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# Equilibrium risk shifting and interest rate in an opaque financial system

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## Abstract

We analyse the risk-taking behaviour of heterogeneous intermediaries that are protected by limited liability and choose both their amount of leverage and the risk exposure of their portfolio. Due to the opacity of the financial sector, outside providers of funds cannot distinguish “prudent” intermediaries from those “imprudent” ones that voluntarily hold high-risk portfolios and expose themselves to the risk of bankruptcy. We show how the number of imprudent intermediaries is determined in equilibrium jointly with the interest rate, and how both ultimately depend on the cross-sectional distribution of intermediaries’ capital. One implication of our analysis is that an exogenous increase in the supply of funds to the intermediary sector lowers interest rates and raises the number of imprudent intermediaries. Another one is that easy financing may lead an increasing number of intermediaries to gamble for resurrection following a bad shock to the sector’s capital, again raising economywide systemic risk.

## 1. Introduction

The ongoing worldwide financial and economic crisis has rejuvenated the interest in systemic risk in the financial system, its dramatic spill over to the real economy and whether and how it should be addressed by public policies. We contribute to this debate with an analysis of the risk taking behaviour of financial intermediaries that are protected by limited liability and may deliberately choose a level of risk in excess of the social optimum. We show how the level of economywide risk taking depends on the distribution of equity among intermediaries and the level of interest rate in the economy.

Our key assumption is that outside providers of funds cannot tell apart “prudent” and well diversified banks from “imprudent” ones overly exposed to one particular asset, because the balance sheets of individual intermediaries are imperfectly observable, or *opaque*. This assumption is consistent with the view of several commentators of the crisis including Brunnermeier (2009), Acharya and Richardson (2009), and Dubecq et al. (2009). In the decade prior to the crisis, risk transfer instruments, which have reached a very large scale in the U.S., have increased the opacity of banks’ leverage and risk-taking incentives (Acharya and Schnabl, 2009). First, regulatory loopholes allowed banks to evade capital requirements by securitising assets and providing (unregulated) liquidity support to “shadow” (i.e., off-balance-sheet) entities (Acharya

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and Richardson, 2009). Second, the financial sector as a whole effectively repurchased much of the senior tranches of structured products, whose payoff distributions was particularly difficult to assess (see, e.g., Coval et al., 2009). Third, some banks actively relied on “window dressing” to manipulate leverage figures – by selling asset before the books releases to repurchase them at a later date (see, e.g., The Financial Crisis Inquiry Commission, 2011). Last but not least, this opacity may have taken the form of shadow subsidiaries that were used to absorb poorly performing assets, as was revealed by the investigation on Lehman’s bankruptcy. While the opacity of the financial sector may have reached unprecedented levels during the run-up to the crisis and the crisis itself, it has long been recognised as a key issue in that industry and one of the fundamental reasons for why it should be regulated. For example, Morgan (2002) shows that bond raters disagree significantly more about U.S. financial intermediaries than they do over other firms, and interprets this result as evidence that banks are intrinsically more opaque – essentially because their assets are difficult to observe and change at a fast pace.<sup>1</sup> This feature of the industry severely limits the ability of outsiders (investors and rating agencies alike) to assess changes in bank’s capital structure in real time.

When intermediaries’ balance sheets are opaque, those intermediaries with relatively low levels of capital may be tempted to hold high-risk portfolios, or even to gamble for resurrection in the face of worsening economic conditions. In our model, intermediaries’ limited liability creates incentives to increase leverage and hold insufficiently diversified portfolios, which raise intermediaries’ return on equity in case of success while transferring much of their losses to their creditors in case of failure. This tendency, however, is alleviated by the inside equity stake of intermediaries’ shareholders, which disciplines risk-taking and thereby limits leverage and favours portfolio diversification. We show that this trade-off gives rise to an endogenous sorting of intermediaries along the equity dimension, with well capitalised intermediaries holding diversified portfolios and keeping a limited level of leverage (that is, behaving prudently), while poorly capitalised ones heavily resort to leverage and invest in correlated assets (i.e., behaving imprudently). Opacity implies that the former are not readily distinguishable from the latter, so that risk-prone behaviour may prosper without being immediately sanctioned by higher borrowing rates.

One property of our model is that the proportion of imprudent intermediaries and, therefore, the level of systemic risk in the financial system, crucially depend on both the cross-sectional distribution of capital and the prevailing interest rate. The endogenous determination of the number of imprudent intermediaries jointly with the interest rate is our key contribution. Equipped with this joint equilibrium outcome, we analyse the impact on the interest rate and the number of imprudent intermediaries of two exogenous aggregate shocks: a *lending boom* that shifts the loan supply curve rightwards; and an *equity squeeze* that shifts the distribution of banks’ capital leftwards. As we show, the downward pressure on the equilibrium interest rate that follows the lending boom raises the number of imprudent intermediaries and hence the level of economywide risk shifting (the *risk-taking channel* of low interest rates). This mechanism is consistent with the recent microeconomic evidence on the risk-taking channel, which suggests that low interest rates tend favour bank risk-taking. For example Jimenez et al. (2011) present direct evidence that falling short-term interest rates systematically favour risk-taking by Spanish banks, especially those at the lower end of the capital distribution. Relatedly, Ioannidou et al. (2009) identify the exogenous impact of interest rate on bank risk taking by looking at dollarised Bolivia (1999–2003), and find that low interest rates were associated with the granting of loans with a higher probability of default, granted to low-rated borrowers, and commanding lower spreads. Altunbas et al. (2010) and Adrian and Shin (2009) focus on banks operating in the U.S. and Europe and point out to the same relationship between interest rates and bank risk taking. As far as we are aware, our paper is the first to offer a theoretical model wherein falling interest rates cause banks to choose riskier portfolios.

Finally, our model predicts that an equity squeeze (that is, a reduction in the overall equity level of the banking sector after a bad aggregate shock) also raises risk taking when the supply of funds is sufficiently elastic, due to a form of “gambling for resurrection” behaviour. Arguably, both shocks occurred in the run-up to the current crisis. In the first half of the 2000s, capital inflows from China and oil-exporting countries into the U.S. and the accommodative monetary policy of the Fed both contributed to keep the yield curve low. Second, the tightening of U.S. monetary policy in 2004 and the rise in delinquency rates on subprime mortgages from 2006 onwards may have deteriorated the equity position of exposed intermediaries and thereby favoured gambling strategies.<sup>2</sup>

In our model, systemic risk in the financial sector arises from the interaction between (i) intermediaries’ limited liability and option to default (the *risk shifting problem*); (ii) their incentive to correlate their risk exposure (the *endogenous correlation problem*); and (iii) the difficulty for outside lenders to discriminate individual institutions on the basis of their true net worth level (the *opacity problem*). While our model is the first to explicitly connect these three dimensions, we build on many contributions that have studied each of them separately. Our modelling of the risk shifting problem closely follows Allen and Gale (2000) and Acharya (2009), who show that limited liability leads financial institutions to overweight

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<sup>1</sup> See also Iannotta (2006) for similar evidence about European banks, and Flannery et al. (2010) on the increase in the opacity of U.S. banks during the crisis. A simple empirical investigation, presented in the separate technical appendix to the paper, confirms on a sample of 77 French banks that banks’ capital to asset ratios are difficult to forecast accurately at the one-quarter horizon. First, the average standard deviation of these ratios between 1993Q2 and 2009Q1 is 4.42%. Second, standard autoregressive forecasting models are characterised by fairly large standard errors, 1.43% on average. The standard error of these forecasting models is greater than 1% for 36% of the banks, and greater than 3% for 17% of the banks. Such a large uncertainty about the one-quarter ahead values of individual bank capital to asset ratios is remarkable.

<sup>2</sup> Landier et al. (2010) provide direct evidence of this behaviour for New Century Financial Corporation, a major subprime originator prior to its bankruptcy in 2007.

risky assets in their portfolio, relative to the first best.<sup>3</sup> There are two main differences between earlier models of risk shifting and ours. In [Allen and Gale \(2000\)](#), market segmentation and limited-liability debt contracts twist intermediaries' risk-taking incentives and lead to an overvaluation (or "bubble") in the price of the risky asset. In both [Allen and Gale \(2000\)](#) and [Acharya \(2009\)](#), intermediaries' excessive risk-taking is ubiquitous in that all banks expose themselves to the risk of bankruptcy. We see this property as somewhat extreme, which leads us to emphasise the disciplining role of shareholders' equity stake and to endogenise each intermediary's (discrete) choice of adopting or not a bankruptcy-prone behaviour.<sup>4</sup>

This asset correlation problem has been the focus of several recent contributions, both empirical and theoretical. [Acharya and Richardson \(2009\)](#) notably document the overexposure of the U.S. banking sector to securitised mortgages prior to the current crisis, with the risk associated with those securities being effectively kept within the sector (via the use of unregulated liquidity enhancements or the repurchase of CDO tranches) rather than transferred to other investors and disseminated throughout the economy. [Greenlaw et al. \(2008\)](#) had reached similar conclusions. The dominant explanation for this excessive correlation, apparently at odds with standard finance theory, is that it is natural consequence of the time-inconsistency of ex post bail-out or interest rate policies; namely, it is optimal to save banks ex post when a large number of them fails, which precisely occurs when they have chosen correlated portfolios in the first place – see [Acharya and Yorulmazer \(2007\)](#) and [Farhi and Tirole \(2012\)](#). Our model differs from those in the source of moral hazard that leads to excess portfolio correlation, i.e., limited liability rather than time-inconsistent policies. In [Acharya \(2009\)](#), the economywide correlation of risks arises from systemic failure externalities amongst intermediaries. The main difference between Acharya's endogenous correlation mechanism and ours is that in his framework banks are assumed to hold undiversified portfolio (because they are industry-specific lenders), and the puzzle to be explained is why correlation occurs *across* banks (i.e., why they tend to lend to the same industries). By contrast, in our model banks are unspecialised and choose the correlation of their portfolio at the individual level; but since those who opt for highly correlated portfolios favour the stochastically dominated asset, the very same asset is overinvested in at the aggregate level, hence more risk-taking at the individual level directly translates into greater systemic risk.

Finally, several authors have discussed how opacity may jeopardise financial stability. The difficulty for unsophisticated outside lenders to perfectly observe banks' assets is a traditional argument for why banks must be supervised (e.g., [Dewatripont and Tirole, 1994](#)). More recently, [Biais et al. \(2010\)](#) have argued that financial innovations create asymmetric information problems that worsen the opacity of the financial sector. Our model focuses on a specific implication of opacity: the fact that outside providers of funds may find it difficult to accurately measure bank shareholders' stake, and thereby their risk-taking incentives.

The rest of the paper is organised as follows. [Section 2](#) presents the model. [Section 3](#) derives the equilibrium level of interest rate and systemic risk in the opaque economy. [Section 4](#) studies the effect on the equilibrium of a lending boom and an equity squeeze. [Section 5](#) extends the baseline model in several dimensions.

## 2. The model

### 2.1. Timing, states and assets

There are two dates,  $\mathbf{t} = \{1, 2\}$ , two possible states at date 2,  $\mathbf{s} = \{s_1, s_2\}$ , and two (supply elastic) real assets available for purchase at date 1,  $\mathbf{a} = \{a_1, a_2\}$ . At date 1, loan contracts are signed and investment in the real assets take place; at date 2, the state is revealed, asset payoffs are collected and financial contracts are resolved – possibly via one party's default. Any unit of investment in  $a_1$  pays  $R_1 = R_1^h > 0$  if  $s = s_1$  and 0 otherwise, while any unit of investment in  $a_2$  pays  $R_2 = R_2^h > 0$  if  $s = s_2$  and  $R_2^l > 0$  otherwise.<sup>5</sup> State  $s_1$  ( $s_2$ ) occurs with probability  $p(s_1) \equiv p = 0.5 - \xi$ ,  $\xi \in [0, 0.5]$  ( $p(s_2) = 1 - p$ ). Finally the two assets are assumed to have identical expected payoffs, i.e.,

$$pR_1^h = pR_2^l + (1-p)R_2^h. \quad (1)$$

This payoff distribution has the following properties: when considered in isolation,  $a_1$  is more risky (in the sense of a mean-preserving spread) than  $a_2$ ; however, the strict negative correlation between the two assets implies that one of them may be used as a hedge against the portfolio risk generated by the other. In particular, a suitably diversified portfolio pays the certain gross return  $pR_1^h$  – thereby entirely eliminating bankruptcy risk for a leveraged investor. This simple payoff structure allows us to focus on the joint choice of leverage and portfolio correlation as the ultimate source of endogenous aggregate risk in the economy.<sup>6</sup>

<sup>3</sup> See also [Rochet \(1992\)](#) for an early analysis of bank risk taking under limited liability. As pointed out in, e.g., [Chevalier and Ellison \(1997\)](#) and [Palomino and Prat \(2003\)](#), the convex reward structures often enjoyed by money managers are another potential mechanism conducive to excess risk-taking in the financial sector.

<sup>4</sup> Another difference with these contributions is that in our model excessive risk taking takes the form of insufficient portfolio diversification in equilibrium, rather than overexposure to a risky asset relative to a safe one.

<sup>5</sup> Assuming that asset  $a_1$  has no liquidation value in state  $s_2$  greatly simplifies the analysis in [Section 4](#), where we need to sum up debt repayments by heterogeneous intermediaries. Alternatively, one may assume that the asset has some liquidation value that is lost in case of default (due, e.g., to bankruptcy costs). Finally, it is straightforward to check that the correlation coefficient between the two returns is  $-1$ .

<sup>6</sup> Since the two real assets are perfectly negatively correlated, they can be combined to create a safe asset with expected payoff  $pR_1^h$ . Our main results would thus be identical if we assumed the existence of a risk-free asset and a mean-preserving risky asset (with payoff  $R_1^h$  or 0). Both specifications are tractable, but that chosen here stresses the importance of insufficient portfolio diversification.

## 2.2. Agents and market structure

There are two types of agents in the economy: “lenders” and “intermediaries”, both risk-neutral and in mass one. Our market structure (and implied decisions) is similar to that in [Allen and Gale \(2000\)](#) and [Acharya \(2009\)](#). In particular, markets are segmented, in the sense that intermediaries have exclusive access to the menu of assets  $\mathbf{a}$  (due, for example, to asymmetric information, difference in asset management abilities, regulation etc.). Intermediaries may borrow from the lenders to achieve their desired level of asset investment, and are protected by limited-liability debt contracts. Once lending has taken place, the portfolio chosen by the intermediaries is out of the control of the lenders. We modify this basic framework in two directions. First, we assume that an intermediary's funding partly comes out of *inside equity*, which will serve both to buffer the intermediary's balance-sheet against adverse shocks and to discipline its shareholders' risk-taking attitude.<sup>7</sup> Second, we study the equilibrium of an economy populated by a large number of intermediaries with heterogeneous equity levels that are imperfectly observed by outside providers of funds.

*Intermediaries.* Intermediaries' shareholders maximise value, given their (exogenously given) initial equity stake  $e > 0$ . Denoting by  $(x_i)_{i=1,2} \geq 0$  the portfolio of an intermediary, its balance sheet constraint may be written as

$$\sum x_i \leq e + b, \quad (2)$$

with  $i=1, 2$  and where  $b$  is the intermediary's debt. Following [Allen and Gale \(2000\)](#) and [Acharya \(2009\)](#), we assume that intermediaries face a convex, nonpecuniary investment cost  $c(\sum x_i)$ , which satisfies  $c'(\cdot) > 0$ ,  $c''(\cdot) > 0$  and  $c(0) = 0$ . In particular, we follow [Acharya's \(2009, p. 230\)](#) interpretation of this cost function as a simple and tractable representation of the costs associated with loan initiation, monitoring and administration, not explicitly modelled but plausibly convex in the number of loans.<sup>8</sup> For the sake of tractability, our analysis in the body of the paper is carried out under the assumption that  $c(\cdot)$  is quadratic, but we show in the technical appendix that all our results carry over to the more general isoelastic case. More specifically,  $c(\cdot)$  takes the form

$$c(\sum x_i) = (2\theta)^{-1} (\sum x_i)^2, \quad \theta > 0. \quad (3)$$

Given its initial equity stake  $e$  and a contracted gross interest rate  $r$  on borrowed funds, an intermediary chooses  $(x_i)$  and, by implication,  $b$  – i.e., it chooses both the size and the structure of the balance sheet. Limited liability implies that an intermediary's payoff net of debt repayment is bounded below by zero, so the ex post net payoff generated by the portfolio  $(x_i)$  is  $\max[\sum x_i R_i - rb, 0]$ . Substituting (2) (with equality) into the latter expression, we find the date 1 value of an intermediary with initial equity  $e$  to be

$$V(e) = \max_{x_i \geq 0} \sum_{s=1,2} p(s) (\max[re + \sum x_i (R_i - r), 0]) - c(\sum x_i). \quad (4)$$

In solving (4), intermediaries differ in the amount of the inside equity stake of their shareholders,  $e$ . The cross-sectional distribution of equity levels is assumed to be characterised by a continuous density function  $f(e; \epsilon)$  with support  $[0, e_{\max}]$  and c.d.f.  $F(e; \epsilon) = \int_0^e f(i; \epsilon) di$ . Since the number of intermediaries is normalised to one we have  $F(e_{\max}; \epsilon) = 1$ , while  $E \equiv \int_0^{e_{\max}} ef(e; \epsilon) de$  is the total capital of the intermediary sector. The parameter  $\epsilon$  indexes the location of the density function, with an increase in  $\epsilon$  being associated with a rightward shift in the distribution of equity level (so that  $F_\epsilon(e; \epsilon) < 0$ .) Whilst the density  $f(e; \epsilon)$  and hence the aggregate equity level  $E$  are taken as exogenous in the baseline version of the model, we provide an extension in [Section 5.3](#) wherein  $E$  responds endogenously to macroeconomic conditions.

*Lenders.* Funds are supplied by the households, or “lenders”, who lend their funds to intermediaries at date 1 and collect repayments at date 2. Each lender enjoys labour income  $w > 0$  at date 1 and maximises  $u(c_1) + c_{2s}$ , where  $c_1$  is date 1 consumption,  $c_{2s}$  consumption at date 2 in state  $s$ , and  $u(\cdot)$  a twice continuously differentiable, strictly increasing and strictly concave function. Let  $\rho_s$  denote lenders' ex post date 2 return in state  $s$  from lending to the intermediary sector, and  $\rho \equiv \sum_s p(s) \rho_s$  the corresponding ex ante return. (Note that both in general differ from the face lending rate  $r$  due to the possibility of intermediaries' default.) Lenders choose their loan supply  $B^s$ , where  $B^s = \arg \max u(c_1) + \sum_s p(s) c_{2s}$ , subject to  $c_1 = w - B$  and  $c_{2s} = B \rho_s$ . The implied loan supply curve is

$$B^s(\rho; w) = w - u'^{-1}(\rho), \quad (5)$$

which is continuous and strictly increasing in both arguments. In short, risk neutrality implies that lenders value the expected return on loans,  $\rho$ , with the implied loan return curve being shifted by date 1 income,  $w$ . We impose specific parameter restrictions later on ensuring that  $B^s(\rho; w) > 0$  in equilibrium.

<sup>7</sup> This paper focuses on agency problems between the intermediary's owner-manager and its creditors, and hence abstracts from incorporating inobservability and conflict of interest between the owners and the managers. See [Acharya et al. \(2010\)](#) for a model of risk-shifting that explicitly incorporates both dimensions.

<sup>8</sup> Alternatively, one may think of a rising unit cost of search for new projects as a foundation for a convex costs function. Surveying earlier evidence on the degree of returns to scale in the financial industry and carrying out their own tests, [Stimpert and Laux \(2011\)](#) conclude that “increasing size is associated with higher costs that increase at an increasing rate, inevitably resulting in diseconomies of scale”.

### 2.3. First best

The key contractual friction in this economy is that an intermediary maximises the expected terminal payoff to its shareholders, who are protected by limited liability and hence transfer losses to the debtors in case of default. Before analysing the implications of this distortion, we compute the first-best outcome where this distortion is removed.

*Planner's problem.* Let  $\tilde{C}_s$  denote the date 2 consumption of intermediaries' shareholders in state  $s=1, 2$ ,  $C_1$  the consumption of lenders at date 1 and  $C_{2s}$  the consumption of lenders at date 2 and in state  $s=1, 2$ . Denoting the planner's portfolio by  $(\hat{x}_i)$ , the planner chooses  $(\hat{x}_1, \hat{x}_2)$  so as to maximise  $u(C_1) + \sum_{s=1,2} p(s)(C_{2s} + \tilde{C}_s) - c(\sum \hat{x}_i)$  subject to

$$\sum \hat{x}_i + C_1 \leq w + E, \quad (6)$$

$$C_{21} + \tilde{C}_1 \leq \hat{x}_1 R_1^h + \hat{x}_2 R_2^l, \quad (7)$$

$$C_{22} + \tilde{C}_2 \leq \hat{x}_2 R_2^h, \quad (8)$$

where (6) is the date 1 resource constraint and (7) and (8) are the state-contingent date 2 resource constraints. Let  $\lambda_1, \lambda_{21}$  and  $\lambda_{22}$  denote the Lagrange multipliers associated with (6)–(8). The first-order conditions with respect to  $C_1, C_{21}, C_{22}, \hat{x}_1$  and  $\hat{x}_2$  are, respectively,

$$\begin{aligned} u'(C_1) - \lambda_1 &= 0, & p - \lambda_{21} &= 0, & 1 - p - \lambda_{22} &= 0, \\ -c'(\sum \hat{x}_i) - \lambda_1 + \lambda_{21} R_1^h &= 0, & -c'(\sum \hat{x}_i) - \lambda_1 + \lambda_{21} R_1^l + \lambda_{22} R_2^h &= 0. \end{aligned}$$

Eliminating the Lagrange multipliers, using (1) and (3) and rearranging, we find that the first-best level of aggregate investment  $\sum \hat{x}_i$  is determined by the following expression:

$$\sum \hat{x}_i = \theta [p R_1^h - u'(w + E - \sum \hat{x}_i)]. \quad (9)$$

Note that Eq. (9) uniquely characterises aggregate investment but not the optimal portfolio  $(\hat{x}_1, \hat{x}_2)$ , since all portfolios belonging to the  $\hat{x}_1 + \hat{x}_2 = \sum \hat{x}_i$  line, with  $\sum \hat{x}_i$  given by (9), are optimal in a first-best sense. This is because (i) the risk differential across the two assets is mean-preserving, and (ii) agents are risk neutral (i.e., second-period utility is linear). Hence, any reallocation of  $(\hat{x}_1, \hat{x}_2)$  that holds  $\sum \hat{x}_i$  at its optimal value yields the same aggregate welfare.

*Decentralisation.* Since the limited-liability constraint is the only friction affecting intermediaries' portfolio choice, the planner's problem can be decentralised by removing this constraint—or equivalently, by punishing default sufficiently severely. When the option to default is not operative, the value of an intermediary in (4) becomes

$$\hat{V}(e) = \max_{\hat{x}_i \geq 0} re + (\sum \hat{x}_i)(p R_1^h - r) - c(\sum \hat{x}_i), \quad (10)$$

where we have used the fact that  $\sum_s p(s) \sum \hat{x}_i (R_i - r) = (\sum \hat{x}_i)(p R_1^h - r)$  under the payoff distribution (1). From (10), the efficient portfolio satisfies  $\sum \hat{x}_i = \theta(p R_1^h - r)$ . As in the planner's problem, this condition determines the optimal investment level  $\sum \hat{x}_i$  but not the optimal portfolio  $(\hat{x}_1, \hat{x}_2)$ —since, given  $r$ , a continuum of such portfolios are consistent with  $\sum \hat{x}_i = \theta(p R_1^h - r)$ . To further characterise the set of feasible portfolios consistent with the first-best level of investment, we note that intermediaries must be *solvent* in both states for the limited liability constraint not to bind. The solvency conditions impose that a given portfolio  $(\hat{x}_i)_{i=1,2}$  never generates a negative net payoff ex post, i.e.,

$$re + \sum \hat{x}_i (R_i - r) \geq 0, \quad \text{for } s = s_1, s_2, \quad (11)$$

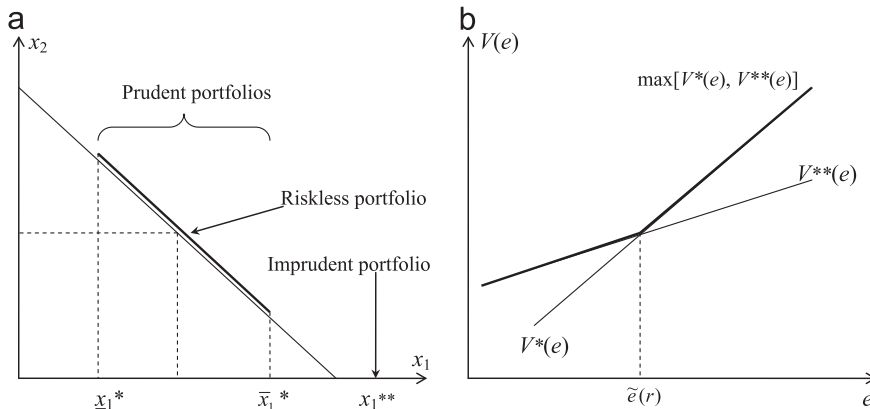


Fig. 1. Intermediaries' optimal portfolios (a) and value (b).

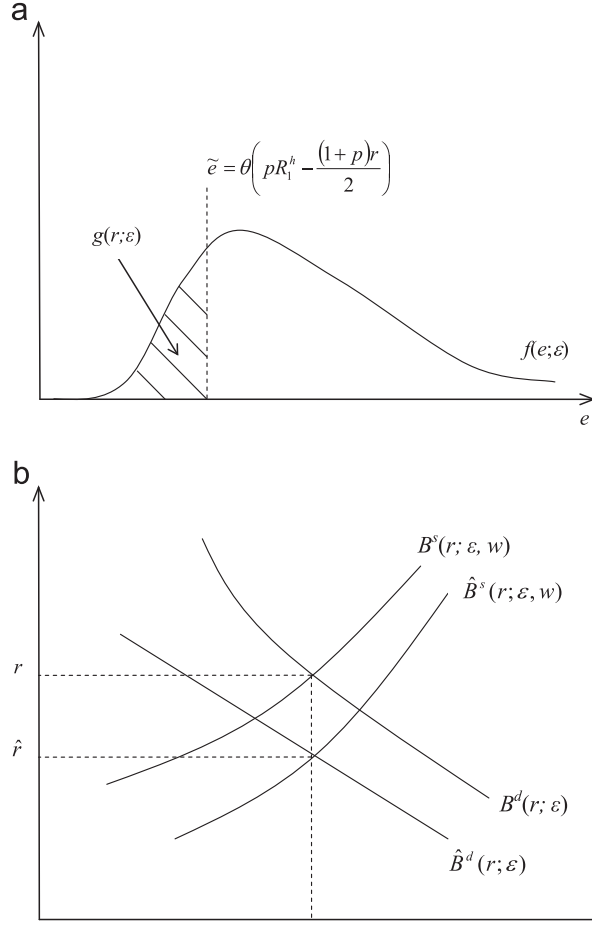


Fig. 2. Share of imprudent intermediaries (a) and loanable funds equilibrium (b).

where  $(R_1, R_2) = (R_1^h, R_2^l)$  if  $s = s_1$  and  $(0, R_2^h)$  if  $s = s_2$ . Combining (11) with the optimal balance-sheet size  $\sum \hat{x}_i = \theta(pR_1^h - r)$ , we find that a solvent portfolio must satisfy  $\hat{x}_1 \in [\underline{x}_1^*, \bar{x}_1^*]$ ,  $0 < \underline{x}_1^* < \bar{x}_1^* < \infty$ , with

$$\underline{x}_1^* = \frac{\theta(pR_1^h - r)(r - R_2^l) - re}{R_1^h - R_2^l}, \quad \bar{x}_1^* = \frac{\theta(pR_1^h - r)(R_2^h - r) + re}{R_2^h}$$

where  $0 < \underline{x}_1^* < \bar{x}_1^* < \theta(pR_1^h - r)$  whenever the intermediary is leveraged. In short, given the optimal balance-sheet size  $\theta(pR_1^h - r)$ ,  $\hat{x}_1$  cannot be too low, otherwise the intermediary would default in state  $s_1$ ; it cannot be too high either (and hence  $\hat{x}_2$  too low), otherwise default would occur in state  $s_2$ . All portfolios satisfying  $\hat{x}_1 + \hat{x}_2 = \theta(pR_1^h - r)$  and  $\underline{x}_1^* < \hat{x}_1 < \bar{x}_1^*$  are consistent with (11) and provide the same utility to the intermediary (due to risk neutrality). These portfolios, which we refer to as “prudent”, lie along a closed subinterval of the  $\hat{x}_2 = \theta(pR_1^h - r) - \hat{x}_1$  – see Fig. 1(a) below. Finally, note that the negative correlation between the two underlying assets implies that among the prudent portfolios a riskless one may be constructed by appropriately weighting the two assets in the portfolio. The riskless portfolio has the property of paying identical payoffs across aggregate states, i.e.,

$$\hat{x}_1 R_1^h + \hat{x}_2 R_2^l = \hat{x}_2 R_2^h, \quad (12)$$

where the left and right hand sides are the portfolio payoffs in states  $s=1$  and  $s=2$ , respectively. Eqs. (1) and (12), together with the fact that  $\sum \hat{x}_i = \theta(pR_1^h - r)$ , uniquely pin down the safe portfolio, which is given by

$$(\hat{x}_1, \hat{x}_2) = \left( \theta(pR_1^h - r) p \frac{R_2^h - R_2^l}{R_2^h}, \theta(pR_1^h - r) p \frac{R_1^h}{R_2^h} \right). \quad (13)$$

*Equilibrium.* To complete the characterisation of the first-best outcome, we must compute the equilibrium interest rate  $\hat{r}$  that results from the equality of the aggregate demand and supply for loanable funds. Since  $\sum \hat{x}_i = \theta(pR_1^h - r)$ , it follows that the leverage of an intermediary with inside equity  $e$  and facing the interest rate  $r$  is given by  $\hat{b}(r, e) = \theta(pR_1^h - r) - e$ .



The aggregate demand for funds is obtained by summing up the demands for debt by all intermediaries, i.e.,

$$\hat{B}^d(r; \epsilon) = \int_0^{e_{\max}} \hat{b}(r, e) dF(e; \epsilon) = \theta(pR_1^h - r) - E. \quad (14)$$

On the other hand, since intermediaries never default in the first-best equilibrium, lenders are repaid  $r$  with certainty. Hence,  $\rho = r$  and we may rewrite (5) as

$$\hat{B}^s(r; w) = w - u'^{-1}(r). \quad (15)$$

$\hat{B}^d(r; \epsilon)$  is continuous and linearly decreasing in  $r$ , while  $\hat{B}^s(r; w)$  is continuous and strictly increasing in  $r$  (since  $u''(\cdot) < 0$ ). Hence the two curves cross at most once and, if they do, give a unique equilibrium interest rate  $\hat{r}$ . In the remainder of the paper, we focus on equilibria in which all intermediaries are active and leveraged. **Lemma 1** provides a sufficient condition for the existence of a first-best equilibrium with this property.

**Lemma 1.** *Assume that (i)  $\theta pR_1^h > e_{\max}$  and (ii)  $w > e_{\max} - E + u'^{-1}(pR_1^h - e_{\max}/\theta)$ . Then, the first-best equilibrium is unique and such that  $\hat{b}(r, e) > 0$  for all  $e \in [0, e_{\max}]$ .*

Essentially, a unique equilibrium with all intermediaries being leveraged exists if both expected asset payoffs (i.e.,  $pR_1^h$ ) and lenders' income (i.e.,  $w$ ) are sufficiently large. This equilibrium is depicted in **Fig. 2(b)**.

### 3. Loanable funds equilibrium under risk-shifting

#### 3.1. Intermediaries' behaviour

The presence of limited-liability debt contracts affects investment choices by altering intermediaries' payoffs relative to the first best. Namely, value maximisation under limited liability may lead an intermediary to choose a high risk/high expected payoff strategy, thereby maximising its own payoff in case of success while transferring losses to the lenders in case default.

We work the problem of an intermediary (i.e., Eq. (4)) backwards. Let us refer to as "prudent" an intermediary whose asset portfolio satisfies both solvency constraints in (11), and denote its value by  $V^*(e)$ . Similarly, let us call "imprudent" an intermediary whose portfolio violates one of the two inequalities in (11) – thereby triggering default in one of the two states –, and denote its value by  $V^{**}(e)$ . The intermediary chooses the best option, giving a value to the initial equity holders of  $V(e) = \max[V^*(e), V^{**}(e)]$ .

*Prudent intermediaries.* Trivially, the absence of the option to default implies that the portfolio choice of a prudent intermediary is the same as in the first best:

$$\sum x_i^* = \theta(pR_1^h - r), \quad \underline{x}_1^* < x_1^* < \bar{x}_1^*, \quad (16)$$

$$b^*(r, e) = \theta(pR_1^h - r) - e. \quad (17)$$

Substituting (16) and (17) into (10), we find the value of a prudent intermediary to be

$$V^*(e) = re + (\theta/2)(pR_1^h - r)^2 \quad (= \hat{V}(e)). \quad (18)$$

*Imprudent intermediaries.* Imprudent intermediaries, unlike prudent ones, correlate their portfolio and consequently default in one of the two states. Consider first the optimal portfolio choice of an intermediary having chosen to overweight asset  $a_1$  in its portfolio, and thus defaults at date 2 if state  $s_2$  occurs. Ex ante, this intermediary earns zero with probability  $1 - p$ , so the objective (4) becomes

$$V^{**}(e) = \max_{x_i \geq 0} p(re + x_1(R_1^h - r) + x_2(R_2^l - r)) - c(\sum x_i). \quad (19)$$

Since  $x_1$  and  $x_2$  enter symmetrically in the cost function while  $R_1^h > R_2^l$ , the intermediary must entirely disregard  $a_2$ , leading to the optimal portfolio:

$$(x_1^{**}, x_2^{**}) = (\theta p(R_1^h - r), 0), \quad (20)$$

$$b^{**}(r, e) = \theta p(R_1^h - r) - e. \quad (21)$$

An alternative investment strategy for an imprudent intermediary would be to overweight  $a_2$ , and hence to default if  $s_1$  occurs ex post. However, it is straightforward to show that it is never optimal to do so under our distributional assumptions. Indeed, imprudent behaviour implies that the intermediary earns 0 if the wrong state occurs, and accordingly only values the state corresponding to the asset being invested in. Since the univariate distribution of  $a_1$  is a mean-preserving spread of



that of  $a_2$ ,  $a_1$  has more value to the imprudent intermediary than  $a_2$ .<sup>9,10</sup> Substituting (20) into (19), we find the optimised value of an imprudent intermediary to be

$$V^{**}(e) = pre + (\theta/2)(pR_1^h - r)^2. \quad (22)$$

To summarise, imprudent intermediaries have two distinguishing characteristics, relative to prudent intermediaries. First, they perfectly correlate their asset portfolio (since  $x_2^{**} = 0$ ), thereby maximising both their payoff in case of success and lenders' losses in case of default. Second, they endogenously choose a larger balance sheet size (since  $x_1^{**} > \sum x_i^*$ ), which in turns implies more leverage for any given level of equity  $e$  (i.e.,  $b^{**}(r, e) > b^*(r, e)$ ). This latter property is a direct implication of the fact that imprudent intermediaries avoid repayment with probability  $1-p$ . This effectively lowers the cost of debt ex ante for any given face interest rate  $r$ , relative to the cost faced by prudent intermediaries (who repay in both states). In the  $(x_1, x_2)$  plane, the imprudent portfolio lies on the  $x_1$  axis and the left of the  $x_2 = \theta(pR_1^h - r) - x_1$  line – see Fig. 1(a).

*Value of an intermediary.* Expressions (18) and (22) reflect the joint roles of equity and the borrowing rate in affecting the intermediary's value and thus the incentives to behave prudently or imprudently. For a given level of equity and borrowing rate, imprudent intermediaries buy larger portfolios, consequently earn large payoffs in case of success, which goes towards raising value (see the second term in the right hand side of both expressions); however, they also risk losing their equity (with probability  $1-p$ ), which tends to reduce value for any given initial equity stake (the first term). Comparing (18) and (22) and assuming that indifferent intermediaries behave prudently, we find that an intermediary engages in imprudent behaviour whenever its equity stake is sufficiently low, that is, if and only if

$$e < \tilde{e}(r) \equiv \theta \left( pR_1^h - \left( \frac{1+p}{2} \right) r \right) \quad (23)$$

Eq. (23) implies that a poorly capitalised intermediary, i.e., one with low equity stake and hence relatively little to lose in case of default, will engage in imprudent behaviour, while an intermediary with high shareholders' equity stake, and hence much to lose in case of default, will behave prudently. The implied value of an intermediary as a function of  $e$ , i.e.,  $V(e) = \max[V^*(e), V^{**}(e)]$  is depicted in Fig. 1(b).

A key implication of (23) is that a lower borrowing rate raises the cut-off equity level below which the intermediary chooses to behave imprudently. To further understand why this is the case, compare the impact of a marginal rise in  $r$  on  $V^*(e)$  and  $V^{**}(e)$  – that is, for each strategy, the loss in the intermediary's value associated with a rise in the face financing cost. Using (18) and (22), we find these falls to be  $V_r^*(e) = -b^*(r, e)$  and  $V_r^{**}(e) = -pb^{**}(r, e)$ . These expressions follow from the envelop theorem and have a straightforward interpretation. For the prudent intermediary, who never defaults and hence always repays  $r$  per unit of debt, the loss in value associated with a marginal rise in  $r$  is its total amount debt,  $b^*(r, e)$ . For the imprudent intermediary, who only repays in state 1, the loss in value is the relevant amount of debt,  $b^{**}(r, e)$ , times the probability that it will actually be repaid,  $p$ . For a rise in  $r$  to lower the threshold  $\tilde{e}$ , it must be the case that  $V^*(\tilde{e})$  increases more than  $V^{**}(\tilde{e})$  for the marginal intermediary, i.e., that for whom  $V^*(\tilde{e}) = V^{**}(\tilde{e})$  (i.e., that intermediary must turn prudent, rather than imprudent, following a rise in the interest rate). It must thus be the case that  $V_r^*(\tilde{e}) > V_r^{**}(\tilde{e})$  or, equivalently by using the two expressions above,  $b^*(r, \tilde{e}) < pb^{**}(r, \tilde{e})$ : a switch by the marginal intermediary from the prudent to the imprudent investment strategy must involve a sufficiently large increase in leverage. This property can be shown to hold not only in the quadratic case but also for any isoelastic investment cost function (see the technical appendix for details).

### 3.2. Aggregate demand for funds

Our key assumption here is that while the distribution of equity levels is perfectly known to outside lenders, financial opacity prevents lenders from observing the equity level of any particular intermediary. Hence, lenders cannot condition the loan rate on the specific equity level of an intermediary, so that a single borrowing rate  $r$  applies to the entire market.<sup>11</sup> Then, we may define

$$g(r; \epsilon) \equiv \int_0^{\tilde{e}(r)} f(e; \epsilon) de = F(\tilde{e}(r); \epsilon) \quad (24)$$

as the proportion of imprudent intermediaries in the economy at a given interest rate  $r$ . Note from (24) that  $g_r(r; \epsilon) = -\theta(1+p)f(\tilde{e}(r); \epsilon)/2 < 0$ , that is, a lower face interest rate raises the proportion of imprudent intermediaries in the economy by increasing the threshold equity level  $\tilde{e}(r)$ . Moreover, we have  $g_\epsilon(r, \epsilon) = F_\epsilon(\tilde{e}(r); \epsilon) < 0$ , that is, an increase in  $\epsilon$  lowers the proportion of imprudent intermediary (for any given value of the cut-off  $\tilde{e}(r)$ ). This is illustrated in Fig. 2(a),

<sup>9</sup> An intermediary choosing to default in state  $s_1$  does not value its payoff in that state and hence maximises  $(1-p)(x_1(0-r) + x_2(R_2^h - r) + re) - c(x_1 + x_2)$ , leading to the optimal portfolio  $(\tilde{x}_1^{**}, \tilde{x}_2^{**}) = (0, \theta(1-p)(R_2^h - r))$ . Computing and comparing the ex ante utility levels associated with  $(x_1^{**}, x_2^{**})$  and  $(\tilde{x}_1^{**}, \tilde{x}_2^{**})$  leads the former to be preferred, provided that  $\xi$  is not too large.

<sup>10</sup> Note that an implication of (20) and (21) is that it is always optimal for the imprudent intermediary to invest its entire own equity in the risky asset. In particular, since asset payoffs are seized in case of default regardless of how assets were purchased, it is never worthwhile to invest one's own equity level  $e$  into a safe combination of assets while using leverage  $b$  to invest in the risky asset.

<sup>11</sup> We assume for simplicity that intermediaries are completely identical from the point of view of the lender. Our results carry over in a set-up with partially segmented market involving different groups of intermediaries, with the members of each group facing the same interest rate (see Section 5.2 below for an example of this). What matters for our results is the presence of an unobserved residual heterogeneity in intermediaries' equity stake.

which depicts an example of a distribution of equity levels  $f(e; \epsilon)$  and threshold  $\tilde{e}(r)$ . The share of imprudent intermediaries corresponds to the shaded area below the  $f(e; \epsilon)$ -curve and to the left of the  $\tilde{e}(r)$  line. Holding  $\epsilon$  constant, an increase (decrease) in  $r$  shifts  $\tilde{e}(r)$  to the left (right) and hence shrinks (expands) the shaded area; holding  $r$  (and hence  $\tilde{e}(r)$ ) constant, and increase (decrease) in  $\epsilon$  shifts  $f(e; \epsilon)$  to the right (left) and hence shrinks (expands) the shaded area.

The total demand for funds  $B^d(r; \epsilon)$  aggregates the leverage choices of individual intermediaries and is thus given by

$$B^d(r; \epsilon) = \int_0^{\tilde{e}(r)} b^{**}(r, e) dF(e; \epsilon) + \int_{\tilde{e}(r)}^{\epsilon_{\max}} b^*(r, e) dF(e; \epsilon). \quad (25)$$

Eq. (25) shows that the interest rate will affect the demand for loanable funds in two ways: first, it will affect the demand for funding of every single intermediaries (the ‘intensive’ leverage margin); and second, by shifting the threshold  $\tilde{e}(r)$ , it will cause a discontinuous change in the leverage choice of some of them, from prudent to imprudent or the other way around (the ‘extensive’ leverage margin.) Substituting (17) and (21) into the latter expression, using (24) and rearranging, the total demand for funds is found to be

$$B^d(r; \epsilon) = \theta[pR_1^i - r(1 - (1-p)g(r; \epsilon))] - E. \quad (26)$$

In the  $(B, r)$  plane, the  $B^d(r; \epsilon)$  curve lies to the right of the  $\hat{B}^d(r; \epsilon)$  curve, its first-best counterpart. This is because, for any given value of  $r$ , the risk-shifting equilibrium includes a nonnegative fraction of imprudent intermediaries, whose demand for debt is larger than that of prudent intermediaries at any given interest rate  $r$  (see Fig. 2b).

There are two properties of the aggregate demand for loanable funds that are worth discussing at this stage. First, it is continuous and decreasing in the borrowing rate, i.e.,  $B_r^d(r; \epsilon) = -\theta(1 - (1-p)g(r; \epsilon)) + \theta(1-p)g_r(r; \epsilon) < 0$ . Two factors contribute to make the demand for funds a downward-sloping function of  $r$ . First, a lower interest rate raises the leverage of both prudent and imprudent intermediaries – see the optimal investment rules (16) and (20). Second, a lower interest rate induces “marginal” intermediaries (those which are close to the cut-off equity level  $\tilde{e}$  in (23)) to switch from prudent to imprudent behaviour, and those experience a discontinuous increase in their leverage – again, by (16) and (20). Hence, changes in the borrowing rate affect the intensive (i.e., conditional on not switching behaviour) and extensive (i.e., the number of intermediaries who switch behaviour) leverage margins in the same direction.

The second relevant property of the curve is that, holding  $r$  constant,  $B^d$  increases as the distribution of equity shifts leftwards. That is,  $B_\epsilon^d(r; \epsilon) = \theta r(1-p)g_\epsilon(r; \epsilon) < 0$ . This is because, as the equity level of intermediaries decreases, some of them switch from prudent to imprudent behaviour. As imprudent intermediaries choose higher leverage than prudent ones, this composition effect translates into an upward shift in the aggregate demand for funds.

### 3.3. Aggregate supply of funds

The aggregate supply of funds depends on the expected return on loans,  $\rho$ , which under risk shifting not only depends on the face borrowing rate but also on both the share of imprudent intermediaries and the probability that they go bankrupt. In state 1, which occurs with probability  $p$ , all intermediaries repay the face interest rate  $r$  to the lenders: prudent intermediaries because they are always able to, imprudent ones because their risky bets turned out to be successful. In state 2, which occurs with complementary probability, only prudent intermediaries, which are in number  $1-g(r; \epsilon)$ , are able to repay  $r$ . Imprudent intermediaries’ bets, on the contrary, turn out to be unsuccessful, leaving lenders with no repayment at all. Summing up unit repayments across states and intermediaries types and rearranging, we find the ex ante gross return on loans to be

$$\rho(r; \epsilon) = pr + (1-p)(1-g(r; \epsilon))r = r(1 - (1-p)g(r; \epsilon)). \quad (27)$$

Note that this ex ante return is strictly increasing in the face interest rate, i.e.,

$$\rho_r(r; \epsilon) = 1 - (1-p)g(r; \epsilon) - (1-p)g_r(r; \epsilon) > 0. \quad (28)$$

The increasing of  $\rho(r; \epsilon)$  with respect to  $r$  occurs for two reasons. First, a higher face interest rate increases intermediaries’ repayment if they do not default (the  $1 - (1-p)g(r; \epsilon) > 0$  part of (28)). Second, a higher face interest rate favours prudent rather than risky behaviour by raising the threshold  $\tilde{e}$ , and hence by lowering the probability of default on a loan unit (the  $-(1-p)g_r(r; \epsilon) > 0$  part). It follows that for  $(\epsilon, w)$  given the loan supply function is a nondecreasing, continuous function of  $r$ , which we may express as

$$B^s(r; \epsilon, w) = w - u^{-1}(r(1 - (1-p)g(r; \epsilon))). \quad (29)$$

Let us briefly summarise the properties of the aggregate supply curve, before we analyse the equilibrium in the market for loanable funds. First,  $B^s(r; \epsilon, w)$  is strictly increasing in  $r$ , holding  $(\epsilon, w)$  constant; this follows from (5), the strictly concavity of  $u(\cdot)$ , and the strict monotonicity of  $\rho$  w.r.t.  $r$  (see (28)). Second, from (5) it is strictly increasing in  $w$ , holding  $r$  and  $\epsilon$  constant. Third, it is increasing in  $\epsilon$ , holding  $r$  and  $w$  constant. The reason for this is that a higher overall level of equity in the economy raises the number of prudent intermediaries (i.e.,  $g_\epsilon(r; \epsilon) < 0$ ), and hence the expected return on loans (see (27)).

In the  $(B, r)$  plane, the  $B^s(r; \epsilon, w)$  curve lies to the left of its first-best analogue,  $\hat{B}^s(r; \epsilon, w)$ . This is because in the equilibrium with risk shifting lenders expect a nonnegative fraction of intermediaries to go bankrupt if state  $s_2$  occurs.

Hence, any given value of the face interest rate  $r$  is associated with a lower expected return in the risk-shifting equilibrium than in the first-best – and hence with a lower supply of loanable funds (see Fig. 2(b)).

### 3.4. Market clearing

In equilibrium, the total demand for funds by the intermediary sector must equal the total supply of funds provided by outside lenders. In other words, the face interest rate that clears the market for loanable funds must satisfy

$$B^s(r; \epsilon, w) = B^d(r; \epsilon) \quad (30)$$

Since  $B^d(r; \epsilon)$  is continuously decreasing in  $r$  while  $B^s(r; \epsilon, w)$  is continuously increasing in  $r$ , the equilibrium is unique provided that it exists. Again, we are focusing on risk-shifting equilibria in which all intermediaries are leveraged, the conditions under which this is the case being summarised in the following lemma.

**Lemma 2.** Assume that (i)  $\theta pR_1^h > e_{\max}$ , and (ii)  $w > \max[u^{-1}(pR_1^h - E/\theta), \theta \bar{r}(1-p)g(\bar{r}; \epsilon) - E + u^{-1}(\bar{r}(1-p)g(\bar{r}; \epsilon))]$ , where  $\bar{r} = pR_1^h - e_{\max}/\theta$ . Then, the equilibrium with risk shifting is unique and such that  $b^*(e)$  and  $b^{**}(e)$  are positive for all  $e \in [0, e_{\max}]$ .

To summarise, the equilibrium is well behaved provided that lenders' income,  $w$ , is sufficiently large. The existence conditions stated in Lemma 2 are slightly more stringent than those stated in Lemma 1, so the former also ensure the existence of the first-best outcome characterised in Section 2.3.

The equilibrium in the market for loanable funds is depicted in Fig. 2(b). The intersection of the two curves gives the equilibrium contracted loan rate  $r$ , given the exogenous parameters  $(\epsilon, w)$ . The loan rate in turn determines the equilibrium share of imprudent intermediaries  $g(r; \epsilon)$  (by Eq. (24)), as well as the equilibrium expected return on loans to intermediaries,  $\rho(r; \epsilon)$  (by (27)).

Finally, note that despite differences in the implied equilibrium interest rate in the two economies, the equilibrium amount of aggregate lending is the same. This can be proven as follows. In the first best, intermediaries never default so the face interest rate is equal to the expected return on loans. From Eqs. (14) and (15), this expected return  $\hat{r}$  is the (unique) solution to

$$\theta(pR_1^h - \hat{r}) - E = w - u^{-1}(\hat{r}) \quad (31)$$

In the second best, the aggregate loan supply is  $w - u^{-1}(\rho)$  (see Eq. (5)), while the aggregate loan demand is given by (26), where by (27)  $\rho(r; \epsilon) = r(1-p)g(r; \epsilon)$ . Hence, in the second best the expected return on loans  $\rho(r; \epsilon)$  satisfies

$$\theta(pR_1^h - \rho(r; \epsilon)) - E = w - u^{-1}(\rho(r; \epsilon)) \quad (32)$$

Eqs. (31) and (32) imply that  $\rho(r; \epsilon) = \hat{r}$ , i.e., lenders' expected return on their loans to intermediaries in the same in the first and second best. By implication, they lend the same amount in equilibrium, i.e.,  $B^s(\rho(r; \epsilon); w) = \hat{B}^s(\hat{r}; w)$ . From (27) and the fact that  $\rho(r; \epsilon) = \hat{r}$ , we find the interest rate premium generated by the presence of imprudent intermediaries to be

$$\frac{r}{\hat{r}} = \frac{g(r; \epsilon)}{(1-p)^{-1}g(r; \epsilon)},$$

which is positive and increasing in both the number of such intermediaries,  $g(r; \epsilon)$ , and the probability that they go bust,  $1-p$ .

## 4. Impact of aggregate shocks

We may now state the main predictions of the model about how shifts in the underlying fundamentals (the supply of funds and the distribution of intermediaries' capital) affect the three key equilibrium variables,  $r$ ,  $\rho(r; \epsilon)$  and  $g(r; \epsilon)$ .

**Proposition 1** (*Lending boom*). An exogenous increase in the supply of funds (i.e.,  $dw > 0$ ) (i) lowers the equilibrium contracted loan rate,  $r$ , (ii) lowers the expected return on loans,  $\rho(r; \epsilon)$ , and (iii) raises the share of imprudent intermediaries in the economy,  $g(r; \epsilon)$ .

Proposition 1 essentially states that easier financing conditions for intermediaries tend to fuel systemic risk by inducing an increasing number of intermediaries to take larger and riskier bets; conversely, tighter credit raises the interest rate and disciplines banks' risk-taking behaviour. The effect of the shift in the loan supply curve is depicted in Fig. 3(a). More specifically, the boom is associated with a rightward shift in the  $B^s$  locus, whose direct effect is to lower the equilibrium contracted loan rate. Holding  $\epsilon$  constant, the new value of  $r$  is associated with a lower value of the equity cutoff  $\hat{e}(r)$  in (23), so that an increasing number of intermediaries become imprudent – i.e.,  $g(r; \epsilon)$  rises. Both the lower value of  $r$  and the higher value of  $g(r; \epsilon)$  contribute to lower the expected return on loans,  $\rho(r; \epsilon)$ .

While our analysis remains formal, several interpretations may be given to the shift in credit supply leading to easier financing conditions. According to Bernanke (2005), for example, a supply driven shift in funding occurred in the first half of the last decade due to recycled balance-of-payment surpluses from China and oil-exporting countries; in this interpretation, systemic risk in the U.S. was closely related to the “global imbalances” problem, which was itself rooted in the willingness of

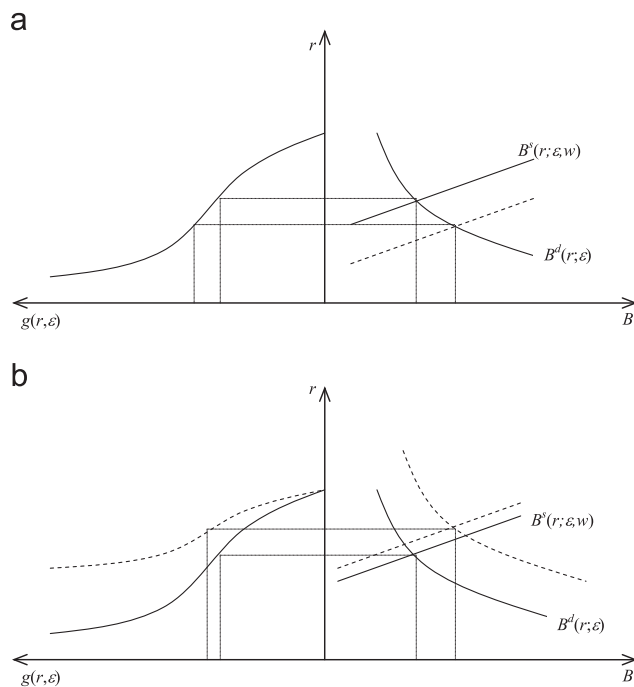


Fig. 3. (a) Lending boom and (b) Equity squeeze (elastic supply of funds).

surplus countries to hoard wealth in the form of U.S. assets. Another view has it that exceptionally loose monetary policy leading to exceedingly low real interest rates in the wake of the 2001 recession in the U.S. would have given rise to a “risk-taking” channel of monetary policy, thereby fostering widespread systemic risk in the U.S. financial sector (see Taylor, 2009; Adrian and Shin, 2009, as well as Altunbas et al. (2010) for a survey and some evidence).<sup>12</sup> Be it the consequence of either or both, the model unambiguously predicts that falling interest rates raise risk-taking by an increasing number of banks and hence the economywide level of risk. Moreover, the model predicts that this increase in aggregate risk is rooted in changes in the portfolio choices of less capitalised intermediaries – i.e., those to the left of, but close to, the equity cutoff  $\bar{e}(r)$ .

Let us turn to the impact of the second aggregate shock under consideration, that on the cross-sectional distribution of intermediaries' equity levels.

**Proposition 2** (Equity squeeze). *A leftward shift in the distribution of equity (i.e.,  $d\epsilon < 0$ ) raises the equilibrium interest rate,  $r$ . If the elasticity of the loan supply with respect to the expected return on loans  $\rho$  is sufficiently high, then it also raises the share of imprudent intermediaries,  $g(r, \epsilon)$ .*

Proposition 2 reflects the three effects at work following a downward shift in the distribution of equity. First, for a given value of the cut-off  $\bar{e}$ , the shift directly increases the number of imprudent banks in the economy by lowering the stake of “marginal” intermediaries (i.e., those who are initially to the right of, but close to,  $\bar{e}$ ); those intermediaries then discontinuously raise their leverage while engaging in imprudent behaviour (see (16) and (20)), thereby raising the demand for loanable funds. Second, to the extent that this shift lowers the overall equity base of the intermediary sector,  $E$ , all intermediaries, which have a target portfolio size, seek to offset the loss in internal funding by external debt, again raising the economywide demand for funding. Both of these effects shift the  $B^d$ -curve rightwards and exert an upward pressure on the equilibrium borrowing rate,  $r$ . Third, this increase in the borrowing rate has a disciplining effect on the intermediary sector by shifting the cut-off equity level  $\bar{e}$  leftwards. Hence, while the effect of the equity squeeze on the borrowing rate is not ambiguous, that on the share of imprudent intermediaries is. However, if the supply of funds is sufficiently elastic, then the adjustment of the borrowing rate after the shock and its disciplining effect will be limited, ultimately causing  $g(r; \epsilon)$  to rise.

This situation is depicted in Fig. 3(b). The initial distributional shift causes the  $g(r, \epsilon)$  curve to move leftwards. The direct impact of higher risk (holding  $r$  fixed) is to lower the expected return on loans,  $\rho(r, \epsilon)$ , which in the  $(B, r)$  plane manifests itself as an exogenous reduction in lending (i.e., an inwards shift of the  $B^s$  curve). Finally, the increase in the demand for funding causes the  $B^d$ -curve to shift rightwards. If the supply of funds is sufficiently elastic (that is, the slope of the  $B^s$  curve is sufficiently low), then the overall effect of the three shifts is to raise the equilibrium value of  $g(r; \epsilon)$

<sup>12</sup> As argued by Obstfeld and Rogoff (2009), these two views are more complementary than substitutes.

## 5. Extensions

### 5.1. Information about intermediaries' balance-sheets and endogenous market segmentation

Our analysis above emphasises intermediaries' balance-sheet opacity as a major source of systemic risk. The key mechanism is that the unobservability of balance sheets makes it possible for bad banks to pretend to be good banks, implying that clearing of the market for loanable funds operates in a single market and at a single interest rate. In order to make this channel as transparent as possible, we derived our results under the somewhat extreme assumption that all intermediaries look alike from the point of view of outside lenders. In reality, some information about intermediaries' balance-sheet is available that may mitigate the opacity problem. In this section, we extend our analysis to allow for (noisy) public signals about intermediaries's capital, which naturally generates a differentiation in the market for loanable funds – in as much a 'good' and 'bad' intermediaries can to some extent be recognised as such. For the sake of tractability we illustrate this possibility by means of a simple parametric example, but we conjecture that the properties that come out of this exercise hold much more generally.

*Distributions, signal structure and parameters.* Let us assume here that the unconditional distribution of intermediaries' inside equity is uniform with support  $[0, 1]$ , so that  $f(e) = 1$ ,  $F(e) = e$  and  $E = 1/2$ . Outside lenders receive the following symmetric binary signal  $\tau$  about every intermediary: if  $e \geq 1/2$ , then  $\tau = g$  ('good') with probability  $\pi \in [1/2, 1)$  and  $\tau = b$  ('bad') w.p.  $1 - \pi$ . Symmetrically, if  $e < 1/2$ , then  $\tau = b$  with probability  $\pi$  and  $\tau = g$  w.p.  $1 - \pi$ . Under these assumptions, the marginal density of the signals is simply  $\Pr(\tau = h) = \Pr(\tau = l) = 1/2$ . From Bayes' rule, the observation of the signal produces the following two conditional distributions and cumulative density functions (both of which are indexed by the signal quality  $\pi$ ):

$$f(e|g; \pi) = \begin{cases} 2(1-\pi) & \text{for } e < \frac{1}{2} \\ 2\pi & \text{for } e \geq \frac{1}{2} \end{cases}, \quad F(e|g; \pi) = \begin{cases} 2(1-\pi)e & \text{for } e < \frac{1}{2} \\ 1-2\pi+2\pi e & \text{for } e \geq \frac{1}{2} \end{cases} \quad (33)$$

$$f(e|b; \pi) = \begin{cases} 2\pi & \text{for } e < \frac{1}{2} \\ 2(1-\pi) & \text{for } e \geq \frac{1}{2} \end{cases}, \quad F(e|b; \pi) = \begin{cases} 2\pi e & \text{for } e < \frac{1}{2} \\ 2\pi - 1 + 2(1-\pi)e & \text{for } e \geq \frac{1}{2} \end{cases} \quad (34)$$

Note that the quality of the signal encompasses two limit cases. When  $\pi = 1/2$ , the signal is uninformative and the two conditional distributions coincide with the unconditional one. When  $\pi \rightarrow 1$ , in the limit the signals exactly identify every intermediary as belonging to the upper or the lower halves of the distribution. Regarding the other deep parameters, we set  $p = w = 1/2$ ,  $\theta = 1$ ,  $R_1^l = 4$ . Finally, we focus on the case where  $u(\cdot) = 0$ , so that lenders only value terminal consumption and inelastically lend  $w$  to the intermediary sector.

The signals identify two categories of intermediaries, and hence two separate markets for loanable funds, each with their own face interest rate. In each market, the problem of an individual intermediary is similar to that described in Section 3, except that they now take their own face borrowing rate  $r^\tau$  as given. Under the assumed parameters, an intermediary borrowing in market  $\tau$  ( $= b, g$ ) behaves prudently if and only if

$$e \geq \tilde{e}(r^\tau) = 2 - 3r^\tau/4, \quad (35)$$

while the share of imprudent intermediaries in that market is given by  $g^\tau(r^\tau; \pi) = F(\tilde{e}(r^\tau)|\tau; \pi)$ .

We may now compute the demand for funds in each market by integrating intermediaries' leverage choices as in (26). Under our parameters, the demand for funds in market  $\tau$  is

$$\begin{aligned} B^{d,\tau}(r^\tau; \pi) &= \int_0^{\tilde{e}(r^\tau)} b^{**}(r^\tau, e) dF(e|\tau; \pi) + \int_{\tilde{e}(r^\tau)}^1 b^*(r^\tau, e) dF(e|\tau; \pi) \\ &= 2 - r^\tau \left( 1 - \frac{1}{2} g^\tau(r^\tau; \pi) \right) - E^\tau, \end{aligned} \quad (36)$$

where, from Eqs. (33) and (34),

$$E^b = \int_0^1 e f(e|b; \pi) de = \frac{3-2\pi}{4} \quad \text{and} \quad E^g = \int_0^1 e f(e|g; \pi) de = \frac{1+2\pi}{4}.$$

*Uninformative signals.* Let us first solve for the equilibrium face interest rate and share of imprudent intermediaries in the uninformative case (i.e.,  $\pi = 1/2$ ), which corresponds to the baseline model analysed in the previous sections (since in this situation both conditional equity distributions coincide with the unconditional one, and we are back to the single-market case.) Under our parameter specification, the (unique) face interest rate  $r$  is determined by the following equilibrium condition:

$$2 - r(1 - g(r; 1/2)/2) - 1/2 = 1/2, \quad (37)$$

where the left hand side is the aggregate demand for loanable funds and the right hand side is the aggregate supply of loanable funds,  $w = 1/2$ . Using (27), we may then explicitly solve for the expected return on loans, which is given by  $\rho(r; 1/2) = 1 \equiv \bar{\rho}$ . By Eq. (27) again, the face interest rate must satisfy  $r(1 - F(\tilde{e}(r))/2) = 1$ . Since the unconditional CDF is

$F(e) = e$  and  $\tilde{e}(r)$  is given by (35), we get the equilibrium interest rate  $r = \sqrt{8/3} = 1.633$ , which in turns produces a share of imprudent intermediaries of

$$g(r; 1/2) = 2 - 3r/4 = 0.775 \quad (38)$$

*Informative signals.* We first note that even when  $\pi > 1/2$  and markets are differentiated, by no-arbitrage and given lenders' risk neutrality, the expected return on loans in the two markets must be identical, i.e., we must have  $\rho^g(r^g; \pi) = \rho^b(r^b; \pi) \equiv \tilde{\rho}(r^g, r^b, \pi)$ . Second, by (27) this common expected rate of return satisfies

$$\tilde{\rho}(r^g, r^b, \pi) = r^\tau(1 - g^\tau(r^\tau; \pi)/2), \quad \tau = b, g. \quad (39)$$

Using (36) and (39), we may express the demand for loanable funds in market  $\tau$  as

$$B^{d,\tau}(r^\tau; \pi) = 2 - \tilde{\rho}(r^g, r^b, \pi) - E^\tau. \quad (40)$$

The marginal density of the signal is  $\Pr(\tau = h) = \Pr(\tau = l) = 1/2$ , so that total equity is  $(E^b + E^g)/2 = E$  while the total demand for loanable funds is  $(B^{d,\tau}(r^\tau; \pi) + B^{d,\tau}(r^\tau; \pi))/2$ . The latter must sum up to the aggregate supply of loanable funds  $w = 1/2$ , so from (40) we get  $\tilde{\rho}(r^g, r^b, \pi) = 1 - \bar{\rho}$ . It can be shown (by contradiction) that for all  $\pi \in (1/2, 1)$  and  $\tau = b, g$ , we always have  $\tilde{e}(r^\tau) > 1/2$ , so the upper halves of the conditional cumulative distribution functions in (33) and (34) determine the shares of imprudent intermediaries in each market. This implies that these shares are given by

$$g^b(r^b; \pi) = F(\tilde{e}(r^b)|b; \pi)_{\tilde{e}(r) > 1/2} = 2\pi - 1 + 2(1 - \pi)(2 - 3r^b/4), \quad (41)$$

$$g^g(r^g; \pi) = F(\tilde{e}(r^g)|g; \pi)_{\tilde{e}(r) > 1/2} = 1 - 2\pi + 2\pi(2 - 3r^g/4). \quad (42)$$

Finally, in both markets we have  $\rho^\tau(r^\tau, \pi) = r^\tau(1 - g^\tau(r^\tau; \pi)/2)$ . Using (41) and (42), the fact that  $\rho^\tau(r^\tau, \pi) = 1$ ,  $\tau = b, g$  and rearranging, we find that  $(r^b, r^g)$  solve

$$1 = r^b(\pi - 1/2 + 3(1 - \pi)r^b/4) \quad \text{and} \quad 1 = r^g(1/2 - \pi + 3\pi r^g/4).$$

Then, with  $(r^b, r^g)$  known, we may compute the shares of imprudent intermediaries in each market from (41) and (42), and that in the whole economy  $(g^b(r^b; \pi) + g^g(r^g; \pi))/2$ . When  $\pi = 1/2$ , we have  $r^b = r^g = r = \sqrt{8/3}$  (the uninformative limit studied above.) As  $\pi$  rises above  $\pi = 1/2$  - i.e., the signal becomes more and more informative-,  $r^b$  goes up and  $r^g$  goes down - since low-equity versus high-equity intermediaries are better and better identified as such. Fig. 4 plots the two face interest rates as a function of  $\pi$ , as well as the shares of imprudent intermediaries in the two markets and the implied proportion of such intermediaries economywide. In this example, the more informative the signal, the higher the share of imprudent intermediaries in the economy. To understand why this is the case, compare the uninformative case ( $\pi = 1/2$ ,  $r = 1.633$  and  $g = 0.775$ ) to the polar opposite ( $\pi \rightarrow 1$ ). Relative to the former, in the latter (i) low-equity intermediaries (i.e., those for whom  $e < 1/2$ ) are perfectly identified as excess risk takers but are charged accordingly (i.e.,  $r = 2$ ,  $g = 1$ , so that  $\rho = 2(1 - 1/2) = 1$ ), and (ii), high-equity intermediaries enjoy lower face interest rates, which induces some of them to behave imprudently (while they would be prudent when charged the high face rate that prevails in the uninformative case.)

## 5.2. Impact of capital requirements

In this section, we explore the effect on systemic risk of imposing capital constraints on intermediaries' behaviour. For tractability, we carry out our analysis under the same parametric specification as in the previous section (i.e.,  $f(\cdot)$  is uniformly distributed over  $[0, 1]$ ,  $p = w = 1/2$ ,  $\theta = 1$ ,  $R_1^h = 4$ , and the supply of loanable funds is inelastic at  $w = 1/2$ .) We consider two simple forms of capital ratios: a 'naïve' capital ratio based exclusively on balance-sheet size, and a risk-based capital ratio that ties the stringency of the ratio to the level of portfolio diversification achieved by the intermediary. As we

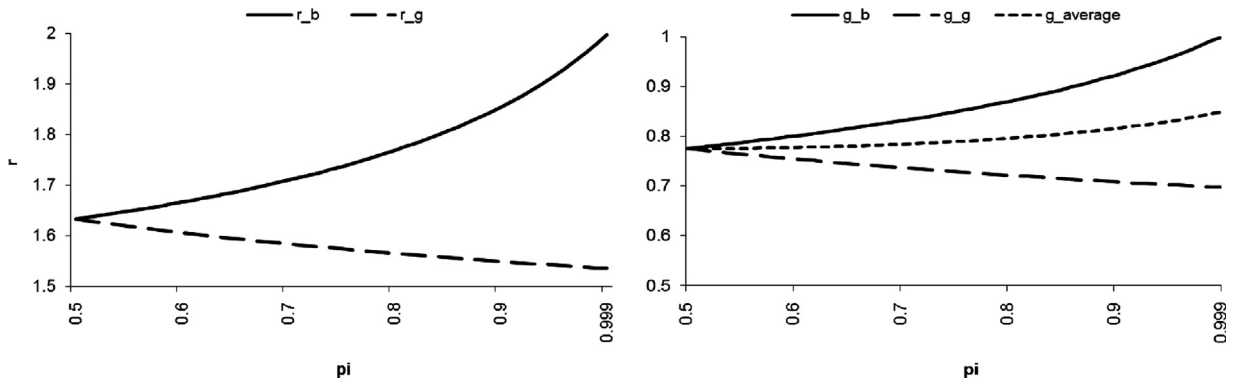


Fig. 4. Face interest rates and shares of imprudent intermediaries in markets b and g as a function of the precision of the signal.



show, the former may turn out to raise rather than lower aggregate risk taking, due to the impact of constrained firms on the equilibrium interest rate. However, in our parametric example risk-based capital ratio are effective at curbing systemic risk (assuming that they are feasible.)

*Asset size-based capital ratios.* We first consider the impact of a simple (naïve) capital ratio prescribing that intermediaries must hold as initial equity at least some pre-specified fraction  $\kappa \in (0, 1)$  of total assets, so that  $e \geq \kappa \sum x_i$ . This constraint might be binding or not, depending on the equity level of each individual intermediary. We focus on the (realistic) case where  $\kappa$  is (i) sufficiently high for the constraint to be binding at least for some intermediaries, given the equity distribution  $f(e)$ ; and (ii) sufficiently low for the constraint not to be binding for intermediaries that would spontaneously choose the prudent portfolio in the absence of a capital constraint (essentially because those are sufficiently capitalised in the first place). This amounts to assuming that  $\kappa$  is positive but small. We first show that the constraint may be effective at limiting the leverage of imprudent intermediaries – i.e., when it is binding–, but *not* at inducing portfolio diversification by those intermediaries. Second, we show that by limiting the leverage of low-equity intermediaries, the capital ratio exerts a downward pressure on the equilibrium face interest rate, which induces some of the originally prudent intermediaries to become imprudent. In consequence, a capital ratio purely based on size may ultimately raise, rather than lowers, the share of imprudent intermediaries in the economy.

*Impact of the ratio on (low-equity) intermediaries.* The leverage of an intermediary facing a binding capital constraint (so that  $e = \kappa \sum x_i$ ) is

$$\check{b}(e) = \kappa \sum x_i - e = e(\kappa^{-1} - 1) > 0. \quad (43)$$

For low values of  $\kappa$ , the constraint is potentially binding for low-equity intermediaries, i.e., those who heavily resort on leverage in the absence of a capital constraint. As discussed above, we focus on the case where  $\kappa$  is sufficiently low for the constraint to be potentially binding only for imprudent intermediaries. Under our parameters, in the absence of capital constraint those imprudent intermediaries would choose a leverage level of (see (21))

$$b^{**}(r, e) = 2 - r/2 - e. \quad (44)$$

The capital ratio is binding if and only if  $\check{b}(e) < b^{**}(r, e)$ , that is, if and only if  $e < \kappa(2 - r/2) \equiv \check{e}(r; \kappa)$ . On the liability side, an intermediary facing a binding capital constraint chooses a lower level of leverage than it would otherwise. On the asset side, does the constraint alter its portfolio choice? The answer is no. To see this, compare the values of a prudent and an imprudent intermediary with total assets given by  $\sum x_i = e/\kappa$  (i.e., the intermediary is constrained.) From our analysis in Section 3 and under our parameters specification, the former and the latter are given by, respectively,

$$\check{V}^*(e) = re + \left(\frac{e}{\kappa}\right)(2-r) - c\left(\frac{e}{\kappa}\right) \quad \text{and} \quad \check{V}^{**}(e) = \frac{1}{2} \left( re + \left(\frac{e}{\kappa}\right)(4-r) \right) - c\left(\frac{e}{\kappa}\right). \quad (45)$$

Since  $\check{V}^{**}(e) > \check{V}^*(e)$ , an intermediary facing a binding constraint always chooses the imprudent portfolio  $(\check{x}_1, \check{x}_2) = (e/\kappa, 0)$ .

*Loanable funds equilibrium.* Under our maintained assumption that  $\kappa$  is small, we have  $\check{e}(r) < \bar{e}(r)$ , so that the capital constraint may only be binding for originally imprudent intermediaries. Then, the demand for loanable funds by the intermediary sector is given by

$$B^d(r; \kappa) = \int_0^{\check{e}(r; \kappa)} \check{b}(e) dF(e) + \int_{\check{e}(r; \kappa)}^{\bar{e}(r)} b^{**}(r, e) dF(e) + \int_{\bar{e}(r)}^1 b^*(r, e) dF(e).$$

Using (43) and (44), the fact that  $dF(e) = de$  and rearranging, we may rewrite the latter expression as

$$B^d(r; \kappa) = \frac{3}{2} - \frac{3}{8}r^2 - \frac{\kappa}{2} \left( 2 - \frac{1}{2}r \right)^2. \quad (46)$$

The face interest rate that clears the market equates  $B^d(r; \kappa)$  with the aggregate supply of funds,  $w = 1/2$ . When  $\kappa = 0$  (our baseline scenario), we again have  $r = \sqrt{8/3}$  (the solution to  $3/2 - 3r^2/8 = 1/2$ ) and  $g(r; \epsilon) = 2 - 3r/4 = 0.775$  (see Eq. (38).) As  $\kappa$  rises and constrains the leverage choices of more and more intermediaries, the aggregate demand curve  $B^d(r; \epsilon)$  shifts down. Given the vertical loan supply curve  $B^s = 1/2$ , the equilibrium face interest rate  $r$  must go down. Solving the equation  $B^d(r; \kappa) = 1/2$  for  $r$ , we indeed obtain the decreasing interest rate function

$$r(\kappa) = \frac{4}{3 + \kappa} \left( \kappa + \sqrt{\frac{3 - 5\kappa}{2}} \right). \quad (47)$$

Finally, since intermediaries facing a binding capital constraint choose the imprudent portfolio  $(\check{x}_1, \check{x}_2) = (e/\kappa, 0)$ , the share of imprudent intermediaries in the economy is given by  $\int_0^{\check{e}(r)} de = \check{e}(r(\kappa)) = 2 - \frac{3}{4}r(\kappa)$ , which is increasing in  $\kappa$ . To summarise, simple capital ratios based on balance-sheet size are (in our example) ineffective at limiting systemic risk. Quite on the contrary, by lowering the equilibrium face interest rate, the capital constraint worsens the risk-taking channel and induces imprudent behaviour by some of those intermediaries that would otherwise behave prudently.

*Risk-based capital ratios.* One key reason for the ineffectiveness of simple capital ratios is that even though the ratio does limit some of the intermediaries' borrowing, it does not curb their risk-taking incentives on the asset side. Suppose now that the regulator (but not an outside lender) is able to observe the riskiness of intermediaries' portfolios and to set the capital



ratio accordingly. For example, assume that the capital ratio  $e/\sum x_i$  is  $\kappa \in (0, 1)$  for a prudent intermediary, but  $\tilde{\kappa} > \kappa$  for an imprudent intermediary. Incorporating this risk-based capital ratios into the values of being prudent or imprudent in (45), we find that a constrained intermediary prefers to be prudent if and only if

$$re + e(pR_1^h - r)/\kappa - c(e/\kappa) > p(re + e(R_1^h - r)/\tilde{\kappa}) - c(e/\tilde{\kappa}).$$

A sufficiently large value of  $\tilde{\kappa}$  (relative to  $\kappa$ ) acts as a deterrent and induces prudent behaviour by constrained intermediaries, so that  $\kappa$  rather than  $\tilde{\kappa}$  effectively applies. Under this regulatory arrangement, the aggregate demand for loanable funds is as in (46), and consequently the equilibrium face interest rate is as in (47). However, since intermediaries facing a binding constraint now behave prudently, the share of imprudent intermediaries in the economy is now  $\int_{e(r, \kappa)}^{\tilde{e}(r)} de = \tilde{e}(r) - e(r, \kappa) = 2 - 3r(\kappa)/4 - \kappa(2 - r(\kappa)/2)$ , where  $r(\kappa)$  is given by (47). The latter expression is decreasing in  $\kappa$ , implying that a risk-based capital requirement is effective at reducing economywide risk-taking.

### 5.3. Endogenous inside equity stake

The analysis in Sections 2–4 is based on the simplifying assumption that the cross-sectional distribution of intermediaries' inside equity levels is exogenous. In reality, inside equity investment adjusts endogenously following changes in the expected gains from investing, and such investments are likely to feed back to aggregate risk-taking. To illustrate this mechanism, we provide a simple extension to our baseline scenario wherein the cross-sectional distribution of inside equity becomes endogenous and dependent on the other deep parameters of the model (including the lending boom parameter  $w$ ). More specifically, we assume that potential inside equity holders are endowed with an initial endowment  $e_0$ , a quantity  $e \in [0, e_0]$  of which may be invested in a bank and the rest being consumed (Stage 1). After this choice is made, the individually optimal portfolio is determined as described in Section 3.2 (Stage 2).<sup>13</sup> Hence, when deciding how much to invest in the bank, potential equity holders value the future utility from owning  $e$  against that from consuming  $e_0 - e$ . Finally,  $e_0$  is distributed with density  $f(e_0)$  and continuous support  $[0, e_{0\max}]$ .

This problem is not tractable in general because  $V(e)$  is convex – see Section 3 –, hence equityholders' programme is not concave and the first-order condition is not sufficient for optimality. One way to solve equityholders' individual problem would be to compute numerically the expected welfare associated with each value of  $e$  on a suitably discretised interval and for a specific set of deep parameters. However, finding the equilibrium would then require solving a complicated fixed-point problem, due to the endogenous state variable of interest ( $e$ ) having a full cross-sectional distribution. Instead, we focus on a simple example that can be solved analytically, but which captures the essence of the endogenous response of bank equity to macroeconomic conditions in the context of our model. More specifically, we assume that equityholders have linear initial consumption utility  $\phi(e_0 - e)$ , where  $\phi > 0$ , and we thus look for the individually optimal policy function  $e(e_0)$  defined as  $e(e_0) = \arg \max_{e \in [0, e_0]} \phi(e_0 - e) + V(e)$ , where  $V(e) = \max[V^*(e), V^{**}(e)]$  is determined as in Section 3, and taking  $r$  (and hence  $\tilde{e}(r)$ ) as given. We then have the following lemma.

**Lemma 3.** *The optimal policy function  $e(e_0)$  is as follows:*

- If  $\phi > r$ , then  $e(e_0) = 0$  for all  $e_0 \in [0, e_{0\max}]$ ;
- If  $\phi < pr$ , then  $e(e_0) = e_0$  for all  $e_0 \in [0, e_{0\max}]$ ;
- If  $\phi \in [pr, r]$  then  $e(e_0) = 0$  for all  $e_0 \in [0, \bar{e}(r)]$  while  $e(e_0) = e_0$  for all  $e_0 \in (\bar{e}(r), e_{0\max}]$ , where  $\bar{e}(r) \equiv (1-p)\tilde{e}(r)/(r-\phi) > \tilde{e}(r)$ . (48)

Intuitively, when  $\phi > r$  the marginal utility of consumption is so high that equityholders prefer to consume their entire endowment; in this case, the economywide inside equity level is  $E = 0$  – and hence independent of all other parameters in the model. At the extreme opposite, when  $\phi < pr$  the marginal utility of consumption is so low that equityholders always prefer to invest their entire endowment  $e_0$ ; then, aggregate inside equity is  $E = \int_0^{e_{0\max}} e_0 f(e_0) de_0$ , i.e., it only depends on the (exogenous) distribution of equityholders initial endowment. We focus on the interesting case where  $pr < \phi < r$ , so that aggregate inside equity is given by  $E(r) = \int_{\bar{e}(r)}^{e_{0\max}} e_0 f(e_0) de_0$  and hence responds endogenously to the equilibrium face interest rate  $r$ . Then, using Leibniz rule and Eq. (48), we find that

$$E'(r) = -\bar{e}(r)f(\bar{e}(r))\frac{\partial \bar{e}(r)}{\partial r} = \frac{(1-p)\bar{e}(r)f(\bar{e}(r))}{(r-\phi)^2} \{ \phi \tilde{e}(r) - (r-\phi)r\tilde{e}'(r) \},$$

which is strictly positive since  $\bar{e}(r) > \tilde{e}(r) > 0$  while  $\tilde{e}'(r) < 0$  – see Eqs. (23) and (48). In other words, as the equilibrium face interest rate  $r$  falls (following, e.g., a lending boom), the threshold level  $\bar{e}(r)$  rises, leading an increasing number of equityholders to consume, rather than save, their initial endowment; this in turn translates into a lower aggregate inside equity level  $E(r)$ . The impact of this effect on the equilibrium for loanable funds is as follows. Recall from (26) that the aggregate demand for loanable funds under risk shifting  $B^d(r; e)$  falls with aggregate inside equity  $E$ . Hence, the dependence of  $E$  on  $r$  *strengthens* the response of the aggregate demand for loans to changes in the face interest rate. This greater

<sup>13</sup> We solve for equityholders' sequential problem under the natural assumption that there is no commitment. Hence, they take  $e$  as given when choosing  $(x_1, x_2)$  in the second stage.

interest-rate elasticity of demand will tend to make the equilibrium face interest rate  $r$  less responsive to aggregate shocks affecting the loan supply curve, such as the lending boom analysed in Section 4. To see this more concretely, consider again the case where lenders only value terminal consumption, so that  $B^s = w$ . Then from (26) the market-clearing condition for the market for loanable funds  $B^d(r; \epsilon) = B^s$  gives  $\theta[pR_1^h - r(1 - (1-p)g(r; \epsilon))] - E(r) = w$ . Total differencing this expression and rearranging, we get

$$-\frac{dr}{dw} = \frac{1}{\theta(1 - (1-p)g(r; \epsilon)) - \theta(1-p)rg_r(r; \epsilon) + E'(r)} > 0,$$

which is strictly decreasing in  $E'(r)$ . Hence, the impact of the lending boom on the interest rate under endogenous equity adjustment is qualitatively similar to that under exogenous equity, although it is smaller in magnitude (in absolute value).

## 6. Concluding remarks

In this paper, we have analysed the portfolio and leverage choices of limited-liability intermediaries and their implications for the level of aggregate risk and the way it responds to changes in economic conditions. The novelty of our framework relative to earlier analyses of intermediaries' risk-shifting behaviour is twofold. First, we emphasise the disciplining role of shareholders' inside equity stake and the heterogeneities that it implies for their equilibrium balance sheets – both on the asset and liability sides. Second, we explicitly model changes in economywide risk-shifting along the extensive margin – i.e., the number of intermediaries that endogenously choose to expose themselves to the risk of default–, in addition to the usual intensive margin –i.e., the change in their individual balance-sheet choices.

A important property of the equilibrium is that it jointly determines the (common) borrowing rate faced by intermediaries and the level of aggregate risk in the economy, due to the endogenous sorting of intermediaries along the equity dimension. Unsurprisingly, intermediaries with low inside equity stake are more likely to behave imprudently than those with high stake. More interestingly, the sorting of intermediaries is itself affected by the interest rate, with falling interest associated with a rising number of imprudent intermediaries and aggregate risk. For this reason, exogenous factors that affect the market for loanable funds (e.g., international capital flows) have a direct impact on the level of risk generated by the financial sector. Similarly, exogenous changes in the distribution of intermediaries' capital affect the equilibrium interest rate, aggregate risk, and the return that ultimate lenders can expect from entrusting the financial sector with their funds. While we have focused on two specific financial fragility channels (the risk-taking channel of low interest rates and the gambling-for-resurrection channel of falling equity), our model could be elaborated further to analyse the impact on intermediaries risk taking of other changes in macroeconomic conditions. For example, it is frequently argued that booms are times of low risk aversion, thereby affecting investors' portfolio choices (e.g., [Bernanke and Kuttner, 2005](#); [Campbell and Cochrane, 1999](#)). Analysing the impact of changes in risk aversion for intermediaries' risk taking would require departing from the risk neutral assumption, with nontrivial implications for both intermediaries' choices and the implied aggregate welfare.

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## Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.eurocorev.2013.07.003>.

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