ for $y < \underline{y}$, and $\tau(y) < y$ for $y > \underline{y}$. It follows that an optimal offer has $p = \rho$ for $y < \underline{y}$ and $p < \rho$ for $y > \underline{y}$. The value \underline{y} is less than ω^* , the unique value that maximizes $H(\omega)\omega$.*

Corollary 1 is included in Proposition B.1 of Appendix B.2, as well as its proof, which relies on the log-concavity of H and on assumptions (11) and (12). Remember that an optimal offer solves a trade-off between selling at a higher price and ensuring a higher probability that the offer is accepted. The corollary shows that when I 's financing needs y increase, his target $\tau(y)$ may increase (thus implying a lower execution probability), but less than y . In

⁹The Online Appendix can be found at <http://sites.google.com/site/jecolliardengl/CP-Appendix.pdf>.

particular, if y is sufficiently small, I chooses $\tau(y) = y$ and thus makes an offer at price $p = \rho$ with the lowest probability of acceptance $H(y)$.

Notice that an optimal offer may not be unique, for two reasons. First, the target may contain several values if β is not concave. Second, the offer characterized by (17) may not be the only optimal one associated to the same threshold when β is locally constant, as is illustrated in the game \mathcal{G}_1 . This second type of multiplicity does not arise when β is always strictly increasing, which will be the case in our general game.

3.3 Equilibrium of the financing game

We now construct an equilibrium of the financing game by considering a sequence of games. Define \mathcal{G}_n as the financing game with the restriction that the game will stop after at most n offers are accepted. We start with \mathcal{G}_1 ; then, knowing the behavior of an intermediary facing a network with at most one layer of other intermediaries, we can iterate and consider the problem of an intermediary facing two layers, three layers and so on. We show that the optimal strategy in the financing game coincides with the optimal one in a game with a finite number of layers.

As we can see on Figure 3 where B_1^* is plotted in an example, when $y \geq \omega^*$ the benefit B_1^* does not increase any more because the proposer extracts the maximum expected cash from a single layer of intermediaries. With a second layer it is possible to reach a higher benefit up to a new threshold, which can be increased again by adding a third layer, and so on. For each game \mathcal{G}_n we can define $B_n^*(y)$ the maximal expected benefit that an intermediary with financing needs y can make and $T_n(y)$ the cash level targeted by an optimal offer.

We show by induction that, for each y , there is a maximal value $N(y)$ for the number of rounds n beyond which the benefit $B_n^*(y)$ stops increasing and stays constant as more rounds are allowed and, furthermore, the optimal target $T_n(y)$ does not change. We denote $B^*(y)$ and $T(y)$ these limit values. Figure 3 shows plots of T and B^* in an example, as well as T_n and B_n^* for the first three iterations.¹⁰

Theorem 1. *The network benefit B^* and the target T do not depend on ρ , ℓ , and r .*

¹⁰The parameterization we use is introduced in Section 4.2.

They are characterized by:

$$B^*(y) = \max_{\omega \leq y} H(\omega)(\omega + B^*(y - \omega)) \quad (21)$$

$$T(y) = \arg \max_{\omega \leq y} H(\omega)(\omega + B^*(y - \omega)). \quad (22)$$

B^* is increasing, bounded above and $B^*(y) \leq hy$. For $y > 0$, $T(y)$ is smaller than ω^* . Given y , the financing needs of the targeted intermediary, denoted $Z(y)$, $Z(y) = y - T(y)$, are weakly increasing in y and an optimal offer satisfies (13) for any ω in $T(y)$. Finally, there exists $\underline{y}_1 \in (0, \omega^*)$ such that for $y < \underline{y}_1$ we have $B^*(y) = B_1^*(y)$ and $T(y) = y$; while for $y > \underline{y}_1$ we have $B^*(y) > B_1^*(y)$ and $T(y) < y$.

[Insert Fig. 3 here.]

The proof is given in the Appendix C, by considering the games \mathcal{G}_n and applying repeatedly Proposition 1. The benefit function B^* is strictly increasing because the benefit functions B_n^* are themselves increasing on intervals that grow with n . As will be studied in more detail in the next section, this reflects the fact that allowing for more layers changes the benefit for large enough financing needs. At the opposite, for small values, smaller than \underline{y}_1 , even one layer is enough since $B^*(y) = B_1^*(y)$ and the target is y : the offered price is ρ and the game surely ends after this offer. But the game goes on with positive probability as soon as $T(y) < y$, equivalently $Z(y) > 0$, as an intermediary with cash close to the target needs extra financing and will make a new offer.

Since the benefit function B^* is strictly increasing, for each target, there is a single optimal offer, which is defined by (13) (as explained in the previous section) and is computed in the following corollary. If B^* is concave, T and Z are single-valued and the optimal offer is always unique. If B^* is not concave, and there are several values in $Z(y)$, the following expressions yield an optimal offer for each value z in $Z(y)$. Note however that, though we have not been proved the concavity of B^* , we have no counterexamples either.

Corollary 2. *Optimal offers are given by*

$$P(y) = \frac{\rho y + r\ell(Z(y) - B^*(Z(y)))}{y + r(Z(y) - B^*(Z(y)))} \quad (23)$$

$$V(y) = \frac{y + r(Z(y) - B^*(Z(y)))}{\rho - \ell} \quad (24)$$

$V(y)$ is weakly increasing in y . For $y \leq \underline{y}_1$, $P(y) = \rho$ and $V(y) = y/(\rho - \ell)$.

These formula are useful in the next section to derive empirical implications on asset prices. The offered price $P(y)$ is maximal and equal to ρ only for an intermediary who has low financing needs and chooses $Z(y) = 0$; the price may not always be decreasing with y however, although we found no example where this was not the case in the simulations of section 4.2. This property always holds at least in the the special case we now study.

Example of a single-valued distribution: we illustrate the theorem with a simple case where all intermediaries (almost surely) have the same level of cash ω^* . Clearly it is then optimal to target exactly this amount of cash, so that for any y we have $T(y) = \omega^*$ and the probability that an offer is accepted is constant and equal to $h = 1 - (1 - q)^d$. The offer itself however depends on the financing needs. Indeed, for $y > \omega^*$ the financing needs of the targeted intermediaries decrease by ω^* at each step. Thus, the proposer relies on a chain with $N(y)$ intermediaries, where $N(y) = \lceil y/\omega^* \rceil$. We showed in the general case that $B^*(y) = H(\omega^*)(\omega^* + B^*(y - \omega^*))$, which by iteration implies here the following expression for the benefit:

$$B^*(y) = [h + h^2 + \dots + h^{N(y)-1}]\omega^* + h^{N(y)}(y - (N(y) - 1)\omega^*). \quad (25)$$

This explains why the optimal offer depends on y even though its chance of being accepted is constant and equal to h : when $y > \omega^*$ an offering intermediary makes an offer to a partner who may not find anybody available to trade, or who may find a partner who will not find a new partner... In order to be accepted, an offer must offer a compensation to the next intermediary for the risk of having is offer rejected, and for the compensation that he himself will have to offer, and so on. As a result, the offered price is decreasing in the financing

needs of the intermediary. As financing needs decrease along an intermediation chain, each intermediary offers a higher price than the previous one.

In the general case of a non degenerate distribution, there is an additional effect, as trading partners with different levels of cash will require different levels of compensation in order to accept an offer and act as intermediaries. With a sufficiently low price, even partners with almost no cash accept to behave as intermediaries, but this is of course a costly offer to make. $T(y)$ solves this trade-off and depends on y , so that in equilibrium the probability $H(T(y))$ that an offer is accepted varies with the financing needs y of the proposer.

3.4 Chains, network benefit and profit

According to Theorem 1, the properties of the OTC network that are relevant to an intermediary are summarized in the *network benefit* function B^* , which includes both the direct benefit an intermediary draws from having access to one layer of intermediaries, and the indirect benefit of this layer having access to another layer of intermediaries, and so on. Solving the financing game thus reduces to solving the problem (21) defining B^* , which we can then use to compute all the other quantities of interest. We show here some results on this function that will be particularly useful.

An intermediary with financing needs y targets an intermediary with cash $T(y)$ and financing needs $Z(y)$. This targeted intermediary chooses a target $T(Z(y))$, and this target will have financing needs $Z(y) - T(Z(y)) = Z^2(y)$. By iteration we can thus define a *targeted chain* starting with an intermediary with financing needs y . The length of this chain, denoted by $N(y)$, is the smallest integer j such that $Z^{j-1}(y) \leq \underline{y}_1$.

Conducting a similar iteration on the network benefit, we have $B^*(y) = H(T(y))(T(y) + B^*(Z(y)))$ and for $Z(y) > 0$, $B^*(Z(y)) = H(T(Z(y)))(T(Z(y)) + B^*(Z^2(y)))$ and so on. We thus obtain:

Corollary 3. *The length of the targeted chain $N(y)$ is increasing in y and*

$$B^*(y) = H(T(y)) \left(T(y) + \sum_{i=1}^{N(y)-1} T(Z^i(y)) \prod_{j=1}^i H(T(Z^j(y))) \right) \quad (26)$$

$$B^{*'}(y) = [H(T(Z^{N(y)-1})) + T(Z^{N(y)-1})H'(T(Z^{N(y)-1}))] \prod_{i=0}^{N(y)-2} H(T(Z^i(y))). \quad (27)$$

Proof: using the envelope theorem, at differentiability points we have:

$$\begin{aligned} B^{*'}(y) &= H(T(y))(B^{*'}(y - T(y))) \text{ for } T(y) < y \\ &= H(y) + yH'(y) \text{ for } T(y) = y \end{aligned} \quad (28)$$

Iterating this relation from y until the last intermediary's financing needs gives us the desired expression. ■

Expression (26) generalizes (25) which was obtained for a degenerate distribution. The network benefit for the first intermediary as expressed in (26) is the sum of the cash used by each targeted intermediary in the chain, weighted by the probability of acceptance at each step. The decision of each intermediary can be understood as determining the targeted chain that maximizes the cash extracted along the chain. Formally, for each y the optimal target $T(y)$ must satisfy the first-order condition associated to (21), which can be written as:

$$\frac{H'(T(y))}{H(T(y))} + \frac{1 - B^{*'}(y - T(y))}{T(y) + B^*(y - T(y))} = 0 \quad (29)$$

A difficulty in investigating the size of the chains comes from the varying values of the target. However, the limit behaviour for large financing needs is easy to determine. From Theorem 1, B^* is weakly increasing in y and bounded above, so that it converges, and the optimal target $T(y)$ converges as well, as is illustrated in Fig. 3.

Proposition 2. *Let*

$$H^\infty(\omega) = H(\omega) + H(\omega)^2 + \dots + H(\omega)^n + \dots = \frac{H(\omega)}{1 - H(\omega)} \quad (30)$$

and ω^∞ be the value that maximizes $H^\infty(\omega)\omega$ over the positive ω . We have:

$$\lim_{y \rightarrow +\infty} T(y) = \omega^\infty, \quad \lim_{y \rightarrow +\infty} B^*(y) = H^\infty(\omega^\infty)\omega^\infty$$

The value of the limits are derived in the proof of Theorem 1. The value $H^\infty(\omega)$ is the expected size of a chain where at each step there is an intermediary with a cash endowment at least equal to ω . Hence $H^\infty(\omega)\omega$ is interpreted as the network benefit a proposer can obtain in an infinite chain where every intermediary targets ω . Using Corollary 2, the convergence of T delivers asymptotic results about the other variables of interest in the model:

Corollary 4. *As y tends to $+\infty$, we have:*

$$\lim_{y \rightarrow +\infty} P(y) = \underline{p}, \quad V(y) \sim \frac{(1+r)y}{\rho - \ell} \quad \text{and} \quad N(y) \sim \frac{y}{\omega^\infty}$$

For large enough financing needs the offered price thus becomes close to its minimum value \underline{p} and the length of the targeted chain becomes proportional to y . For small values of y we can still get a lower bound on $N(y)$: as $T(y) \leq \omega^*$ by Theorem 1, the financing needs of an intermediary along the targeted chain decrease by at most ω^* at each step. To reach financing needs of \underline{y}_1 starting with some y , one needs $N(y)$ to be larger than $\lceil (y - \underline{y}_1)/\omega^* \rceil + 1$.

Finally, notice that $B^*(y)$ is a weighted sum of the cash collected along a targeted chain, but the total expected cash is higher than this quantity: at the first step for example, an accepting intermediary is likely to have more cash than the target, and the extra cash does not accrue to the originator. As a result, such an intermediary expects a positive profit. Let us consider the intermediaries who receive proposal $(P(y), V(y))$. Those with cash $\omega \geq T(y)$ accept the offer and their profit is

$$(\rho - P(y))V(y) - rz + rB^*(z) \quad \text{where } z = \max(y - \omega, 0). \quad (31)$$

As B^* is non-decreasing, we have:

Corollary 5. *The profit of an intermediary accepting $(P(y), V(y))$ is increasing in his cash ω .*

Consider an intermediary with positive financing needs. Each marginal unit of cash ω above $T(y)$ brings a marginal profit of $r(1 - B^*(y - \omega))$, which is positive (assuming differentiability to simplify). When the financing needs become null the profit is constant and equal to $(\rho - P(y))V(y)$.

4 Equilibrium of the full game

4.1 The originator's choice

We now solve for the optimal origination decision. Originating k units of the asset costs $C(k)$. The amount ℓk is borrowed at a null rate by using k as a collateral and the amount $y(k) = C(k) - \ell k$ needs to be borrowed at the unsecured rate r (remember the originator is assumed to have no initial fund for simplicity). Once he has originated k assets, the originator is in the same situation as any other intermediary who can sell up to k units and has financing needs $y(k)$. Thus an optimal offer provides the network benefit $B^*(y(k))$ and the originator's profit is:

$$\Pi_O(k) = \rho k - C(k) - r y(k) + r B^*(y(k)). \quad (32)$$

Before analyzing the optimal investment when the originator has access to a network of intermediaries, let us consider two polar cases. In the first case the originator has no access to a network ($d = 0$) and in the second one he does not need the network because the unsecured interest rate is null ($r = 0$). Thanks to assumption (A1) on C , optimal levels are positive and characterized by first order conditions. In the former case ($B^* = 0$) the optimal level k^{min} is characterized by

$$C'(k^{min}) = \frac{\rho + r\ell}{1 + r} \leq \rho \quad (33)$$

and in the latter case ($r = 0$) the optimal level k^{max} by:¹¹

$$C'(k^{max}) = \rho. \quad (34)$$

¹¹ k^{max} is also the optimal level for an originator who is not liquidity constrained.

Since marginal cost C' is increasing, the level k^{min} is smaller than k^{max} .

With access to a network, the originator has access to the cash resources of other intermediaries and can finance the projects at a possibly lower cost than without a network, but at a higher cost than when the interest rate is null.

Proposition 3. *Let k be the optimal investment for the originator. It satisfies:*

$$C'(k) = \frac{\rho + r\ell(1 - B^{*'}(y(k)))}{1 + r(1 - B^{*'}(y(k)))} \text{ where } y(k) = C(k) - \ell k. \quad (35)$$

k is between k^{min} and k^{max} , respectively the optimal levels without access to a network and with a null unsecured rate r . k is increasing in ρ , weakly decreasing in r , and weakly increasing in ℓ if B^* is concave.

The originator makes an offer with $p < \rho$, and chains have more than two intermediaries with positive probability.

See the Appendix D.1 for the proof. The comparison of (35) and (33) reveals that the marginal network benefit $B^{*'}$ acts like a reduction in the interest rate r , which measures the shadow cost of cash in this model. In the extreme case where $B^{*'}(y)$ is close to 1, condition (35) becomes equivalent to (34) and the originated volume is maximal. $B^{*'}$ depends in particular on the length of the intermediation chain targeted by the originator, as we saw in the previous section (Corollary 3). The next section studies in more detail how the lengths of the chains are affected by the financial conditions, as described by r and ℓ .

4.2 Numerical example

We illustrate the equilibrium with a numerical example, using a set of parameters that we will consider as representing a baseline scenario: $\rho = 100$, $\ell = 60$, $r = 0.2$, $d = 3$, $q = 0.8$, G is a Gamma distribution with a shape parameter $k = 1$ and a scale parameter $\theta = 5$, and the cost function is $C(x) = c_0x + (c_1/2) \times x^2$, with $c_0 = 80$, $c_1 = 40$. We solve for B^* and T and determine the optimal level of origination k .

We first consider the origination stage. Under the baseline parameters, the originator chooses to issue $k = 0.43$ units and targets a chain of size $N(y(k)) = 20$. He offers to sell

$v = 0.32$ at a price of 98.49 per unit, thus selling 74% of the originated volume and conceding a rebate of about 1.5%. To assess the impact of the network on the originator, Table 1 reports his investment and profit as well as those in the two polar cases, when the interest rate is null or when there is no network (referred respectively by a superscript max or min).

Scenario	k	$y(k)$	$N(y(k))$	Π_O	p	v
Baseline	0.43	12.16	20	4.35	98.49	0.32
$r = 0$	0.5	15	0	5	—	—
$d = 0$	0.33	8.89	0	2.7	—	—

Table 1: Simulation results. Originator’s investment and profit.

We now simulate the intermediation chains starting with the originator. We run 1.000 simulations for each set of parameters we consider and record empirical averages, at each level in the realized chain of intermediaries, of the volume v_0 faced by the intermediary, the price p and the volume v he offers, his asset holdings $v_0 - v$, his cash level ω , the targeted cash level T and benefit level B^* . We additionally record for each layer the probability pr_a that an intermediary makes a new offer, pr_r that nobody accepts an offer, and p_c that someone completes the volume and closes the chain. The averages of v_0 , ω and $v_0 - v$ are weighted by the proportion of intermediaries making a new offer or completing the volume at a given layer, while p , v , T and B are weighted by the proportion of intermediaries making a new offer.

Table 2 reports the results for the first five layers.¹²

¹²As shown on the last line, in 8% of the simulations the game continues after the 5th layer, the longest realized chain having a length of 10.

Layer	v_0	p	v	Holdings	ω	T	B	pr_a	pr_r	pr_c
Orig.	0.43	98.46	0.31	0.12	0	0.69	9.44	1	0	0
1	0.31	98.93	0.21	0.13	5.12	0.66	6.80	0.88	0.03	0.09
2	0.21	99.25	0.15	0.10	5.08	0.62	4.92	0.65	0.02	0.21
3	0.15	99.47	0.11	0.08	5.13	0.58	3.66	0.39	0.01	0.25
4	0.11	99.60	0.08	0.07	4.99	0.55	2.87	0.19	0.01	0.19
5	0.08	99.69	0.06	0.05	5.06	0.52	2.31	0.08	0.00	0.11

Table 2: Empirical averages under the baseline parameters, first five layers.

The offered price goes up on average along a chain whereas the offered volume decreases. The target T is close to $\omega^\infty = 0.7$, much lower than the average cash of the accepting receivers (which is slightly larger than the unconditional average equal to 5), and slowly decreases along the chain. This reflects the fact that the origination level is low relative to the network benefit that can be extracted from the chain, which can also be seen by comparing prices with \underline{p} , equal to 93.30. As the originators' financing needs are 12.16, long chains can arise when several accepting intermediaries have little cash, close to the target. In that case their financing needs are close to their purchase and they have to sell a large part of the volume they accepted to buy received. The large discrepancy between the target and the average cash is due to the shape of the distribution, with a rather flat density. In the case of a concentrated distribution, the target and the average cash are much closer, as we saw in Section 3.4.

5 Comparative statics on OTC trading

We review in this section the implications of our model on trading behavior and the building up of intermediation chains. To that purpose, we derive how a change in the environment as reflected by the parameters in the model affects the equilibrium outcome. We detail in the next section how these implications can be tested on particular markets.

5.1 Impact of the haircut and of the unsecured interest rate

We first look at the impact of the financial parameters ρ, ℓ, r . A change in a parameter, say r , affects the originator's investment and thus his financing needs. The other intermediaries' behavior is then affected through two channels: 1. the change in r affects directly all offers starting with an intermediary (possibly the originator) whose financing needs are given, which we call the *trading effect*; 2. it also affects the originator's financing needs, hence his offer and the financing needs of subsequent intermediaries, which we call the *origination effect*. Combining the two gives the overall impact of a parameter and allows us to disentangle the different channels through which each parameter affects observable variables such as originated volumes and chain size.

5.1.1 The trading effect

We consider here an intermediary I with given financing needs $y_0 = (p_0 - \ell)v_0 - \omega_0$ and evaluate the impact of a change in the parameters ρ, ℓ and r on his offers. We also look at the intermediaries' *asset holdings*, the volume they buy and pledge as collateral but do not sell, and their profits. The implications we derive here can be tested empirically by looking at the behaviour of an intermediary, controlling for his financing needs. They apply in particular to an originator with a fixed investment level and whose financing needs $y_0 = C(k) - \ell k$ are not affected by a change in ρ or r . For a change in ℓ there is an additional effect coming from the reduction of financing needs that will be studied in the next subsection.

Offers and asset holdings: From Theorem 1, we know that I makes an offer targeting $T(y_0)$ with a cash value $(p_0 - \ell)v_0 = y_0$, both being independent of the financial parameters. This implies that an intermediary receiving the offer accepts if and only if his cash ω is not less than $T(y_0)$, a decision that is not affected by the financial parameters. His own financing needs $y_0 - \omega$ are also unchanged, as well as his own target, $T(y_0 - \omega)$. By induction, this is also true at all subsequent layers: given any intermediary with fixed financing needs and a realized chain of subsequent partners with given levels of cash, their targets are all independent on ρ, r and ℓ .

However, these parameters still have an impact on the prices and volumes that are offered

and on the intermediaries' profits. We have from (23) and (24):

$$\frac{\rho - P(y)}{\rho - \ell} = 1 - \frac{y}{y + r(Z(y) - B^*(Z(y)))} \quad (36)$$

$$V(y)(\rho - \ell) = y + r(Z(y) - B^*(Z(y))) \quad (37)$$

$\rho - P(y)$ is a *liquidity rebate*, which shows how much an intermediary is ready to concede in order to attract potential buyers; thus, up to the haircut $\rho - \ell$ which reflects the difficulty to borrow against the asset as collateral, the left hand sides of equations (36) and (37) are the liquidity rebate and the volume at the optimal offer. The terms on the right hand side are independent of ρ and ℓ , and depend on the financing needs y and on r . The formulas apply to our first intermediary I with $y = y_0$ and also to all the following intermediaries in a realized chain, as we have seen that their financing needs are not affected by the financial parameters (given the realized cash levels). Assuming that I finds it profitable to make an offer under the parameters considered, we easily derive the following results:

Proposition 4. *1. For given financing needs y_0 of I , the targets, the length of the targeted chain, the acceptance probability of the offers made and the financing needs of all intermediaries following I are unaffected by ρ , ℓ and r .*

2. Offers: prices are weakly decreasing in r and weakly increasing in ρ and ℓ ; the liquidity rebate is directly proportional to the haircut. Volumes are weakly increasing in r and ℓ and weakly decreasing in ρ , and are inversely proportional to the haircut.

3. Asset holdings: for I , they are weakly decreasing in r and ℓ and weakly increasing in ρ , and the reverse is true for each subsequent intermediary.

To understand why an intermediary's offer depends on the financial parameters even though the target and the acceptance probability do not, denote by $(P_r(y), V_r(y))$ the offer made by an intermediary with financing needs y for a given value of r (a similar analysis can be conducted for ρ and ℓ). When r increases, the target $T(y)$ is unchanged and still makes zero profit, so that $(\rho - P_r(y))V_r(y) = r(y - T(y) - B^*(y - T(y)))$, but the financing costs $r(y - T(y) - B^*(y - T(y)))$ increase proportionally to r . The offer must be adjusted to compensate for this increase and restore a null profit: $(\rho - P_r(y))V_r(y)$ must increase proportionally to r . As the net cash value $(P_r(y) - \ell)V_r(y)$ remains equal to y , this results

in a lower price and a higher quantity.

Finally, let us examine the asset holdings. I 's holdings are given by $v_0 - V(y_0)$ and thus vary in ρ, r and ℓ like $-V$. Consider now the subsequent intermediaries who end up buying some units of the asset. Let R with cash ω accept the offer of a partner with financing needs y ; R 's financing needs are $z = y - \omega$. In case R makes a successful offer, his asset holdings are equal to $V(y) - V(z)$. From (37) these holdings are proportional to $[Z(y) - B^*(Z(y)) - [Z(z) - B^*(Z(z))]$, with a factor of proportionality equal to $r/(\rho - \ell)$. This expression is positive since Z is non-decreasing and the function $x - B^*(x)$ is increasing in x , proving point 3.

Profits. Consider again an intermediary I making an offer $(P_r(y), V_r(y))$ to another intermediary R who has ω in cash. Assume $T(y) \leq \omega$, so that R accepts the offer. Using as above that $(P_r(y), V_r(y))$ leaves zero profit to an intermediary with exactly $T(y)$ in cash, R 's profit if he transacts with I and does not have enough cash to finance the whole volume ($\omega < y$) is equal to:

$$\begin{aligned}\Pi_R(\omega) &= (\rho - P_r(y))V_r(y) - r(y - \omega) + rB^*(y - \omega) \\ &= r[\omega - T(y) - B^*(y - T(y)) + B^*(y - \omega)].\end{aligned}$$

If instead R has $\omega \geq y$ then his profit is constant and equal to $\Pi_R(y)$. In both cases Π_R is proportional to r . This is explained as follows. As we have just seen, the offer is adjusted to an increase in r so as to compensate the targeted intermediary for the increase in his funding costs. As a result, any additional unit of cash above the target is fully rewarded at the interest rate r . As for the other parameters, ρ and ℓ , the profits are unchanged, as these parameters affect all intermediaries in the same way.

Iterating these results on intermediaries' profits at any layer, we can evaluate the impact of an increase in r on the whole chain starting from I (possibly the originator). I 's profit writes $(\rho - p_0)v_0 - r(y_0 - B^*(y_0))$ and is thus decreasing in r (since $y_0 - B^*(y_0) > 0$), increasing in ρ and ℓ . Using that the targeted chain and all intermediaries' financing needs are unchanged, we have:

Proposition 5. 1. *I 's profit is decreasing in the unsecured rate r and the subsequent intermediaries' profits are linearly increasing in r .*

2. *I 's profit is increasing in ρ and ℓ , and the following intermediaries' profits are unchanged.*

The impact of a change in r on the distribution of profits between an intermediary who has just accepted an offer (or the originator) and his successors is thus very different from that of ρ or ℓ . Ultimately, cash provides a rent for those intermediaries who receive an offer and have more than the targeted level, and this rent increases with r . But for I , who starts with a given y_0 , an increase in r only affects his funding cost and reduces his profit. The other financial parameters ρ and ℓ do not affect I and the subsequent intermediaries differently and are thus not a source of rent. As a result, a change in these parameters affects only I 's profits.

5.1.2 The overall impact

We now study the impact of the different parameters when the financing needs at each level are endogenized. The originator's financing needs are determined by his investment k , and the financing needs of all the subsequent intermediaries are endogenously determined by the chain of offers they receive, starting with the originator.

It follows from Proposition 3 that the originated volume is weakly increasing in ρ and weakly decreasing in r . Since the financing needs and the trading strategies are unaffected by ρ or r , an increase in k directly translates into an increase in $y(k)$. Hence, arguing recursively at all levels, the size of the chain, the targeted levels and the intermediaries' financing needs (taking as given their endowments in cash) all increase.

The impact of ℓ is more complex: increasing ℓ has a non-ambiguous positive effect on the origination level k when B^* is concave, but simultaneously reduces the financing needs for a given investment. Whether the overall impact on the originator's financing needs $y(k) = C(k) - \ell k$ is positive depends on the flexibility of origination: for a fixed investment the financing needs surely decrease, and we show that for a flexible enough investment they increase. Gathering these results gives the following:

Proposition 6. 1. *The investment of the originator, the size of the targeted chain, and the financing needs of all intermediaries are weakly increasing in the expected return ρ and weakly*

decreasing in the unsecured rate r .

2. The investment of the originator is weakly increasing in ℓ under the concavity of B^* . His financing needs are also increasing in ℓ for a flexible enough investment, as is the case for a linear C , but decreasing for an inflexible investment. The size of the targeted chain and the financing needs of all intermediaries vary with ℓ in the same direction as the originator's financing needs.

See the Appendix D.2 for the proof that the originator's financing needs are increasing in ℓ with a linear C . All the other points have already been proved.

Assets that are expected to be more valuable are originated in larger quantities, making the chain size higher. An increase in r on the contrary increases the originator's costs and thus reduces origination and chain size. The impact of ℓ depends on the convexity of C and the concavity of B^* . Unless the cost function is too convex, an increase in the collateral value of the asset leads to more origination and higher total financing needs, so that more layers are needed to cover them entirely.

The overall impact of the parameters on the offered volumes is ambiguous. Consider for example ρ . The volume sold is increasing in y (Corollary 2) but decreasing in ρ . As the investment is increasing in ρ , it is unclear whether the originator will offer a larger volume or a smaller one. The same is true at all levels. The same remark applies to r (reversing the sense of variation). The impact of ℓ however is unambiguous when the investment is flexible enough:

Proposition 7. *When investment is flexible enough, so that a higher ℓ increases the originators' financing needs, it also increases the volume of all subsequent offers.*

We conclude this section with an illustration based on the same numerical example as in 4.2. We keep our baseline parameters but let ℓ vary between 60 and 80 (i.e. the haircut goes from 40 to 20%). Fig. 4 shows how the originated volume k , the size of the targeted chain $N(y(k))$ and the average realized chain size vary with ℓ . An increase in ℓ leads the originator to issue more assets, but the increase in $C(k)$ is lower than the decrease in financing needs due to the larger ℓk . As a result the originator's financing needs decrease, less intermediaries are needed to provide cash, and the intermediation ends sooner on average.

[Insert Fig. 4 here.]

5.2 Impact of the network structure

The network structure is governed by the parameters d and q and the distribution G . All three have an impact both on the trading behavior of intermediaries for given financing needs, and on the investment level chosen by the originator and thus the equilibrium financing needs of intermediaries.

Impact of d and q . Due to the strategic choice of offers by the intermediaries, a change in the network structure can impact intermediation chains in different directions. This is best seen by looking at extreme cases. First, for a given y , consider the case $d \rightarrow \infty$: for any financing need y an intermediary is almost sure that an offer to sell all his assets at price ρ will be accepted, so that it is always optimal to target $\omega = y$ and the chain has a length of 1. A large enough increase in d thus has a negative trading effect on chain size.

This is a simple yet surprising point, which illustrates well the importance of endogenizing the intermediaries' trading decisions: one may have expected that increasing the number of intermediaries at each layer would always increase the length of intermediation chains, as there is a higher probability to find someone ready to buy some units of the asset. But precisely for this reason, intermediaries tend to endogenously choose offers that are accepted by fewer intermediaries, so that the length of the chain can ultimately decrease if finding a buyer becomes likely enough.

Considering now finite values of d and q , we know from Corollary 4 that as $y \rightarrow \infty$ the size of the targeted chain becomes equivalent to y/ω^∞ . We show in the Online Appendix E.3 that ω^∞ is decreasing in d and q . For large enough financing needs the trading effect of these variables on chain size is thus positive.¹³

The trading effect is thus ambiguous: if an intermediary has access to more trading partners, or knows they have a higher probability of being active, then this changes the probability $H(\omega)$ that an offer targeting type ω is accepted, but it also changes the network

¹³There is no contradiction between these two results, which rely on two different asymptotic cases: fixed y and infinite d , or fixed d and infinite y .

benefit of the target. This can be seen by inspecting the first-order condition (29). We show in the Online Appendix E.3 that higher values of d and q increase $H'(\omega)/H(\omega)$. But they also have a positive impact on B^* , at least for small values of y , which goes in the opposite direction. Depending on the shape of B^* , either the direct impact of these parameters through the acceptance probability or the indirect impact through the target's benefit may dominate.

The origination effect is also ambiguous for the same reason. According to the originator's first-order condition (35), the impact of a change in d or q on the originated volume k depends on how these parameters impact B^* . In the single-layer game this analysis is equivalent to studying how H'/H is affected by d and q , and as we have seen the impact is positive, implying that a higher d or q increases origination. For the general game the indirect impact through other intermediaries' benefits may be stronger.

Longer chains can be studied through simulations. Fig. 5 shows how the origination level, the length of the targeted chain and the average realized chain length vary when changing d and q while keeping the other parameters equal to their baseline values. In both cases a more dense network increases the length of the chain, mostly through a positive trading effect as investment is quite inelastic. Notice also that the average realized chain length tends to react much less than the targeted chain. An increase up to $d = 6$ for instance yields a targeted chain of size 52, while the average chain has a length of only 4. Since intermediaries have a very high chance to find at least one partner with enough cash to buy at a low rebate, even intermediaries with low cash endowments are ready to buy at high prices. This can result in long targeted chains with many intermediaries with low cash having to sell most of the volume they buy. At each step, however, there is a high probability that an intermediary has enough cash to buy the whole volume, thus ending the chain long before its maximum size.

[Insert Fig. 5 here.]

Impact of the distribution of cash G . The distribution G also impacts both the originator's incentives and the intermediaries' trading decisions. A natural question to ask is how intermediation chains are affected when all intermediaries tend to have more cash. In our numerical example we use $G \hookrightarrow \Gamma(k, \theta)$ with $k = 1, \theta = 5$. An increase in cash endowments

can be obtained by increasing the scale parameter θ . In addition, we also simulate a change in the distribution that increases or decreases its standard deviation while keeping the mean constant. Fig. 6 shows our results.

[Insert Fig. 6 here.]

Increasing the scale parameter θ , thus simulating a situation in which all intermediaries are more likely to have higher cash endowments, increases the origination level, but still reduces chain size due to the trading effect: as an institution's partners are more likely to have cash, targets are higher and the chain is more likely to end rapidly. We prove in the Online Appendix E.4 in the particular case of a single layer of intermediaries that switching to a higher distribution in the sense of likelihood ratio dominance (which is achieved here by increasing θ) has more generally a positive impact on origination. Increasing the standard deviation has a negative origination effect, but the total impact is ambiguous.

6 Applications and extensions

We detail in this section how our model applies to several markets and contexts and discuss the related empirical and applied literatures.

6.1 Dealer networks

Our theory of the building-up of chains of cash-constrained intermediaries demanding liquidity to other intermediaries can be applied to dealer networks, which have been studied by the empirical literature. Each intermediary can be seen as a dealer and his ω is given by a flow of buy orders coming from customers. It is less costly for a dealer facing strong demand from customers to buy the asset, while a dealer facing a lower demand will have to either hold the asset or sell it to a different dealer.

Green, Hollifield, and Schuerhoff (2007) study the market for municipal bonds and provide evidence on interdealer transactions that matches our model fairly well. They document in particular that in interdealer transactions the bonds are traded at a discount in the first five days after issuance, a mispricing that decreases over time while interdealer transactions

and volumes simultaneously get lower. This corresponds to the fact that financing needs and volumes decrease along an intermediation chain in our framework, while prices tend to get closer to the fundamental value. [Li and Schuerhoff \(2012\)](#) study more specifically intermediation chains between dealers standing between two final customers. They show in particular that the interdealer price rises along the intermediation chain, which is expected from (36), and also that dealers at the end of the chain derive higher profits, which is consistent with our analysis as the last intermediary in a chain has a surplus of cash (or demand from customers), which is rewarded at rate r at the margin.

Our results in section 5.1 give additional implications that could be tested on dealer markets, typically the impact of the unsecured interest rate on dealers' mark-ups, profits, and the length of intermediation chains. The primary market for municipal bonds is particularly interesting as the "origination" level is probably quite constrained and not very elastic, so that only the trading effect matters. On other markets, our results suggest that it is necessary to control for the origination level.

It is interesting in this respect to compare [Li and Schuerhoff \(2012\)](#), who show that on the municipal bonds market more connected dealers charge higher mark-ups, with [Hollifield, Neklyudov, and Spatt \(2012\)](#), who find the opposite result on the market for securitized products. To explain this difference, we can extend our model to feature a stylized core-periphery structure and thus dealers with different degrees of connectivity. Imagine that a central intermediary has access to d_1 intermediaries, who themselves have access to a network in which all intermediaries have $d < d_1$ partners. We can compute the network benefit function B^* faced by a dealer with d partners. The benefit for the central dealer making a proposal can then be easily derived, assuming she expects correctly the benefit of her partners. Indeed, she behaves as if she faced a single layer game as in section 3.2 in which her partners receive an extra benefit described by B^* . Of course if she had d partners, her network benefit would be given by B^* ; as she has more partners, this increases the chances for her offers to be accepted and her benefit is higher. For a given y , she makes more aggressive offers with a higher price and a smaller volume, reflecting the fact that a central dealer has more trading partners, so that her offers have more chances to be accepted. At the same time, she can offer better prices to a customer than a non-central dealer and will tend to get

larger volumes to sell and larger financing needs y , an origination effect which goes in the opposite direction. Differences in connectivity between the central and non-central dealers and on how much volume they originate on the different markets considered may thus explain why the two mentioned papers observe different types of dealers to offer lower spreads.

6.2 Securitization and the trading of toxic assets

It is particularly interesting to apply the model to banks originating subprime loans, securitizing them and selling them to other financial intermediaries who then disseminate the securitized products through the shadow banking sector. The length of the intermediation chain can then be interpreted as a measure of how widely an asset gets disseminated in the financial system. Such asset dissemination through the shadow banking system seems to have been an important mechanism through which problems with securitised products based on U.S. subprime mortgages spread to other constituencies, including Europe, thereby creating an important international systemic risk.

In the current model, longer chains and more dissemination are a good thing, as they reduce the financing needs for the originator and thus make k closer to the maximum level k^{max} . However, [Rajan, Seru, and Vig \(2013\)](#) argue that the quality of mortgage loans was severely underestimated in the run-up to the subprime crisis. In the terms of our model this would mean that the agents base their decisions on irrationally high values of ρ and ℓ , leading to too much origination and to a large dissemination of these assets through intermediation chains (Propositions 6 and 7). [Gennaioli, Shleifer, and Vishny \(2013\)](#) also elaborate on this assumption of optimism in a model of shadow banking.

Fluctuations in ℓ also have an impact on the price at which the asset is traded. Fig. 7 below illustrates Proposition 4 by plotting $P(y)$ for different values of ℓ . [Gorton and Metrick \(2012\)](#) document that haircuts in bilateral repo agreements reached 50% or higher for several securitised products, which would imply a severe increase in the liquidity rebate in our model.

[Insert Fig. 7 here.]

Another possibility is to enrich the origination stage of the model with moral hazard considerations, the findings of [Keys *et al.* \(2010\)](#) suggesting that they played an important role

on the subprime market. While still facing a network benefit $B^*(y)$, the originator would choose whether he screens loan applicants, making ρ endogenous. As they anticipate moral hazard, potential buyers do not accept high prices, reducing the network benefit.¹⁴ We could investigate whether the availability of more cash in the system increases the incentives for lax screening, and how toxic assets get disseminated.

6.3 Haircuts and collateral policy

The asset's collateral value ℓ and the associated haircut $\rho - \ell$ are exogenously given in the model, as would be the case for instance if there is a pool of competitive lenders who determine the haircut at which they accept to lend, based on the asset's riskiness.¹⁵ However, two types of economic agents, central counterparties and central banks, also set haircuts and cannot be seen as small competitive players.

Central counterparties and payment delays. Consider an intermediary I who bought v_0 units and then sold $v < v_0$. His asset holdings are equal to $v_0 - v$. As I sold exactly enough assets to cover his financing needs and did not keep any cash, he ended up with $v_0 - v$ units of the asset and pledged them as collateral against a debt of $\ell(v_0 - v)$. The same is true at each layer in the realized intermediation chain, except the last one. The asset's final value is random and equal to $\tilde{\rho}$. The value ℓ is supposed to be such that the probability to have $\tilde{\rho} < \ell$ is negligible. When such a bad shock realizes however, the value of each intermediary's asset holdings is lower ex post than what he has to pay back to creditors. When $\tilde{\rho} < \ell$, *all* intermediaries in a chain typically have to delay the repayment of their debt in order to have time to find the necessary cash, by selling other assets or asking their parent company. The risk associated with such a delay motivates the presence of a haircut in the first place.

The search for liquidity by intermediaries and their common exposure to the same asset endogenously lead to correlated payment delays in the market for collateralized funding. A central counterparty setting the haircut on an asset should thus take into account that the inability of a participant to make a payment, an event that can trigger the use of the

¹⁴See Li, Rocheteau, and Weill (2012), who introduce moral hazard in a search model of OTC trading.

¹⁵Simsek (2013) and Gorton and Ordonez (2014) provide theories of which assets serve as collateral and at what haircuts.

collateral, can be heavily correlated among market participants.

Central banks' collateral framework. Another interesting line of research would be to use our model to analyze the collateral policy of central banks. Since the outbreak of the financial crisis, several central banks have started accepting more assets as collateral when lending to banks, which can be seen as a change in haircut from 100% to some lower number. Interpreting secured borrowing as borrowing at the central bank against collateral, the haircut is a choice variable for the central bank. Our model could be tested by examining how the decisions on which assets are eligible or on their haircut levels affect the volume of assets traded and the level of origination.¹⁶

Of particular interest is our result that lowering haircuts may encourage intermediaries to all sell larger volumes to their partners (Proposition 4). If the origination effect is small, this implies chains where intermediaries will actually use less collateralized funding, and sell most of the assets they receive until an institution with a high cash endowment is found. Central bank liquidity is then less needed and the total risk exposure of the central bank decreases. At particular times, lower haircuts may thus both provide more liquidity to the banking sector and reduce the risk faced by the central bank in its role as a lender of last resort, a point also made by [Bindseil and Jablecki \(2013\)](#). In our model this is more likely to happen when for a given y the derivative $\partial V/\partial \ell$ is higher, which is the case when r is high, for instance due to a malfunctioning of the interbank market.

Fig. 8 illustrates these points. We plot for different values of ℓ the average total volume pledged by all intermediaries as a proportion of the original volume, and the average amount borrowed at the unsecured rate r . We see in particular that increasing the collateral value ℓ above its baseline value actually leads to less usage of collateralized funding, because it is more likely that an intermediary with a high cash holding will be found and completes the volume without needing to borrow.

[Insert Fig. 8 here.]

¹⁶The same assets can be accepted as collateral by the central bank and by a CCP. [Mancini, Ranaldo, and Wrampelmeyer \(2013\)](#) study the European interbank repo market and provide evidence on how the collateral policies of both the ECB and the CCP affect repo pricing.

7 Conclusion

Securing a cheap access to funding has been an increasingly important determinant of profits for financial firms (see e.g. [Hanson, Kashyap, and Stein \(2011\)](#)). When cash is in scarce supply, reducing funding costs is a powerful motive to trade besides traditional ones such as diversification. We propose a model in which financial intermediaries differ only in their access to liquidity and show how their positions in an OTC market for a risky asset are determined by their liquidity needs and the ease with which they can sell to partner intermediaries. Intermediation chains arise naturally from such a model and assets get disseminated among many intermediaries, depending on how cash is distributed among them.

Our approach delivers two sets of empirical implications.

First, we can relate OTC transaction prices and trading volumes to the funding liquidity of intermediaries (at which rate they can borrow and what haircut they face on the collateralized market) and to how much liquidity is available in the network of intermediaries. The quantity in which the asset is originated is endogenized, so that market liquidity and the investment level are simultaneously determined in equilibrium.

Second, we study how assets are disseminated over intermediation chains of endogenous lengths, which gives a rich crop of new predictions concerning the impact of an asset's risk, the funding liquidity of intermediaries and the properties of the network on how many intermediaries are involved in the intermediation process. We show in particular that it is key to control for the total volume to be distributed, as parameters that add liquidity to the system typically lead to shorter chains, but also to higher originated volumes, which goes in the opposite direction.

While the paper applies well to chains of pure intermediaries such as dealers, it can also be applied to the dissemination of securitized products through the shadow banking system. Intermediation chains then endogenously generate systemic risk in the form of common exposures, and can explain in which circumstances an asset can “contaminate” a large number of intermediaries. The role played by the asset's haircut in the model also has implications for the collateral policies of central banks and CCPs. Other applications such as the formation of rehypothecation chains are left for future research.

A Figures

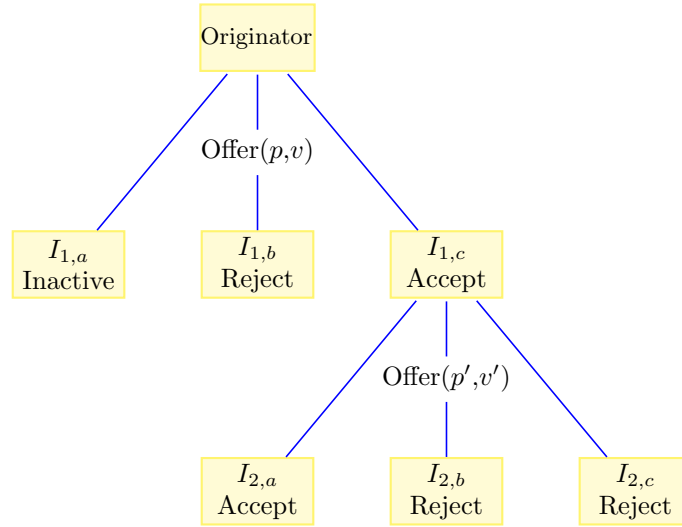


Figure 1: Two first layers of a network, example.

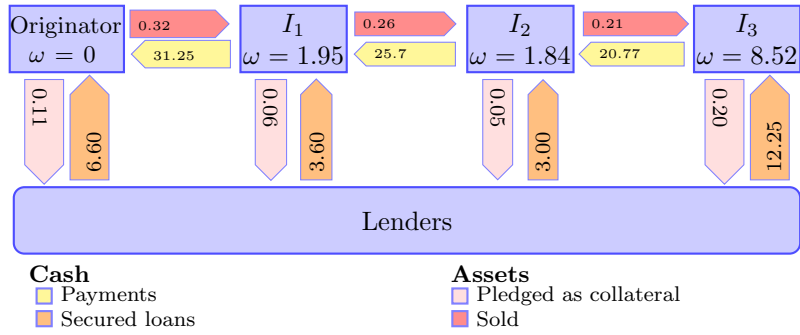


Figure 2: Realized intermediation chain, example.

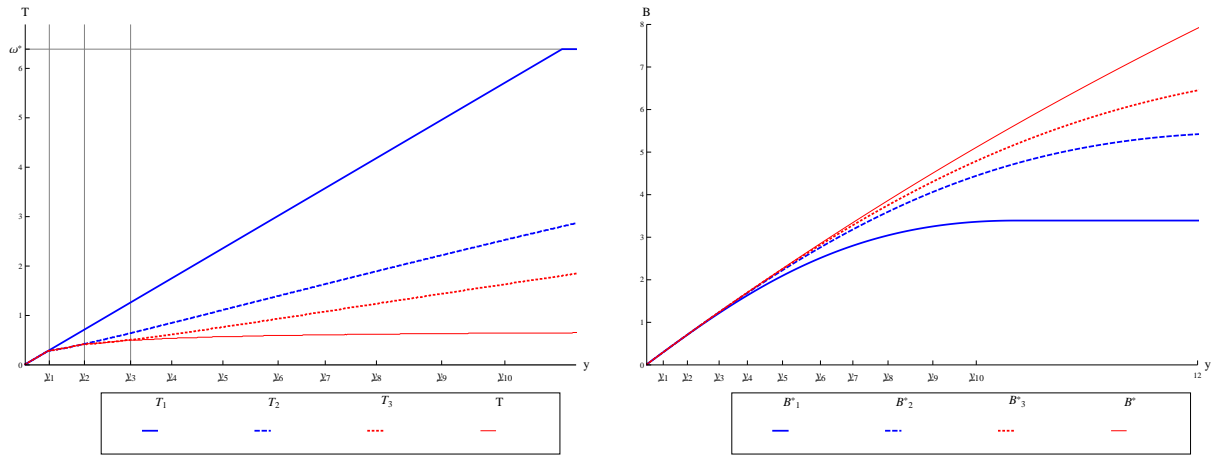


Figure 3: Equilibrium T , T_n (left), B^* and B_n^* (right), for $n \leq 3$.

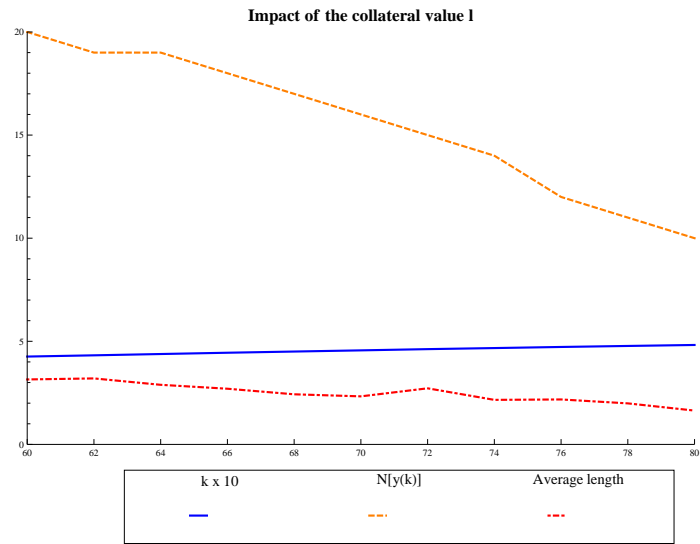


Figure 4: Comparative statics: impact of the collateral value on origination and chain size.

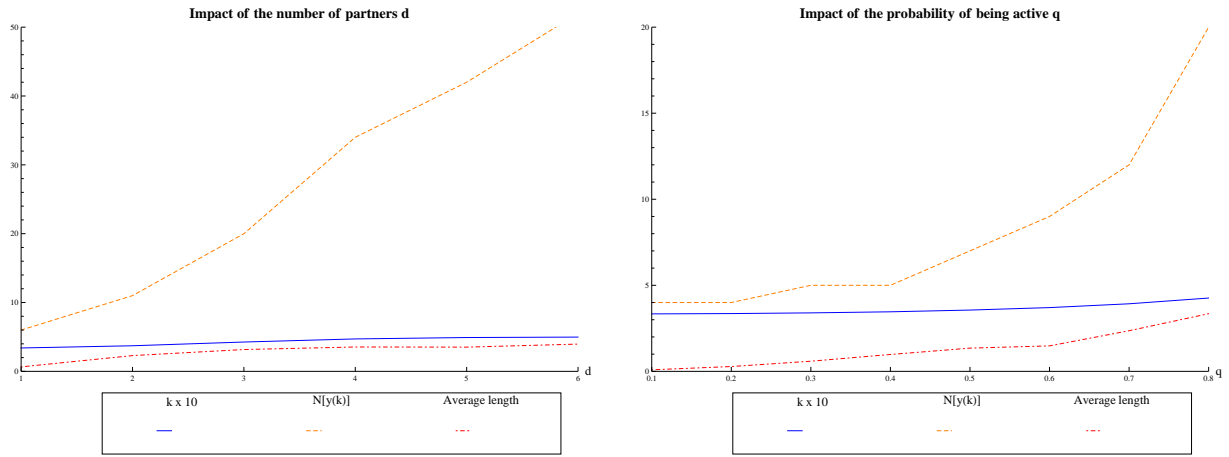


Figure 5: Comparative statics: impact of network parameters on origination and chain size.

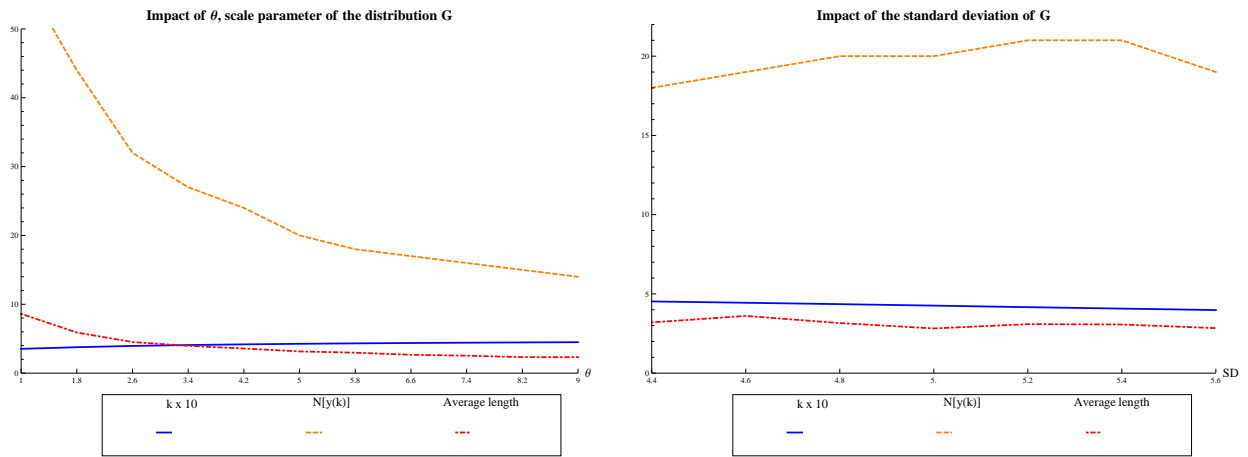


Figure 6: Comparative statics: impact of the distribution G on origination and chain size.

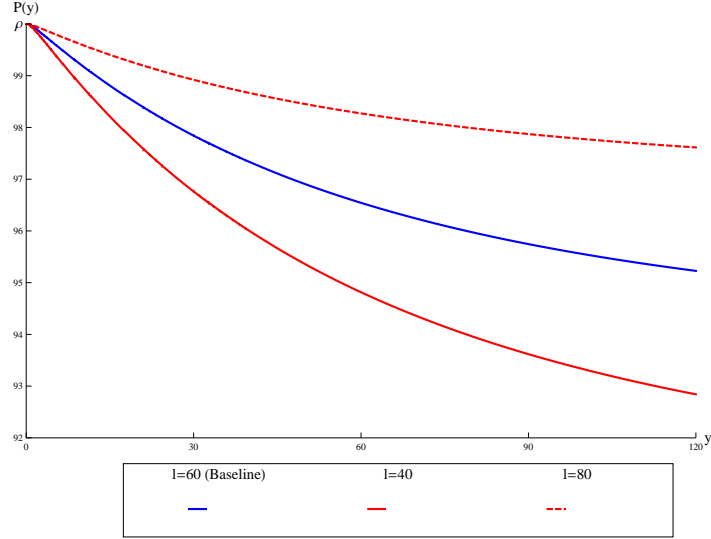


Figure 7: Equilibrium offered price $P(y)$, for different values of ℓ .

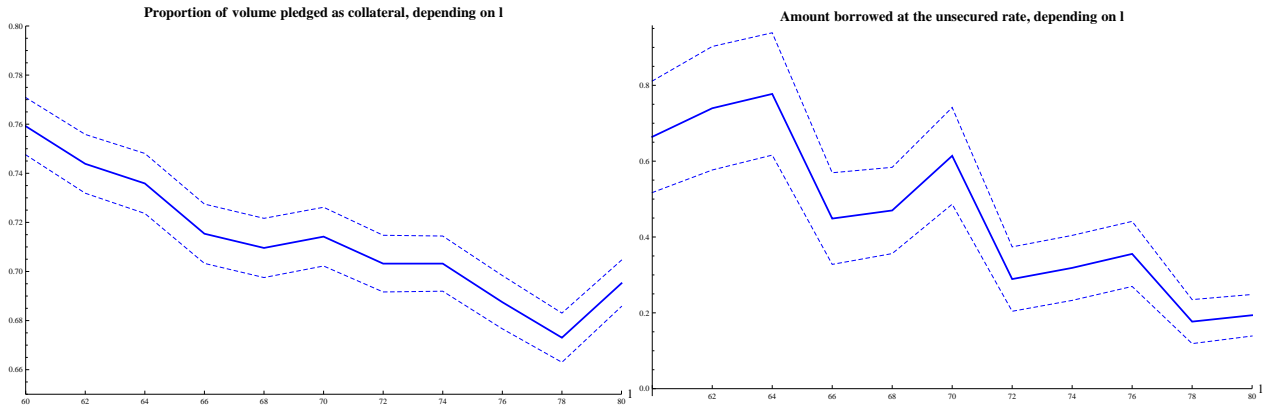


Figure 8: Average volume pledged as collateral (left), and average amount borrowed at the unsecured rate (right), along realized chains, depending on ℓ .

B Appendix: Proofs related to the single layer game

B.1 Proof of Proposition 1

We first prove that if I 's offer is optimal then the profit of a receiver R whose cash is equal to the threshold $w(p, v)$ is null. Consider two cases. In the first case, $w(p, v) > 0$. Hence $\pi_R(p, v, 0) < 0$ by definition of the threshold; it follows that $\pi_R(p, v, w(p, v)) = 0$ by continuity of β and H . In the second case, $w(p, v) = 0$. Assume by contradiction $\pi_R(p, v, 0) > 0$; then surely $p < \rho$. I can thus increase the price by a small amount so that R 's profit is still positive

for any level of cash: I 's transaction value is increased and the probability of acceptance is kept equal to $H(0)$: a contradiction.

The proof relies on Lemmas B.1 and B.2. Lemma B.1 shows that an optimal offer never over-finances the needs and Lemma B.2 that I 's expected transaction value is bounded by $r\beta^*(y)$. As in the statement of Proposition 1, I faces offer (p_0, v_0) and has positive financing needs y in Γ_β .

Lemma B.1. *An optimal offer (p, v) has a cash value no greater than y : $(p - \ell)v \leq y$. Thus I 's transaction value is: $Q_I(y; p, v) = -(\rho - p)v + r(p - \ell)v = (1 + r)(p - \underline{p})v$, and the profit of a receiver R who has positive needs writes*

$$\pi_R(p, v, \omega) = -Q_I(y; p, v) + r\omega + r\beta((p - \ell)v - \omega). \quad (38)$$

Proof of Lemma B.1. By contradiction let us assume $(p - \ell)v > y$. I 's transaction value satisfies $rQ(y; p, v) = ry - (\rho - p)v$. Let us distinguish two cases.

In the first case, $p = \rho$. Decreasing v marginally to v' and keeping the price equal to ρ is beneficial: as $w(\rho, v') = (\rho - \ell)v'$, the threshold decreases and I 's transaction value is constant and equal to y , provided that $y < (\rho - \ell)v'$. Therefore, decreasing v allows to increase the chances of success while keeping the transaction value constant: (p, v) is not optimal.

In the second case, $p < \rho$. I 's transaction value is $ry - (\rho - p)v$. Let us first adjust marginally the offer to (p', v') by increasing p and decreasing v (which is feasible) so as to keep $(\rho - p')v'$ equal to $(\rho - p)v$. $(p - \ell)v$ is decreased but for a marginal change $(p' - \ell)v' > y$ still holds. Thus I 's transaction value is kept constant and, in addition, the financing needs of a receiver with positive financing needs $z = (p - \ell)v - \omega$ are decreased to a smaller z' . Since the function $z - \beta(z)$ is increasing by (12), R 's profit satisfies

$$\pi_R(p', v', \omega) = (\rho - p)v - rz' + r\beta(z') > (\rho - p)v - rz + r\beta(z) = \pi_R(p, v, \omega)$$

Applying this inequality to the cash target $\omega = w(p, v)$ which has $\pi_R(p, v, w(p, v)) = 0$, we obtain $\pi_R(p', v', w(p, v)) > 0$. It follows that p' can be increased into p'' so as to make R 's profit $\pi_R(p'', v', \omega)$ null (as we saw that π_R strictly decreases with the offered price). As a result, $w(p'', v') = w(p, v)$. Furthermore, since $(p'' - \ell)v' > (p' - \ell)v' > y$, I 's transaction value at (p'', v') is given by $ry - (\rho - p'')v'$. Since $(\rho - p'')v' < (\rho - p')v' = (\rho - p)v$, I 's transaction value at (p'', v') is thus larger than at (p, v) . Hence, at (p'', v') the acceptance

probability is the same as at (p, v) but the transaction value is larger: (p, v) is not optimal.

The expression for Q_I then follows straightforwardly from (6) and that for π_R from (13). ■

Lemma B.2. *I 's expected transaction value from offering (p, v) satisfies*

$$H(w(p, v))Q_I(y; p, v) = rH(w(p, v))(w(p, v) + \beta((p - \ell)v - w(p, v))) \quad (39)$$

$$\leq rH(w(p, v))(w(p, v) + \beta(y - w(p, v))). \quad (40)$$

It follows that $\max_{p, v, p \leq \rho, v \leq v_0} H(w(p, v))Q_I(y; p, v) \leq r\beta^(y)$, i.e. $r\beta^*(y)$ is an upper bound to I 's expected transaction value.*

Proof of Lemma B.2. At the target, R 's profit is null. Thus (38) in Lemma B.1 implies $Q_I(y; p, v) = r\omega + r\beta((p - \ell)v - \omega)$ at $\omega = w(p, v)$. This proves equality (39). The inequality (40) follows since $(p - \ell)v \leq y$ and β is non-decreasing. The last statement straightforwardly follows from the definition (14) of β^* . ■

End of the proof of Proposition 1. We know from Lemma B.2 that $\beta^*(y)$ is an upper bound to I 's benefit so that $\pi_I^*(p_0, v_0, y)$ is an upper bound to I 's profit. Hence I surely refuses (p_0, v_0) when $\pi_I^*(p_0, v_0, y) < 0$. We now show that conversely I accepts the offer when $\pi_I^*(p_0, v_0, y) \geq 0$ by proving that I can achieve $\beta^*(y)$. Consider the offer (p, v) that satisfies

$$(p - \ell)v = y \text{ and } (\rho - p)v = r(y - \tau(y) - \beta(y - \tau(y))). \quad (41)$$

These equations characterize a unique pair. Step 1 shows that the upper bound to I 's benefit $\beta^*(y)$ is achieved at this offer (p, v) and step 2 that the offer is feasible under the non-negativity of $\pi_I^*(p_0, v_0, y)$.

Step 1 shows that the upper bound to I 's benefit $\beta^*(y)$ is achieved at the offer (p, v) and step 2 that the offer is feasible under the non-negativity of $\pi_I^*(p_0, v_0, y)$.

Step 1. I 's expected benefit at (p, v) is $\beta^*(y)$ and I 's profit is $\pi_I^*(p_0, v_0, y)$.

The first equation in (41) requires that I exactly covers his needs and the second one that the profit of R with cash equal to $\tau(y)$ is null. Thus the transaction value for I satisfies

$$Q_I(y; p, v) = -(\rho - p)v + r(p - \ell)v = ry - (\rho - p)v = r(\tau(y) + \beta(y - \tau(y))) \quad (42)$$

and the threshold $w(p, v)$ is equal to $\tau(y)$. Applying (42) and using $w(p, v) = \tau(y)$, I 's expected benefit at (p, v) is $H(\tau(y))(\tau(y) + \beta(y - \tau(y)))$, which is equal to $\beta^*(y)$ by definition

of τ . It follows that I 's expected profit when offering (p, v) is

$$(\rho - p_0)v_0 - ry + r\beta^*(y).$$

which is defined as $\pi_I^*(p_0, v_0, y)$ by (16).

Step 2. (p, v) is feasible if $\pi_I^*(p_0, v_0, y) \geq 0$.

Solving for (41) yields the unique pair (p, v)

$$p = \frac{\rho y + r\ell(z - \beta(z))}{y + r(z - \beta(z))}, v = \frac{y + r(z - \beta(z))}{\rho - \ell}$$

where $z = y - \tau(y)$. This implies $p \leq \rho$ and $v \geq 0$ since $z - \beta(z) \geq 0$. The offer is thus feasible if furthermore $v \leq v_0$. Using (42) the profit also writes as

$$\pi_I^*(p_0, v_0, y) = (\rho - p_0)v_0 - ry + H(\tau(y))(ry - (\rho - p)v).$$

We show that $\pi_I^*(p_0, v_0, y) \geq 0$ implies $(\rho - p_0)v_0 > (\rho - p)v$. Since $ry - (\rho - p)v = r(\tau(y) + \beta(y - \tau(y)))$ by the second equation in (41), this quantity is non-negative and as moreover $H(\tau(y)) < 1$ we have $0 \leq \pi_I^*(p_0, v_0, y) \leq (\rho - p_0)v_0 - (\rho - p)v$, the desired inequality.

It follows that $v > v_0$ can hold only if $p > p_0$. But then $y = (p_0 - \ell)v_0 - \omega_0 < (p - \ell)v$, which contradicts Lemma B.1 which shows that $y \geq (p - \ell)v$. Note that the same argument holds if I is the originator by replacing p_0 with the average cost $C(v_0)/v_0$. Thus $v \leq v_0$.

This ends the proof: when $\pi_I^*(p_0, v_0, y)$ is non-negative, I can achieve this amount, hence accepts the offer. \blacksquare

B.2 Additional properties of the target τ and benefit β^*

The paper's main theorem relies on additional properties of the single-layer game, which are summarized in the following proposition:

Proposition B.1. *1. The target τ is bounded above by ω^* : $\omega \leq \omega^*$ for any $\omega \in \tau(y)$. The financing needs of the target $\phi(y) = y - \tau(y)$ are increasing in y : let $y < y'$, $z \in \phi(y)$ and $z' \in \phi(y')$, then $z < z'$.*

2. The benefit function β^ satisfies assumptions (11) and (12). In particular, let $\bar{\omega}$ be the (unique) value that maximizes $H(\omega)(\omega + \bar{\beta})$ (where $\bar{\beta}$ denotes the maximum of $\beta(z)$). Define $\bar{y} = \bar{z} + \bar{\omega}$. β^* is increasing for y less than \bar{y} and constant equal to $\bar{\beta}^* = H(\bar{\omega})(\bar{\omega} + \bar{\beta})$ for y larger than \bar{y} .*

3. $\bar{\omega}$ is positive if $\frac{H'}{H}(0) + \frac{1}{\beta} > 0$ and in that case $\frac{H'}{H}(0) + \frac{1}{\beta^*} > 0$.

Proof of Proposition B.1

Point 1. The target τ is bounded above by ω^* : As β is non negative and non decreasing, the derivative of $H(\omega)(\omega + \beta(y - \omega))$ with respect to ω is bounded by $H'(\omega)\omega + H(\omega)$, which is strictly negative for $\omega > \omega^*$.

The problem (14) of maximizing $H(\omega)(\omega + \beta(y - \omega))$ with respect to $\omega, 0 \leq \omega \leq y$ can be stated equivalently in terms of the financing needs $z = y - \omega$, as that of maximizing $H(y - z)[y - z + \beta(z)]$ with respect to $z, 0 \leq z \leq y$. Define

$$F(z, y) = H(y - z)[y - z + \beta(z)].$$

To prove the monotonicity of ϕ we show that F satisfies the following (ordinal) single-crossing property:

$$F(z, y) - F(z', y) \geq 0 \text{ for } y \geq z > z' \Rightarrow F(z, y') - F(z', y') > 0 \text{ for } y' > y. \quad (43)$$

Let $z \in \phi(y)$; then $z \leq y$ and $F(z, y) - F(z', y) \geq 0$ for any $z' \leq y$, and in particular any $z' < z$. Hence $F(z, y') - F(z', y') > 0$ for any $y' > y$ and $z' < z$: z' is surely not in $\phi(y')$.

It remains to prove that F satisfies (43). Observe that

$$F(z, y) - F(z', y) \geq 0 \Leftrightarrow \frac{y - z + \beta(z)}{y - z' + \beta(z')} - \frac{H(y - z')}{H(y - z)} \geq 0.$$

Thus it suffices to show that the function on the right hand side is increasing in y . The first term writes as

$$\frac{y - z + \beta(z)}{y - z' + \beta(z')} = 1 + \frac{z' - \beta(z') - z + \beta(z)}{y - z' + \beta(z')},$$

hence is increasing in y because $z' - \beta(z') < z - \beta(z)$ for $z' < z$ (by (12)). The second term is non-decreasing in y by the log-concavity of H : the log derivative w.r.t. y of $-\frac{H(y - z')}{H(y - z)}$ is $-\frac{H'}{H}(y - z') + \frac{H'}{H}(y - z)$, which is non-negative since $z' < z$. This proves (43). \blacksquare

Point 2. Let $\bar{\omega}$ maximize $H(\omega)(\omega + \beta)$ over $\omega \geq 0$. As H is log-concave, the log of $H(\omega)(\omega + \beta)$ is strictly concave with a derivative given by $\frac{H'}{H}(\omega) + \frac{1}{\omega + \beta}$, which is negative for $\omega > \omega^*$. It follows that $\bar{\omega}$ is unique and not larger than ω^* .

β^* satisfies (11). Clearly, $\beta^*(0) = 0$ and β^* is non-decreasing. Observe that $\bar{\beta}^*$ is an

upper bound to β^* : using $\beta(z) \leq \bar{\beta}$ implies

$$\beta^*(y) = \max_{\omega \leq y} H(\omega)(\omega + \beta(y - \omega)) \leq \max_{\omega} H(\omega)(\omega + \bar{\beta}) = \bar{\beta}^*.$$

Furthermore $\beta^*(y)$ can be equal to $\bar{\beta}^*$ only if $\beta(y - \omega) = \bar{\beta}$ and $\omega = \bar{\omega}$. This is clearly possible for $y \geq \bar{y} = \bar{z} + \bar{\omega}$ and only for these. This proves that $\beta^*(y)$ is less than $\bar{\beta}^*$ for $y \leq \bar{y}$ and constant equal to the maximum for $y \geq \bar{y}$.

It remains to show that β^* is increasing in y for $y < \bar{y}$. Pick $y < \bar{y}$; the targeted financing needs, $y - \omega$ for $\omega \in \tau(y)$, are lower than those for \bar{y} , by point 1, hence lower than \bar{z} . This implies that the targeted financing needs are in the range of values for which β is increasing. It follows that for $\omega \in \tau(y)$ and $y' > y$:

$$\beta^*(y) = H(\omega)(\omega + \beta(y - \omega)) < H(\omega)(\omega + \beta(y' - \omega)).$$

As $\omega \leq y < y'$, the right hand side is less than or equal to $\beta^*(y')$, which proves the desired result.

β^* satisfies (12). Let $y' < y$. Let $\omega \in \tau(y)$ be a maximizer of $H(\omega)(\omega + \beta(y - \omega))$ and $\omega' \in \tau(y')$.

Assume $\omega \leq y'$. Then ω is a feasible target for y' ; hence, by definition of β^* , $\beta^*(y') \geq H(\omega)(\omega + \beta(y' - \omega))$. Thus

$$0 \leq \beta^*(y) - \beta^*(y') \leq H(\omega)(\beta(y - \omega) - \beta(y' - \omega)) \leq H(0)^2(y - y').$$

Assume $\omega > y'$. Then $\omega > \omega'$; it follows that $H(\omega) \leq H(\omega')$ and

$$0 \leq \beta^*(y) - \beta^*(y') \leq H(\omega')[\omega + \beta(y - \omega) - \omega' - \beta(y' - \omega')].$$

Since $y - \omega \geq y' - \omega'$ by Point 1, $\beta(y - \omega) - \beta(y' - \omega') \leq y - \omega - y' + \omega'$ by the contraction assumption of β , hence the term in square brackets is less than $y - y'$. This finally gives $0 \leq \beta^*(y) - \beta^*(y') \leq H(0)(y - y')$. \blacksquare

Point 3. The maximizer $\bar{\omega}$ of $H(\omega)(\omega + \beta)$ is characterized by the first order condition. Thus $\bar{\omega}$ is null if $\frac{H'}{H}(0) + \frac{1}{\bar{\beta}} \leq 0$ and is positive otherwise characterized by:

$$\frac{H'}{H}(\bar{\omega}) + \frac{1}{\bar{\omega} + \beta} = 0. \tag{44}$$

As $\bar{\beta} = H(\bar{\omega})(\bar{\omega} + \beta) \leq \bar{\omega} + \beta$, this implies $\frac{H'}{H}(\bar{\omega}) + \frac{1}{\bar{\beta}} > 0$ and finally $\frac{H'}{H}(0) + \frac{1}{\bar{\beta}} > 0$ by the log-concavity of H . \blacksquare

C Appendix-Proof of Theorem 1

We prove Theorem 1 by considering the auxiliary games \mathcal{G}_n and using recursively Proposition 1. Starting with $B_0^* = 0$, let us define recursively the functions B_n^* for $n \geq 1$ by:

$$B_n^*(y) = \max_{\omega \leq y} H(\omega)(\omega + B_{n-1}^*(y - \omega)) \quad (45)$$

and let $T_n(y)$ denote the cash level(s) that solve the above maximization program. As already observed, the game \mathcal{G}_1 coincides with the auxiliary game Γ_β when β is equal to 0. In particular,

$$B_1^*(y) = H(y)y, \text{ for } y \leq \omega^*, H(\omega^*)\omega^* \text{ for } y \geq \omega^*. \quad (46)$$

In \mathcal{G}_2 , if I makes an offer (p, v) , then a receiver R is playing game \mathcal{G}_1 as a proposer. Thus, if R accepts and has positive financing needs z , he anticipates the benefit $B_1^*(z)$. Note that this benefit depends only on his financing needs and satisfies assumptions (11) and (12) (Proposition B.1). Anticipating R 's benefit and behavior, I plays the game Γ_β in which $\beta = B_1^*$. Hence I 's optimal behavior yields him the benefit $B_2^*(y)$, which also satisfies assumptions (11) and (12). We can repeat the argument and obtain that $B_n^*(y)$ is the maximal benefit that an intermediary with financing needs y can make in \mathcal{G}_n .

The sequence B_n^* is non-decreasing, $B_n^*(y) \leq B_{n+1}^*(y)$ for each y , because a player with needs y playing \mathcal{G}_{n+1} can always play as in \mathcal{G}_n and hence secure at least $B_n^*(y)$ (alternatively this is shown by induction using (45)).

Let us denote by \bar{b}_n the maximum value of B_n^* , $\bar{\omega}_n$ the maximizer of $H(\omega)(\omega + \bar{b}_{n-1})$ and \bar{y}_n the values above which B_n^* is constant, defined by $\bar{y}_n = \bar{y}_{n-1} + \bar{\omega}_n$.

We first show by induction that the sequence $\bar{\omega}_n$ is positive for each n . For $n = 0$ we have $\bar{\omega}_0 = \omega^* > 0$ (Corollary 1). It suffices to use repeatedly point 3 of Proposition B.1 to obtain that $\frac{H'}{H}(0) + \frac{1}{\bar{b}_n} > 0$, hence that $\bar{\omega}_n$ is positive for any n . Thus the sequence $(\bar{b}_n, \bar{\omega}_n)$ is characterized by

$$\bar{b}_{n+1} = H(\bar{\omega}_{n+1})(\bar{\omega}_{n+1} + \bar{b}_n) \text{ and } \frac{H'}{H}(\bar{\omega}_{n+1}) + \frac{1}{\bar{\omega}_{n+1} + \bar{b}_n} = 0. \quad (47)$$

It is easy to show that sequence \bar{b}_n is increasing, $\bar{\omega}_n$ is decreasing and the limits $(b^\infty, \omega^\infty)$

satisfy:

$$b^\infty = H(\omega^\infty)(\omega^\infty + b^\infty) \text{ and } \frac{H'}{H}(\omega^\infty) + \frac{1}{\omega^\infty + b^\infty} = 0. \quad (48)$$

The positivity of $\bar{\omega}_n$ implies that the sequence \bar{y}_n is strictly increasing (because $\bar{y}_n = \bar{y}_{n-1} + \bar{\omega}_n$) and diverges to ∞ since $\bar{\omega}_n \geq \omega^\infty$.

The next Lemma shows that, for each n , there is a positive \underline{y}_n such that I with financing needs less than \underline{y}_n behaves as if he had access to at most n layers even if he has access to more. Furthermore, the sequence \underline{y}_n increases to $+\infty$.

Lemma C.3. *There exists \underline{y}_n , $\underline{y}_n < \bar{y}_n$, such that $B_n^*(y) = B_{n-1}^*(y)$ for $y \leq \underline{y}_n$ and $B_n^*(y) > B_{n-1}^*(y)$ for $y > \underline{y}_n$. For $y < \underline{y}_n$, I 's strategy in \mathcal{G}_m , $m \geq n$ is the same as in \mathcal{G}_n . The sequence \underline{y}_n is increasing and goes to $+\infty$.*

We end the proof. According to Lemma C.3, the benefit $B_m^*(y)$ and the optimal target $T_m(y)$ stay constant on the interval $[0, \underline{y}_n]$ as more rounds m are possible. We denote $B^*(y)$ and $T(y)$ these values on this interval, which thus gives $B^*(y) = B_m^*(y)$, $T(y) = T_m(y)$. In particular, $B^*(y) = B_n^*(y)$ and $B^*(y - \omega) = B_n^*(y - \omega)$ for any $\omega \leq y$. Thus, applying (45) we obtain $B^*(y) = \max_{\omega \leq y} H(\omega)(\omega + B^*(y - \omega))$, i.e. (21) holds for y in the interval $[0, \underline{y}_n]$. Since the sequence \underline{y}_n increases to $+\infty$, this proves that (21) is valid for each y .

Proof of Lemma C.3. The proof is by induction.

n = 1. We show that for some positive \underline{y}_1 , I chooses to play as in \mathcal{G}_1 for y less than \underline{y}_1 and only for those. I plays as in \mathcal{G}_1 if he targets $T_2(y) = y$ or equivalently if $Z_2(y) = 0$. Define \underline{y}_1 as the infimum value such that $Z_2(y) > 0$. We show that \underline{y}_1 is positive and finite. This will imply that $B_2^*(y) = B_1^*(y)$ for $y < \underline{y}_1$, and since Z_2 increases, $Z_2(y) > 0$ for $y > \underline{y}_1$, hence $B_2^*(y) > B_1^*(y)$.

By the definition (45), B_2^* satisfies

$$B_2^*(y) = \max_{\omega \leq y} H(\omega)(\omega + B_1^*(y - \omega)). \quad (49)$$

Let us show that \underline{y}_1 is finite and smaller than \bar{y}_1 . Recall that $y = \bar{y}_1$ maximizes $\omega H(\omega)$, hence satisfies $H'(y)y + H(y) = 0$. The derivative of $H(\omega)(\omega + B_1^*(y - \omega))$ at $\omega = y$ is $H'(y)y + H(y)(1 - H(0))$, hence is negative for $y = \bar{y}_1$. Surely decreasing ω improves I 's profit: $Z_2(y) > 0$ and $B_2^*(\bar{y}_1) > B_1^*(\bar{y}_1)$.

Let us show that \underline{y}_1 is positive.¹⁷ Since $B_1^*(z) \leq H(0)z$ we obtain $B_2^*(y) \leq \max_{\omega \leq y} H(\omega)(\omega +$

¹⁷If the function $yH(y)$ is concave, the proof is simpler: \underline{y}_1 is characterized by $H'(y)y + H(y)(1 - H(0)) = 0$.

$H(0)(y - \omega)$. For y small enough, the maximum over $\omega \leq y$ of the function $H(\omega)(\omega + H(0)(y - \omega))$ is reached at $\omega = y$: the function is log-concave with a derivative at $\omega = y$ equal to $H'(y)y + H(y)(1 - H(0))$, which is positive for y small enough. This implies $B_2^*(y) \leq yH(y)$. Now recall that $yH(y) \leq B_1^*(y)$ and $B_1^*(y) \leq B_2^*(y)$. We thus obtain that for y small enough $B_2^*(y) = B_1^*(y) = yH(y)$, which implies $T_2(y) = T_1(y) = y$ and $Z_2(y) = 0$. Hence surely $\underline{y}_1 > 0$.

Finally, we show that $B_m^*(y) = B_1^*(y)$ for $y \leq \underline{y}_1$ and any positive m . For $m = 3$ the result follows from the definition of B_3^* : since $B_3^*(y) = \max_{\omega \leq y} H(\omega)(\omega + B_2^*(y - \omega))$ and $B_2^*(y - \omega) = B_1^*(y - \omega)$ for $y - \omega \leq y \leq \underline{y}_1$, surely $B_3^*(y) = B_2^*(y)$. The argument can be repeated for any m .

Induction argument. Assume the property is true for a given n . Define \underline{y}_{n+1} as the infimum of the y such that $Z_{n+1}(y) \geq \underline{y}_n$ in the sense that $z > \underline{y}_n$ for any $z \in Z_{n+1}(y)$. For $y \leq \underline{y}_{n+1}$, $B_{n+1}^*(y) = B_n^*(y)$ because $B_n^*(z) = B_{n-1}^*(z)$ for financing needs z less than \underline{y}_n . Since $Z(y) \leq y$, we surely have $0 < \underline{y}_n \leq \underline{y}_{n+1}$.

Let us show $\underline{y}_{n+1} < \bar{y}_{n+1}$. Recall that by definition $\bar{y}_n \in Z_{n+1}(\bar{y}_{n+1})$; since $\underline{y}_n < \bar{y}_n$ by the induction assumption we obtain $\underline{y}_{n+1} > \bar{y}_{n+1}$.

Let us show that $\underline{y}_{n+1} > \underline{y}_n$. At an optimal target $\omega \in T_n(y)$: $B_{n+1}^*(y) = H(\omega)(\omega + B_n^*(y - \omega))$. This implies the following inequalities:

$$B_{n+1}^*(y) \leq H(0)(\omega + B_n^*(y - \omega)) \leq H(0)(\omega + B_n^*(y)).$$

The first inequality holds because H is non-increasing and the second one because B_n^* is non-decreasing. Since $B_n^*(y) \leq B_{n+1}^*(y)$, these inequalities imply $\omega \geq B_n^*(y)(1 - H(0))/H(0)$ for $\omega \in T_n(y)$.

Finally let us show that the sequence \underline{y}_n strictly increases and goes to infinity. For $y \geq \underline{y}_1$, $B_n^*(y) \geq H(\underline{y}_1)\underline{y}_1$. This provides a lower bound ω_{min} to any target: $\omega_{min} = H(\underline{y}_1)\underline{y}_1(1 - H(0))/H(0)$. Thus, $z \leq y - \omega_{min}$ for $z \in Z_{n+1}(y)$. As a result, surely $z \leq y - \omega \leq \underline{y}_n$ for $y \leq \underline{y}_n + \omega_{min}$. This implies that $\underline{y}_n + \omega_{min} \leq \underline{y}_{n+1}$. This yields the desired result \blacksquare

D Appendix-Other proofs

D.1 Proof of Proposition 3

The originator's profit if he chooses k is $\Pi_O(k) = \rho k - C(k) - r(C(k) - \ell k - B^*(C(k) - \ell k))$ which gives the derivative :

$$\frac{\partial \Pi_O}{\partial k} = \rho - C'(k) - r[1 - B^{*'}(C(k) - \ell k)](C'(k) - \ell).$$

Thus, the first-order condition associated to the optimal k satisfies (35). The second derivative is negative at the optimal k . The cross derivative $\frac{\partial^2 \Pi_O}{\partial k \partial \rho}$ is equal to 1. The monotonicity of the optimal k with respect to ρ follows.

$p = \rho$ is optimal if $y(k) = C(k) - \ell k$ is less than \underline{y}_1 . An optimal k must necessarily be such that $C'(k) > \ell$, so that increasing ρ will increase the chosen k until \underline{y}_1 is reached. This shows that surely for ρ large enough the optimal k will give $y(k) > \underline{y}_1$ and thus an offered price $p < \rho$.

The derivative $\frac{\partial \Pi_O}{\partial k}$ decreases with r , hence the optimal investment is non-increasing in r . Similarly, when B^* is concave, the derivative increases with ℓ , hence the optimal investment is non-decreasing in ℓ . ■

D.2 Proof of Implication 6

We only need to show that the financing needs are increasing in ℓ for a linear C : $C(x) = cx$. We have:

$$\frac{\partial \Pi_O^2}{\partial k \partial \ell} = r[1 - B^{*'}(y(k)) - B^{*''}(y(k))(c - \ell)k], \quad \frac{\partial \Pi_O^2}{\partial^2 k} = rB^{*''}(y(k))(c - \ell)^2.$$

At the optimal solution the second-order condition is satisfied and thus $B^{*''}(y(k)) \leq 0$. This gives:

$$\frac{\partial k}{\partial \ell} = \frac{k}{(c - \ell)} - \frac{1 - B^{*'}(y(k))}{B^{*''}(y(k))(c - \ell)^2} > 0.$$

As expected, the originated volume increases when ℓ is higher. Moreover,

$$\frac{\partial y(k)}{\partial \ell} = (c - \ell) \frac{\partial k}{\partial \ell} - k = -\frac{1 - B^{*'}(y(k))}{B^{*''}(y(k))(c - \ell)} > 0.$$

This proves that $y(k)$ is increasing in ℓ . ■

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