A Note on Cooperative Strategies in Gladiators' games

Jérôme Ballet, Damien Bazin, Radu Vranceanu

HAL Id: halshs-00927106

https://shs.hal.science/halshs-00927106

Submitted on 10 Jan 2014

Article

A Note on Cooperative Strategies in Gladiators' Games

Jérôme Ballet [1], Damien Bazin [2],* and Radu Vranceanu [3]

[1] Fund for Research on Ethics and Economics (FREE) and French Institute for Research on Development (IRD), UMI RESILIENCES, France

[2] University of Nice Sophia-Antipolis, UFR Higher Institute of Economics and Management (ISEM), National Centre for Scientific Research GREDEG UMR7321 UNS-CNRS, 250, rue Albert Einstein, 06560 Valbonne, France

[3] ESSEC Business School and THEMA (UMR 8184), 95021 Cergy, France

* Author to whom correspondence should be addressed; E-Mail: damien.bazin@gredeg.cnrs.fr (D.B.); Tel.: +33-0-4-89-88-14-03/04.

Abstract: Gladiatorial combat was in reality a lot less lethal than it is depicted in the cinema. This short paper highlights how cooperative strategies could have prevailed in the arenas, which is generally what happened during the Games. Cooperation in the arena corresponded to a situation of the professionalization of gladiators, who been trained in gladiatorial schools. This case provides an analogy of the conditions under which cooperation occurs in a context of competition between rival companies.

Keywords: sustainable competition; cooperation rule; gladiatorial combat
JEL Codes: A19, B11, C79, Z13

1. Introduction

The *munera* (gladiatorial displays) often form part of the received image of Ancient Greece and of Ancient Republican Rome. Many big-budget movies have added to the barbarous image of these fights. However, in reality the *munera* were much more subtle than they are usually depicted in these films. These combats were obviously bloody affairs, but they were not so very remote from present–day sports [1] and also corresponded to religious, social and political needs [2]. Far from the image of unregulated fights to the death between gladiators who had no choice in the matter, the combats were

in fact highly codified and orderly affairs. Two "Referees of the Games" (the *summa rudis* and his deputy, the *secunda rudis*) had the task of enforcing the rules of the Games. Their job was not the same as that of the *munerarius*. The task of the *munerarius*, or Sponsor of the Games, was to act as an umpire-adjudicator, and in the end to decide the outcome of the combat. The combats were of course extremely violent, but this ferocity masked the complexity of the encounters. The combats were above all about entertainment rather than about slaughter [3]. To achieve this, the combatants had to put on a good show, often before an audience of connoisseurs, which required a mastery of their art and some degree of motivation [4]. The motivation was provided by the fame and riches that "good" fights could lead to. The diversity of the weapons used and the attacking and defensive techniques used contributed to the quality of the display. The objective was therefore to avoid killing ones opponent, but to put on a good show. To do this, one obviously should not kill ones adversary too quickly. Cutting blows were usually preferred. The weapons were not sharp enough to kill the adversary outright, and usually the aim was to produce injuries that would in the end lead one of the combatants to admit defeat. Fatal blows were in fact very rare, and virtually banned in these combats. If death did ensue, it usually occurred after the fight as a result of the injuries sustained, rather than during the actual match. Surgery actually progressed during this period of history as a result of these combats. Two Greek doctors contributed particularly to improving reparative surgery as a result of practicing on gladiators: Celsus in the 1st century CE and Galen in the 2nd century CE [5].

The purpose of this article is to highlight the fact that there are two possible types of gladiatorial combat. The first corresponds to the situation depicted in the movies, in which fights between slaves end with one being slaughtered by the other. This outcome does not correspond to the historical reality of these Games. The second possibility is that in which the outcome of the Games is that quite often defeated gladiators survive. This does reflect the historical reality, and is associated with the existence of professional gladiators.

We therefore propose an analysis of cooperation during gladiatorial combats corresponding to the situation in which the outcome is generally the survival of the defeated gladiator. We point out that in combats of this type, there was indeed fierce competition, with a chance of gaining fame and fortune, but set against this a risk of death. However, this dramatic pursuit of selfish interests did not exclude forms of cooperation, which had the effect of limiting the risk of death. In turn, a lower attrition rate was consistent with a sustainable economic model of the Games, reducing the need to train large numbers of fighters.

The analysis should be seen as an application of Robert Axelrod's [6] theory of cooperation. He emphasized that cooperation can emerge in repeated games when choices today influence not only the payoff today but also the later choices and payoffs. In that book and subsequent studies, Axelrod demonstrated the welfare improving properties of the tit-for-tat strategy: "cooperate at the first move, than do what the other player does". In a gladiator game, this strategy must be qualified, since if a bloody gladiator kills his opponent, the later will no longer be able to reciprocate. Hence, we need to consider a different retaliation strategy that brings into the picture a plausible norm of conduct pertaining to gladiator profession.

In a first section, we introduce the main assumptions and notations. In the second section, we argue that cooperative behavior in the Arena combat could be a Nash equilibrium if a collective retaliation

strategy is at work. Finally, we conclude by considering what this case contributes to an analysis of cooperation and competition.

2. Assumptions and Notations

2.1. Assumptions

Let us suppose that there is a large population of gladiators with identical fighting skills. The pairs of gladiators who are going to fight are selected at random. Since all the gladiators have similar fighting skills, the outcome of the fight depends on "chance"; in this state of doubt, the ex-ante probability to win a fight is 0.5. We further assume that fighters know whether the opposing gladiator has a reputation as a killer or not. In other words, gladiators know whether the opponent killed his defeated adversaries at the end of his previous fights or not.

The fight ends by the death or capitulation of one of the gladiators. We assume that if the fight ends with the capitulation of one of the gladiators, the loser's life is in the gift of the winner. We therefore assume that the umpire-adjudicator automatically validates the winner's decision. This hypothesis may seem to be rather extreme, but it corresponds to the reality in many cases of the combats known as combats with *Missio*, in which the decision whether to put the loser to death or to spare his life was indeed in the hands of the winner. If there had been a good fight, the umpire-adjudicator usually validated the winner's decision.

Numerous fights were scheduled, and there were enough gladiators for the pairs to be different on each occasion, and for a gladiator who was not killed in a fight to be able to go on fighting numerous times, approximating to infinity.

2.2. Notations

Let:

- v: denote the immediate "tangible" reward from of a victory (fame and wealth, the incremental raise in chances to be freed from slavery), this parameter is under the control of the Administration of the Games;
- δ: denote the value of the satisfaction obtained by killing ones adversary (for example, because fighting induces a certain bloodthirstiness). This parameter should be seen as a trait of character, but the Administration of the Games can tune it through the screening and training of the Gladiators.
- We also denote $(-u)$ the disutility of getting killed, with $u > 0$ and a value of u that is not infinite (otherwise there would be no game). If the gladiator loses the fight, but is not put to death, it is accepted that his immediate utility is 0.
- Finally, β is a factor of temporal discount, with $\beta \in (0,1)$. The more *β* tends towards 0, the more the gladiator "lives for the present moment" (*carpe diem*). Conversely when *β* tends towards 1, the gladiator gives increasing importance to the future.

To sum up, v and to some extent δ can be seen as under the control of the organisers of the Games; u and β are related to the character of the fighters.

2.3. Strategies

Face to a defeated opponent, the winner can undertake one of the two actions: kill his opponent (*k*), or to let him live, *i.e.*, be merciful (*m*).

Given the choice of his past strategies, the opponent can hold one of the two identities: he can be a Killer, or a Merciful person.

By definition, a gladiator who always kills his defeated opponent is a Killer. Once that he killed a Merciful gladiator, he will keep his identity for the rest of his professional life (even if he repents and does not kill another Merciful person, he will have the stigma of a Killer).

A Merciful would normally spare the life of his opponent. However, when opposed to a Killer, we consider the retaliation strategy where, should he win the fight, he will kill him. A Merciful gladiator who kills a Killer preserves his Merciful identity.

Thus, the set of strategies is, depending to the identity (type) of the Gladiator:

- Killer: play (*k*) whatever the identity of his opponent;
- Merciful: play (*m*) when his opponent is Merciful: play (*k*) when the opponent is a Killer.

The fact that a Merciful gladiator preserves his identity after killing a Killer introduces a credible treat. According to the standard result in repeated games, the punishment for non-cooperative behavior will explain the emergence of cooperation.

2.4. Equilibria

We would like to know firstly under which condition "generalized no-killing" can be a Nash equilibrium. In this case, all gladiators preserve the life of the defeated opponents, *i.e.*, are Merciful, and no gladiator deviates by killing a Merciful gladiator.

Let $G_M(m)$ denote the expected inter-temporal utility of a gladiator in the Merciful equilibrium, before the fight starts, who adopts the strategy *m* (merciful). With a probability 0.5 he wins the combat, and with a probability 0.5 he loses. In this equilibrium, no defeated fighter is killed, so both winners and defeated gladiators will fight again in the future. We can write:

$$G_M(m) = 0.5[v + \beta G_M(m)] + 0.5\beta G_M(m)$$

or

$$G_M(m) = 0.5v / (1 - \beta)$$

Notice that this expression is positive.

What would be now, in the same equilibrium, the gain of a gladiator who decides to deviate from the equilibrium (*m*) strategy, and kill his defeated mate? For sure, in a first time, he will get the "killing benefit" δ. But from now on he will hold the identity of a Killer, and, should ever he lose a fight, he will be killed. His intertemporal expected utility, denoted by $G_M(k)$, is:

$$G_M(k) = 0.5(v + \delta + \beta G_K(k)) + 0.5 \, \beta G_M(k)$$

where $G_K(k)$ is the intertemporal utility of a killer. The latter is given by:

$$G_K(k) = 0.5(v + \delta + \beta G_K(k)) + 0.5(-u)$$

We have:

$$G_K(k) = [v + \delta - u] / (2 - \beta)$$

Thus:

$$G_M(k) = [2(v + \delta) - \beta u] / (2 - \beta)^2$$

The condition for the Merciful equilibrium to exist is thus:

$$G_M(k) < G_M(m)$$

$$[2(v + \delta) - \beta u] / (2 - \beta)^2 < 0.5v / (1 - \beta)$$

That ultimately can be written as:

$$\delta < \delta_1 = [\beta^2 / 4(1 - \beta)]v + (\beta / 2)u$$

where $\delta_1 > 0$. In line with what intuition would suggest, all things equal, the no-killing equilibrium can exist provided that the bloodiness of the fighters is kept under control; a large disutility form dying (large u) or a large reward (v) raise chances that this condition is fulfilled. Notice that even if Gladiators do not care about dying ($u = 0$), the cooperative equilibrium can still exist if the "tangible" reward from winning (v) is large enough.

Obviously, the game presents also a "generalized killing" Nash equilibrium, where all gladiators are killers. The expected intertemporal utility of a Killer who enters the arena and knows that chances to win are 0.5 is:

$$G_K(k) = 0.5(v + \delta + \beta G_K(k)) + 0.5(-u)$$

or

$$G_K(k) = (v + \delta - u) / (2 - \beta)$$

In a world made up of killers, it makes little sense for a player to be Merciful, since in all future combats he will have to fight against other killers, so he would get no benefit in the future. Hence, the basic condition for this equilibrium to exist is merely:

$$G_K(k) > 0$$

or

$$\delta > \max\{0, (u - v)\}$$

We can show that the Merciful equilibrium is Pareto dominant from the Gladiators' perspective.

We have shown that $G_M(m) = 0.5v / (1 - \beta)$ and $G_K(k) = (v + \delta - u) / (2 - \beta)$. $G_M(m) > G_K(k)$ is equivalent to $\delta < \delta_2 = [\beta v / (2(1 - \beta)) + u]$. But the Merciful equilibrium exists for $\delta < \delta_1$. It can be easily shown that $\delta_1 < \delta_2$. Thus, for $\delta < \delta_1$ we also have $G_M(m) > G_K(k)$.

2.5. The Multiple Equilibria Condition

The most interesting case occurs when $(u - v) < \delta_1$, which is tantamount to $v > [4(1 - \beta) / (2 - \beta)]u$ $\Leftrightarrow (u - v) > u\beta / (2 - \beta) > 0$, *i.e.*, the reward ($v$) connected to fame, wealth, perspective to once become a free man, is large enough compared to the disutility of dying.

In this case the game presents two pure strategy equilibria, both the Killing and the Merciful one, the latter being Pareto dominant. However, even if the cooperative equilibrium is Pareto dominant, there is no way to guarantee its emergence spontaneously. If the cost of capturing slaves and training them to become good fighters is brought into the picture, the intervention of the administration is required to secure the emergence of the Merciful equilibrium. However, once that the Merciful equilibrium is in place then it is self-sustainable; no Gladiator has the incentive to unilaterally deviate from the equilibrium strategy *m*.

In the opposite case, the configuration is trivial, the equilibrium is unique (either Killing or Merciful).

One remark applies to the retaliation strategy in the Merciful equilibrium. We have seen that this strategy allows for the existence of the "good" equilibrium, preferred by the Gladiators and probably by the Administration of the Games. However, its enforcement is a matter of collective action. Normally, the retaliation strategy in a tit-for-tat setting is implemented by the harmed player. In this game, the harmed player is not in a position to retaliate (he is dead); but the other similar players would collectively benefit from implementing the punishment. We have here an evolutionist perspective on retaliation as a device for preserving the Gladiator "specie", very much in line with Axelrod's approach to cooperation.

2.6. Interpreting the Results

The type of equilibrium depends on the values of *v*, δ, *u* and β, *i.e.*, on the reward from a victory, the disutility of death and the preference for the present.

The Killing equilibrium corresponds to the stereotypes of the big-budget Hollywood movies on gladiatorial combats between slaves, who have nothing to hope for beyond dying with dignity in the arena. However, as we have already pointed out, these depictions are very far from an accurate representation.

In contrast, gladiators were professionals whose goal is to earn money and become famous and who most often would spare the life of their adversaries. This solution is socially preferable, because it ensures that the gladiatorial profession survives and indeed that the gladiators themselves survive. We have shown how this cooperative equilibrium can prevail if a collective retaliation strategy is adopted by the group of gladiators. In particular, this equilibrium can exist even if gladiators are individuals who have nothing to lose ($u = 0$).

This solution dominates particularly in the case of professional gladiators, because fame and riches do not generally accrue from a single fight, but from numerous repeated fights that have won public approval.

As a corollary to this equilibrium, in order to benefit from fame and riches, the professional fighters must provide the public with a good show. In other words, this result makes it clear that the fight must last for long enough, and display the various techniques that will provide an aesthetic show, whilst still looking "convincingly" violent so that it does not look like a sham fight. This solution explains why the violence of the fights often leads to the death of some combatants outside the arena, as a result of their injuries, even though death was not the intended aim of the fight. Should we introduce in our model an exogenous probability of dying from injuries, the scope for cooperative behavior would further increase.

To summarise, our game highlights the two possible outcomes of gladiatorial combats: one in which killing is the equilibrium strategy, which corresponds to the image portrayed in films of gladiatorial combats between slaves who have nothing to lose, but which is not true to the historical reality; and the other, in which the dominant outcome is the survival of the defeated fighters, a situation which was the historic reality for professional gladiators. In a multiple equilibria configuration, the Administration of the Games had probably an important role in implementing the cooperative equilibrium in the first place. Once that the cooperative equilibrium is instated, it is self-perpetrating.

3. Conclusion

The game that we have described highlights a possible outcome of fights between gladiators that is compatible with very little loss of life. The analysis accounts for the situation that prevailed after the development of Games involving professional gladiators. Many imperial (and also private) gladiatorial schools were set up to train novice gladiators how to handle their weapons and put on good shows. The best known were the *Ludus Magnus*, *Dacius*, *Martutinus* and *Gallicus*. At first these were located in the Flavian amphitheatre in Rome, but others were subsequently organized outside Rome in the Italian provinces.

Gladiators had to combine putting on a good display (the duration and elegance of the fight) with survival. It would seem that fatal blows were not the usual rule, and that if the fight met to the public's expectations, the winner spared the life of the loser. In our case, the fact that the game was repeated *ad infinitum*, combined with a collective retaliation strategy according to which Killers can be Killed, reveals the possibility of cooperation. Here we find, therefore, a variant of the *folk theorem*. The game that we have presented here makes it possible to understand that gladiatorial combats disappeared as a result of a political and social decision, and not to the disappearance of gladiators themselves (as a result of being killed) [7].

With regard to economics, this model yields some conclusions that are relevant for analyzing competition between firms. Even fierce competition equivalent to a fight that can result in death (as a result of injuries) is not incompatible with a minimum degree of cooperation (the losers are not put to death on purpose). In the Ancient Gladiator Games, such a socially preferable solution constitutes a Nash equilibrium in a repeated game provided that a collective retaliation strategy can be enforced. By analogy, we can take that "liquidation" of competitors in business world is not the principle underlying the market economy, or at least it should not be. Our game highlights the fact that on the contrary, it is not only conceivable that the strongest competitors will allow the weaker ones to survive, but that this is actually desirable, because such a strategy ensures that a satisfactory supply flow can be sustained over time.

References

1. Flobert, P. Quelques survivances de la gladiature. *Voces* **1990**, *1*, 71–76 (in French).
2. Ville, G. Religion et politique: Comment ont pris fin les combats de gladiateurs. *Ann. Econ. Soc. Civilis*. **1979**, *34*, 651–671 (in French).
3. Veyne, P. Les gladiateurs ou la mort en spectacle. *L'Histoire* **2004**, *290*, 77–83 (in French).
4. Teyssier, E.; Lopez, B. *Gladiateurs: Des Sources à L'expérimentation*; Editions Errance: Paris, France 2005, pp. 154 (in French).
5. Régnier, C. Les plaies de la Rome Antique: entre légionnaires et gladiateurs. *Journal des Plaies et Cicatrisations* **2004**, *41*, 45–48 (in French).
6. Axelrod, R. *The Evolution of Cooperation*; Basic books: New York, NY, USA, 1984; pp. 241.
7. Beschaouch, A. Nouvelles recherches sur les sodalités de l'Afrique romaine. *Comptes-rendus des Séances de l'Académie des Inscriptions et Belles-Lettres* **1977**, *121*, 486–503 (in French).