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Price dynamics, financial fragility and aggregate volatility[*][†]

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Abstract

Within a standard framework à la Arrow-Debreu, we investigate the dynamics emerging from the interactions of heterogeneous households and firms that are adaptive price setters and financially constrained. We show that depending on the stringency of the financial constraints the model can settle in two very different regimes: one characterized by equilibrium, the other by disequilibrium and financial fragility. We then investigate how the structure of the production network affects the emergence of aggregate volatility from micro-level price and financial shocks, hence providing a dynamical counterpart to recent results of Acemoglu and al (2012).

1 Introduction

A series of recent studies, in particular Gabaix (2011) and Acemoglu et al. (2012), analyzes macroeconomic volatility as a deviation from the central limit theorem in the diversification of idiosyncratic productivity shocks occurring at the firm or at the sectoral level. Quantitative estimates performed by these authors on the basis of US data indicate that the induced fluctuations would be of the same order of magnitude as the actual fluctuations of GDP.

The work by Gabaix shows that in an economy where the distribution of firms is fat-tailed, volatility brought about by independent productivity shocks decays logarithmically with the number of firms. Acemoglu and co-authors

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[†]The source code for simulations reported in this paper is available online at https://sites.google.com/site/antoinedavidmandel/codes.

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adopt an approach similar to this of Fisher and Vega-Redondo (2006) or Battiston et al. (2007) and treat the input-output structure of a multi-sectoral economy as a weighted network. They show that the rate of decay of aggregate volatility in response to sectoral productivity shocks depends crucially on the degree distribution of the production network, i.e on inter-sectoral linkages. To take an extreme example, in a star-shaped network, a shock on the central sector triggers an equivalent variation at the aggregate level.

These results potentially represent major advances in our understanding of macro-economic fluctuations and the unfolding of crisis. Yet, they abstract away from the actual mechanics of crisis and the temporal dimension embedded in the idea of fluctuation, as they are essentially static and asymptotic: they describe the limit properties of a sequence of economies as the number of firms or of sectors tends towards infinity. Therefore, their relevance is conditional on the existence of a model that provides a micro-economic description of the generation and the propagation of shocks and allows to reproduce dynamically the emergence of aggregate volatility. Agent-based models, which allow to simulate the evolution of complex economic systems formed by heterogeneous interacting agents (see LeBaron and Tesfatsion, 2008, for an introduction), are the ideal candidate to provide micro-foundations to these results that crucially rely on non-linearity and deviation from Gaussian standards.

As far as Gabaix (2011) is concerned, there are strong empirical evidence on the fat-tailed distribution of firms' size (see Axtell, 2001, and the subsequent literature) and a number of recent contributions provide micro-founded agent-based models that reproduce the emergence of both fat-tailed distribution of firms and macroeconomic volatility (see in particular Delli Gatti et al., 2005; Dosi et al., 2010).

Concerning the contribution by Acemoglu et al. (2012), it is grounded in a static version of the equilibrium real-business cycle model of Long and Plosser (1983). Consequently it lacks a temporal dimension and ought to measure "aggregate volatility" through a comparative static analysis of the level of GDP at different equilibria. We shall argue that volatility is much more likely to materialize out of equilibrium, e.g when the economy transits between two equilibria following a productivity shock. However, performing such a disequilibrium analysis requires a dynamical model that can represent both processes of convergence to and of divergence from an equilibrium.

The efforts of general equilibrium theory in that direction were based on centralized price adjustment processes, first the walrasian tâtonnement and then non-tâtonnement processes (see Bénassy, 2005, for a recent account). These efforts were almost entirely stopped by the impossibility results put forward in the Sonnenschein-Mantel-Debreu theorem. As far as agent-based models are concerned, they have until now focused mainly on disequilibrium situations and neglected to establish an explicit link to general equilibrium. Yet, the paradigmatic shift to a decentralized and disaggregated approach of economic dynamics can also shed new light on the issue of price dynamics. As a matter of fact, recent work of ours show that evolutionary learning of heterogeneous price setting agents provide a viable alternative to the Walrasian tâtonnement (see

Gintis, 2007; Gintis and Mandel, 2012). Brought together with previous work on financial fragility developed in a series of models with heterogeneous interacting agents by Delli Gatti, Gallegati and co-authors (most recently Delli Gatti et al., 2010; Battiston et al., 2012a), this approach allows us to propose, in this paper, a model of an economy à la Long and Plosser that accounts on the one hand for processes of convergence towards equilibrium and on the other hand for the endogenous creation of shocks, disequilibrium and aggregate volatility. The model is stock-flow consistent and dynamically complete.

We then use this model to investigate wether dynamic counterparts of Acemoglu et al. (2012) results hold, that is how the structure of the production network affects the speed at which the model relaxes to equilibrium and the magnitude of fluctuations thereabout. Additionnaly, our model sheds light on the relationships between agent-based models à la Delli Gatti et al. and equilibrium: it allows to track how financial fragility comes about and propagates in a stock-flow consistent model. For example, basic accounting shows that if the monetary mass is constant, the financial profits of a sector are the losses of another. This implies that the deviation of prices from their (zero-profit) equilibrium values are likely driver of losses, financial fragility, bankruptcies and that endogenous monetary creation is necessary to sustain an economy out-of-equilibrium. Conversely, equilibrium appears as the only sustainable state of the economy in absence of monetary creation.

There is a substantial literature on the origin of aggregate fluctuations with close connections to our work. (Horvath, 1998, 2000) and Dupor (1999) discuss the aggregation of sectoral shocks in a general equilibrium setting. Bak et al. (1993) looks at self-organized criticality in production networks and show that independent shocks fail to cancel in the aggregate if interaction are local and technologies non-convex. A wealth of agent-based models investigates further the interplay between local interactions and aggregate fluctuations (see among others Dosi et al., 2010; Dawid et al., 2011; Mandel et al., 2010; Wolf et al., 2013). Our own contribution is closely related to the work of Delli Gatti, Gallegati and co-authors (see Delli Gatti et al., 2005, and further references) emphasizing scaling laws and financial fragility as sources of macroeconomic volatility. Among those contributions, Battiston et al. (2007) is the closer to ours as it allows for an arbitrary number of sectors.

With respect to the equilibrium literature, our contribution is to ground existing results on the non-diversification of shocks in a dynamical setting with heterogenous interacting agents and to allow for the endogenous generation and propagation of shocks via bankruptcies, defaults and network-based financial accelerator mechanisms (see Delli Gatti et al., 2010) thanks to the introduction of out-of-equilibrium dynamics and of an explicit financial structure. In particular, we address some of the research questions put forward in Acemoglu et al. (2012): *"another important area for future research is a systematic analysis of the relationship between the structure of financial networks and the extent of contagion and cascading failures."*

With respect to the agent-based literature on financial fragility (Delli Gatti et al., 2005, and further references), our contribution is to explicit the link of this class of models to equilibrium and thereupon to provide a stock-flow consistent model of financial fragility in which losses and bankruptcies are brought about by disequilibrium rather than by randomness. Indeed, these previous contributions completely abstract away from the price formation process: in (Delli Gatti et al., 2005, and further contributions) prices are drawn randomly. Therefore, the underlying equilibrium of the model is not explicitly identified nor is the fact that perturbations are actually drawn from a distribution centered on this equilibrium. This missing link to equilibrium partly explains the lack of stock-flow consistency in the model: financial shocks are caused by violations of budgetary balance conditions rather than by disequilibrium in the economy. Our approach is, perhaps in a metaphorical sense, more Schumpeterian: it is out-of-equilibrium that financial fragility and bankruptcy materialize.

We use computational methods. We implement our model numerically and analyze its properties via Monte-Carlo simulations. This is the standard approach in agent-based computational economics (see LeBaron and Tesfatsion, 2008). Yet, an important innovation of our model is its hybrid nature. Part of the dynamics are defined by behavioral rules and local interactions as is standard in the agent-based literature, part are defined in a more axiomatic mode by assuming some form of efficiency, and then implemented as solutions to linear programs. This allows to reduce the number of free parameters by abstracting away from the details of processes whose time-scale or magnitude is below these of concern in the model (this perspective on time-scales is akin to this of subscale parametrization used in climate modeling (see Edwards, 2010)). However, this approach involves solving large and complex optimization problems on networks. Therefore it raises, for the first time as far as we understand, the issue of computational capacity in the field of agent-based computational economics. The simulations presented in this paper altogether required months of computation time[1] although none of them involve a large number of sectors or of firms .

The remaining of the paper is organized as follows. Section two describes the model. In section three, we investigate the interplay between equilibrium, financial fragility and disequilibrium. Section four analyzes the relationship between the structure of the production network, convergence to equilibrium and aggregate volatility. Section five concludes.

2 A multisectoral model

2.1 The general equilibrium framework

We investigate the dynamics of a multi-sectoral economy built up by a large number of households, firms and banks, interconnected via input-output, commercial credit, and financial credit networks.

We represent firms as financially constrained and boundedly rational agents, which combine intermediary inputs and labor in view of production, and adap-

[1]For a single-core machine.

tively search for a profit-maximizing pricing policy. Households are inelastic labor suppliers and, for some of them, entrepreneurs. Their wages and entrepreneurial profits are the only source of final demand (i.e consumption). Banks are rule-based suppliers of short-term credit.

Intermediary consumption is financed by commercial credits and hence defines two networks linking firms: one of physical/commodity flows, one of financial obligations. Wages are anticipated so that leveraged firms possibly require credit from the banks in order to finance production: firms' liabilities towards banks define a third, financial, network linking firms and banks.

Our representation of the firm is built upon two fundamental features: firms are equity constrained (see Greenwald and Stiglitz, 1993) and adaptive price setters (see Gintis, 2007). As demonstrated in the recent literature on financial fragility (e.g Battiston et al., 2007; Delli Gatti et al., 2010), accounting for imperfect markets and financial constraints is key to understand the genesis of the financial network and the spreading of shocks in the economy through bankruptcy cascades and, more generally, financial accelerator mechanisms. Recognizing the adaptive nature of decision making and the decentralized nature of prices allows to develop micro-founded approaches of out-of-equilibrium dynamics (see Gintis and Mandel, 2012). Combining both insights allows to situate the literature on economic fragility *vis-à-vis* the standard general equilibrium framework and in particular to demonstrate that the propagation of shocks and volatility are essentially out-of-equilibria phenomena.

In order to perform this comparison, we root our model in a standard Cobb-Douglas technological infrastructure, identical to these in which Long and Plosser (1983) develop their real business cycle model and Acemoglu et al. (2012) perform their equilibrium analysis of the decay of shocks in production networks. Namely, firms are grouped in N distinct sectors producing homogeneous goods. The production possibilities of firms in sector g are described by a production function $F_g : \mathbb{R}_+^{N+1} \to \mathbb{R}_+$, such that:

$$F_g(l_g, x_g) = \gamma_g l_g^{\alpha_g} \prod_{h=1}^{N} x_{g,h}^{(1-\alpha_g)w_{g,h}} \tag{1}$$

where the parameters $\gamma_g > 0$, $\alpha_g \in]0,1[$ and $w_{g,h} \geq 0$ respectively represent productivity, the share of labor and the share of good h in the total intermediate input use of firms in sector g while the variables l_g and $x_{g,h}$ respectively correspond to the amount of labor hired and the amount of commodity h used in the production process. The coefficient $w_{g,h}$ can be related to the entry of an input-output table measuring the value of spending on input h per dollar of production of good g.

Each household inelastically supplies a unit of labor. Whenever individual preferences have to be introduced, in order to ensure comparability with the equilibrium literature, we shall assume that individual are characterized by utility functions of the form

$$u(c_1, \cdots, c_N) = \prod_{h=1}^{N} c_h^{v_h}$$

where the variable c_h stands for the consumption of good g and the parameter v_h empirically corresponds to the share of expenses devoted to good h.

We will consider there is the same finite number $M \in \mathbb{N}$ of households and of firms in each sector[2] and denote the economy by $\mathcal{E}(M, N, \alpha, \gamma, \omega, v)$, the set of goods, by $\mathcal{G} = \{1, \cdots, N\}$, the set of firms by $\mathcal{F} = \{(g, j) \mid g \in \mathcal{G}, j \in \{1, \cdots M\}\}$ and the set of households by $\mathcal{H} = \{1, \cdots, M\}$.

2.2 The State Space

In order to extend the static results of Acemoglu et al. (2012) on the decay of aggregate volatility and to clarify the relationships between the agent-based financial fragility literature (see Battiston et al., 2007; Delli Gatti et al., 2010) and general equilibrium, we undertake an exploration of out-of-equilibrium dynamics in this economy.

In order to accurately account for financial interactions, we shall introduce M financial agents or banks whose set is denoted by $\mathcal{B}$. Bank k will be characterized throughout by a level of net worth a_k, corresponding to its financial capital .

Two key state variables will govern the behavior of firms and households: a private vector of prices and a level of net worth. More precisely, we will denote by $p_{g,j} \in \mathbb{R}_+^N$ and $a_{g,j} \in \mathbb{R}_+$ (resp $p_i \in \mathbb{R}_+^N$ and $a_i \in \mathbb{R}_+$) the private price and the net worth of the jth firm[3] in sector g (resp. of the ith household) and let $(p, a) \in P \times A := (\mathbb{R}_+^N)^{(N+1)M} \times (\mathbb{R}^+)^{(N+2)M}$ denote the complete state of the system (it also includes the net worths of banks).

The private price for good h of firm (g, j) (resp. of the ith household), $p_{g,j,h}$ (resp. $p_{i,h}$), is a reserve price, the maximum price the firm (resp. the household) is willing to pay for that good. The private price of firm (g, j) for its own good, $p_{g,j,g}$, is its actual selling price. On top of being an accurate representation of the fact that firms actually are price setters, private prices have good asymptotic properties in terms of convergence to general equilibrium when updated according to evolutionary dynamics, see Gintis and Mandel (2012).

The net worths of households correspond to cash holdings to be spent on consumption. The net worth of firm (g, j), $a_{g,j}$, represents the amount of financial capital it can autonomously employ in the production process. In line with the idea of having credit-constrained firms à la Greenwald and Stiglitz (1993), we use financial capital as a measure of the firm's financial robustness and determine accordingly the extent of a firm's access to credit and hence its production capacity. Namely, we shall assume following previous work by

[2]The number of firms per sector and of households is set identically to simplify the exposition and the normalization of variables

[3]Firm (g, j) hereafter.

Delli Gatti and al., that the production capacity of firm (g, j) is given by the financially constrained output function:

$$f(a_{g,j}) = \phi a_{g,j}^{\beta} \tag{2}$$

where $a_{g,j} \in \mathbb{R}_+$ is the net worth the firm holds, $\phi > 1$, and $0 < \beta < 1$. The parameters ϕ and β condition the intensity at which capital is put in motion in the economy. More precisely, β measures the extent to which there are "decreasing returns" to financial robustness in terms of credit leverage, and eventually production, it can yield. These parameters also implicitly define the time-scale of the model through the production to capital ratio.

The initial monetary mass in the model consists in the sum of the net worths of banks, firms and households. Permanent addition to the monetary mass can occur at runtime in case a bank goes bankrupt (see below).

2.3 Initialization and transition

Initialization of the model consists in the choice of an initial value for the net worth and private prices of each agent, i.e of an element $(p^0, a^0) \in P \times A := (\mathbb{R}_+^N)^{(N+1)M} \times (\mathbb{R}^+)^{(N+2)M}$. Then, every period the following operations take place sequentially:

- *Real Step.* The quantities of goods produced, exchanged and consumed are determined as a function of net worths and private prices.
- *Financial Step.* The evolution of agents' net worths are determined, on the one hand by the outcome of the production and exchange processes and on the other hand by the inflows and outflows of money between the productive and the financial sector induced by the credit scheme according to which production is financed. At this stage, firms and banks can go bankrupt if their net worth becomes negative. New firms and banks are then funded from existing capital in the financial sector.
- *Price Step.* The evolution of agents' private prices is determined according to stochastic evolutionary dynamics that use as measures of fitness the profit of firms and the utility of households.

A time step of the model hence consists in the sequential application of the real, financial and price steps. It returns updated values on one hand for the net worth and private prices of each agent and on the other hand for the economic variables (production, consumption, profits,...) that have been correlatively determined. Our actual assumptions about economic dynamics are embedded within the description of each of these steps that is given below.

2.4 Real Dynamics

In order to capture the dynamic interactions between prices, quantities, and credit conditions, we must first specify how a private price and net worth profile

$(p, a) \in P \times A$ determines the level of production and its allocation in the economy. In agent-based models, (e.g Dosi et al., 2010; Dawid et al., 2012; Wolf et al., 2013; Mandel, 2012), this process is generally emerging from a very detailed representation of the firms' decisions and of their interactions on the different markets. We consider that our analysis can abstract away from part of these details as the time-scale at which they matter is below this that is of concern when one focuses on aggregate volatility and equilibration. This perspective on time-scales is akin to this of subscale parametrization used in climate modeling (see Edwards, 2010).

We shall also consider as a first approximation that production and allocation of goods take place in a frictionless [4] way given the constraints implied by the compatibility of private prices and the financial capacity. Finally, we shall assume the agents are "computationally" rational in the sense that they are able to perform cost minimization operations. These assumptions yield the following representation of the production and allocation processes.

Firms use the input mix that minimizes production costs according to their private prices. That is firm (g, j) uses an input combination proportional to $(\mu_{g,j}, \nu_{g,j}) := \text{argmin}_{F_g(l_g, x_g)=1}\ p_{g,j}x_g + wl_g$ where w is the unit labor cost. Hence firm (g, j) uses $\mu_{g,j}$ units of labor and a vector $\nu_{g,j}$ of goods per unit of output produced.

Households choose their preferred input mix given their private prices. That is household i consumes proportionally to $\gamma_i := \text{argmax}_{p_i \cdot x_i \leq 1}\ u_i(x_i)$. In other words, household i will consume $\gamma_{i,h}$ units of good h for every unit of income spent.

Let us then denote by $z_{(g,j),(g',j')}$ the flow of good g from firm (g, j) to firm (g', j'), $z_{(g,j),i}$ the flow of good g from firm (g, j) to household i, $z_{i,(g,j)}$ the flow of labor from household i to firm (g, j), $l_{g,j}$ the quantity of labor employed by firm (g, j), $x_{g,j,g'}$ the quantity of good g' used as input by firm (g, j), $x_{i,g'}$ the quantity of good g' consumed by household i, $y_{g,j}$ the quantity of output produced by firm (g, j) and w_i the income of household i.

We shall assume that the production and allocation processes are efficient in the sense that the maximal share of aggregate demand for each good is fulfilled[5] under the constraints that agents only trade with peers using compatible prices (i and ii below), labor and goods are conserved during exchanges (iii to vi), production satisfies the technological (vii) and financial ($viii$) constraints, consumption satisfies the budget constraints (ix and x). That is production, consumption, labor and goods' flows are determined as a solution of the following maximization problem.

[4] It would be straightforward to introduce frictions in the process, e.g by considering that a certain share of production dissipates instead of being consumed. It seems reasonable to focus first on the frictionless case and to delay generalization to further work.

[5] More precisely we maximize the minimal ratio of consumption to demand among goods.

$$
\begin{cases}
\max & \min_{g\in\mathcal{G}} \dfrac{\sum_{i=1}^{M} x_{i,g}}{\sum_{i=1}^{M}\gamma_{i,g}} \\
\text{s.t} & \\
(i) & \forall (g,j),(g',j') \in \mathcal{F},\ z_{(g,j),(g',j')} > 0 \Rightarrow p_{g,j,g} \leq p_{g',j',g} \\
(ii) & \forall (g,j), \in \mathcal{F}, \forall i \in \mathcal{H}\ z_{(g,j),i} > 0 \Rightarrow p_{g,j,g} \leq p_{i,g} \\
(iii) & \forall (g,j) \in \mathcal{F},\ l_{g,j} = \sum_{i\in\mathcal{H}} z_{i,(g,j)} \\
(iv) & \forall (g,j) \in \mathcal{F}, \forall g' \in \mathcal{G}\ x_{g,j,g'} = \sum_{\{j' \mid (g',j')\in\mathcal{F}\}} z_{(g',j'),(g,j)} \\
(v) & \forall (i) \in \mathcal{H}, \forall g \in \mathcal{G}\ x_{i,g} = \sum_{\{j \mid (g,j)\in\mathcal{F}\}} z_{(g,j),i} \\
(vi) & \forall (g,j) \in \mathcal{F},\ \sum_{\{(g',j')\in\mathcal{F}\}} z_{(g,j),(g',j')} + \sum_{\{i\in\mathcal{H}\}} z_{(g,j),i} \leq y_{g,j} \\
(vii) & \forall (g,j) \in \mathcal{F},\ y_{g,j} \leq F_g(l_{g,j}, x_{g,j}) \\
(viii) & \forall (g,j) \in \mathcal{F},\ y_{g,j} \leq f(a_{g,j}) \\
(ix) & \forall i \in \mathcal{H},\ w_i = \lambda w \sum_{\{(g,j)\in\mathcal{F}\}} z_{i,(g,j)} \\
(x) & \forall i \in \mathcal{H}, \forall g \in \mathcal{G},\ x_{i,g} \leq (w_i + a_i)\gamma_{i,g}
\end{cases}
\tag{3}
$$

Hence production, income and consumption are completely determined by the profile of private prices and net worth $(p,a) \in P \times A$. These being given, the economy functions in an "efficient" way in the sense that as large a share as possible of the final demand is fulfilled given the financial and price compatibility constraints. However, there are two potentially major sources of inefficiency related respectively to prices and to the distribution of financial capital. Concerning prices they might be dispersed, hence preventing trading, or they might be away from their equilibrium values, what leads to rationing, misallocation of goods, abnormal profits and losses and eventually bankruptcies. Note in particular that unless agents all use the same equilibrium price, supply and demand can't balance exactly and hence constraints (vi) and (vii) can't bind simultaneously: there must be excess supply or inefficient use of inputs. Both can trigger losses. Concerning financial capital, it might be misallocated, what leads to capacity shortage and rationing in certain sectors and to capacity underutilization in others.

Though it is standard in economic theory to focus on equilibrium situations only, it is necessary here to introduce explicitly out-of-equilibrium trading as we focus on the (possible) emergence of equilibrium (see Fisher, 1989). Consideration of such disequilibrium situations is standard in agent-based models, which aim at dynamical completeness (see LeBaron and Tesfatsion, 2008). Alternatively, our model can also be viewed as a concrete implementation of disequilibrium/temporary equilibrium theories à la Benassy-Grandmont (see Grandmont, 1988; Bénassy, 2005). In any case, it shall be made clear that our approach is not inconsistent with the law of one price nor with the notion that economic agents are price-takers under perfect competition as both of these conditions can be obtained as long-run properties of our model (in the equilibrium regime described below).

Remark 1 *The cost of labor w is fixed and used as a numeraire throughout the paper. However the labor cost differs from the wage actually paid upfront* *to employees in exchange of labor. The actual wage is set equal to λw where $\lambda \in [0, 1]$ is a parameter representing approximately the share of labor in the value added. The remaining of the labor cost $(1 - \lambda)w$ is the surplus that shall accrue to the owner/entrepreneur of the firm in case the production is actually profitable (see subsection* 2.5.3 *below) and be spent on consumption next period.*

2.5 Financial Dynamics

2.5.1 Credits and Interests

The financial counterparts of the production and exchange processes are determined by the magnitude of these exchanges as well as by the scheme according to which they are financed. We shall consider here a setting à la Delli Gatti et al. (2010), which account both for commercial credit between firms and financial credit extended to firms by banks. Namely, we shall assume that sales of productive inputs are financed by a commercial credit extended by the seller to the buyer while wages are anticipated (i.e paid cash by the firms) and the excess of the wage bill over the net worth of the firm is financed by a credit extended by its bank to the firm (For each $g \in \mathcal{G}$, firm (g, j) is initially linked to bank j ; the dynamics of the financial network are specified in subsection 2.6.1).

Following Delli Gatti et al. (2010), we consider the interest rates on both commercial and financial credit are increasing with the financial soundness (i.e the net worth) of the lender and decreasing with the leverage (i.e the ratio between the level of debt and the net worth) of the borrower. More precisely the interest rate charged by lender ℓ (be it a bank or a firm) to borrower/firm (g, j) is:

$$r_{\ell,(g,j)} = \rho a_\ell^{-\rho} + \rho \left(\frac{d_{g,j}}{a_{g,j}} \right)^\rho$$

where the parameter $\rho \in \mathbb{R}_+$ controls the sensitivity of the interest rate (see Delli Gatti et al., 2010, for details). The debt of firm (g, j), $d_{g,j} \in \mathbb{R}$ refers to the total debt computed as:

$$d_{g,j} := -a_{g,j} + wl_{g,j} + \sum_{\{(g',j') \in \mathcal{F}\}} p_{g',j',g'} z_{(g',j'),(g,j)}.$$

2.5.2 Clearing

After production and allocation operations have taken place, the financial status of a firm (g, j) is given by:

- Its net worth updated by wage payments in cash to households and cash payments of consumption by households during the period. That is the updated net worth of firm g, j is given by:

  $$a_{g,j} = \underline{a}_{g,j} - wl_{g,j} + \sum_{i \in \mathcal{H}} p_{g,j,g} z_{(g,j),i}$$

  where $\underline{a}_{g,j}$ refers to the value of $a_{g,j}$ at the beginning of the period.

- Its liabilities towards other firms, which consist in the commercial credit it has been granted plus interest payments. That is the debt of firm (g, j) towards firm (g', j') is given by:

$$d_{(g,j),(g',j')} := (1 + r_{(g',j'),(g,j)})p_{g',j',g'}z_{(g',j'),(g,j)}$$

- Its liabilities towards its bank, which consist in the credit it has been granted to finance its wage bill plus the interest payments. Namely, the debt of firm (g, j) towards its bank k is given by:

$$d_{(g,j),k} := (1 + r_{k,(g,j)})\underline{d}_{(g,j),k}$$

  where the principal of the debt is given by

$$\underline{d}_{(g,j),k} = \lambda w l_{g,j} - \underline{a}_{g,j}$$

Debts are then cleared by compensation. If complete clearing is impossible because of default, that is if there are firms for which the value of liabilities excess this of assets (net worth plus debts from other firms), defaulting firms are bankrupted/liquidated and their assets shared uniformly among creditors. More precisely, the following clearing algorithm is implemented.

1. If for all $(g, j) \in \mathcal{F}$, one has:

$$a_{g,j} + \sum_{\{(g',j')\in\mathcal{F}\}} d_{(g',j'),(g,j)} \geq \sum_{\{(g',j')\in\mathcal{F}\}} d_{(g,j),(g',j')} + \sum_{\{k\in\mathcal{B}\}} d_{(g,j),k}$$

   then all the debts can be cleared by having each firm (g, j) adding to its financial capital $a_{g,j}$ a net transfer of

$$\sum_{\{(g',j')\in\mathcal{F}\}} d_{(g',j'),(g,j)} - \sum_{\{(g',j')\in\mathcal{F}\}} d_{(g,j),(g',j')} - \sum_{\{k\in\mathcal{B}\}} d_{(g,j),k}$$

   and each bank k adding to its financial capital a_k a net transfer of

$$\sum_{\{(g',j')\in\mathcal{F}\}} d_{(g',j'),k} - \sum_{\{(g',j')\in\mathcal{F}\}} \underline{d}_{(g',j'),k}.$$

2. Otherwise, each firm $(g_0, j_0) \in \mathcal{F}$ such that the condition 1. above does not hold goes bankrupt. Its assets are allocated to its creditors proportionally to the amount of outstanding debt. That is, the net worth and the claims of each firm and bank are updated as follows (the symbol $+=$ denotes an incrementation, that is the variable on the left-hand side is updated by adding the value of the expression on the right-hand side):

   For all $\ell \in \mathcal{F} \cup \mathcal{B}/\{(g_0, j_0)\}$:

$$
\begin{cases}
a_\ell \mathrel{+}= \dfrac{d_{(g_0,j_0),\ell}}{\sum_{\{(g',j')\in\mathcal{F}\}} d_{(g_0,j_0),(g',j')} + \sum_{\{k\in\mathcal{B}\}} d_{(g_0,j_0),k)}} a_{g_0,j_0} \\
\forall (g',j') \in \mathcal{F}, \\
d_{(g',j'),\ell} \mathrel{+}= \dfrac{d_{(g_0,j_0),\ell}}{\sum_{\{(g',j')\in\mathcal{F}\}} d_{(g_0,j_0),(g',j')} + \sum_{\{k\in\mathcal{B}\}} d_{(g_0,j_0),k)}} \; d_{(g',j'),(g_0,j_0)}
\end{cases}
$$

For (g_0, j_0) :

$$
\begin{cases}
a_{(g_0,j_0)} := 0 \\
\forall (g',j') \in \mathcal{F}, \\
d_{(g',j'),(g_0,j_0)} := 0
\end{cases}
$$

3. 1. and 2. are repeated until all firms are sound in the sense of 1.

It is relatively easy to check that this algorithm stops after a finite number of iterations as there is at least one bankruptcy per (non-terminal) iteration and at most $N \times M$ firms can go bankrupt. Note also that each creditor is treated in a purely symmetric manner but possibly for the order according to which bankrupted firms are litigated, and this latter point does not affect the outcome of the algorithm (see Eisenberg and Noe, 2001).

A key implication of the presence of commercial credit in the model and of the clearing mechanism is that the actual/financial profit of the firm does not only depend on its commercial performance, which it can "control" through its pricing policy, but also on the financial soundness of its partners, through which it is in fact exposed to the whole credit network. Indeed the debt clearing mechanism can propagate default far away from its source. The firm is hence exposed to a systemic risk whose manifestations appear as random (at least from the firm's point of view). The only influence the firm has on this risk is through the interest rate it charges. However, because of systemic effects, the idea of mitigating risk by a higher interest rate might be self-deceiving (see Battiston et al., 2012a).

2.5.3 Accounting and financial flows

Actual/financial profits of firms are computed as the net increase in wealth after the clearing of debts. Profits might differ from the value of sales minus production costs because some of the firm's creditors might have gone bankrupt and default on their commercial debt. Part of these profits are retained by the firm in order to increase its financial capital, part are distributed to the household sector. More precisely, for each $g = 1 \cdots N$ and each $j = 1 \cdots M$, firm (g, j) distributes to household j a dividend equal to the minimum between its financial profit and $(1 - \lambda)$ times its value added, that is:

$$
\min(a_{g,j} - \underline{a}_{g,j}, (1-\lambda)(p_{g,j} y_{g,j} - \sum_{h=1}^{N} \sum_{k=1}^{M} p_{h,k} z_{(h,k),(g,j)}))
$$

The remaining share of the profit is retained in the firm's capital. As far as the banks are concerned they retain all their profits.

As far as losses are concerned, they are covered by the financial capital of the agents (see clearing algorithm above) until the losses excess the financial capital available, in which case the corresponding agent goes bankrupt. The bankrupted firms are determined during the clearing algorithm. The bankrupt banks are those whose financial capital becomes negative after the clearing algorithm, that is banks k such that $a_k < 0$. Note that when a bank goes bankrupt, the amount of its outstanding deficit has already passed into the general circulation (it was used to pay wages) and given that we do not represent the interbank clearing system, the loss can't be affected to a counterpart so that the monetary mass increases of the corresponding amount. Each bankrupted firm and bank is then replaced by a new entrant that is capitalized as follows.

- For each new firm, a bank is drawn at random to capitalize it. The new capital of the firm is set equal to to the minimum between half the bank's capital and the mean wealth of firms in the economy. The corresponding amount is subtracted from the capital of the bank that finance the capitalization.
- For each new bank, another bank is drawn at random to capitalize it. The outstanding deficit is foregone and the target new capital of the bank is drawn uniformly at random in $[0, 2]$ (so that the expected capital of a newly funded bank is one). However, as for the firms, the actual capitalization can not exceed half the capital of the bank financing the operation.

These assumptions correspond to an institutional setting in which firms are managed by entrepreneurial households and owned by banks. The managers capture part of the profit (the part that is distributed), the other part is saved by banks within the firm's capital. The assumption that each household manages the same number of firm (he receives compensation from all firms whose index is the same as his, see above) is certainly over-simplistic but the distribution of wealth among households shall have little impact on the dynamics of our model given that consumption behavior is independent of the level of income (see subsection 2.4). The assumption that firms are owned by banks rather than by households is consistent with the fact that there are no motives nor means for savings in a setting where there is no capital accumulation and the only financial assets are intra-period loans. As a matter of fact, in absence of capital accumulation, aggregate budgetary balance implies that all income should be spent on consumption.

The bankruptcy rules for firms are such that the monetary mass is conserved and hence the model is stock-flow consistent with the caveat that whenever a bank goes bankrupt, the amount of its outstanding deficit yields a net monetary creation of the same amount. The aggregate flows of capital between the productive and the financial sectors are determined by the level of debt and interests and by the financial fragility of the system. On the one hand, interests payments induce a transfer of capital from the productive to the financial sector.

On the other hand, default and recapitalization of firms following bankruptcies induce a net transfer from the financial to the productive sector.

In this setting, the key feature characterizing the financial system is the speed and the magnitude at which it can exchange capital with the productive sectors. This will be mainly determined by the total capital in the financial sector and the distribution of wealth in the productive sector (see next section). In many other respects, our representation of the financial system is over-simplistic. In particular, the capital of banks is here more of a buffer than a truly operative variable. For example, leverage of banks is unbounded in our setting (banks' capital only affects the interest rate) and the only rationale for banks to invest in firms are capital gains (they never receive dividends) although we lack the representation of the market for firms' stocks.

2.6 Price Dynamics

Our representation of prices' dynamics is in line with the core assumptions in general equilibrium theory that firms are profit maximizers and households are utility maximizers. Yet, our approach is more behavioral, in the sense that we consider that this optimization takes place adaptively through stochastic evolutionary process in which firms update their competitiveness and their profitability through their private prices and households update their consumption plan through a monetary evaluation again based on their private prices. It turns out that this approach has much better dynamical properties than the Walrasian tâtonnement (see Gintis and Mandel, 2012).

Stochastic evolutionary processes are based on a measure of fitness that determine which strategies get imitated and which disappear. In our case, the fitness of an household is computed during consumption operations as the ratio between utility and income. The fitness of a firm is computed after commercial credit clearing as the ratio between the profit and the net worth at the beginning of the period (i.e return on equity). Agents are then pooled according to their types: all households together and producers by sectors. There are $N+1$ such pools, each consisting of $m = 1 \cdots M$ agents characterized by a fitness f_m and a private price p_m. Prices are updated independently for each pool of agents according to the following algorithm (from Gintis, 2007).

1. Normalized fitness $\overline{f}_m$ are computed according to $\overline{f}_m = \dfrac{f_m - f_{\min}}{f_{\max} - f_{\min}}$.

2. Until a fraction τ_{copy} of agents have been selected as imitators, an agent is drawn at random and selected as an imitator with probability $1 - \overline{f}_m$. Similarly, until a fraction τ_{copy} of agents have been selected as models, an agent is drawn at random and selected as a model with probability $\overline{f}_m$.

3. Each imitator is randomly paired with a model and copies its private price. That is the imitator m, if paired with the model m', sets the value of its private price to $p_{m'}$,

Then prices mutate (see again Gintis, 2007), that is τ_{mutate} agents are randomly drawn and independently for each good divide or multiply (each with probability one half) their price by a factor μ. That is if $p_{k,g}$ is the private price of good g for agent k before mutation, the mutation turns it to $\mu p_{k,g}$ with probability $\frac{1}{2}$ and to $\frac{p_{k,g}}{\mu}$ with probability $\frac{1}{2}$. Eventually, each firm checks the consistency of its prices by ensuring its selling price is at least equal to its unit production cost.

Mutations might account for errors in the imitation process or random innovation. Their structural importance comes from the fact that they make the evolutionary process ergodic.

2.6.1 Dynamics of the financial network

For each $g \in \mathcal{G}$, firm (g, j) is initially linked to bank j. The dynamics of the financial network are identical to those in Delli Gatti et al. (2010). In every period, after accounting took place, each firm observes the interest rates offered by a randomly selected sample of 30% of the population of banks. If the interest rate r_{old} offered by the current bank is less than the minimum interest rate r_{new} offered in the sample of banks observed, the firm sticks to its current bank. Otherwise, with probability $1 - e^{\frac{r_{new} - r_{old}}{r_{new}}}$ the firm shifts to the bank offering the interest rate r_{new}.

2.7 Simulation setting

We investigate the dynamics of the model via numerical simulations. A period of the model corresponds to the sequential execution of the real, financial and price steps and a simulation corresponds to the iteration of the model for a finite number of periods. The model is implemented in Matlab and the linear program in subsection (2.4) is solved using IBM ilog cplex optimization studio[6]. The default parameters for simulations are reported in table 1. Some remarks about the relation between simulation and the nature of the dynamics are also in order at this stage.

- The real step is formally non-deterministic as it involves picking-up a solution to the linear program in subsection (2.4) that can a priori admit many such solutions. Yet, our implementation is deterministic as long as the behavior of the optimization algorithms we use are. This shall be the case[7].

- The financial step is deterministic.

[6] Made freely available for academic use by IBM. This IBM academic initiative is gratefully acknowledged.

[7] According to the software provider: see CPLEX user's manual on Advanced programming techniques/Parallel optimizers/Determinism of results

- The price step is stochastic but is implemented on the basis of a pseudo-random number generator in order to ensure reproducibility of results.

Symbol	Interpretation	Default value
N	nb. of sectors	3
M	nb. of agents per sector	50
γ	productivity parameter	$2N$
α	share of labor in production	0.5
ω	input-output table	$\forall g, g' \in \mathcal{G}, \omega_{g,g'} = \frac{1}{N}$
v	expenditure shares	$\forall g, \in \mathcal{G}, v_{g,} = \frac{1}{N}$
λ	share of labor in value-added	0.95
ϕ	magnitude of leverage	2
β	returns to financial robustness	0.9
ρ	interest rate parameter	0.015
τ_{copy}	price imitation rate	0.25
τ_{mutate}	price mutation rate	0.05
μ	mutation factor	0.1

Table 1: simulations' default parameters

3 General equilibrium and financial fragility.

The general equilibrium of the economy $\mathcal{E}(M, N, \alpha, \gamma, \omega, v)$ will play a central role in our analysis as most of our results are stated *vis-à-vis* this equilibrium: convergence to equilibrium, mean residence time in or away of equilibrium, effects of the structure of the production network or of financial constraints on equilibrium. It is therefore fundamental to characterize equilibrium in our model: both analytically and as a reference point of the dynamics of simulations.

3.1 Analytic derivation of equilibrium

Analytically, an equilibrium of the economy $\mathcal{E}(M, N, \alpha, \gamma, \omega, v)$ is any collection of prices $p^* \in \mathbb{R}_+^N$, labor cost $w^* \in \mathbb{R}_+$, consumption $(x_i^*)_{i=1,\cdots,M} \in \mathbb{R}_+^{N \times M}$ and production plans $(l_{g,j}^*, y_{g,j}^*)_{g=1,\cdots,N, j=1,\cdots,M} \in (\mathbb{R}_+ \times \mathbb{R}_+^N)_+^{N \times M}$ such that:

1. For all $i = 1 \cdots M$, household i maximizes utility:

$$x_i^* := \text{argmax}_{p_i^* \cdots x_i \leq w^*} u_i(x_i)$$

2. For all $g = 1 \cdots N$, $j = 1 \cdots M$, firm (g, j) maximizes profit:

$$y_{g,j}^* := \text{argmax}_{F_g(l_{g,j}, y_{g,j,-g}) = y_{g,j,g}} p_g^* \cdot y_{g,j,g} - p_{-g}^* \cdot y_{g,j,-g} - w^* l_{g,j}$$

.

3. Good markets clear:

$$\sum_{g=1}^{M}\sum_{j=1}^{N} y^*_{g,j} = \sum_{i=1}^{M} x^*_i$$

4. Labor market clears

$$\sum_{g=1}^{M}\sum_{j=1}^{N} l^*_{g,j} = M$$

Once labor is defined as the *numéraire* and has its price fixed[8] it is straightforward to show that the equilibrium of this economy is unique up to the allocation of production among firms. In particular there is a unique equilibrium price for goods, p^* such that[9]:

$$\log(p^*) = (Id - \omega)^{-1} \begin{pmatrix} -\log(\gamma_1) - \alpha_1 \log(\alpha_1) - \sum_{h=1}^{N} \log(w_{1,h}) \\ \dots \\ -\log(\gamma_1) - \alpha_1 \log(\alpha_1) - \sum_{h=1}^{N} \log(w_{N,h}) \end{pmatrix}$$

Therefrom, one can compute the equilibrium consumption of each household and the equilibrium production aggregated at the sectoral level. However, as there are constant returns to scale, profit maximization yields zero profits. Hence the firm can only determine an optimal input mix, not an optimal production level, on the basis of prices. In this sense, coordination through prices is incomplete and the equilibrium is indeterminate.

3.2 Financial viability and Equilibrium

In our setting, indeterminacy on the firm's production level is reduced thanks to the financially constrained output function, which provides a cap on the production level of each firm. It is then a matter of basic accounting to realize that general equilibrium and financial stability are intrinsically linked. Indeed, unless there is a permanent inflow of money from the financial sector to the productive ones, the total financial capital held by firms must remain constant if production is to be sustained.

In a setting with constant returns, at a general equilibrium aggregate profits shall be zero in each sector, so that the financial capital and the production capacity shall remain constant. Hence, the equilibrium production and consumption as well as the corresponding distribution of financial capital can be reproduced period after period without additional inputs. Therefore, the general equilibrium shall be a viable state of our dynamical system that induces stability of both the financial and the real spheres.

Away from equilibrium, a sector that makes positive profits sees his production capacity growing. Now, in absence of financial inflows from the financial to

[8] As a matter of fact, we set the labor cost in such a way that the mean price of commodities equals one at equilibrium

[9] We denote by Id the identity matrix of size n. A detailed derivation is given in the appendix of Acemoglu et al. (2012).

the production sphere, the profits of a sector are the losses of another. Hence the counterpart of these out-of-equilibrium profits shall be out-of-equilibrium losses in another sector whose production capacity must then shrink. Given the sectoral interdependencies in the production structure, such a situation isn't viable as the shrinking of the unprofitable sector would eventually yield some rationing of the profitable one and a general decrease of production. The only ways forward are technological innovation (that we do not consider here) or inflow of additional financial capital, i.e the built-up of further leverage. In other words, disequilibrium is for us a symptom of financial fragility.

We will further analyze these relationships between real and financial instability below, but our initial inquiry is wether an economy can self-organize into a viable equilibrium state. That is, do economic agents' reaction to disequilibrium such as price or capacity changes, form a strong enough feedback mechanism to induce the convergence of the system towards equilibrium? A positive answer to this question is implicitly assumed in most of the existing economic theory. Previous simulation and analytical results of ours (see respectively Gintis (2007) and Gintis and Mandel (2012)) suggest that in a setting where competition is actually implemented through private updating of prices, evolutionary dynamics do lead the system to equilibrium. Here, the issue is revisited in a more realistic setting with intermediary production and financial constraints. It turns out that financial constraints actually matter. More precisely the stability of equilibrium and the dynamical regime crucially depend on the relative strength of resource and financial constraints. When the resource (labor) constraints dominate, the economy efficiently plays its role of allocation of a scarce resource and equilibrium prevails. When financial constraints dominate, financial fragility is the key driver of the dynamics.

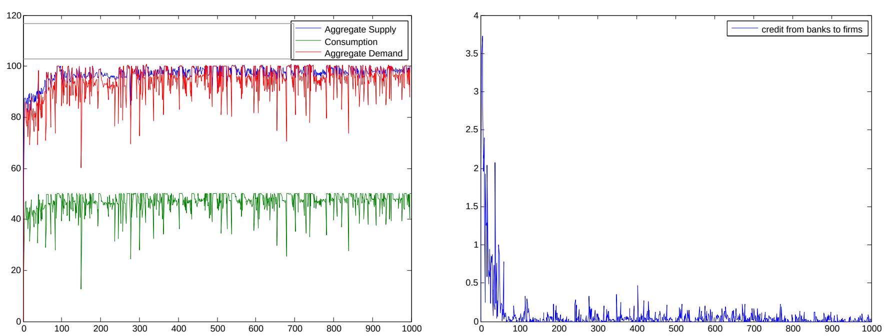
3.3 Convergence properties

Resources constraints in our economy are determined by the labor supply. Indeed, in absence of technological innovation, labor supply defines an upper bound on the aggregate production throughout time.

Conversely, given the leverage parameters ϕ and β, the financial constraints are defined by the initial wealth of firms and banks $a^0 \in A$. These determine on the one hand, the initial monetary mass and the initial production capacity. On the other hand, they will affect the capital of firms and banks created at runtime. Hence, they will determine the potential inflow of money in the production sector during a period (see section 2.5.3).

When these financial constraints are weak, i.e when the stock and the potential inflow of capital in the economy is large with respect to the labor resources, the model has very robust properties of convergence to equilibrium. To illustrate this point, we first consider a version of the model with three productive sectors ($N = 3$), fifty agents per sector ($M = 50$), and the remaining parameters set as in table 1. In this setting, the equilibrium price is equal to 1 for every good and the aggregate equilibrium production (resp. consumption) level is 100 (resp. 50). We initialize the model by drawing each price uniformly at random in $[0, 2]$, allocating an initial wealth of 1 to each firm and of 100 to each bank. Then, resource constraints dominate as the equilibrium production level (100 units) is much less than what firms can finance without recourse to credit (300 units). Hence, as illustrated in the right panel of figure 1, total credit outstanding is negligible meaning that wages are mainly financed by firms' net worth. Constraints on credit would anyhow be very weak: initial capital of banks is much greater than the recurrent financing needs of the production sector.

In this economy, the dynamics of the model are very clearcut: in about hundred periods, the mean private price of firms converges to its equilibrium value and the standard deviation of the price among firms become negligible (see figure 2) It is also the case that aggregate production and consumption reach their equilibrium values and that demand and supply balance each other (see left panel of figure 1). Hence the economy reaches equilibrium in approximately hundred periods and rests there for the remaining of the simulation.



(a) Aggregate Demand (final plus intermediary), Aggregate Consumption and Supply

(b) total credit to firms

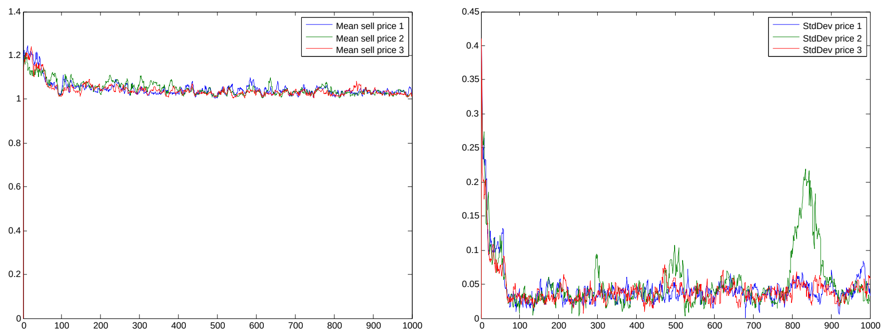
Figure 1: Markets

To provide a quantitative assessment of the convergence properties of the model, we perform 250 Monte-Carlo simulations where we let the number of sectors vary from 2 to 6 and randomly draw 50 distinct production networks (the other parameters being set as in table 1). To assess the results of these simulations, we use a concept of approximate equilibrium. We shall say that the model is in an ϵ-equilibrium in period t if:

1. The euclidian distance between the mean selling price and the equilibrium price is less than ϵ;

2. The standard deviation of prices is less than ϵ;

3. The excess demand (i.e total production minus total consumption minus total intermediary consumption as determined in 2.4.) is less than ϵ times the total production.



(a) Mean of prices

(b) standard deviation of prices

Figure 2: Prices

indicator\nb of sectors	2	3	4	5	6	mean
mean nb of ϵ-equilibria in periods 1-500	431	324	267	265	184	294
mean nb of ϵ-equilibria in periods 100-500	384	314	261	262	184	281
mean nb of ϵ-equilibria in periods 400-500	97	82	67	79	62	78

Table 2: nb of periods in equilibrium

We then measure for the 250 Monte-Carlo simulations the mean number of periods in which the model is in an ϵ-equilibrium (with $\epsilon = 0.1$). The results are reported in table 2. Though the convergence time increases with the number of sectors, the model shows very robust properties of convergence to equilibrium. In the last 100 periods of the simulation, the model spends in average 80% of the time in equilibrium and for small number of sectors, the model is almost always in equilibrium.

Hence, we do obtain converge to equilibrium when resource constraints dominate. The small source of volatility introduced by the random mutations in the price formation process do not get amplified into aggregate volatility nor fragility. These good convergence properties are a first contribution of the model: we extend to an economy with intermediary production, the results of Gintis (2007) about the convergence to general equilibrium of private prices under evolutionary dynamics. It is worth noting in this respect that the introduction of efficient dynamics for the allocation of goods in 2.4 is perfectly in line with the axiomatic characterization of exchange processes inducing evolutionary stability of equilibrium obtained in Gintis and Mandel (2012).

3.4 Disequilibrium and financial fragility

The previous picture of economic equilibrium and financial stability seems at odds with the results reported in Delli Gatti et al. (2010) where *"prices are important determinants of profits, which in turn affect the accumulation of net worth and financial fragility. The financial vulnerability of an agent therefore is affected by the dynamics of prices."*

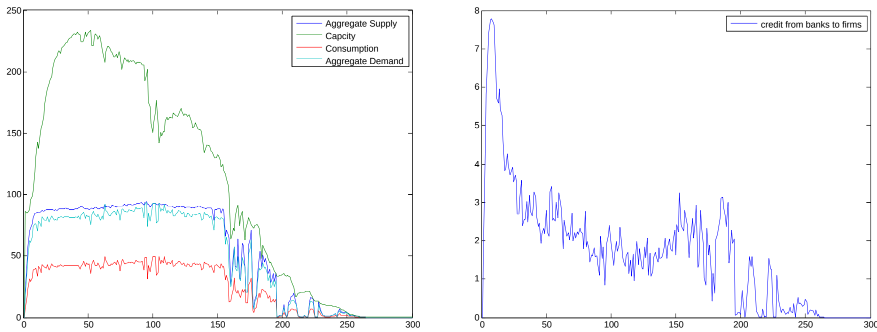
Yet, the economic environment considered in Delli Gatti et al. (2010) is a particular case of ours with two sectors and intermediary consumption by the second sector only, something that corresponds to $\alpha_1 = 1, \alpha_2 = 0.5$ and $\omega = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. The dynamics we consider are also very similar to these of Delli Gatti et al. (2010) but for the evolution of prices, which is purely random and exogenous in Delli Gatti et al. (2010) while it is endogenous and directed by evolutionary learning in our case. It is also the case that the sales of produced quantities at a purely random price in Delli Gatti et al. (2010) induce net inflows or outflows of money in the economy, whereas our model is stock-flow consistent as final consumption is entirely financed by wages and dividends.

One could therefore claim that it is the lack of stock-flow consistency and the exogenously imposed price volatility that induces financial fragility, network-based financial accelerator mechanisms, bankruptcy avalanches and aggregate volatility in Delli Gatti et al. (2010). We shall show this is not the case.

Indeed, even small shocks on the the price system induce variation in sales and profits and eventually of firms' net worths. As soon as the financial constraints are binding, downward variations in net worths lead to a decrease of productive capacity. Decreased capacity in turn lowers the competitive pressure, so that prices start leaving the equilibrium paths. Rationing and losses follow, financial fragility increases and the way is paved for bankruptcy avalanches *à la* Delli Gatti et al. (2010).

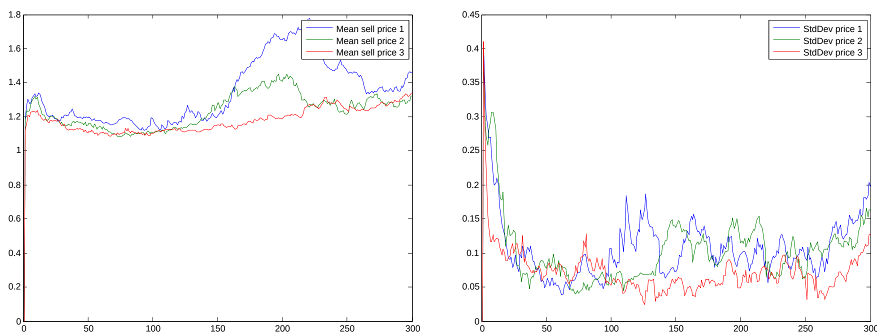
In order to illustrate, the processes at play, we consider a three-sector version of the model with parameters set as in table 1 but where the initial net worths of firms is set equal to 0.25 and this of banks equal to 2 (recall that these values also condition the capital of firms and banks created at runtime and hence the potential inflow of capital into the system). This economy is financially constrained, on the one hand because the aggregate production the firms can self-finance (75 units) is less than the equilibrium production (100 units) and on the other hand because the additional capital the banks can provide to the productive sector is also very limited. Indeed, as underlined in section 2.5.3, the only source of increase of the aggregate capital of the productive sector is the funding of new firms and the write-off of the claims on bankrupt firms, which are both financed by the banks.

As illustrated in figures 3 and 4, the first hundred periods of the simulation are rather similar to the ones leading to equilibrium in section 3.3. A slight difference being that in the very first periods, a series of bankruptcies induce a massive transfer of capital from the financial to the productive sector (see figure 5). It then seems that the productive sector reaches a level of capital



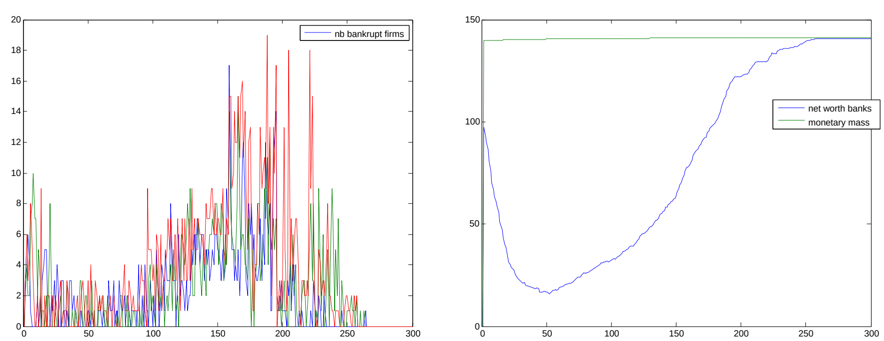
(a) Aggregate Demand and Supply (b) credit from banks to firms

Figure 3: Markets



(a) Mean of prices (b) Standard deviation of prices

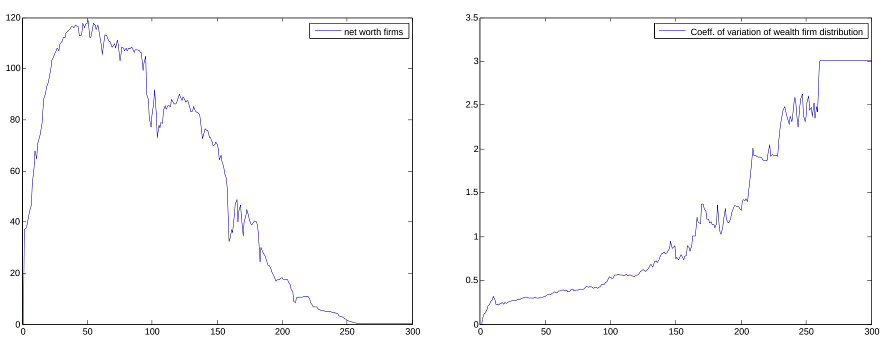
Figure 4: Prices



(a) Number of bankruptcies (b) Bank wealths and monetary mass

Figure 5: Finance

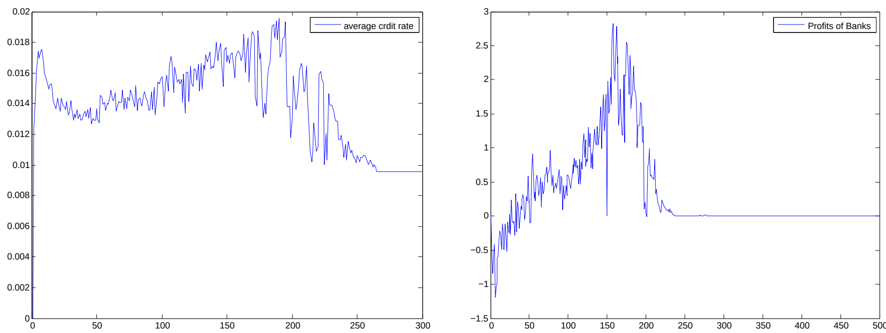
compatible with the equilibrium production level. Yet this high capitalization is unsustainable as it is out of proportion with the financing flow that the financial sector can maintain towards the productive sector (see the right panel in figure 5). As a matter of fact, a shock to the price system around period 100 (see figure 4, most notably the right panel) triggers a temporary increase of the mean and of the standard deviation of the price of good 1, which induces a drop in demand (see figure 3) in sales, in profits and eventually in the net worth and productive capacity of firms (see figure 6). Observing figure 5, one can remark that this loss of financial capital in the production sector is not compensated by a sizable inflow of capital from the financial to the production sector ; quite the contrary, the total net worth of banks is increasing. This is due to the fact that, as in Delli Gatti et al. (2010), the distribution of wealth among firms evolves during the simulation (see the evolution of the coefficient of variation in figure 6) and becomes much more dispersed. Larger firms face losses without going bankrupt and hence these losses aren't passed to the financial sector (as it was the case at the beginning of the simulation when the size distribution of firms was uniform). Hence productive capacity does not recover, competition becomes less stringent and prices exit the equilibrium paths (see figure 4 from period 100 and onwards). As underlined in section 3.2, out-of-equilibrium, financial losses become structural. Consequently, the wealth of firm further decreases, interest rates increase with the financial fragility of borrowers (see figure 7) and a positive feedback loop ensues which eventually leeds to a bankruptcy avalanche around period 170 (see figure 5).



(a) Total firms' wealth (b) Coefficient of variation for firms' wealth

Figure 6: Firms' wealth

The mechanisms at play are very similar to these described in Delli Gatti et al. (2010) *"The bankruptcy of a borrower creates a negative externality because the bad debt recorded on the lender's balance sheet yields an increase of the interest rate charged to all the other borrowers. This is the starting point of the financial accelerator. If the surviving borrowers experience an increase of leverage due to the interest rate hike, the lender will react by raising the interest rate even further. Financial fragility will spread to the neighborhood and may*



(a) Mean interest rate

(b) Profits of banks

Figure 7: Prices

spill over to the entire economy. An avalanche of bankruptcies may ensue." Yet, in our setting two other feedback mechanism are at play. First the decrease of financial capital and production capacity reduces the strength of competition and hence the stability of the price system. Second the sectoral interlinkages propagate rationing shocks throughout the system. These rationing shocks play the role of the idiosyncratic productivity shocks described in Acemoglu et al. (2012) as the drivers of aggregate volatility.

3.5 Financial constraints and phase transition

The simulations' results reported above suggest that, depending on the strength of the financial constraints, the model exhibits two very different regimes. One is characterized by general equilibrium, low aggregate volatility and financial stability. The other is characterized by market disequilibrium and financial fragility, which are linked by a positive feedback loop that yield high aggregate volatility and eventually crisis and crashes.

In order to test the assumption that financial constraints are the determinants of these two regimes, we run a series of 750 Monte-Carlo simulations (MC) where we let the default wealth of firms and banks vary (the other parameters being fixed as in table 1). More precisely, the default wealth of firms, which we denote by wf, takes the values (0.1250, 0.25, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2), this of banks, which we denote by wb, takes the values (0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 3, 3.5, 4, 5, 10, 20, 50) and we run 5 MC over 500 periods for each possible combination of the variables wf and wb. Table 3 reports the mean (over the five MC) of the number of periods for which the model is in an ϵ-equilibrium (with $\epsilon = 0, 1$) as a function of the default wealth of firms and banks, while Table 4 reports the mean volatility of GDP for the same set of

$wf\backslash wb$	0.5	1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4	5
0.125	0	0	0	0	0	29.4	0	17.6	180	106	246	233
0.25	0	0	0	0	0.6	109	203	138	170	237	266	299
0.4	0	8.8	3.2	130	129	227	76.5	227	296	310	272	312
0.5	0	8.2	85.4	209	212	248	254	239	184	261	248	308
0.6	15	68.6	246	163	245	200	253	267	243	281	271	315
0.7	63.6	131	260	242	197	309	246	264	313	237	308	347
0.8	130	236	249	284	227	241	301	333	265	326	297	321
0.9	144	279	296	264	273	287	243	212	212	340	268	325
1	236	294	367	229	234	278	334	199	272	266	328	313
2	269	305	241	289	286	293	277	293	338	270	323	366

Table 3: nb of equilibria (values of wf\wb in line\column)

$wf\backslash wb$	0.5	1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4	5
0.125	22.7	16.8	20.8	21.2	11.2	12.6	8.75	12.8	12.3	4.27	4.34	4.5
0.25	23.5	11.6	17.3	12.8	8.79	8.35	17.8	8.84	4.12	3.97	4.08	4.13
0.4	23.3	10.4	8.24	3.93	10.8	6.22	4.88	4.46	6.17	3.81	3.92	3.91
0.5	15.3	18.7	3.84	8.06	3.92	6.76	3.85	3.82	3.82	3.79	3.74	3.65
0.6	12.4	10.5	3.75	7.71	6.46	3.8	3.54	3.68	3.71	3.66	3.82	3.75
0.7	5.9	6.8	3.73	3.57	3.62	3.44	3.83	3.55	3.7	3.69	3.48	3.67
0.8	3.33	3.45	3.48	3.8	3.38	3.44	3.35	3.75	3.4	3.42	3.44	3.62
1	3.76	3.74	3.55	3.29	3.44	3.53	3.73	3.47	3.56	3.31	3.27	3.17
2	3.06	3.09	3.26	3.2	2.95	3.2	2.86	2.97	2.94	3.22	3.14	3.25

Table 4: volatility of GDP (values of wf\wb in line\column)

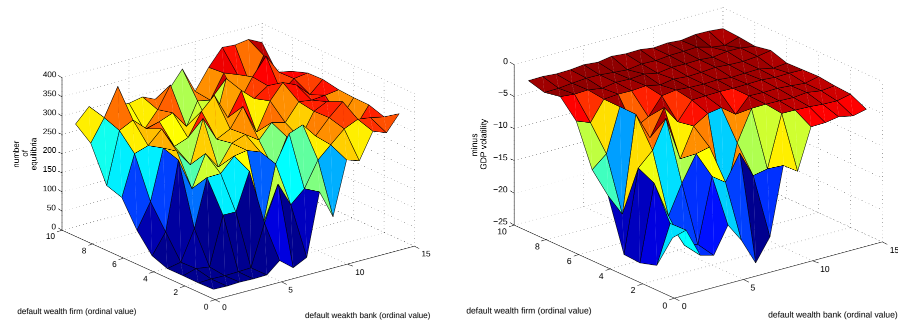
simulations[10]. Figure 8 provides a graphical illustration of these results[11].

The hypothesis that the financial constraints govern the regime of the model is confirmed by the identification of a phase transition between the equilibrium and the disequilibrium states of the model, which materializes as a critical line (highlighted in red in table 2) in the (wf, wb) plane. For small values of the pair(wf, wb) the model almost never is in equilibrium and exhibits high volatility, for large values of the pair (wf, wb) the model settles in equilibrium and exhibits low volatility. When the critical line is crossed, there is an abrupt transition between the two regimes.

A preliminary exploration of the data through linear regression suggests that the relationship between default wealths and number of equilibria is exponential and that the influence of both variables, wf and wb is of a similar magnitude. Figure 9a illustrates this point by showing a scatter plot of the number of ϵ-equilbria against $\log(wf * wb)$. It suggests a logistic dependence between the two quantities, which is confirmed in figure 9b and by a series of

[10]Results corresponding to wealth banks of 10,20,50 are omitted in these tables as they do not change the qualitative nor the quantitative features of the results.

[11]The values on the x and y axis of figure 8 are ordinal, i.e 1 (resp. 15) corresponds to the smallest (reps. largest) value of wealth bank

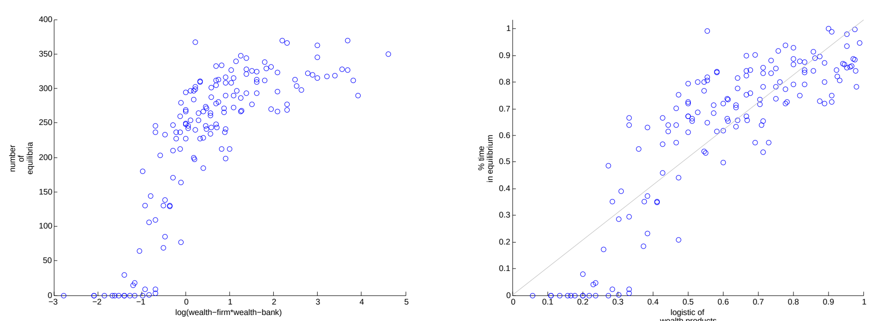


(a) Number of ϵ-equilibria

(b) minus GDP volatility

Figure 8: Phase transition under financial constraints

regression analysis reported in appendix A. There is statistical evidence that the financial constraints explains the regime in which the model lies and that the transitions between the two regimes is well-described by a logistic function, what is consistent with the idea of a phase-transition between equilibrium and non-equilibrium regimes. In this respect, our results are very similar and inspired by those obtained in Gualdi et al. (2013): these authors show that phase transitions between equilibrium and disequilibrium can be driven by small shifts in firms' employment policy.



(a) Nb of equilibria vs log of wealths

(b) Equilibrium index vs logistic of wealths

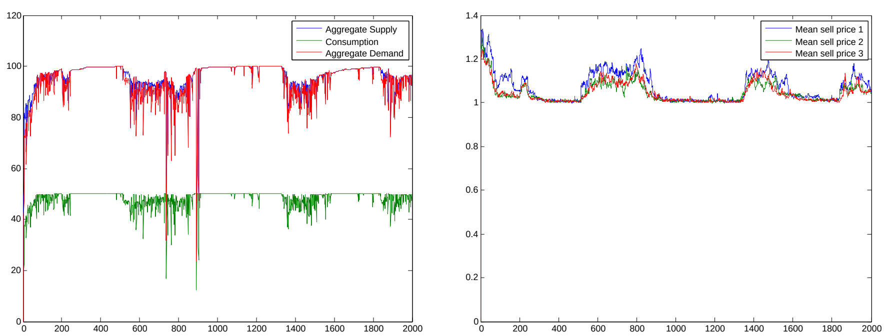
Figure 9: Logistic model of equilibrium transition

These results are robust with respect to change in the value of the other key financial parameter that is the interest rate. As reported in appendix B, results of simulation for a default interest rate varying between 0 and 6 percents show a picture that is qualitatively equivalent to the one presented in this section.

3.6 A financial management experiment

In the simulations presented in the previous subsection, crisis are so acute that they lead to the collapse of the economy. From a theoretical point of view, this helps contrasting the equilibrium and disequilibrium regimes of the model. Still, this prevents any application of the model to questions other than theoretical. In order to overcome this limitation and address issues such as the origins of aggregate volatility in a dynamical setting, we investigate in this subsection remedies to this systemic fragility.

The roots of fragility eventually lay with the individual firm. Therefore, increasing individual firms' resilience shall lead to a decrease in the magnitude of systemic crisis and ease the recovery therefrom. The default financial policy of the firm is to distribute a dividend as soon as it makes a profit in the period (see 2.5.3). This very myopic behavior might be conductive to fragility as it prevents the firm from reconstituting a strong enough capital basis after it has been affected by a negative shock. In order to further investigate this point, we consider an alternative profit distribution policy which consists in distributing profits only if the current capital of the firm is greater than a benchmark, which is set equal to the initial value of the capital. If this condition holds, the totality of the profit is distributed. The use of this alternative financial management rule does not affect the behavior of convergence to equilibrium reported in subsection 3.3 for parameter values that do not induce financial constraints. However in a setting with financial constraints, using the same parameter setting than in subsection 3.4, the results are qualitatively different (see figure 10). The model now exhibits shifts between an equilibrium regime where prices stay around their equilibrium value and aggregate volatility is very low, and a disequilibrium regime where prices fluctuate away from their equilibrium value and aggregate volatility is high.



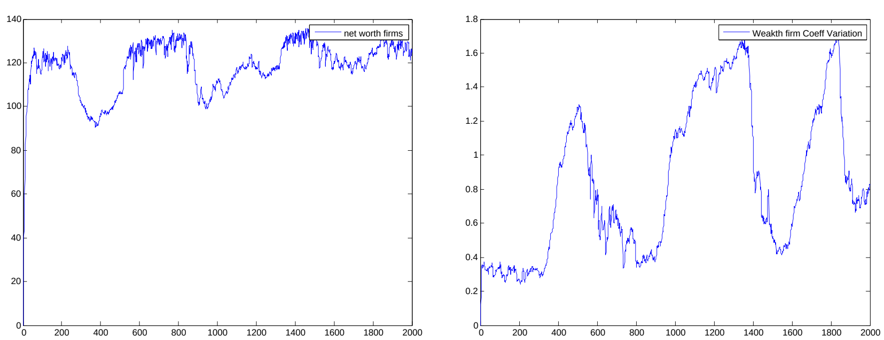
(a) Aggregate Demand and Supply (b) Mean of prices

Figure 10: Markets

The mechanisms that trigger a crisis are similar to these described in subsection 3.4. In a context where the distribution of wealth is dispersed (see figure

11), a shock to the price system triggers a positive feedback loop from price fluctuations to losses and bankruptcies then to reduced competition and further price fluctuations (see figures 10 and 12). The main differences with the processes that lead to collapse in section 3.4 is that the productive sector is now more resilient: bankruptcy crisis are less acute and the economy eventually recovers. This is due to the fact that firms now have time to recapitalize themselves after a shock. As can be observed in figures 11 and 12, the total wealth of firms increase after bankruptcies thanks to funding of new firms plus disequilibrium profits. One also observes a reduction in the dispersion of the distribution of wealth, which restores competition and eventually equilibrium in the economy. This dynamical pattern for the financial capital of the productive sector is in strong contrast with this observed in 3.4 where a downward spiral starts as soon as financial fragility bites. Nevertheless, once the equilibrium regime is restored, financial constraints foster an increase in the dispersion of firms' wealth and pave the way for the next crisis.

Hence, one observes long-term fluctuations between an equilibrium regime characterized by stability of price and production and a disequilibrium one characterized by price instability and financial fragility. This crisis regime is the carrier of aggregate volatility in the model. In other words, local disequilibrium in a decentralized price system and networked financial fragility are the micro-economic origins of aggregate volatility in our model. In the next section, we investigate the influence of the structure of the production network on the magnitude of these aggregate fluctuations.

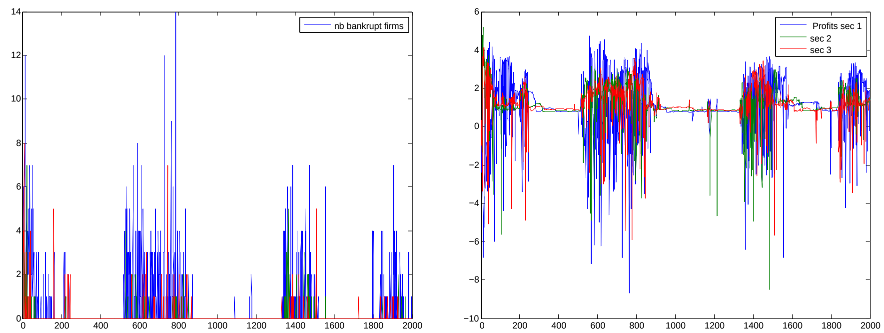


(a) Total firms' wealth (b) Coefficient of variation for firms' wealth

Figure 11: Firms' wealth

4 Network structure, equilibrium, and aggregate volatility

The model introduced in subsection 3.6 allows to represent dynamic fluctuations between equilibrium and disequilibrium. In this section, we use it to analyze in



(a) Number of bankruptcies

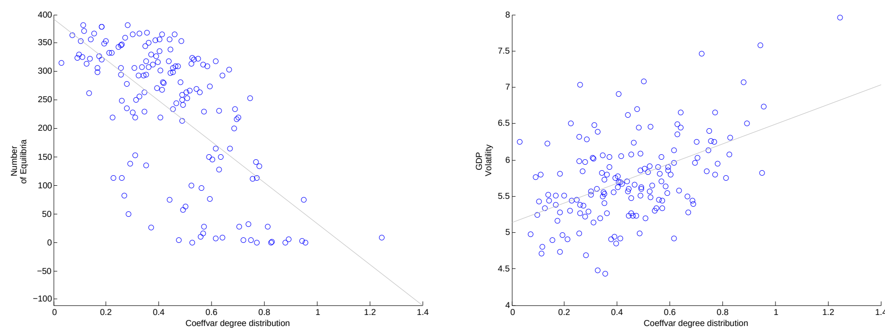
(b) Profits of firms

Figure 12: Firms' finance

a dynamical setting the relationships between network structure and aggregate volatilty put forward by Fisher and Vega-Redondo (2006), Battiston et al. (2007) and Acemoglu et al. (2012). This analysis is performed along two dimensions. First we analyze the relationship between network structure and the process of relaxation to equilibrium as well as equilibrium stability. In order to do so, we measure, for a sample of production networks, the average number of periods in which the model is in an ϵ-equilibrium (for $\epsilon = 0.1$). This is an indirect measure of aggregate volatility based on the equivalence established in the previous section between disequilibrium and aggregate volatility. Second, we perform a direct analysis of the relationship between network structure and aggregate volatility. There, we measure, for the same sample of production networks, the volatility of GDP observed during the timeframe of a simulation.

Our characterization of the network's structure is based among others on the degree sequence, which associates to a sector g the sum $\delta_g := \sum_{h=1}^{N} \omega_{h,g}$ corresponding to the share of sector g's output in the input supply of the entire economy. We then focus on the following network statistics: the mean of the degree sequence $\overline{\delta} := \frac{1}{N} \sum_{g=1}^{N} \delta_g$; the coefficient of variation of the degree sequence $\upsilon_1 := \frac{\sqrt{\sum_{g=1}^{N}(\delta_g - \overline{\delta})^2}}{\sqrt{N}\overline{\delta}}$, which is used as a first-order measure of the asymmetry of the network in Acemoglu et al. (2012); the second order interconnectivity coefficient $\upsilon_2 := \sum_{g=1}^{N} \sum_{h \neq g} \sum_{k \neq g,h} \omega_{h,g}\omega_{k,g}\delta_h\delta_k$, introduced by Acemoglu et al. (2012) to measure the extent to which sectors with high degrees are interconnected to one another through common suppliers; and the clustering coefficient à la Grindrod-Zhang-Horvath (see Grindrod (2002) and Zhang et al. (2005)) defined as $\kappa := \sum_{g=1}^{N} \frac{\delta_g}{\overline{\delta}} \frac{\sum_{h \neq g} \sum_{k \neq g,h} \hat{\omega}_{g,h}\hat{\omega}_{h,k}\hat{\omega}_{k,g}}{(\sum_{h \neq g} \hat{\omega}_{g,h})^2 - \sum_{h \neq g} \hat{\omega}_{g,h}^2}$ where the $\hat{\omega}$ are normalized versions of ω obtained by dividing by the largest coefficient in ω.

Our analysis is based on a series of 750 Monte-Carlo simulations where we let the network structure vary (the initial wealth of firm is set to 0.5, this of banks to 2.5 and the other parameters are fixed as in table 1). More precisely, we run 5 MC over 500 periods for 150 distinct random production networks (the coefficients are drawn at random according to a lognormal distribution and then normalized to ensure input shares of all sectors add up to one). We then analyze the dependencies between the number of ϵ-equilibria and the GDP volatility on the one hand and the network statistics introduced above on the other hand.



(a) Number of equilibria

(b) GDP volatility

Figure 13: Variations with the degree

Figure 13 illustrates our results by plotting the number of periods in ϵ-equilibrium (resp. gdp volatility) obtained in simulations as a function of the coefficient of variation of the degree sequence of the network. We perform a linear regression[12] for both indicators using as independent variables first the coefficient of variation of the degree sequence only and then together with the second order interconnectivity coefficient. Using the standard deviation instead of the coefficient of variation and/or the clustering coefficient instead of the second order interconnectivity coefficient give similar results. Results are reported in appendix C.

Our results confirm these of Acemoglu et al. (2012) in the sense that there is a significant statistical dependence between aggregate volatility of GDP and both the coefficient of variation and the second order interconnectivity of the degree sequence. The same is true if we use our indirect measure of volatility based on number of ϵ-equilibria/ mean residency time in equilibrium. A very puzzling fact however is that though the coefficient of variation has a positive impact on aggregate volatility as in Acemoglu et al. (2012), we find that the second order interconnectivity has a negative one, oppositely to the results of these authors. The second order interconnectivity coefficient, or similarly the clustering coefficient, measures the extent to which sectors with high degrees

[12]We use the applied econometrics toolbox made freely available by James P. LeSage, see LeSage (1999), who is gratefully acknowledged.

(those that are major suppliers to other sectors) are interconnected to one another through common suppliers. This points towards a possible explanation of the opposed relationship we find. In Acemoglu et al. (2012), the main influence of the coefficient of variation seems to be on the transmission of shocks between sectors. In our setting, this feature of transmission of shocks seems to be more than compensated by the fact that increased interconnectivity implies larger markets for intermediary consumption and hence further price-stabilizing competition that fosters convergence to equilibrium. The process of convergence isn't represented in Acemoglu et al. (2012). This might explain the difference.

We shall argue that our results have a stronger empirical content as we measure the actual volatility of GDP in a dynamical setting rather than its asymptotic properties in a static setting. Additionally, our results about the number of equilibria show that the network structure has a significative impact on the average residency time in equilibrium. These results somehow transcend these of Acemoglu et al. (2012) as they show that the network structure matters even before equilibrium is established, which is the starting point of Acemoglu et al. (2012). Moreover, we do not need to resort to exogenous productivity shocks to explain the origin of aggregate fluctuations. In our setting, the origin of fluctuations can eventually be traced to the microscopic shocks to the price system that come along evolutionary dynamics and that bring about disequilibrium and volatility when they are amplified by financial accelerator mechanisms.

5 Conclusion

This paper confirms previous findings (see Gintis, 2007; Gintis and Mandel, 2012) that evolutionary dynamics applied to decentralized systems of private prices have strong properties of convergence towards general equilibrium. We have built on these properties to equip the standard real-business cycle model à la Long and Plosser (1983) with agent-based dynamics. In particular, we have introduced financially constrained production and a primitive financial architecture à la Delli Gatti et al. (2010). We have revisited in this framework the relationships between equilibrium, financial fragility and aggregate volatility.

Our first contribution is to show that the stability of equilibrium and the dynamical regime of the economy crucially depend on the relative strength of resource and financial constraints. When financial constraints are weak, the economy behaves as an efficient mechanism for the allocation of scarce resources thanks to the emergence of a general equilibrium. When financial constraints bite, i.e outside a barter economy, there is a correspondence and a positive feedback loop between disequilibrium and financial fragility: small price variations can trigger financial imbalances that get amplified by financial accelerator mechanisms, these imbalances lead to reduced productive capacity and competitive pressure, less competition favors out-of-equilibrium excursions of the price system, what leads to additional financial imbalances. These findings allow to explicit the link between the theory of financial fragility (see Greenwald and Stiglitz, 1993; Delli Gatti et al., 2010, and earlier contributions) and general equilibrium.

Our second contribution is to provide dynamic and endogenous foundations to the findings of Acemoglu et al. (2012) on the network origins of aggregate fluctuations. In our dynamical setting, we can measure aggregate volatility along dimensions that are consistent with empirical observations, i.e through time rather than as a static response to a distribution of shocks. We can also trace the origin of aggregate fluctuations to the microscopic shocks to the price system that come along evolutionary dynamics and that bring about disequilibrium and volatility when they are amplified by financial accelerator mechanisms. More generally, we show that volatility materializes mainly out-of-equilibrium and that the network structure affects aggregate volatility because it impacts the speed of convergence to and the stability of equilibrium.

Though the financial constraints play a central role in our analysis, our representation of the financial system is rather primitive. There are only interim lived financial assets, no precautionary savings, no investment. There is no interbank clearing mechanism, no bound on banks' leverage, no monetary policy. These gaps to be filled provide elements of a research program that could aim at a better understanding of the transmission channel of monetary policy thanks to refined models of the financial system (for initial contributions in that directions, see Battiston et al. (2012b) or Geanakoplos et al. (2012)). There are two key differences between this research program and this initiated by Arrow and Debreu (see Arrow (1964) and Debreu (1959)) on general equilibrium with financial markets. First, the focus is on the behavior of financial actors (i.e financial institutions) rather than on financial assets. Second, the inquiry is about the impact financial markets have on the creation of resources rather than on their allocation.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Arrow, K. J. (1964). The role of securities in the optimal allocation of risk-bearing. *The Review of Economic Studies*, 31(2):91–96.

Axtell, R. L. (2001). US firm sizes are zipf distributed. *Science*, 93:1818–1820.

Bak, P., Chen, K., Scheinkman, J., and Woodford, M. (1993). Aggregate fluctuations from independent sectoral shocks: self-organized criticality in a model of production and inventory dynamics. *Ricerche Economiche*, 47(1):3–30.

Battiston, S., Delli Gatti, D., Gallegati, M., Greenwald, B., and Stiglitz, J. E. (2007). Credit chains and bankruptcy propagation in production networks. *Journal of Economic Dynamics and Control*, 31(6):2061–2084.

Battiston, S., DelliGatti, D., Gallegati, M., Greenwald, B., and Stiglitz, J. E. (2012a). Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, 36(8):1121 – 1141.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012b). Debtrank: too central to fail? financial networks, the FED and systemic risk. *Scientific reports*, 2.

Bénassy, J.-P. (2005). *The macroeconomics of imperfect competition and non-clearing markets: a dynamic general equilibrium approach*. The MIT Press.

Dawid, H., Gemkow, S., Harting, P., and Neugart, M. (2012). Labor market integration policies and the convergence of regions: the role of skills and technology diffusion. *Journal of Evolutionary Economics*, 22(3):543–562.

Dawid, H., Gemkow, S., Harting, P., van der Hoog, S., and Neugart, M. (2011). The eurace@ unibi model: An agent-based macroeconomic model for economic policy analysis. Technical report, Working paper. Universität Bielefeld.

Debreu, G. (1959). *Theory of value: An axiomatic analysis of economic equilibrium*. Number 17. Yale University Press.

Delli Gatti, D., Gallegati, M., Greenwald, B., Russo, A., and Stiglitz, J. E. (2010). The financial accelerator in an evolving credit network. *Journal of Economic Dynamics and Control*, 34(9):1627–1650.

Delli Gatti, D., Guilmi, C. D., Gaffeo, E., Giulioni, G., Gallegati, M., and Palestrini, A. (2005). A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility. *Journal of Economic Behavior & Organization*, 56(4):489–512.

Dosi, G., Fagiolo, G., and Roventini, A. (2010). Schumpeter meeting keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, 34(9):1748–1767.

Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics*, 43(2):391–409.

Edwards, P. N. (2010). *A Vast Machine: Computer Models, Climate Data, and the Politics of Global Warming (Infrastructures)*. The MIT Press.

Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Fisher, E. and Vega-Redondo, F. (2006). The linchpins of a modern economy. In *AEA Annual Meeting, Chicago, IL*.

Fisher, F. M. (1989). *Disequilibrium foundations of equilibrium economics*, volume 6. Cambridge University Press.

Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica*, 79(3):733–772.

Geanakoplos, J., Axtell, R., Farmer, J. D., Howitt, P., Conlee, B., Goldstein, J., Hendrey, M., Palmer, N. M., and Yang, C.-Y. (2012). Getting at systemic risk via an agent-based model of the housing market. *American Economic Review*, 102(3):53–58.

Gintis, H. (2007). The dynamics of general equilibrium. *Economic Journal*, 117(523):1280–1309.

Gintis, H. and Mandel, A. (2012). The stability of walrasian general equilibrium. Documents de travail du Centre d'Economie de la Sorbonne 12065, Université Panthéon-Sorbonne (Paris 1), Centre d'Economie de la Sorbonne.

Grandmont, J.-M. (1988). *Temporary Equilibrium: selected readings*. Academic Press.

Greenwald, B. C. and Stiglitz, J. E. (1993). Financial market imperfections and business cycles. *The Quarterly Journal of Economics*, 108(1):77–114.

Grindrod, P. (2002). Range-dependent random graphs and their application to modeling large small-world proteome datasets. *Physical Review E*, 66(6):066702.

Gualdi, S., Tarzia, M., Zamponi, F., and Bouchaud, J.-P. (2013). Tipping points in macroeconomic agent-based models. *arXiv preprint arXiv:1307.5319*.

Horvath, M. (1998). Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics*, 1(4):781–808.

LeBaron, B. and Tesfatsion, L. (2008). Modeling macroeconomies as open-ended dynamic systems of interacting agents. *American Economic Review*, 98(2):246–50.

LeSage, J. P. (1999). Applied econometrics using matlab. *Manuscript, Dept. of Economics, University of Toronto*.

Long, John B, J. and Plosser, C. I. (1983). Real business cycles. *Journal of Political Economy*, 91(1):39–69.

Mandel, A. (2012). Agent-based dynamics and the general equilibrium model. *Complexity Economics*, 1(1):105–121.

Mandel, A., Jaeger, C., Fürst, S., Lass, W., Lincke, D., Meissner, F., Pablo-Marti, F., and Wolf, S. (2010). Agent-based dynamics in disaggregated growth models. Documents de travail du Centre d'Economie de la Sorbonne 10077, Université Panthéon-Sorbonne (Paris 1), Centre d'Economie de la Sorbonne.

Wolf, S., Fuerst, S., Mandel, A., Lass, W., Lincke, D., Pablo-Marti, F., and Jaeger, C. (2013). A multi-agent model of several economic regions. *Environmental Modelling and Software*, 44:25 – 43.

Zhang, B., Horvath, S., et al. (2005). A general framework for weighted gene co-expression network analysis. *Statistical applications in genetics and molecular biology*, 4(1):1128.

Appendix A: Statistical analysis of the phase transition

Regressions ran on the results of the 750 MC simulations described in subsection 3.5 confirm a logistic dependence between the product of default wealths and the number of equilibria.

- If we use as dependent variable a binary variable "*equilibrium*", which is 1 if and only if the number of periods in which the model is in an ϵ-equilibria is greater than 200, and as independent variable the logarithm of the wealth products, $\log(wb * wf)$ we obtain the following results for logit and probit regressions:

```
Logit Maximum Likelihood Estimates
Dependent Variable = equilibrium binary
McFadden R-squared     =    0.6087
Estrella R-squared     =    0.6543
LR-ratio, 2*(Lu-Lr)    =  103.3550
LR p-value             =    0.0000
Log-Likelihood         =  -33.2176
# of iterations        =      9
Convergence criterion  =     3.106804e-09
Nobs, Nvars            =    150,     2
# of 0's, # of 1's     =     38,   112
***************************************************************
Variable                Coefficient      t-statistic    t-probability
constant                   1.229688         3.589071         0.000451
log wealth                 3.593064         5.292620         0.000000


Probit Maximum Likelihood Estimates
Dependent Variable = equilibrium binary
McFadden R-squared     =    0.6022
Estrella R-squared     =    0.6478
LR-ratio, 2*(Lu-Lr)    =  102.2491
LR p-value             =    0.0000
Log-Likelihood         =  -33.7706
# of iterations        =      8
Convergence criterion  =    1.2064068e-08
Nobs, Nvars            =    150,     2
# of 0's, # of 1's     =     38,   112
***************************************************************
Variable                Coefficient      t-statistic    t-probability
constant                   0.642919         3.642938         0.000372
```

```
log wealth                     1.933429         5.983258          0.000000
```

- If we fit directly a logistic function by using as dependent variable an index of the number of periods in which the model is in an ϵ-equilibrum (normalized so that the maximum value is 1) and as independent variable the value of the logistic function $\frac{(wb * wf)}{1 + wb * wf}$, we obtain the following results for an OLS regression:

```
Ordinary Least-squares Estimates
Dependent Variable = equilibrium index
R-squared      =    0.7220
Rbar-squared   =    0.7201
sigma^2        =    0.0234
Durbin-Watson  =    1.1419
Nobs, Nvars    =    150,     2
***************************************************************
Variable                Coefficient      t-statistic    t-probability
constant                   0.001668         0.048724         0.961205
logistic wealth            1.034190        19.604267         0.000000
```

Appendix B: Sensitivity with respect to the interest rate

In order to check the robustness of our results with respect to variation of the interest rate parameter, we have ran 800 MC simulations where the interest rate take values in $(0, 0.05, 0.01, 0.015, 0.02, 0.025, 0.03, 0.04, 0.05, 0.06)$, the wealth of firms in $(0.25, 0.5, 075, 1)$, this of banks in $(1, 2.5, 5, 10)$. For each possible combination of the parameters, we run 5 MC of the model over 500 periods. We give below the mean (over the 5 MC) of the number of periods in which the model is in an ϵ-equilibrium, the lines being indexed by the default wealth of firms, the columns by this of bank. The qualitative picture is identical to this described in section 3.5 and regression analysis gives similar results.

$$\rho = 0 : \begin{pmatrix} 0 & 230.0 & 275.0 & 351.0 \\ 1.4 & 306.0 & 290.0 & 304.0 \\ 280.0 & 220.0 & 297.0 & 266.0 \\ 339.0 & 272.0 & 339.0 & 308.0 \end{pmatrix} \rho = 0.005 : \begin{pmatrix} 0 & 143.2 & 316.6 & 301.8 \\ 43.2 & 268.0 & 277.2 & 329.2 \\ 208.2 & 246.6 & 283.6 & 341.2 \\ 217.8 & 310.2 & 302.6 & 338.4 \end{pmatrix}$$

$$\rho = 0.01 : \begin{pmatrix} 0 & 137.8 & 298.8 & 316.2 \\ 8.2 & 239.4 & 308.2 & 324.0 \\ 124.8 & 330.4 & 357.8 & 309.2 \\ 294.4 & 198.6 & 313.0 & 277.4 \end{pmatrix} \rho = 0.015 : \begin{pmatrix} 0 & 187.75 & 308.8 & 330.2 \\ 42.2 & 254.2 & 274.6 & 299.4 \\ 191.4 & 259.8 & 274.0 & 312.4 \\ 292.2 & 269.8 & 280.6 & 281.6 \end{pmatrix}$$

$$\rho = 0.02 : \begin{pmatrix} 0 & 184.0 & 261.8 & 319.2 \\ 47.8 & 163.6 & 268.2 & 339.6 \\ 219.4 & 328.8 & 313.6 & 309.6 \\ 243.4 & 263.2 & 293.8 & 317.8 \end{pmatrix} \rho = 0.025 : \begin{pmatrix} 0 & 142.0 & 297.2 & 264.2 \\ 0 & 233.0 & 321.0 & 318.6 \\ 174.8 & 328.6 & 326.4 & 315.4 \\ 200.0 & 295.4 & 316.2 & 333.8 \end{pmatrix}$$

$$\rho = 0.03 : \begin{pmatrix} 0 & 130.6 & 339.8 & 342.8 \\ 27.4 & 263.2 & 272.8 & 279.2 \\ 183.0 & 310.6 & 344.4 & 328.8 \\ 230.4 & 305.4 & 268.8 & 364.2 \end{pmatrix} \rho = 0.04 : \begin{pmatrix} 0 & 108.8 & 171.6 & 343.0 \\ 24.8 & 141.0 & 322.2 & 369.2 \\ 235.4 & 373.6 & 269.0 & 344.2 \\ 229.8 & 268.2 & 228.2 & 292.0 \end{pmatrix}$$

$$\rho = 0.05 : \begin{pmatrix} 0 & 28.6 & 260.6 & 341.4 \\ 0 & 274.4 & 343.4 & 313.8 \\ 152.8 & 214.4 & 287.0 & 329.0 \\ 230.0 & 275.0 & 285.0 & 297.0 \end{pmatrix} \rho = 0.06 : \begin{pmatrix} 0 & 90.8 & 251.8 & 277.0 \\ 0 & 243.0 & 280.8 & 293.0 \\ 285.2 & 332.2 & 331.8 & 355.0 \\ 278.8 & 219.6 & 309.6 & 356.8 \end{pmatrix}$$

Appendix C: Statistical analysis of the impact of the production network structure

```
Ordinary Least-squares Estimates
Dependent Variable =  Nb_Equilibria
R-squared      =    0.4209
Rbar-squared   =    0.4170
sigma^2        = 8435.1251
Durbin-Watson  =    1.8847
Nobs, Nvars    =    150,     2
***************************************************************
Variable             Coefficient      t-statistic    t-probability
constant              392.004497        22.791586         0.000000
coeffvar_degree      -358.865149       -10.370942         0.000000


Ordinary Least-squares Estimates
Dependent Variable =  Nb_Equilibria
R-squared      =    0.3811
Rbar-squared   =    0.3769
sigma^2        = 9014.4068
Durbin-Watson  =    1.9492
Nobs, Nvars    =    150,     2
***************************************************************
Variable             Coefficient      t-statistic    t-probability
constant              -42.785244        -1.437832         0.152593
sqrt_snd_degree       528.325294         9.546409         0.000000


Ordinary Least-squares Estimates
Dependent Variable =  Nb_Equilibria
R-squared      =    0.6583
Rbar-squared   =    0.6536
sigma^2        = 5011.2727
Durbin-Watson  =    1.8730
Nobs, Nvars    =    150,     3
```

```
***************************************************************
Variable              Coefficient      t-statistic    t-probability
constant               143.146573         5.118240         0.000001
coeffvar_degree       -298.450721       -10.919076         0.000000
sqrt_snd_degree        427.330089        10.105348         0.000000


Ordinary Least-squares Estimates
Dependent Variable = GDP_volatility_mean
R-squared      =    0.2451
Rbar-squared   =    0.2400
sigma^2        =    0.2698
Durbin-Watson  =    1.8292
Nobs, Nvars    =    150,     2
***************************************************************
Variable                  Coefficient      t-statistic    t-probability
constant                     5.135981        52.799750         0.000000
coeffvar_degree              1.356440         6.931263         0.000000


Ordinary Least-squares Estimates
Dependent Variable = GDP_volatility_mean
R-squared      =    0.3057
Rbar-squared   =    0.3010
sigma^2        =    0.2481
Durbin-Watson  =    1.9886
Nobs, Nvars    =    150,     2
***************************************************************
Variable                  Coefficient      t-statistic    t-probability
constant                     6.959425        44.576605         0.000000
sqrt_snd_degree             -2.343758        -8.071810         0.000000


Ordinary Least-squares Estimates
Dependent Variable = GDP_volatility_mean
R-squared      =    0.4527
Rbar-squared   =    0.4452
sigma^2        =    0.1969
Durbin-Watson  =    1.9817
Nobs, Nvars    =    150,     3
***************************************************************
Variable                  Coefficient      t-statistic    t-probability
constant                     6.288718        35.868118         0.000000
coeffvar_degree              1.076593         6.283068         0.000000
sqrt_snd_degree             -1.979440        -7.466842         0.000000
```