A sharing mechanism for superadditive and non-superadditive logistics cooperation

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A sharing mechanism for superadditive and non-superadditive logistics cooperation ⋆

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Abstract

The lack of a stable, fair and generally applicable sharing mechanism is one of the most noticeable impediments to the implementation of logistics cooperation. Most of the current literature on the sharing mechanism in logistics cooperation focuses on superadditive logistics cooperation games, neglecting the probable occurrence of other types of games resulting from coordination cost and unequal partners. In this work, we propose a sharing model based on game theoretic solutions, taking account of the bargaining power of players to identify a fair in-Core allocation. Under full cooperation assumption, we generalize this model for non-superadditive logistics cooperation games with coordination costs at different levels. The games with empty Core are also studied within the model.

Key words: Logistics cooperation, game-theoretical approach, bargaining power, non-superadditive game in logistics

1 Introduction

Today cooperation is becoming more and more crucial to improve the global performance of logistics. As the complement of traditional vertical cooperation, a new cooperation mode, the *horizontal cooperation* was proven efficient to reduce global cost and improve service rate in logistics. However, despite the advantages and the available planning tools, horizontal cooperation is not a must-have in logistics due to barriers to implementation. One main obstacle is the absence of appropriate sharing mechanism, which guarantees the incentive and the stability of the cooperative relationship by fairly allocating the common gain in different modalities of cooperation. In particular the situation is more complex when extra *coordination cost* (CC) is needed to build up the cooperation, or when the *bargaining power* is taken into account in the allocation. To overcome the barrier, we adopt the cooperative game theory to investigate the logistics horizontal cooperation as cooperative games. By proposing fair and stable gain allocations to the players (the horizontal logistics cooperators), we promote the implementability of such cooperation.

The contribution of this paper is as follows. Firstly, we identify different cooperation types of supply chains pooling under full cooperation assumption, for example cooperation with negligible CC or significant CC, with or without a global satisfaction solution, etc. Secondly, we investigate the gain sharing and coalition stability issues in different types of cooperative pooling game, and then propose a set of generally applicable gain

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sharing mechanisms that take account of the players' bargaining power. Finally, noticing the scarcity of non-superadditive game investigation in current logistics cooperation literatures, we generalize our sharing model to non-superadditive pooling games to improve its general applicability.

This paper is organized as follows. In Section 2, we present the supply chains pooling concept, and then identify different cooperation types and the barrier to the implementation. In Section 3, we model the supply chains pooling as a cooperative game, examine the existing sharing rules, and then introduce the sharing mechanisms developed in this paper. Then in Section 4, we apply this sharing mechanism in an experimental pooling case with real world data. The conclusion of this work is drawn and some insights are highlighted in Section 5.

2 Questions raised by supply chains pooling

2.1 Supply chains pooling: a modality of horizontal cooperation in logistics

A great emphasis was put on cooperation in supply chains, but mostly between suppliers and customers, a practice also known as vertical cooperation. Since few years another type of cooperation is studied and experienced within supply chains: *horizontal cooperation* [4,5]. This form of cooperation takes place between companies operating at the same level of market. In this paper, we will focus on a specific application of horizontal cooperation in logistics: the supply chains pooling concept. As defined and studied by Ballot and Fontane [3] and Pan et al. [20], the idea of pooling is the co-design of a communal logistics network for partners (suppliers, clients, carriers, etc.) with a common objective in order to share logistics resources and to improve the logistics performance as a whole. The motivation is the consolidation of flows on shared facilities. Figure 1 illustrates an example of pooling. In this example, all partners (suppliers 1, 2 and retailers 1, 2, 3, 4) plan their logistics network cooperatively by sharing facilities.

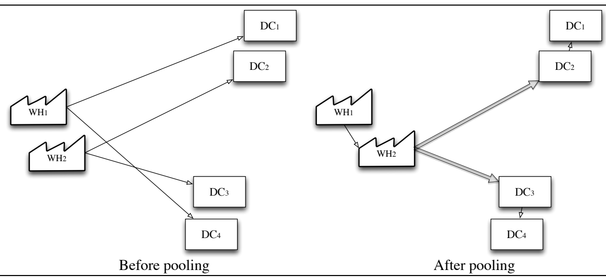


Fig. 1. Illustration of the supply chains pooling (WH_i: Warehouse of suppliers i; DC_j: Distribution Center of retailer j)

It has been proven in the literature that the horizontal cooperation or pooling practice can be seen as an efficient solution to improve the freight transportation [17,16,12,10,19,20]. However, we see only few realizations until now. The gap lying between the pooling initiative and its implementation is the lack of appropriate cooperation model, in particular sharing mechanism for different pooling cases [4]. In following sections, we firstly identify different pooling cases, and then examine why current sharing mechanisms are not sufficient to support horizontal pooling.

2.2 Identification of pooling cases

In this work, we focus on the logistics cooperation organized by *Logistics Service Providers* (LSPs), where a global optimum yielding highest cost efficiency is the most preferable. We broadly divide pooling cooperation into 2 categories with respect to negligible or significant CCs arising in cooperation, as shown in Table 1.

Table 1
Different categories of LSP-initiated pooling

Coordination cost	Cooperation categories
Negligible	C1: superadditive cooperation with global optimum incentive
Significant	C2: non-superadditive cooperation with global optimum incentive

One of the most important criteria to distinguish pooling cases is the existence of CC (coordination cost). Broadly it is the extra costs engaged in the cooperation, i.e. the communication cost, the investment in IT or in facility etc. In previous works on game-theoretic investigation of logistics cooperation, the costs of the coordination among players for coalition formation and maintaining are always considered negligible [13,1]. We notice that this holds in some cases, for example when the logistics cooperation occurs among the customers of a common LSP. Since the LSP has already got the prerequisite (information, facility, etc.), for the cooperation among his customers, the CC is negligible. Otherwise, the CC may become significant.

When CC is negligible, the coalition containing all participants (i.e. the grand coalition) will generate the highest common gain (C1 in Table 1). But when the CC arises, the game is becoming non-superadditive. The synergy lying in the cooperation between certain participants may be less than the CC needed to form the corresponding cooperation relationship. In this case, a cooperation scheme based on a partition of grand coalition (cooperation occurs only within sub-groups of participants, namely, the sub-coalitions) may be more preferable (C2 in Table 1). And the partition generating the highest global profit is called the *optimal coalition structure* (optimal CS). Note that potentially the grand coalition could be optimal CS in non-superadditive game.

Most of the existent research works on logistics cooperation focus mainly on the C1 cooperation cases, where the CC is negligible, and on the corresponding superadditive cooperation game, where the grand coalition will always be the optimal CS [16,14,6]; but very few on non-superadditive cases [9]. Having identified different categories of horizontal cooperation, we intend to investigate different cooperations cases from a general viewpoint, and propose feasible game-theoretic solutions. In the following section, we investigate the sharing mechanism issue in pooling from a game-theoretic viewpoint.

3 The sharing mechanism

3.1 Modeling of logistics pooling game and some preliminaries

We denote our supply chains pooling game by $G_p = (N, v, P)$. N is the set of all players in the game (potential pooling cooperators). v is the value function of a given coalition S defined as $v(S) = B(S) - M(S) - CC(S)$, where $B(S) = \sum_{i \in S} B(i)$, and $B(i)$ denote the individual logistics cost of player $i \in N$; $M(S)$ denote the optimized logistics cost of a coalition S after pooling; and $CC(S)$ denote the coordination costs for pooling arisen in the coalition S. Note that when $CC(S) = 0$, $\forall S \subseteq N\, and\, |S| > 1$, we can easily proof that the game is superadditive since we always have $B(S) \geq M(S)$ and then $v(S+T) \geq B(S) + B(T) - M(S) - M(T) \geq 0$. But when $CC(S) > 0$, $\forall S\, that\, |S| > 1$, the latter inequation is no longer always validated due to the presence of CC in it. The game thus is non-superadditive. P is a given CS (recall that coalition Structure) that is a partition of coalitions of all players N, that $P = \{S_1, S_2, ...S_k\}$ where for all $l \in \{1, 2, ...k\}$, we have $S_l \subseteq N, \sum_{l=1}^{k} S_l = N, (i \neq j) \rightarrow S_i \cap S_j = \emptyset$. If $\mathscr{P}$ is the set of all possible partitions, for the given $P \in \mathscr{P}$, we have $v(P) = \sum_{i \in \{1,...k\}} v(S_i)$. Particularly P^* is an optimal CS that $v(P^*) \geq v(P)$ for all $P \in \mathscr{P}$. Note that we also take $\{N\}$ as a special CS. Thus specially, we have game $G_p = (N, v, \{N\})$, abbreviated by (N, v), as the game with the grand coalition being the optimal CS. Aumann and Dreze [2] prove that a necessary condition for a game with coalition structure (CS) being stable (i.e. having a non-empty CS Core) is that the CS formed is the optimal one. In this sense the optimal CS is the most "promising" CS in the cooperation, in terms of the possibility to be stable and the total gain that can be generated. So we defined our pooling game based on the optimal CS, noted as $G_p = (N, v, P^*)$.

An allocation x is a vector with elements x_i that indicate the corresponding payoff for each player. We have $x(S) = \sum_{i \in S} x_i$ and $x(P) = \sum_{i \in P} x_i$. A payoff vector x is called an imputation if x satisfies two criteria: the *individual rationality*, i.e. $x_i \geq v(i)$, $\forall i \in N$; and *efficiency*, i.e. $\sum_{i \in S_k} x_i = v(S_k)$, $\forall S_k \in P^*$. The set of all imputations of a game $G = (N, v, P^*)$ is denoted by $I(N, v, P^*)$, and especially $I(N, v)$ for game (N, v).

With regard to the outcome of a game, we introduce two important concepts of allocation solution: the *Core* and the *CS Core*. The Core of a game is firstly defined by Gillies [11] for game $G = (N, v)$:

$$Core(N, v) = \{x | x(S) \geq v(S), \forall S \subseteq N, \, and \, x \in I(N, v)\}. \tag{1}$$

It contains all stable allocations, which guarantee that there is no coalition can benefit by jointly deviating from the grand coalition. Drechsel and Kimms [7] present an algorithm to compute Core allocations, which makes the Core concept more applicable. Aumann and Dreze [2] generalize the Core solution concept to games with

CS, i.e. the CS Core. Similarly, given a CS, the CS Core contains all allocations that can guarantee no incentive to deviate from this CS. The CS Core of a game $G = (N, v, P)$ is defined by:

$$CS\,Core(N, v, P) = \{x|x(S) \geq v(S),\ \forall S \subseteq N, ,\ and\, x \in I(N, v, P)\} \tag{2}$$

The Core and CS Core are credible stability criteria that are valid in both myopic and farsighted points of view [15]. Many authors use the Core concept to guarantee the stability of their allocation [13,8]. Similarly in this paper, the stability examinations of different sharing rules are all based on these two concepts.

3.2 A sharing mechanism for superadditive pooling games

We start investigating the sharing mechanism in pooling cases with negligible CC (Coordination Cost), namely superadditive game. Since in this case the grand coalition N is always the optimal CS, we study the game $G = (N, v)$. A sharing model, which is feasible for superadditive games with non-empty Core, will be introduced in Section 3.2.1. For superadditive games with empty Core, we propose a variation of this model in Section 3.2.2.

3.2.1 A sharing model for superadditive pooling games with non-empty Core

We suppose that a fair sharing model in the pooling games should take into account following factors: contribution to the common profit, bargaining power that impacts the negotiation result, and stability consideration for the long-term cooperative relationship. Since the *Shapley value* (SV) is based on the average marginal contribution of player, so it can be regarded as the measure of players' contribution to the common profit. Bargaining power should be modeled into the construction of such a fair sharing model by weight, so that with the player's weight increasing while that of the others unchanged, his payoff increases. Further, the allocation rule should integrate stability consideration alongside its construction. Taking account of all these criteria, we propose the following linear programming (LP) as a sharing model to compute a fair allocation, named *contribution-and-power weighted value* (CPWV).

$$\begin{aligned}
MIN:\ & \theta \\
s.t.:\ & \frac{x_i}{s_i w_i} - \frac{x_j}{s_j w_j} \leq \theta, \forall i, j \in N; && (3)\\
& \sum_{i \in S} x_i \geq v(S), \forall S \subset N; && (4)\\
& \sum_{i \in N} x_i = v(N); && (5)\\
& x_i \geq 0, \forall i \in N. && (6)
\end{aligned}$$

x_i in this LP is the CPWV payoff to the player i; s_i is the SV payoff of player i; w_i is the factor denotes the bargaining power of player i, e.g. player i's total weight/volume of goods transported in this cooperative logistics system, or its revenue, etc. However, the bargaining power of player is usually a composite factor whose determination is decided by negotiation. This adjustment factor plays an important role in the allocation solution according to outcome of negotiation. Since the SV in a superadditive game with no dummy player will always be positive (dummy player having no contribution is not considered in the game), and the bargaining power factors are positive, thus it is guaranteed that $s_i > 0$ and $w_i > 0$. This LP can identify a payoff vector x that minimize the maximum difference between any two players' payoff rates defined as $x_i/(s_i \cdot w_i)$. The other constraints guarantee that the solution is in the Core.

From the LP of CPWV, we can easily prove that CPWV of a game with non-empty Core satisfies following axioms:

Axiom 1 *Proportionality: if for* $i, j \in N$, $v(S \cup \{i\}) = v(S \cup \{j\})\, \forall S \subseteq N$ *and* $i, j \notin S$, *then* $x_i/w_i = x_j/w_j$ *when* $\{x_1, ...x_i, ...x_j, ...x_n\} \in Core(N, v)$*;*

Axiom 2 *Efficiency:* $\sum_{i \in N} x_i = v(N)$*;*

Axiom 3 *Individual rationality:* $x_i \geq v(i)$, $\forall i \in N$*;*

Axiom 4 *Collective rationality:* $\sum_{i\in S} x_i \geq v(S)$, $\forall S \subset N$;

Axiom 5 *Weak monotonicity: if* $w'_i > w_i$, *and* $w'_j = w_j \, \forall j \in N \, and \, j \neq i$, *then* $x'_i \geq x_i$;

The proportionality axiom means that if two players i, j can be replaced by each other in any coalitions without changing the worth of the coalition, they will get payoffs proportional to their weights, provided this payoff allocation is a Core-stable one; the efficiency axiom means that the common gain achieved by cooperation will be shared out among all cooperators; the individual and collective rationality axioms mean that the CPWV allocation is immune to unilateral or multilateral deviation; and the weak monotonicity axiom means that as a player i's weight increases while keeping those of the others unchanged, player i's payoff will increase or stay the same, depending on if the Core-stability can be satisfied. Using this sharing mechanism, the payoff vector calculated satisfies the previous axioms, and takes players' contribution and bargaining power into account, thus is fair and reasonable for players, and can be applied to model the complicated multilateral bargaining process. Once all players arrive at a consensus on the appropriate set of bargaining-power factors, the CPWV model can propose a reasonable allocation.

This sharing model can be applied to C1 pooling case in Table 1, when the Core is non-empty. In such superadditive pooling games, players will prefer the cooperation in the grand coalition, and the payoff vector computed by CPWV model will be always in the Core.

3.2.2 A sharing model for superadditive pooling games with empty Core

The CPWV model studies only the superadditive logistics games with non-empty Core. But this kind of game can also have an empty Core. For example in a game with 3 players, with value function $v(1) = v(2) = v(3) = 0, v(\{1,2\}) = 6; v(\{1,3\}) = 7; v(\{2,3\}) = 5; v(\{1,2,3\}) = 8$, the Core is empty though the grand coalition is still the optimal CS. When the cooperation organizers have incentive to achieve the global optimality in the optimal grand coalition (i.e. C1 in Table 1), we examine an alternative to the Core solution to construct a feasible sharing mechanism for C1 cooperation with an empty Core. Shapley and Shubik [23] introduce the weak ϵ-Core that is defined as:

$$Weak\,\epsilon\text{–}core(N,v) = \{x|\, x(S) \geq v(S) - |S|\epsilon \;\forall (S \subseteq N \,and\, S \neq \emptyset),\; and\, x \in I(N,v)\}. \tag{7}$$

When the Core of the game is empty, with sufficiently large ϵ value, a non-empty weak ϵ-Core can always be found. The ϵ value in this non-empty weak ϵ-Core can be directly interpreted as the highest individual sacrifice/give-up that players would like to afford for achieving the cooperation in the grand coalition. Maschler et al. [18] formally defines the *weak least Core* (WLC) as being the non-empty weak ϵ-Core with smallest possible ϵ value, noted by ϵ^*. For games with empty Core, the WLC can be interpreted as the allocation set that contradicts the least with Core stability. Based on the CPWV model above, we propose an alternative solution by replacing the Core-stability constraint in the previous model by the WLC constraints. This solution denoted by CPWV in WLC can be computed by following LP.

$$MIN: \; \theta$$

$$s.t.: \; \frac{x_i}{s_i w_i} - \frac{x_j}{s_j w_j} \leq \theta, \forall i,j \in N \tag{8}$$

$$\sum_{i\in S} x_i \geq v(S) - |S|\epsilon^*, \forall S \subset N \tag{9}$$

$$\sum_{i\in N} x_i = v(N). \tag{10}$$

$$x_i \geq 0, \forall i \in N \tag{11}$$

This solution is suitable for C1 pooling case with an empty Core. By using this solution, we can always find appropriately compromised allocation for the players with full-cooperation preference, and this allocation satisfies *efficiency*, *individual rationality*, *weak monotonicity* and the following two axioms:

Axiom 6 *Modified proportionality:* $if\, for\, i,j \in N$, $v(S \cup \{i\}) = v(S \cup \{j\}) \,\forall S \subseteq N \,and\, i,j \notin S$, $then\, x_i/w_i = x_j/w_j \; when\, \{x_1, ...x_i, ...x_j, ...x_n\} \in Weak\, Least\, Core(N,v)$;

Axiom 7 *Weak collective rationality:* $\sum_{i\in S} x_i \geq v(S) - |S|\epsilon^*$, $\forall S \subset N$.

3.3 Generalization of CPWV in non-superadditive games

As the coordination cost (CC) arises, the pooling game is no longer superadditive and the optimal CS may not be the grand coalition. In this case (i.e. C2 in Table 1), a sharing mechanism in a game with CS will be needed. At first place it is important to highlight the two peculiarities of non-superadditive pooling games: the occurrence of non-positive SVs and the determination of the optimal CS. These peculiarities should also be taken into account in the CPWV model.

Firstly, when the pooling game is non-superadditive, for some coalitions whose synergy is lower than the CC to pay, the value function v in game $G_p = (N, v, P^*)$ may be negative. And due to negative marginal contribution to some coalitions, some players may have negative SVs, even when the grand coalition is still global optimal and stable. In this case, the SVs computed with v defined in the original game cannot represent the real contribution of players, and CPWV model cannot calculate reasonable payoffs.

To always generate reasonable non-negative SVs, we propose the use of the value function in the *superadditive cover* of the game as the input of SV computation [2]. The superadditive cover of a cooperative game (N, v) is denoted by $(N, \hat{v})$. The value $\hat{v}(S)$, is defined by: $\hat{v}(S) = \max\{\sum_{S \in P_S} v(S) | P_S$ is a partition of $S\}$. The intuitive meaning of superadditive cover is that when new members join a coalition, only the most beneficial cooperation will actually occur, and the thus-formed coalition will cooperate in an optimal partition of this coalition. The SV computed with $\hat{v}$ in the superadditive cover will always be positive, provided that there is no dummy player in the pooling game. Further, Perez-Castrillo and Wettstein [21] find that their dynamic non-cooperative approach implements the SV of the superadditive cover as an equilibrium outcome for games with CS, which is a support for the use of superadditive cover SV in our work. Note that the superadditive cover of a superadditive game is itself, so it is a generally feasible way to compute the SV.

Secondly, another question is to determinate the optimal CS, demoted by P^*, in which the pooling games should be carried out. This questions is well studied at non-superadditve games branch. And in this paper we adopt the model developed by Rahwan and Jennings [22] to compute the optimal CS. Once the latter is decided, we have to integrate into the CPWV model the corresponding generalizations of the SV and the Core, i.e. the CS SV and the CS Core, which is introduced by Aumann and Dreze [2].

We adapt the CPWV to non-superadditive pooling games by integrating the CS SV (computed with superadditive cover $\hat{v}$ and corresponding to the optimal CS) and the CS Core. This solution named CS CPWV is helpful to support pooling cases as C2 in Table 1 when the CS Core is non-empty. The CS CPWV allocation can be computed by following LP.

$$\begin{aligned}
MIN: \ & \theta \\
s.t.: \ & \frac{x_i}{s'_i w_i} - \frac{x_j}{s'_j w_j} \leq \theta, \ \forall i, j \in S_k, \forall \, (S_k \in P^* \, and \, v(S_k) > 0) && (12) \\
& \sum_{i \in S} x_i \geq v(S), \forall S \subset N && (13) \\
& \sum_{i \in S_k} x_i = v(S_k). \, \forall \, S_k \in P^* && (14) \\
& x_i \geq 0, \forall i \in N && (15)
\end{aligned}$$

In this linear program, s'_i is the CS SV of player i computed by the value function of the superadditive cover $\hat{v}$ with respect to the optimal CS P^*. To implement this allocation rule, the coalitions S_k that $v(S_k) = 0$ should be excluded from this cooperation group to make sure that $s'_i > 0$. Note that in C3 and C4 cooperations, the optimal CS may or may not be the grand coalition. When the optimal CS is the grand coalition, the CS CPWV model is equivalent to the CPWV model, with $P^* = \{N\}$ and $S_1 = N$ being the only sub-coalition in P^*.

Similarly, from the formulation of the CS CPWV, we can see that CS CPWV of a game in coalitional form with non-empty CS Core satisfies *efficiency, individual rationality, collective rationality*, and *weak monotone axioms*.

When the CS Core is empty in the pooling cooperation of type C3, a compromise can be made by replacing constraints (13) in previous model with following constraint set:

$$\sum_{i \in S} x_i \geq v(S) - |S|\epsilon^{**}, \forall S \subset N, \tag{16}$$

where ϵ^{**} is the corresponding ϵ value in the weak least CS Core (WLCSC).

Thus we construct a set of CPWV solutions that correspond with both superadditive and non-superadditive pooling cases, listed in Table 2. When players are interested on global profit, whenever the Core (CS Core) is empty/non-empty, our CPWV solutions constructed for identified feasible pooling cooperation categories provide fair and reasonable gain-sharing solutions with general applicability. The coalition stability in different games, the players' contributions and the players' bargaining powers are considered in the model.

Table 2
Gain-sharing solutions for different supply chains pooling cases

	Coordination cost	Core	Solution
Pooling cases	Negligible	Non-empty	CPWV
		Empty	CPWV in WLC
	Significant	Non-empty	CS CPWV
		Empty	CS CPWV in WLCSC

4 Application of CPWV model to experimental pooling cases

4.1 Presentation of the case

The aim of this part is to investigate the practicability of the developed gain sharing models. To this end, a pooling cooperation based on FMCG (Fast-Moving Consumer Goods) supply chains real data in France has been investigated. By our partners of this research works, we are provided an original database, which contains one-week flows of one retailer and his four suppliers in food sector, from the suppliers' Warehouse (WH) to eight national Distribution Centers (DC) of the retailer with knowing the location of all WHs and DCs. The characteristics of the flows are described in Table 3.

Table 3
Characteristics of flows of the pooling case during the week studied (NS:Num of sites; SF:Sum of flows in pallet; NL:Num of links; AF:Average flows/link; SDF: Standard deviation of flows/link; AK:Average KM/link; SDK:Standard deviation of KM/link)

	NS	SF	NL	AFL	SD	AK	SDK
Supplier 1	1	77	1	77	-	511	-
Supplier 2	1	714	8	89,25	32,63	519	148
Supplier 3	1	55	4	13,75	20,63	491	187
Supplier 4	1	63	2	31,5	37,48	476	71

There are four players in the game (N, v, P^*), where $N = \{1, 2, 3, 4\}$ represents the set of four suppliers. In this pooling case, we focus mainly on the impact of different CCs on the cooperation scheme: superadditive game when $CC(S) = 0 \ \forall S \subseteq N$ and non-superadditive game when CC increases.

Recall that here we define the value of coalition S $v(S)$ as the saving of transportation cost in S. And it is calculated as $v(S) = B(S) - M(S) - CC(S)$ where $B(S)$ is the transportation cost of coalition S before pooling, $M(S)$ is the optimized transportation cost after pooling, and $CC(S)$ is the coordination cost of coalition S.

In this work, the coordination cost $CC(S)$ is defined as:

$$CC(S) = \begin{cases} 0 & \text{if} |S| < 2 \\ cc \cdot (|S| - 1) & \text{if} |S| \geq 2. \end{cases} \tag{17}$$

We can further change the coefficient cc (the extra cost for adding a new player to the game) to study the impact of CC on the optimal CS of the game.

The transportation cost of coalition S after pooling $M(S)$ is obtained by an adapted optimization model of Mixed Integer Linear Programming (MILP) based on the former works concerning the pooling logistics network design problem. For more detail refer to [19].

4.2 Sharing schemes in the pooling game

4.2.1 Supperadditive pooling scenarios

This pooling case with real-world data will be investigated under two assumptions: with coefficient $cc = 0$ (C1 or C2 in Table 1) or $cc > 0$ (C3 or C4). At first, we investigate the pooling games with $cc = 0$.

When $cc = 0$, this game is superadditive, and has a non-empty Core, thus the grand coalition will be stable. By cooperating in the grand coalition, 15.9% logistics cost saving (6313 Euros for one week) can be achieved. Then we consider how to divide the common gain. We simply adopt an arbitrary bargaining power vector $\{1, 2, 1, 3\}$ to compute the CPWV allocation in this game, and compare it with the SV allocation. These two allocations are illustrated in Figure 2.We can see that our sharing mechanism further adjusts the gain allocation according to different bargaining powers, for example the payoff of S3 with power weighted 1 has clearly decreased, in contrast to that of S4 with power weighted 3. While the payoff vector will be changed by modifying the bargaining power vector, the Core stability of the allocation is always guaranteed.

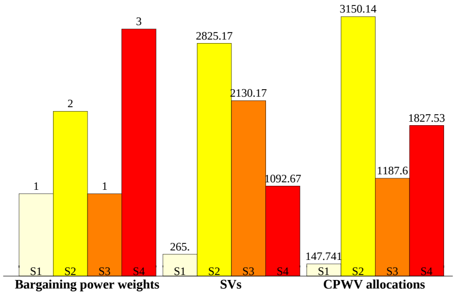


Fig. 2. Comparison of allocations in the pooling case with null CC

4.2.2 Non-supperadditive pooling scenarios

As cc increases, coalitions' profits decrease, and the player whose logistics network is of least synergy potential will leave the grand coalition the first. Only coalition with high synergy may survive. And at the end, when cc increases to a certain level that makes none of the players feels profitable to cooperate, the optimal CS will be singletons. In our pooling game, when $cc \geq 530$, the grand coalition is no longer stable, and when $cc > 3929$, players tend to singletons. Figure 3 shows how the optimal CS changes as the coefficient cc increases. The vertical axe denotes the scale of the largest coalition in the optimal CS.

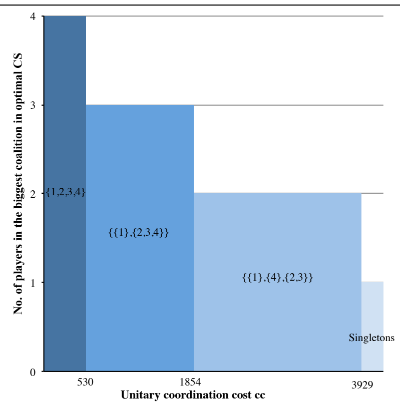


Fig. 3. Optimal coalition structure submitted to different coordination cost

And as we discussed in Section 3.3, we demonstrate in this non-superadditive scenario of pooling the occurrence of negative SVs , presented in Figure 4. We illustrated the (CS) SV allocations when cc increase by 100 from 0 to 4200. We can see that in the game with $cc = 500$, where the grand coalition is the optimal CS and the Core is non-empty, player 1's SV is negative. And so does the game with $cc = 1800$ where player 2 has a negative SV. In these cases, another allocation rule is necessary to achieve globally optimal solution. Hence the CS CPWV model in Section 3 based on supperadditive cover concept is employed.

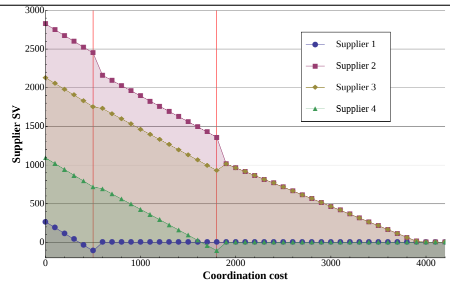
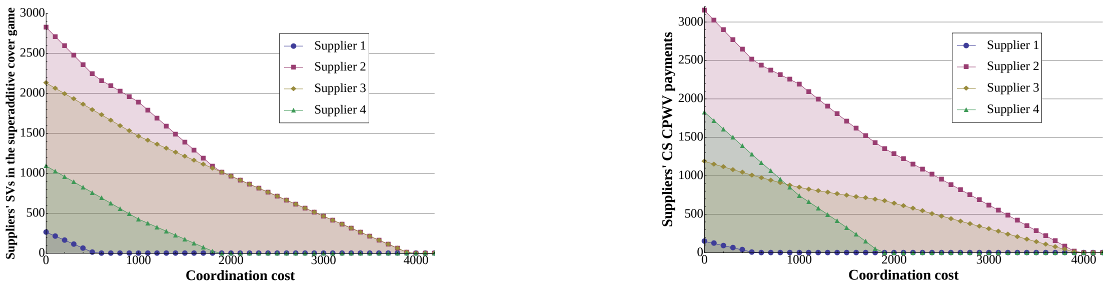


Fig. 4. Player SVs/CS SVs as coordination cost increases

We computed different SV allocations of the superadditive cover when cc increases and the corresponding CPWV allocations with bargaining power weight vector $w = \{1, 2, 1, 3\}$. The two sharing schemes are illustrated in Figure 5.



(a) The SV/CS SV allocations of players

(b) The CPWV/CS CPWV allocations of players

Fig. 5. Comparison of players' SV and CPWV allocations in the pooling game computed by superadditive cover value function $\hat{v}$

There is an apparent difference between the two sharing schemes. With the integration of players' bargaining power and game-theoretic solutions, the CPWV sharing model can propose theoretical feasible solutions and captures more factors that is important for real-world cooperation implementation at the same time.

There are three remarks for Figure 5. Firstly, there is no more negative SV in all scenarios due to superadditive cover. Secondly, overall the SVs before or after superadditive cover has not significantly changed, comparing to Figure 4. However, the payoff to player 3 is very different from SV to CPWV, but less obvious for the other players. Thirdly, in a coalition having only two players, for instance in the sub coalition {2,3} when $cc = 1854$, the CPWV model allocates to player 2 and 3 payoffs according to their power, while in SV allocation they equally share the gain. Overall, the allocation solution computed by CS CPWV model is more appropriate for non-superadditive pooling cases.

As an example, we investigate the case with $cc = 1500$, which corresponds to the C3 pooling case in Table 1. In this case, the game has an optimal CS $P^* = \{\{1\}, \{2, 3, 4\}\}$ and a non-empty CS Core. The supplier 1 in this game stays in singleton state due to low synergy, and suppliers 2, 3, 4 cooperate. Figure 6 shows the comparison between the SV allocation of the superadditive cover and the CS CPWV allocation.

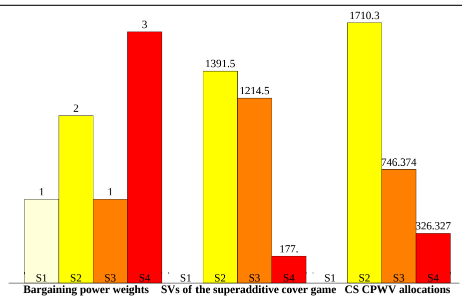


Fig. 6. Comparison of allocations in the pooling case with $cc = 1500$

In this case, the SV allocates to suppliers 2, 3 the payoffs at almost the same level, while the CS CPWV allocation makes distinction between payoffs according to both the contribution and the bargaining power. And in addition, in a game with non-empty (CS) Core, CPWV solution will always propose Core-stable allocations.

5 Conclusion

One of the main impediments lying in the implementation of logistics pooling is the lack of a fair and generally applicable sharing mechanism. In this paper, we identify different cooperation types corresponding to actual supply chains pooling cases. We propose the CPWV sharing mechanisms and its variations and generalizations adapted for different cooperation types. This solution set can construct fair and stable allocations by considering at the same time the *Core stability*, the *contribution*, and the *bargaining power* for both *superadditive* and *non-superadditive* games.

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