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It is Not Just Confusion!
Strategic Uncertainty in an Experimental Asset Market

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It is not just confusion!
Strategic uncertainty in an experimental asset market∗

Eizo Akiyama† Nobuyuki Hanaki‡ Ryuichiro Ishikawa§

August 8, 2013

Abstract
To what extent is the observed mis-pricing in experimental asset markets caused by strategic uncertainty (SU) and by individual bounded rationality (IBR)? We address this question by comparing subjects initial price forecasts in two market environments – one with six human traders, and the other with one human and five computer traders. We find that both SU and IBR account equally for the median initial forecasts deviation from the fundamental values. The effect of SU is greater for subjects with a perfect score in the Cognitive Reflection Test, and it is not significant for those with low scores.

Keywords: Bounded rationality, Strategic uncertainty, Experiment, Asset markets, Computer traders, Cognitive Reflection Test

JEL Code: C90, D84

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1 Introduction

To what extent are the observed price deviations from the fundamental values in experimental asset markets (Smith et al., 1988) caused by strategic uncertainty (uncertainty about the behavior of others) and by individual bounded rationality (or confusion)? First we describe the context in which this question is posed.

Consider an asset with a finite life of $T$ periods. Each unit of the asset pays a constant dividend $D$ at the end of each period, and becomes worthless after the final dividend payment at the end of period $T$. Under these conditions, the fundamental value of a unit of the asset during period $t$ ($t = 1, 2, ..., T$), $FV_t$, is the sum of the remaining dividend payments, i.e., $FV_t = (T + 1 - t)D$. When these conditions are commonly known, the common knowledge of rationality implies that if rational traders trade this asset, it will only be at its fundamental value.

Following the seminal study by Smith et al. (1988), it has been repeatedly shown for a variety of experimental conditions and subject pools, that the market prices of assets deviate substantially from their fundamental values in these experimental asset markets.\footnote{In the original experiment in Smith et al. (1988) as well as in many other studies, the value of the dividend payment in each period, $d_t$, is determined randomly from a known i.i.d. distribution. Thus, the fundamental value of the asset is defined as $FV_i = \sum_{t=1}^{T} E(d_t)$, where $E(d_t)$ is the expected dividend payment. Porter and Smith (1995) eliminated the uncertainty about dividend payments in investigating the effect of varying degrees of risk aversion among subjects. No significant difference was found in the observed pattern of mis-pricing in the experiments with uncertain dividend payments. While most of the studies, including Smith et al. (1988), considered continuous double auction markets, van Boening et al. (1993) and Haruvy et al. (2007) considered call markets. They reported that prices deviate substantially from the fundamental values in call markets as well. King et al. (1993) investigated the effects of short-selling, margin-buying, equal endowment, and circuit breakers. They also conducted experiments with corporate executives and stock market dealers to see the effect of different subject pools. “Bubbles” and “crashes” were observed in most of their experiments, except in those where transaction fees were introduced or where subjects had experienced the same market conditions twice. Haruvy and Noussair (2006) showed that allowing short-selling can cause prices to deviate substantially below the fundamental values. Noussair et al. (2001) reported bubbles in markets with a constant fundamental price, i.e., the expected value of the dividend per period is zero and an asset is converted into a fixed sum of money at the end of the final trading period. Dufwenberg et al. (2005) mixed experienced and inexperienced subjects to investigate whether the presence of inexperienced subjects among experienced subjects induced greater price deviation. They showed that the presence of two (respectively four) inexperienced subjects in a market with four (resp. two) experienced subjects (who had experienced the same market three times) did not produce larger price deviations than in a market with six twice-experienced subjects. Hussam et al. (2008) investigated whether (twice) experienced subjects, when facing a new market environment with a large variance in dividend payments and higher initial cash holdings, would avoid creating bubbles. The answer was negative, and therefore, learning to trade close to the fundamental values in one market condition does not carry over to a different market condition. Deck et al. (2011) considered overlapping generations of traders to study the effect of the arrival of inexperienced traders and the departure of experienced traders. They found that bubbles form when inexperienced traders arrive and bring liquidity to the market, and that crashes occur when experienced traders leave and withdraw liquidity from the market. See Stöckl et al. (2010) and the references therein for details of other experiments.}

Such deviations in observed prices from the fundamental values were initially considered to result from strategic uncertainty (i.e., uncertainty about others’ behavior) by Smith et al. (1988). They noted that “what we learn from the particular experiments reported here is that a common dividend, and common knowledge thereof, is insufficient to induce initial common expectations. As
we interpret it, this is due to agent uncertainty about the behavior of others” (Smith et al., 1988, p.1148).

An implication of this is what Lei et al. (2001) called a “speculative hypothesis.” That is, “traders are uncertain that future prices will track the fundamental value, because they doubt the rationality of the other traders, and therefore speculate in the belief that there are opportunities for future capital gains” (Lei et al., 2001, p.832). Such speculation results in observed price deviations from the fundamental values. Lei et al. (2001) tested the “speculative hypothesis” by performing a set of experiments in which capital gains were not possible because resale of the assets was prohibited. Based on the observed deviations of prices from the fundamental values even in the absence of capital gain possibilities, Lei et al. (2001) rejected the speculative hypothesis and noted that “the hypothesis that the traders are rational, and that the bubble is due to the fact that this rationality is not common knowledge, cannot be the whole story behind the bubbles” (p. 857). Instead, they suggested the “active participation hypothesis,” i.e., subjects in these experiments who are trained to engage in trading simply want to trade (even if such trading results in making losses) because there are no other activities available to them during the experiment.

In two recent papers, Huber and Kirchler (2012) and Kirchler et al. (2012) suggested that these bubbles are mainly due to subjects being confused about the nature of declining fundamental values of the asset in the experiment. Kirchler et al. (2012) also showed that mis-pricing due to confusion is further fueled by increasing the amount of cash relative to the number of assets available in the market as a result of dividend payments during the experiment.2

These studies showed that the magnitude of mis-pricing, that is, the deviation of prices from the fundamental values, becomes much smaller if in the instructions, a figure of declining fundamental values is presented to the subjects, rather than a table containing the same information (Huber and Kirchler, 2012), or if the word “stock” (the value of which many subjects assume does not decline constantly) is explained as “stocks of a depletable gold mine” (Kirchler et al., 2012).

Is it just confusion (i.e., subjects not understanding the declining fundamental value in these experiments that is not natural in real markets) that is causing the mis-pricing? It would be quite surprising, however, if all the subjects in the experiments were equally confused given the widely reported heterogeneity in the depth of strategic thinking among subjects in laboratory experiments.

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2This conclusion is in contrast with an earlier study by Noussair et al. (2001) that showed substantial mis-pricing under a constant fundamental value.
In addition, several studies have demonstrated, both theoretically (Haltiwanger and Waldman, 1985, 1989) and experimentally (Fehr and Tyran, 2008; Sutan and Willinger, 2009; Heemeijer et al., 2009; Bao et al., 2012), that such heterogeneity can manifest as a large deviation from the equilibrium in the presence of positive feedback. Because it is very likely that positive feedback exists in asset markets, confusion or bounded rational behavior of a few subjects in the market can be amplified owing to strategic responses by more sophisticated subjects. This consideration leads us to our main question: to what extent is mis-pricing caused by confusion (or some kind of individual bounded rationality) and strategic uncertainty? The above mentioned studies do not investigate these questions, and thus the relevance of strategic uncertainty in these experiments remains an open question.

In this paper, we address this open question by eliminating strategic uncertainty as far as possible by introducing computer traders, and quantify the potential impact of strategic uncertainty as well as individual bounded rationality (or confusion) on these asset market experiments. That is, we consider two types of markets (treatments) consisting of six traders: one human and five computers (1H5C), and six humans (6H). Our computer traders follow the equilibrium strategy (under the usual set of assumptions of profit maximization and the common knowledge of rationality). All subjects are informed of which treatment they are involved in, and those in 1H5C are clearly told how computer traders behave. Therefore, in the 1H5C treatment, the single human trader does not face any strategic uncertainty. Let us mention a couple of additional design aspects before discussing how our experimental design allows us to achieve our objective.

To facilitate the introduction of computer traders, we employ a call market rule similar to those used by van Boening et al. (1993) and Haruvy et al. (2007) rather than the continuous double auction employed by Smith et al. (1988) and in various other studies. In a call market, traders submit buy (sell) orders by specifying the maximum (minimum) price they are willing to pay (accept) for a unit of asset in each period. This means that our computer traders, in each period, submit their buy orders (sell orders) by specifying the fundamental value of the asset in that period as the

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3The theoretical developments that followed these experimental findings suggest that considering interaction among heterogeneous boundedly rational agents helps us to better understand experimental outcomes. See Camerer (2003, Ch.5) and Crawford et al. (2013) for further details.

4These experimental analyses considered different games; Fehr and Tyran (2008) and Sutan and Willinger (2009) considered price setting and beauty contest games, while Heemeijer et al. (2009) and Bao et al. (2012) investigated price forecasting games. What these studies also showed is that in the presence of strategic substitution (or negative feedback), the observed outcomes are much closer to the Nash or rational expectation equilibrium.

5van Boening et al. (1993) and Haruvy et al. (2007) reported that prices deviate substantially from the fundamental values in call markets as well.
maximum (minimum) price they are willing to pay (accept) for a unit of asset. Once all the traders in the market have submitted their orders, the price that clears the market is calculated, and all transactions take place at that price among traders who submitted a maximum buying price no less than, or a minimum selling price no greater than, the market clearing price.

Note, however, that because of the way computer traders behave, the market prices in 1H5C follow the fundamental values very closely. Thus, comparing the realized market prices for the two treatments, 6H and 1H5C, is not very informative for our purposes. Therefore, we elicit subjects’ expectations about future prices as in Haruvy et al. (2007). It has been shown that expected future prices deviate quite substantially from the fundamental values in all human markets (Haruvy et al., 2007, Fig.3, p.1909), and that the deviations disappear gradually as subjects gain more experience from trading under the same market conditions. This is similar to what is observed from the realized prices. Thus, our focus on price forecasts is informative for studying the cause of realized price deviations.

How then does our design allow us to achieve our objective; that is, to identify the effect of strategic uncertainty as well as individual bounded rationality (or confusion) in asset market experiments? Let us imagine a rational human trader. In the 1H5C treatment, s/he does not face any uncertainty regarding the behavior of the other traders in the market. And given the behavior of the computer traders, s/he will expect that the prices follow the fundamental values. Therefore, if we observe any deviation from this expectation in our data, it must be due to some kind of individual bounded rationality (or confusion). On the contrary, in the 6H treatment, the rational human trader is unsure about the behavior of the other traders in the market and can expect a variety of outcomes. Of course, we should not eliminate the possibility that subjects are confused or boundedly rational. Thus, the observed deviations of price forecasts from the fundamental values in the 6H treatment are due to both strategic uncertainty and individual bounded rationality. A comparison of the subjects’ price expectations in the 1H5C and 6H treatments, therefore, gives us a direct measure of the extent to which strategic uncertainty explains the deviation of price forecasts from the fundamental values.

Several experiments have introduced computer agents that follow equilibrium behavior in laboratory experiments to reduce strategic uncertainty. Cason and Friedman (1997), in their experiments on price formation in a simple market institution, introduced robot traders that follow a Bayesian-
Nash equilibrium strategy to facilitate learning by human subjects.\textsuperscript{7} Fehr and Tyran (2001) introduced robots that utilize a Nash equilibrium strategy in their investigation of nominal (money) illusion to decompose the reasons for non-immediate adjustment against negative nominal shocks into (i) those arising from individual irrationality, and (ii) those due to a lack of common knowledge of rationality.\textsuperscript{8} They reported that both individual irrationality (or bounded rationality) and the lack of common knowledge of rationality (or strategic uncertainty) account equally for the failure of immediate adjustment to the new equilibrium after a negative nominal shock in the game.\textsuperscript{9}

Our experimental results suggest that strategic uncertainty accounts for about 50\% of the median initial forecasts deviation from the fundamental values; the remaining 50\% is due to bounded rationality. Therefore, it is not just confusion that causes mis-pricing in an experimental asset market; strategic uncertainty also plays an important role. To further investigate the effect of heterogeneity among subjects, we categorized subjects based on their scores in the Cognitive Reflection Test (Frederick, 2005, CRT) and analyzed the data separately based on the CRT scores. We found that the effect of strategic uncertainty is much larger (about 70\% of the median initial forecasts deviation from the fundamental values) for those with a perfect CRT score. For those with very low CRT scores, on the other hand, no significant effect of strategic uncertainty was observed.

Our finding complements the recent finding by Cheung et al. (2012). They investigated the effect of uncertainty regarding how others understood the nature of fundamental values in asset markets by (1) training subjects extensively on the nature of fundamental values, and (2) manipulating the subjects knowledge about whether all the other players in the same market have undergone the same extensive training. The latter manipulation is done by implementing the following two treatments: (a) everyone in the room is trained and they are all told that everyone in the room is trained, and (b) training only half of the subjects in the room and telling everyone that not everyone in the room is trained.

\textsuperscript{7}It should be noted that when human subjects play against other human subjects who are also learning, the learning process can be very slow. In addition to robots that follow the Bayesian-Nash equilibrium strategy (BNE robots), Cason and Friedman (1997) also introduced “revealing robots” whose behavior is different from BNE robots, to investigate whether the convergence to equilibrium is due to human subjects mimicking the behavior of the BNE robots or to their best response against the BNE robots. Their results suggest the latter.

\textsuperscript{8}Fehr and Tyran (2001) considered price-setting games and varied two aspects of the game: (a) whether negative nominal shocks are present, and (b) whether a human subject plays the game with other human subjects or with rational computer programs that assume all the players are rational. This two-by-two design allowed them to achieve the objective of the experiments. This is also discussed quite extensively in Fehr and Tyran (2005).

\textsuperscript{9}In addition to rational robots that follow the equilibrium strategy, Costa-Gomes and Crawford (2006) introduced boundedly rational robots that follow Level-1,2,3 or Dominance 1,2 strategies in their experiments on a two-person guessing game to better analyze the responses of human subjects who were informed about the behavioral rules of various opponents. In a different strand of the literature, computer agents were introduced to investigate whether deviations of observed behavior from the equilibrium prediction are due to bounded rationality (confusion) or kindness in games such as an alternative offer bargaining game (Johnson et al., 2002) or public good games (Houser and Kurzban, 2002). These studies identified substantial impact of bounded rationality.
room is trained. In (b), however, the subjects are not told that groups are created in such a way that either all the subjects in a group are trained or none of them is. Treatment (b) makes trained subjects believe that not all the other subjects in their group understand the declining nature of fundamental values when, in fact, their group consists only of trained subjects. The authors found that the magnitude of mis-pricing is substantially smaller when all the subjects are trained and know that they are all trained than the case in which subjects are trained but do not know that they are all trained. They also reported that, in the latter case, the mis-pricing is as great as in the case where training is absent. Based on these results, Cheung et al. (2012) concluded that individual confusion alone cannot account for the observed mis-pricing, but that uncertainty about how well others understand the nature of fundamental values can. An interesting question is how the effect of strategic uncertainty we have identified via price forecasts (which accounts for about 50% of the median deviation) is amplified so much that it accounts for most of the price deviations as reported by Cheung et al. (2012). Unfortunately, our experiment does not address this question and it is left for future research.

The rest of the paper is organized as follows. The experimental design is discussed in detail in Section 2. Section 3 presents the results of our experiments, and Section 4 concludes the paper.

2 Experimental design

We set up an experimental call asset market consisting of six traders, who were either human subjects or computer programs, and considered two treatments. In the first, referred to as the 6H treatment, all six traders were human subjects. In the other, referred to as the 1H5C treatment, only one of the six traders was a human subject, and the other five traders were computer traders who submitted orders at the fundamental values. In each treatment, subjects were told explicitly about the composition of the six traders in the market in which they were participating. Moreover, in the 1H5C treatment, subjects were also informed about the behavior of the computer traders. Our main focus was to compare the data for these two treatments to separate the effect of strategic uncertainty and individual bounded rationality.

In each market, traders can trade an asset with a life of ten periods. Initially, all traders were endowed with 4 units of asset and 520 experimental currency units (ECUs, which we called Marks). Subjects were also asked to submit their expectations regarding the future prices of a unit of the
asset. We first describe the trading rule employed, and then proceed to explain how subjects’ expectations about future prices were elicited.

We used a call market rule similar to that in Haruvy et al. (2007). In each period, each trader can submit at most one buy order and one sell order. An order consists of a pair of values: a price and a quantity. When submitting a buy order, a trader must specify the maximum price, $PD$, at which s/he is willing to buy a unit of asset, and the maximum quantity, $QD$, s/he is willing to buy. In the same manner, when submitting a sell order, a trader must specify the minimum price, $PS$, at which s/he is willing to sell a unit of asset, and the maximum quantity, $QS$, s/he is willing to sell.

We attached three constraints: the admissible price range, a budget constraint, and the relationship between $PD$ and $PS$ in the case that a subject submits both buy and sell orders. The admissible price range is set so that, when $QD \geq 1$ ($QS \geq 1$), $PD$ ($PS$) must be an integer between 1 and 2000, i.e., $PD \in \{1, 2, \ldots, 2000\}$ ($PS \in \{1, 2, \ldots, 2000\}$). The budget constraint simply means that neither borrowing of cash nor short-selling of an asset is allowed. The final constraint is such that when a trader is submitting both buy and sell orders, i.e., $QD \geq 1$ and $QS \geq 1$, the maximum buying price must not be greater than the minimum selling price, i.e., $PS \geq PD$. Once all the traders in the market have submitted their orders, the price that clears the market is calculated, and all transactions are processed at that price among traders who submitted a maximum buying price no less than, or a minimum selling price no greater than, the market clearing price.

At the end of each period, for each unit of the asset 12 ECUs is paid as a dividend. We selected a fixed dividend payment, instead of a stochastic dividend as commonly considered in the literature, to eliminate all uncertainty other than strategic uncertainty from the experiment. The dividend can be used to purchase the asset in subsequent periods. After the final dividend is paid at the end of period 10, the asset has no value. Other than this stream of dividend payments, the asset has no intrinsic value. Thus, the fundamental value of a unit of asset at the beginning of period $t$ is $FV_t = 12 \times (11 - t)$. We distributed a table showing the sum of the remaining dividends after the

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10 Of course, a trader can choose not to submit any orders by specifying zero as the quantities to buy and sell. We imposed a 60 second, non-binding, time limit for submitting orders. When the time limit was reached, the subjects were simply informed, though a flashing message in the upper right corner of their screen, to submit their orders as soon as possible.

11 Thus, the budget constraint implies (i) $QD \times PD \leq$ cash holding at the beginning of the period, and (ii) $QS \leq$ units of asset on hand at the beginning of the period.

12 When there are several such prices, the lowest one is chosen as the market clearing price. This is important to ensure the price does not jump up in the absence of transactions at the market clearing price.

13 Any ties among the last accepted buy or sell orders are resolved randomly. It is possible that no transaction will take place given the computed market clearing price.

14 Eliminating uncertainty about dividend payments did not significantly lower the magnitude of price deviations from the fundamental values according to the study by Porter and Smith (1995).
dividend for each period has been paid out, a value we called the “next value” in the experiment. Thus, subjects had a table showing $FV_t$ for $t = 1, 2, ..., 10$ and could refer to it any time during the experiment. This table is given in the Appendix A.

After the explanation of the setup of the call market, as well as the table of fundamental values (called “next value” in the experiment), subjects in the 1H5C treatment were told the following about the behavior of computer traders: “In each period, each computer trader places buy and sell orders by setting both the maximum price it is willing to pay and the minimum price at which it is willing to accept to the next value at the beginning of the period.” Thus, subjects were informed about the exact trading strategy employed by the computer traders.

Next we explain how expectations about future prices were elicited. At the beginning of each period, subjects were asked to submit their price forecasts for all the remaining periods in the market. That is, in period $t$, each subject submitted $10 - t + 1$ forecasts. Therefore, subjects submitted a total of 55 price forecasts over the 10 periods. Subjects were informed that they would receive the following bonus payment based on how accurate their forecast prices were:

$$\text{Bonus (in ECU)} = 0.5\% \times (\text{number of forecasts that were within } \pm 10\% \text{ of the actual market price}) \times \text{final cash holding in period 10}.$$  

Therefore, if all 55 forecasts were within 10% of the realized prices, the subject would receive 27.5% of his/her final cash holding as a bonus payment. This incentive scheme for accurate forecasts was chosen to reduce subjects’ incentive to influence the prices to move closer to their forecasts by making losses. When submitting price forecasts, all previous market clearing prices are shown on the screen. Our design is closely related to that used by Haruvy et al. (2007), who showed substantial deviations of both realized price and price forecasts from the fundamental values. The call market

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15 Although this was not stated explicitly in the instructions, each price forecast takes the form of an integer value between 0 and 2000. We set this range to match the admissible values of orders. If subjects tried to submit a value outside this range, an error message stating that the forecast must be in the above range would be displayed on their computer terminal.

16 We imposed a 120 second, non-binding, time limit for submitting price forecasts. When the time limit was reached, the subjects were simply told, through a message flashing in the upper right corner of their screen, to submit their forecasts as soon as possible.

17 As noted by Haruvy et al. (2007), there is a trade-off between an incentive for accurate transactions and an incentive for maximizing profit from trading. In other words, because we ask subjects to submit their forecasts before submitting their order, it is possible that if the incentive for accurate forecasts is too great, subjects may submit potentially loss-making orders to influence the prices to be closer to their forecasts. In our design, since the bonus for accurate forecasts is a fraction of the final cash holding, this incentive is reduced. It is, of course, best to have both accurate forecasts and high profit from trading.
rule has several advantages: (i) the prices subjects have to forecast are clear, (2) learning based on observing orders submitted by other traders within a period is not possible, and (3) it is easier to introduce computer traders because all orders are submitted simultaneously.

As the end of each period, subjects were informed about the market clearing price for the period, the units of asset they have traded,\textsuperscript{18} their cash and asset holdings, the number of price forecasts that were within 10\% of the actual market prices up to that period, and the next value of a unit of the asset.\textsuperscript{19}

As noted above, each trader was given an endowment of 520 ECUs of cash and four units of the asset before the market opened in period 1. The same group of traders, with identical initial endowments of cash and assets, repeated the same 10-period market three times as one experiment. We call a 10-period market a round. Thus, the experiment consisted of 3 rounds of a 10-period market with identical initial endowments and the same group of subjects. The purpose of repeating the round three times was to compare how quickly the price forecasts and the market clearing prices converge to the fundamental values.\textsuperscript{20}

At the end of the experiment (after participating in 3 rounds of the 10-period market), subjects were paid in cash the sum of their final cash holdings (including the bonus payment for accurately predicting future market prices) for each round plus a participation fee of 500 yen. We used an exchange rate between ECUs and Japanese yen of 1 ECU = 1 Japanese yen. The experiment lasted about two and a half hours including the explanation of the instructions and completion of a questionnaire after the experiment.\textsuperscript{21} The questionnaire consisted of the Cognitive Reflection Test (Frederick, 2005). Subjects earned on average about 4000 yen.

3 Results

A series of computerized experiments were conducted at the University of Tsukuba between May and July 2013.\textsuperscript{22} 173 subjects who had never participated in a similar experiment were recruited

\textsuperscript{18}In the presentation of this information, a positive (resp. negative) number denotes that they had bought (resp. sold) a certain number of units of asset.

\textsuperscript{19}The next value of an asset at the end of period \( t \) is \( 12 \times (10 - t) \).

\textsuperscript{20}Before entering round 1, there was a practice period to allow subjects to familiarize themselves with the user interface of the software. Subjects were given their initial endowment of cash and assets, and asked to enter their price forecasts for the 10 periods and their orders for period 1. Information regarding the resulting market clearing price and so on were not shown to the subjects.

\textsuperscript{21}See Appendix B to obtain the English translation of the instructions.

\textsuperscript{22}The experiments were implemented using z-tree (Fischbacher, 2007).
Table 1: Summary of experimental sessions

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Dates</th>
<th>No. of subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>6H</td>
<td>May 25, 2013</td>
<td>24</td>
</tr>
<tr>
<td>6H</td>
<td>June 1, 2013</td>
<td>24</td>
</tr>
<tr>
<td>6H</td>
<td>June 15, 2013</td>
<td>24</td>
</tr>
<tr>
<td>1H5C</td>
<td>May 26, 2013</td>
<td>25</td>
</tr>
<tr>
<td>1H5C</td>
<td>June 2, 2013</td>
<td>25</td>
</tr>
<tr>
<td>1H5C</td>
<td>July 7, 2013</td>
<td>51</td>
</tr>
</tbody>
</table>

We begin our discussion by presenting the results of our main analysis: initial deviation of price expectations from fundamental values. We then move on to discuss the realized prices and the dynamics of forecasts deviations. We also discuss the orders submitted by subjects.

3.1 Deviations of forecasts

In each period, subjects forecast prices for all the remaining periods within the round. To better summarize the magnitude of forecast deviations from the fundamental values for each subject, we introduce two measures of forecast deviations based on the measures of price deviations from the fundamental values: the relative absolute deviation (RAD) and the relative deviation (RD), proposed by Stöckl et al. (2010).

For subject $i$, the magnitudes of the deviations of the forecasts submitted in period $t$ of round $r$ from the fundamental values are measured as the relative absolute forecast deviation ($RAFD_{i,t,r}$) and relative forecast deviation ($RFD_{i,t,r}$) defined as:

$$RAFD_{i,t,r} = \frac{1}{T - t + 1} \sum_{p=t}^{T} \frac{|f_{i,p,r} - FV_p|}{|FV_p|}$$

Subjects had to register on our database before the experiment. We confirmed their lack of participation in past experiments by checking their names, student ID numbers, and e-mail addresses.

After conducting an in-depth analysis of the data for 1H5C from the May 26th and June 2nd sessions, we realized that there was an error in a parameter value affecting how subjects were grouped in these sessions. As a result of the error, subjects in these 1H5C sessions were in groups of 4 (or 3) humans and 20 (or 15) computer traders, and not 1 human and 5 computers as intended. We ran an additional session on July 7th for 1H5C with the correct grouping. While this error should not have any effect on the initial forecasts (because the instructions were identical), it could affect the outcome once subjects had submitted their orders and prices were realized. For our analysis of the initial forecasts deviation, we decided to pool the data from the May 26th, June 2nd, and July 7th sessions, providing a larger number of observations. However, in our analysis of prices, subsequent forecasts, and orders submitted by subjects, we discarded the data for 1H5C from the sessions on May 26th and June 2nd, and used only data from the session on July 7th. An analysis of the initial forecasts deviation by discarding the data from the sessions on May 26th and June 2nd is given in the Appendix C for completeness.
Distributions of \( RAFD_{1,1} \)

\[
p = 0.029 (KS) \\
p = 0.030 (MW)
\]

Distributions of \( RFD_{1,1} \)

\[
p = 0.009 (KS) \\
p = 0.032 (MW)
\]

Figure 1: Distributions of \( RAFD_{1,1} \) and \( RFD_{1,1} \) (period 1, round 1) for 1H5C (dashed line, \( N=101 \)) and 6H (solid line, \( N=72 \)). In all the figures given below, \( p \)-values are given for a Kolmogorov-Smirnov (KS) 2-tailed test as well as a Mann-Whitney (MW) 2-tailed test.

\[
RFD_{t,r}^{i} = \frac{1}{T-t+1} \sum_{p=t}^{T} \frac{f_{t,p,r}^{i} - FV_{p}}{|FV|},
\]

where \( T \) is the number of periods (\( T = 10 \) in our experiments), \( f_{t,p,r}^{i} \) is the forecast of the asset price in period \( p \) submitted by subject \( i \) in period \( t \) of round \( r \), \( FV_{p} \) is the fundamental value of the asset in period \( p \), and \( |FV| \) is the absolute value of the average fundamental value of the asset over all periods.\(^{25}\) The only difference between \( RAFD \) and \( RFD \) is the numerator. The former uses absolute values, while the latter does not. As noted by Stöckl et al. (2010), these two measures are complementary in that, whereas \( RAFD \) shows the magnitude of the forecast deviations, \( RFD \) shows the direction of these deviations.

Figure 1 shows the distributions of the two measures of initial forecasts deviation, \( RAFD_{1,1} \) (left) and \( RFD_{1,1} \) (right), for 6H (solid lines) and 1H5C (dashed lines). The observed distributions as well as the median values of \( RAFD_{1,1} \) and \( RFD_{1,1} \) for the two treatments differ significantly as shown in the figure.

Recall that the effect of individual bounded rationality (or confusion) is measured by the forecast deviations from the fundamental values in the 1H5C treatments. And the effect of strategic uncertainty is measured as the difference between the forecast deviations for the 1H5C and 6H treatments. Since the median \( RAFD_{1,1} \) values are 0.583 for 6H and 0.315 for 1H5C, about 54% (=0.315/0.583) of the median initial forecasts deviation from the fundamental values is due to individual bounded

\(^{25}\) We omit subscript \( r \) for \( FV_{p}, |FV| \), and \( T \) because these values remain constant across all three rounds of our experiment. One could also consider normalizing the measure using the average fundamental value of the asset over the remaining periods after period \( t \). We avoid this to keep the denominator constant for all \( t \).
rationality, while the remaining 46% (0.268/0.583) is due to strategic uncertainty.26

3.2 Cognitive ability and strategic uncertainty

We have identified a significant effect of strategic uncertainty based on the price forecasts submitted at the beginning of round 1 in each experiment. While the identified effect of strategic uncertainty is large, it is not as large as the effect of individual bounded rationality. If subjects are heterogeneous in terms of their depth of strategic thinking or cognitive ability, one may wish to investigate beyond the median effect, because it is possible that some subjects make random forecasts without thinking much about the experimental environment (especially at the beginning of the experiment) while others reflect upon the instructions and make more careful forecasts. If this is the case, we should not observe a significant effect of strategic uncertainty in the former group of subjects, while we should in the latter group of subjects.

To gather information about the cognitive ability of the subjects, we implemented the Cognitive Reflection Test (Frederick, 2005, CRT) as part of the questionnaire at the end of the experiment. This test consists of the following three simple questions, structured in such a way that intuitive or “impulsive” (Frederick, 2005, p. 26) answers are incorrect:

1. A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost? ____ cents.27

2. If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets? ____ minutes.

3. In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? ____ days.

In all three questions, some cognitive reflection is necessary to overcome impulse and arrive at the correct answers. The score of this test, computed simply as the number of correct answers to these

26Akiyama et al. (2012) reported the results of a comparison of a market with a single human and five computer traders and that with all human traders when everyone is told that “Each computer trader assumes that all traders maximize their profits without making any mistakes. Given this assumption about the others, the computer trader maximizes its profits without making mistakes. If the computer trader is indifferent between trading and not trading, it prefers to trade.” Akiyama et al. (2012) failed to reject the null hypothesis that distributions of initial (period 1, round 1) forecast deviations from the fundamental values in two treatments are drawn from the same underlying distribution. The difference between the finding reported in the current paper and that by Akiyama et al. (2012) indicates that subjects have difficulty inferring the behavior of profit maximizing computers.

27In translating this question into Japanese, we changed $1.10 and $1.00 to 11,000 and 10,000 yen, respectively.
<table>
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<tr>
<th>Treatment</th>
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<th>CRTS=2</th>
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<td>101</td>
<td>8</td>
<td>19</td>
<td>26</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 2: CRT scores

<table>
<thead>
<tr>
<th>CRT score ≤ 1</th>
<th>CRT score = 2</th>
<th>CRT score = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p = 0.744 (KS)</td>
<td>p = 0.733 (KS)</td>
<td>p = 0.009 (KS)</td>
</tr>
<tr>
<td>p = 0.819 (MW)</td>
<td>p = 0.365 (MW)</td>
<td>p = 0.056 (MW)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRT score ≤ 1</th>
<th>CRT score = 2</th>
<th>CRT score = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p = 0.626 (KS)</td>
<td>p = 0.285 (KS)</td>
<td>p = 0.038 (KS)</td>
</tr>
<tr>
<td>p = 0.327 (MW)</td>
<td>p = 0.089 (MW)</td>
<td>p = 0.509 (MW)</td>
</tr>
</tbody>
</table>

Figure 2: Distributions of $RAFD_{1,1}$ (top) and $RFD_{1,1}$ (bottom) in 6H (solid lines) and 1H5C (dashed lines) for subjects with CRT score 0 or 1 (left), CRT score 2 (middle), and CRT score 3 (right).

three questions, has been shown to correlate negatively with lower incidences of the conjunction fallacy and conservatism in updating probabilities (Oechssler et al., 2009). On the other hand, those with higher CRT scores tend to choose numbers closer to the Nash equilibrium in beauty contest games (Brañas-Garza et al., 2012). Corgnet et al. (2013), in their investigation of the effect of “house money” on the magnitude of price deviations, reported that subjects with low CRT scores tend to buy (sell) an asset at prices above (below) fundamental values while the opposite is true for those with high CRT scores. Here we are interested in the correlation between the CRT score and the magnitude of the initial forecasts deviation. Table 2 shows the frequencies of subjects with various CRT scores in each treatment. The average score of the 173 subjects is 2.01. Since the number of subjects with a CRT score equal to 0 is small, we included them in the group with a CRT score of 1 in our analysis.

Figure 2 shows the distributions of $RAFD_{1,1}$ (top) and $RFD_{1,1}$ (bottom) in the 6H (solid
lines) and 1H5C (dashed lines) treatments for subjects with CRT score 0 or 1 (left), CRT score 2 (middle), and CRT score 3 (right). While the distributions of $RAFD_{1,1}$ in the two treatments are not significantly different for subjects with CRT score 0 or 1 and 2, they are significantly different for subjects with CRT score 3.\footnote{P-values are reported in the figure.} For those with CRT score 3, the median $RAFD_{1,1}$ values are 0.570 in 6H and 0.183 in 1H5C. Thus, about 32% (0.183/0.570) of the median initial forecasts deviation stems from individual bounded rationality and the remaining 68% is due to strategic uncertainty. Compared with the result based on all the data, the effect of strategic uncertainty is much more pronounced for those with a perfect CRT score. For those with CRT scores 0 or 1, on the other hand, no significant effect of strategic uncertainty was found.

In the bottom-left and bottom-middle panels of Figure 2, the dashed curves (that is, the results for 1H5C) lie to the left of the solid curves (the results for 6H), although the two distributions of $RFD_{1,1}$ for those subjects with CRT scores 0 or 1 are not significantly different.\footnote{For those with CRT score 2, the median $RFD_{1,1}$ values are marginally significantly different.} For those with a perfect CRT score (bottom-right panel), this is not the case. The forecasts of these subjects participating in the 1H5C market were closer to the fundamental values than those of subjects participating in the 6H market.

It is quite clear from Figure 2 that the magnitude of $RAFD$ in the 1H5C treatment is much smaller for subjects with higher CRT scores than those with lower CRT scores. Thus, CRT score is clearly related to the degree of bounded rationality that we identified through the magnitude of forecast deviations from the fundamental values ($RAFD$) in the 1H5C treatment. On the other hand, the magnitude of $RAFD$ in the 6H treatment does not seem to vary across subjects with varying CRT scores.

This is confirmed by Figure 3, which shows the distributions of $RAFD_{1,1}$ for subjects with CRT score 3 (solid lines) and those with CRT score 1 or 0 (dashed lines) in the 6H (left) and 1H5C (right) treatments. We omitted subjects with CRT score equal to 2 to focus on the two extreme groups. The distributions of $RAFD_{1,1}$ for the two groups of subjects basically lie on top of each other, except for those with very high $RAFD$, and are not significantly different in the 6H treatment. On the other hand, the two distributions are quite different in the 1H5C treatment, although this difference is not statistically significant.\footnote{The distributions of $RFD_{1,1}$ for subjects with CRT score less than 1 and CRT score 3 are not statistically different in either the 6H ($p$-value = 0.469 (KS) and 0.427 (MW), both 2-tailed tests) or 1H5C treatment ($p$-value = 0.457 (KS) and 0.539 (MW), both 2-tailed tests).}
Figure 3: Distributions of $RAFD_{1,1}$ for subjects with CRT score 3 (solid lines) and those with CRT score 1 or less (dashed lines) in 6H (left) and 1H5C (right).

The insignificant difference between the two distributions of $RAFD_{1,1}$ in the 6H treatment shows that the total initial effects of bounded rationality and strategic uncertainty are not significantly different for these two groups of subjects. As shown above, whereas for subjects with CRT 0 or 1, it is mainly individual bounded rationality that explains the initial deviation of price forecasts from the fundamental values, for subjects with CRT score 3, the effect of strategic uncertainty accounts for more than half of the initial forecasts deviation.

3.3 Dynamics: prices and forecasts

So far, we have only considered the forecasts submitted before subjects observed any prices. In what way did subjects change their forecasts after observing the realized prices? To address this question, we first summarize the realized price data and then move on to the dynamics of forecast deviations.

Figure 4 shows the dynamics of the realized prices in the three rounds in 6H (top) and 1H5C (bottom). As expected the prices in the 1H5C treatment follow the fundamental values except for a few rare cases in which the human subject dominates one side of the market by placing a large quantity of a buy order at a price above the fundamental value (period 9 in round 2 and period 10 in round 3). In the 6H treatment, on the other hand, prices deviate from the fundamental values in round 1. Such deviations of prices from the fundamental values gradually disappear in rounds 2 and 3 as subjects gain experience from trading in the same group as reported in the literature.

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31 If anything, the distribution of $RAFD_{1,1}$ for subjects with CRT score 3 lies to the right of that for subjects with CRT scores 1 or 0.

32 As noted in footnote 24 above, we only used the data from July 7th for the 1H5C treatment.
Figure 4: Dynamics of realized prices in 6H (top) and 1H5C (bottom) for three rounds.

Figure 5: Distributions of $RAD^g_r$ (left) and $RD^g_r$ (right) in the 6H treatments for round 1 (thick solid lines), round 2 (dashed lines), and round 3 (thin solid lines).

Figure 5 shows the distributions of the two measures of price deviations from the fundamental values, the relative absolute deviation (RAD) and the relative deviation (RD), proposed by Stöckl et al. (2010) for three rounds of 6H treatments. For group $g$ in round $r$, these two measures are defined as follows:

$$RAD^g_r = \frac{1}{T} \sum_{p=1}^{T} \frac{|P_{p,r}^g - FV_p|}{|FV|}$$

$$RD^g_r = \frac{1}{T} \sum_{p=1}^{T} \frac{P_{p,r}^g - FV_p}{|FV|}$$

where $P_{p,r}^g$ is the realized price for group $g$ in period $p$ of round $r$ and the other terms are the same as those used in defining $RAFD$ and $RFD$.

The left panel of the figure clearly shows that the magnitude of mis-pricing (measured by RAD)
Figure 6: Dynamics of the median $RAFD_{t,r}$ (left), and distributions of $RAFD_{i,1}^{r,1}$ (middle) and $RFD_{i,1}^{r,1}$ (right) in period 1 of the next round for 1H5C (dashed lines) and 6H (solid lines) in round 1 (top) and round 2 (bottom). The $p$-values are given for 2-tailed KS and MW tests. In conducting the statistical tests, for 6H we take the mean $RAFD$ and $RFD$ values for the six traders in the same group and use these as independent observations.

One can also see from the distribution of $RD$ (shown in the right panel) that most of the large price deviations from the fundamental values are on the positive side. In other words, there is much more over-pricing than under-pricing.

Now we turn to the dynamics of the forecasts deviation. Did subjects in 1H5C learn to forecast prices to follow the fundamental values? How about the subjects in 6H? The left panel of Figure 6 shows the dynamics of the median $RAFD_{t,r}$ over 10 periods in rounds 1 (top) and 2 (bottom). It is clear that the subjects in 1H5C (dashed lines) learnt much more quickly than those in 6H (solid lines) to forecast prices that follow the fundamental values. As can be seen in the middle panel of the figure, by the beginning of round 2 (3), close to 80% (90%) of the subjects in the 1H5C treatment (dashed lines) learnt to forecast prices to follow the fundamental values, whereas this is not the case for subjects in the 6H treatment (solid lines). These differences are quite natural given the differences in the realized prices in the two treatments. While subjects in the 1H5C treatment see that the price is exactly the same as the fundamental value in every period, those in the 6H

The difference between the RAD of two consecutive rounds, $\Delta RAD_{t,r}^{g} \equiv RAD_{t,r}^{g} - RAD_{t,r+1}^{g}$, is significantly different from zero both between rounds 1 and 2 ($p$-value = 0.003, signed-rank (SR) two-tailed test) and between rounds 2 and 3 ($p$-value = 0.007, SR two-tailed test.)
treatment see the prices deviate from the fundamental values, as shown in Figure 4. Comparing $RFD_{1,1}$ (round 1) and $RFD_{1,2}$ (round 2) shown in the rightmost panels of Figure 10 and Figure 6, respectively, we also note that fewer subjects submit forecasts that are below the fundamental values in later rounds in both treatments.

3.4 Submitted orders

We have seen that the median initial forecasts deviation from the fundamental values are significantly smaller in 1H5C than in 6H, i.e., the observed forecasts deviations are significantly smaller in the absence of strategic uncertainty. We have also shown that the differences in the initial forecasts deviation between the two treatments correlate positively with the CRT scores. While we did not find any significant effect of strategic uncertainty for those subjects with low CRT scores, we observed a significant effect of strategic uncertainty for those with a perfect CRT score. In addition, we have seen that as subjects gain experience trading in the same market environment, they learn to trade at prices closer to the fundamental values and also to forecast prices that follow the fundamental values more closely.

We now ask some different questions. Did subjects in the 1H5C and 6H treatments behave differently in terms of the orders they submitted? Is there a significant difference between orders submitted by subjects with high and low CRT scores?\(^{34}\) This is important because the market prices follow the fundamental values in the 1H5C treatment regardless of the orders submitted by the subjects, except in a few rare cases in which a human subject dominates one side of the market as we have seen above. This is due to the call market structure we have employed. In continuous double auctions, on the other hand, if a subject submits a buy order above, or a sell order below, the fundamental value of the current period, it will be executed at those prices as computer traders respond quickly because they are constantly looking out for such profit making opportunities.

In this subsection, we first consider potentially loss making orders, i.e., buy orders at prices above and sell orders at prices below the fundamental value of the period submitted by subjects without taking their price forecasts into account.\(^{35}\) This is to quantify the deviation of subjects’ trading behavior (orders) from the equilibrium one. It should be noted that subjects may “rationally”

\(^{34}\)As noted in footnote 24 we only used the data from July 7th for 1H5C. There were 51 subjects in 1H5C. See the Appendix for the CRT scores for these 51 subjects.

\(^{35}\)As noted above, this may not result in any losses in our experiment because the actual trading prices can differ from those submitted by the subjects.
deviate from the equilibrium behavior in the 6H treatment because of strategic uncertainty. To allow for such possibilities, we then move on to incorporate subjects’ price forecasts into the analysis.

As before, let $f^t_{i,p,r}$ represent the forecast period $p$ asset price submitted by subject $i$ at the beginning of period $t$ in round $r$. Let $pd^t_{i,r}$ and $ps^t_{i,r}$ be the maximum price at which $i$ is willing to buy and the minimum price at which $i$ is willing to sell an asset, respectively, specified in subject $i$’s orders submitted in period $t$ of round $r$. Let $d^t_{i,r}$ and $s^t_{i,r}$ be the maximum quantity demanded and supplied associated with $pd^t_{i,r}$ and $ps^t_{i,r}$, respectively.

We define the potential losses, normalized by the value of the initial endowments (1000 ECUs, consisting of 4 units of the asset and 520 ECUs in cash), for subject $i$ in round $r$ as:

$$PL^t_i \equiv \frac{1}{1000} \sum_t (d^t_{i,r} \max(pd^t_{i,r} - FV_t, 0) + s^t_{i,r} \max(FV_t - ps^t_{i,r}, 0)).$$

Here we are considering buy prices that are above and sell prices that are below the fundamental values of the asset in each period.

As noted above, “potential losses” simply captures the magnitude of the deviation (in one direction) of orders from the equilibrium behavior, but does not incorporate subjects’ expectations about future prices. To do so, we now extend “potential losses” slightly. Note that the maximum capital gain $i$ can expect based on his/her price forecasts and submitted buy order in period $t$ is $\max_{p > t}(f^t_{i,p,r} - pd^t_{i,r} + FV_t - FV_p)$. Here $FV_t - FV_p$ is included to account for the difference in the fundamental value across periods. Therefore, the maximum potential loss for subject $i$ from buying a unit of asset above the fundamental value in period $t$ in round $r$ given his/her expectation about future asset prices, $plb^t_{i,r}$, can be defined as

$$plb^t_{i,r} \equiv \max \left(\max_{p > t}(pd^t_{i,r} - FV_t, f^t_{i,p,r} + FV_p - FV_t), 0\right).$$

Another way of looking at this expression is that the loss from buying above the fundamental value in period $t$, $pd^t_{i,r} - FV_t$, can be recovered in the future by, at most, $\max_{p > t}(f^t_{i,p,r} - FV_p)$, given the expectation of the subject.

Similarly, the maximum potential loss for subject $i$ from selling a unit of asset below the fundamental value in period $t$ in round $r$ given his expectation about future asset prices, $pls^t_{i,r}$, can be
Distribution of $P L$ (top) and $\bar{P}L$ (bottom) over three rounds. Dashed line: 1H5C (N=51). Solid line: 6H (N=72). Please note that the scale of the x-axis differs between the top (maximum 1.0) and bottom (maximum 0.5) panels. $P$-values are based on KS and MW 2-tailed tests. The within group means of $P L$ and $\bar{P}L$ for 6H are used as independent samples to conduct the statistical tests.

defined as

$$pls_{t,r}^i \equiv \max \left( \max_{p>t} (f_{t,p,r}^i - ps_{t,r}^i + FV_t - FV_p), 0 \right).$$

Again, one way of interpreting this expression is that the loss from selling below the fundamental value in period $t$, $FV_t - ps_{t,r}^i$, can be recovered in the future, at most, $\max_{p>t} (FV_p - f_{t,p}^i)$, by buying the asset back more cheaply in the future.

Based on the above, belief adjusted potential losses, normalized by the value of the initial endowments, for subject $i$ in round $r$ can be defined as

$$\bar{PL}_r^i \equiv \frac{1}{1000} \sum_t (d_{t,r}^i plb_{t,r}^i + s_{t,r}^i pls_{t,r}^i).$$

The measure of belief adjusted potential losses, therefore, captures the degree of boundedly rational behavior given one’s subjective belief.

Figure 7 shows the empirical distribution of $PL_r^i$ (top) and $\bar{PL}_r^i$ (bottom) for rounds 1 (left), 2 (middle), and 3 (right) in the 6H (solid lines) and 1H5C (dashed lines) treatments. It is clear from
Figure 8: Distributions of $PL_i$ for subjects with CRT score 3 (solid lines) and CRT score less than or equal to 1 (dashed lines) in rounds 1 (left), 2 (middle), and 3 (right) of 6H (top) and 1H5C (bottom). For the 6H treatment, we used the average of the subjects in the same market as an independent sample when conducting the statistical tests.

The top panel (distribution of $PL_i$) that subjects in the 1H5C treatment submitted orders that were potentially less damaging than those in the 6H treatment. With increasing rounds, we see that the fraction of subjects who submitted potentially loss generating orders decreases in both treatments, and for those that continued submitting such orders, the magnitude of the potential loss decreases.

If the price forecasts are taken into account, the magnitude of the potential loss decreases. In addition, we see that the distributions of $PL_i$ for 1H5C are to the left of those for 6H. This is simply due to the fact that subjects in the 1H5C treatment quickly learnt to forecast prices that follow the fundamental values. It should, however, be noted that even adjusting for their price forecasts, there were more than 40% (30%) of the subjects in 1H5C (in 6H) who submitted orders that could result in losses in round 1. As the number of rounds increases, this fraction decreases, although it remains non-negligible (more than 20% of the subjects) even in round 3.

Do subjects with high and low CRT scores submit similar orders? Here we focus on “potential losses,” $PL$. Figure 8 shows the distributions of $PL_i$ over three rounds for the two treatments (6H at the top and 1H5C at the bottom) for subjects with CRT scores 0 or 1 (dashed lines) and 3 (solid lines). The p-values for the Kolmogorov-Smirnov (KS) and Mann-Whitney (MW) tests are shown for each round.

36 Please note that the scale of the x-axis is different for the subfigures for $PL_i$ (top) and $\overline{PL}_i$ (bottom) in Figure 7.
lines). In round 1 (leftmost graph), for both treatments, the distribution of $PL_i$ for those with low CRT scores lies to the right of that for subjects with a perfect CRT score and the differences are statistically significant. This is in line with the finding by Corgnet et al. (2013) that subjects with lower CRT scores tend to trade at prices that can result in losses.

In round 2 (middle graph), the difference is insignificant for both the 1H5C and 6H treatments and remains so in round 3 (rightmost graph) for the 6H treatment. While we do not have a good explanation for the significant difference in the distribution of $PL_i$ in round 3 for 1H5C, it may be due to some subjects trying to see whether they can manipulate the prices by submitting large numbers of buy orders at a price above the fundamental value. Of course, if they succeed in manipulating the price in this way, they will make a loss in the 1H5C market.

4 Conclusion

In this paper, we investigated to what extent the deviations of price forecasts from the fundamental values in experimental asset markets are caused by individual bounded rationality (or confusion) and by strategic uncertainty (uncertainty about others behavior) to better understand the source of mis-pricing in these experimental markets. We investigated this by comparing the initial, as well as subsequent, price forecasts submitted by subjects in two market environments - one in which all six traders were human subjects (6H), and the other where one human subject interacted with five computer traders submitting orders at the fundamental values (1H5C). Subjects in the 1H5C treatment were all told that the computer traders submitted orders at the fundamental values. All the subjects were also clearly informed about the composition of traders in their group. Our analysis shows that about 50% of the median initial forecasts deviation from the fundamental values is due to some kind of individual bounded rationality and the remaining 50% to strategic uncertainty. We also found that the effect of strategic uncertainty is greater for those subjects with a higher cognitive ability that manifests as a higher score in the CRT (Frederick, 2005). For those with a perfect CRT score, individual bounded rationality accounts for about 30%, while strategic uncertainty accounts for the remaining 70% of the median initial forecasts deviation from the fundamental values. For those with CRT scores of 0 or 1, we did not observe any significant effect of strategic uncertainty.

Our results show that both strategic uncertainty and individual bounded rationality (or confusion) have significant power in explaining the initial deviation of price forecasts from the fundamental
values. While individual bounded rationality (or confusion) clearly plays an important role in causing prices to deviate from the fundamental values in these experimental asset markets as recently emphasized by Huber and Kirchler (2012) and Kirchler et al. (2012), so too does strategic uncertainty as conjectured by Smith et al. (1988). Subjects being confused cannot be the whole story behind the bubble.

Our finding complements the recent finding by Cheung et al. (2012) who reported that reducing individual confusion alone cannot account for the observed mis-pricing, although the lack of common knowledge of how well everyone understands the nature of the experiment (i.e., the fundamental values) can. It is interesting to think about the difference in the magnitude of the effect of strategic uncertainty in our result and that in Cheung et al. (2012). Our result, which is based on initial price forecasts, shows that strategic uncertainty plays a major role, but cannot account for all the initial deviations of price forecasts from the fundamental values. On the other hand, Cheung et al. (2012) showed that, based on the realized prices, the lack of common knowledge of understanding of the nature of the fundamental values accounts for most of the price deviations from the fundamental values. One may speculate that the effect of strategic uncertainty we identified is possibly amplified by trading owing to the presence of strategic complementarity or positive feedback in these markets. While this is a very interesting and plausible hypothesis, a different set of experiments would need to be carried out to test it. We leave this to future research.

Let us make another remark before closing the paper. It is quite well known that experiences in trading in the same market environment help subjects “learn” to trade at prices closer to the fundamental values and to forecast prices to do so. Haruvy et al. (2007) reported that this dynamic is due to subjects adjusting their expectation adaptively, i.e., based on the pattern of price changes observed in the previous rounds and in previous periods in the same round. However, it is not clear why these subjects adjust their forecasts in this manner. For example, do subjects in 1H5C realize that prices follow the fundamental values because of the way the five computer traders behave? If this is the case, what kind of price dynamics will they expect when these experienced subjects are recruited back to the experiment and are told that they will be in the same market with five subjects who have never participated in similar experiments (i.e., inexperienced subjects)? If the subjects with experience in 1H5C understood the effect of the behavior of the computer traders on the prices, and expected inexperienced subjects to behave quite differently from the computer traders, they would expect the prices to deviate substantially from the fundamental values.
The forecasts as well as the behavior of experienced subjects are of interest in the light of findings by Dufwenberg et al. (2005) and Hussam et al. (2008). Dufwenberg et al. (2005) considered an asset market with six traders and mixed two (four) subjects with experience of the same market condition three times with four (two) inexperienced subjects to see whether the presence of inexperienced subjects caused the prices to deviate more from the fundamental values than in the markets consisting only of subjects who had experienced the same market twice. A surprising finding is the lack of a significant difference between the magnitudes of price deviations in the two kinds of markets. Hussam et al. (2008) allowed subjects to trade in one market condition twice and then changed the market condition quite drastically by increasing the variance of the distribution of possible dividend payments as well as the cash/asset ratio in the endowment. While subjects learnt to trade at prices close to the fundamental values after two iterations in the first market environment, under the new market condition, the prices once again deviated substantially from the fundamental values. What these studies leave us to ponder on is: (1) whether experienced subjects learn something fundamental about the experiment in the first few rounds or whether they just respond to prices they have observed without thinking much about the reason for observing such prices, and (2) how experienced subjects expect others to behave in the face of a new condition. By recruiting subjects who have obtained experience in the 1H5C or 6H treatment and placing them in a market with inexperienced subjects or in a market with quite different fundamental values and endowments, we may be able to shed some light on these questions. This remains to be investigated in future research.
References


**A Table of fundamental values given to the subjects**

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<th>No. of remaining periods</th>
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<th>Next value</th>
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<td>108</td>
</tr>
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<td>At the end of period 2</td>
<td>8</td>
<td>12</td>
<td>96</td>
</tr>
<tr>
<td>At the end of period 3</td>
<td>7</td>
<td>12</td>
<td>84</td>
</tr>
<tr>
<td>At the end of period 4</td>
<td>6</td>
<td>12</td>
<td>72</td>
</tr>
<tr>
<td>At the end of period 5</td>
<td>5</td>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td>At the end of period 6</td>
<td>4</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>At the end of period 7</td>
<td>3</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>At the end of period 8</td>
<td>2</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>At the end of period 9</td>
<td>1</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>At the end of period 10</td>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Next value before the beginning of period 1 is 120.

The value of stock is zero after the final dividend payment in period 10.

**B Instructions**

English translations of the instructions, the script and the slides shown, can be downloaded from:

- [http://www.vcharite.univ-mrs.fr/~nobi/assetM2/slide6Hv2.pdf](http://www.vcharite.univ-mrs.fr/~nobi/assetM2/slide6Hv2.pdf) (slides for 6H)
- [http://www.vcharite.univ-mrs.fr/~nobi/assetM2/slide1H5Cv2.pdf](http://www.vcharite.univ-mrs.fr/~nobi/assetM2/slide1H5Cv2.pdf) (slides for 1H5C)

The set of instructions in Japanese is available upon request.
C Analysis of initial forecasts deviation using only data from July 7th

In this Appendix, we first discuss the comparison between the initial forecasts deviation in 1H5C for the May 26th and June 2nd sessions and the July 7th session. We then report the results from comparing the initial forecasts deviation between the 1H5C and 6H treatments by discarding the data for 1H5C from the May 26th and June 2nd sessions in which subjects were not grouped correctly. Table 3 shows the distribution of CRT scores for this restricted set of subjects.

As noted in footnote 24, there is no reason for us to expect the distribution of the initial forecasts deviation in 1H5C to differ across sessions because the instructions were identical. However, as shown in Figure 9, it turns out that the distributions of $RFD_{1,1}$ (left panel) are significantly different for the May-June sessions (dashed lines) and the July sessions (solid lines). The distribution of $RAFD_{1,1}$ (right panel), on the other hand, does not differ significantly between these two sessions. This difference may be due to the difference in the composition of subjects in the two sessions. While only 10 of the 50 subjects in the May-June sessions were students of engineering related colleges, 37 of the total 51 in the July sessions were students from these colleges. Given the between subject

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Table 3: CRT scores

<table>
<thead>
<tr>
<th>Treatment</th>
<th>N</th>
<th>CRTS=0</th>
<th>CRTS=1</th>
<th>CRTS=2</th>
<th>CRTS=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6H</td>
<td>72</td>
<td>8</td>
<td>17</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>1H5C (May 26th and June 2nd)</td>
<td>50</td>
<td>4</td>
<td>9</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>1H5C (July 7th)</td>
<td>51</td>
<td>4</td>
<td>10</td>
<td>14</td>
<td>23</td>
</tr>
</tbody>
</table>
nature of the experiment, we decided to pool the data in analyzing the initial forecasts deviation reported in the main text because results based on a larger number of observations tend to be more reliable.

Let us now proceed with analyzing the comparison of the initial forecasts deviation for the 6H and 1H5C treatments using only the data from July 7th for 1H5C. Figure 10 shows the distributions of $RADF_{1,1}$ (left) and $RFD_{1,1}$ (right) for 1H5C (dashed lines) and 6H (solid lines). As reported in the main text, the two distributions are statistically significantly different. The median $RADF_{1,1}$ is 0.583 for 6H and 0.303 for 1H5C. Thus, roughly 52% of the initial forecasts deviation is due to bounded rationality (or confusion) and the remaining 48% to strategic uncertainty.

Figure 11 shows the distribution of $RADF_{1,1}$ (top) and $RFD_{1,1}$ (bottom) for 6H (solid lines) and 1H5C (dashed lines) treatments for subjects with CRT score 0 or 1 (left), CRT score 2 (middle), and CRT score 3 (right). As in the main text, the distributions of $RADF_{1,1}$ for the two treatments are not significantly different for subjects with low CRT scores. However, they are significantly different for subjects with CRT score 3 according to the KS test (but not the MW test). The median $RADF_{1,1}$ for those with CRT score 3 is 0.570 for 6H and 0.182 for 1H5C. Thus, as reported in the main text, about 32% of the initial forecasts deviation is due to individual bounded rationality and the remaining 68% to strategic uncertainty for these subjects.

What is observed from this restricted sample and not when pooling the data, is a significant difference in the distributions of $RFD_{1,1}$ (shown in the bottom panel). In the bottom-left and bottom-middle panels of Figure 2, the dashed curves (for 1H5C) lie to the right of the solid curves (for 6H). This is the same tendency observed when comparing the May-June sessions of 1H5C and
Figure 11: Distribution of $RAFD_{1,1}$ (top) and $RFD_{1,1}$ (bottom) for 6H (solid lines) and 1H5C (dashed lines) for subjects with CRT score 0 or 1 (left), CRT score 2 (middle), and CRT score 3 (right).

Finally, Figure 12 shows the distributions of $RAFD_{1,1}$ for subjects with CRT score 3 (solid line) and CRT score 1 or less (dashed line) for the 1H5C treatment using our restricted sample. The main difference from what was reported in the main text is that for this restricted sample, the two distributions of $RAFD_{1,1}$ are much more similar to each other.

Figure 12: Distributions of $RAFD_{1,1}$ for subjects with CRT score 3 (solid line) and CRT score 1 or less (dashed line) for the restricted sample from the 1H5C treatment.